Title: TBA

Date: Aug 20, 2015 09:00 AM

URL: http://pirsa.org/15080075

Abstract: TBA

Most of us are here because we are interested in the following types of questions:
And because we have a particular hope of how to answer them:

1. What states have good geometric duals?

2. How do we understand holographic RG and the emergence of the radial direction?

3. What is the meaning of area in spacetime and the relation to black hole entropy?

Most of us are here because we are interested in the following types of questions:
And because we have a particular hope of how to answer them:

1. What states have good geometric duals?
   - To what extent is this geometry determined by information theoretic and entropic CFT quantities? What information about the state is necessary?

2. How do we understand holographic RG and the emergence of the radial direction?
   - Is there an interpretation in terms of an information theoretic coarse-graining scheme?

3. What is the meaning of area in spacetime and the relation to black hole entropy?
   - Is this entropy actually counting something?

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It's useful to look back at where we were at the same time last year at *Quantum Information in Quantum Gravity I* and see what progress we have made (and where we're still hopelessly confused).

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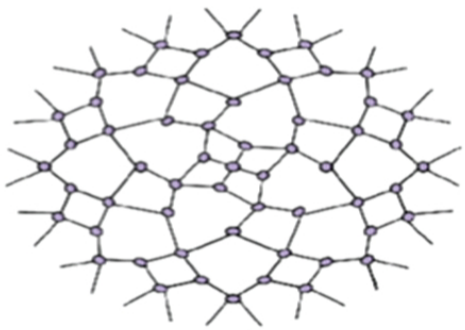
So let's pull up some slides from last year and see how well (poorly?) we've done.

Note: This presentation is a reenactment, not history.
Past slides may have been edited for continuity and brevity, but I promise not to have cleaned up our misconceptions.

2) The bulk geometry is perhaps associated to a MERA lattice [Swingle]
One can efficiently represent certain low-energy states by a lattice of unitary operators. Swingle has suggested that the structure of this lattice for the vacuum state of a CFT mimics the coarse structure of AdS. **(Q1)**



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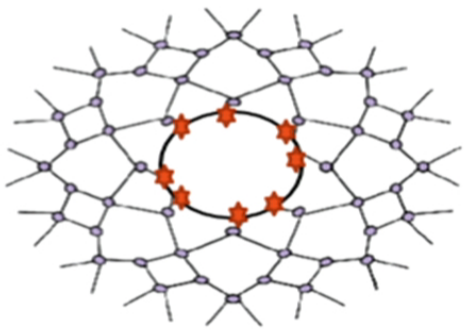

- When we cut out a region of the MERA lattice, we remove a number of unitary operators proportional to the area of the cut.
- The number of possible states that fills in the lattice is proportional to the area of the cut. **(Q3)**

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**This is a nice story for the ground state, but what is the general story?**

1. **What is the analog of the MERA network for non-vacuum states?**
   What are the rules for building the 'right' type of network?
   What boundary data determines this network?
   When is the network geometric?

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   What boundary data determines this network?
   When is the network geometric?

2. **How do we interpret the holographic RG in terms of a tensor network?**

3. **What is the entropic meaning of subregions of a tensor network?**
   What is this entropy actually counting?

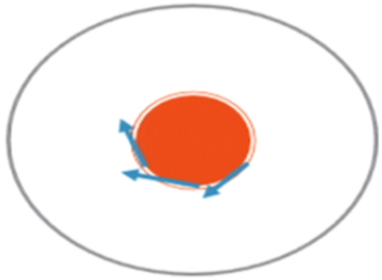
4. **How do we translate between the tensor network and continuum language?**

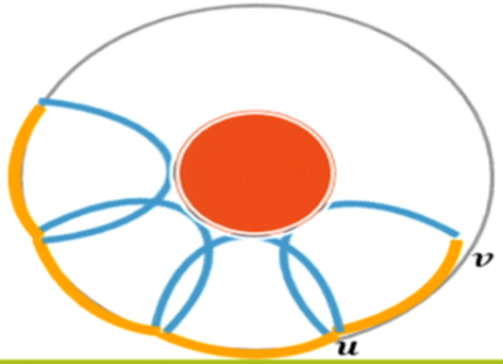
# Our Starting Point: Differential Entropy

**Differential entropy** is the continuum analog of our procedure of assigning an entropy to a length in the MERA.

[BALASUBRAMANIAN, CHOWDHURY, CZECH, DE BOER, HELLER]

- We begin by noting that we can write the integral for the length of the boundary of a region in terms of it's tangent vectors.
- We can do a (non-local) transformation from the tangent space to the boundary-anchored geodesics that the tangents lie on.
- These geodesics are (sometimes) minimal. RT tell us their length is actually the Entanglement entropy of the region.





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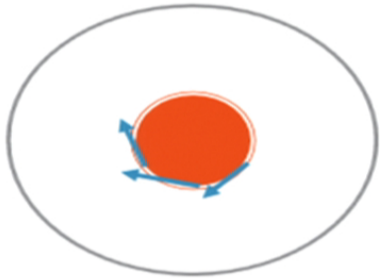
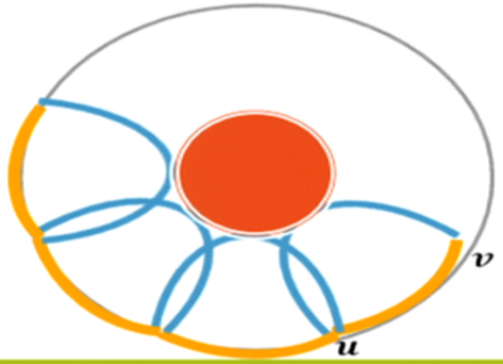
- We can do a (non-local) transformation from the tangent space to the boundary-anchored geodesics that the tangents lie on.
- These geodesics are (sometimes) minimal. RT tell us their length is actually the Entanglement entropy of the region.
- The integral takes a miraculously simple form:

$$S_{\text{diff}} = \int \mathrm{d}\lambda \frac{du(\lambda)}{d\lambda} \frac{\partial S[v(\lambda), u(\lambda)]}{\partial u}$$





# 1) Is there a relation to integral geometry?

- Integral geometry studies measures on geometric spaces that are invariant under the action of the symmetry group

- Two classic results of integral geometry:
  1. Radon Transform: [Lin et al; Faulkner et al.]
     The Radon Transform of a function f on the real plane is another function on the space of straight lines in the plane :

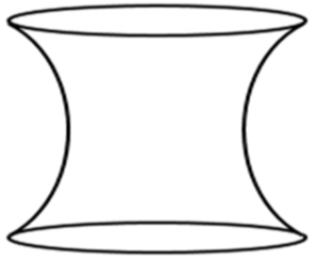
$$Rf(l) = \int_l f ds$$

  2. Crofton Formula:
     The area of a plane curve can be written as an integral of the intersection number over the space of lines:

$$S(C) = \frac{1}{4} \int_L \#(l \cap C)\, d^2 l$$

- The Radon transform and the Crofton formula can be naturally extended to hyperbolic space:
  - Let $\Gamma$ be the space of planes in $\mathbb{H}^2$, with the unique invariant measure. Then:

$$S(C) = \frac{1}{4}\int_\Gamma \#(\gamma \cap C)\, d^2\gamma$$



  - If we call the Crofton Form a volume form, then it is the metric on Lorentzian de Sitter space.
- This appears distinct from the differential entropy formula, but they are in fact equivalent: Differential Entropy = Crofton Formula
  - We can first integrate over the interval size coordinates at each point (with support starting at $\alpha(\theta)$ ) to give the differential entropy formula:

$$\int_\Gamma \#(\gamma \cap C)\, d^2\gamma \quad \rightarrow \quad \int \frac{\partial S}{\partial u}(u,v)|_{v(u)} du$$

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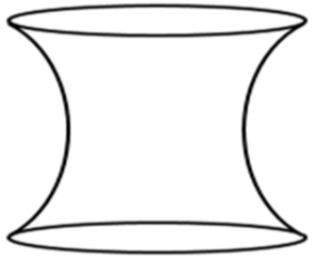

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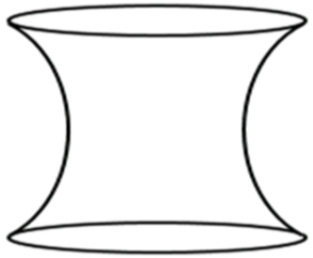

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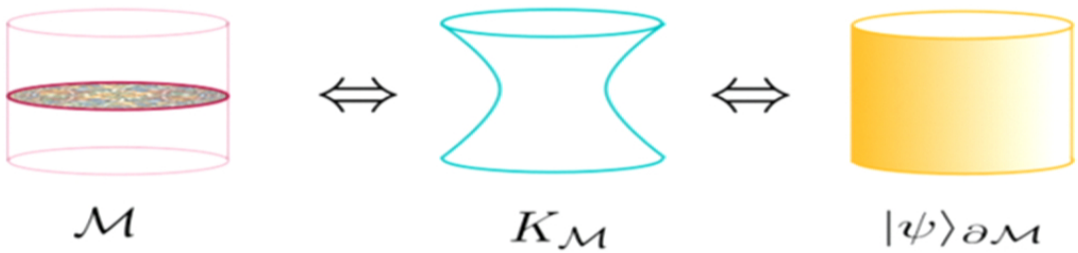
# A Review of Kinematic Space

# Kinematic Space

- We would like to describe geometry in terms of information theoretic quantities.
  - These are not local observables in the geometry, but are integrated quantities.
- It would be natural, then, to describe a geometry indirectly, by some sort of integral transform.
- In the context of 2 dimensional bulk spatial geometry, we want a dual space whose points correspond to geodesics. Let's call this space the associated **Kinematic Space** for our geometry.



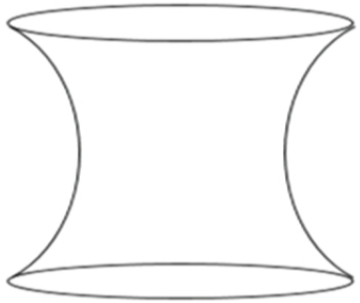
# General Geometries

- Is there a measure on the space of geodesics so the Crofton Formula works more generally?

- Yes. There is a natural Crofton Form that reproduces the lengths of any curve in any geometry whose tangent space is covered by boundary-anchored geodesics.
- The Crofton form is given by

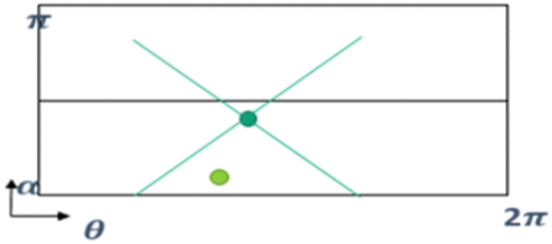
$$\mathrm{d}^2\gamma = \partial_u \partial_v S(u,v)\mathrm{d}u\mathrm{d}v$$

- It will be natural to interpret this density as the **volume form** of our Kinematic Space.

