Title: Tensor Network Renormalization and the MERA

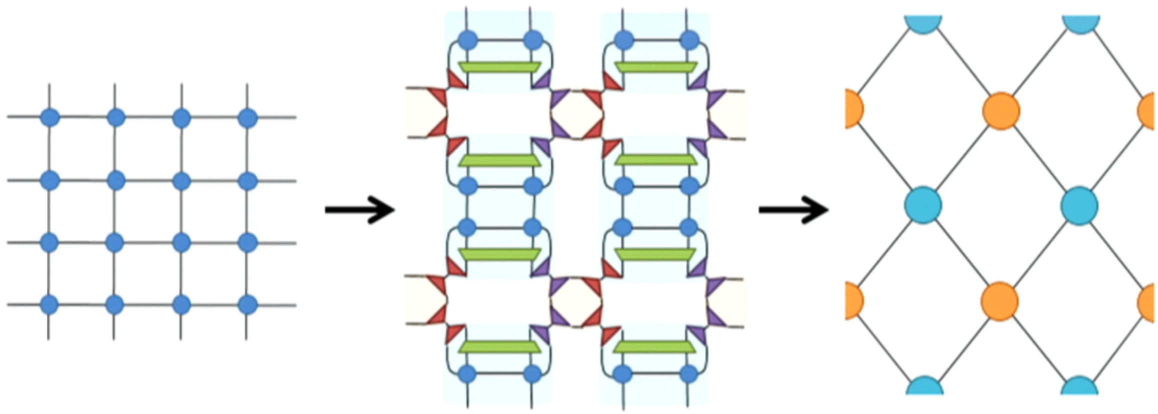
Date: Aug 18, 2015 09:00 AM

URL: http://pirsa.org/15080065

Abstract: I describe a class of non-perturbative renormalization group (RG) transformations which, when applied to the (discrete time) Euclidean path integrals of a quantum systems on the lattice, can give results consistent with conformal transformations of quantum field theories. In particular, this class of transformation, which we call Tensor Network Renormalization (TNR), is shown to generate a scale-invariant RG flow for quantum systems at a critical point. Applications of TNR towards study of quantum critical systems, and its relationship to the multi-scale-entanglement renormalization ansatz (MERA) for ground and thermal states of quantum systems, will be discussed.

“Quantum Information and Quantum Gravity II”,
Perimeter Institute, August 2015

Tensor Network Renormalization and the MERA



Glen Evenbly

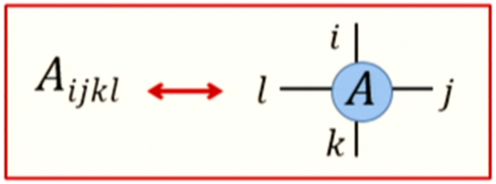
Guifre Vidal

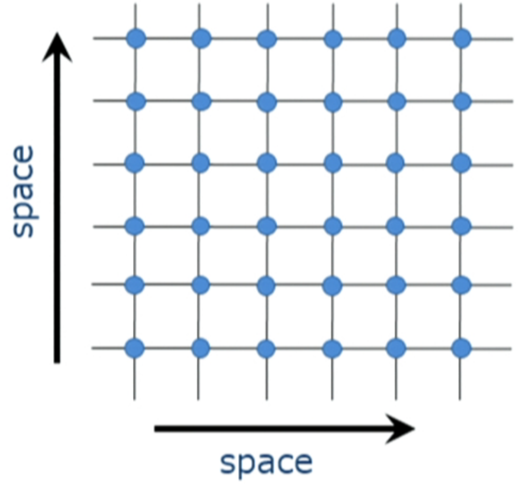
Tensor Network Renormalization, arXiv:1412.0732

Tensor Network Renormalization yields the MERA, arXiv:1502.05385

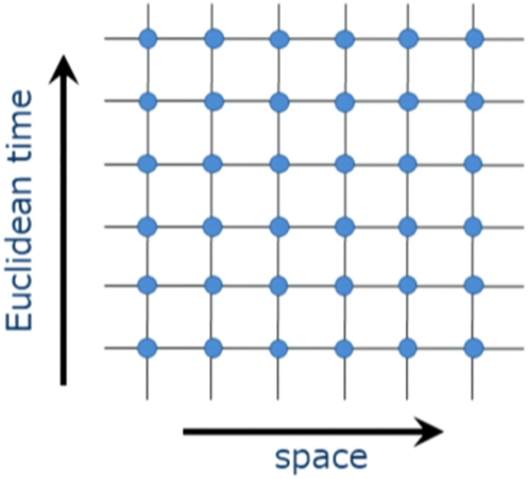
Overview

express many-body system
as a tensor network:





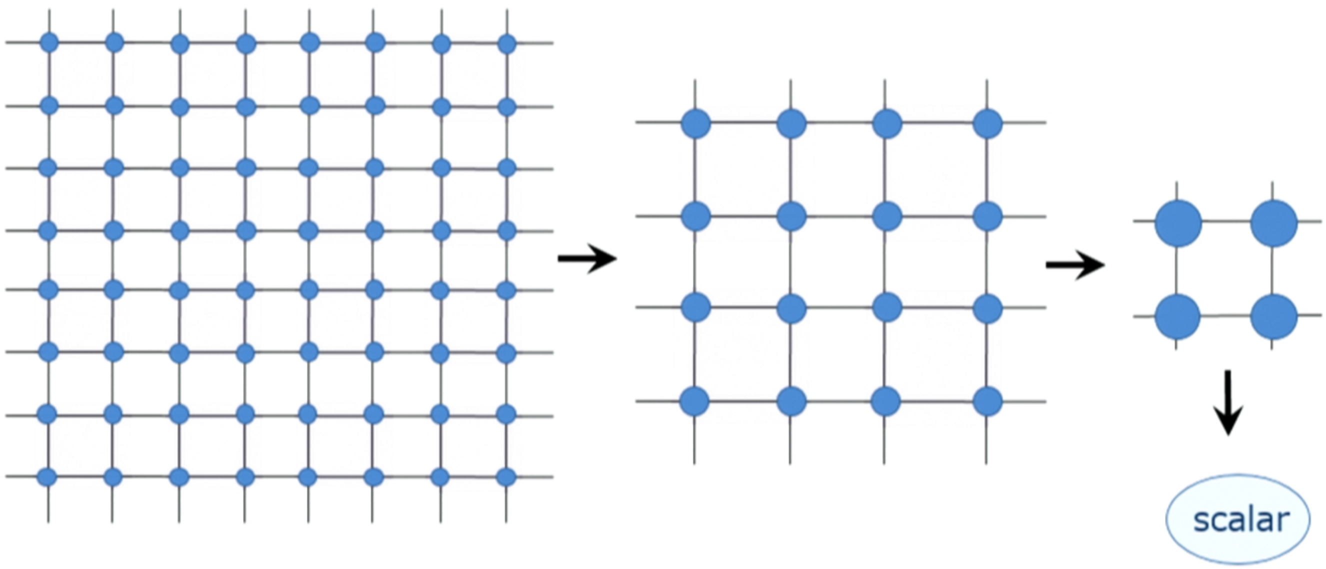
- tensors encode Boltzmann weights
- contraction of tensor network equals weighted sum over all microstates



- row of tensors encodes small evolution in imaginary time
- contraction of tensor network equals weighted sum over all trajectories

Overview

local scale transformations on the lattice:



Practical goal: efficient and accurate contraction of a tensor network to a scalar

Conceptual goal: achieve a proper RG flow

i.e. could represent an expectation value in the quantum system $\langle\psi|o|\psi\rangle$

Overview: real-space RG

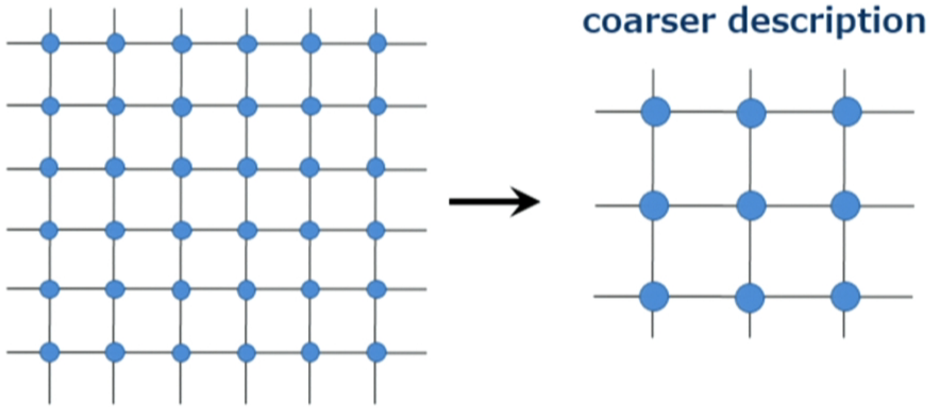
Basic idea of RG:

description in terms of very many microscopic degrees of freedom → Iterative RG transformations → description in terms of a few effective (low-energy, long distance) degrees of freedom

each transformation removes short-range (high energy) degrees of freedom

initial description

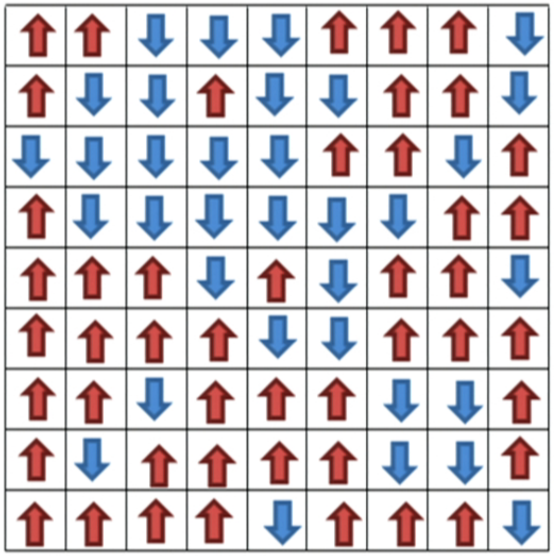
coarser description



Overview: real-space RG

Early real-space RG: Kadanoff's "spin blocking" (1966)

lattice of classical spins



initial description: $H(T, J)$

Overview: real-space RG

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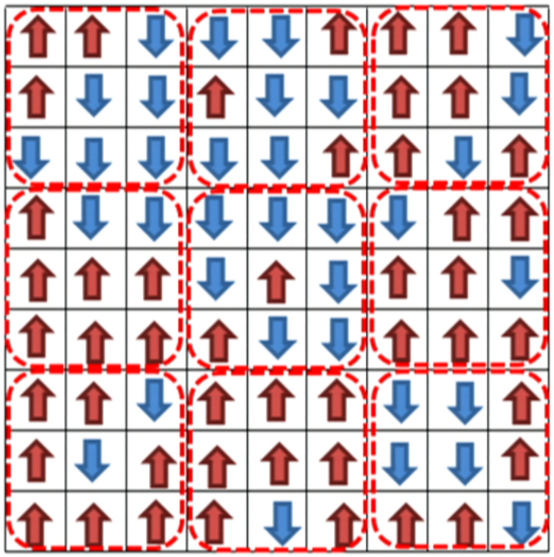
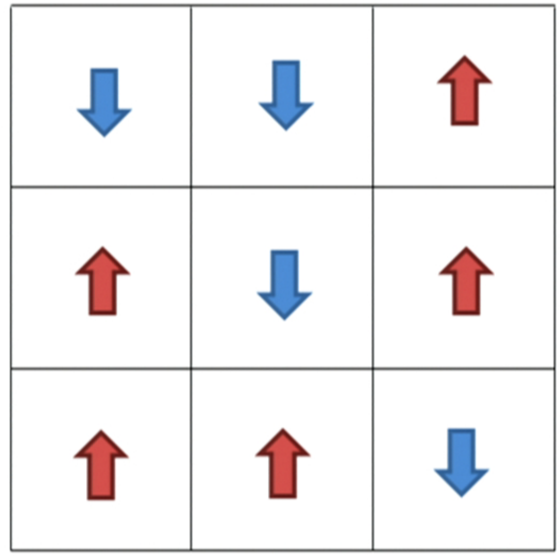


majority vote blocking

coarser lattice



initial description: $H(T,J)$

renormalized parameters: $H(T',J')$

$\cdots$successful only for certain systems

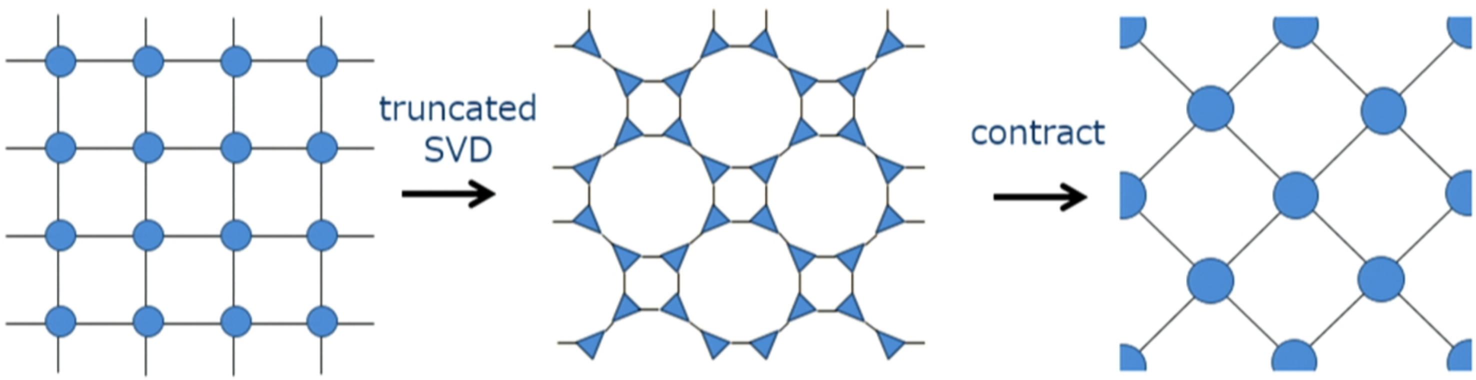
Overview: real-space RG

L.P. Kadanoff (1966): "Spin blocking"

spiritual successor ↓

Key change: a more general prescription for deciding which degrees of freedom can safely be removed at each RG step

Levin, Nave (2006) : "Tensor renormalization group (LN-TRG)"



Overview: real-space RG

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spiritual successor ↓

Key change: a more general prescription for deciding which degrees of freedom can safely be removed at each RG step

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+ many improvements and generalizations:

Xie, Jiang, Weng, Xiang (2008): "Second Renormalization Group (SRG)"

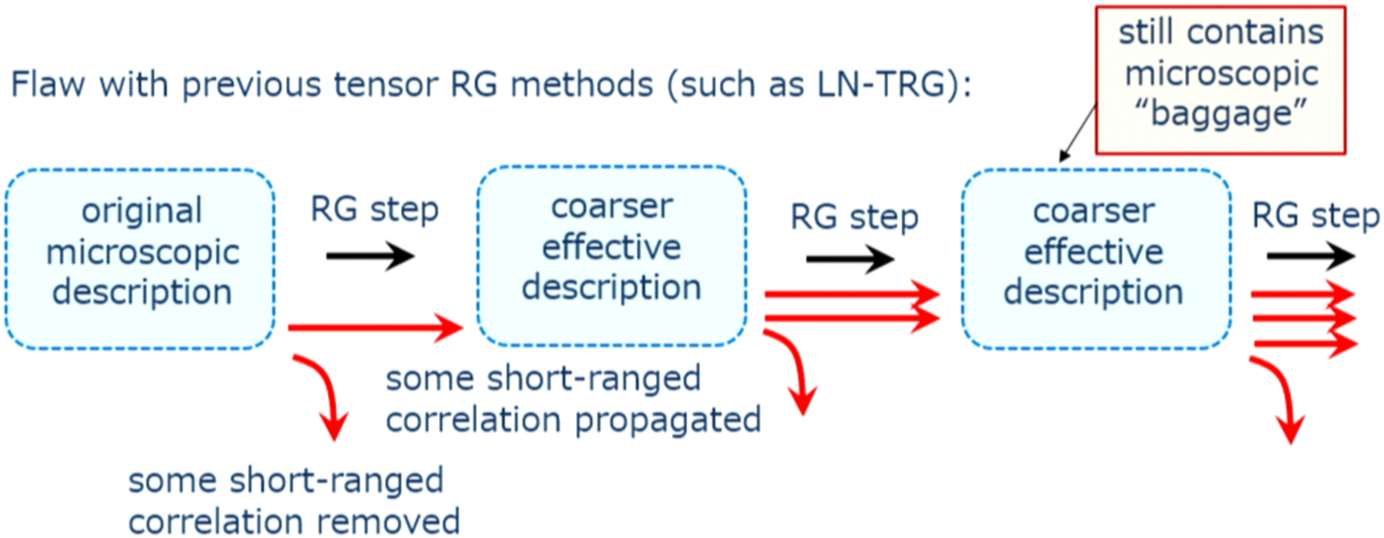
Gu, Levin, Wen (2008): "Tensor Entanglement Renormalization Group (TERG)"

Gu, Wen (2009): "Tensor Entanglement Filtering Renormalization(TEFR)"

Xie, Chen, Qin, Zhu, Yang, Xiang (2012): "Higher Order Tensor Renormalization Group (HOTRG)"

Overview

Flaw with previous tensor RG methods (such as LN-TRG):



Flaw: each RG step removes some (but not all) of the short-ranged degrees freedom

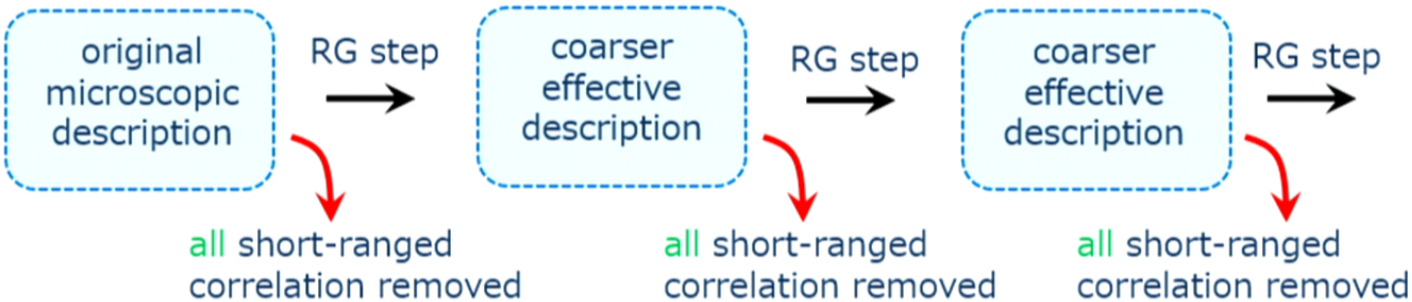
Consequences:

- Computational breakdown near (or at) a critical point ✗
- Effective theory still contains unwanted microscopic detail; one does not recover proper structure of RG fixed points ✗

Overview

New approach: **"Tensor Network Renormalization (TNR)"** arXiv:1412.0732

A way of implementing real-space RG that addresses all short-ranged degrees of freedom at each RG step



Advantages:

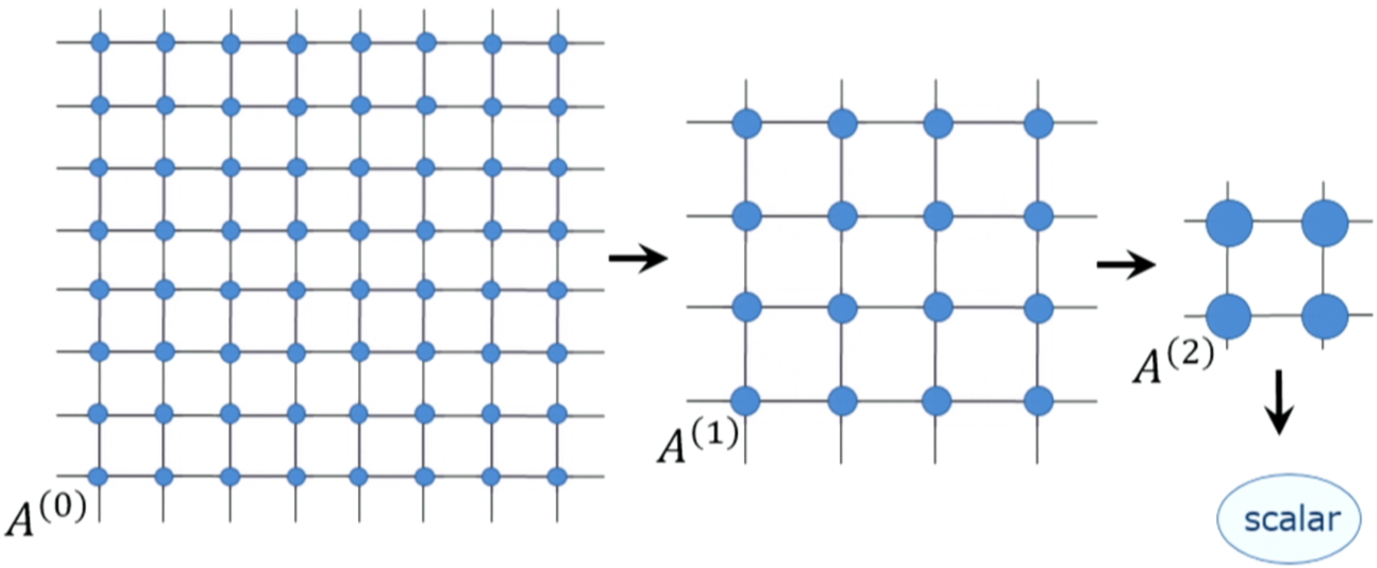
- **Proper RG flow is achieved**, TNR reproduces the correct structure of RG fixed points (including critical fixed points)
- Prevents harmful accumulation of short-ranged detail, allowing for a computationally sustainable RG flow

exponential cost scaling (previous tensor RG) → constant cost (new approach, TNR)

Overview

local scale transformations on the lattice:

RG flow in the space of tensors: $A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{(s)} \rightarrow \cdots$



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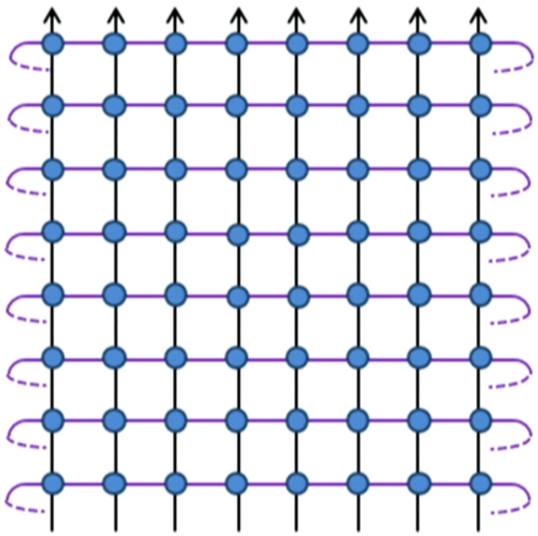
i.e. could represent an expectation value in the quantum system $\langle\psi|o|\psi\rangle$

Overview

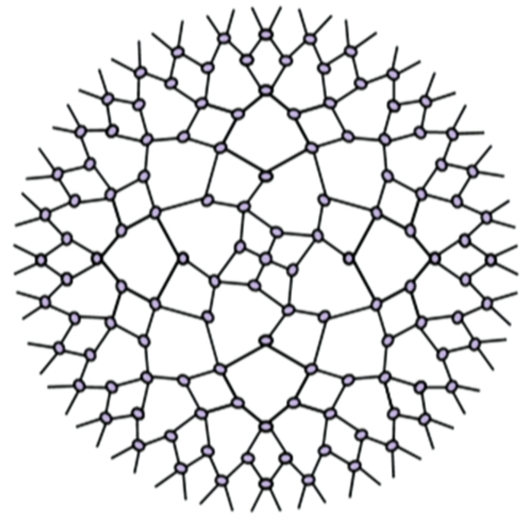
Local scale transformation can be applied inhomogeneously
to do interesting things...

