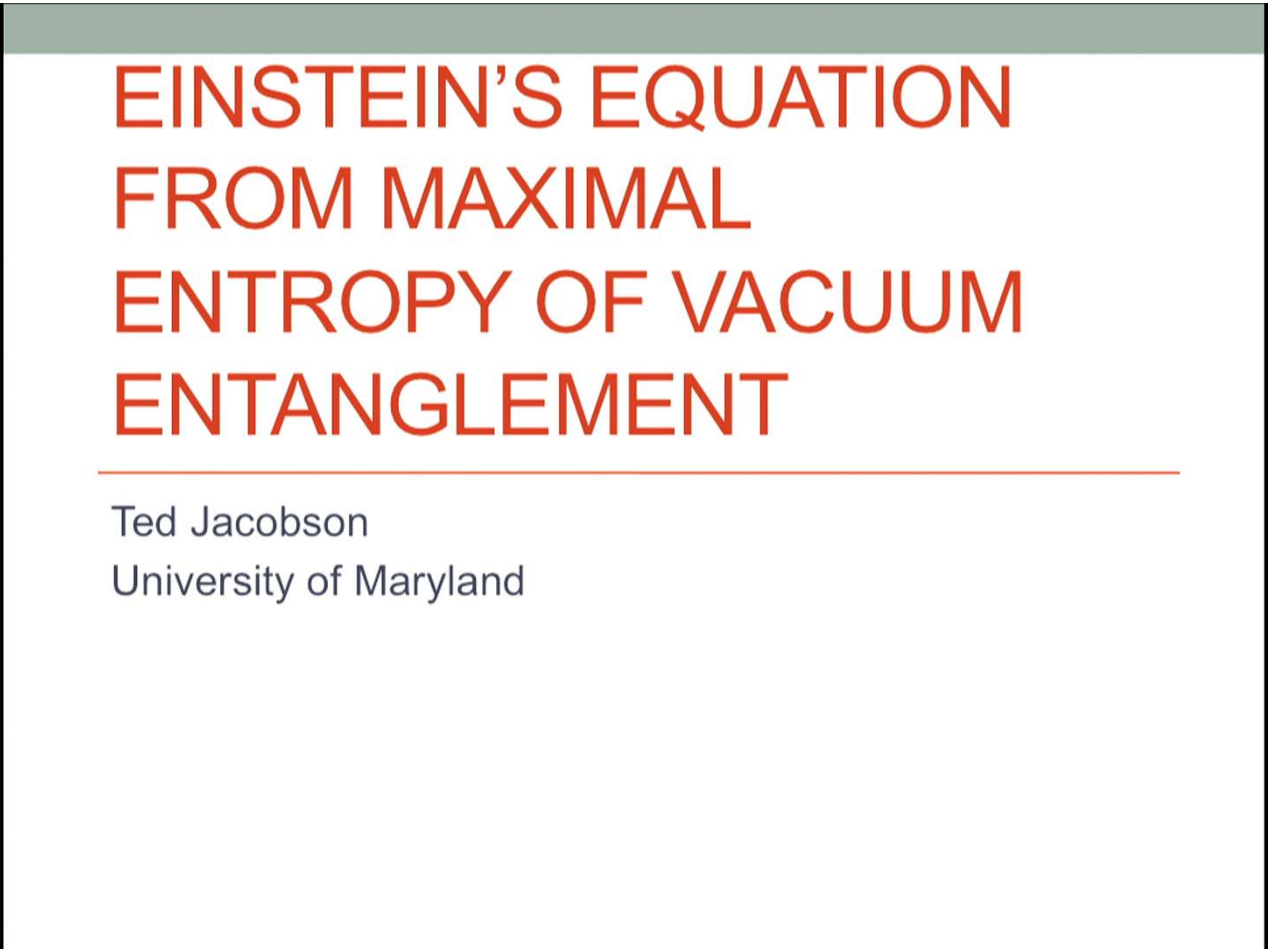
Title: Einstein's equation from maximal entropy of vacuum entanglement

Date: Aug 19, 2015 02:00 PM

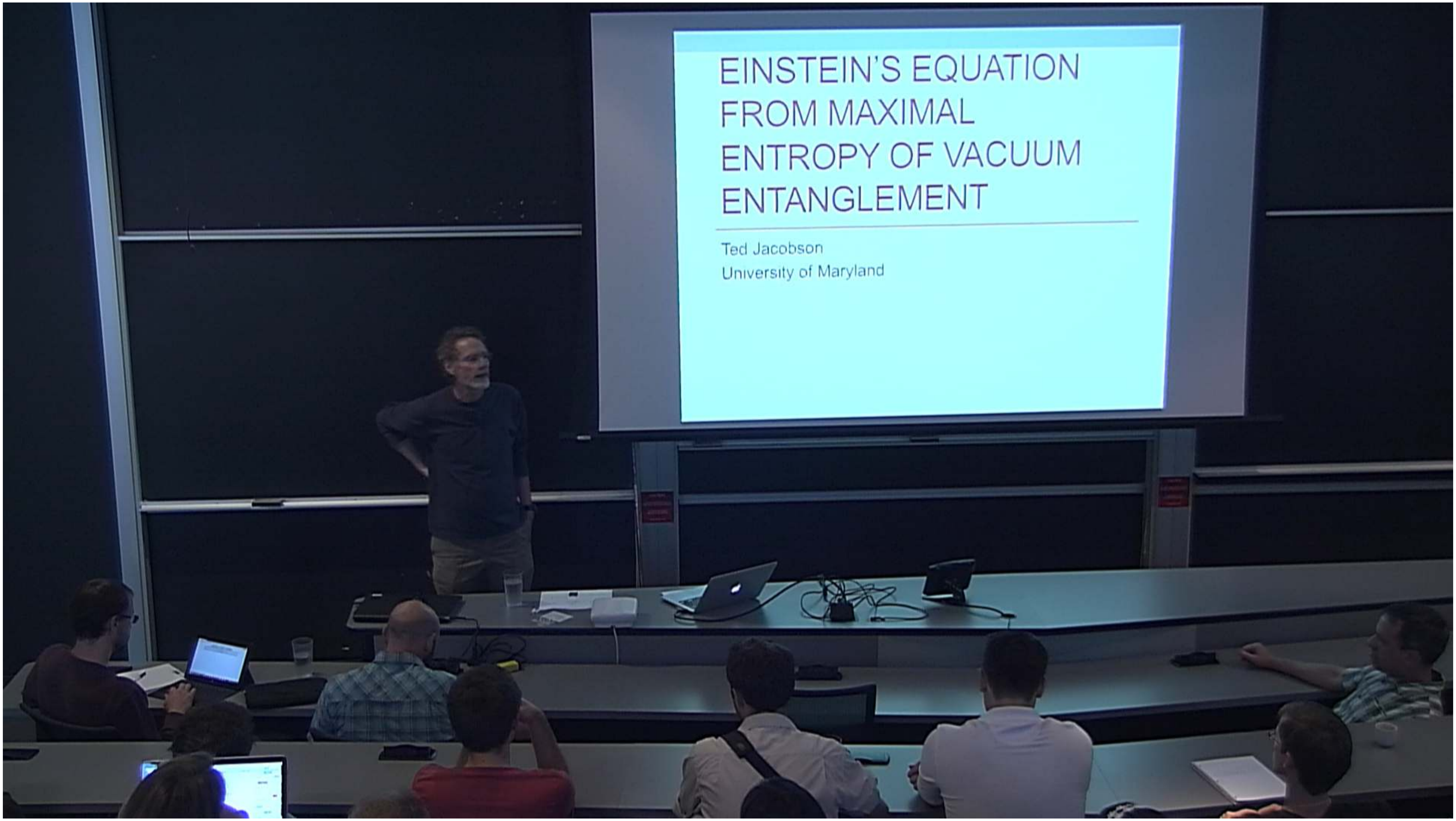
URL: http://pirsa.org/15080072

Abstract: If entanglement entropy in a small geodesic ball is maximized at fixed volume in the vacuum, then it should be stationary under variation to a nearby state. I will show that this stationarity condition is equivalent to the semiclassical Einstein equation. If the matter QFT is not conformal, then the derivation requires a further assumption about QFT, whose validity is currently under investigation. [Based on http://arxiv.org/abs/1505.04753]



