Title: Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence

Date: Aug 18, 2015 02:00 PM

URL: http://pirsa.org/15080066

Abstract: In this talk I will introduce a family of exactly solvable toy models of a holographic correspondence based on a novel construction of quantum error-correcting codes with a tensor network structure. The building block for these models are a special type of tensor with maximal entanglement along any bipartition, which gives rise to an exact isometry from bulk operators to boundary operators. The entire tensor network is a quantum error-correcting code, where the bulk and boundary degrees of freedom may be identified as logical and physical degrees of freedom respectively. These models capture key features of entanglement in the holographic correspondence; in particular, the Ryu-Takayanagi formula and the negativity of tripartite information are obeyed exactly in many cases. I will describe how bulk operators may be represented on the boundary regions mimicking the Rindler-wedge reconstruction.

Holographic quantum error correcting codes

(Toy models for bulk-boundary correspondence)

Fernando Pastawski @ PI 2015

Joint work with: Beni Yoshida, Daniel Harlow & John Preskill

(HaPPY) JHEP 1506 (2015) 149



Prepared for submission to JHEP

Bulk Locality and Quantum Error Correction in AdS/CFT

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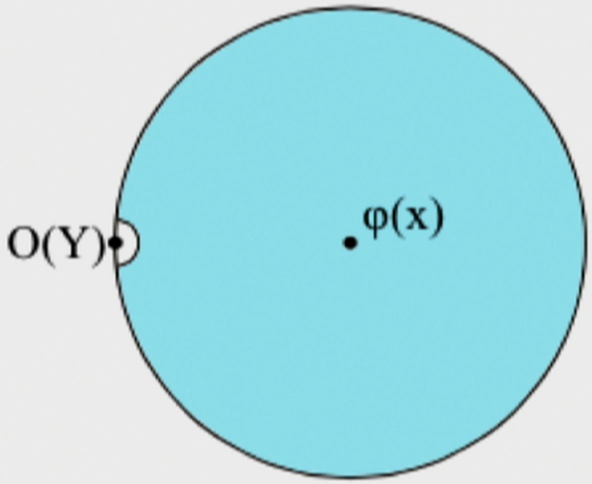
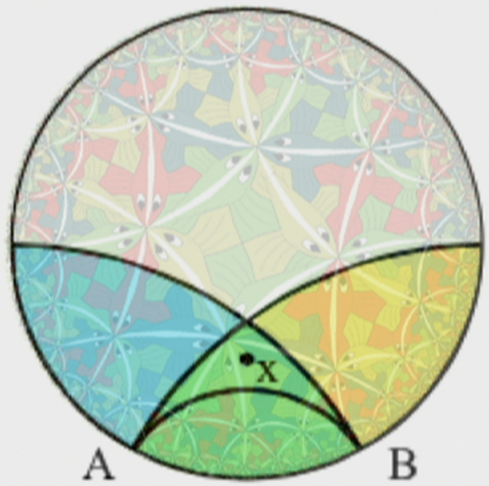
Abstract: We point out a connection between the emergence of bulk locality in AdS/CFT and the theory of quantum error correction. Bulk notions such as Bogoliubov

Bulk-Boundary correspondence & the AdS-Rindler reconstruction

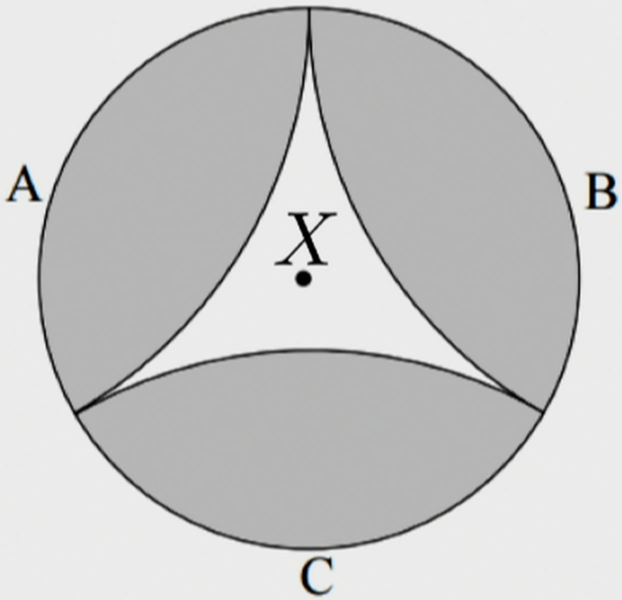
$$\lim_{r\to\infty} r^{\Delta}\phi(r,x) = \mathcal{O}(x)$$





Multiple AdS-Rindler reconstructions on CFT

Sharpening the paradox



$$A \cup B \cup C = Boundary$$

$$\Omega \qquad X\Omega X^\dagger$$

$$\updownarrow \qquad \updownarrow$$

$$\rho \neq \tilde{X}\rho\tilde{X}^\dagger = \sigma$$

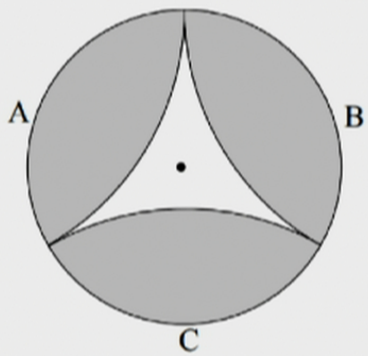
$$\rho_A = \sigma_A \qquad \rho_B = \sigma_B \qquad \rho_C = \sigma_C$$

$$\rho_{BC} \neq \sigma_{BC} \quad \rho_{CA} \neq \sigma_{CA} \quad \rho_{AB} \neq \sigma_{AB}$$

Information in non-local correlations like QECC.

Example QECC: 3 qutrit code

$$\mathcal{E}nc : \mathcal{H}_{logical} \rightarrow \mathcal{H}_{physical}$$



$$|0\rangle \rightarrow |\tilde{0}\rangle = |000\rangle + |111\rangle + |222\rangle$$
$$|1\rangle \rightarrow |\tilde{1}\rangle = |012\rangle + |120\rangle + |201\rangle$$
$$|2\rangle \rightarrow |\tilde{2}\rangle = |021\rangle + |102\rangle + |210\rangle$$

Encoder = Isometry: $E^\dagger E = I_{logical}$

$$\mathcal{E}nc(\rho) = E\rho E^\dagger \qquad E = \sum_j |\tilde{j}\rangle\langle j|$$

No local information: $Tr_{A,B}\left[|\tilde{\psi}\rangle\langle\tilde{\psi}|\right] = \frac{\mathbb{1}_3}{3}$

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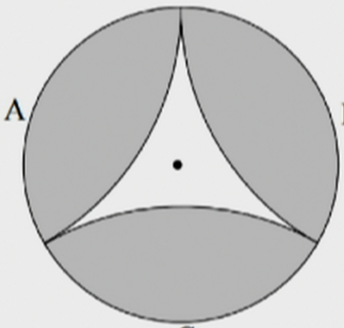

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Error correction condition

$$\mathcal{E}nc : \mathcal{H}_{logical} \to \mathcal{H}_{physical}$$

$$\mathcal{N}oise : \mathcal{H}_{physical} \to \mathcal{H}_{physical}$$

$$\mathcal{D}ec : \mathcal{H}_{physical} \to \mathcal{H}_{logical}$$

$$\mathcal{D}ec \circ \mathcal{N}oise \circ \mathcal{E}nc(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi|$$

Example: erasure noise

$$\mathcal{E}nc := \begin{array}{l} |0\rangle \to |\tilde{0}\rangle = |000\rangle + |111\rangle + |222\rangle \\ |1\rangle \to |\tilde{1}\rangle = |012\rangle + |120\rangle + |201\rangle \\ |2\rangle \to |\tilde{2}\rangle = |021\rangle + |102\rangle + |210\rangle \end{array}$$

$$\mathcal{N}oise_C(\rho) = \mathrm{tr}_C(\rho)$$

$$\mathcal{D}ec_C(\rho) = \mathrm{tr}_A(U_{AB}\rho U_{AB}^\dagger)$$

$$U_{AB}|a, b\rangle = |a, b-a\rangle$$

Example: Restricted support operators

$$|0\rangle \to |\tilde{0}\rangle = |000\rangle + |111\rangle + |222\rangle$$

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$$\mathcal{N}oise_C(\rho) = \mathsf{tr}_C(\rho)$$

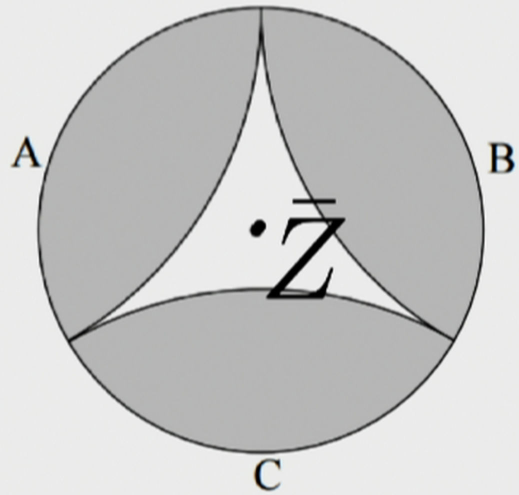
$$Z|j\rangle = \omega^j |j\rangle \qquad X|j\rangle = |j+1\rangle \qquad \omega = e^{\frac{2i\pi}{3}}$$

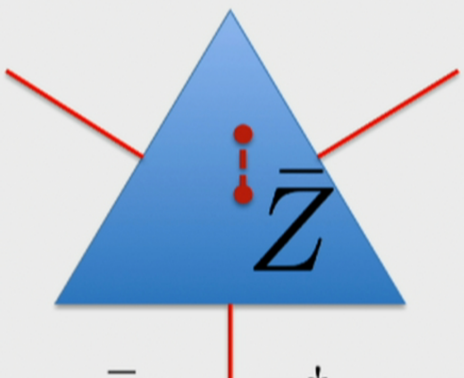
$$\bar{Z} \equiv Z_a^\dagger \otimes Z_b \otimes \mathbb{1} \; = \mathcal{D}ec_C^\dagger(Z)$$

$$\bar{X} \equiv X_a \otimes X_b^2 \otimes \mathbb{1} = \mathcal{D}ec_C^\dagger(X)$$

AdS/CFT ⇔ QECC

AdS/CFT	Quantum Error Correcting Code
State mapping: AdS -> CFT	Encoding: Logical -> Physical
Bulk operators	Logical operators
CFT operators	Physical operators
AdS-Rindler reconstruction	Systematic cleaning a physical realization of logical operators.

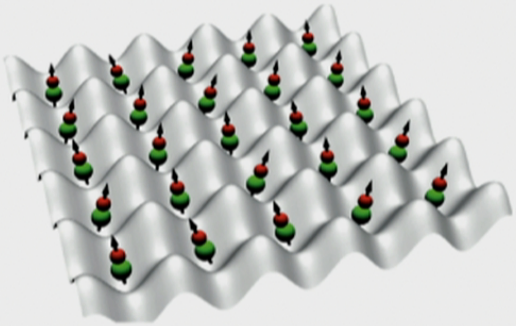


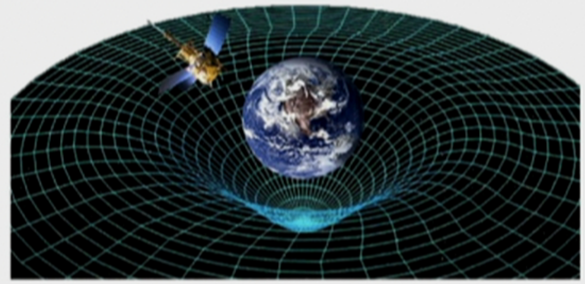


$$\bar{Z} \equiv Z_a^\dagger \otimes Z_b \otimes \mathbb{1}$$
$$\bar{Z} \equiv \mathbb{1} \otimes Z_b^\dagger \otimes Z_c$$

Motivation

- Construct new (better ?) QECC





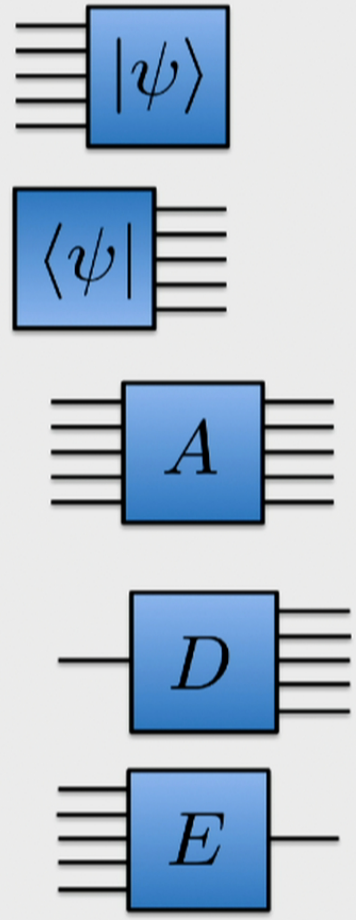
- Concrete realizations of Almheiri, Dong & Harlow
- Gain insight on the information structure of holographic models such as AdS/CFT
- QI Tools: Quantum error correcting codes
  tensor network states

Outline

- Bulk locality and Quantum error correction
- This outline
- Tensors and perfect tensors
- Bulk geometry from tensor network geometry
- Greedy bulk reconstruction
- Saturating Ryu-Takayanagi
- Conclusions + Open problems