Euclidean path integral (1D quantum system, PBC)



→

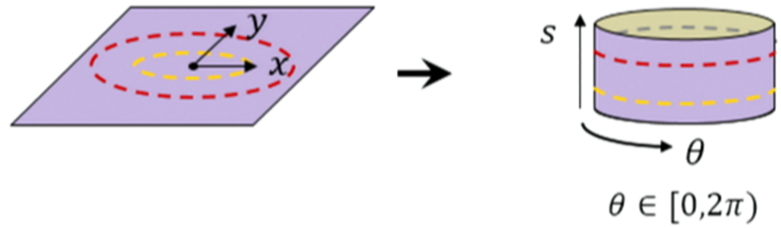
Multi-scale entanglement renormalization ansatz (MERA)



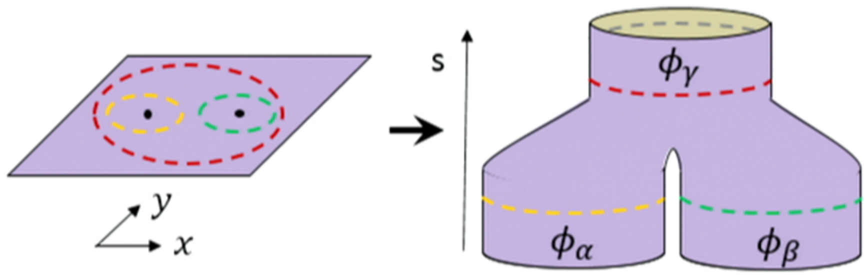
Local scale transformations on the lattice with TNR

(**G. E.**, D. Gaiotto, R. Myers, G. Vidal, *in prep.*)

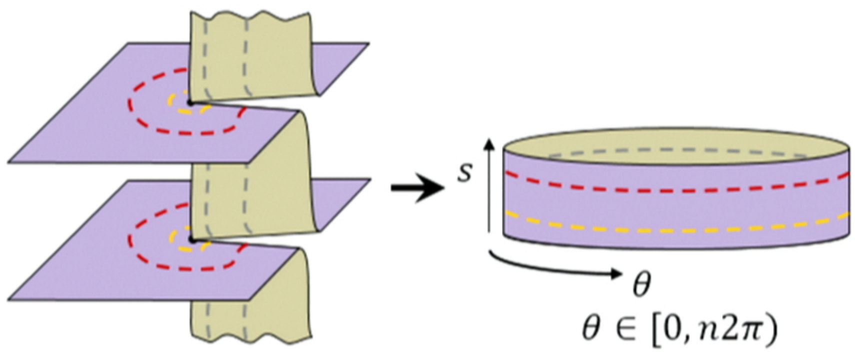
punctured plane to cylinder



plane with two punctures into pair of pants



n – sheeted Riemann surface to cylinder



Overview: Tensor Networks

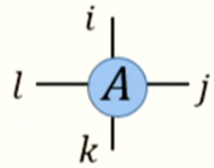
bond dimension

Let A_{ijkl} be a four index tensor with $i, j, k, l \in \{1,2,3,\ldots,\chi\}$

i.e. such that the tensor is a $\chi \times \chi \times \chi \times \chi$ array of numbers

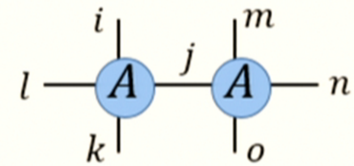
Diagrammatic notation:

A_{ijkl} ⟷



Contraction of two tensors:

$\sum_j A_{ijkl} A_{mnoj}$ ⟷



Overview: Tensor Networks

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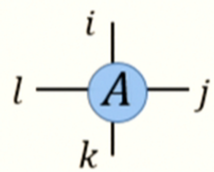
A_{ijkl} ⟷ 

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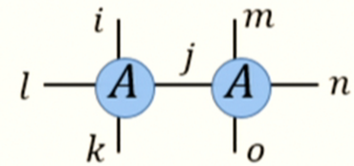
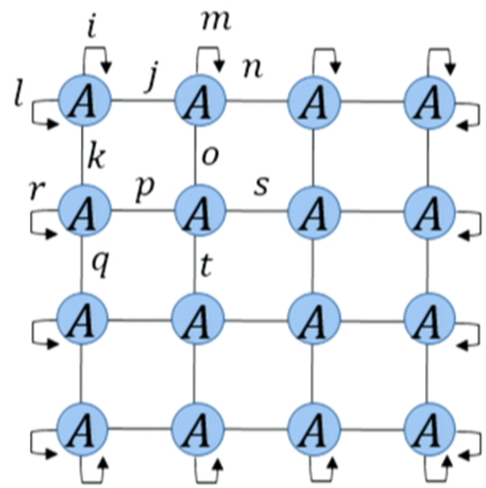
$\sum_j A_{ijkl} A_{mnoj}$ ⟷ 

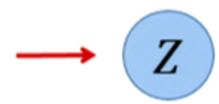
Square lattice network (PBC):

$$\sum_{ijklmn\dots} A_{ijkl} A_{mnoj} A_{kpqr} A_{ostp} \cdots$$

$$\equiv \mathrm{tTr}\left(\bigotimes_{x=1}^{N} A\right) = \mathrm{Z}$$

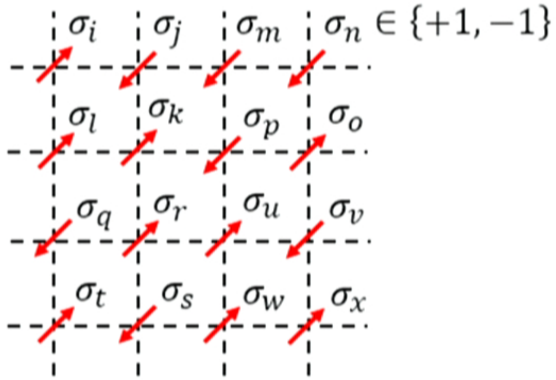


Contracts to a scalar:

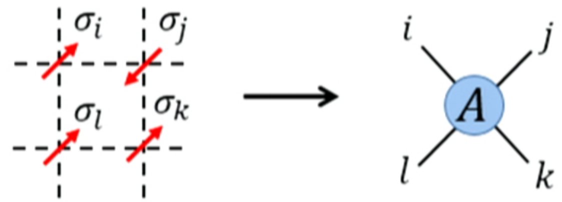


Partition functions as Tensor Networks

Square lattice of Ising spins:



Encode the Boltzmann weights of a plaquette of spins in a four-index tensor



Hamiltonian functional for Ising ferromagnet:

$$H(\{\sigma\}) = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

Partition function:

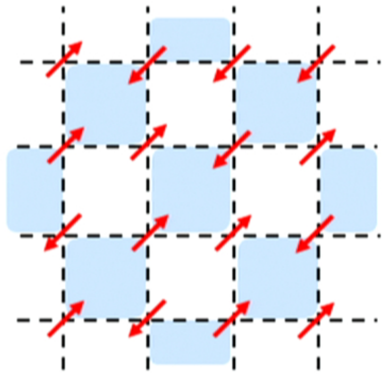
$$Z = \sum_{\{\sigma\}} e^{-H(\{\sigma\})/T}$$

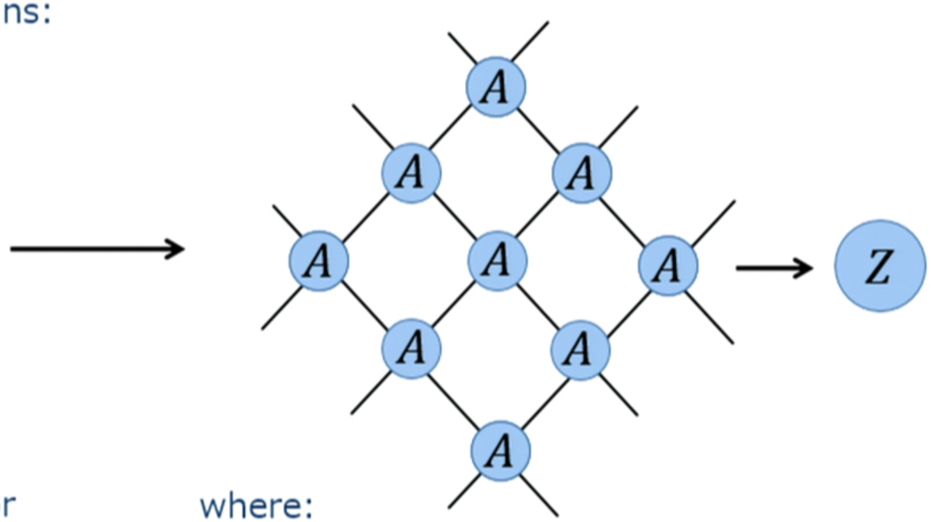
where:

$$A_{ijkl} = e^{(\sigma_i\sigma_j + \sigma_j\sigma_k + \sigma_k\sigma_l + \sigma_l\sigma_i)/T}$$

Partition functions as Tensor Networks

Square lattice of Ising spins:





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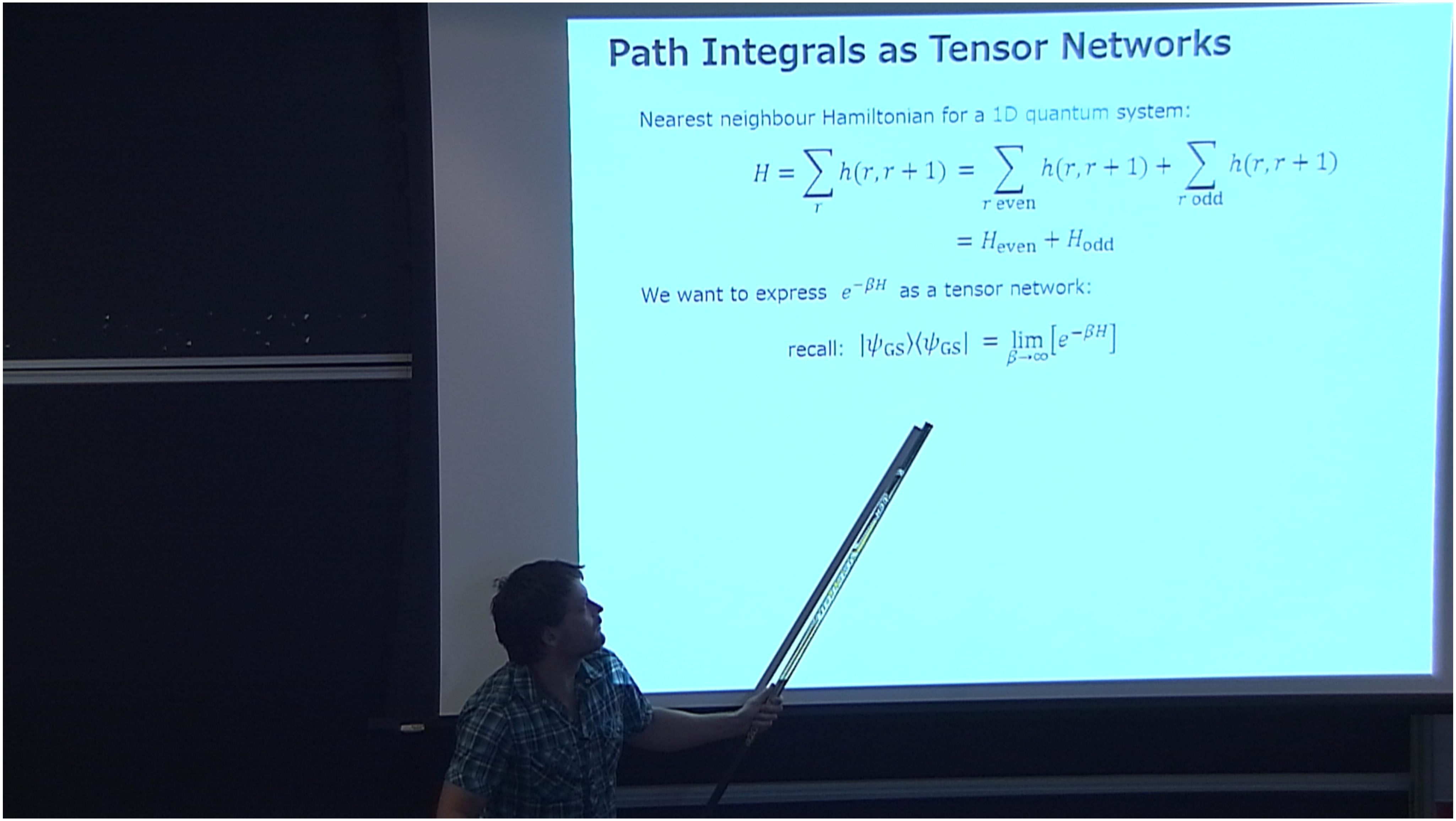
Partition function:

$$Z = \sum_{\{\sigma\}} e^{-H(\{\sigma\})/T} = \mathrm{tTr}\left(\bigotimes_{x=1}^{N} A \right)$$

← Partition function given by contraction of tensor network


Path Integrals as Tensor Networks
Nearest neighbour Hamiltonian for a 1D quantum system:
$H = \sum_r h(r, r+1) = \sum_{r \text{ even}} h(r, r+1) + \sum_{r \text{ odd}} h(r, r+1)$
$= H_{\text{even}} + H_{\text{odd}}$
We want to express $e^{-\beta H}$ as a tensor network:
recall: $|\psi_{\text{GS}}\rangle\langle\psi_{\text{GS}}| = \lim_{\beta \to \infty} [e^{-\beta H}]$

Path Integrals as Tensor Networks

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We want to express $e^{-\beta H}$ as a tensor network:

recall: $|\psi_{\text{GS}}\rangle\langle\psi_{\text{GS}}| = \lim_{\beta\to\infty}\left[e^{-\beta H}\right]$

Expand in small time steps:

$$e^{-\beta H} = e^{-\tau H} e^{-\tau H} e^{-\tau H} e^{-\tau H} \ldots$$

Suzuki-Trotter expansion:

$$e^{-\tau H} = e^{-\tau H_{\text{even}}} e^{-\tau H_{\text{odd}}} + o(\tau^2)$$

Path Integrals as Tensor Networks

Separate Hamiltonian into even and odd terms:

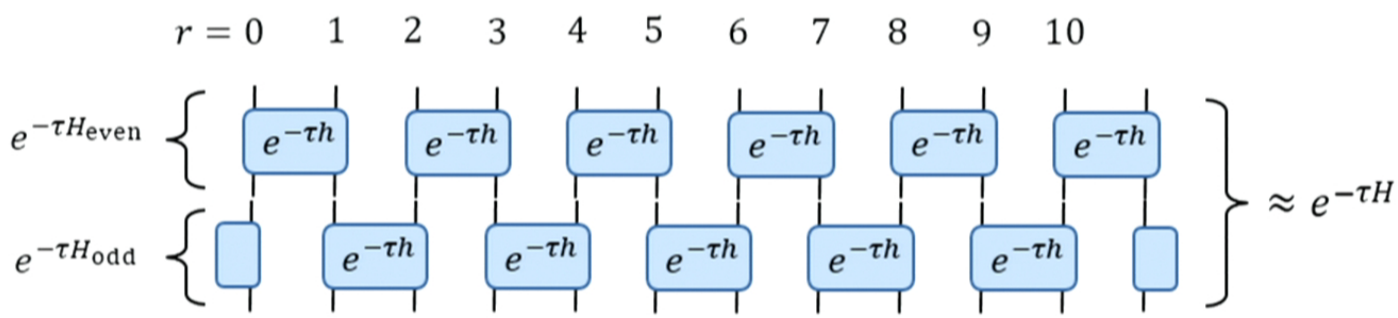
$$H = \sum_{r\ \mathrm{even}} h(r, r+1) + \sum_{r\ \mathrm{odd}} h(r, r+1) \; = H_{\mathrm{even}} + H_{\mathrm{odd}}$$

Expand path integral in small discrete time steps:

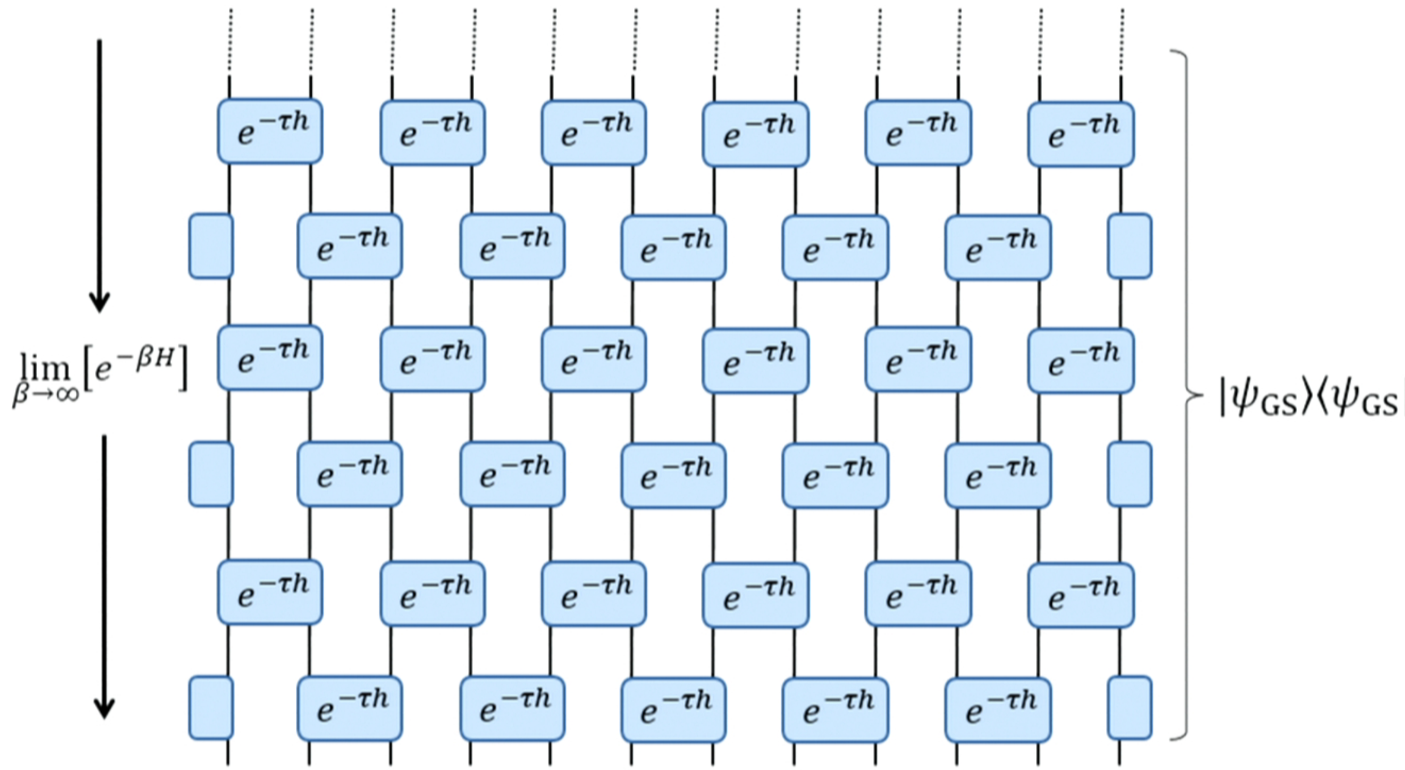
$$e^{-\beta H} = e^{-\tau H} e^{-\tau H} e^{-\tau H} e^{-\tau H} \ldots$$

$$e^{-\tau H} = e^{-\tau H_{\mathrm{even}}} e^{-\tau H_{\mathrm{odd}}} + o(\tau^2)$$

Exponentiate even and odd separately :



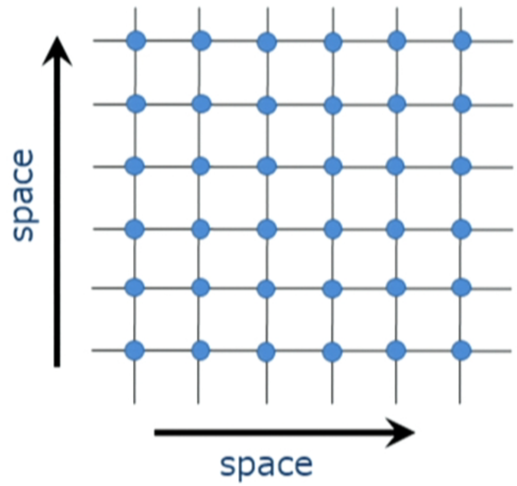
Path Integrals as Tensor Networks



Overview

encode many-body systems as a tensor network:

partition function of 2D classical statistical model



- tensors encode Boltzmann weights
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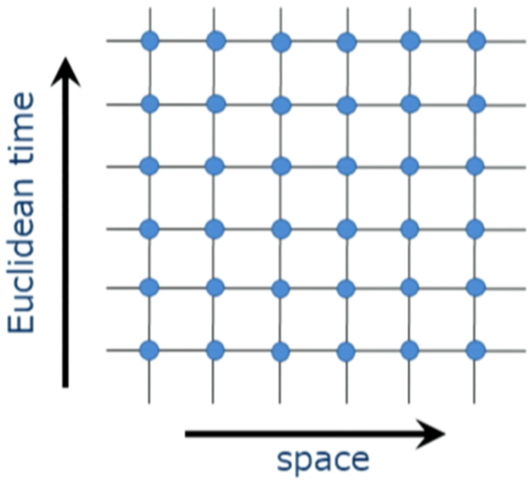
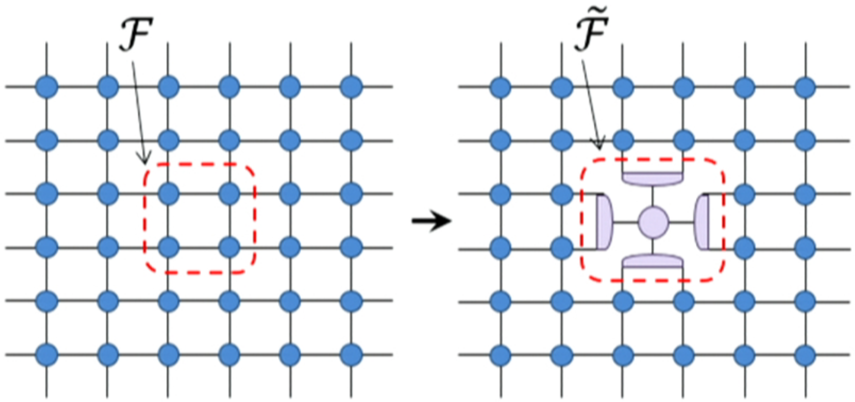
Euclidean path integral of 1D quantum model



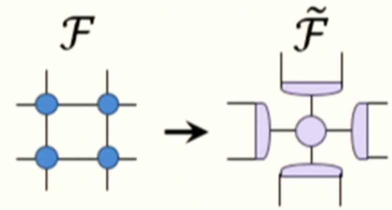
- row of tensors encodes small evolution in imaginary time
- contraction of tensor network equals weighted sum over all trajectories

Local Approximations:

want to replace a local subnetwork of tensors (e.g. a 2 x 2 block) with something else...