EINSTEIN'S EQUATION FROM MAXIMAL ENTROPY OF VACUUM ENTANGLEMENT

Ted Jacobson
University of Maryland

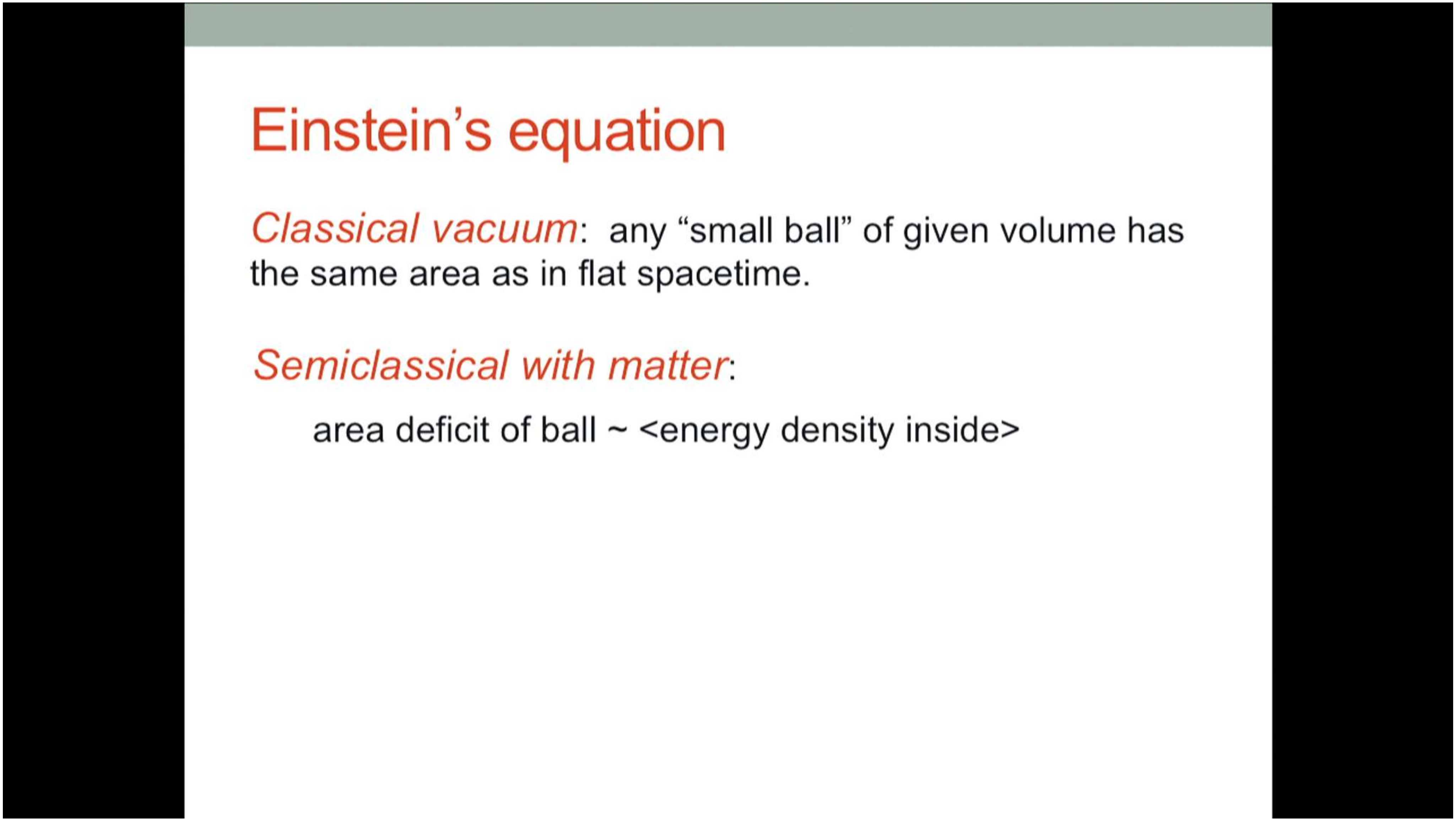


EINSTEIN'S EQUATION FROM MAXIMAL ENTROPY OF VACUUM ENTANGLEMENT

Ted Jacobson
University of Maryland

Einstein's equation

Classical vacuum: any "small ball" of given volume has the same area as in flat spacetime.



Einstein's equation

Classical vacuum: any "small ball" of given volume has the same area as in flat spacetime.

Semiclassical with matter:

area deficit of ball ~ <energy density inside>

Einstein's equation

Classical vacuum: any "small ball" of given volume has the same area as in flat spacetime.

Semiclassical with matter:

area deficit of ball ~ <energy density inside>

Eintanglement: under some assumptions,

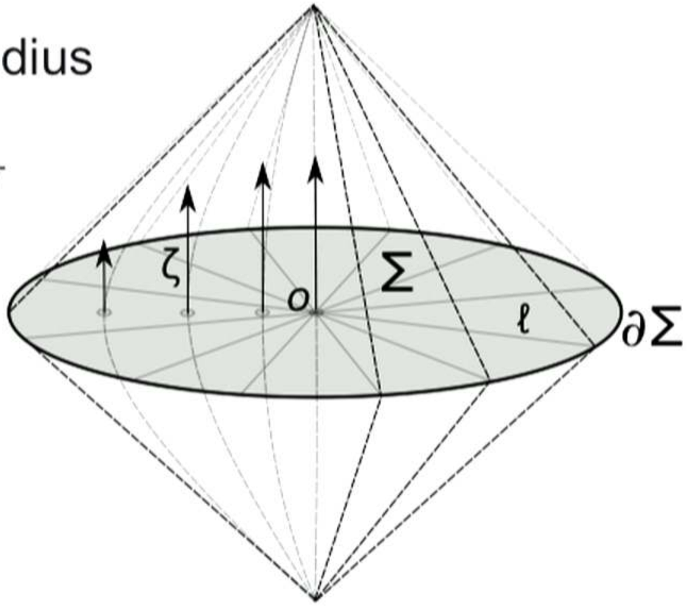
the above is equivalent to

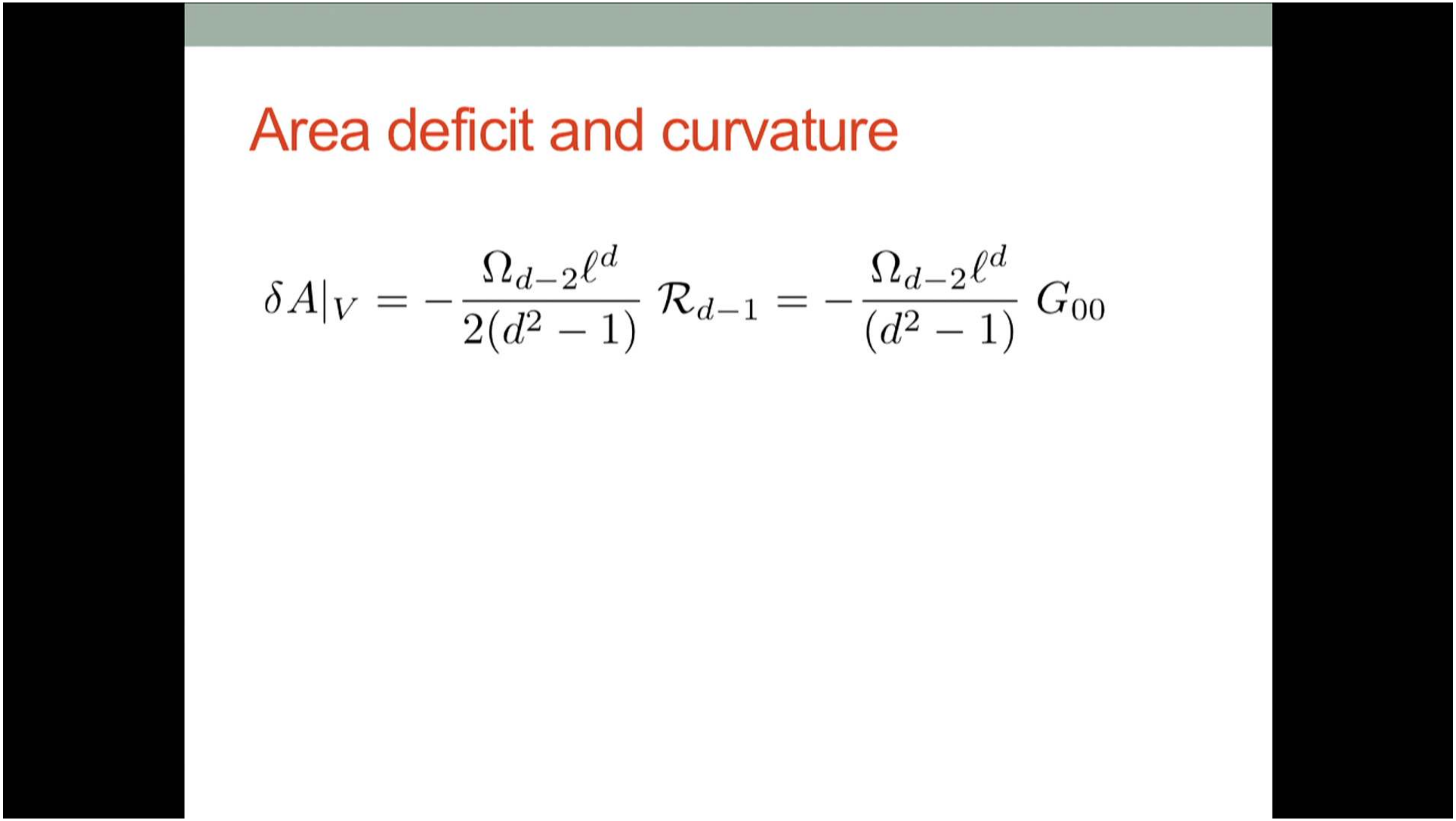
vacuum entanglement in any small ball of given volume is maximal

"Small Ball"

- Spacelike geodesic ball of radius

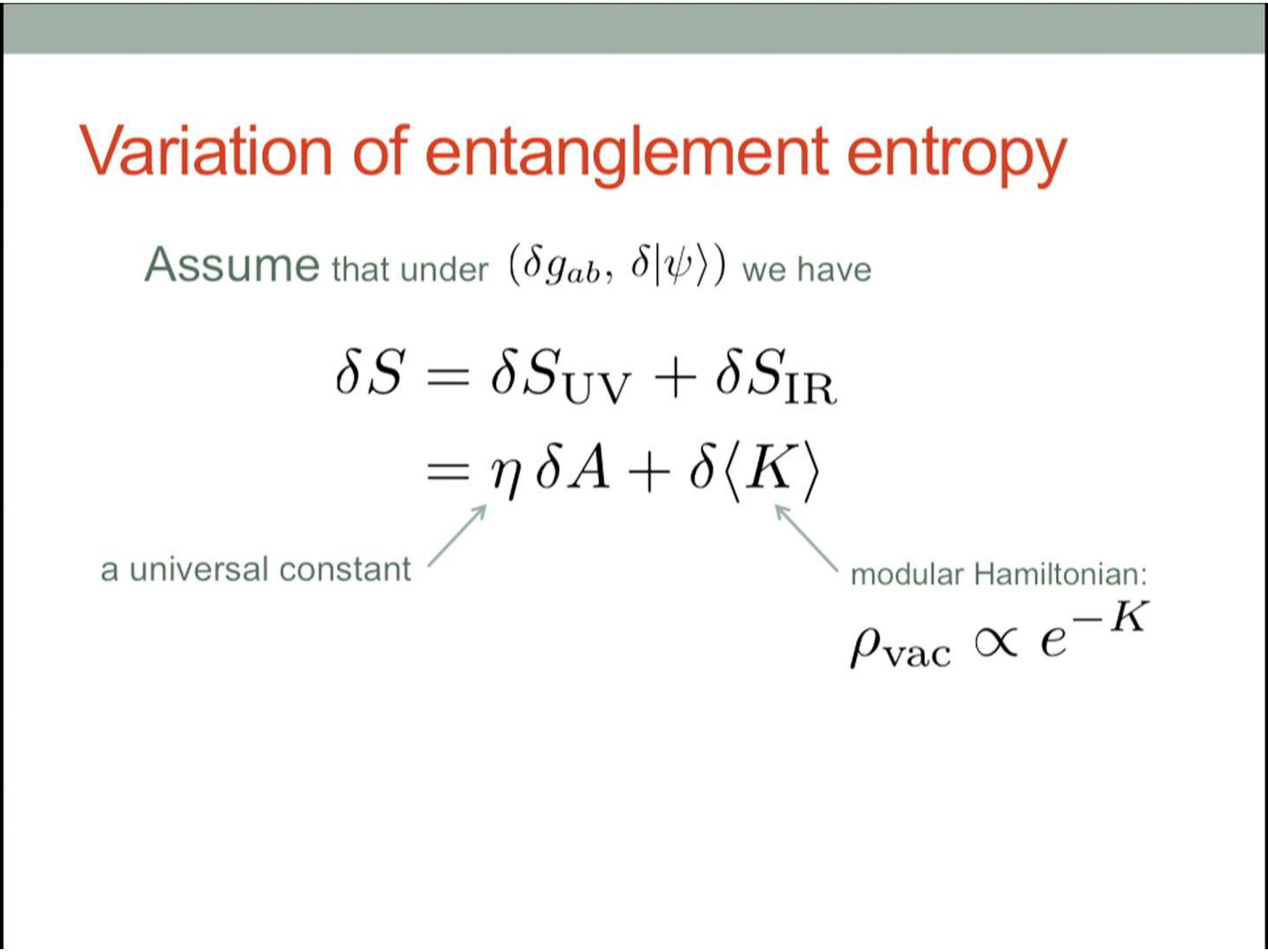
$\ell << L_{curvature}, L_{excitations}, L_{QFT}$





Area deficit and curvature

$$\delta A|_V = -\frac{\Omega_{d-2}\ell^d}{2(d^2-1)}\,\mathcal{R}_{d-1} = -\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\,G_{00}$$