Tensors

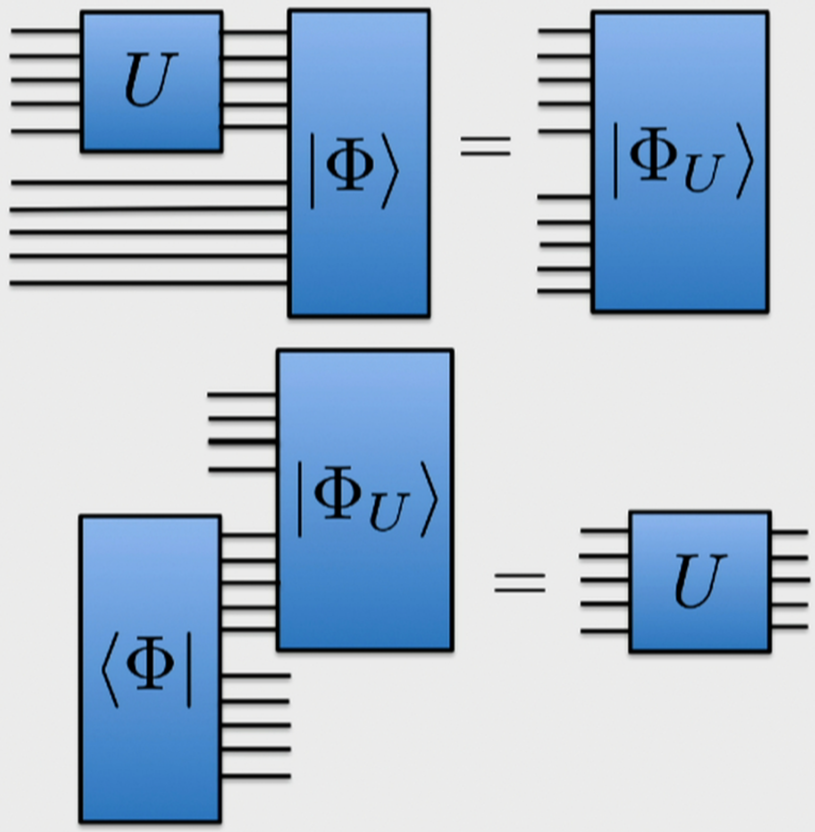
- Ket
- Bra
- Operator
- Decoder
- Encoder



Maximally entangled = Unitary

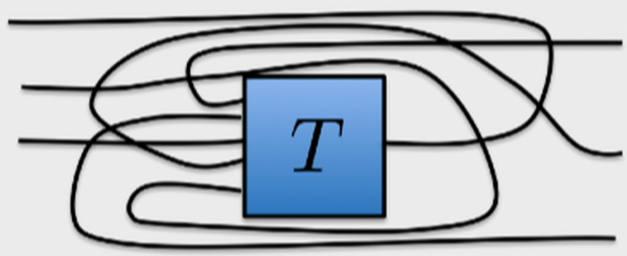
Choi-Jamiolkowski Isomorphism

$$|\Phi\rangle = \sum_j |j, j\rangle$$



Perfect tensors = absolutely maximally entangled (AME)

- Maximally entangled along **all possible** cuts
- Always proportional to unitary or isometry



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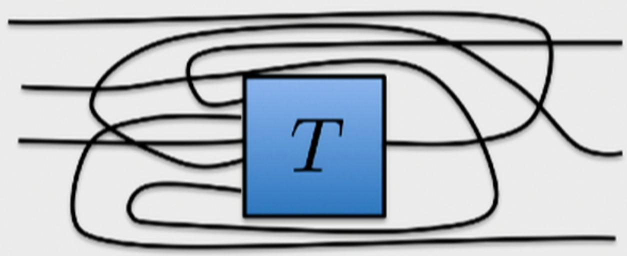

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Example:

$[[3,1,2]]_3$ (3 qutrit 1-error detecting code)

$$|0\rangle \rightarrow |\tilde{0}\rangle = |000\rangle + |111\rangle + |222\rangle$$

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$$X|j\rangle = |j+1\rangle$$
$$Z|j\rangle = \omega|j\rangle \quad \omega = e^{\frac{2i\pi}{3}}$$

$$P \in \mathcal{S} \Rightarrow T|\psi\rangle = PT|\psi\rangle$$
$$\mathcal{S} = \langle X \otimes X \otimes X, Z \otimes Z^{-2} \otimes Z \rangle$$

$[[3,1,2]]_3$ Encoder

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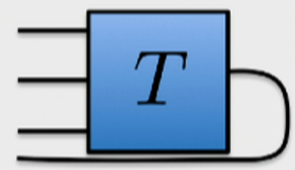
$[[3,1,2]]_3$ Encoder

$$P \text{ implements } L \Rightarrow T|\psi\rangle = P^{\dagger}TL|\psi\rangle$$
$$\mathcal{S}_{state} = \left\langle \begin{matrix} X \otimes X \otimes X \otimes I, Z \otimes Z^{-2} \otimes Z \otimes I \\ I \otimes X \otimes X^2 \otimes X^*, Z^{\dagger} \otimes Z \otimes 1 \otimes Z^* \end{matrix} \right\rangle$$

$[[4,0,3]]_3$ State



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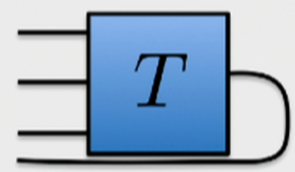
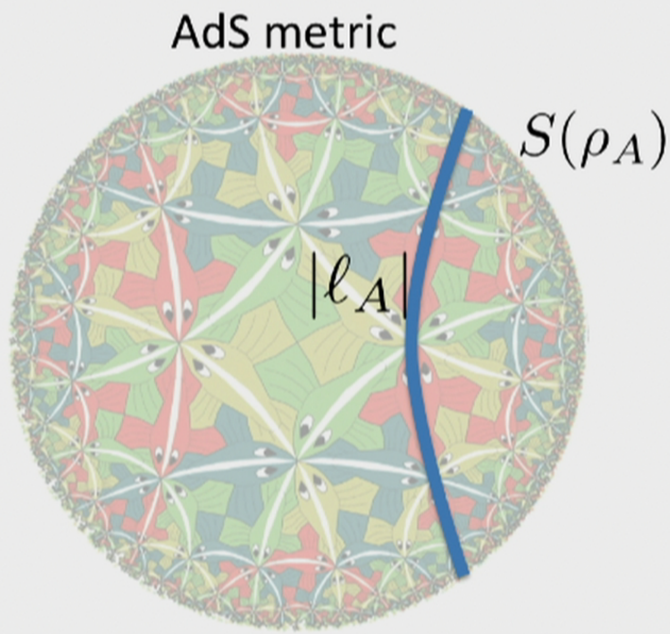


Ryu-Takayanagi

Holographic Derivation of Entanglement Entropy from AdS/CFT (2006)

Distance = entanglement entropy



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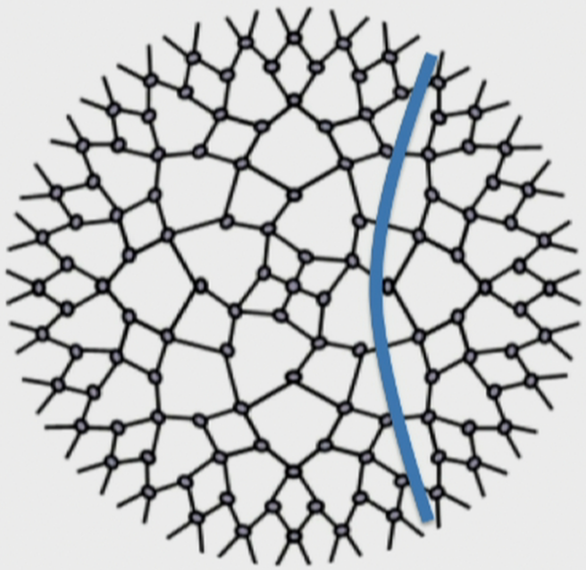
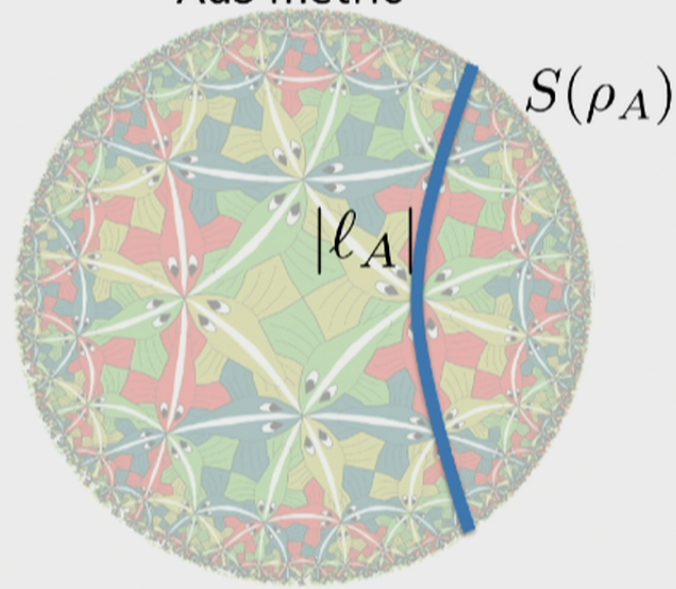
MERA



AdS metric



Class of Quantum Many-Body States That Can
Be Efficiently Simulated Guifre Vidal (2008)
Can represent CFT ground states efficiently.

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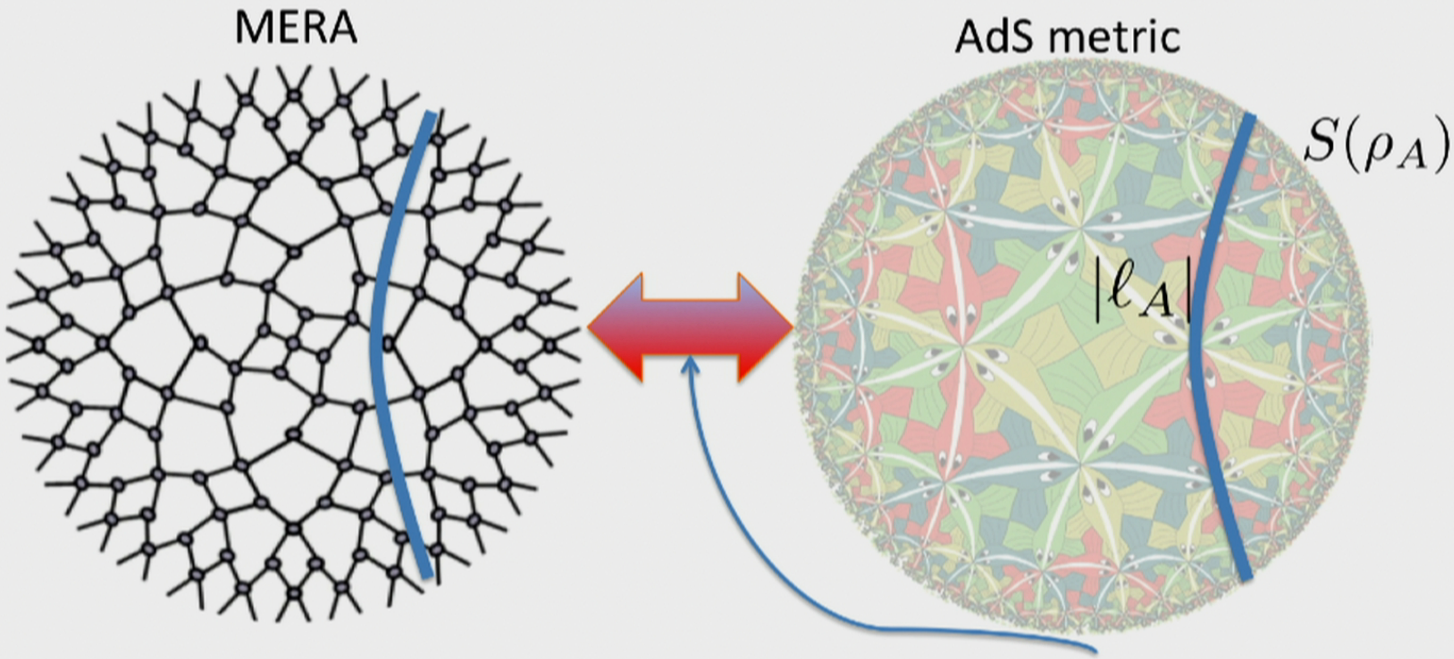


Class of Quantum Many-Body States That Can Be Efficiently Simulated Guifre Vidal (2008)
Can represent CFT ground states efficiently.