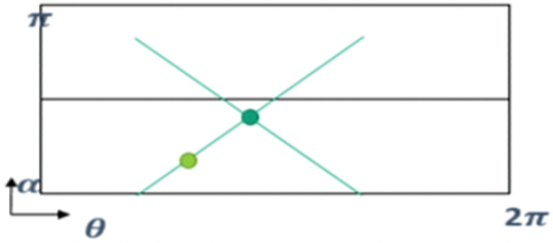


# Causal Structure

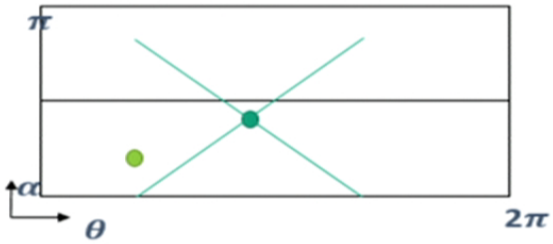
- What is the meaning of the causal structure in Kinematic space?
- Time-like:



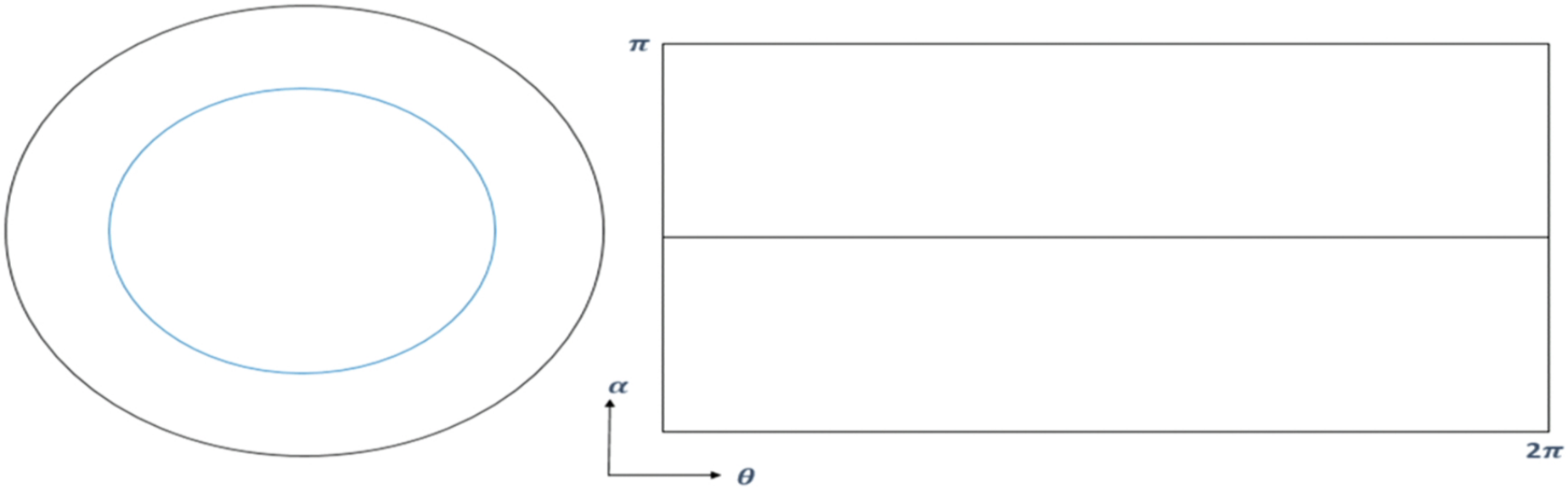

- Light-like:




- Space-like:



# What do length computations look like?



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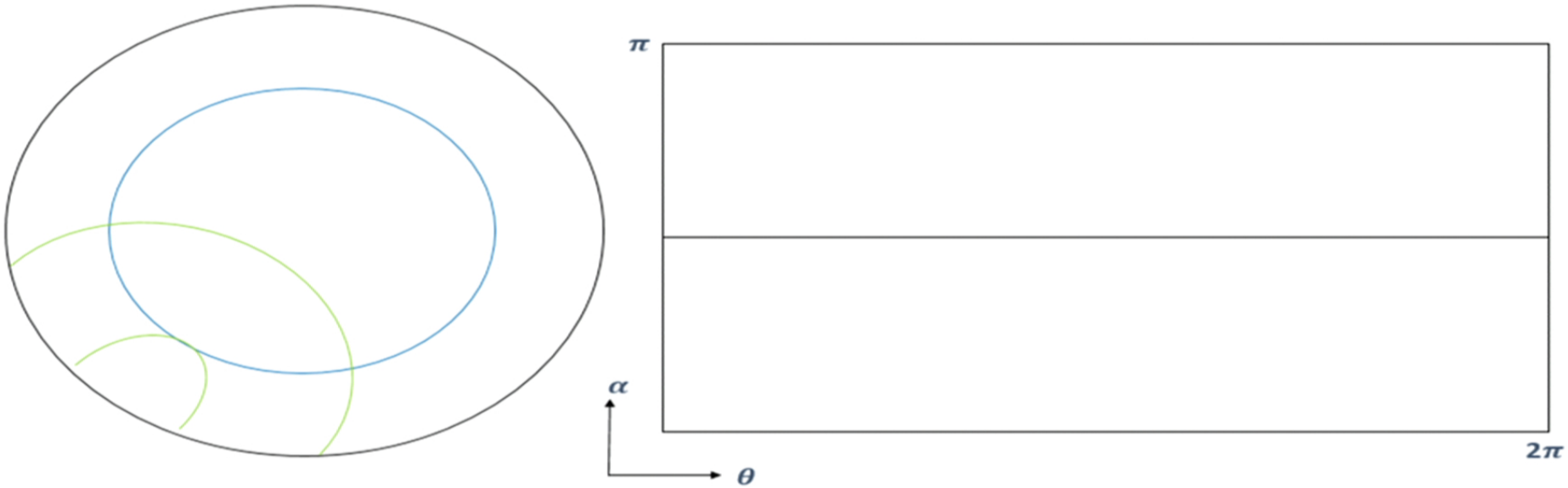

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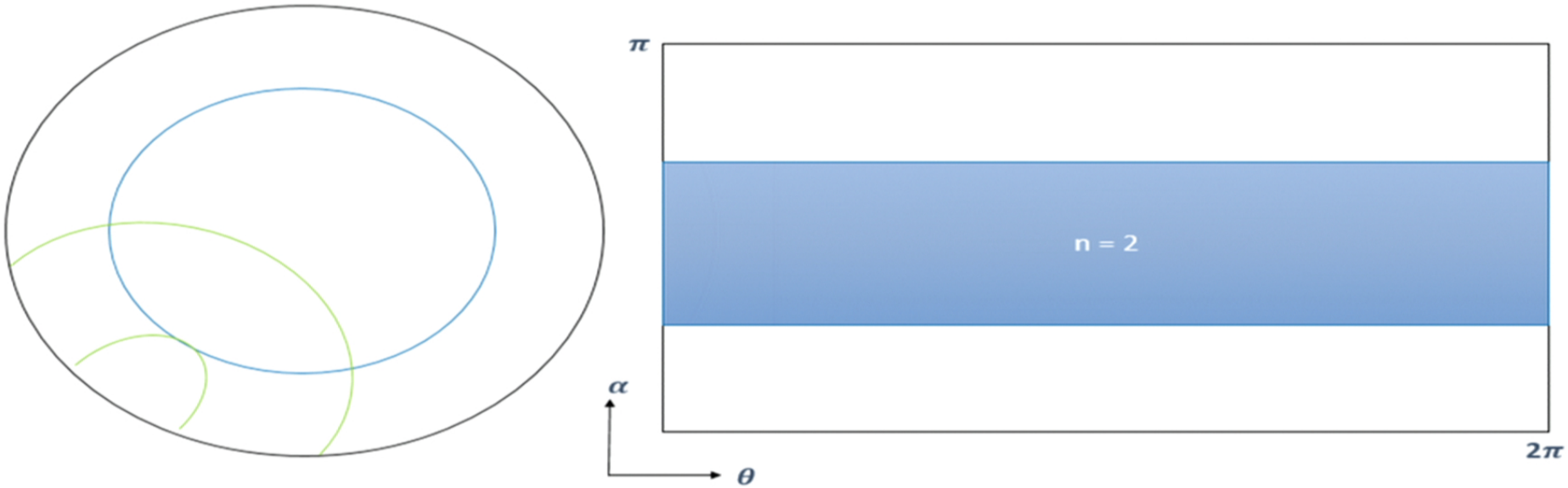
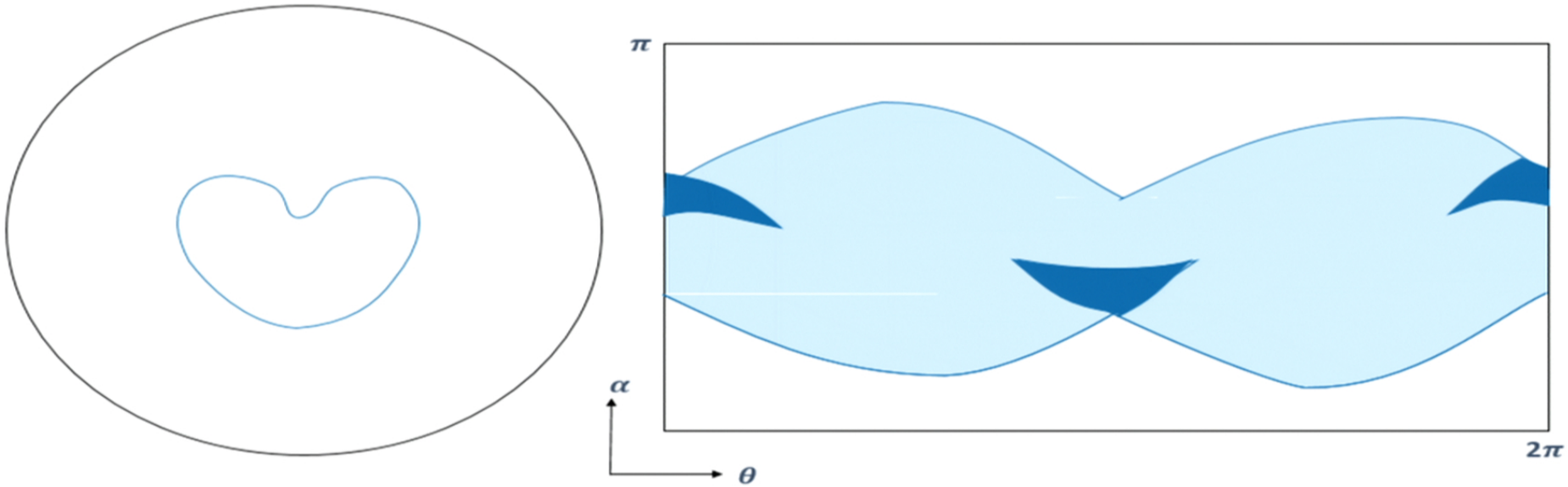
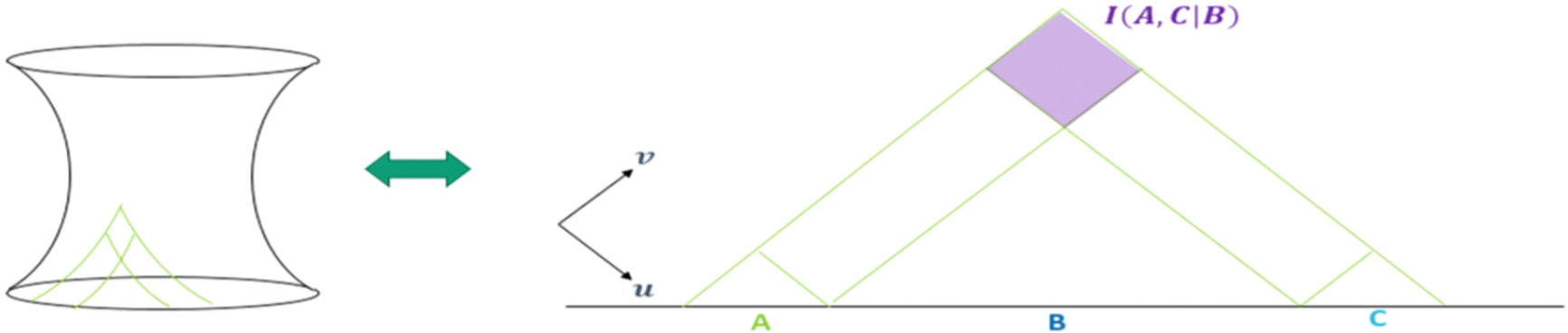


- Cusps from non-convex curves are just regions with higher intersection number because geodesics can enter and leave a region multiple times.