Rule: any local replacement is potentially valid...

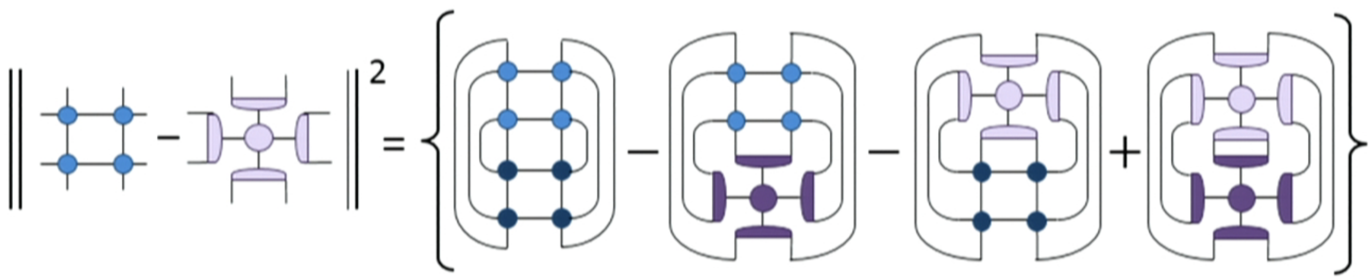


...so long as the difference is small:

$$\delta = \|\mathcal{F} - \tilde{\mathcal{F}}\|$$

Replacement error:

$$\delta^2 = \|\mathcal{F} - \tilde{\mathcal{F}}\|^2 = \|\mathcal{F}\|^2 - \mathrm{tTr}\left(\mathcal{F} \otimes \tilde{\mathcal{F}}^\dagger\right) - \mathrm{tTr}\left(\tilde{\mathcal{F}} \otimes \mathcal{F}^\dagger\right) + \|\tilde{\mathcal{F}}\|^2$$



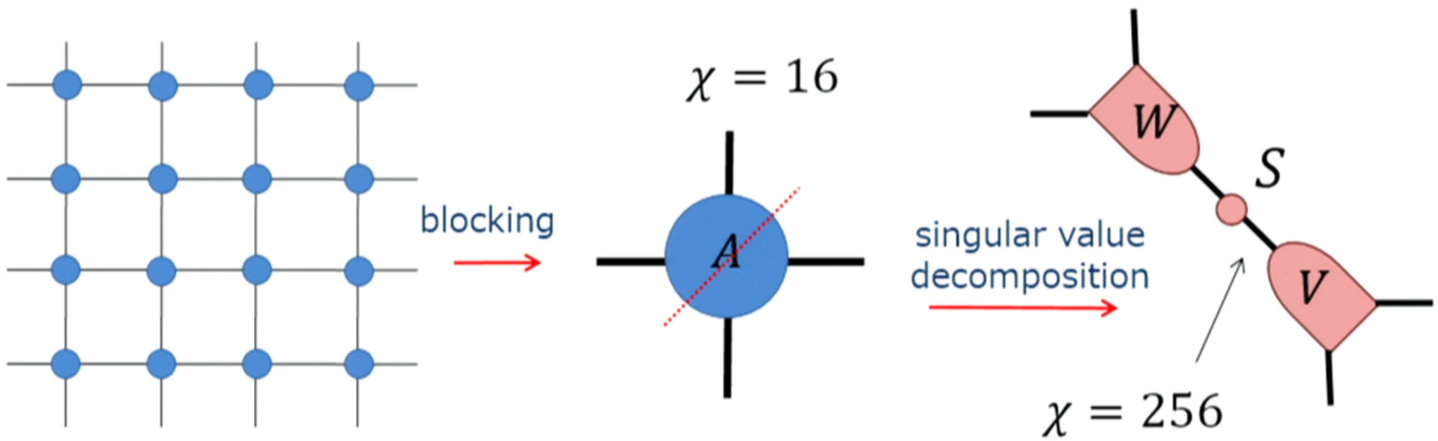
Tensor Renormalization Group (LN-TRG)

Levin, Nave (2006)

Tensor renormalization group (LN-TRG) is a method for coarse-graining tensor networks based upon **blocking** and **truncation steps**

Example of blocking + truncation: 2D classical Ising (critical temp)

- take a (4 x 4) block of tensors from the partition function
- contract to a single tensor; each (16-dim) index describes the state of four classical spins
- can the block tensor be truncated?



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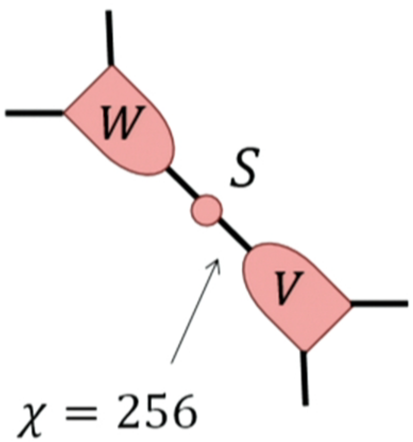

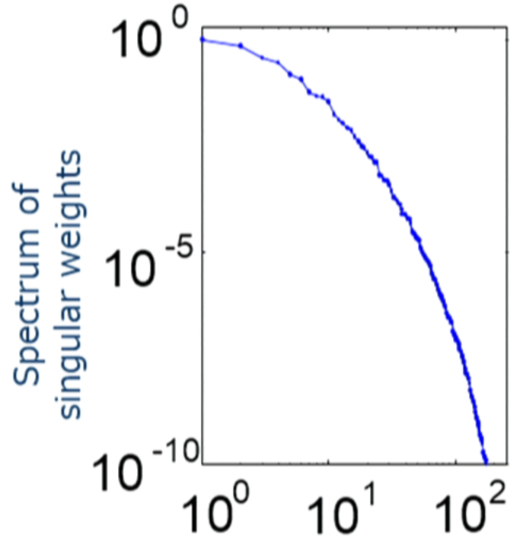

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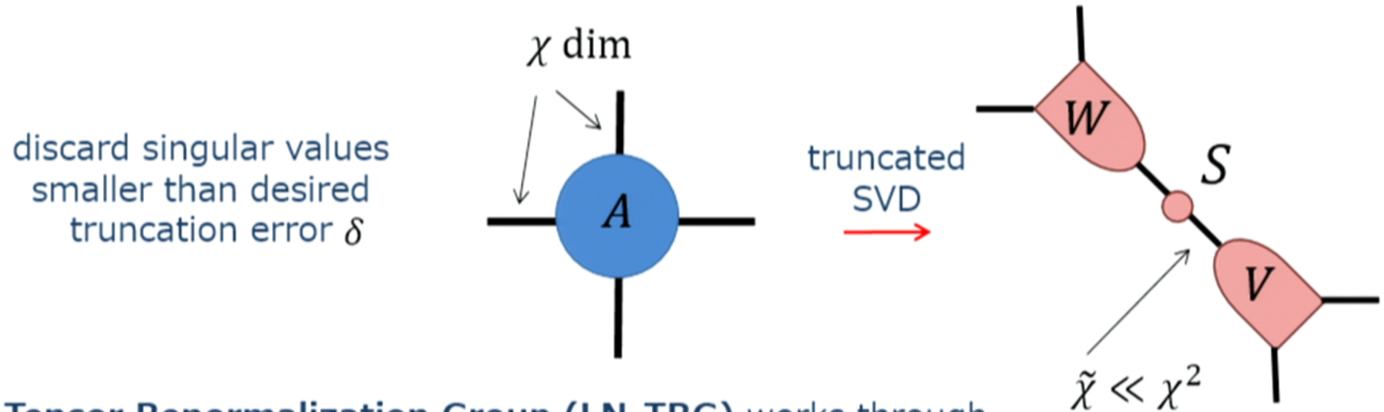
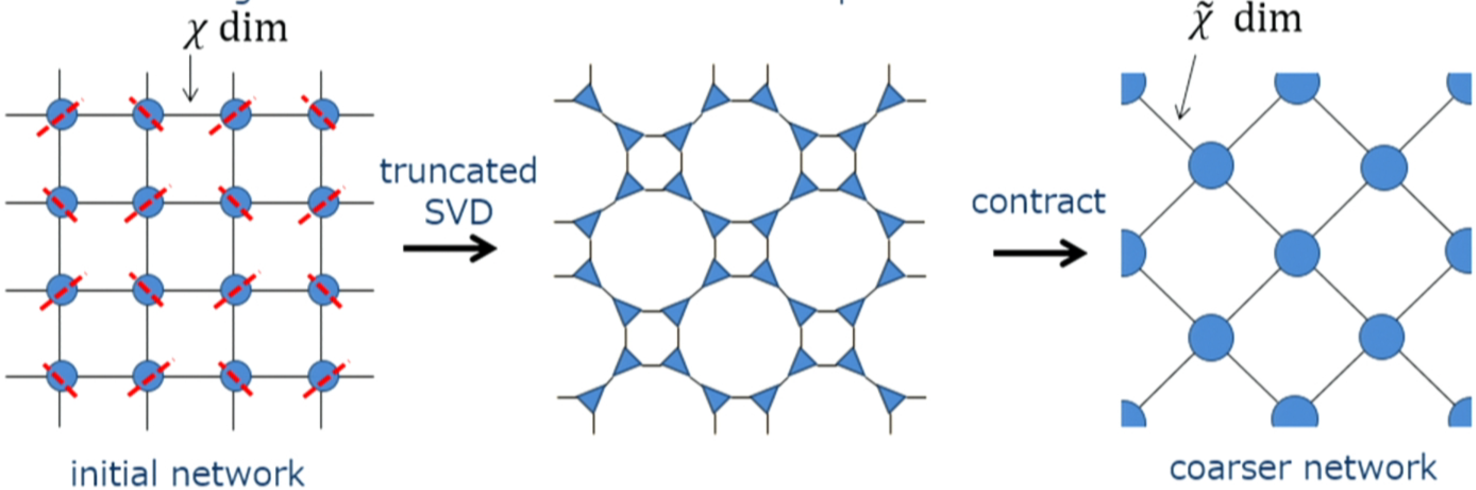
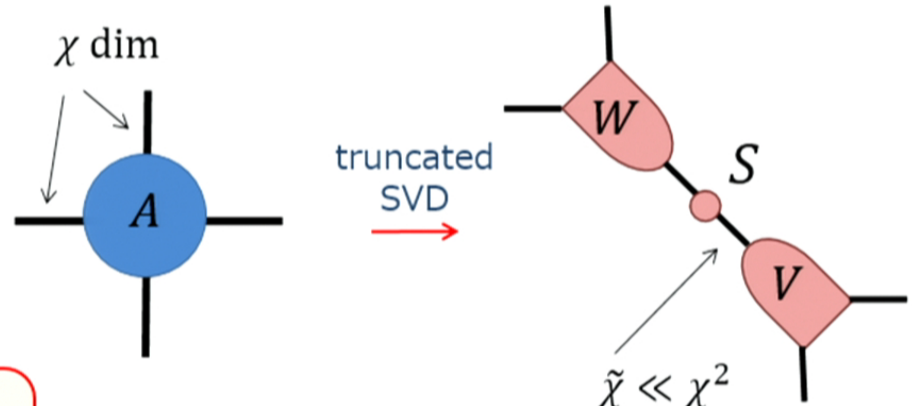


Tensor Renormalization Group (LN-TRG) works through alternating truncated SVD and contraction steps:



Tensor Renormalization Group (LN-TRG)

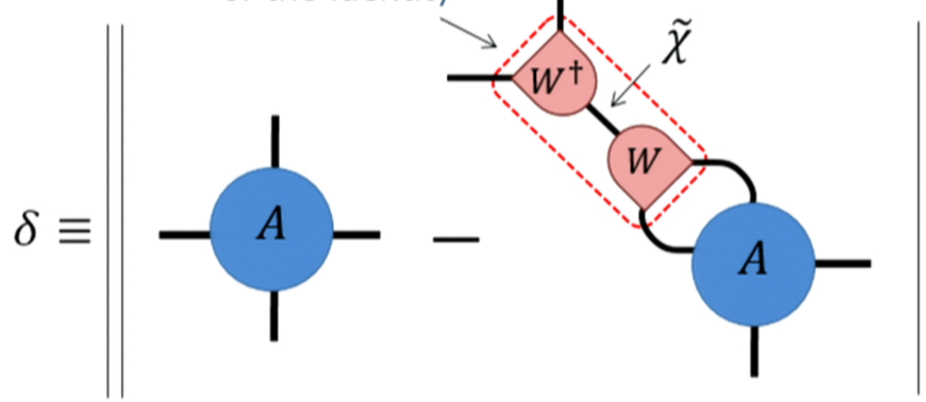
discard singular values smaller than desired truncation error δ



equivalent approach:
implement truncation through projector of the form $W^\dagger W$ for isometric W

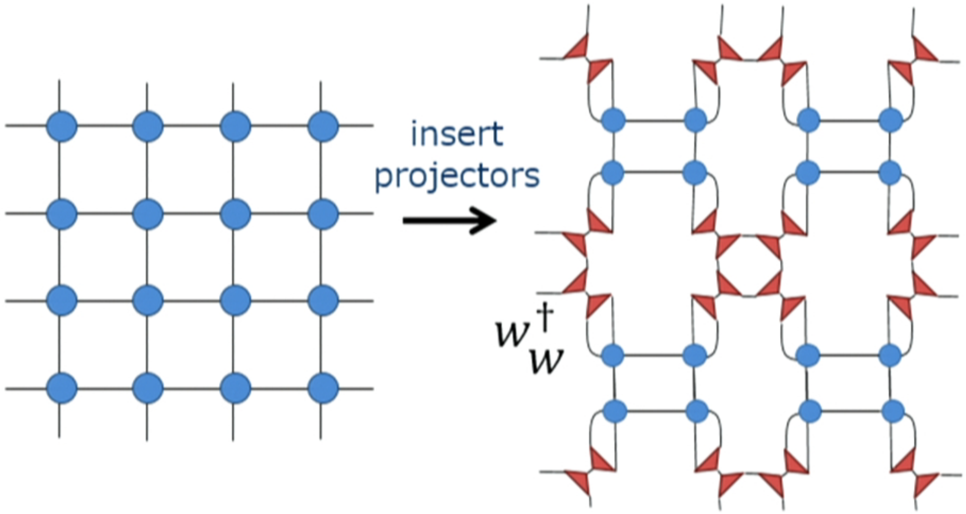
projector P acts as a (approximate) resolution of the identity

i.e. choose isometry W to minimise truncation error δ



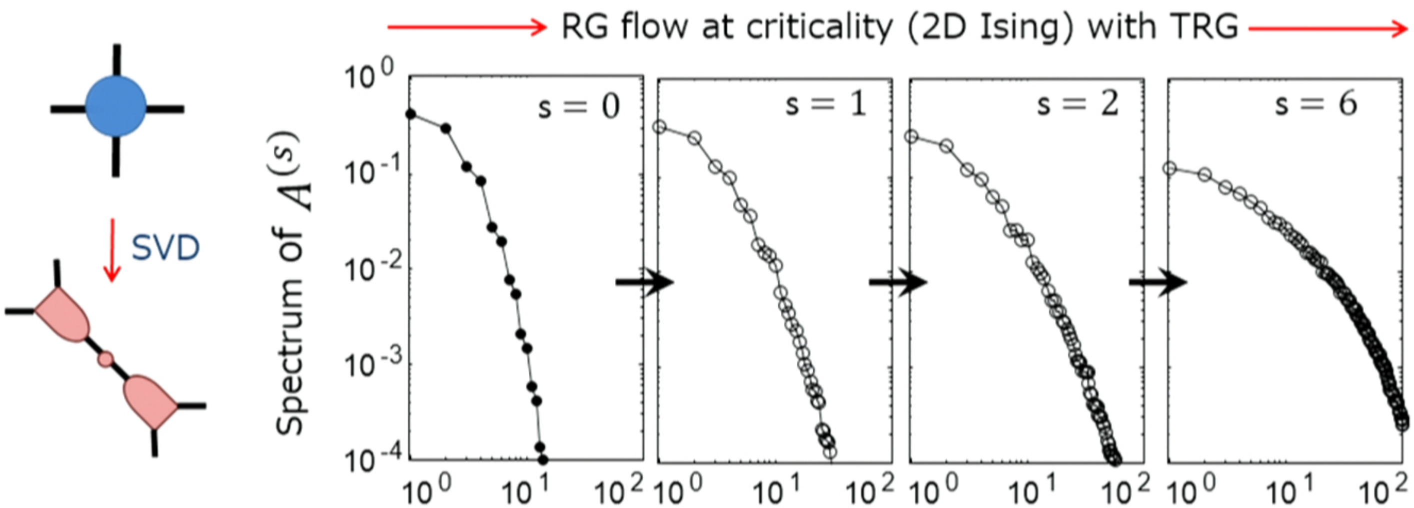
Tensor Renormalization Group (LN-TRG)

Levin, Nave (2006)



Tensor Renormalization Group (LN-TRG)

RG flow in the space of tensors: $A^{(0)} \to A^{(1)} \to A^{(2)} \to \cdots \to A^{(s)} \to \cdots$

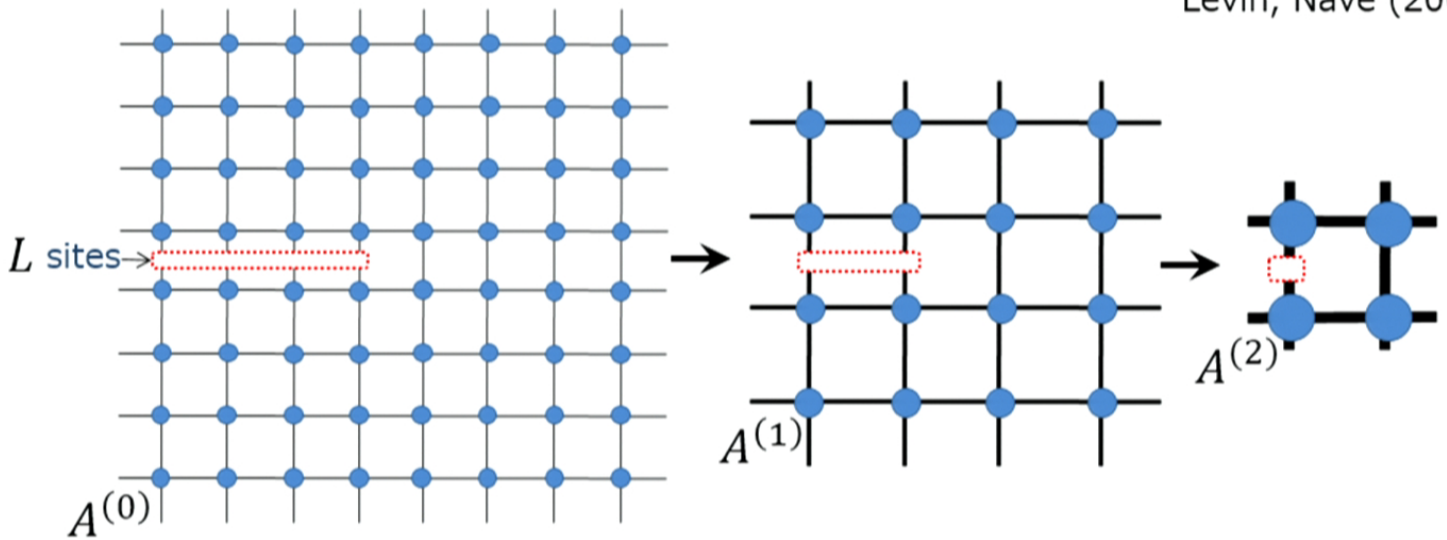


Bond dimension χ required for truncation error < 10^{-3}: ~10 → ~20 → ~40 → >100

Spectra become increasingly flat with each RG step! ✗

Tensor Renormalization Group (LN-TRG)

Levin, Nave (2006)



Is LN-TRG a good RG transformation for critical systems?