Variation of entanglement entropy

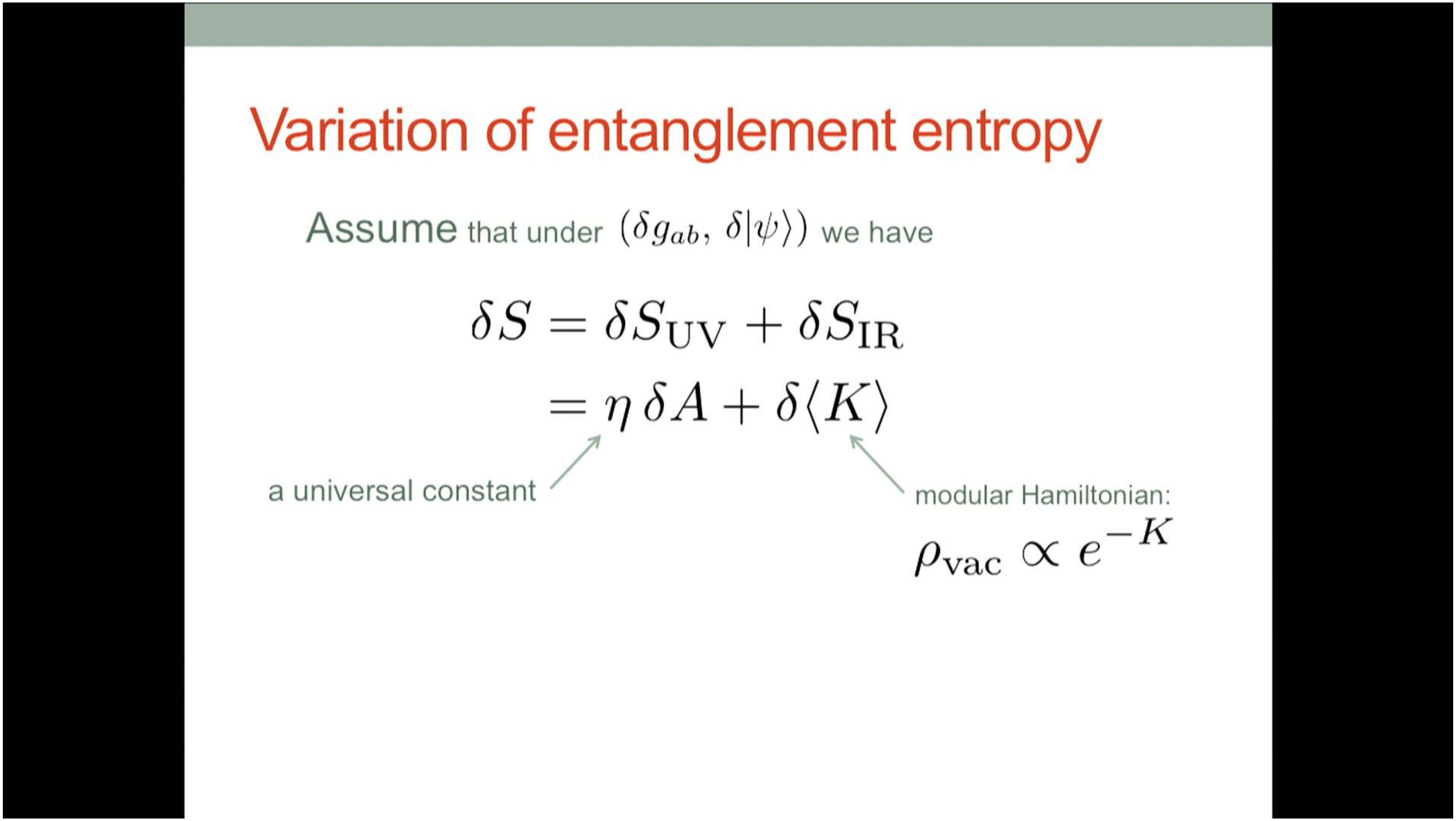
Assume that under $(\delta g_{ab},\, \delta|\psi\rangle)$ we have

$$\begin{aligned}\delta S &= \delta S_{\mathrm{UV}} + \delta S_{\mathrm{IR}} \\ &= \eta\, \delta A + \delta\langle K\rangle\end{aligned}$$

a universal constant

modular Hamiltonian:

$$\rho_{\mathrm{vac}} \propto e^{-K}$$



Variation of entanglement entropy

Assume that under $(\delta g_{ab},\, \delta|\psi\rangle)$ we have

$$\begin{aligned}\delta S &= \delta S_{\mathrm{UV}} + \delta S_{\mathrm{IR}} \\ &= \eta\, \delta A + \delta\langle K\rangle\end{aligned}$$

a universal constant

modular Hamiltonian:

$$\rho_{\mathrm{vac}} \propto e^{-K}$$

Variation of entanglement entropy

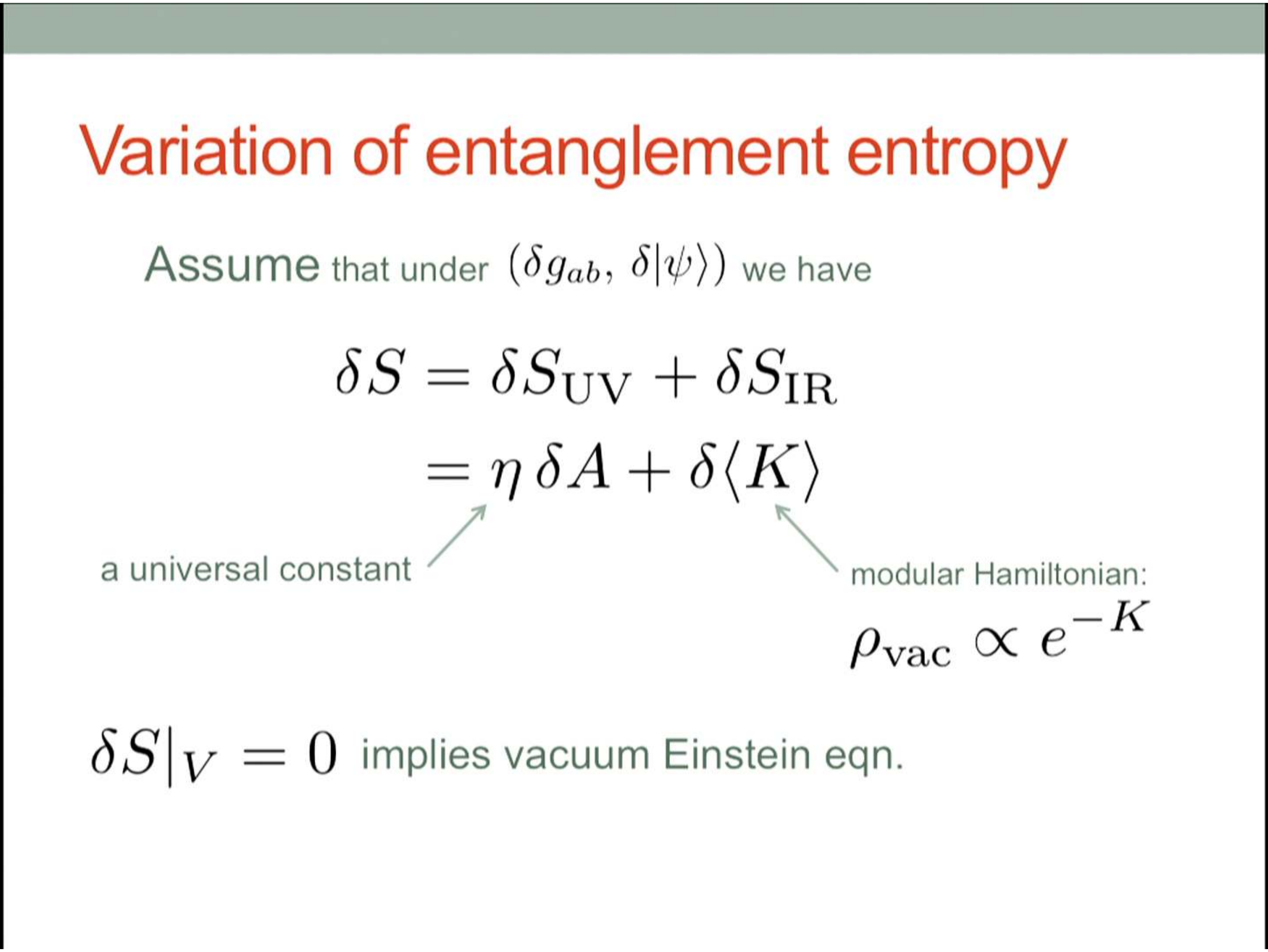
Assume that under $(\delta g_{ab},\, \delta|\psi\rangle)$ we have