# Interpreting Kinematic Geometry

- The volume form was chosen to be geometrically useful.
- But it also has a very simple information theoretic interpretation:



# MERA and Holography

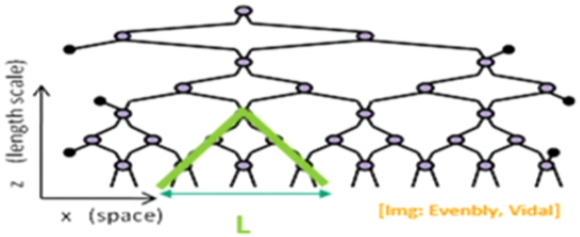
[Swingle; Evenbly, Vidal]

# MERA and Holography

[Swingle; Evenbly, Vidal]

- The MERA network has an additional spatial dimension to the boundary state.
  - It is related to the length-scale of entanglement coarse-graining.
- It is natural to view the MERA as related to a holographic geometry.

- This can be made precise:
  - Associate a fixed distance to crossing each line of the lattice.
  - The corresponding metric is that of a spatial slice of AdS2.



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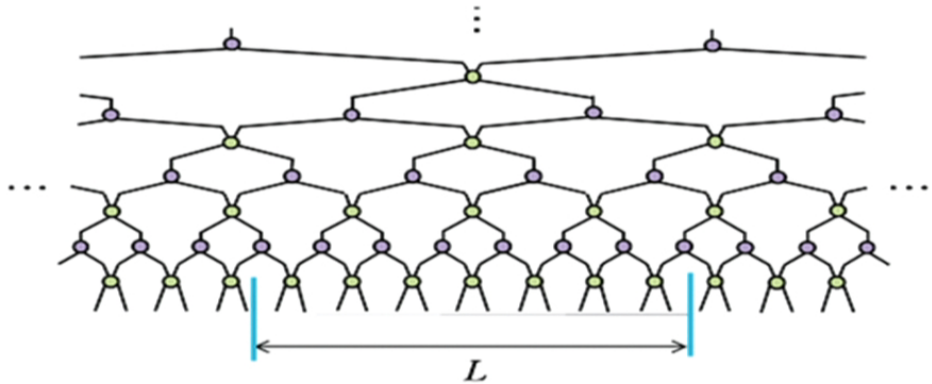
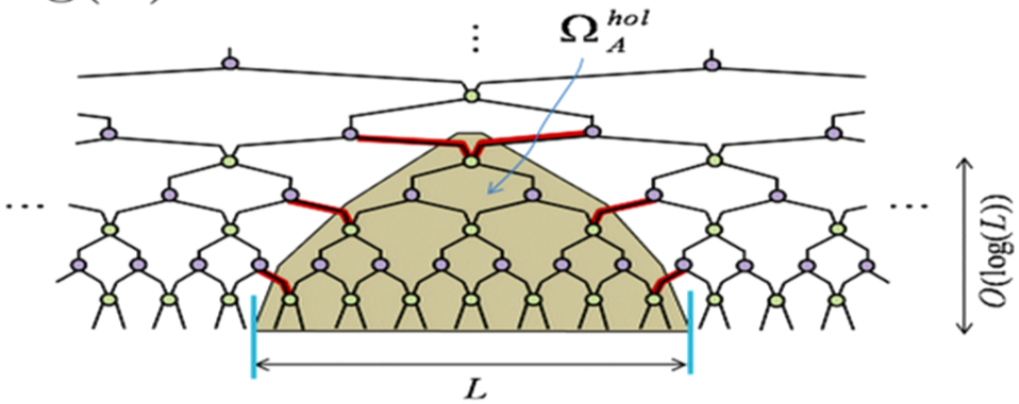



[Img: Evenbly, Vidal]

- MERA viewed as a discretized AdS2 geometry also realizes the Ryu-Takayanagi procedure for calculating entanglement entropies:
  - Consider a minimal cut in the MERA network that is homologous to some boundary region of length L (measured in lattice sites)
  - The length of the cut is then given by:

$$d \sim \log(L)$$

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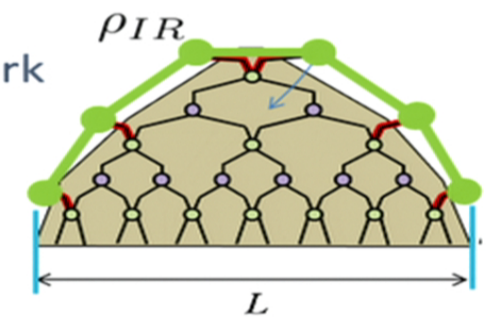


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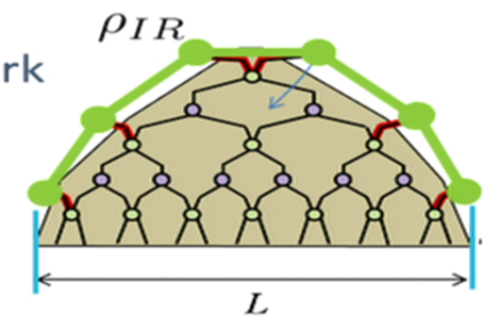
  - The density matrix for the boundary regions is determined by the IR density matrix and part of the network
  - The dimension of the IR Hilbert space gives an upper-bound on the EE:

$$S \leq k \log(L)$$

(scaling realized for the ground state)

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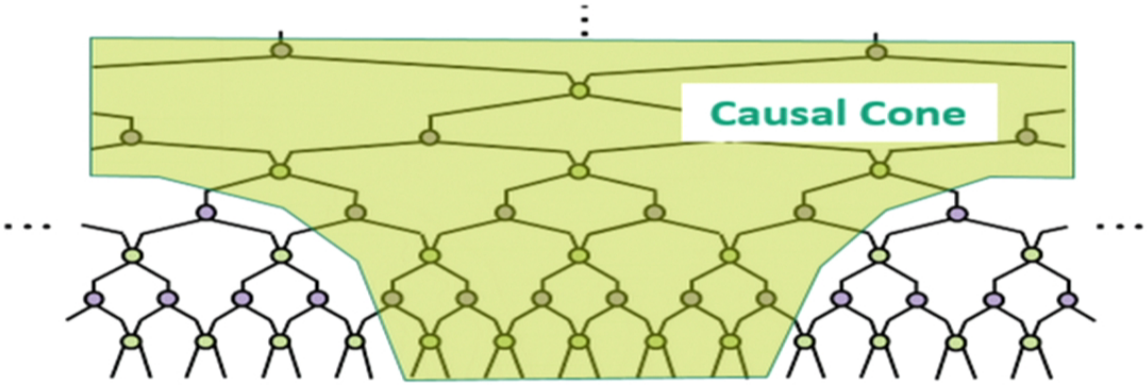
# Incongruities with the Standard Picture

1. In assigning a metric to the MERA network, a Euclidean signature was assumed, not derived
   - The network can have a natural causal structure associated to it that is suggestive of Lorentzian signature:



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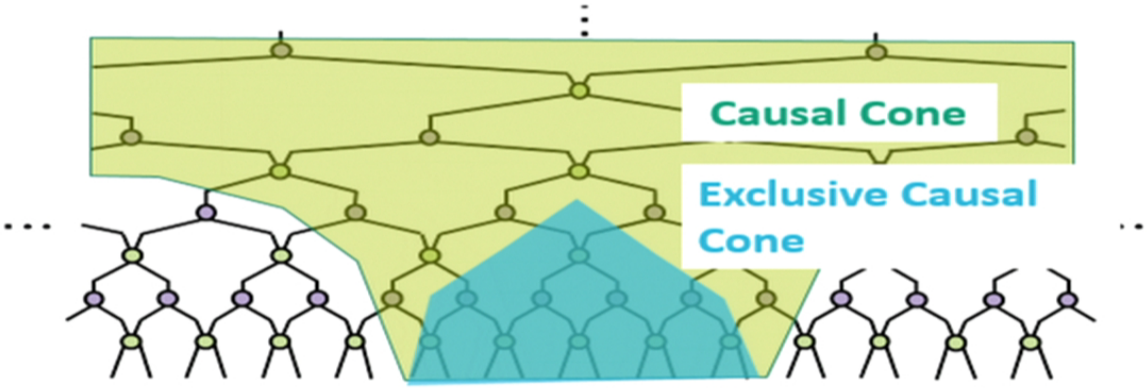

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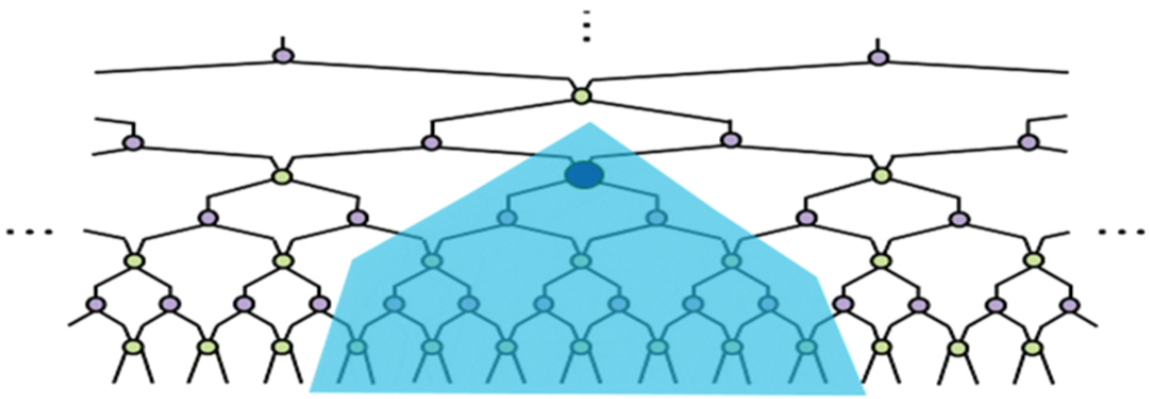
2. A particular node in the network corresponds to a unitary transformation that affects a region of a certain size, but that can't be localized as a unitary on any smaller interval.