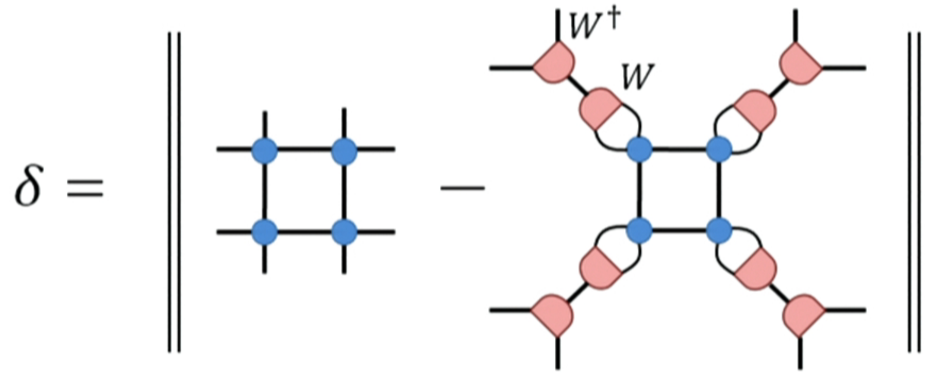
- need to grow bond dimension χ with each RG step
- does not converge to scale-invariant fixed point

Can be understood logarithmic scaling of entanglement entropy in 1D quantum systems: $S_L = \frac{c}{3}\log(L) + k$

Tensor Renormalization Group (LN-TRG)

Levin, Nave (2006)

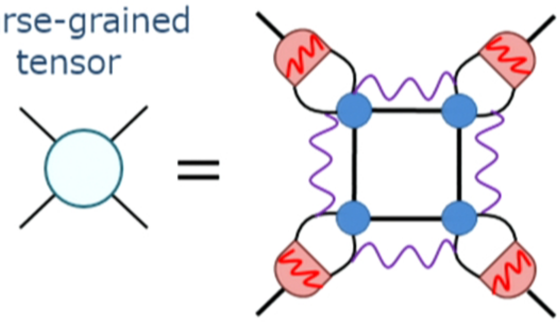
basic step of LN-TRG
(projective truncation):



- isometries remove some (but not all!) short-ranged correlated degrees of freedom
- LN-TRG fails to remove some short-ranged correlations, which propagate to next length scale

Example: corner-double line (CDL) tensors

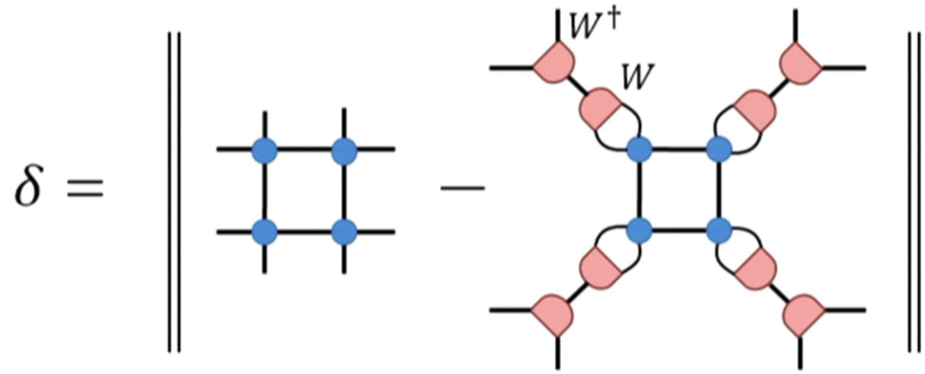
coarse-grained tensor



Tensor Renormalization Group (LN-TRG)

Levin, Nave (2006)

basic step of LN-TRG
(projective truncation):



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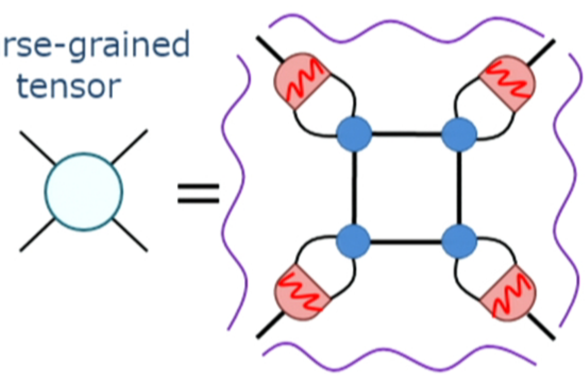
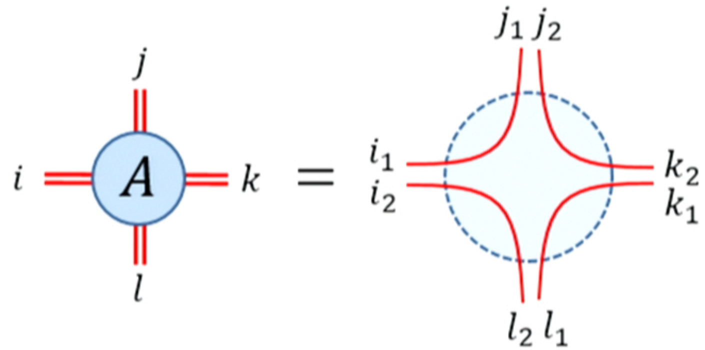


Fixed points of LN-TRG



Imagine "A" is a special tensor such that each index can be decomposed as a product of smaller indices,

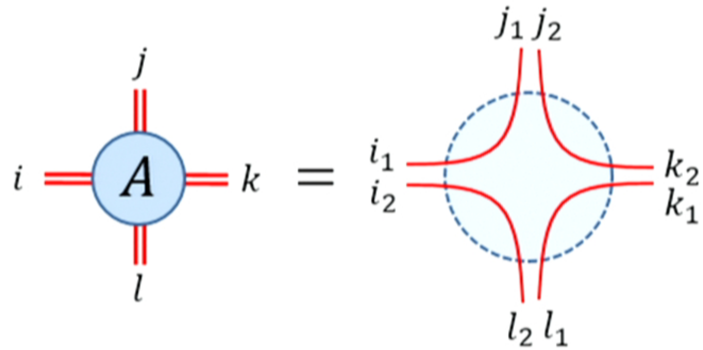
$$A_{ijkl} = A_{(i_1 i_2)(j_1 j_2)(k_1 k_2)(l_1 l_2)}$$

such that certain pairs of indices are perfectly correlated:

$$A_{(i_1 i_2)(j_1 j_2)(k_1 k_2)(l_1 l_2)} \equiv \delta_{i_1 j_1} \delta_{j_2 k_2} \delta_{k_1 l_1} \delta_{l_2 i_2}$$

These are called corner double line (CDL) tensors. CDL tensors are fixed points of TRG.

Fixed points of LN-TRG



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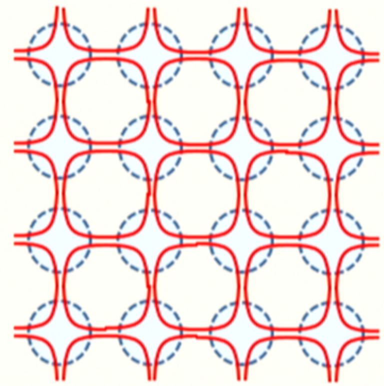
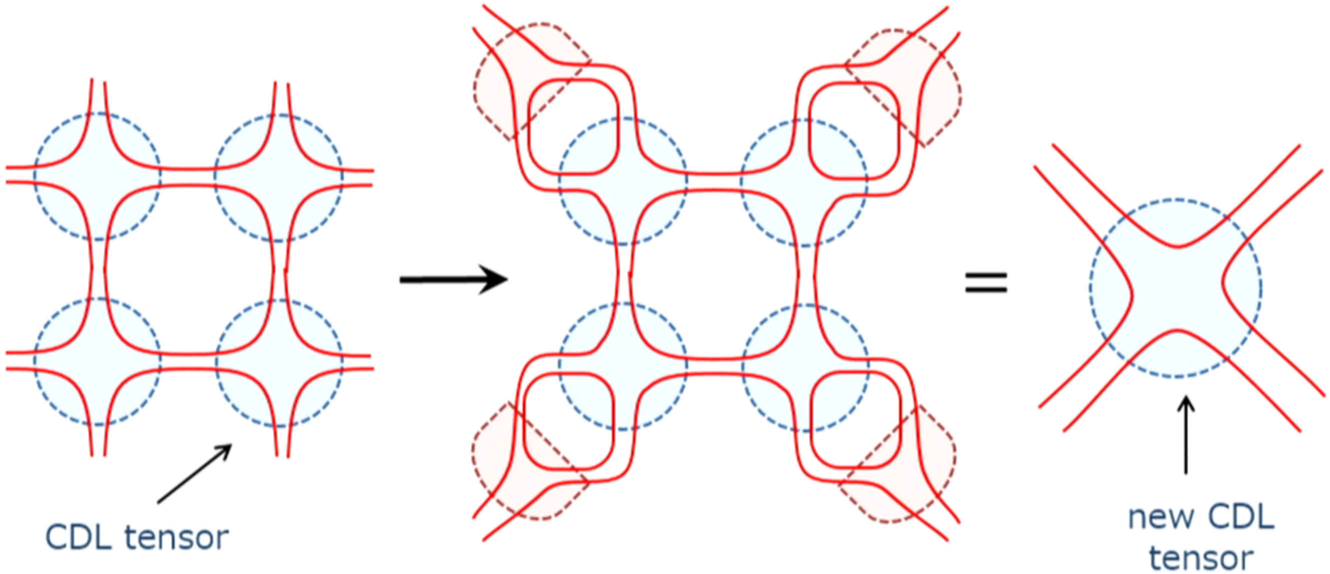
These are called corner double line (CDL) tensors. CDL tensors are fixed points of TRG.

Partition function built from CDL tensors represents a state with short-ranged correlations



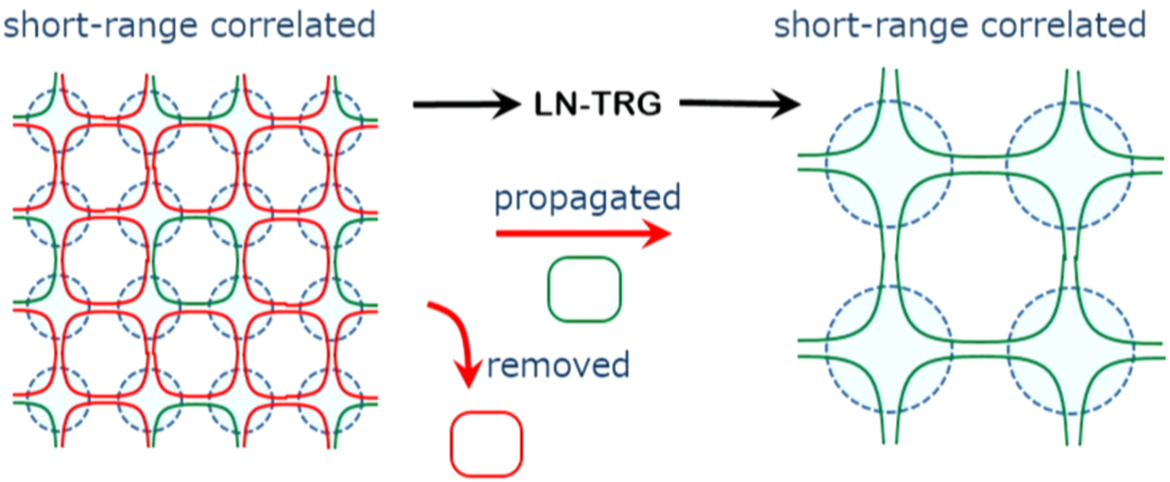
Fixed points of LN-TRG

single iteration of LN-TRG:



Some short-ranged always
correlations remain under LN-TRG!

Fixed points of LN-TRG



TRG removes some short ranged correlations, but...
others are artificially promoted to the next length scale

Accumulation of short-ranged details causes computational breakdown when near (or at) criticality

Is there some way to 'fix' tensor renormalization such that all short-ranged correlations are addressed?

Outline: Tensor Network Renormalization

The set-up: Representation of partition functions and path integrals as tensor networks

Previous approaches: Levin and Nave's Tensor Renormalization Group (LN-TRG), conceptual and computation problems.

New approach: Tensor network renormalization (TNR): proper removal of all short-ranged degrees of freedom via disentanglers

Benchmark results

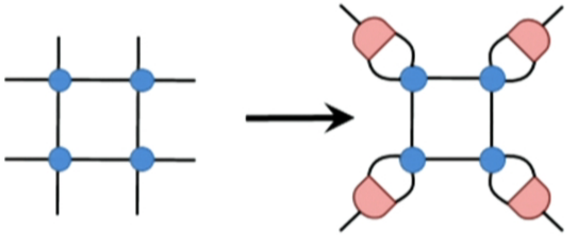
Extensions

Tensor Network Renormalization

arXiv:1412.0732

previous RG schemes for tensor networks based upon **blocking:**

i.e. isometries responsible for combining and truncating indices



but blocking alone fails to remove short-ranged degrees of freedom...

...can one incorporate some form of unitary disentangling into a tensor RG scheme?

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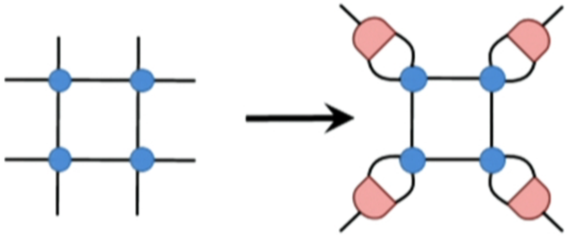

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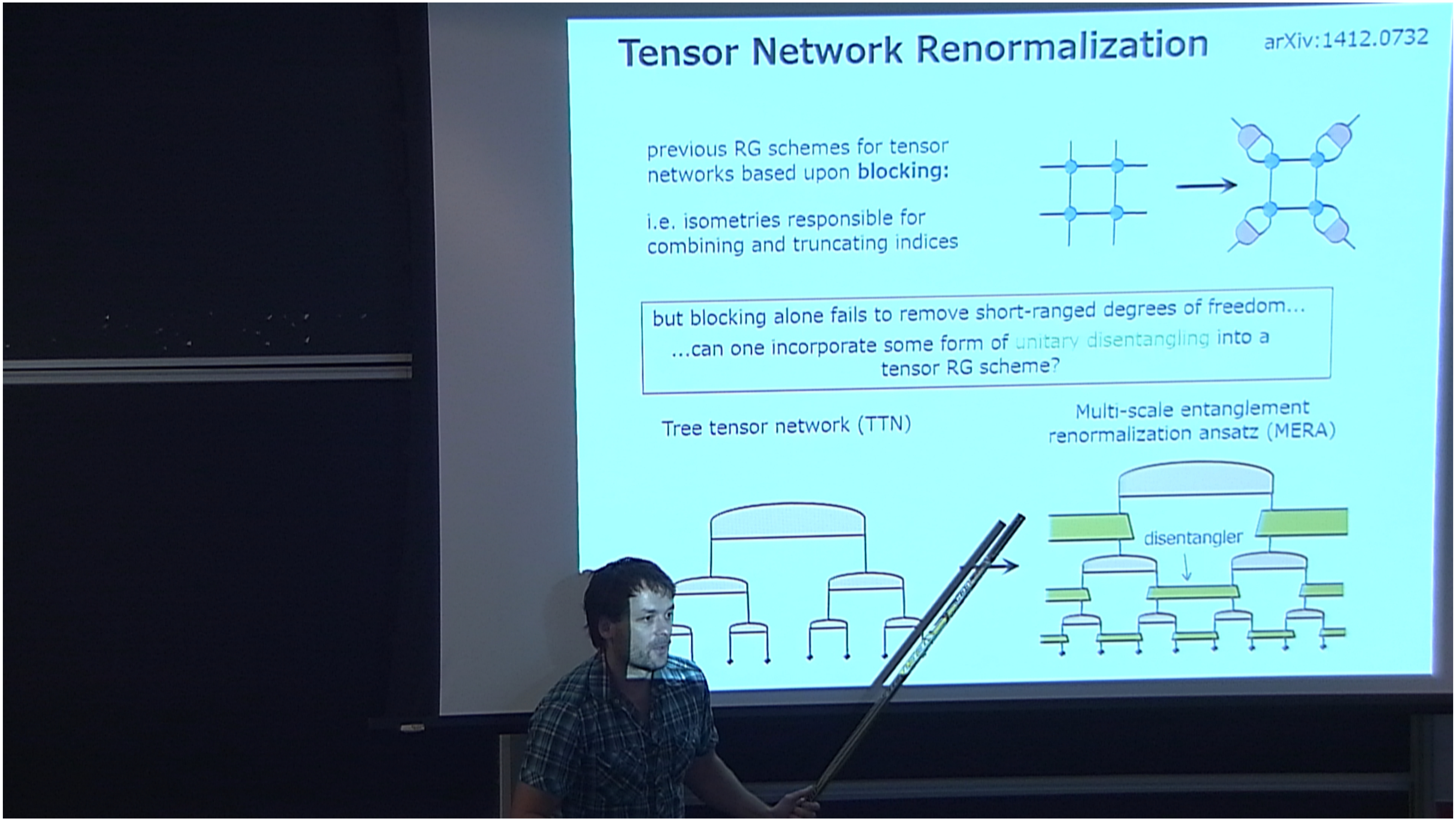
Tensor Network Renormalization
arXiv:1412.0732
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i.e. isometries responsible for combining and truncating indices
but blocking alone fails to remove short-ranged degrees of freedom...
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Tree tensor network (TTN)
Multi-scale entanglement renormalization ansatz (MERA)
disentangler

Tensor Network Renormalization

arXiv:1412.0732

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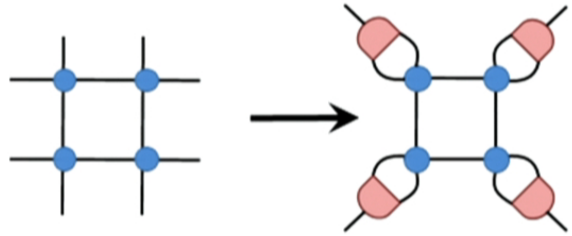

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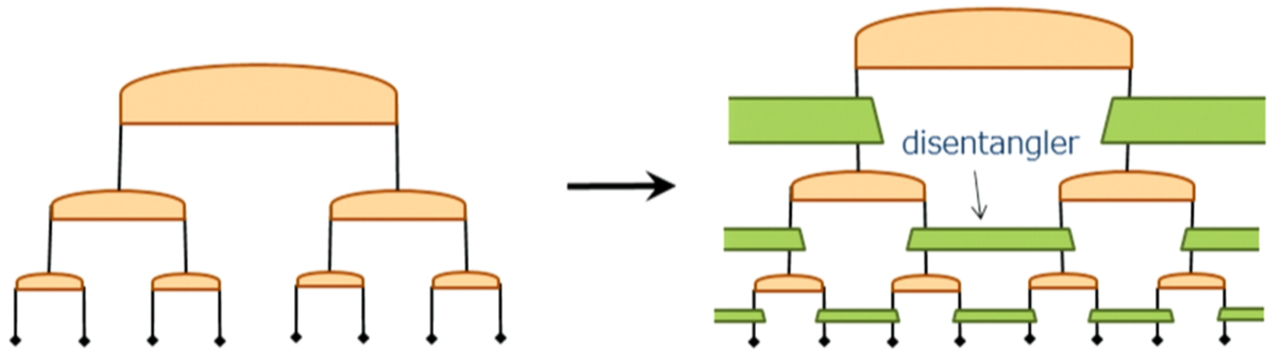
Tree tensor network (TTN)

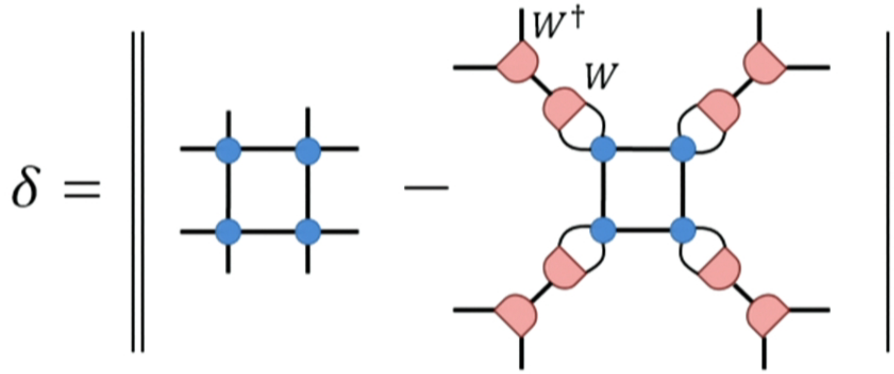
Multi-scale entanglement renormalization ansatz (MERA)



Tensor Network Renormalization

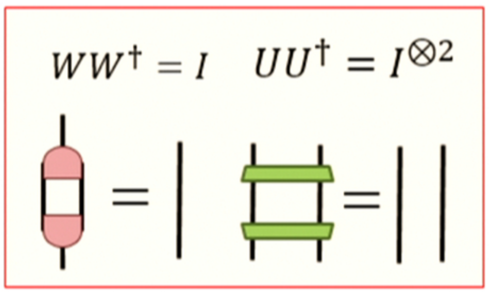
previous approach

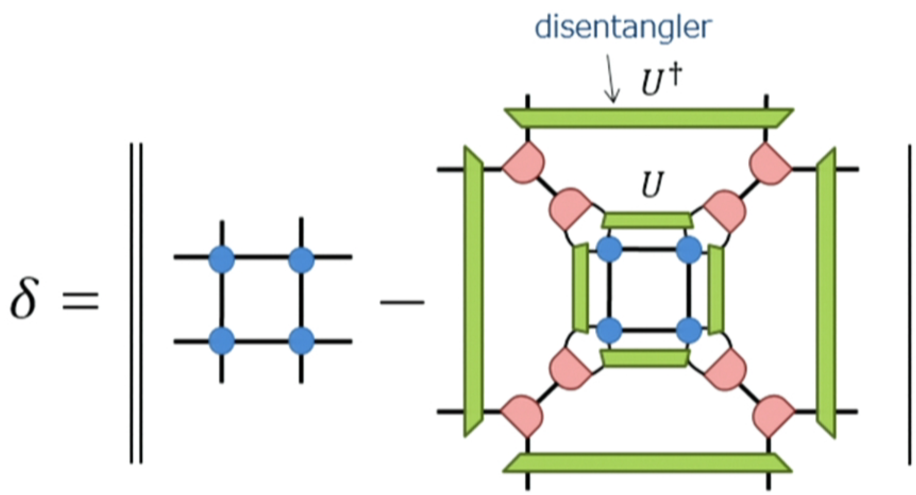
basic step of LN-TRG
(projective truncation):



new approach

basic step of Tensor Network Renormalization
(projective truncation):

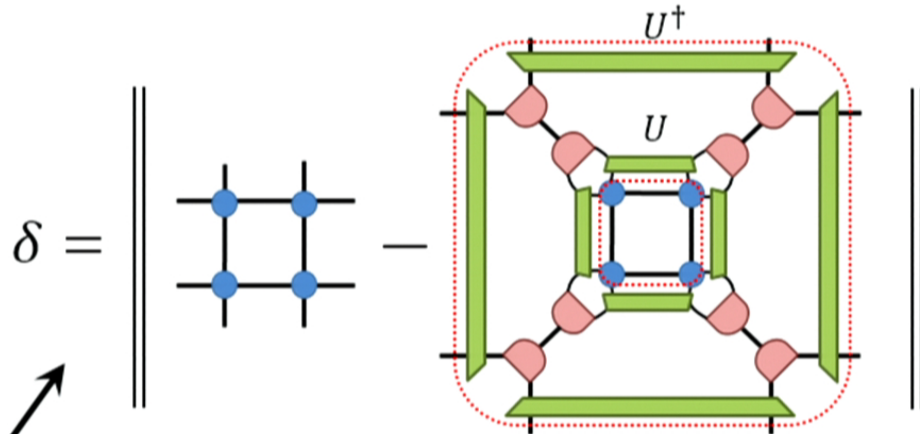




Tensor Network Renormalization

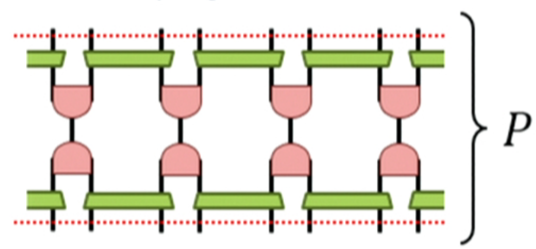
new approach

basic step of Tensor Network Renormalization (projective truncation):



use iterative methods for optimizing isometry W and disentangler U

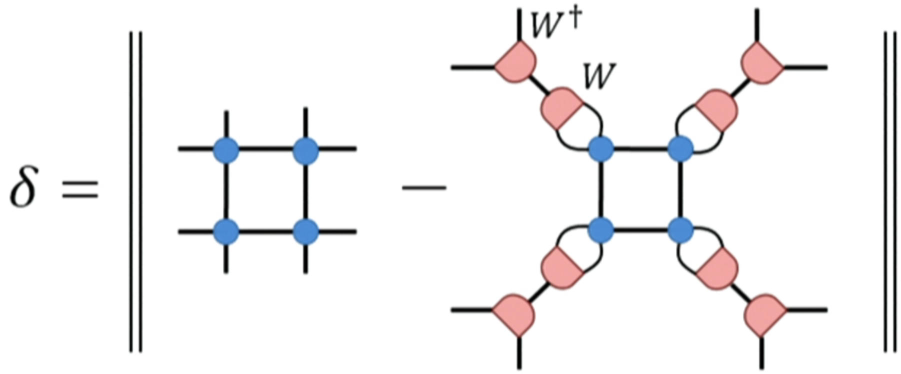
projector



Tensor Network Renormalization

previous approach

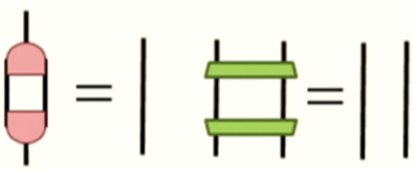
basic step of LN-TRG (projective truncation):