$$\begin{aligned}\delta S &= \delta S_{\mathrm{UV}} + \delta S_{\mathrm{IR}} \\ &= \eta\, \delta A + \delta\langle K\rangle\end{aligned}$$

a universal constant

modular Hamiltonian:

$$\rho_{\mathrm{vac}} \propto e^{-K}$$



Variation of entanglement entropy

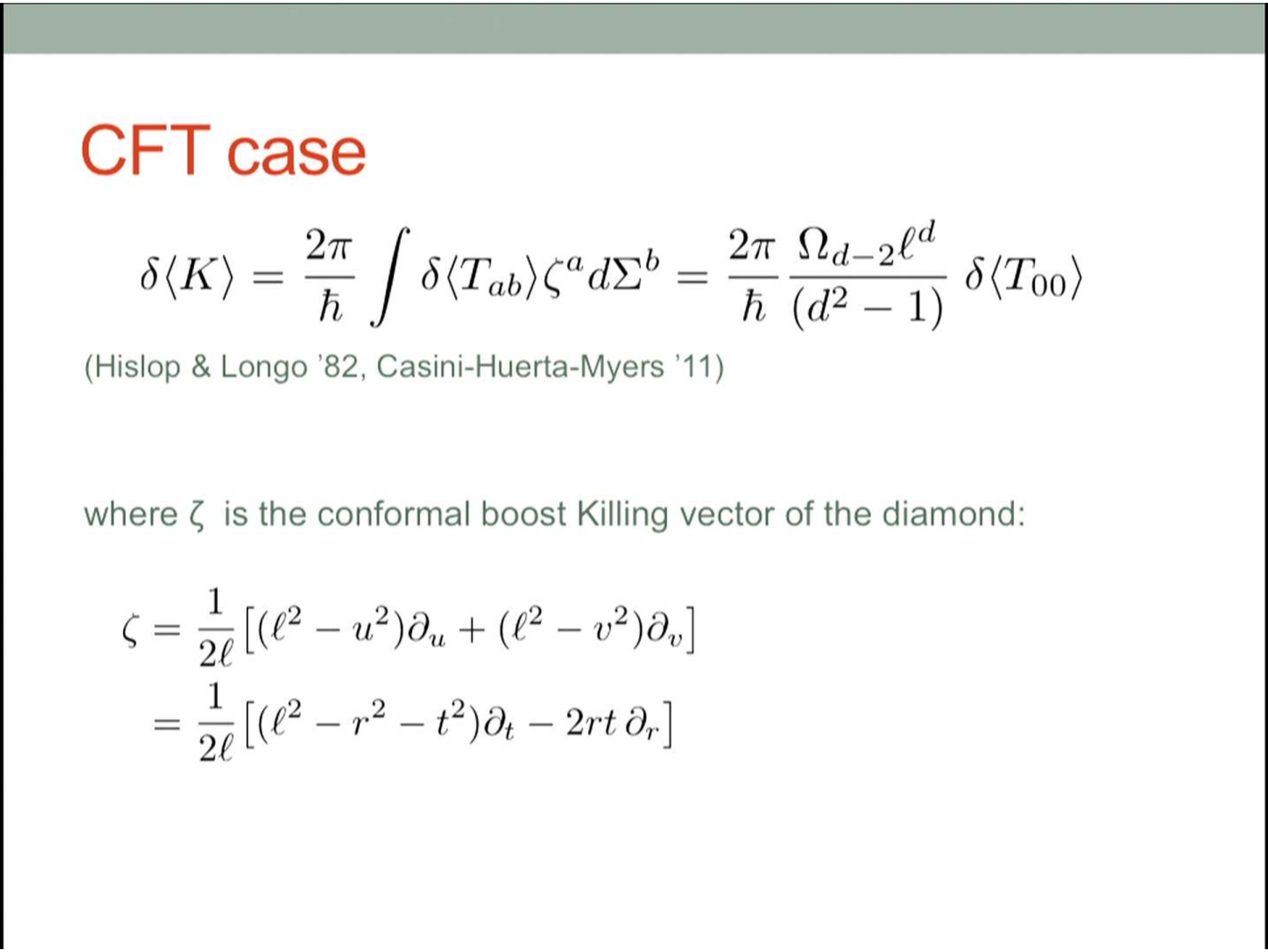
Assume that under $(\delta g_{ab},\ \delta|\psi\rangle)$ we have

$$\delta S = \delta S_{\text{UV}} + \delta S_{\text{IR}}$$
$$= \eta\, \delta A + \delta\langle K\rangle$$

a universal constant

modular Hamiltonian:
$\rho_{\text{vac}} \propto e^{-K}$

$\delta S|_V = 0$ implies vacuum Einstein eqn.



CFT case

$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \zeta^a d\Sigma^b = \frac{2\pi}{\hbar}\,\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\;\delta\langle T_{00}\rangle$$

(Hislop & Longo '82, Casini-Huerta-Myers '11)

where ζ is the conformal boost Killing vector of the diamond:

$$\begin{aligned}\zeta &= \frac{1}{2\ell}\left[(\ell^2-u^2)\partial_u + (\ell^2-v^2)\partial_v\right]\\ &= \frac{1}{2\ell}\left[(\ell^2-r^2-t^2)\partial_t - 2rt\,\partial_r\right]\end{aligned}$$

CFT case

$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \zeta^a d\Sigma^b = \frac{2\pi}{\hbar}\,\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\,\delta\langle T_{00}\rangle$$

(Hislop & Longo ’82, Casini-Huerta-Myers ’11)

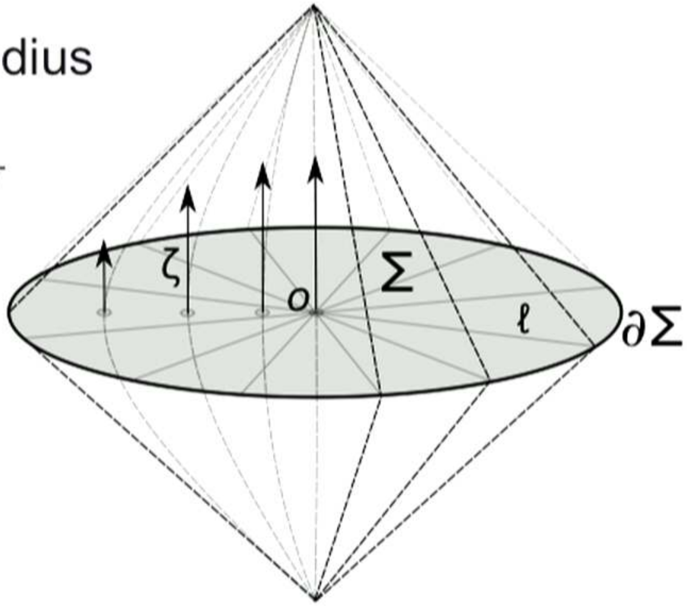
where ζ is the conformal boost Killing vector of the diamond:

$$\begin{aligned}\zeta &= \frac{1}{2\ell}\left[(\ell^2-u^2)\partial_u + (\ell^2-v^2)\partial_v\right]\\ &= \frac{1}{2\ell}\left[(\ell^2-r^2-t^2)\partial_t - 2rt\,\partial_r\right]\end{aligned}$$