Entanglement renormalization and holography
Brian Swingle (2012)

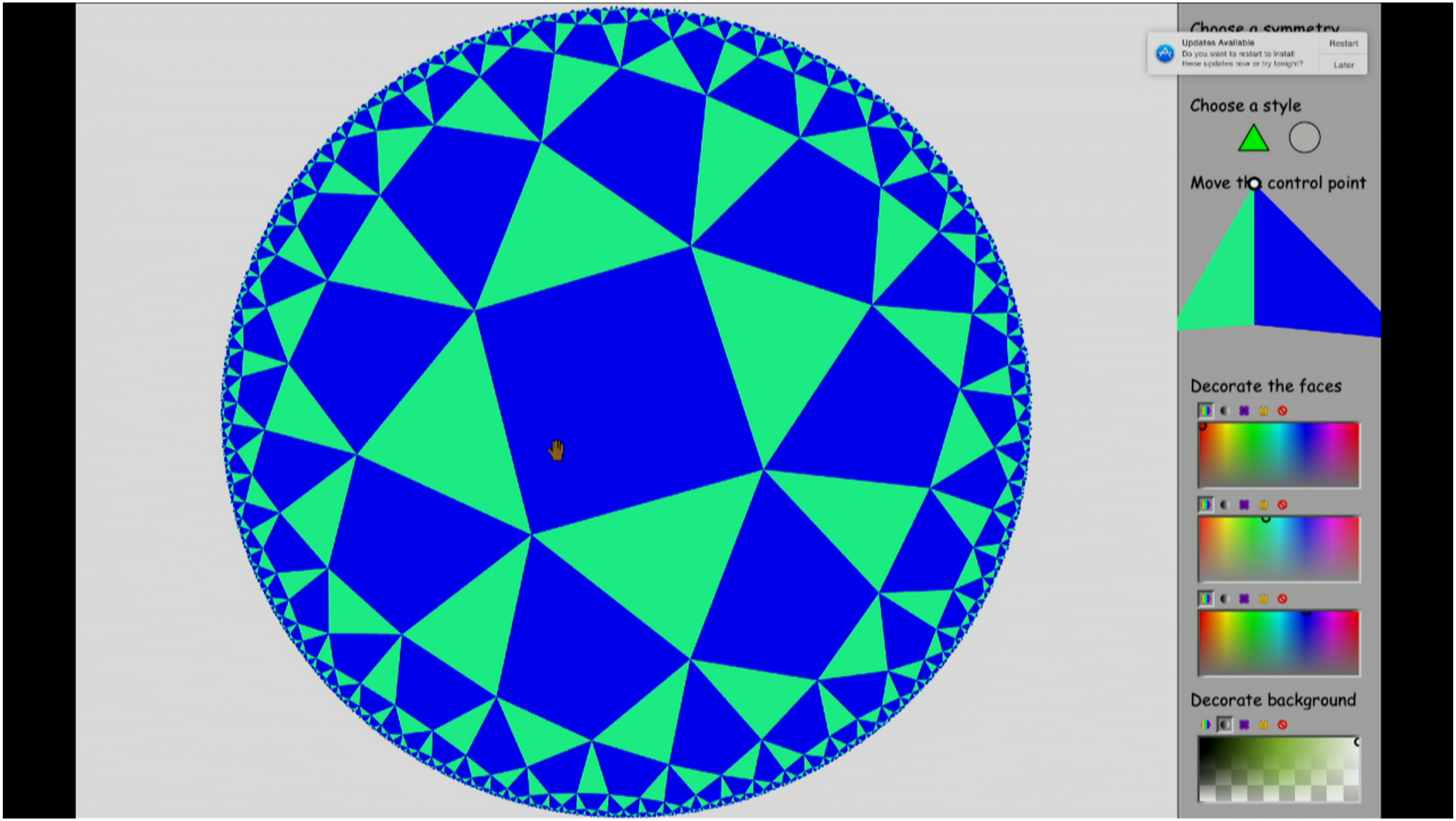
Constructing a bulk geometry

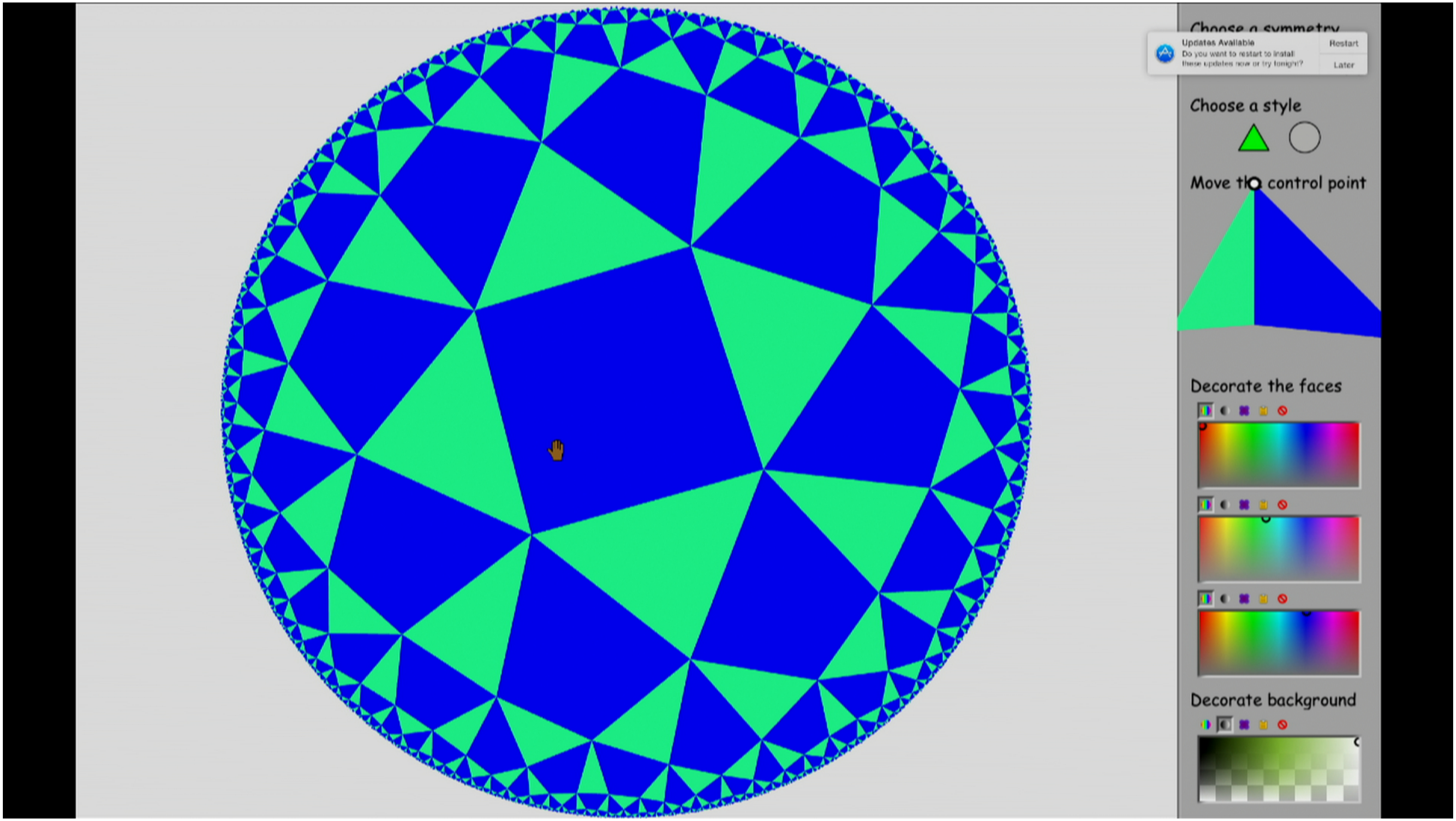
Test 1) Local bulk reconstruction

Test 2) Ryu-Takayanagy entropy

Digression to Kaleidotile

(on drawing more than 6 polygons on a hyperbolic lattice)


Choose a symmetry
Updates Available
Do you want to restart to install these updates now or try tonight?
Restart
Later
Choose a style
Move the control point
Decorate the faces
Decorate background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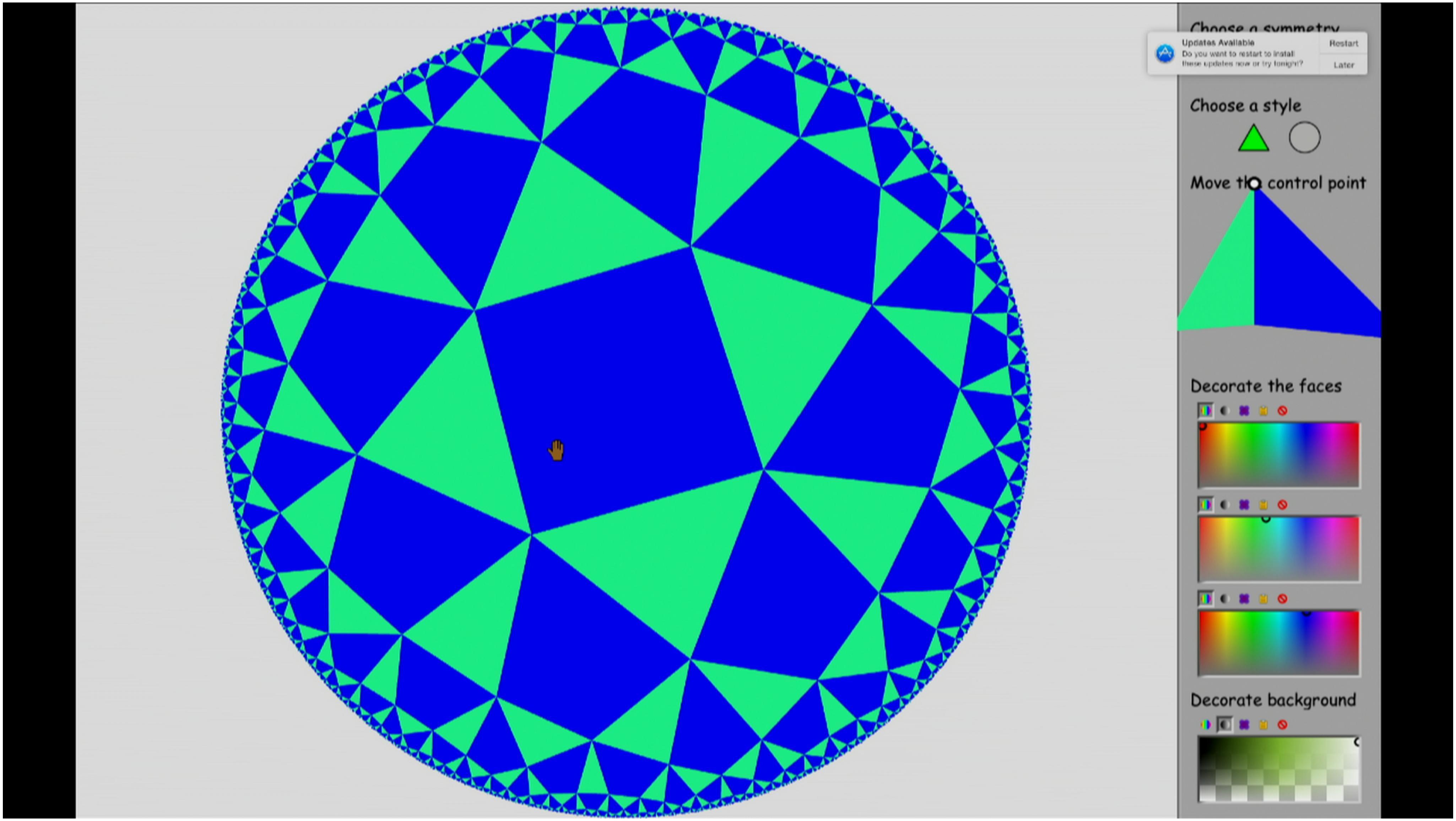
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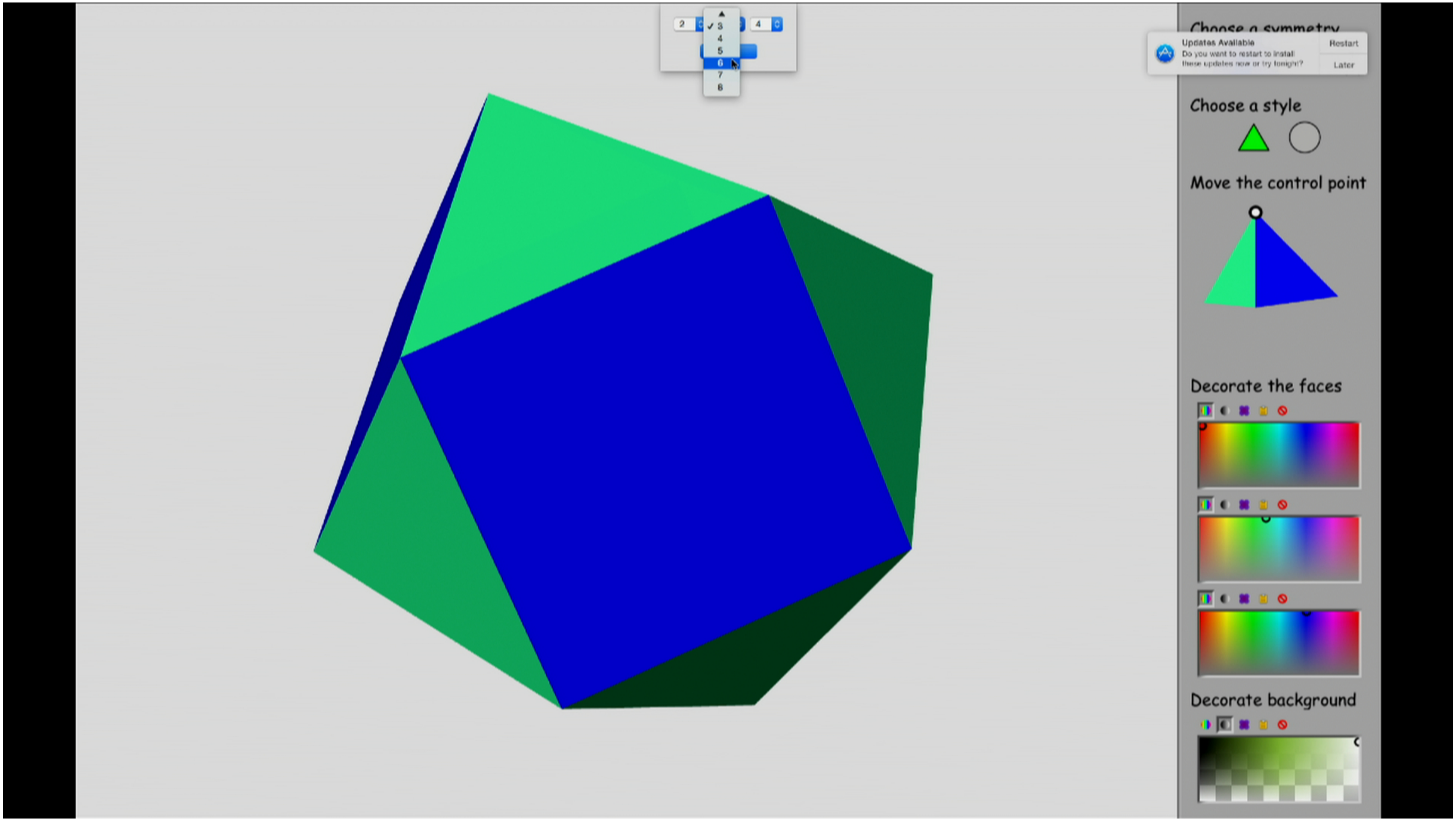