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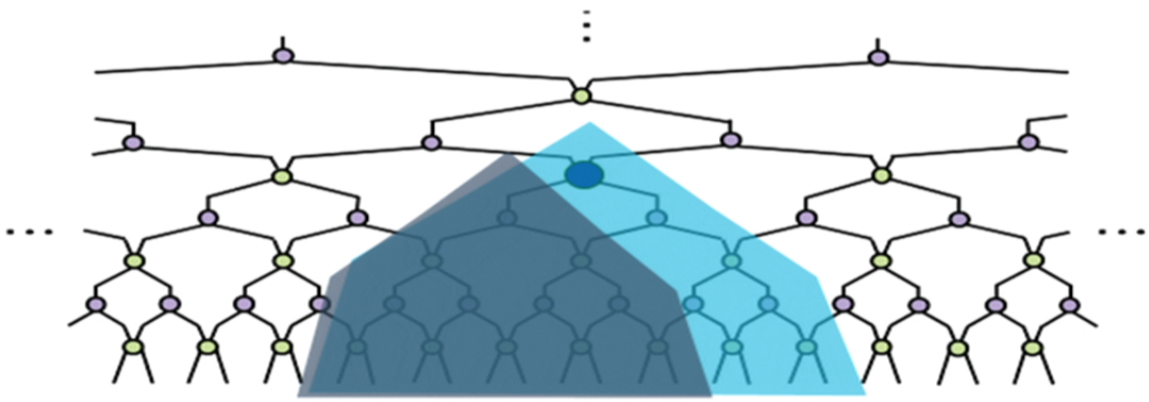

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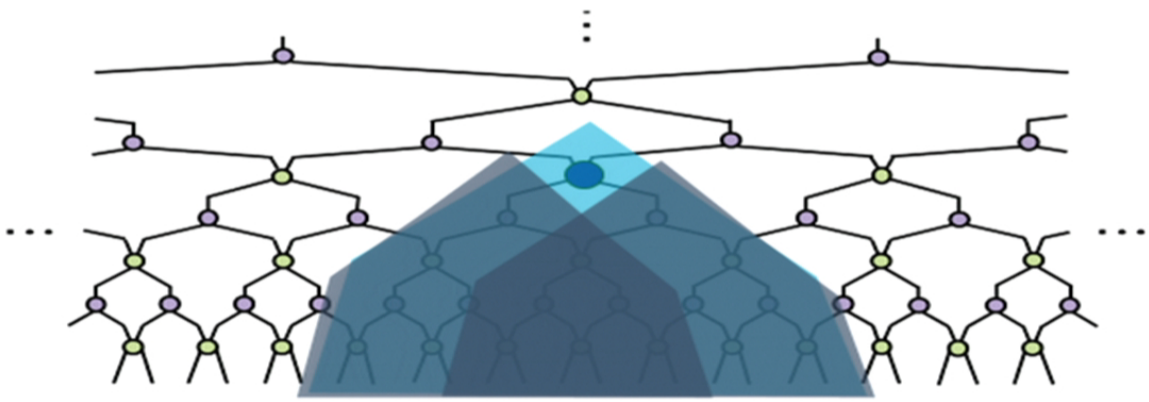

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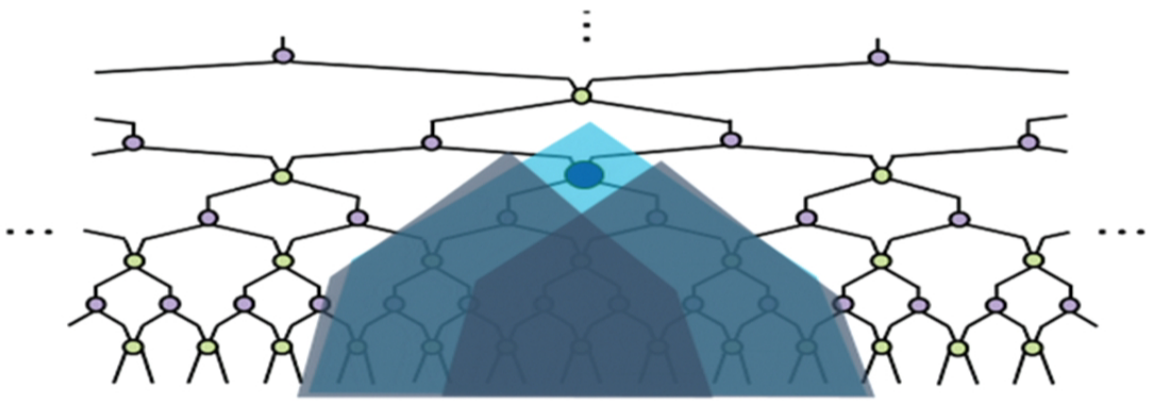

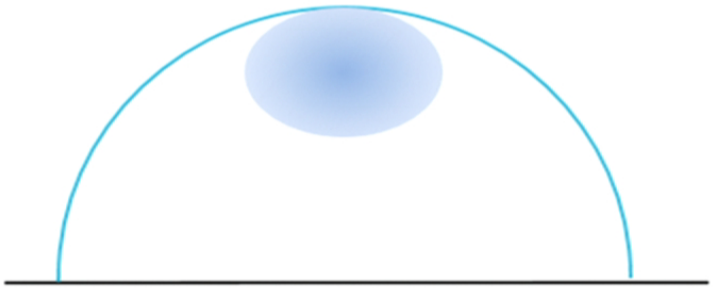

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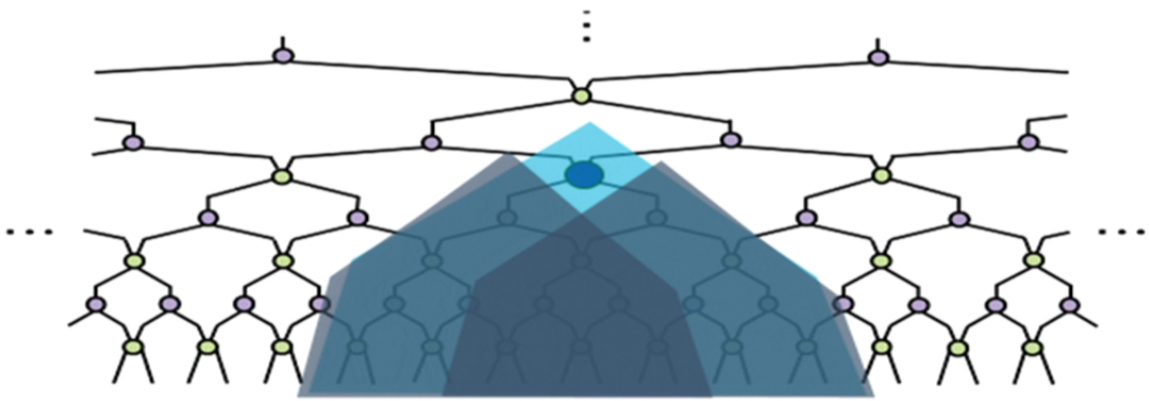
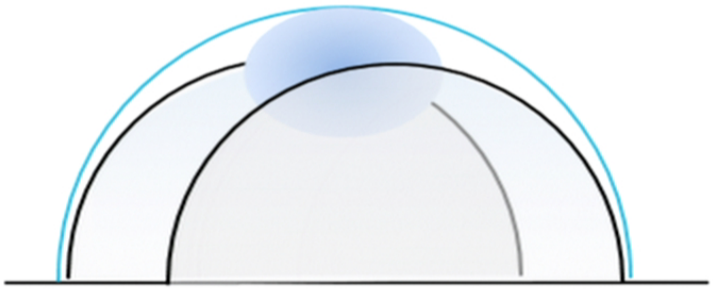
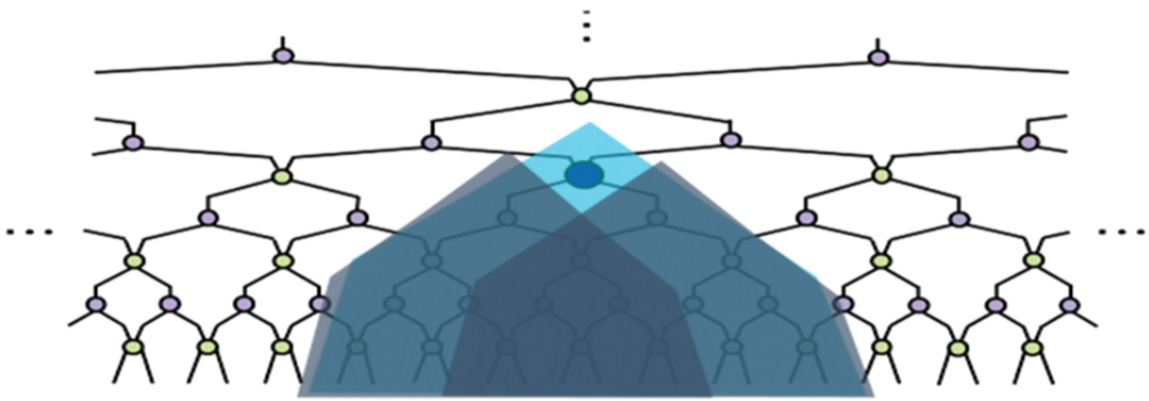
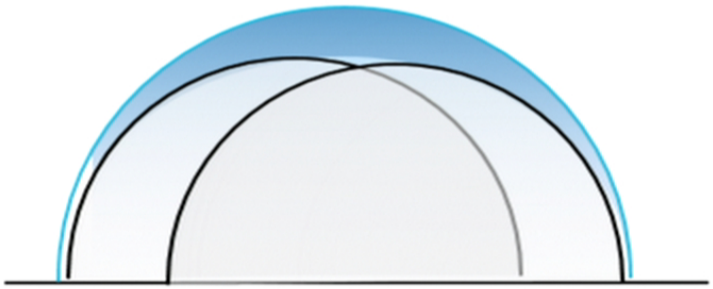




# Incongruities with the Standard Picture

2. A particular node in the network corresponds to a unitary transformation that affects a region of a certain size, but that can't be localized as a unitary on any smaller interval.
   - Seems more natural to associate this with a symmetric region in a narrow shell of the RT surface.





# Lessons from MERA for general TN

- Picture from MERA is not perfect. But, I would like to translate the workings of MERA into general principles for constructing a tensor network. What is MERA ***almost*** doing that's useful?
- MERA only realizes the RT procedure insofar as the links cut are assumed to be uncorrelated, so distances and cuts are both additive and locally measurable. **This is the heart of the connection between tensor networks and geometry.**
  - This is equivalent to saying that the RT surface in the TN encodes a **compressed state** of the interval, where we have removed all mutual information of edges of the network.
- We argue this should be elevated to a principle:

**Principles of Tensor Networks for Gravitational States:**

1. **The TN should encode compressed states along the 'RT' surfaces.**

# Lessons from MERA for general TN

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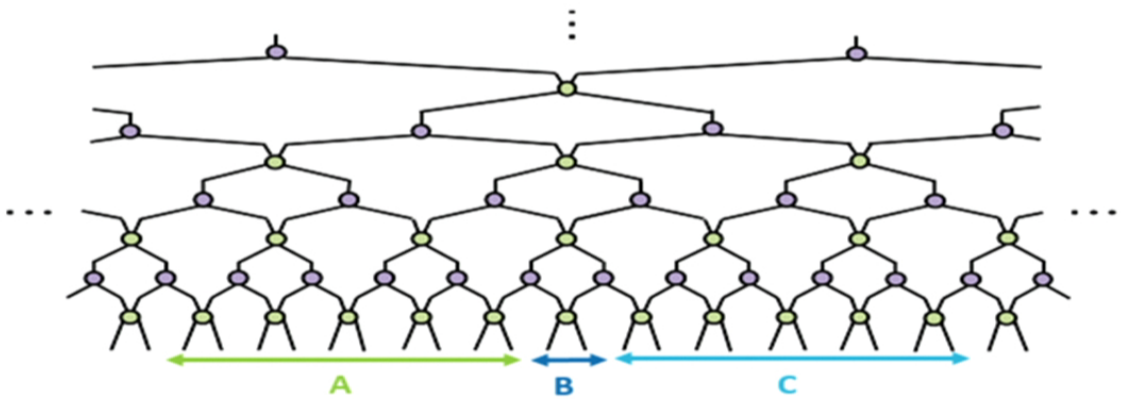

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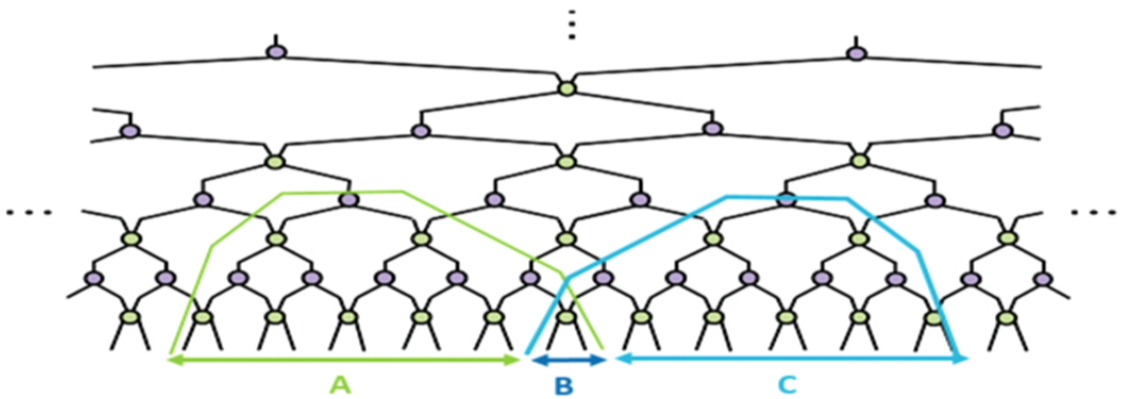