“Small Ball”

- Spacelike geodesic ball of radius

$\ell << L_{curvature}, L_{excitations}, L_{QFT}$



CFT case

$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \zeta^a d\Sigma^b = \frac{2\pi}{\hbar}\,\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\;\delta\langle T_{00}\rangle$$

(Hislop & Longo ’82, Casini-Huerta-Myers ’11)

where ζ is the conformal boost Killing vector of the diamond:

$$\begin{aligned}\zeta &= \frac{1}{2\ell}\left[(\ell^2-u^2)\partial_u + (\ell^2-v^2)\partial_v\right]\\ &= \frac{1}{2\ell}\left[(\ell^2-r^2-t^2)\partial_t - 2rt\,\partial_r\right]\end{aligned}$$

Variation of entanglement entropy

Assume that under $(\delta g_{ab},\ \delta|\psi\rangle)$ we have

$$\begin{aligned}\delta S &= \delta S_{\mathrm{UV}} + \delta S_{\mathrm{IR}} \\ &= \eta\, \delta A + \delta\langle K\rangle\end{aligned}$$

a universal constant

modular Hamiltonian:

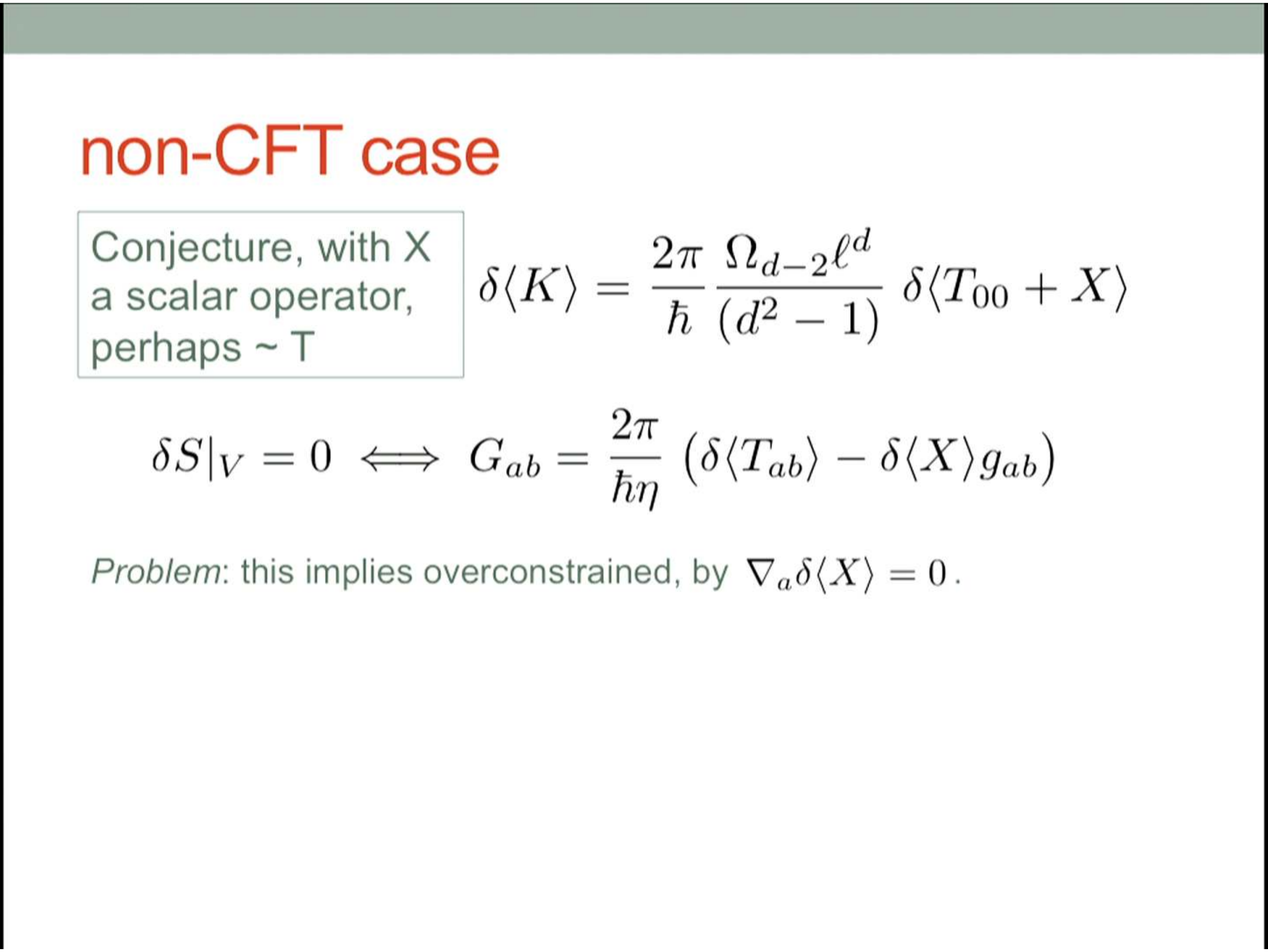
$$\rho_{\mathrm{vac}} \propto e^{-K}$$

$\delta S|_V = 0$ implies vacuum Einstein eqn.

non-CFT case

Conjecture, with X a scalar operator, perhaps ~ T

$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\,\delta\langle T_{00}+X\rangle$$