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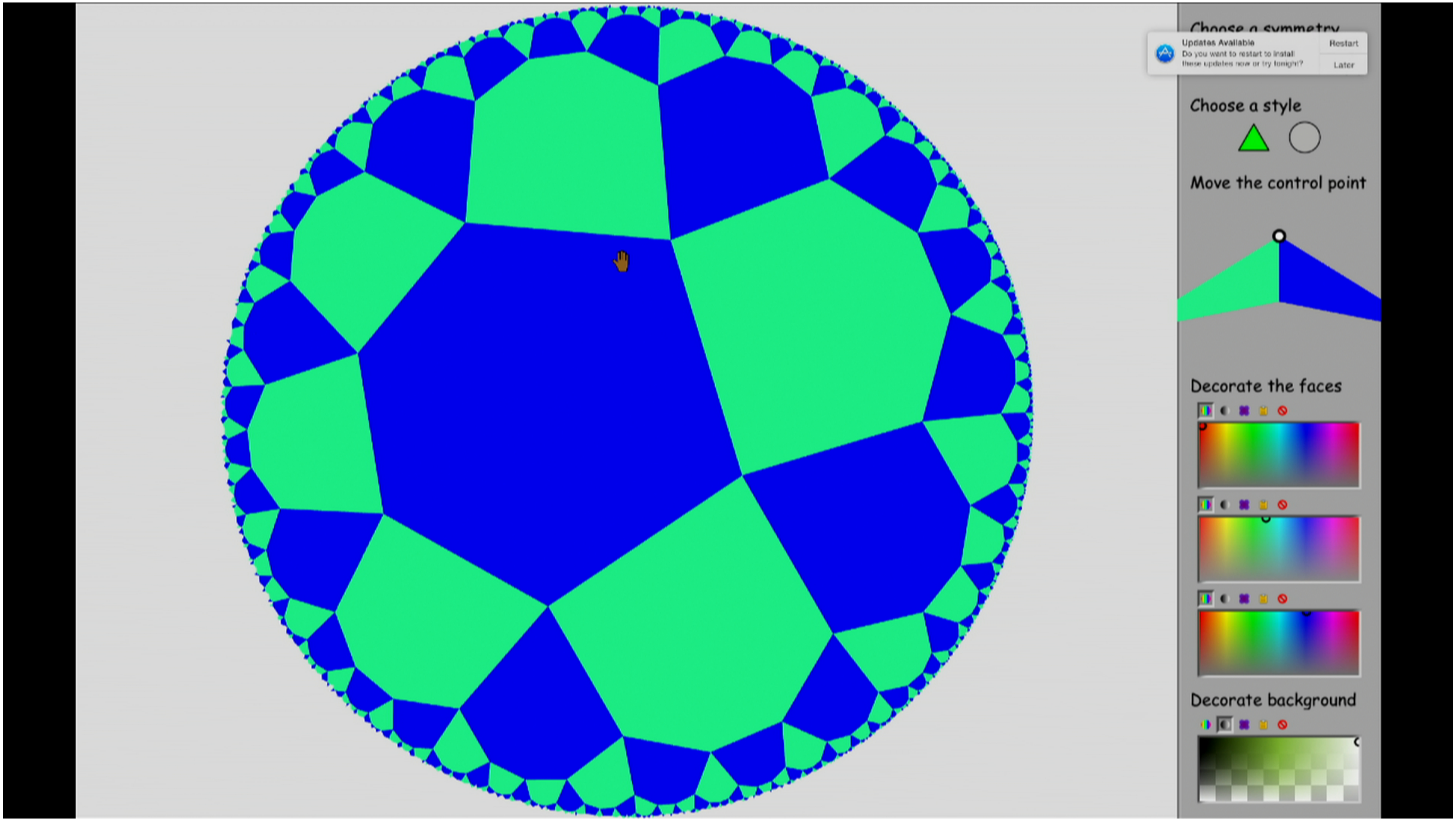
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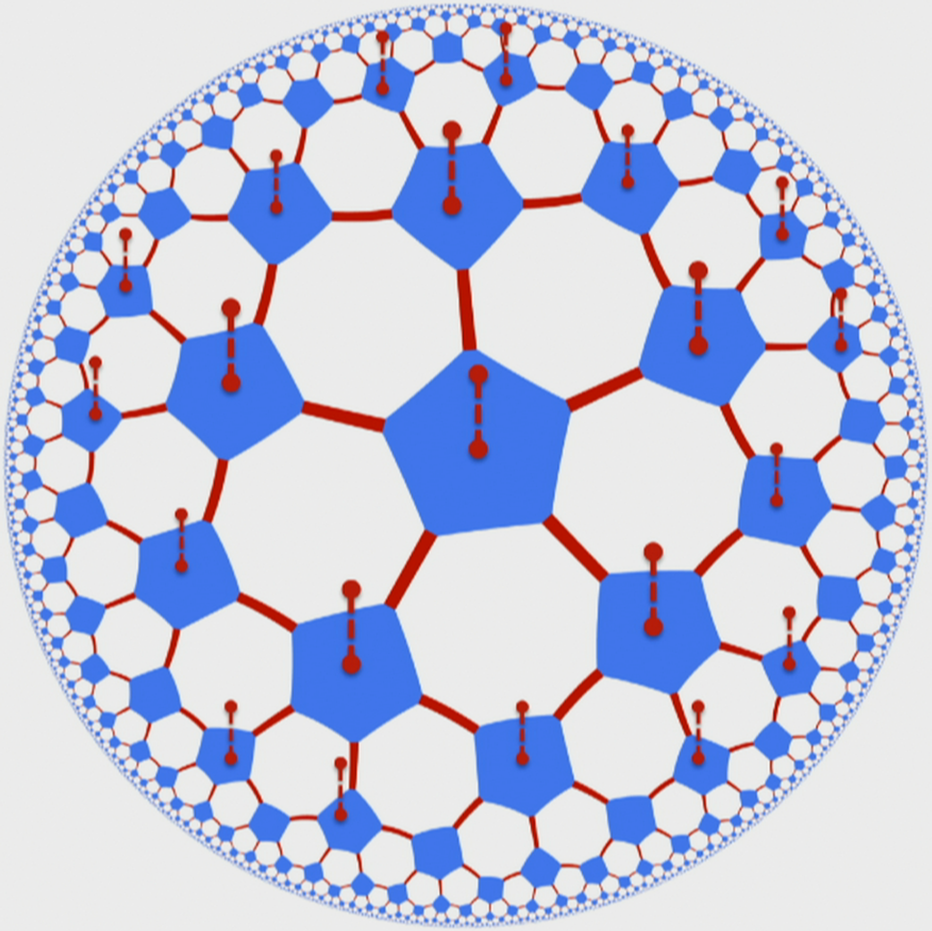
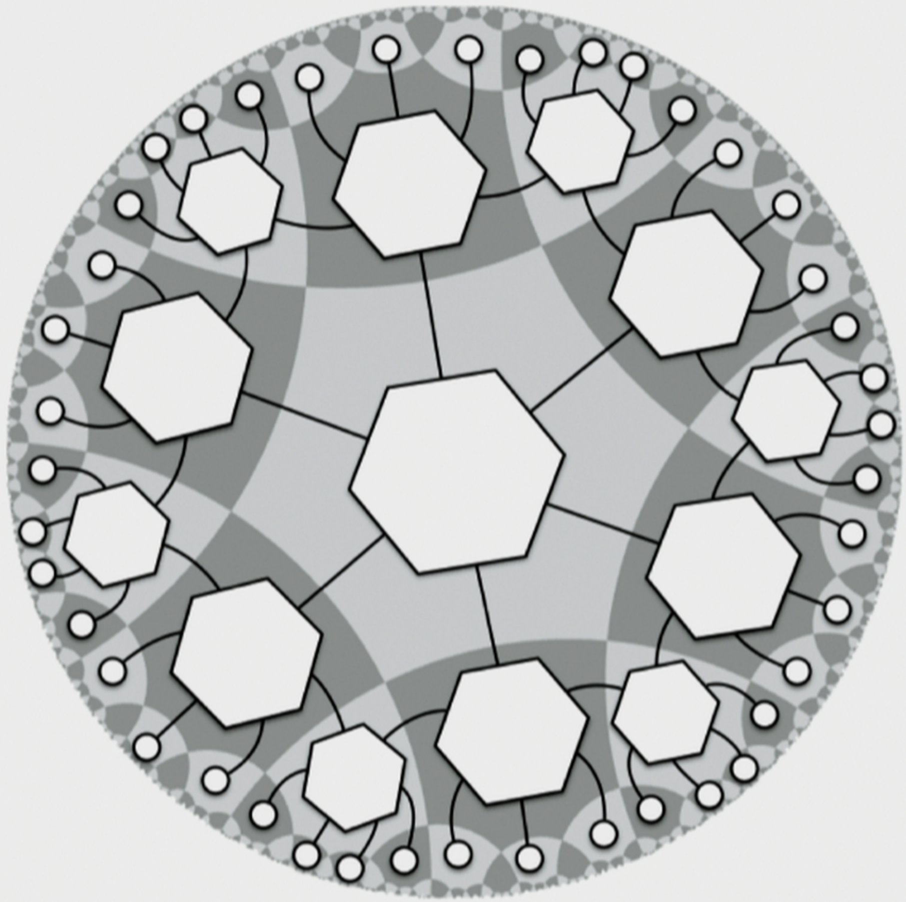
Holographic Code



Holographic state

No bulk/logical legs.



Ryu-Takayanagi -> Entanglement entropy = length of bulk geodesics.

Holographic state

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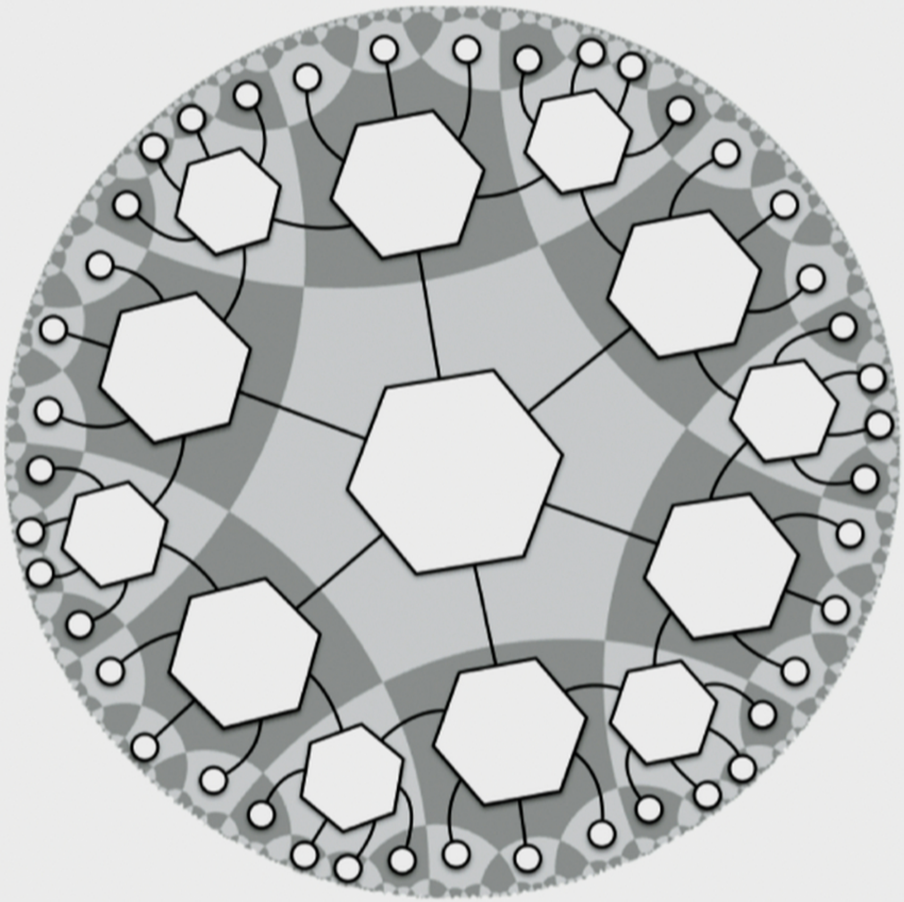

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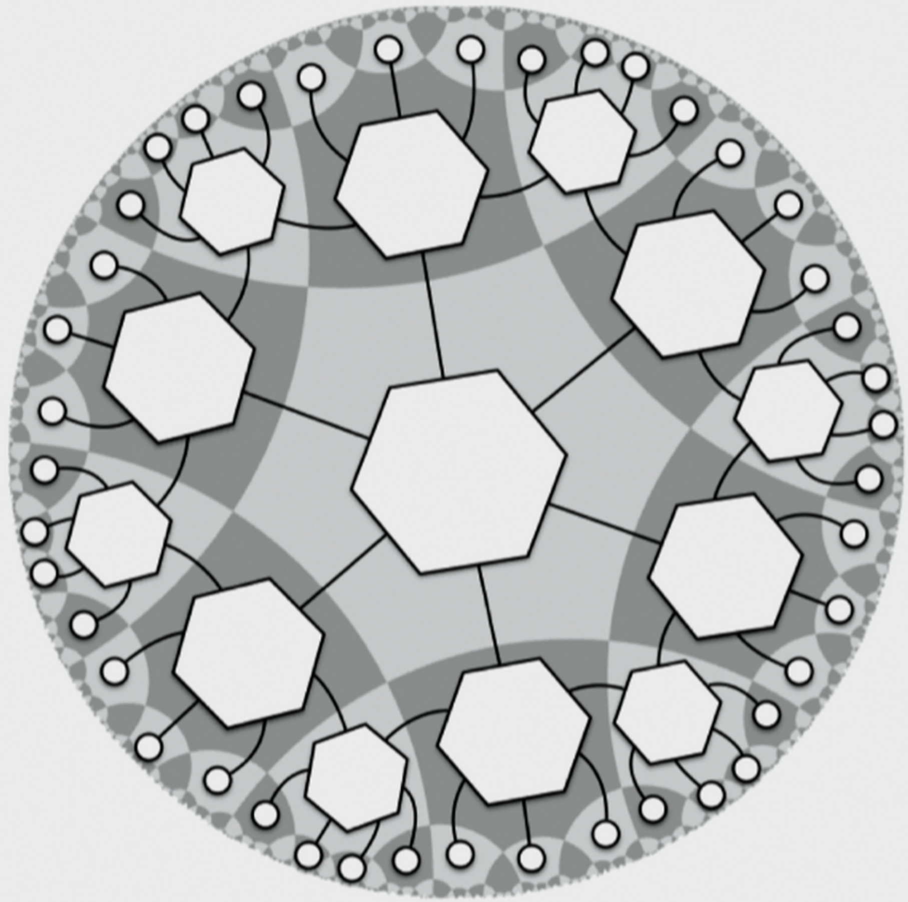