$S_{AB} + S_{BC}$

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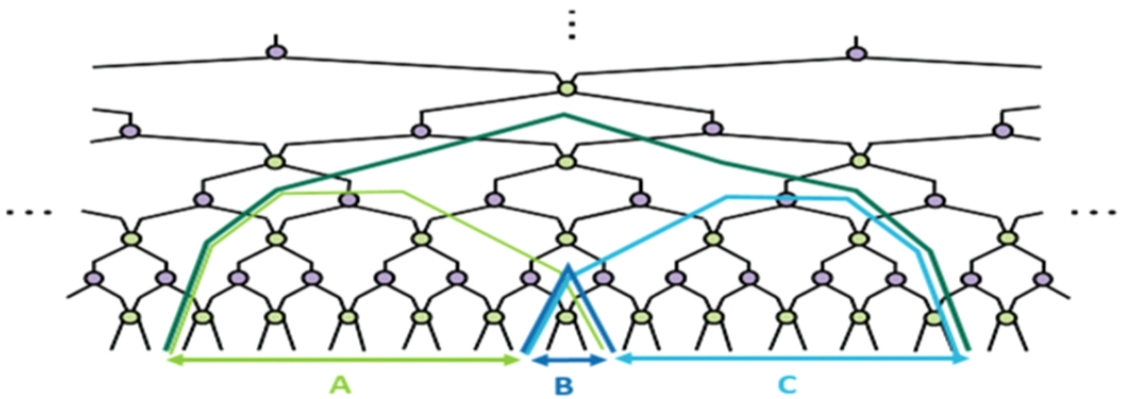


$$S_{AB} + S_{BC} - S_{ABC} - S_B$$

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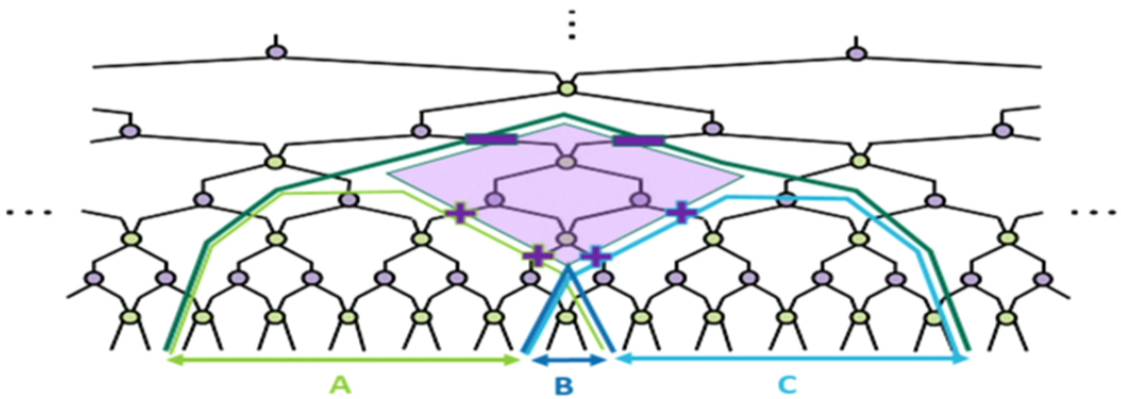


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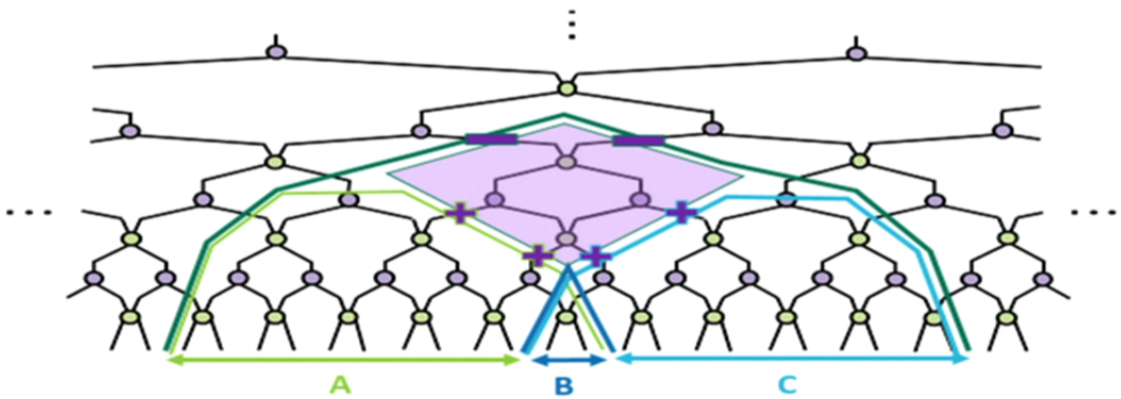
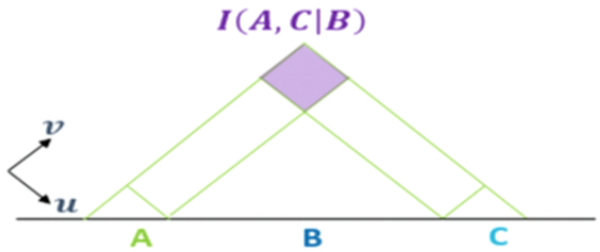
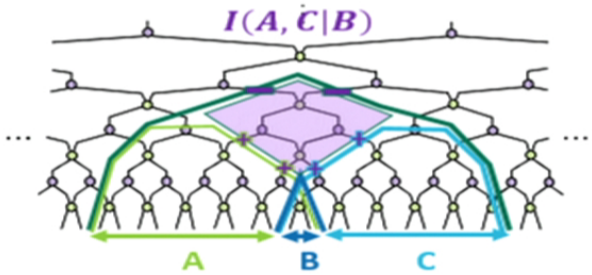


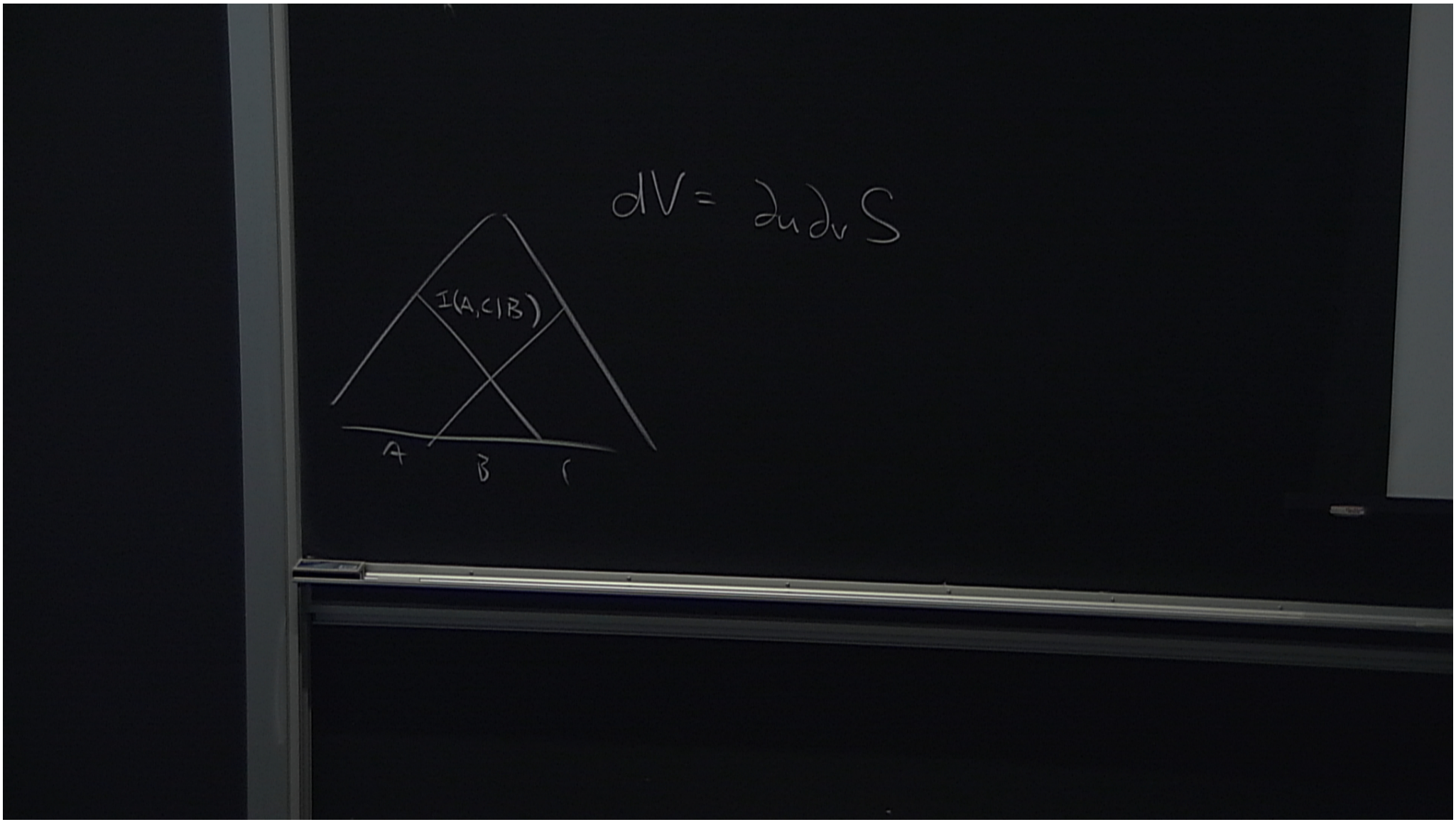

$$S_{AB} + S_{BC} - S_{ABC} - S_B = I(A, C|B)$$

# The Basics

- Notice that an area element in Kinematic space describes the exact same quantity as does the marked region in MERA for the ground state of a CFT
- It is just a small leap now to suggest that the MERA network is actually describing not hyperbolic space, but its dual de Sitter space!
  - The 'RT' surfaces should actually be understood as the causal domain of dependence of the boundary region of dS2
  - The tensor nodes aren't describing a point of discretized H2, but are points of discretized dS2 associated to a particular geodesic/boundary interval




dV = ∂u ∂v S
I(A,C|B)
A
B
C

- **But, we didn't just find a Kinematic space for the AdS vacuum.**
- Using our principled approach to understand what MERA is doing, and our knowledge of Kinematic geometry, can we find efficient tensor networks for more general states?
- In hindsight, let's modify our principles:

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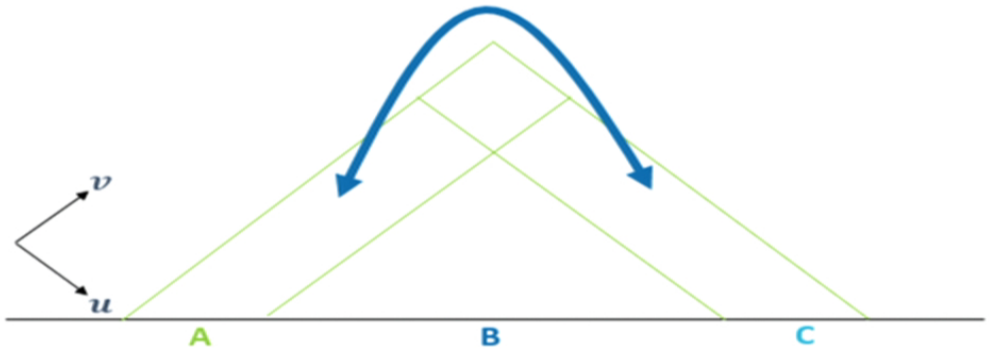
2. **Conditional Mutual Information is encoded locally by the volume of Kinematic space.**

and see how far we can go with just these ideas...