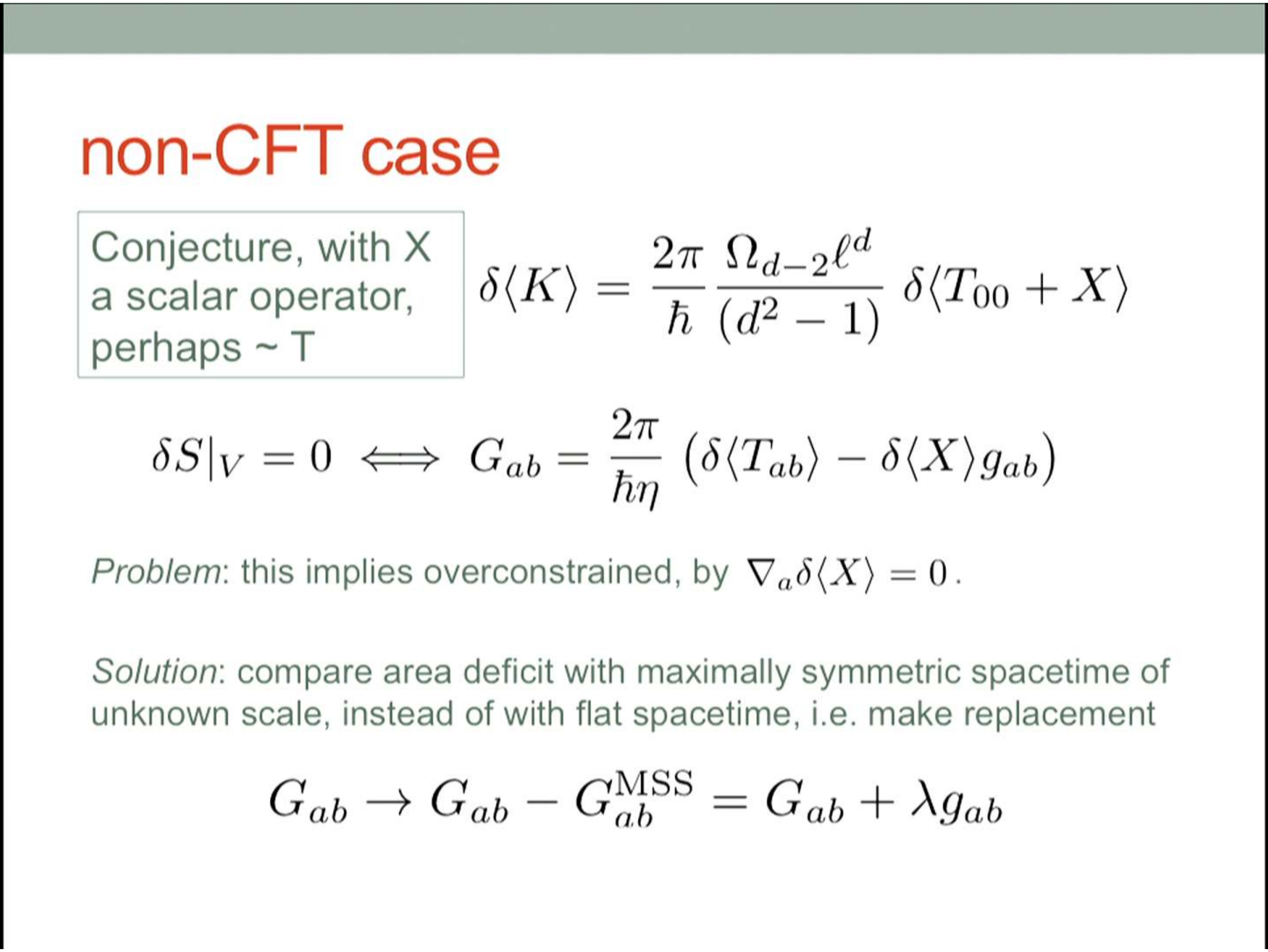
non-CFT case

Conjecture, with X a scalar operator, perhaps ~ T

$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\,\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\,\delta\langle T_{00}+X\rangle$$

$$\delta S|_V = 0 \iff G_{ab} = \frac{2\pi}{\hbar\eta}\left(\delta\langle T_{ab}\rangle - \delta\langle X\rangle g_{ab}\right)$$

Problem: this implies overconstrained, by $\nabla_a\delta\langle X\rangle = 0$.



non-CFT case

Conjecture, with X a scalar operator, perhaps ~ T

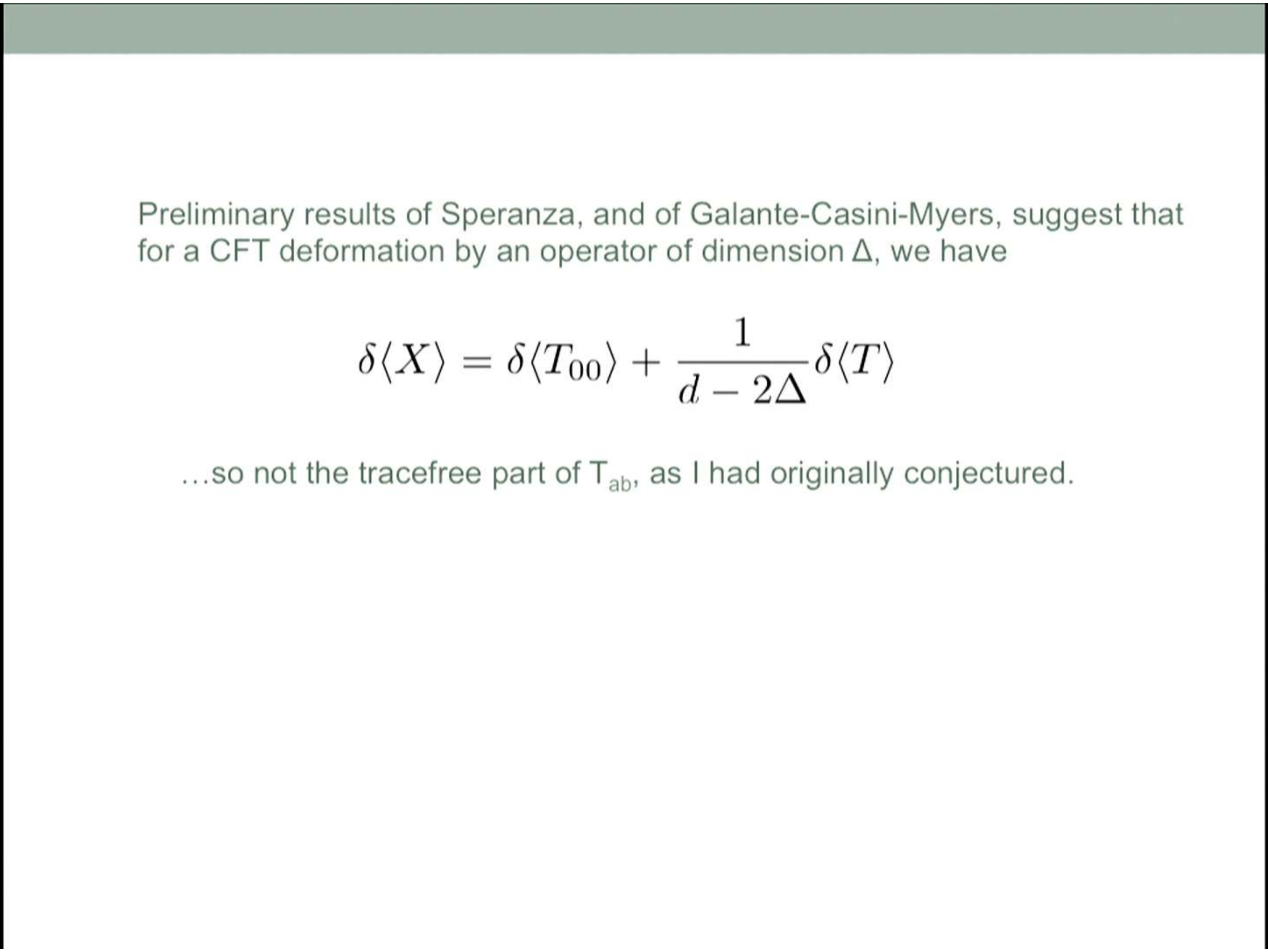
$$\delta\langle K\rangle = \frac{2\pi}{\hbar}\frac{\Omega_{d-2}\ell^d}{(d^2-1)}\,\delta\langle T_{00}+X\rangle$$

$$\delta S|_V = 0 \iff G_{ab} = \frac{2\pi}{\hbar\eta}\left(\delta\langle T_{ab}\rangle - \delta\langle X\rangle g_{ab}\right)$$

Problem: this implies overconstrained, by $\nabla_a\delta\langle X\rangle = 0$.