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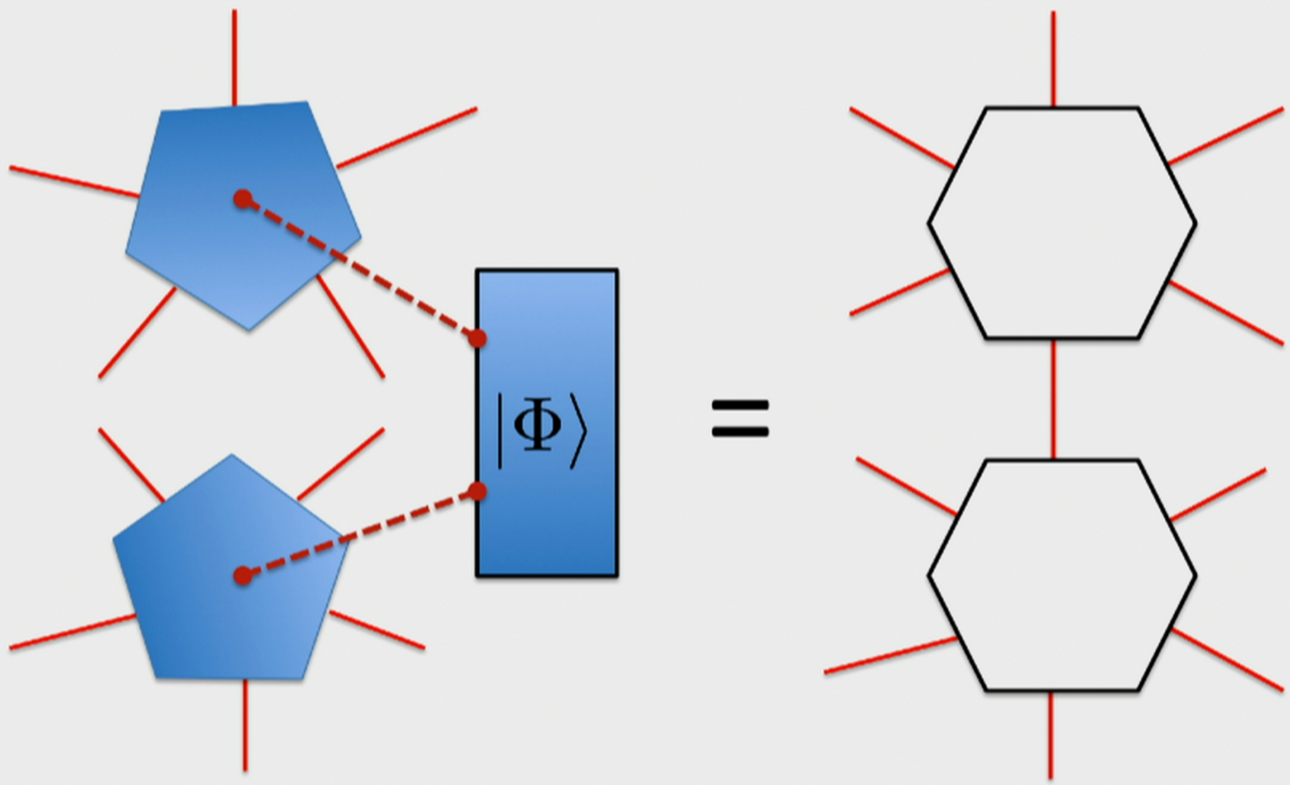
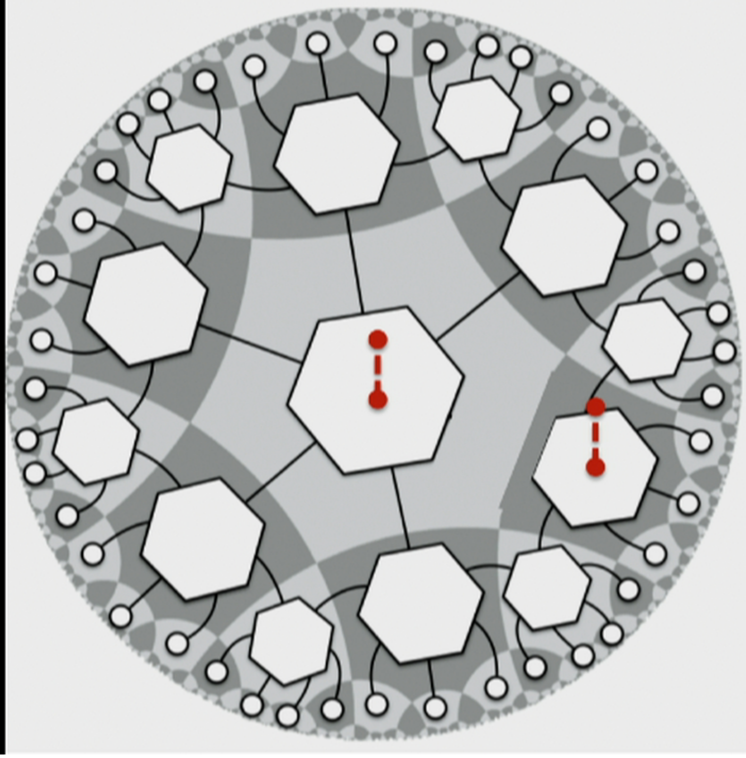
Connection between state & code

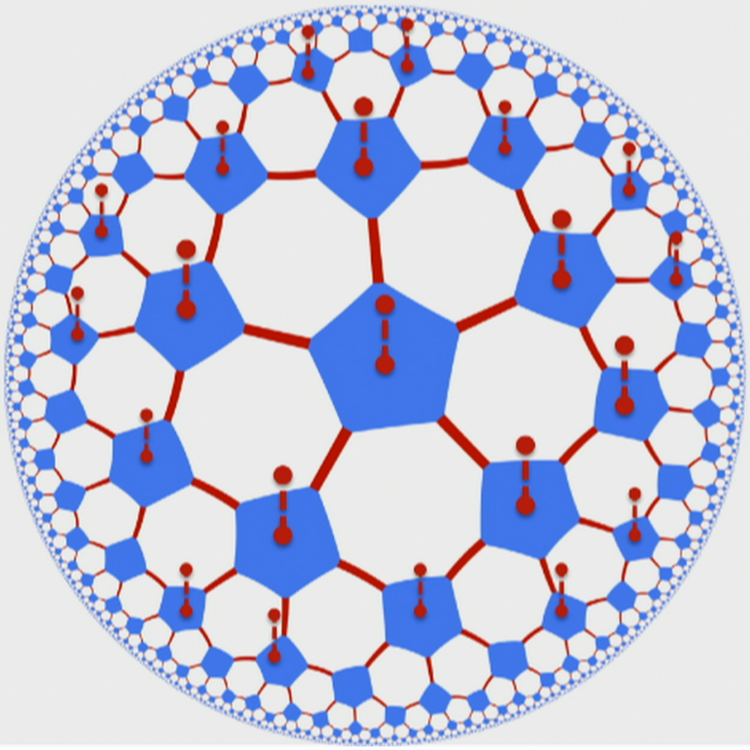


Connection between state & code

State -> Code
Cut virtual bonds

Code -> State
Input EPR pairs





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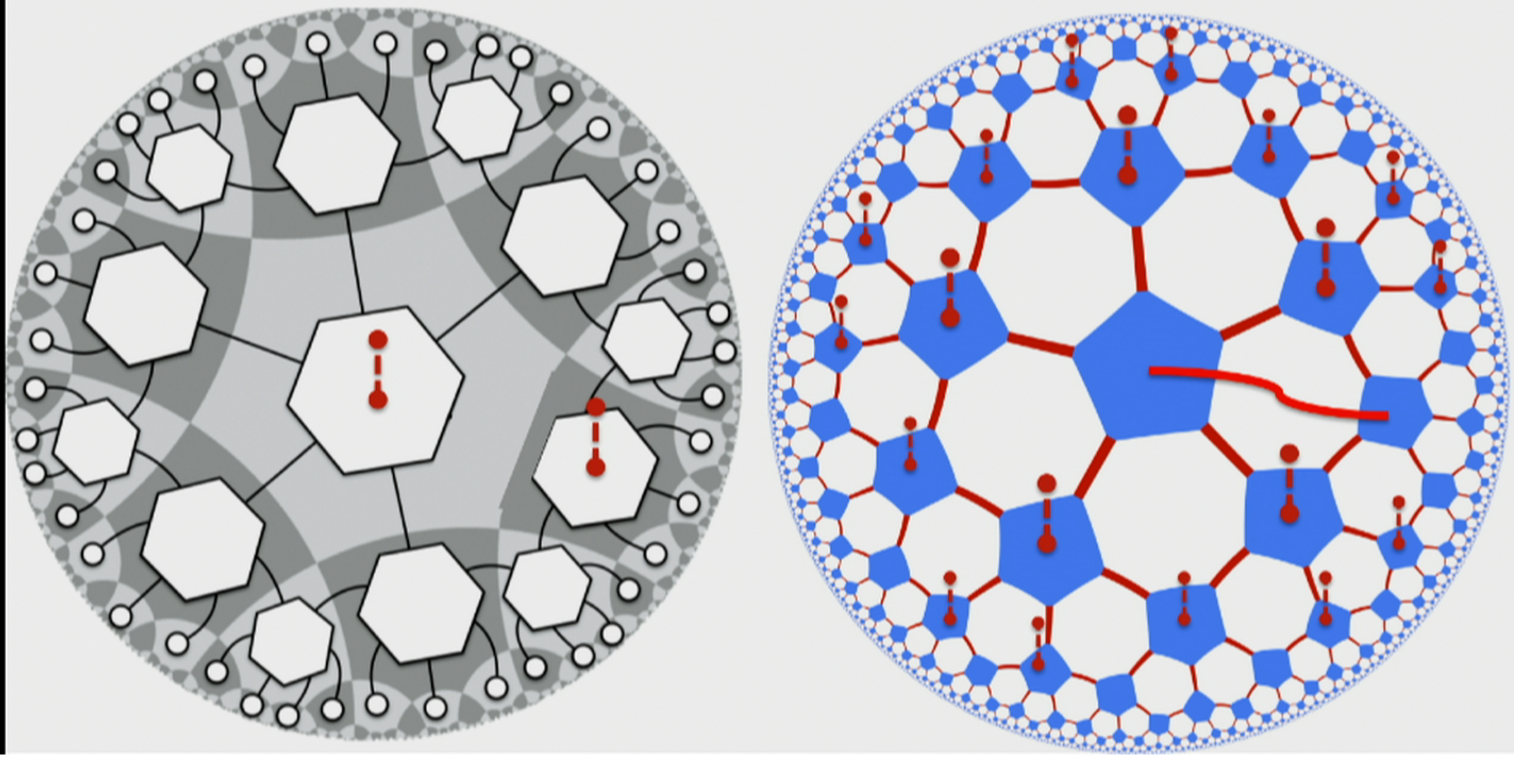
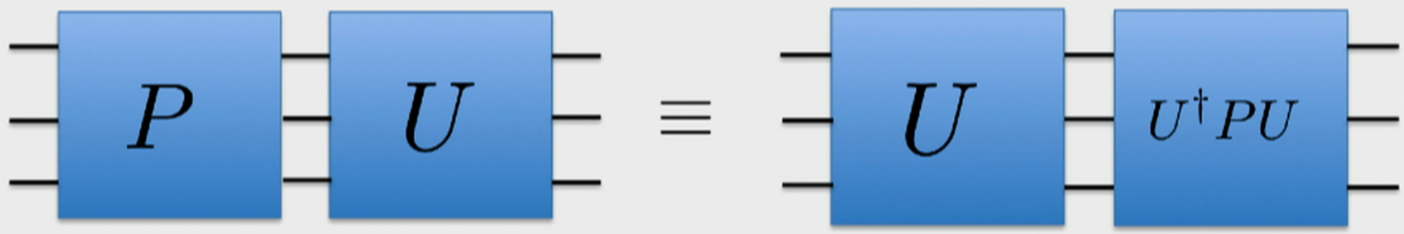
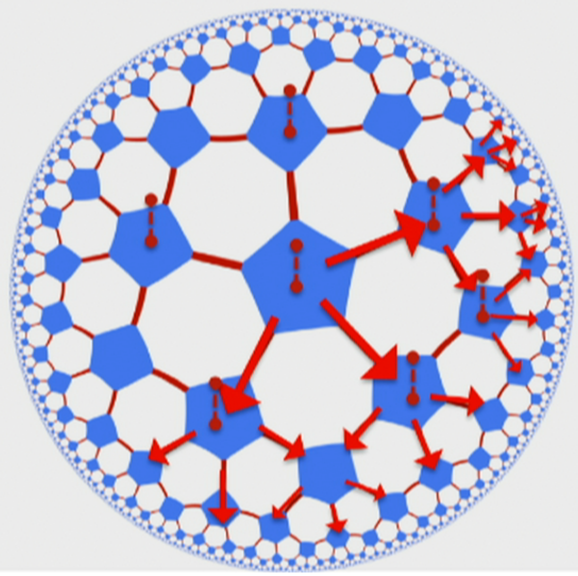