- The requirement that our network performs compression along the future boundary is extremely powerful:
  - Correlations between subsystems A and C cannot be created outside the future boundary of ABC because this is a compression surface,



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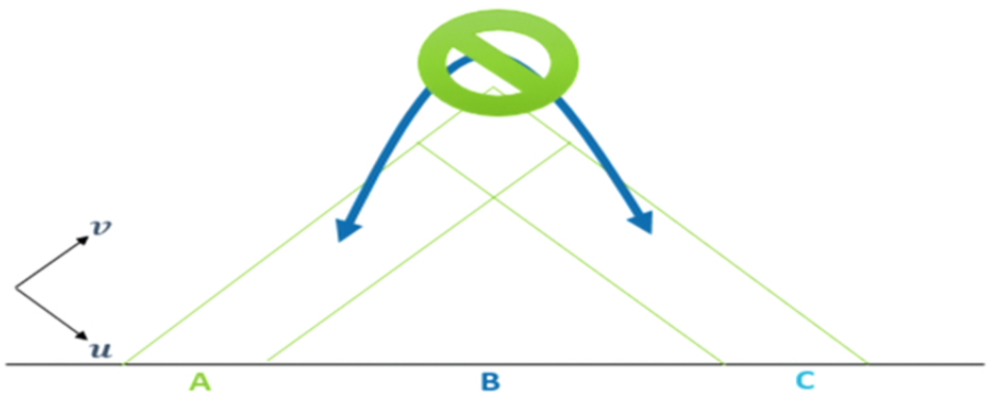

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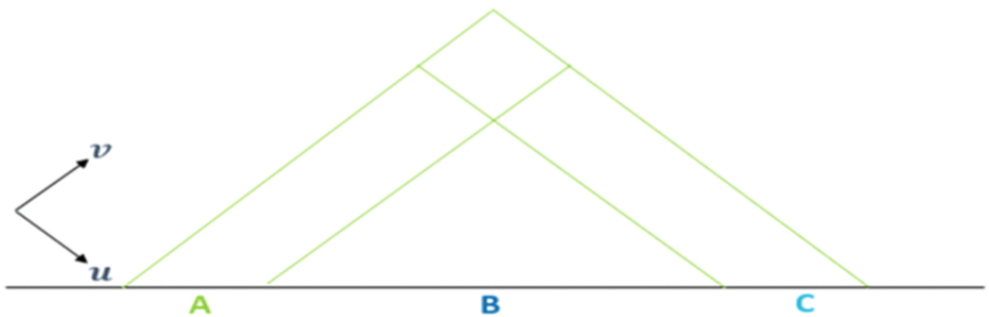

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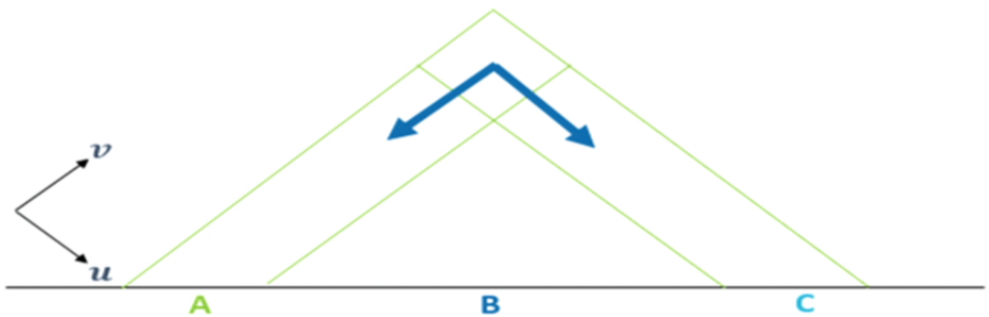
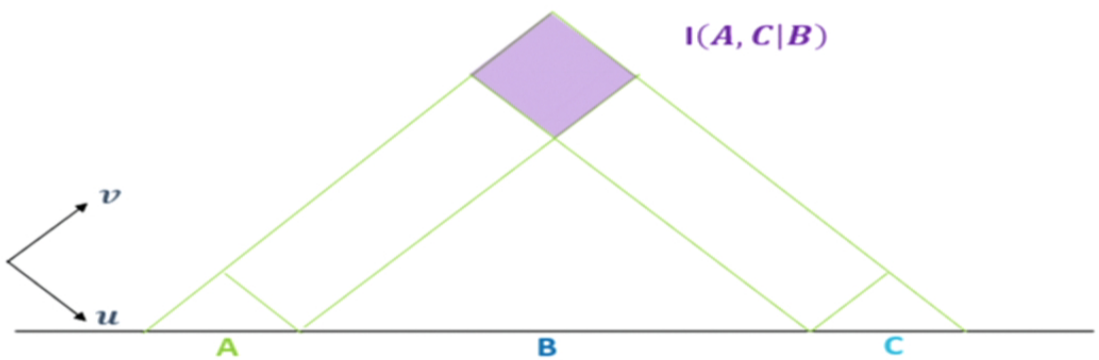
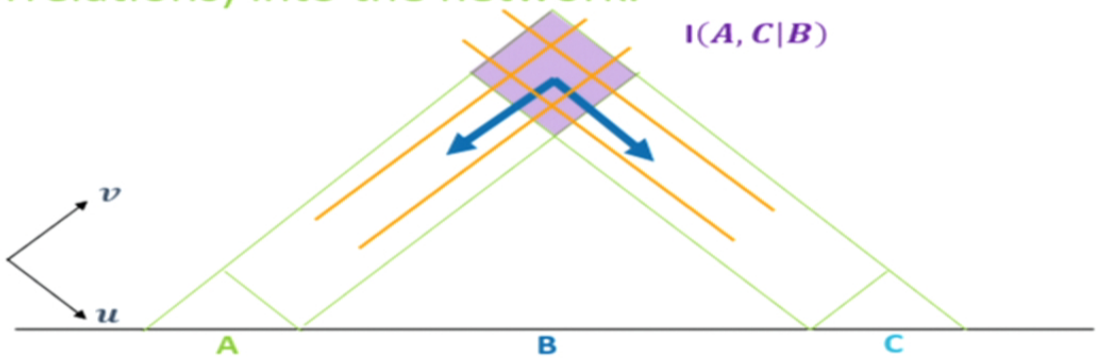
- So all correlations must arise from the part of the network in the top diamond



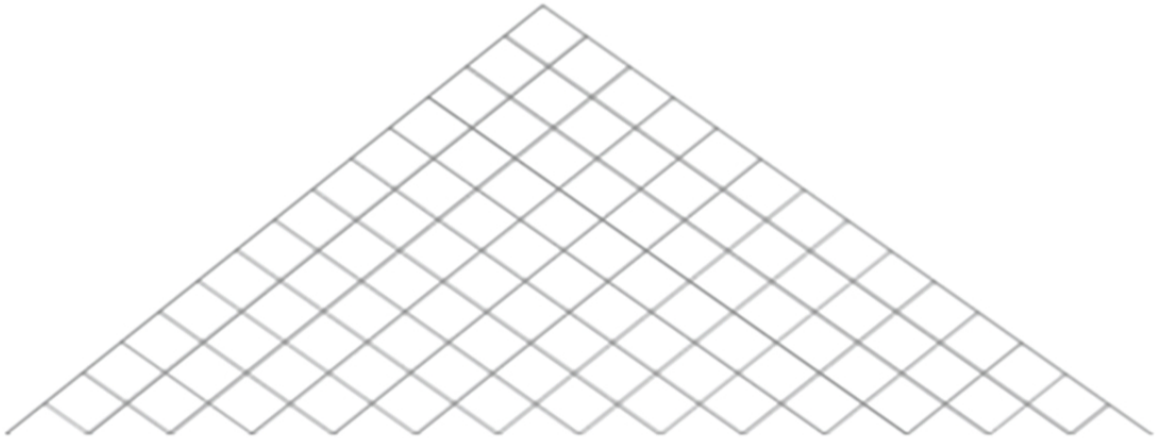
- Consider the Conditional Mutual Information I(A,C|B):



- Consider the Conditional Mutual Information I(A,C|B):
  - If all boundaries are uncorrelated, each line in the network gives an additive contribution.
    - Then CMI is just the difference in lines cutting the top and bottom and the colored region.
    - But the difference in the number of lines crossing the bottom and the top is just the number of lines inserted into the network in this region.
  - The volume integral measures the density of isometries that insert new lines (correlations) into the network.



- So for an arbitrary choice of entanglement entropies $S(u, v)$, we can use the corresponding volume form to add lines to the network that prepare a state with exactly these entropies



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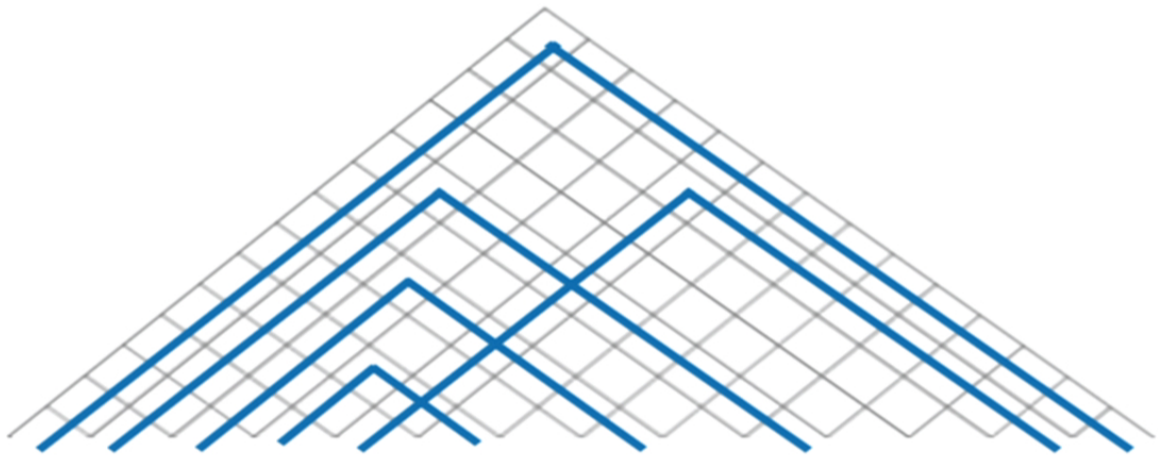