Solution: compare area deficit with maximally symmetric spacetime of unknown scale, instead of with flat spacetime, i.e. make replacement

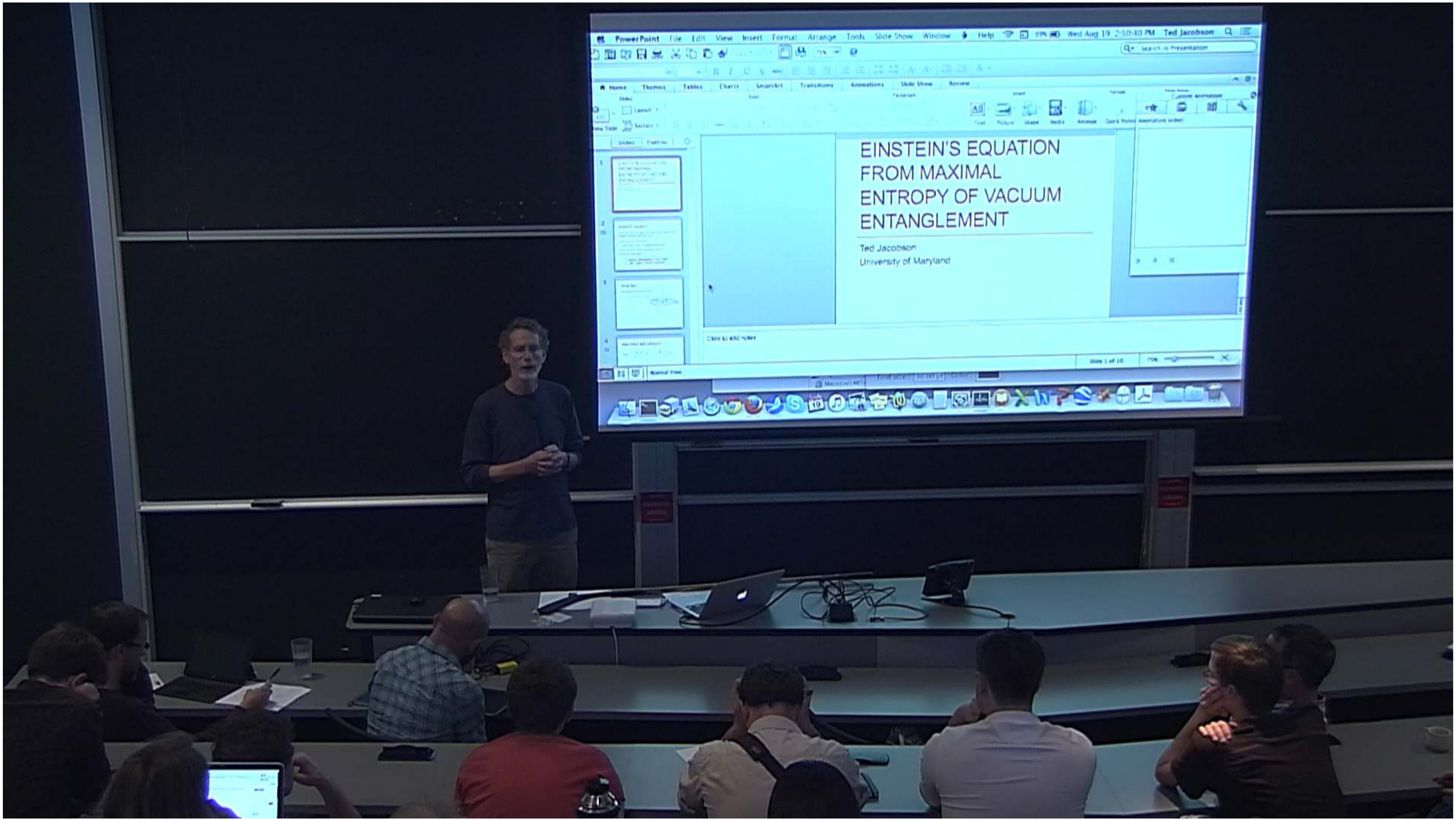
$$G_{ab} \to G_{ab} - G_{ab}^{\mathrm{MSS}} = G_{ab} + \lambda g_{ab}$$



Preliminary results of Speranza, and of Galante-Casini-Myers, suggest that for a CFT deformation by an operator of dimension Δ, we have

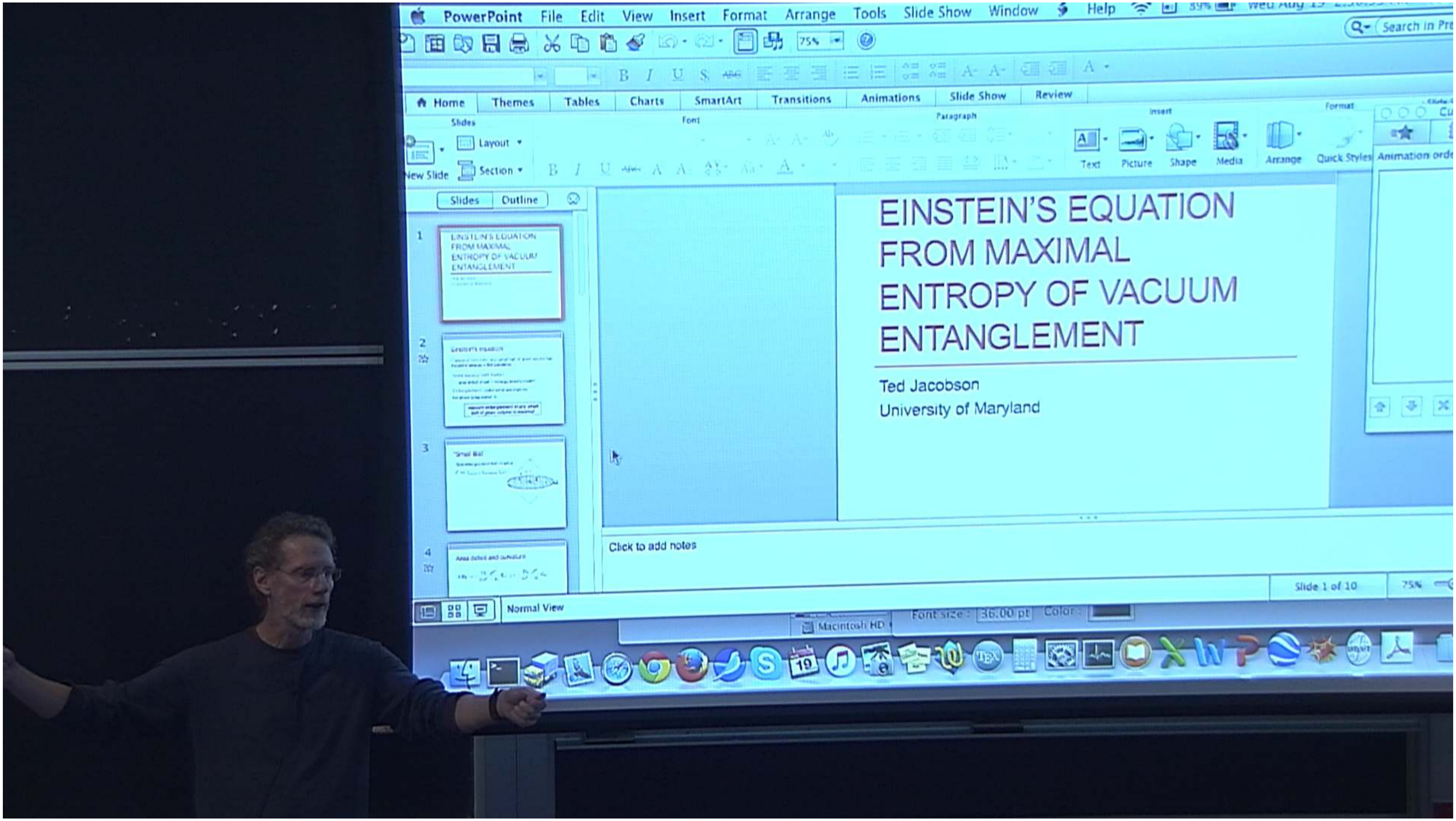
$$\delta\langle X\rangle = \delta\langle T_{00}\rangle + \frac{1}{d-2\Delta}\delta\langle T\rangle$$

…so not the tracefree part of T_{ab}, as I had originally conjectured.



EINSTEIN'S EQUATION FROM MAXIMAL ENTROPY OF VACUUM ENTANGLEMENT

Ted Jacobson
University of Maryland



EINSTEIN'S EQUATION FROM MAXIMAL ENTROPY OF VACUUM ENTANGLEMENT

Ted Jacobson
University of Maryland