Bulk reconstruction & the greedy wedge

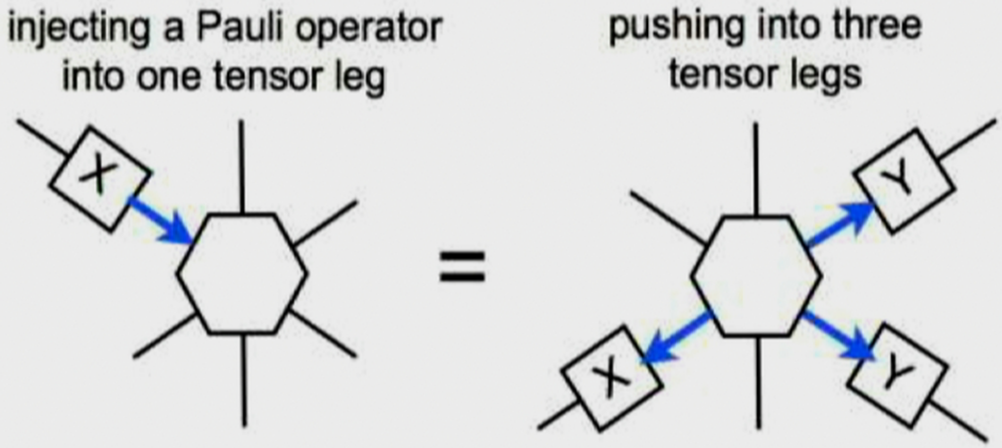
Holographic QECC
Operator pushing



More out than in.
=>
Hyperbolic lattice

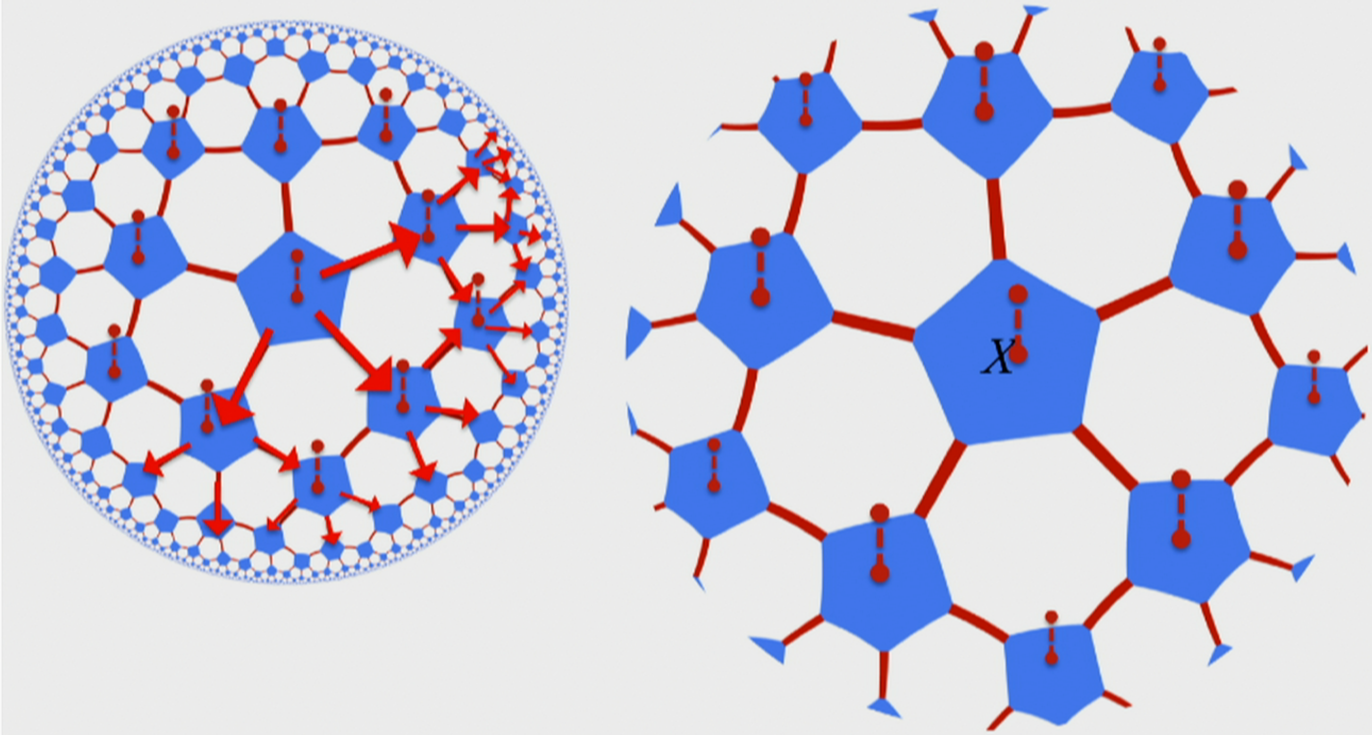


Pauli pushing on Holographic Stabilizer tensors



- Perfect Pauli stabilizer states act like Clifford gates.
- Holographic code is Pauli stabilizer if all tensors are.
- Holographic code is CSS if all tensors are.

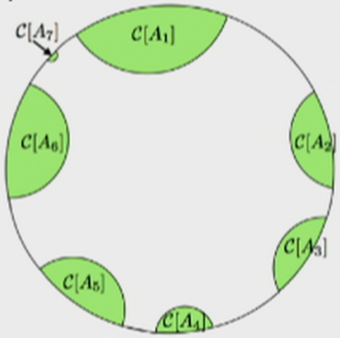
Pauli pushing



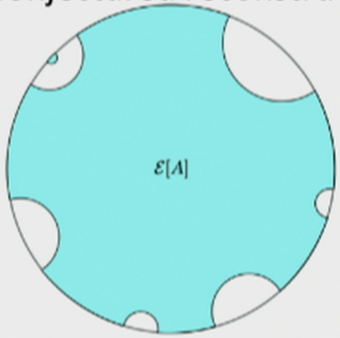
- Pauli group stable under Clifford gates and transposition.
- Clifford isometries are closed under (partial) composition.

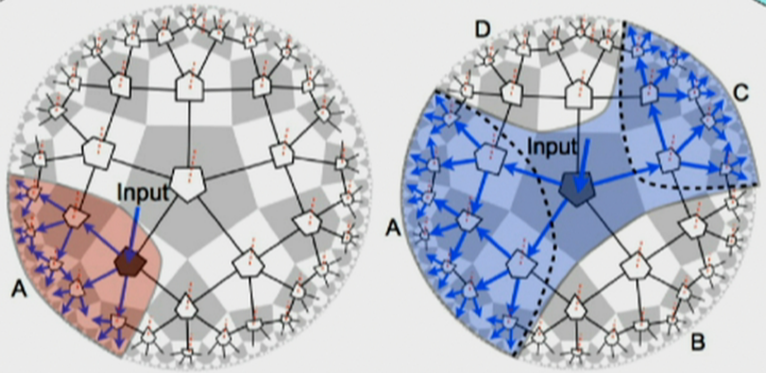
Causal and entanglement wedge reconstruction as isometries

Causal wedge
(Systematic reconstruction)

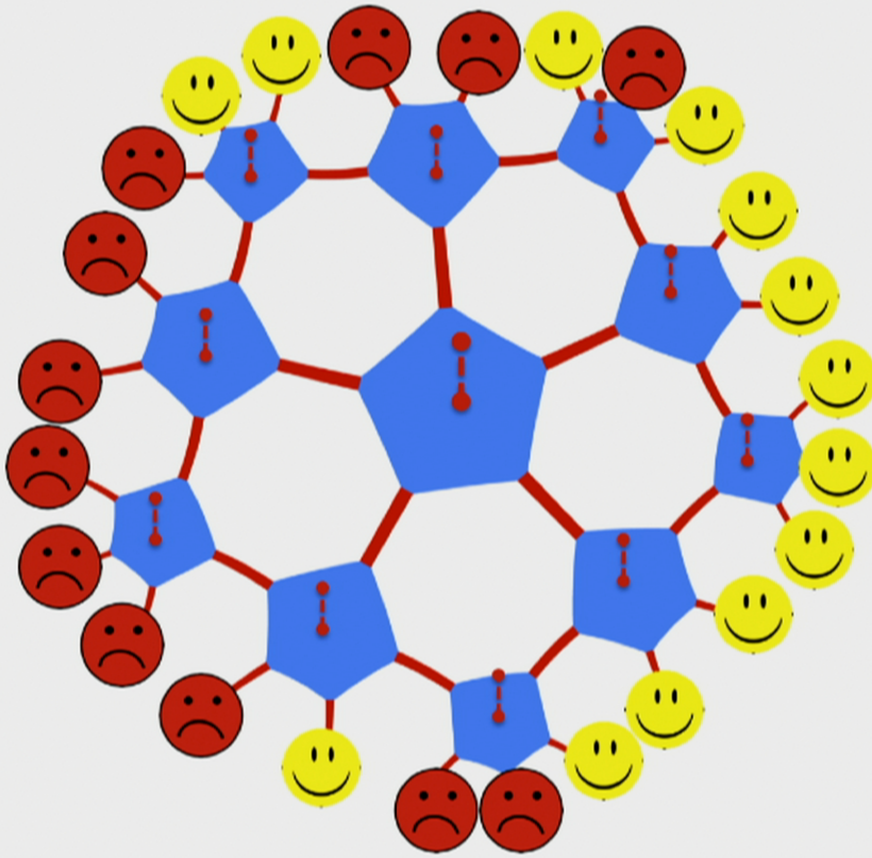


Entanglement wedge
(Conjectured reconstruction)