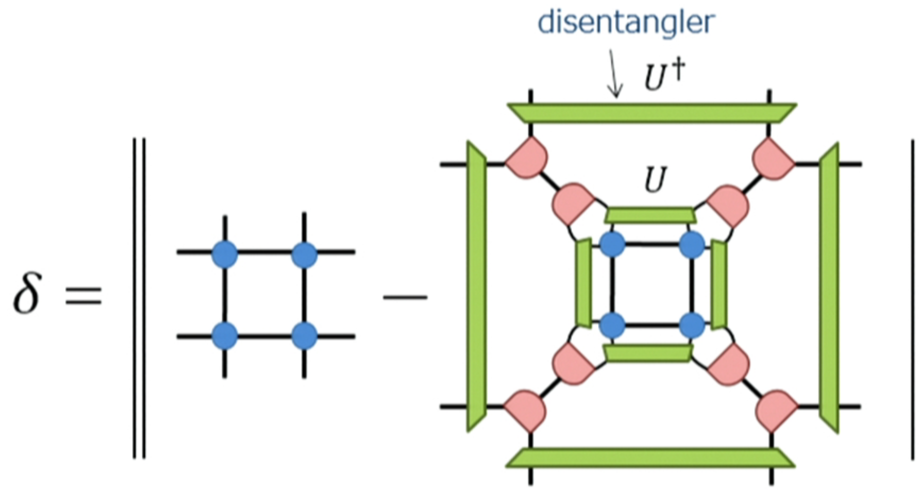


new approach

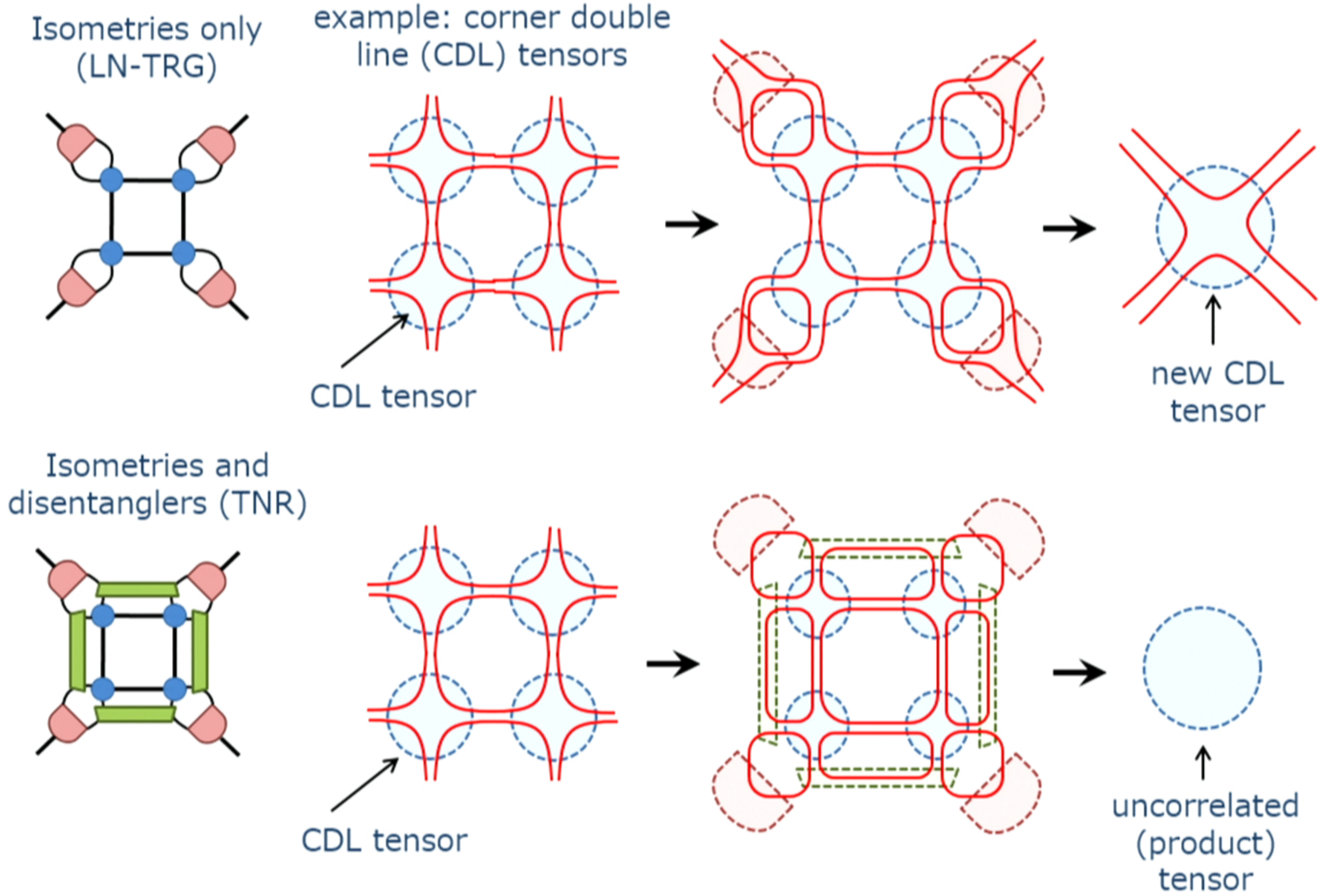
basic step of Tensor Network Renormalization (projective truncation):

$ww^\dagger = I \quad UU^\dagger = I^{\otimes 2}$





Corner double line tensors revisited



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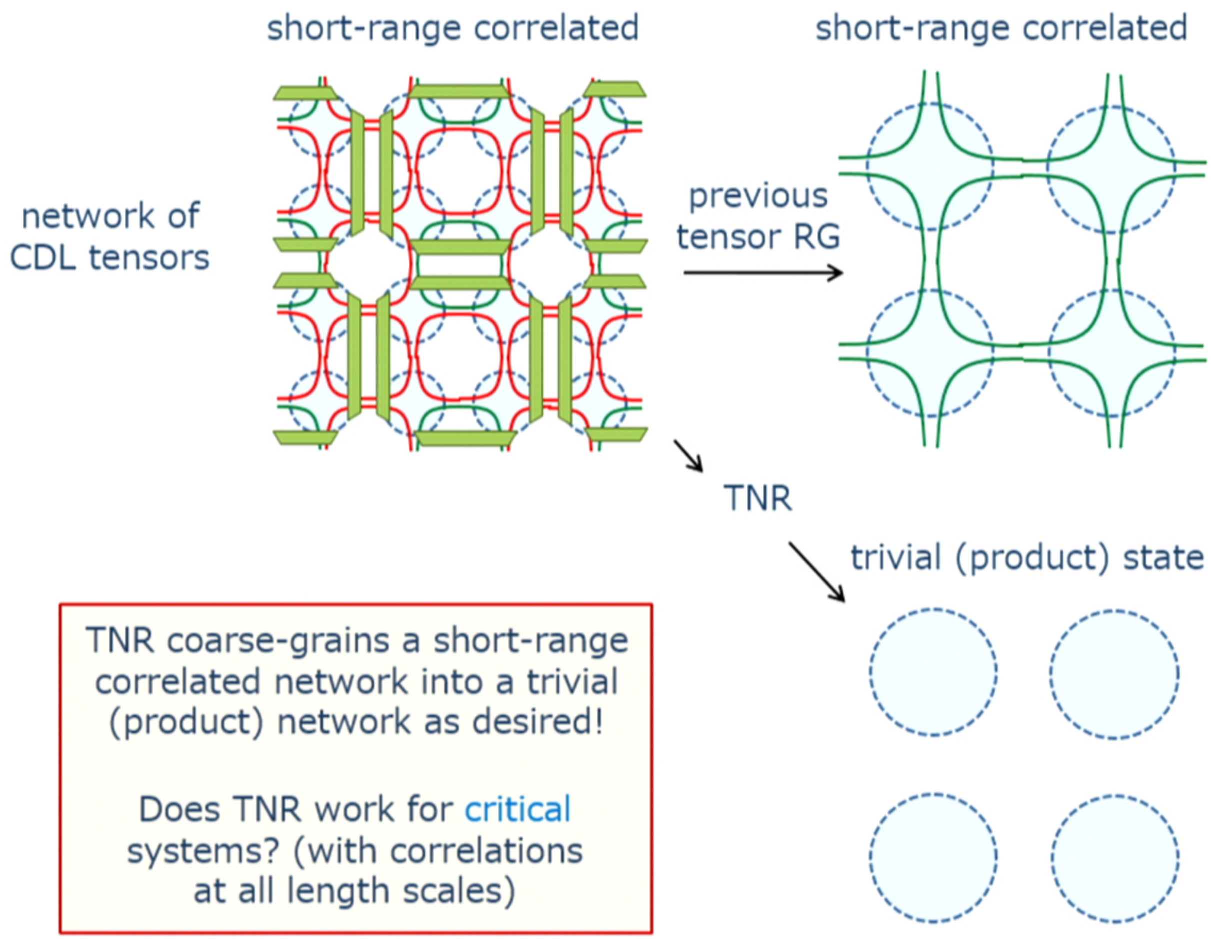
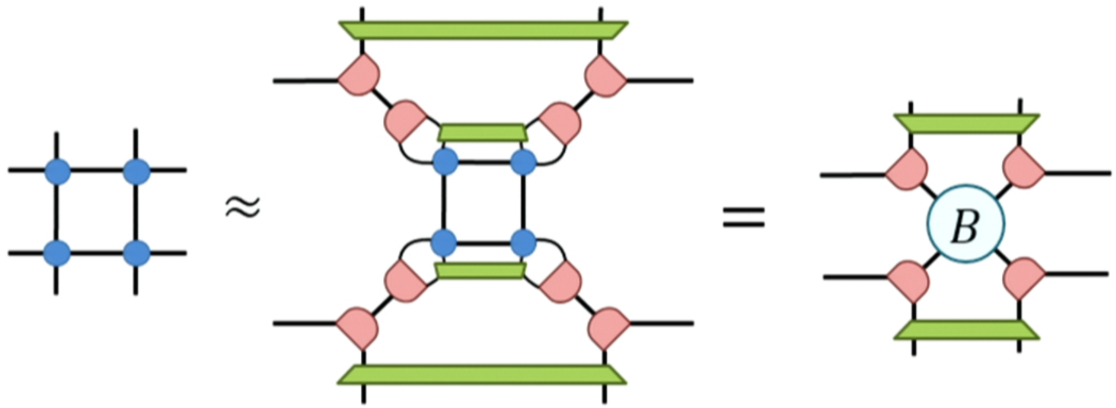


TNR coarse-grains a short-range correlated network into a trivial (product) network as desired!

Does TNR work for critical systems? (with correlations at all length scales)

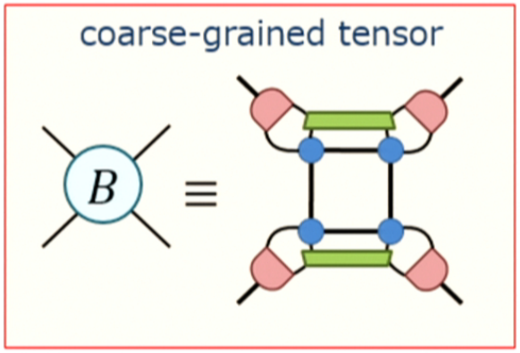
Tensor Network Renormalization

key coarse-graining step of TNR:

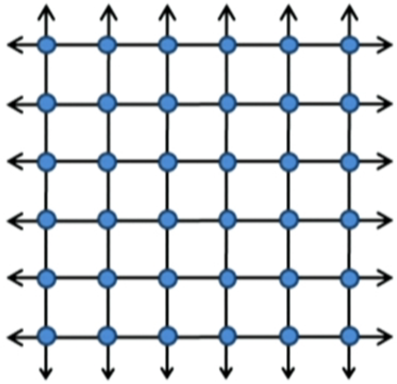


Simplification: disentangling is expensive... are all disentanglers necessary? No!

Can still address all short-range degrees of freedom...



TNR iteration



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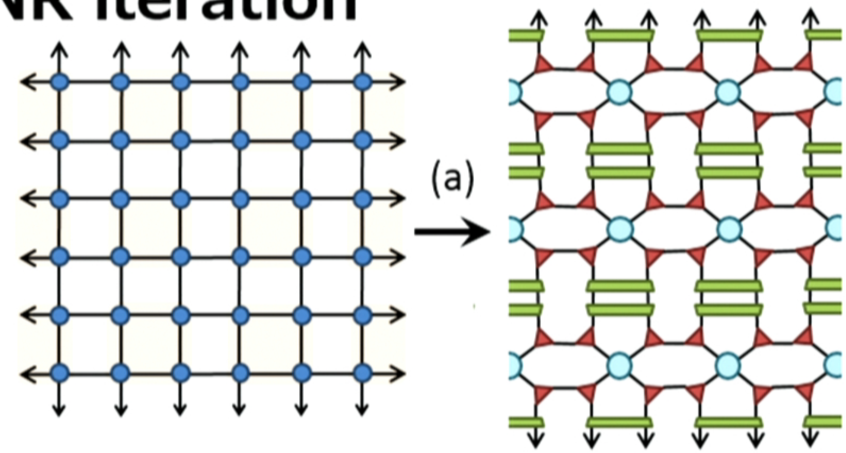

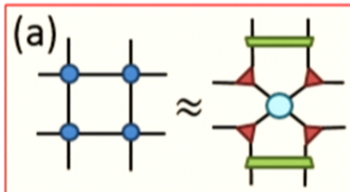

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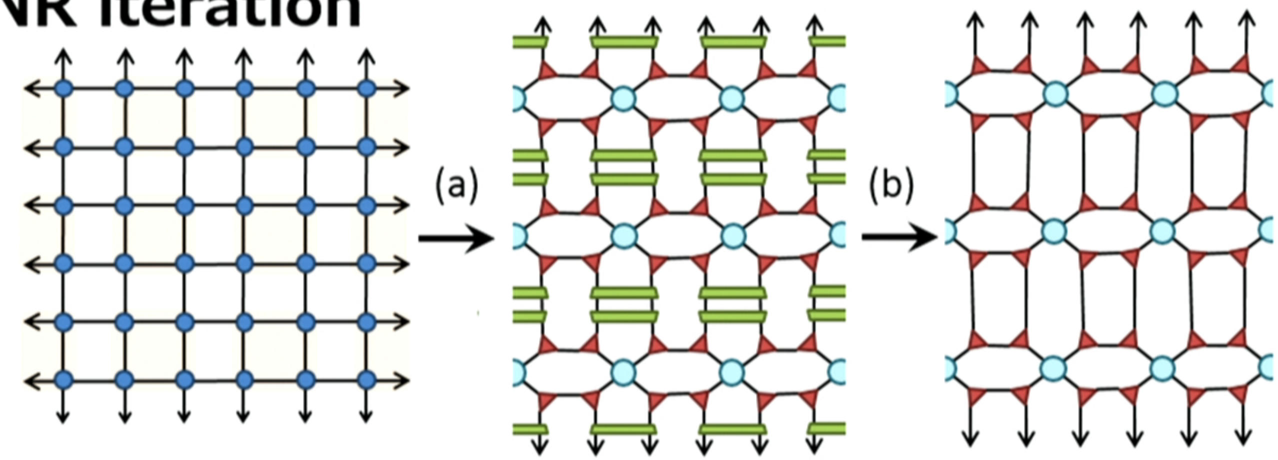
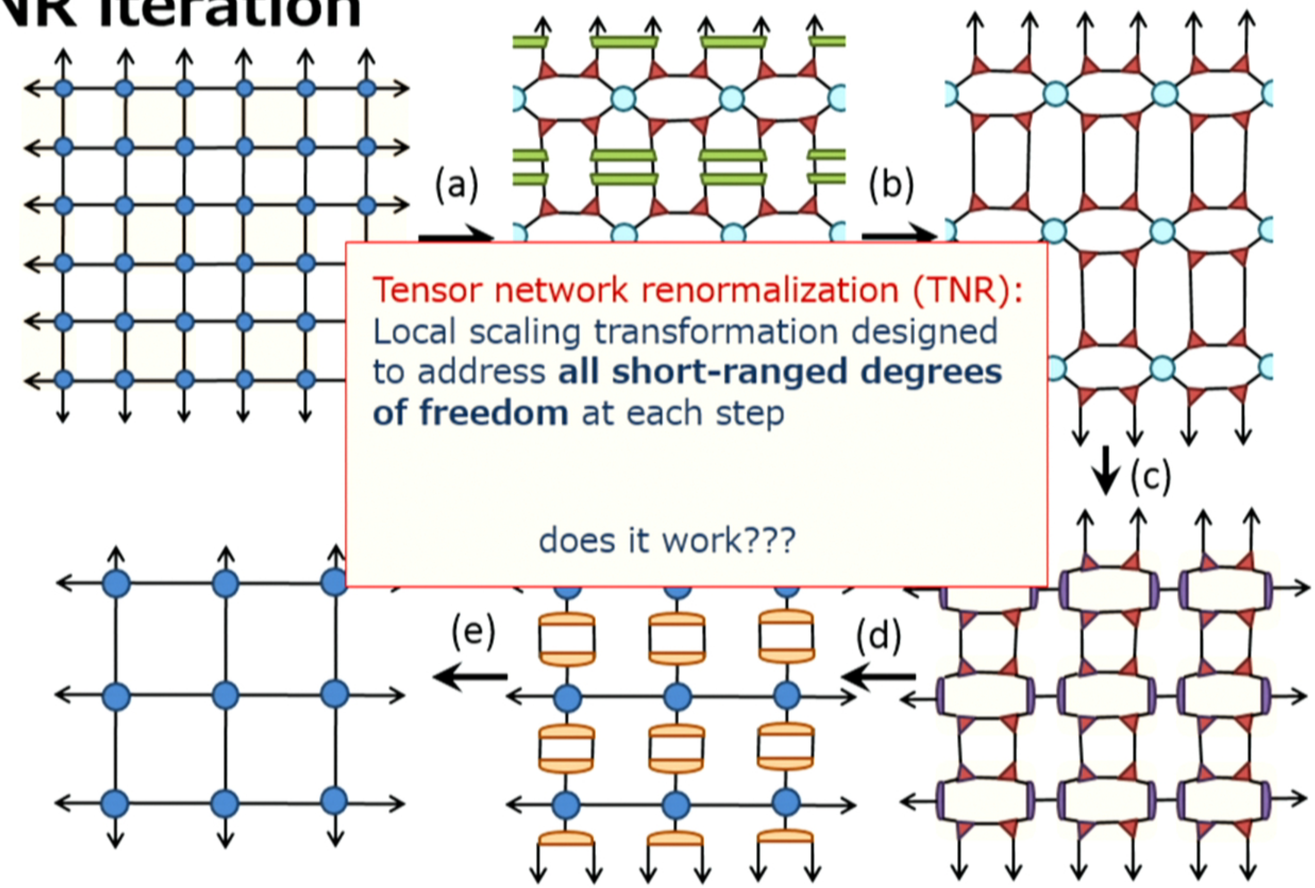
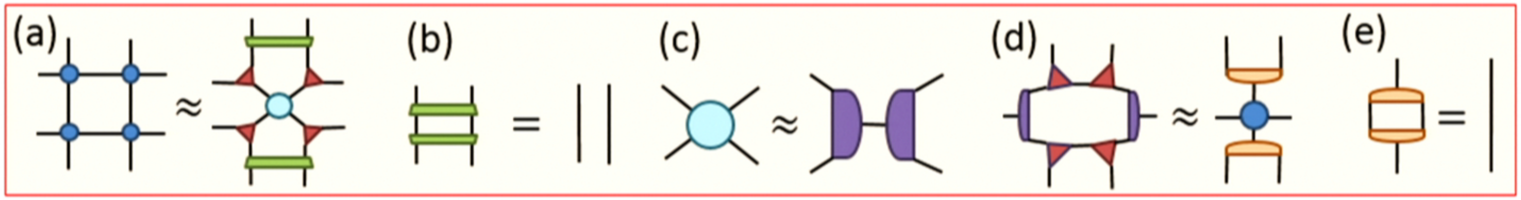


TNR iteration



Tensor network renormalization (TNR):
Local scaling transformation designed to address **all short-ranged degrees of freedom** at each step

does it work???

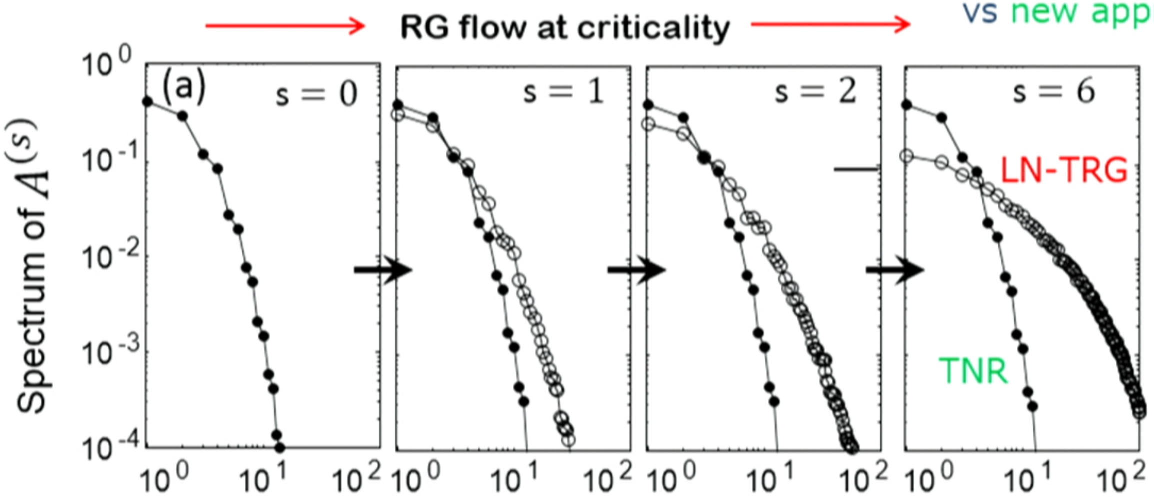


Sustainable RG flow

RG flow in the space of tensors: $A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{(s)} \rightarrow \cdots$

Does TRG give a sustainable RG flow?