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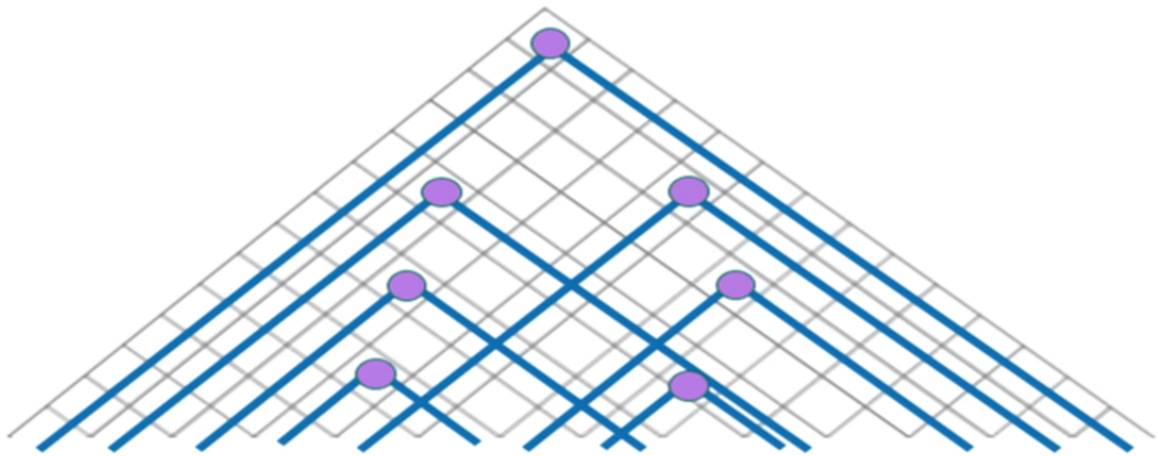

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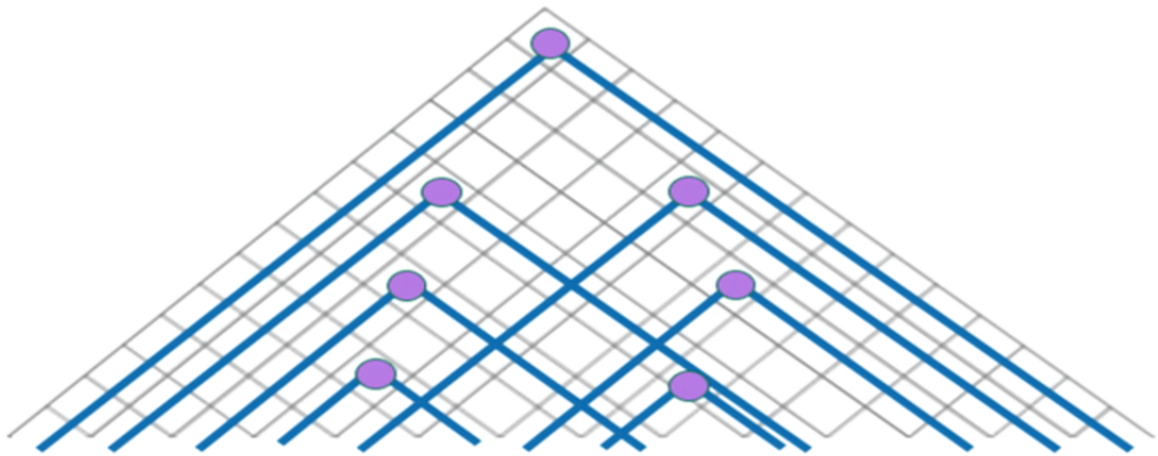
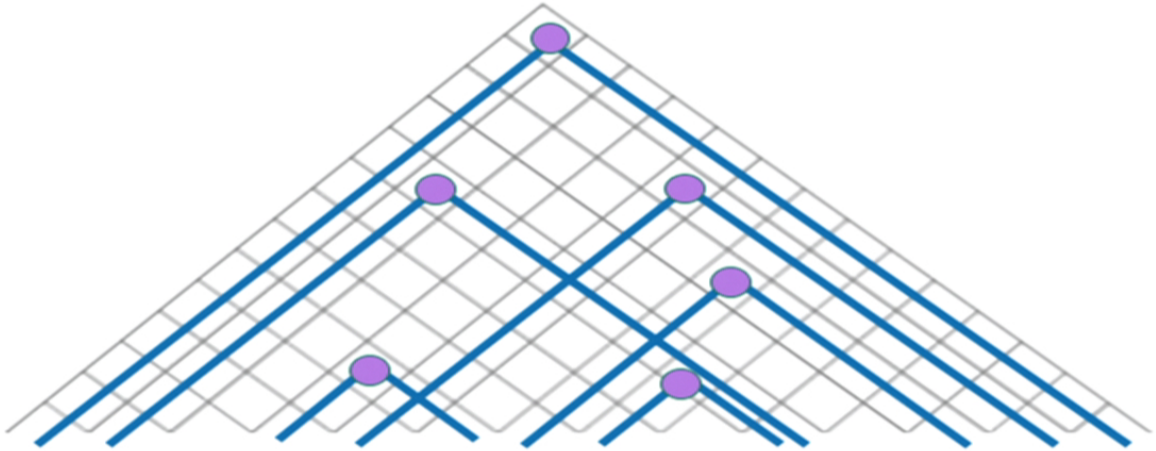
- Have we succeeded in reproducing the desired state with our tensor network?

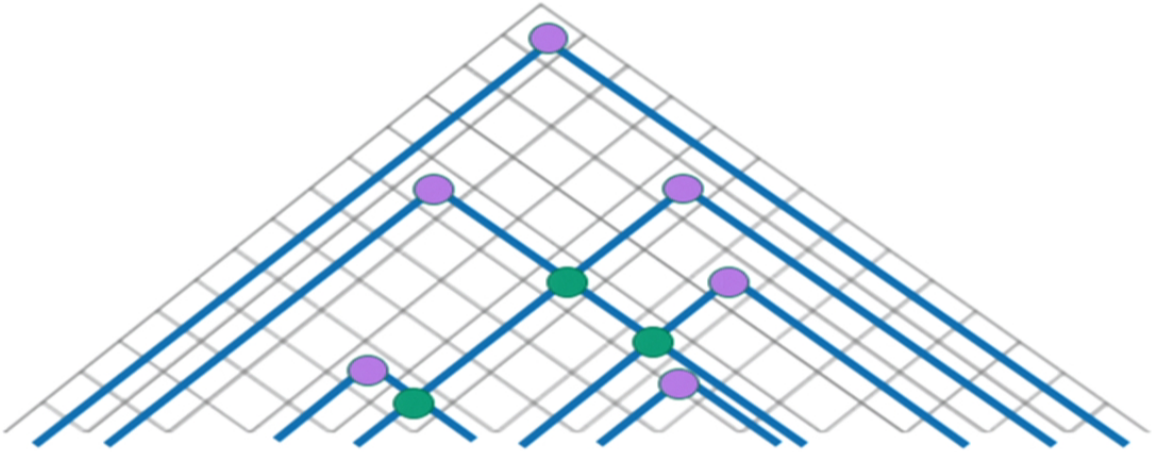


NO.

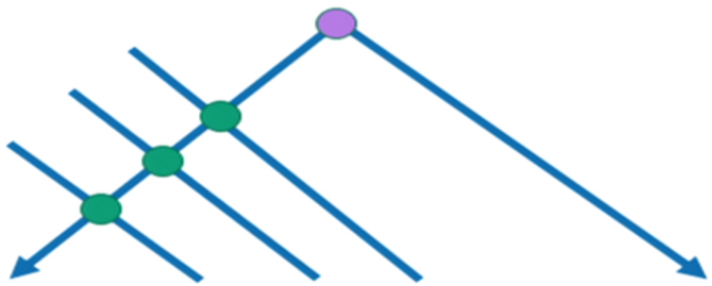


NO.

- While we have fixed the role of isometries (the insertion of correlation) in the network, we haven't said anything yet about the role of **disentanglers**.
  - The state we have is also far from generic---there is long range entanglement, but it is still localized. This is not at all like the geometric states in which we're interested.



- Reading the network from the top down, it's clear the disentanglers (now entanglers!) role is to distribute information across the interior of the light-cone:



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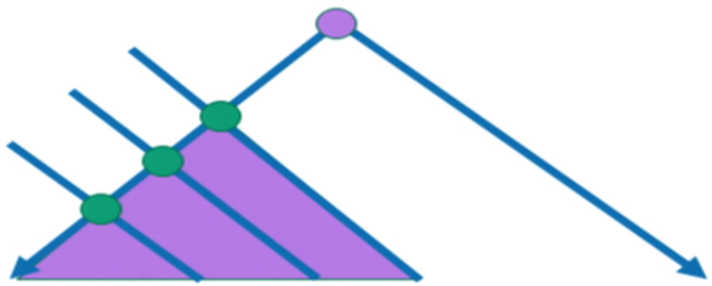

- To do so, note that the non-local mixing we have been describing is precisely what is measured by the tripartite information:

$$I_3(A, B, C) = I(A, C) + I(B, C) - I(AB, C)$$

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- Their equality is the condition that every disentangler is maximally mixing.

# Summary

- We constructed a dual Kinematic Space whose points are boundary-anchored geodesics in position space.
- We demonstrated that Kinematic Space is the natural setting for formulating differential entropy in 2-dimensions.
- We showed that natural geometric structures on Kinematic Space encode simple information theoretic quantities about a CFT state.
- We gave an explicit procedure to use these information theoretic quantities to construct an iterative compression network for the CFT state. **(Q4)**
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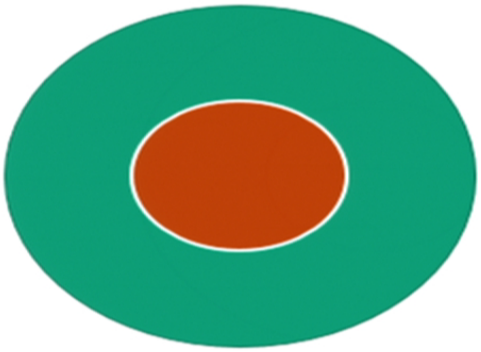
- We argued that these tensor networks give a non-perturbative definition for spacetime entropy that is markedly different for the perturbative one. **(Q3)**

# Desiderata:
# Implications, Work in Progress, Open Questions

# Kinematic Space and Entropy

- Differential entropy was a partial answer to this question, but there were some confusions.



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$$S_{\text{diff}} = \int \mathrm{d}x \frac{\delta S[R(x)]}{\delta R(x)}$$

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[Swingle, Kim]:

Assume the ground state is unique.

The collection of marginal density matrices $\{\rho_{x,R}\}_{x\in\partial B}$ is sufficient to calculate the expectation value of the local Hamiltonian.

Thus, there is a unique state $\rho_{\text{vac}}$ that is consistent with our collection of density matrices. It's impossible to "fill in" the hole in the spacetime without propagating energy out the boundary. *(This may be a problem for other definitions of 'residual' entropy as well.)*

To see where we went wrong, it's helpful to return to the KS/TN picture:

- Cutting out a hole in the interior of the TN does **NOT uniquely determine** the collection of density matrices.
  - The density matrix depends on the complete causal cone of the boundary region.
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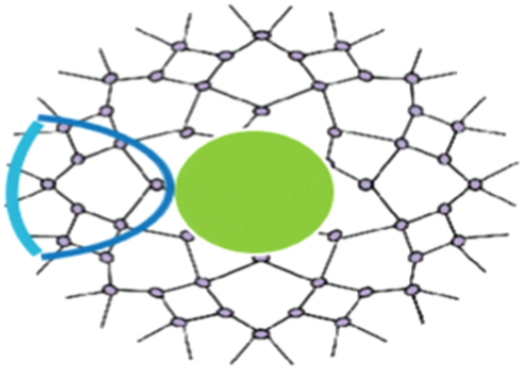

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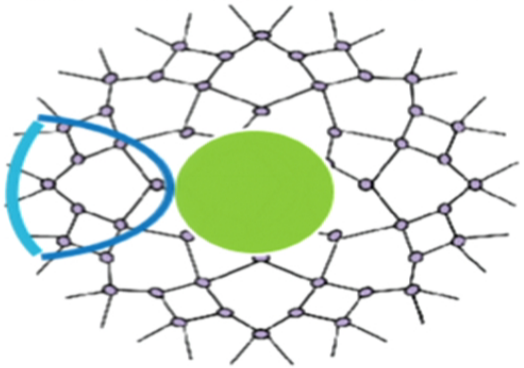
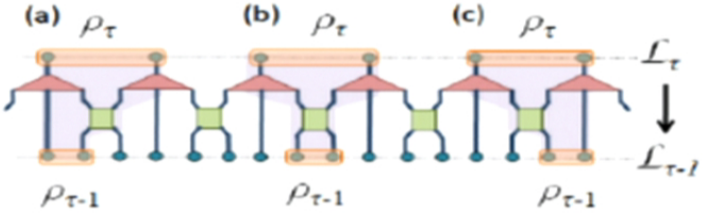
  - The exterior region cuts the causal cone at its shortest width.
- The information encoded in the exterior of the TN is really a 'Descending Superoperator' that maps the IR density matrix specified at the hole to UV density matrix at the boundary.
- The Descending Superoperator can be thought to propagate the conserved charges of interior excitations to the UV boundary; it enforces gravitational Gauss' Law.