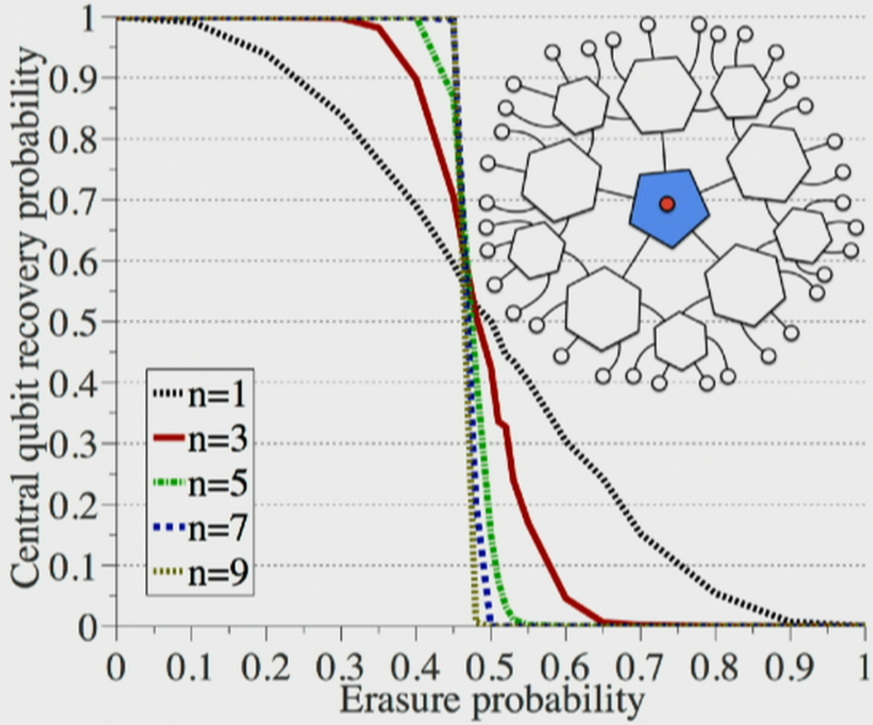


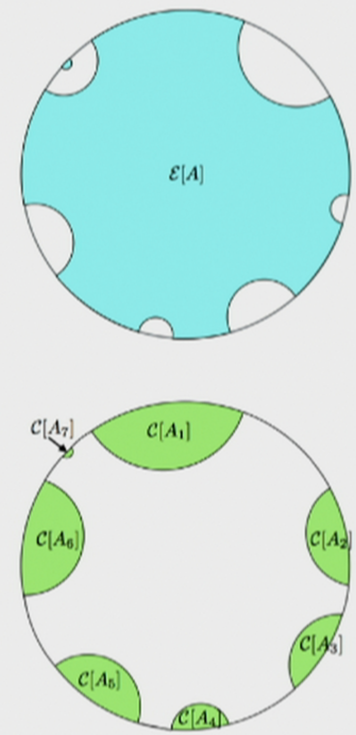
Erasure recovery



Deep beyond the causal wedge

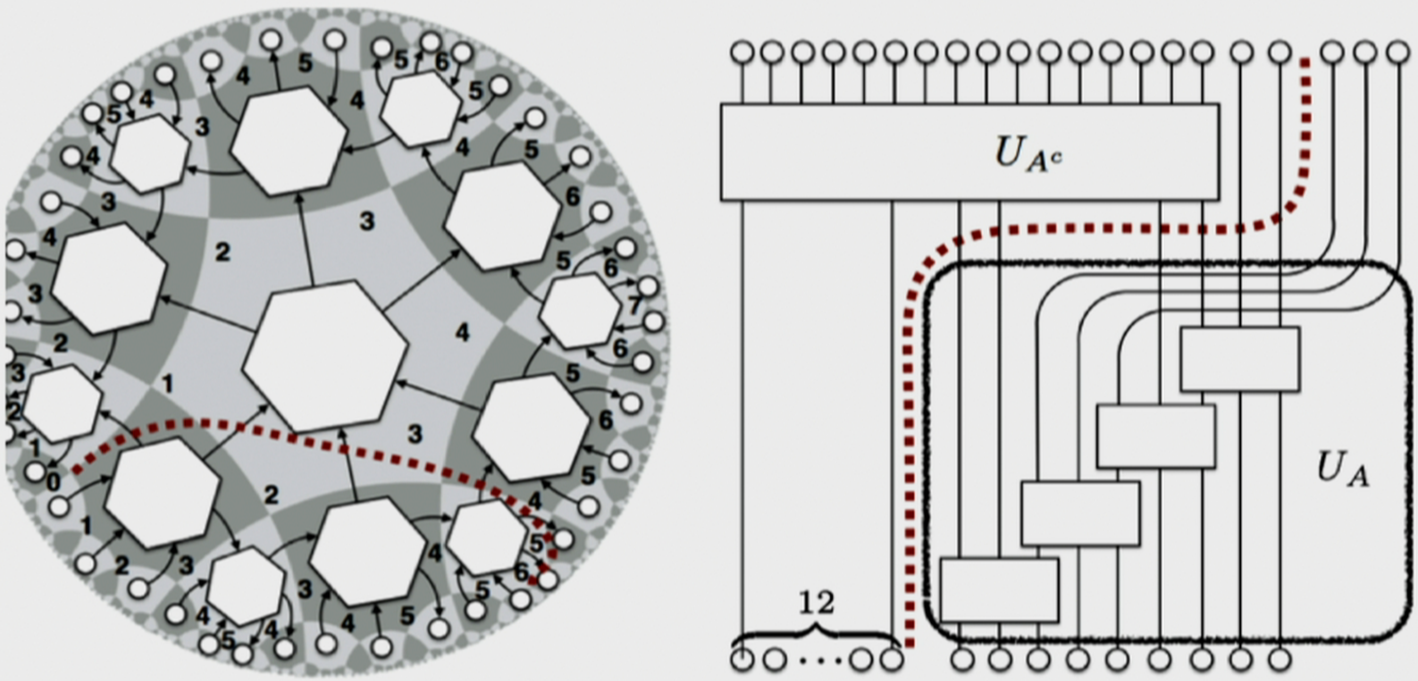
- Numerical greedy recovery threshold ~0.52.
- Actual threshold of 0.5?





Holographic states and Ryu-Takayanagi

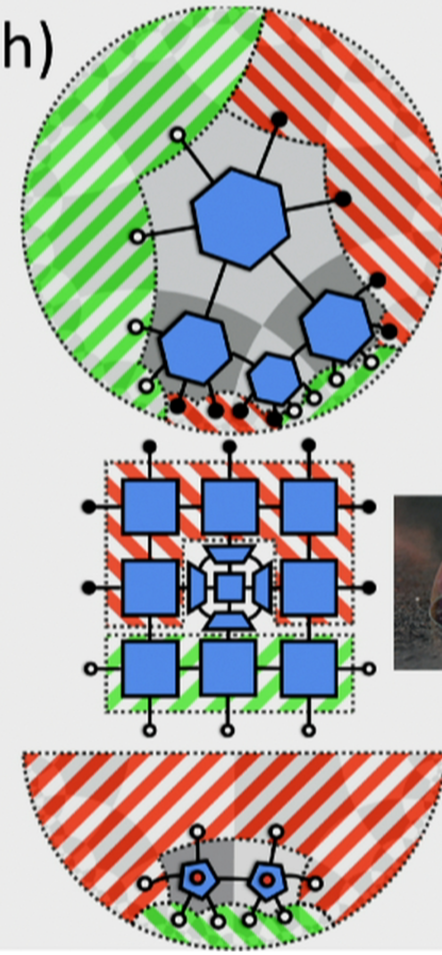
Ryu-Takayanagi is saturated



(a) Circuit interpretation construction (b) Bulk operator reconstruction circuit

What are the technical hypothesis?

(Planar tensor network graph)



- Simply connected boundary
- Non-positive curvature (AdS)
- Holographic **STATE**

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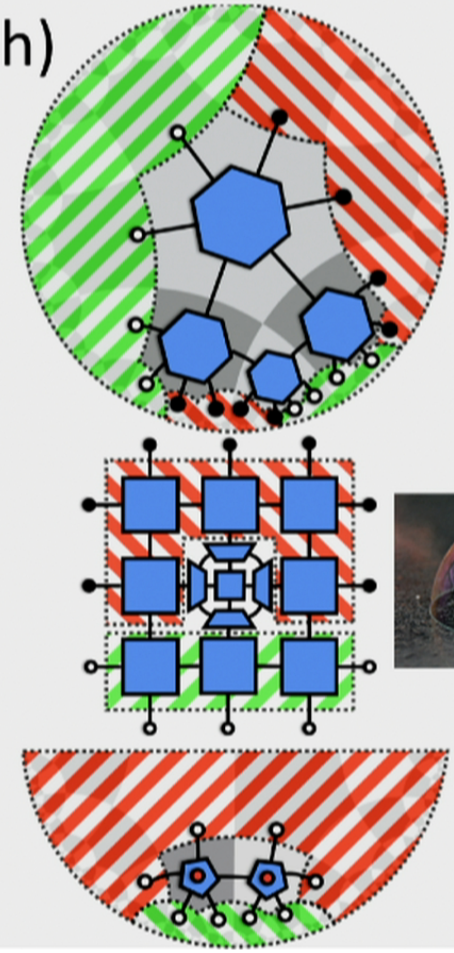

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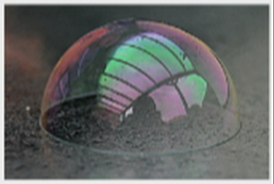
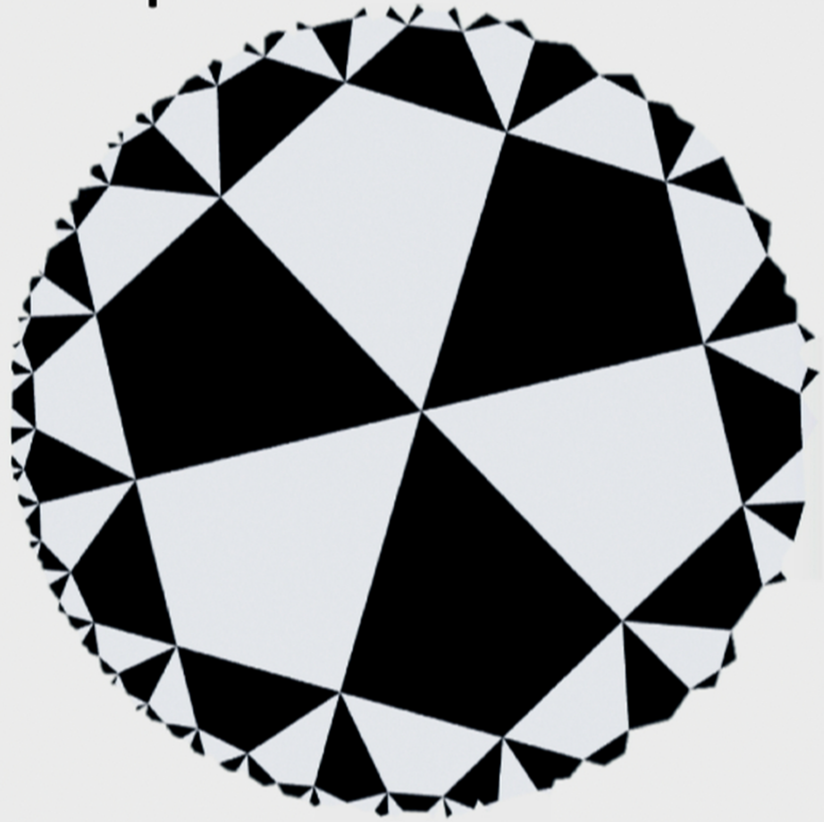


Circuit interpretation

Flux: #Incoming = #Outgoing

DAG: Directed acyclic graph
No closed time-like curves



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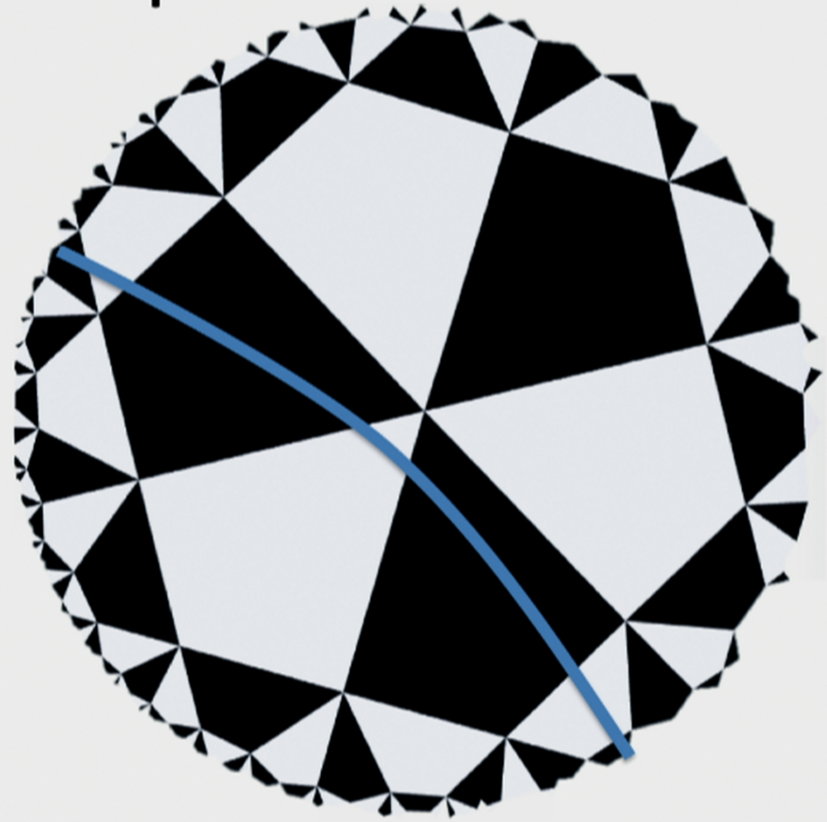

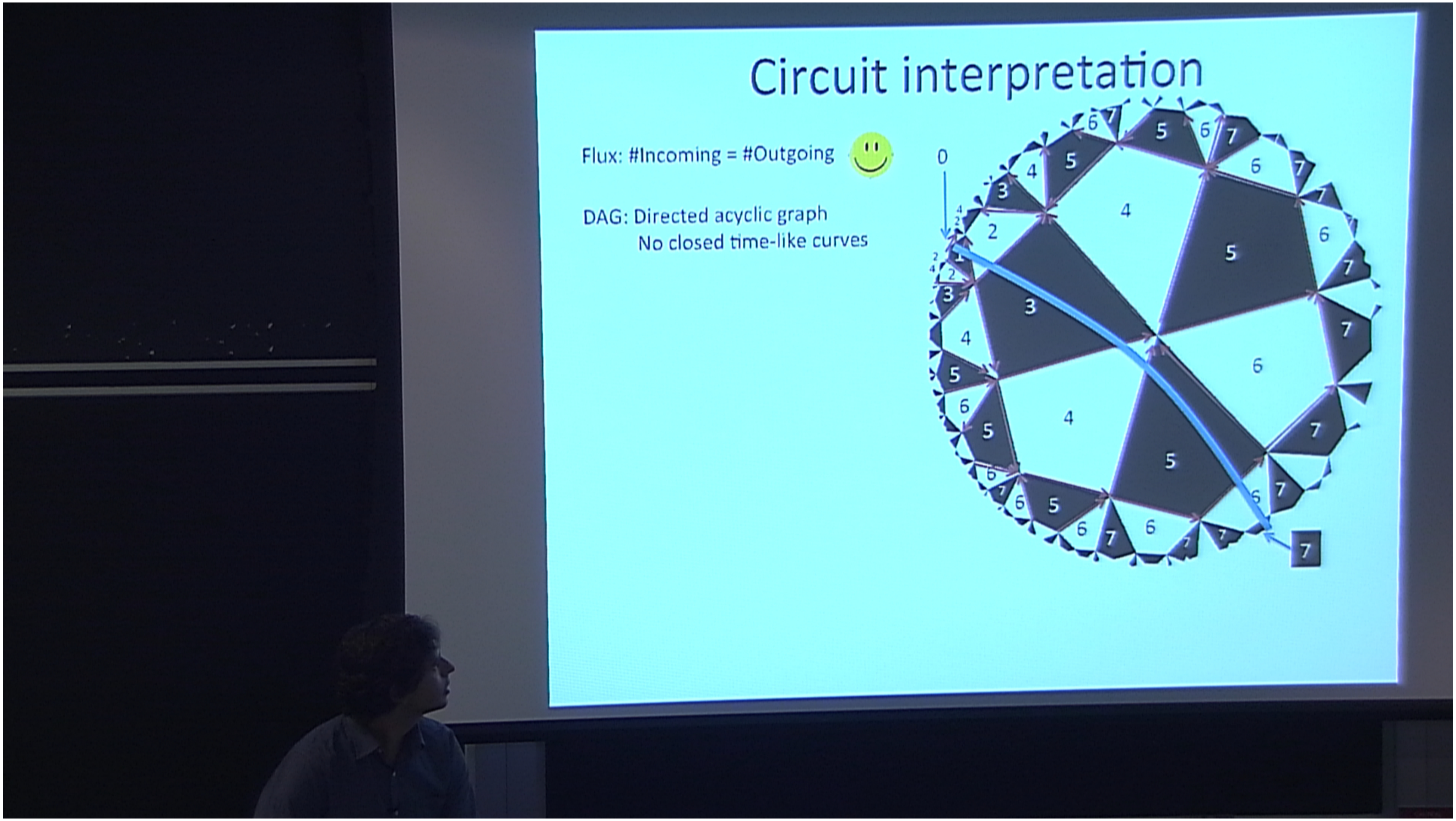
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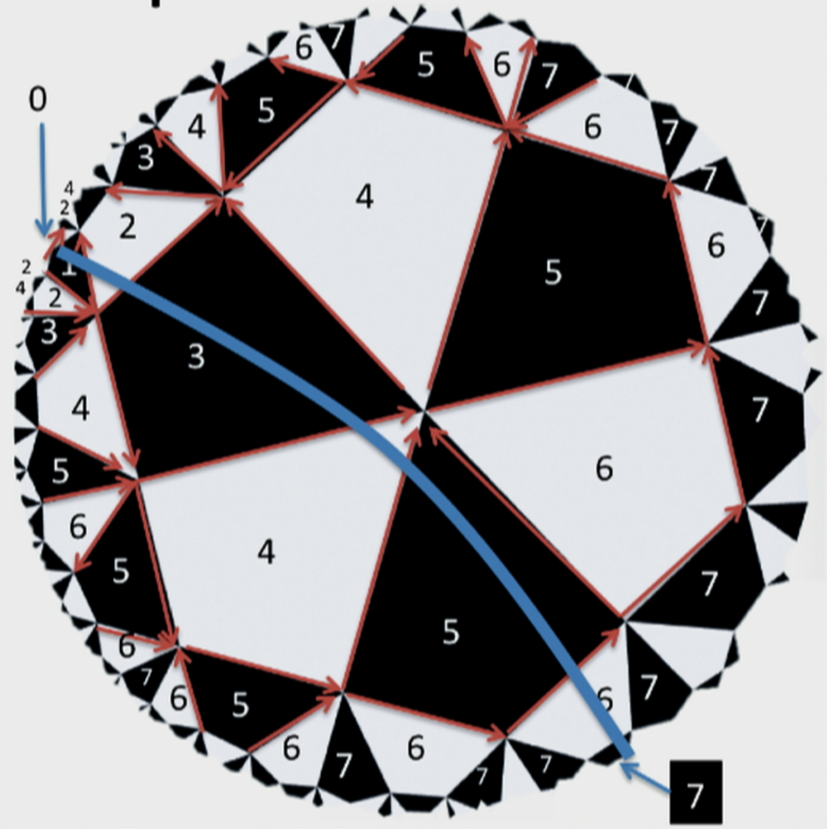