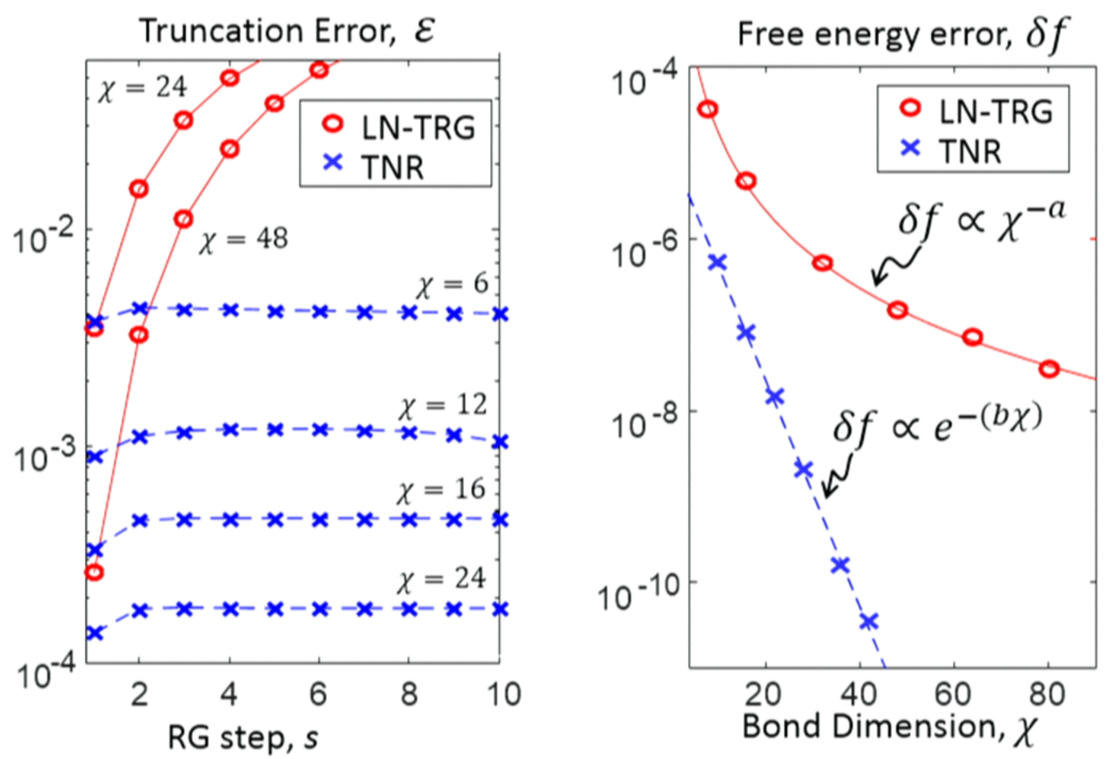
Old approach (LN-TRG) vs new approach (TNR)



LN-TRG: Spectra become increasingly flat with each RG step! ✗

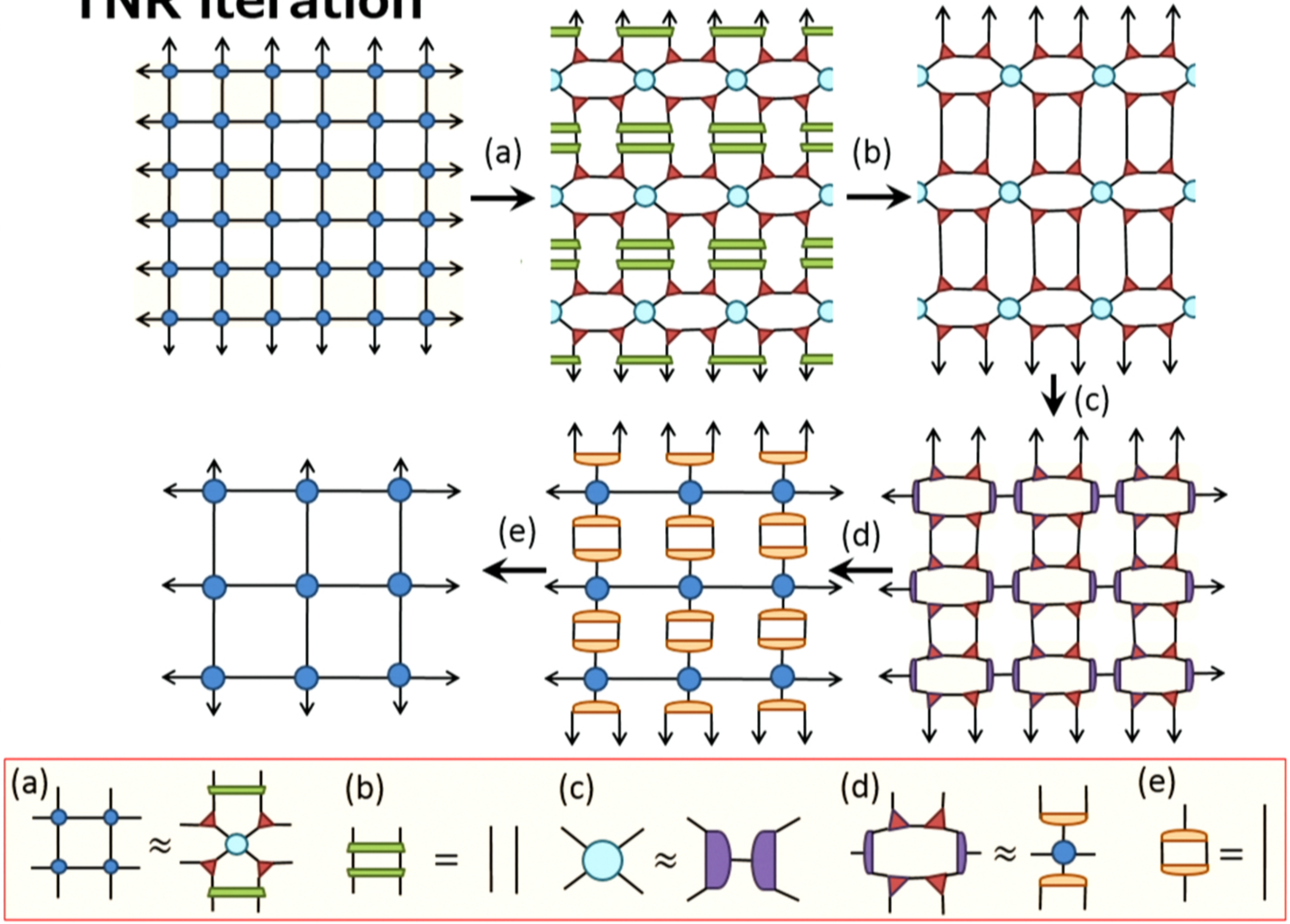
TNR: Spectra unchanged over many RG steps ✓

Benchmark results, 2D classical Ising at criticality



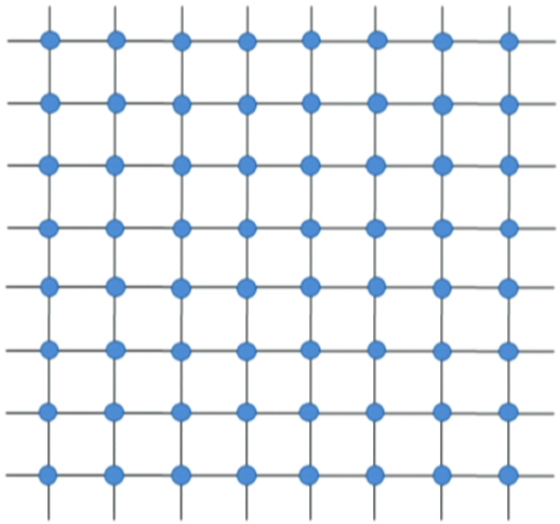
Old approach (LN-TRG):	truncation error **grows** with RG step (fixed χ)	free energy converges **polynomially** in χ
New approach (TNR):	truncation error **remains constant**	free energy converges **exponentially** in χ

TNR iteration



Overview

RG flow in the space of tensors: $A^{(0)} \to A^{(1)} \to A^{(2)} \to \cdots \to A^{(s)} \to \cdots$



partition function

contraction → Z

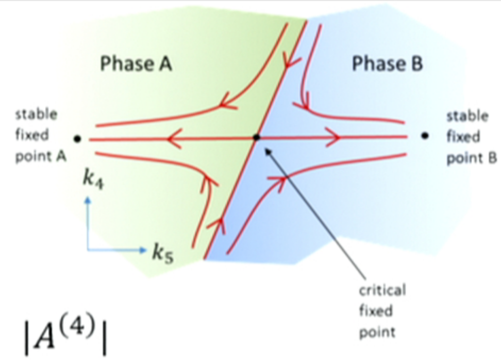
free energy (per site) $f = \frac{-kT\log(Z)}{N}$

Practical goal: efficient and accurate contraction of a tensor network to a scalar

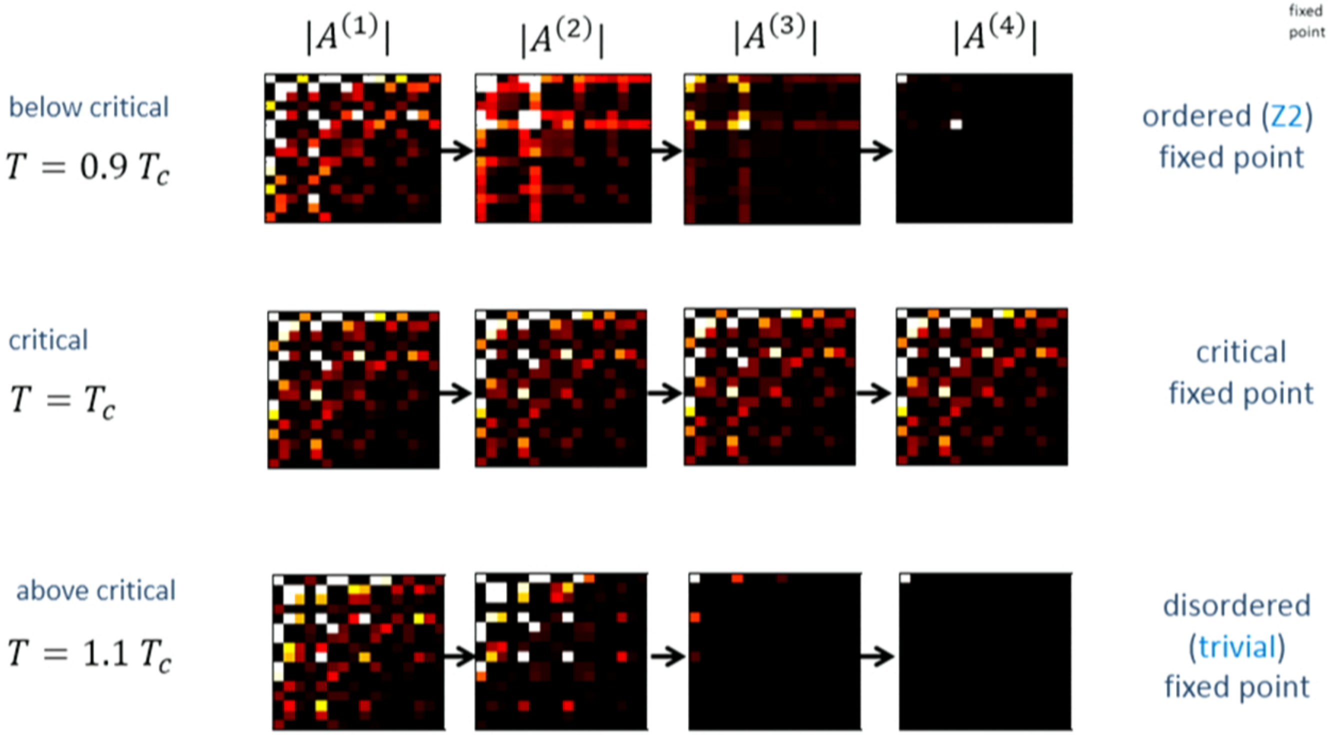
Conceptual goal: achieve a proper RG flow

TNR: proper RG flow

2D classical Ising, $\chi = 6$ TNR

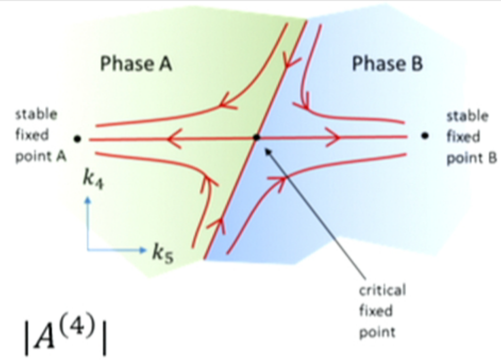


RG flow in the space of tensors: $A^{(0)} \to A^{(1)} \to A^{(2)} \to \cdots \to A^{\mathrm{fp}}$



TNR: proper RG flow

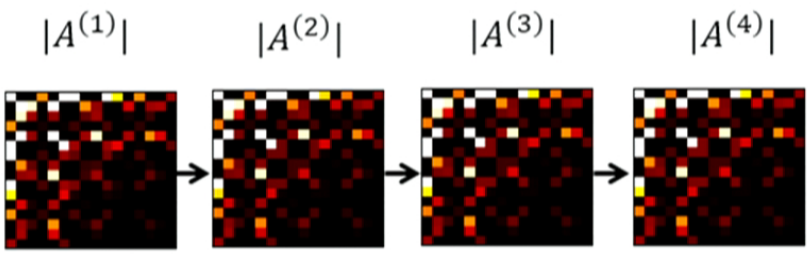
2D classical Ising



RG flow in the space of tensors: $A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{\text{fp}}$

critical

$T = T_c$



critical fixed point

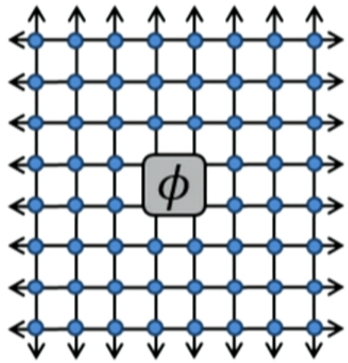
Previous approach (LN-TRG): RG flow does not converge at criticality

New approach (TNR): RG flow converges at criticality (to arbitrarily high precision)

Is the critical fixed point RG map given by TNR a good approximation to the Ising CFT???

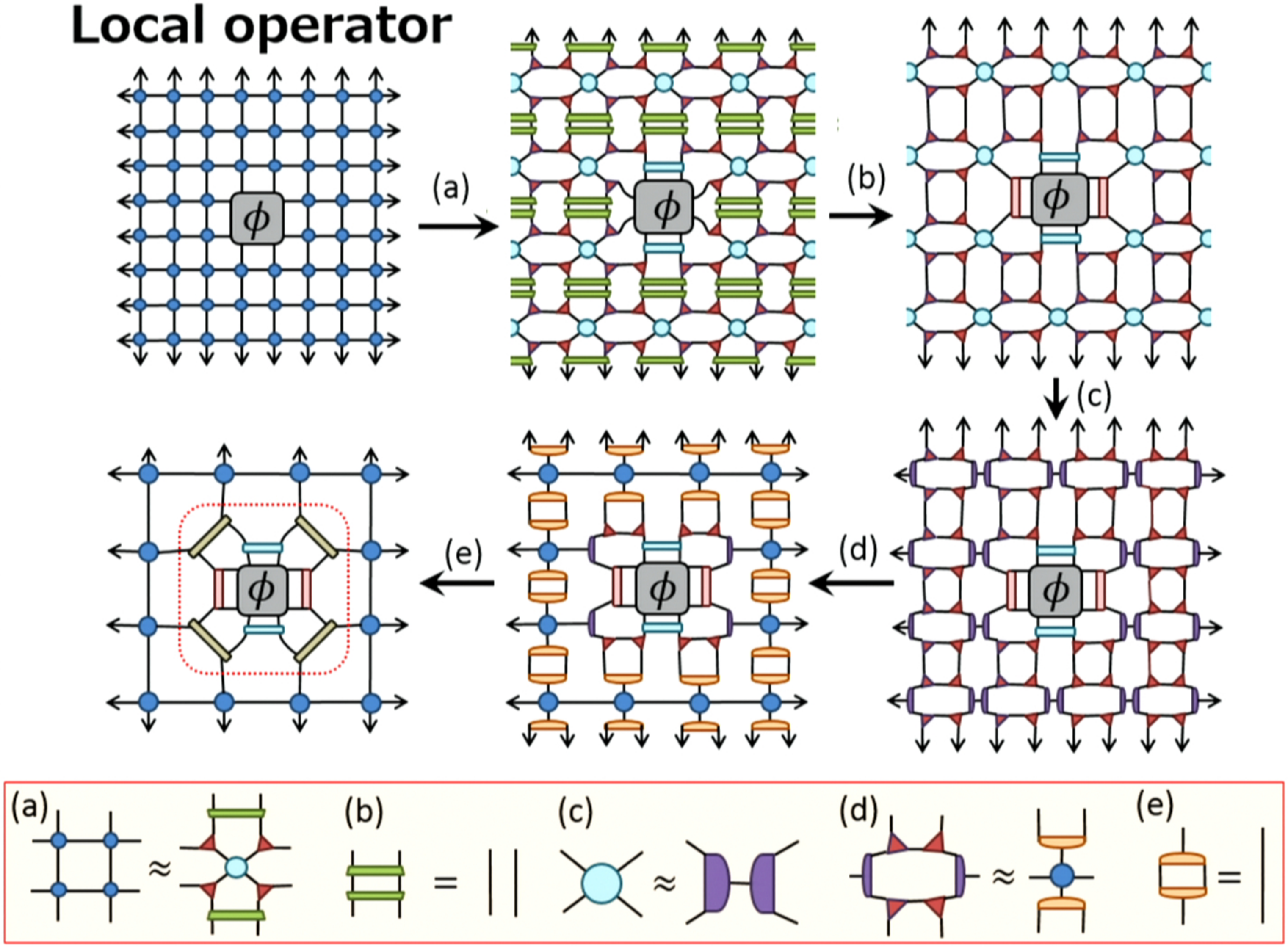
...it should be if the truncation error has been kept small, but let's check anyway

Local operator

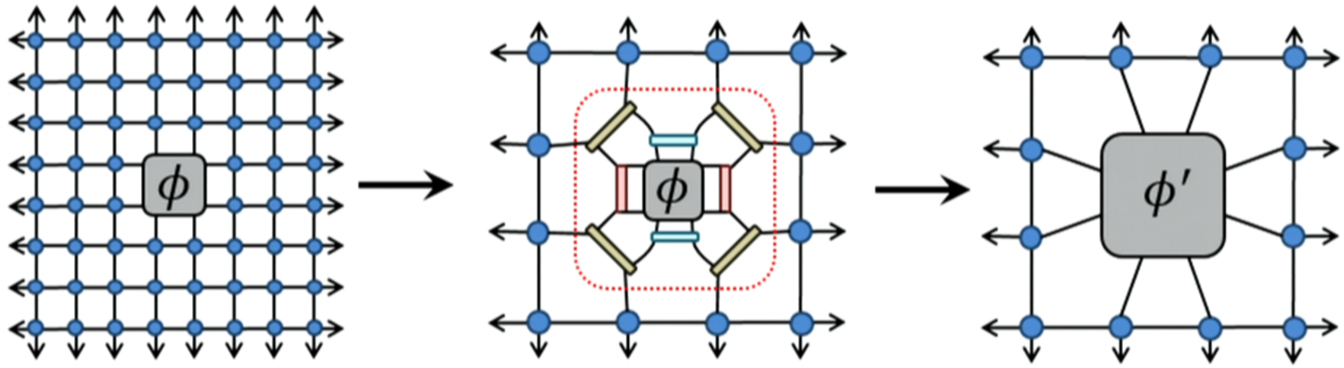


how does a local operator (inserted as in impurity) transform under the fixed point RG map?

Local operator



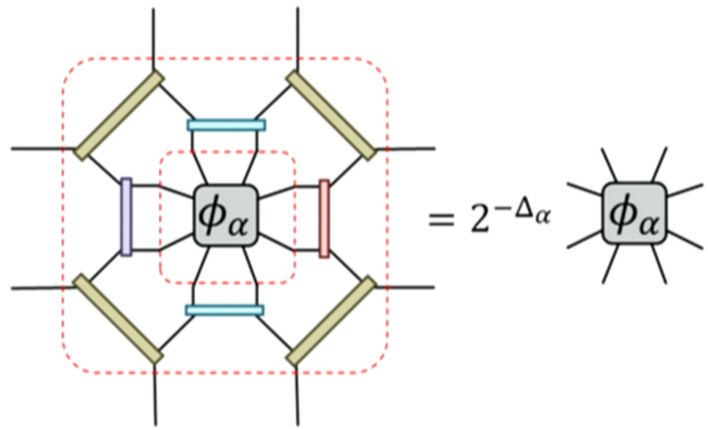
Local operator



local operators coarse-
grain to local operators!