**Descending Superoperator:**

(a') $\rho_{\tau-1} = \mathcal{D}_{\mathcal{L}}(\rho_\tau)$

[Vidal, Evenbly]

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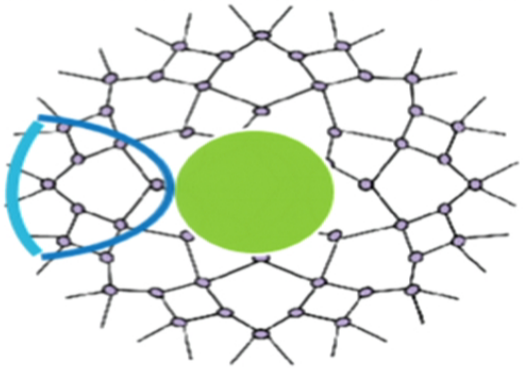
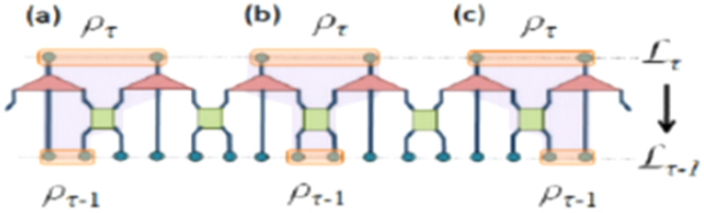
  - The exterior region cuts the causal cone at its shortest width.
- The information encoded in the exterior of the TN is really a 'Descending Superoperator' that maps the IR density matrix specified at the hole to UV density matrix at the boundary.
- The Descending Superoperator can be thought to propagate the conserved charges of interior excitations to the UV boundary; it enforces gravitational Gauss' Law.



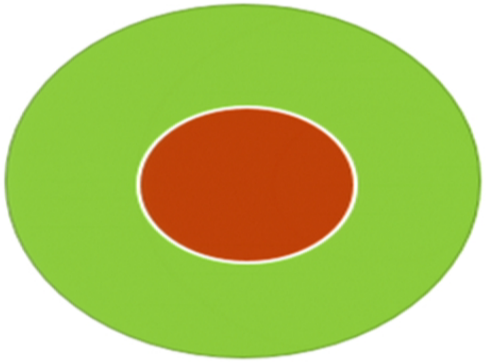



**Descending Superoperator:**

(a') $\rho_{\tau-1} = \mathcal{D}_{\mathcal{L}}(\rho_\tau)$

[Vidal, Evenbly]

- So we find that while perturbative QFT associates the entropy to states...



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- It's clear that the TN can propagate information about the IR state to the boundary. But what about the geometry?
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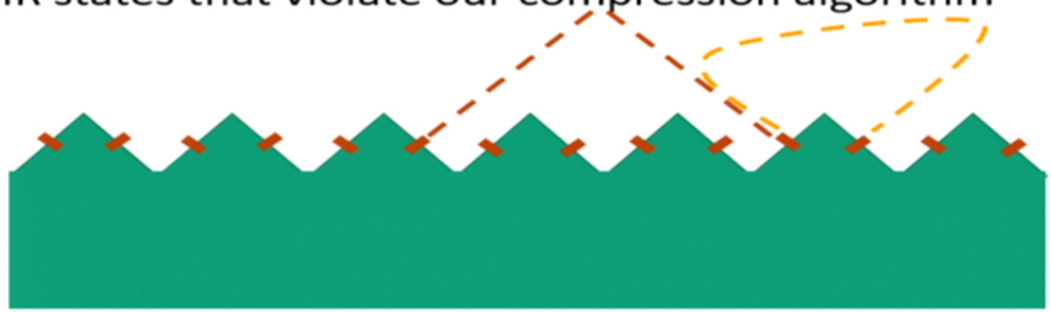

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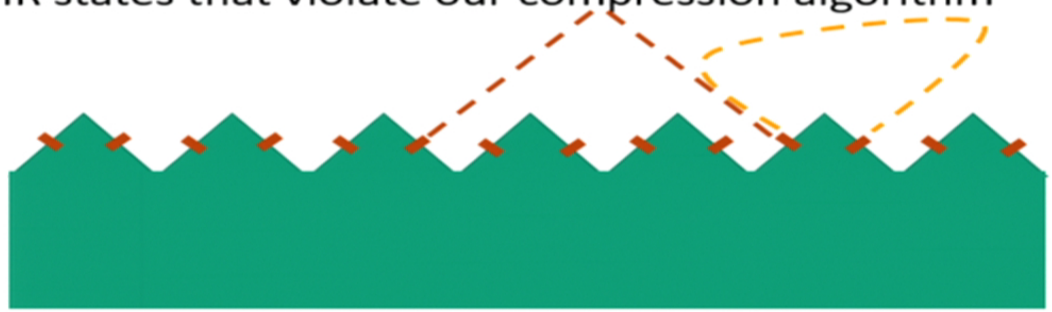

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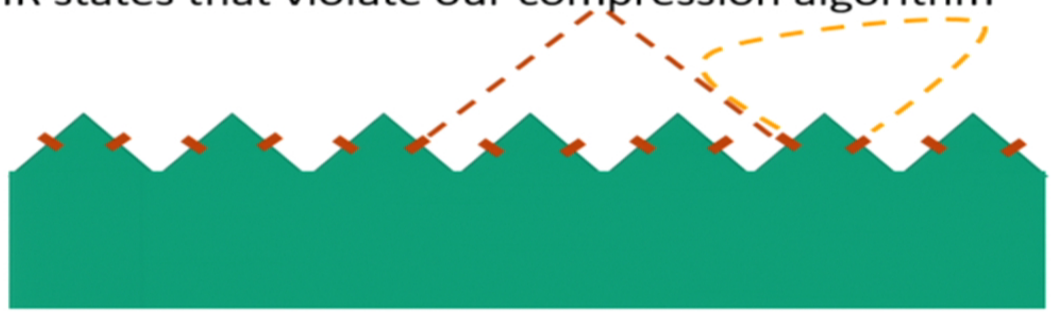

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Is there an elegant rewriting of EE in this language?

# The Bianchi-Myers Conjecture

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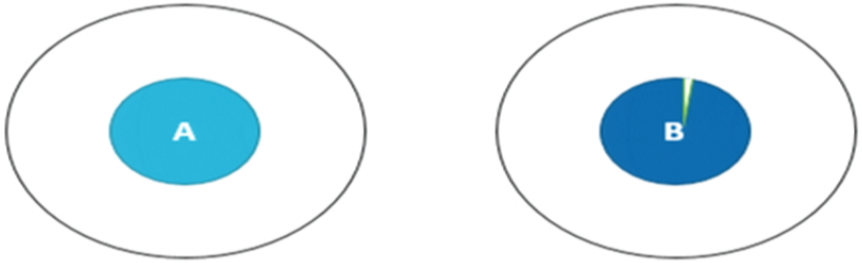
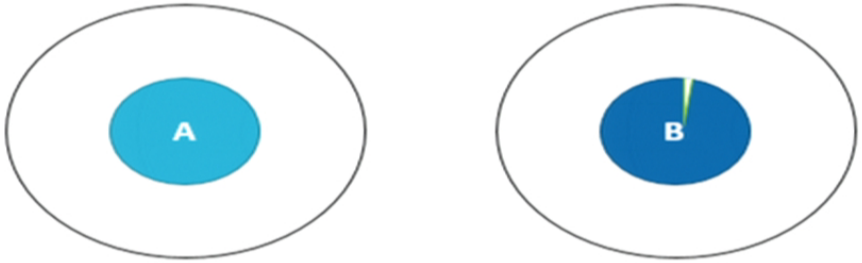
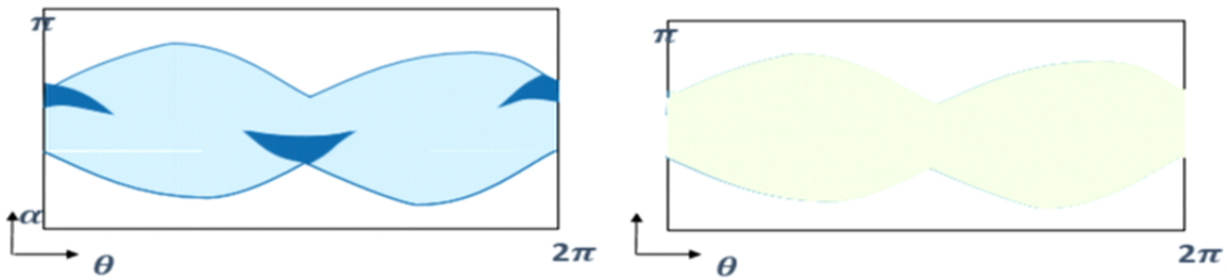


# The Bianchi-Myers Conjecture

- Recall one of our aims was to better understand what holographic entropy is counting.
- The BM conjecture suggests that we should assign some entanglement entropy A to the factorization of the Hilbert space corresponding to the light blue region. The entropy A is given by the length of its boundary.
  - Similarly, we assign an entropy B>A to the dark blue region.



- From the tensor network perspective: Is this latter region encoded by a part of the network that prepares a much larger number of states?
  - **No: the non-perturbative perspective given to us by MERA and Kinematic Space sees this counting very differently**
- The spike makes our surface non-convex. Such a surface is not encoded by a simple region of KS/ TN because we double-count regions.



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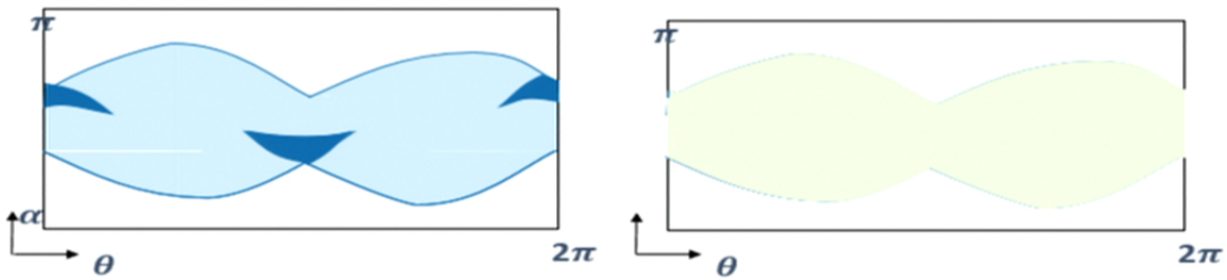
- The spike makes our surface non-convex. Such a surface is not encoded by a simple region of KS/ TN because we double-count regions.
- Can't assign a state to this double-counted portion of the network in the same way that we can the single-counted region that encodes it's convex hull



- Only in special situations (convex regions) should we associate an entropy to a region of spacetime that is proportional to the area.

- **MERA and the more general Kinematic Space agree on a refined spacetime entanglement proposal that can be markedly different from the perturbative viewpoint.**

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**Q: What are the implications of this new counting?**