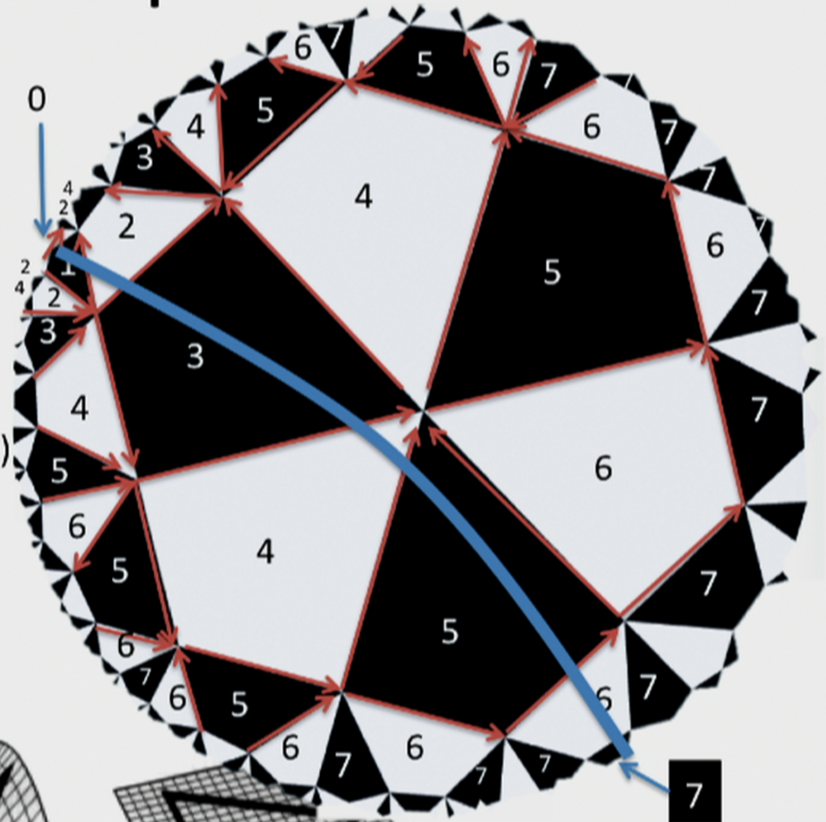
Circuit interpretation

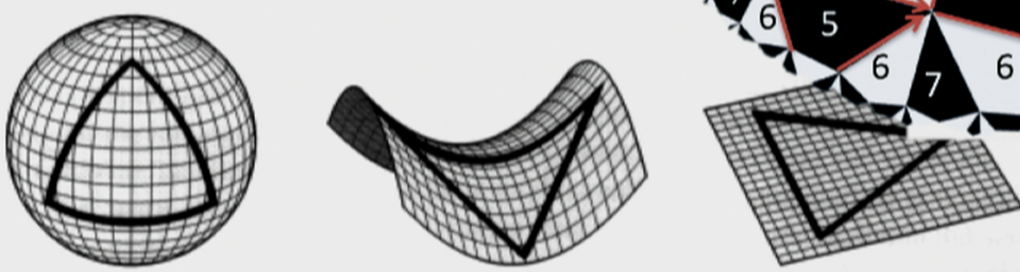
Flux: #Incoming = #Outgoing

DAG: Directed acyclic graph
No closed time-like curves

No local maxima for distance (in bulk)

Non-positive curvature

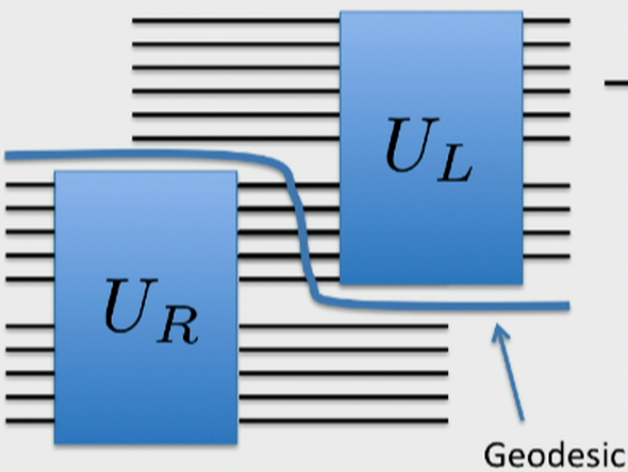


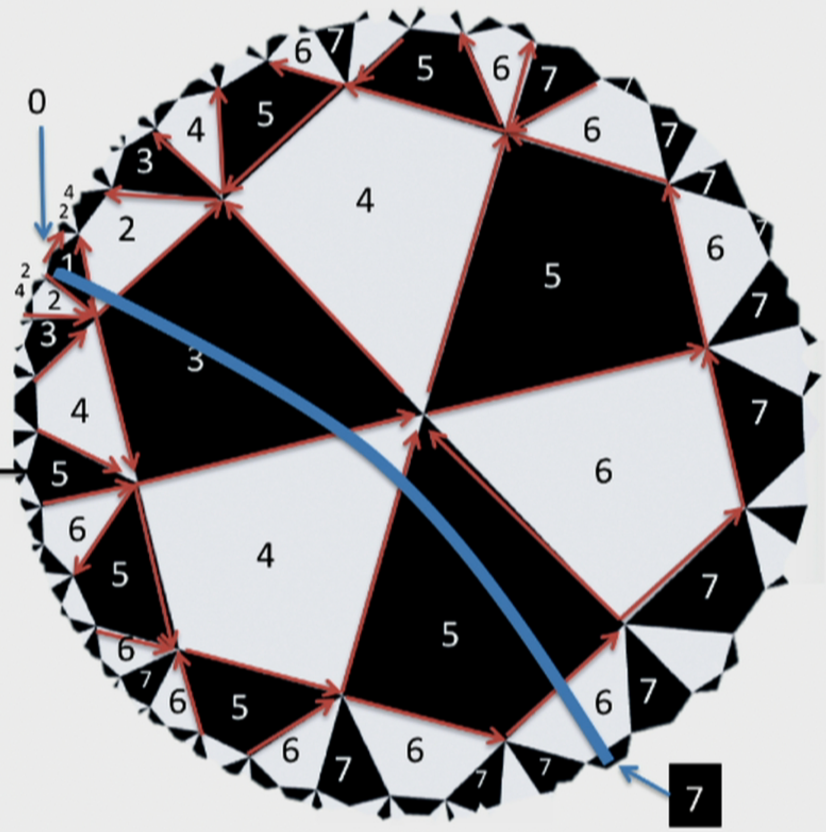


Positive Curvature **Negative Curvature** **Flat Curvature**

Proof of RT for Holographic states

$$T = U = U_L U_R$$

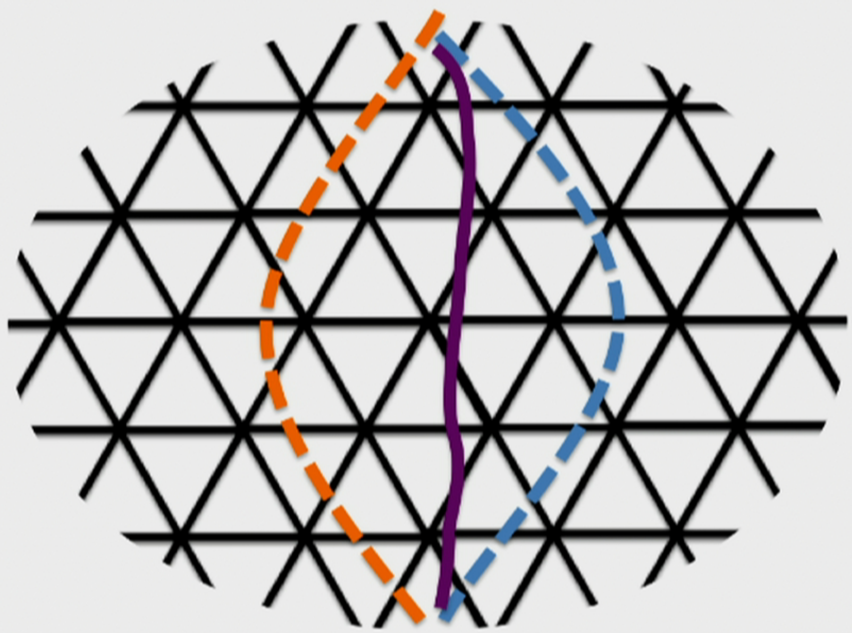




Geodesics and the greedy algorithm

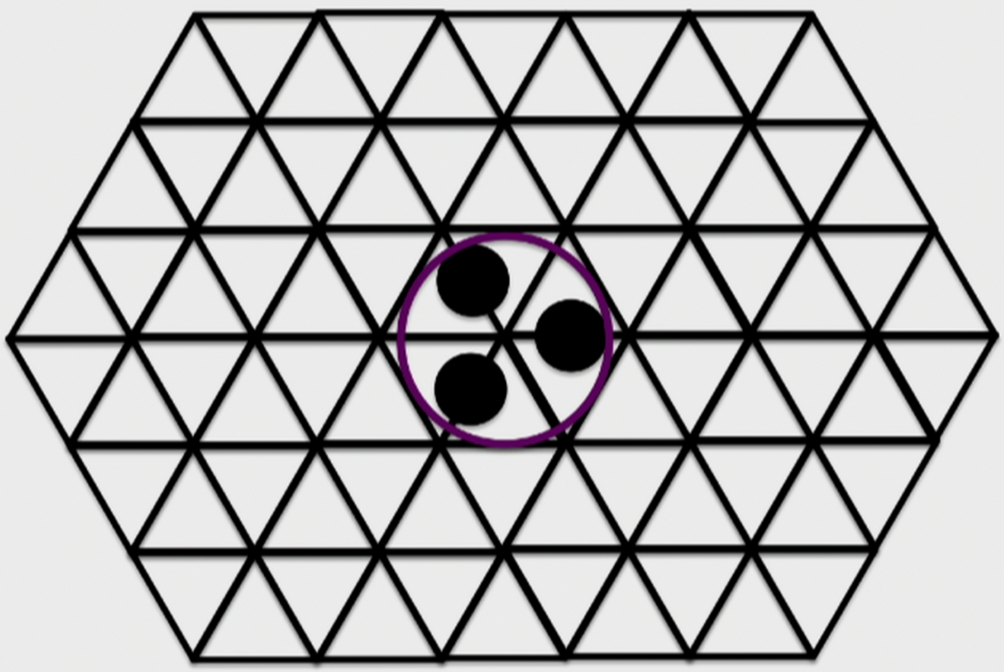
Greedy geodesics region R_A for A contains all geodesics.

Proof: For holographic states, just follow the arrows.



Black holes and Bekenstein-Hawking

Consider the perimeter of an encoded region



Conclusions

- Illustrated power of perfect tensors
  - For constructing `flexible' isometries
  - Connecting entanglement and geometry
- Constructed a family of holographic codes
  - Bulk locality
  - Erasure threshold possible (beyond causal wedge)
- Constructed holographic “vacuum” states
  - Proved exact Ryu-Takayanagi entanglement entropy

Open problems

- Analyze and optimize code parameters.
- General error decoding algorithms!!
- Non-positive curvature from TNR
- Identifying bulk/boundary dynamics
- Sub-AdS bulk locality (lattice continuum limit)
- Bounding of RT corrections
- Generalize RT proof to higher D and relevant assumptions.