Local impurity

diagonalize transfer operator
to find scaling operators



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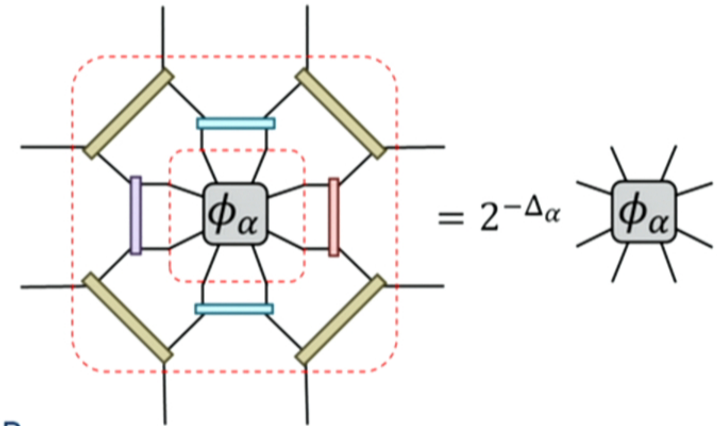
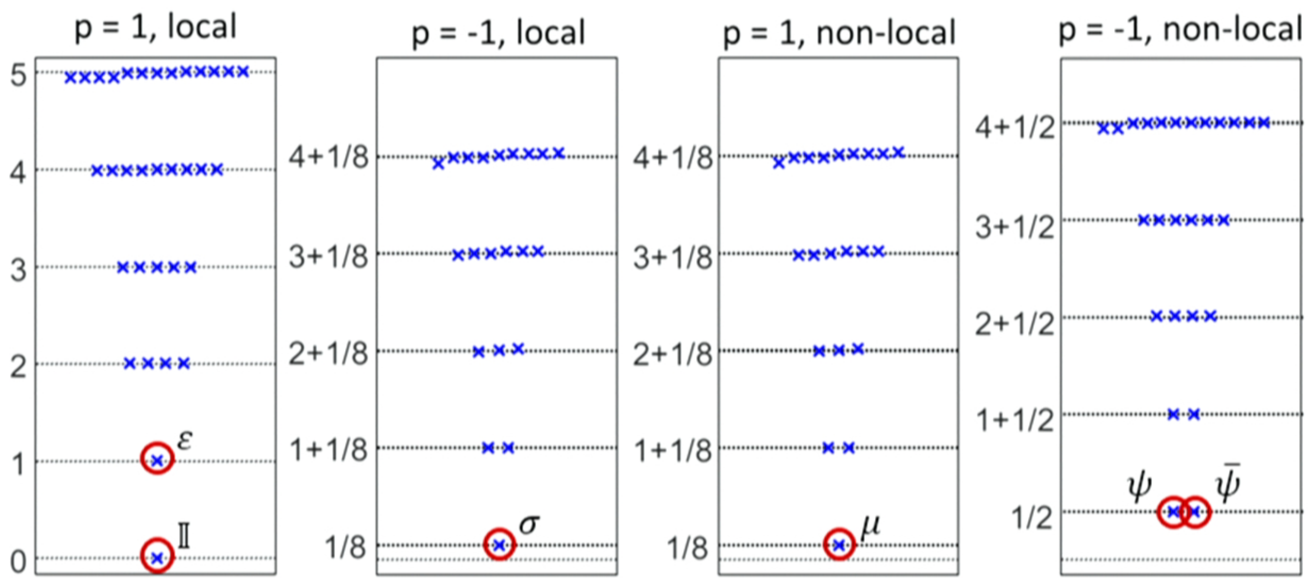
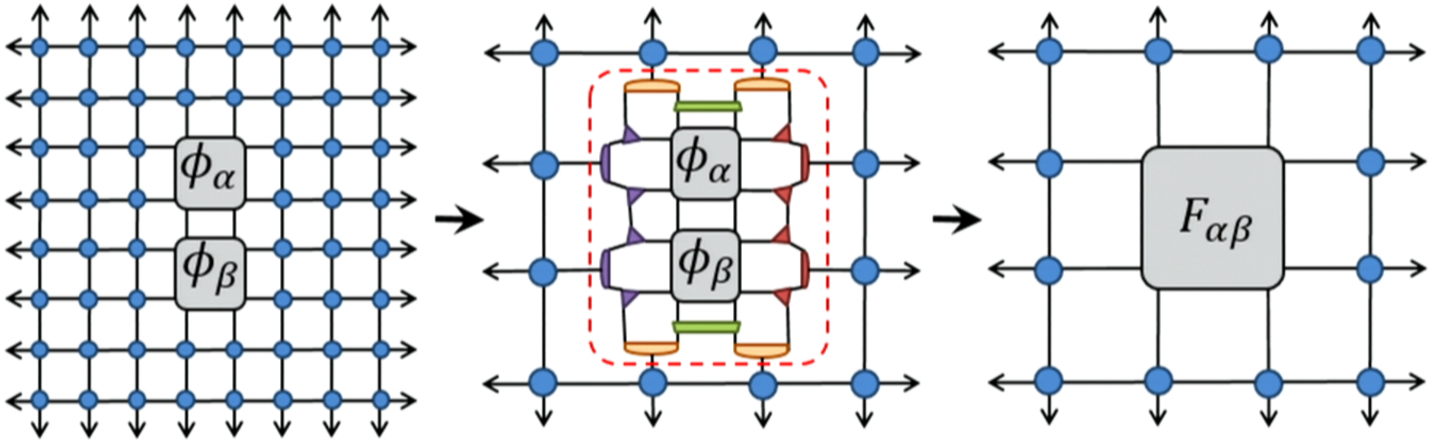


2D classical Ising at criticality, $\chi = 6$ TNR



Fusion of operators



expand fused operator in basis of scaling operators:

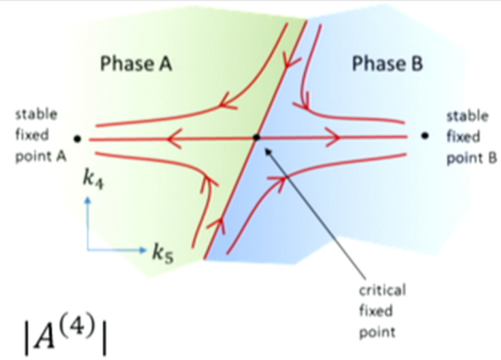
$$F_{\alpha\beta} = \sum_{\gamma} C_{\alpha\beta\gamma}\, \phi_\gamma$$

operator product expansion (OPE) coefficients

	exact	$\chi = 6$ TNR
$C_{\varepsilon,\sigma,\sigma}$	$1/2$	0.50105
$C_{\varepsilon,\mu,\mu}$	$-1/2$	-0.49941
$C_{\psi,\mu,\sigma}$	$(1-i)/2$	$0.49957 - 0.49957i$
$C_{\bar{\psi},\mu,\sigma}$	$(1+i)/2$	$0.49957 + 0.49957i$
$C_{\varepsilon,\psi,\bar{\psi}}$	i	$1.00009i$
$C_{\varepsilon,\bar{\psi},\psi}$	$-i$	$-1.00009i$

TNR: proper RG flow

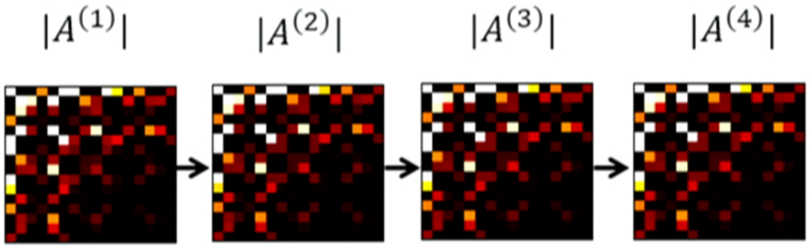
2D classical Ising



RG flow in the space of tensors: $A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{\text{fp}}$

critical

$T = T_c$



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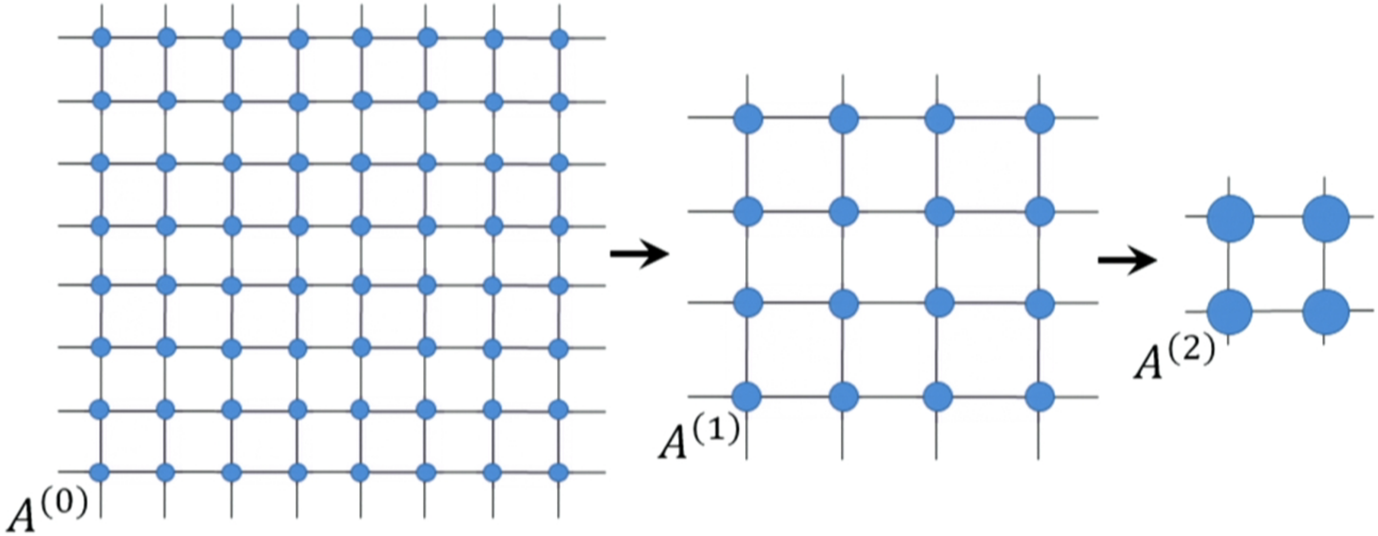
Overview

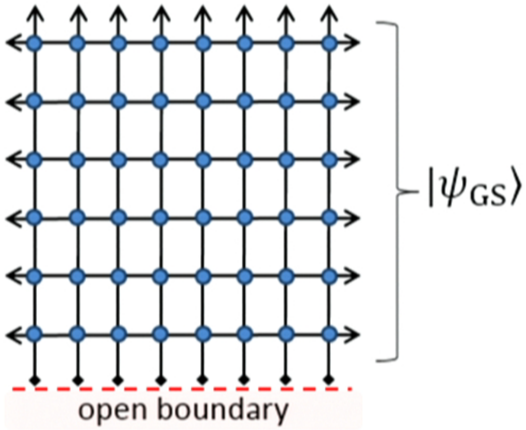
local scale transformations on the lattice:

RG flow in the space of tensors: $A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{(s)} \rightarrow \cdots$



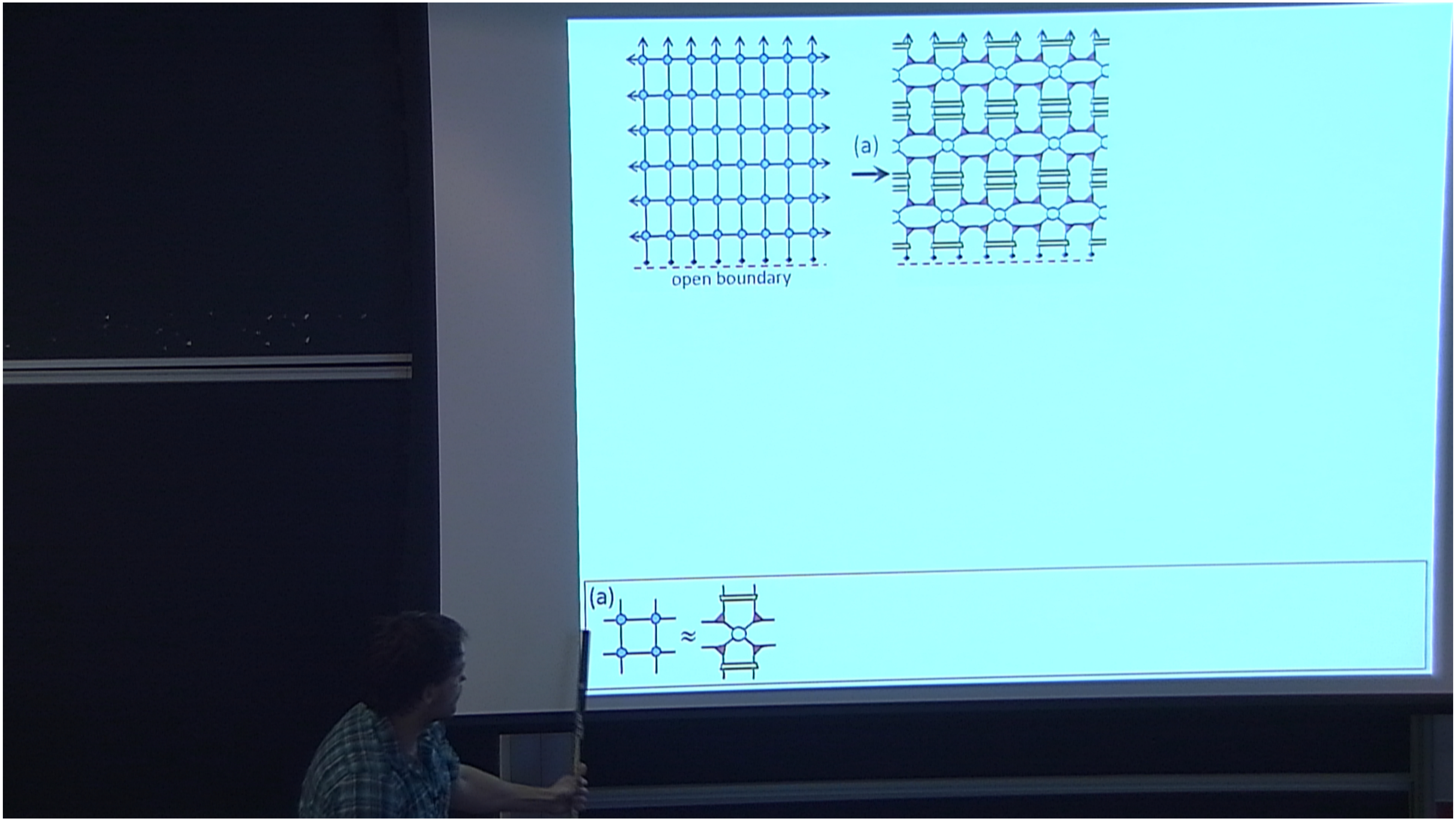
Practical goal: efficient and accurate contraction of a tensor network to a scalar ✓

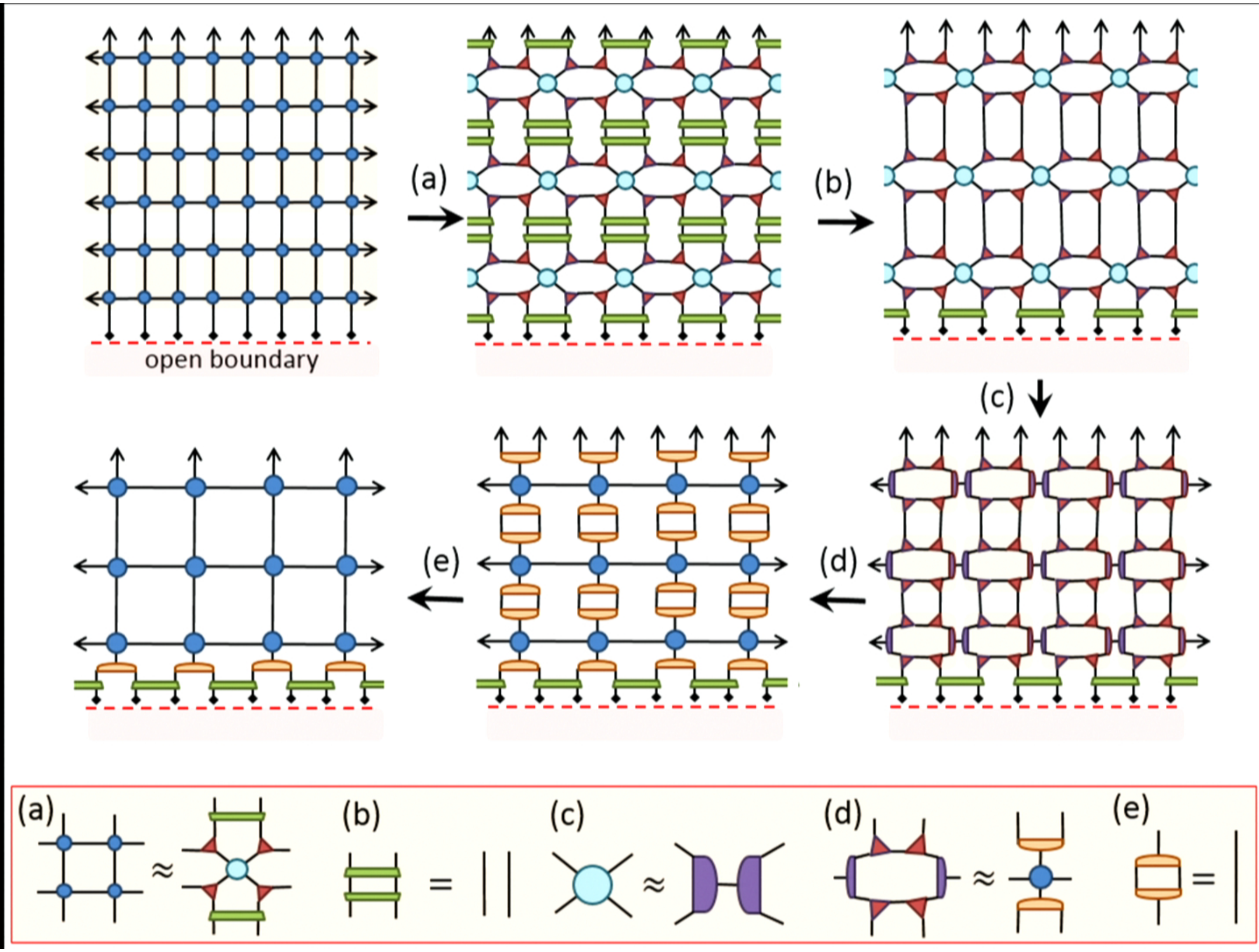
Conceptual goal: achieve a proper RG flow ✓



Euclidean path integral for 1D quantum system

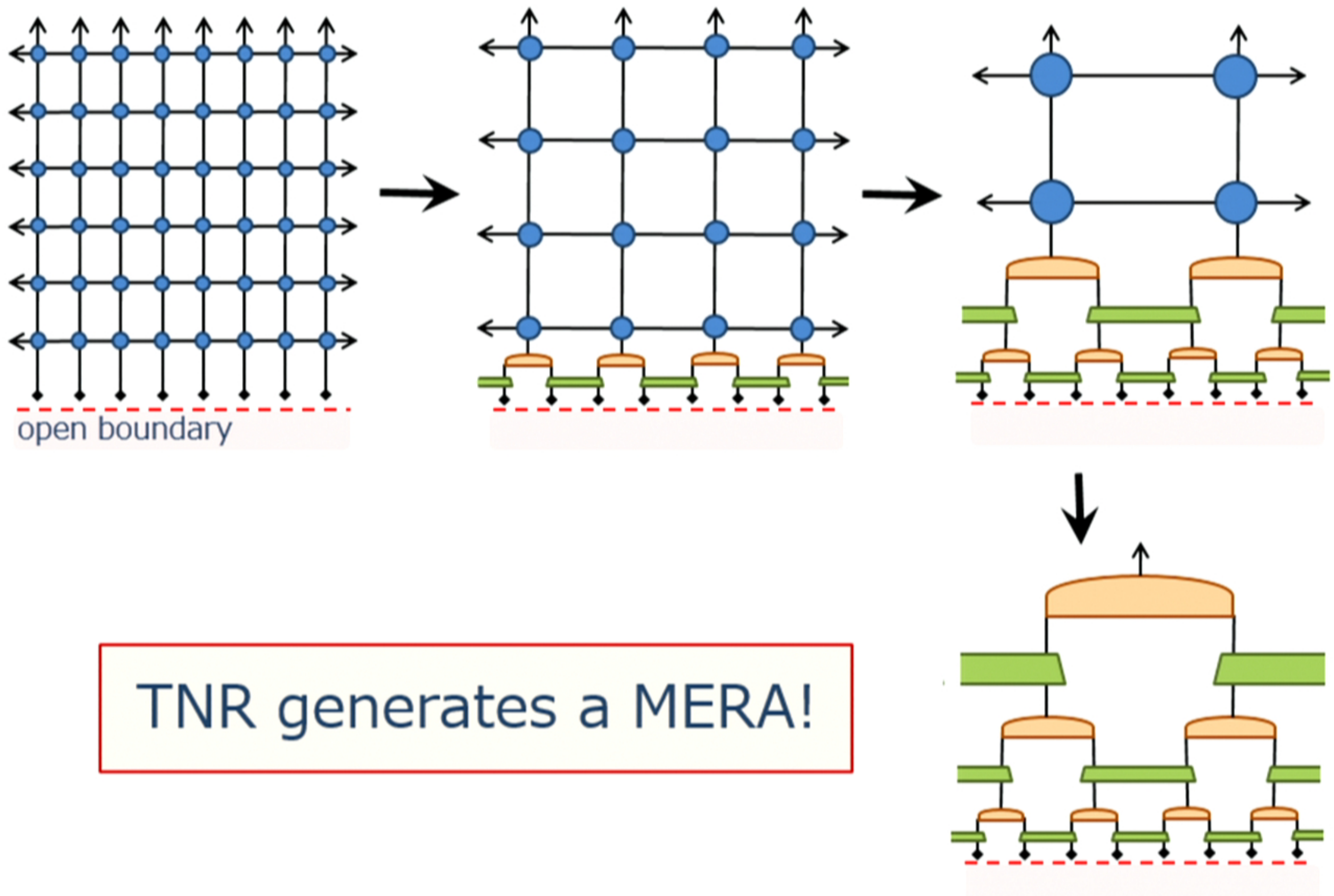
Apply TNR to the half plane???


open boundary
(a)
(a)
≈



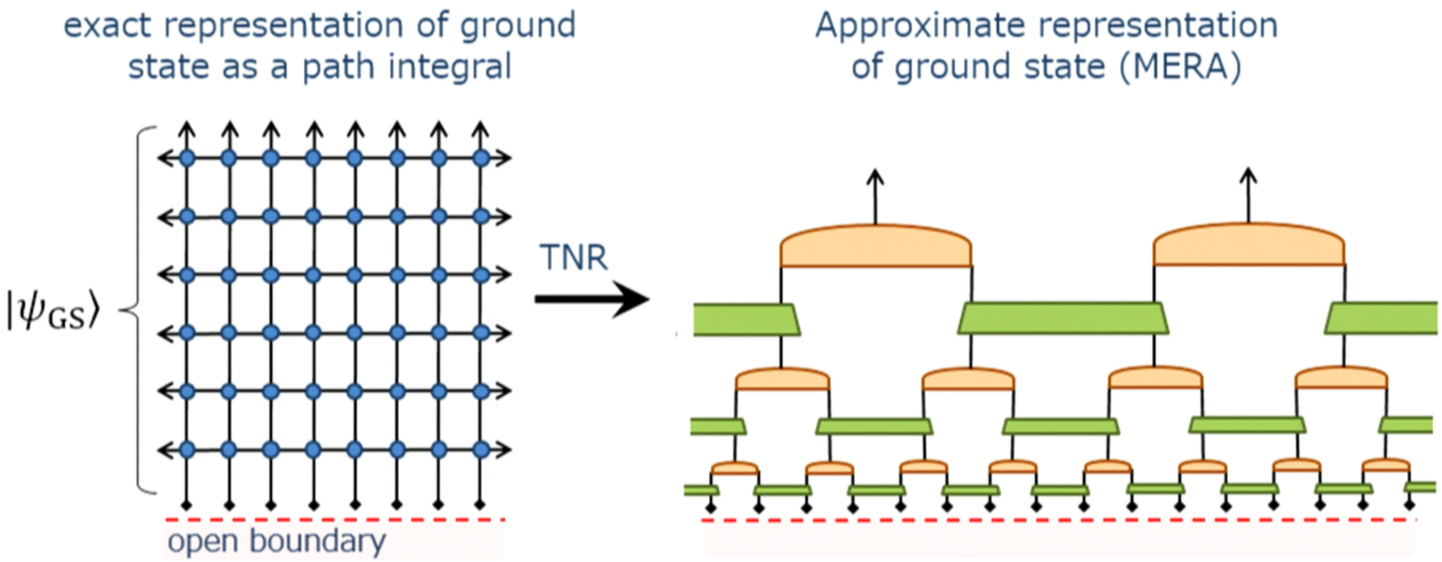
open boundary
(a)
(b)
(c)
(d)
(e)

TNR yields the MERA (Evenbly, Vidal, arXiv:1502.05385)

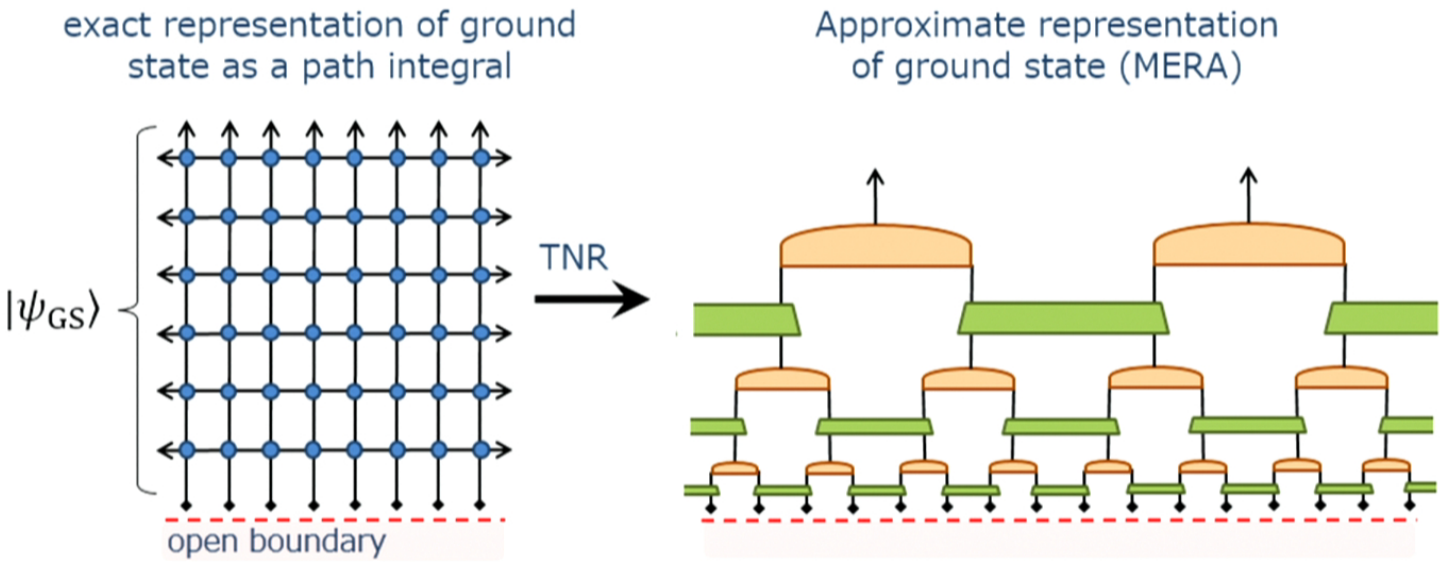


TNR generates a MERA!

TNR yields the MERA (Evenbly, Vidal, arXiv:1502.05385)

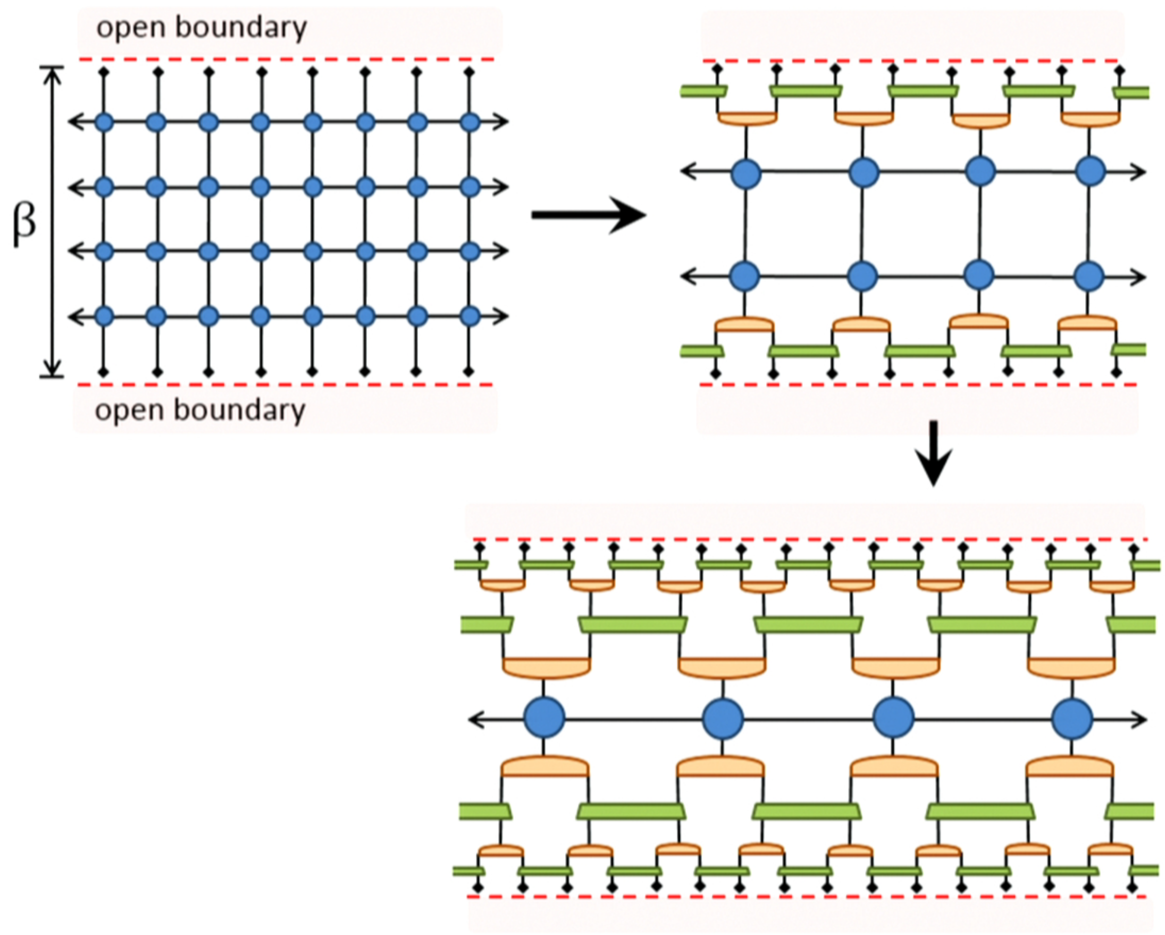


TNR yields the MERA (Evenbly, Vidal, arXiv:1502.05385)



scale-invariant fixed point of TNR $\Rightarrow$ scale-invariant MERA

MERA for a thermal state (or black hole in holography)



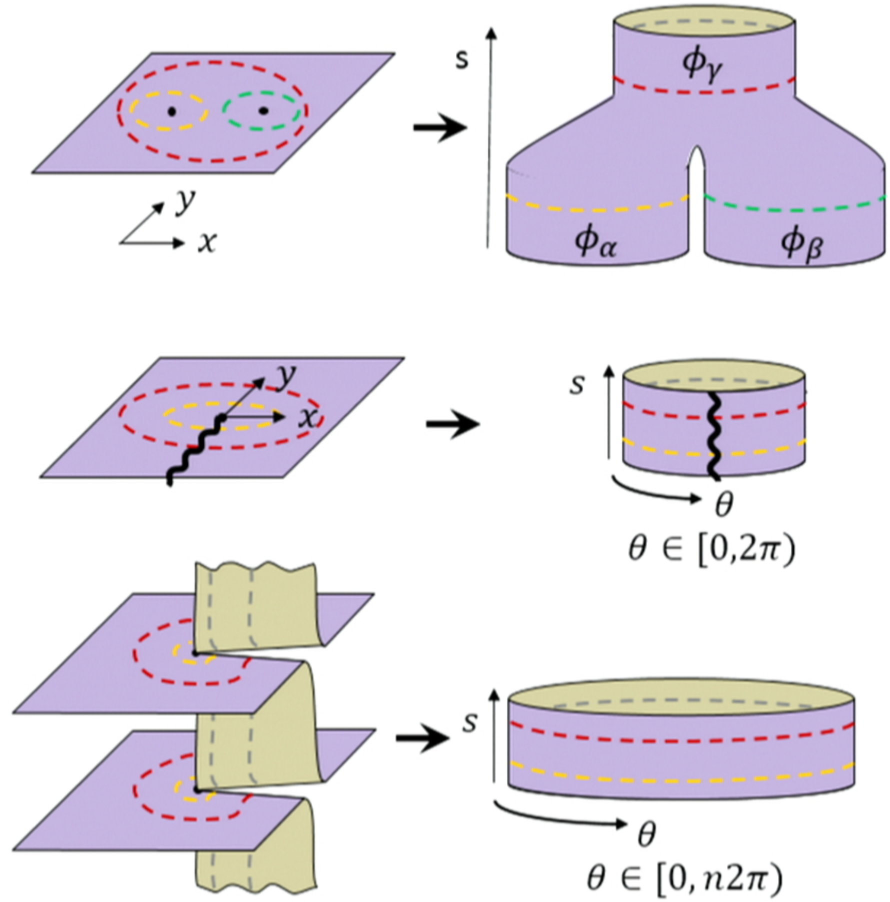
Local scale transformations on the lattice with TNR

(**G. E.**, D. Gaiotto, R. Myers, G. Vidal, *in prep.*)

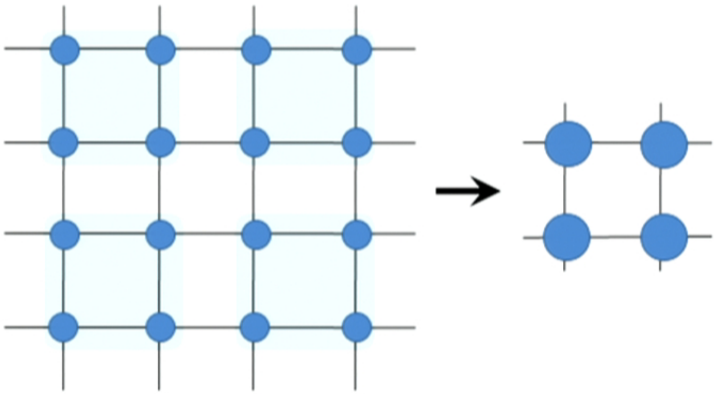
plane with two
punctures into
pair of pants

plane with semi-
infinite defect line
to cylinder

n – sheeted
Riemann surface
to cylinder



Summary

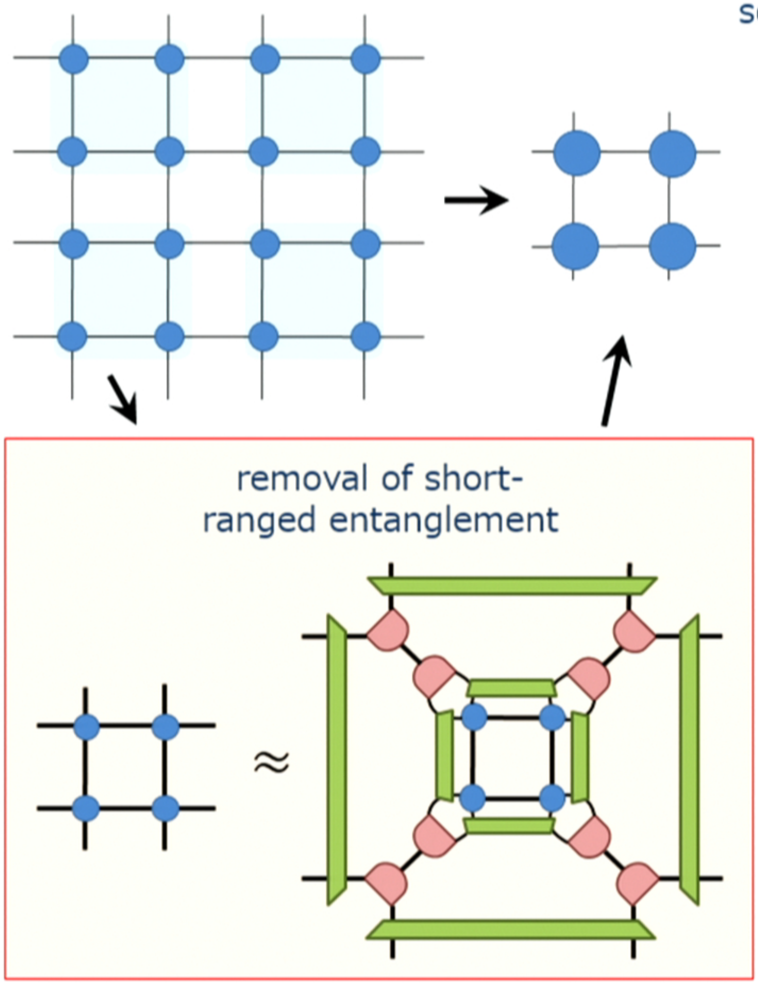


Introduced `proper' way to do local scale transformations for lattice systems

- partition functions of classical systems on the lattice

- (discrete time) Euclidean path integrals of quantum systems on the lattice

Summary



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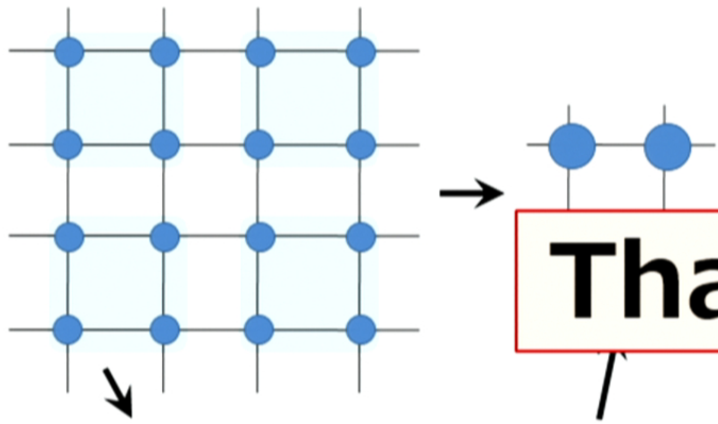
Summary



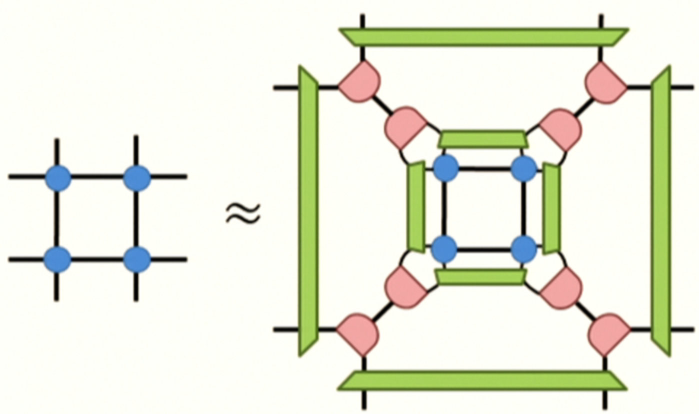
Introduced `proper' way to do local scale transformations for lattice systems

- partition functions of classical systems on the lattice

- (discrete time) Euclidean path ... quantum systems on the

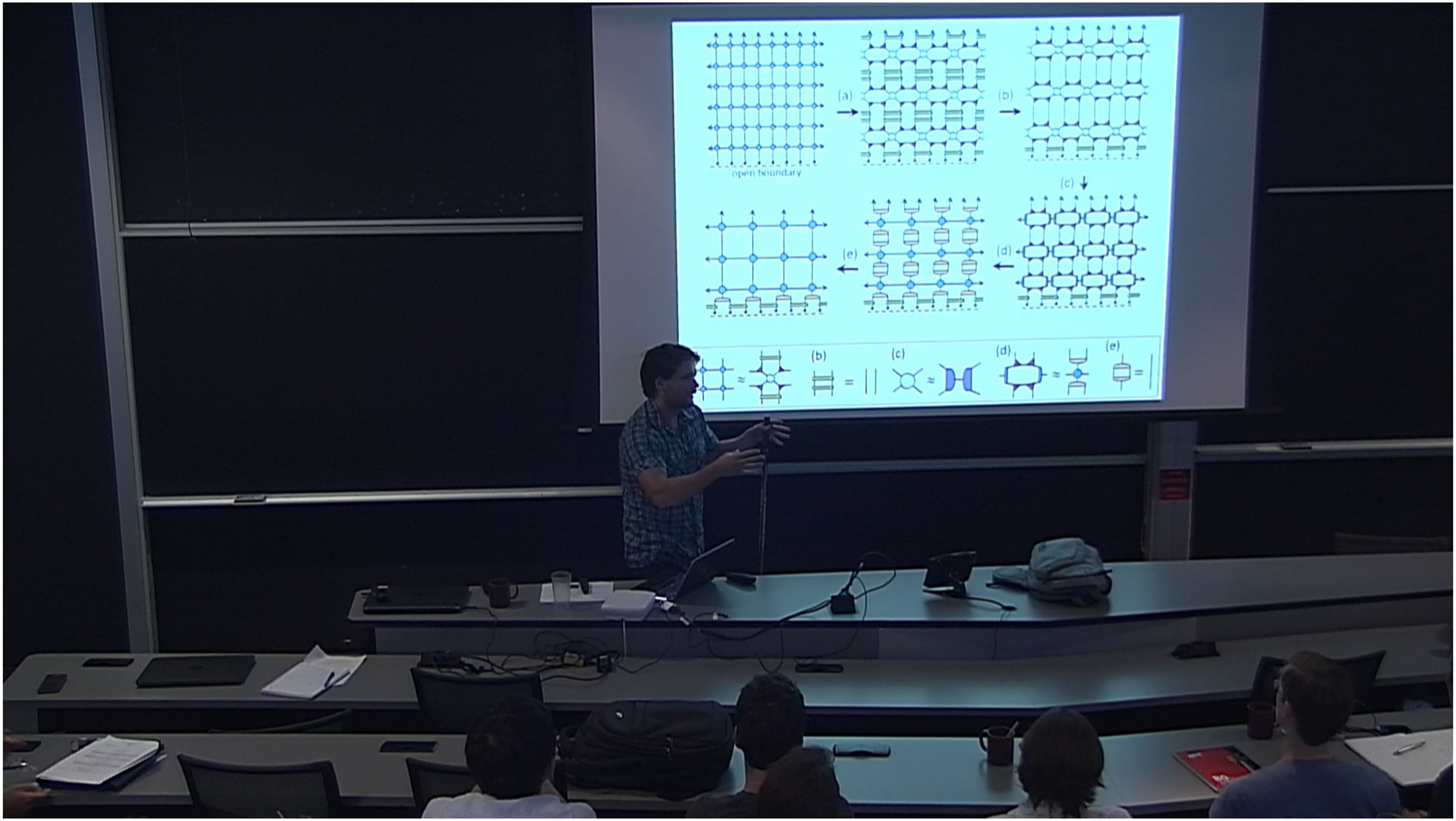
Thanks!

removal of short-ranged entanglement

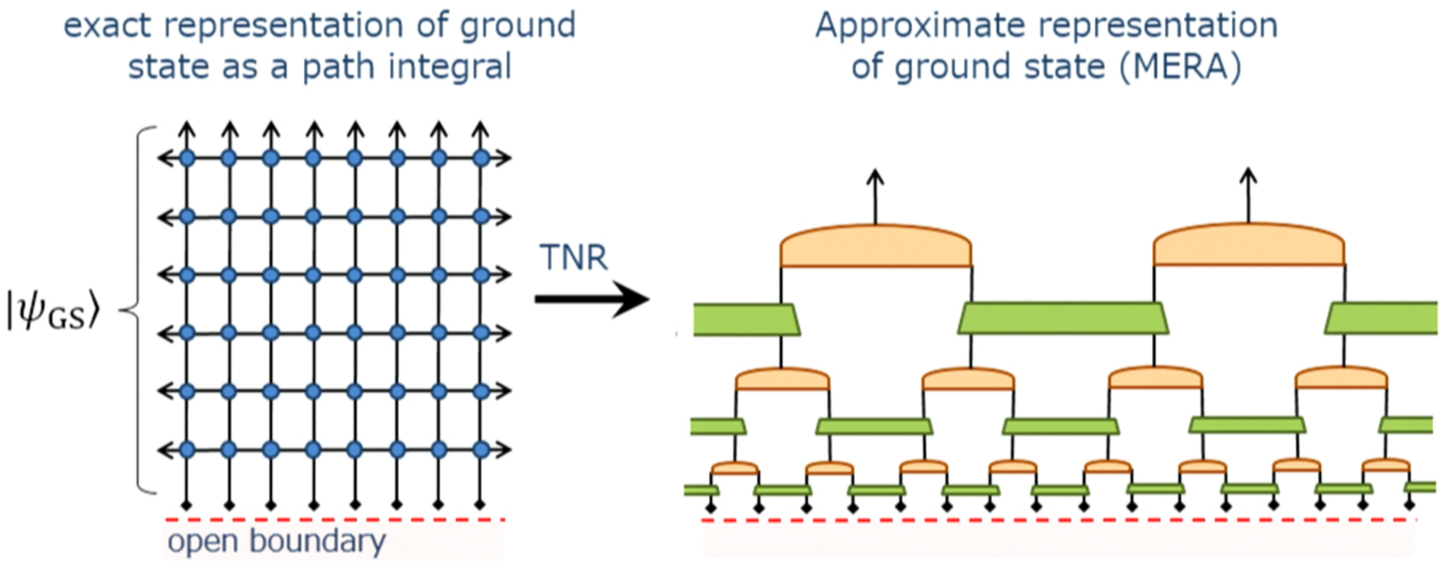


- Base for powerful numeric algorithms
- Nice conceptual connections

Future: higher dimensions, other applications, blah blah blah...


(a)
(b)
(c)
(d)
(e)
open boundary

TNR yields the MERA (Evenbly, Vidal, arXiv:1502.05385)



scale-invariant fixed point of TNR $\Rightarrow$ scale-invariant MERA