Title: Universal holographic description of CFT entanglement entropy

Date: Aug 17, 2015 02:00 PM

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Abstract: The Ryu-Takayanagi proposal (and generalizations) for holographic entanglement makes predictions for geometric CFT entanglement entropy (EE) that continue to hold for any CFT, regardless of existence of large-N limit or strong coupling. We establish this using a direct field theory calculation, thus providing a non-trivial check of the holographic proposal. This universality emerges for small perturbations of the EE of a ball shaped region. Einsteinâ€™s equations arise from the field theory calculation as a way to efficiently encode this perturbative CFT entanglement holographically in the geometry of a dual space-time.

UNIVERSAL HOLOGRAPHIC ENTANGLEMENT FOR CFTS

Tom Faulkner, University of Illinois Urbana-Champaign

Based on arXiv:1412.5648 + …

MOTIVATION: Entanglement ⟷ Geometry

Learned from AdS/CFT that gravity can emerge from a purely quantum system. Relating:

- Ryu-Takayanagi; EE = Area
- First law of EE = Einstein's equations
- EPR = ER
- Error-correcting codes = bulk/boundary correspondence
- etc.

RYU-TAKAYANAGI

Entanglement Entropy in holographic CFTs computed via geometric quantity in dual gravity theory

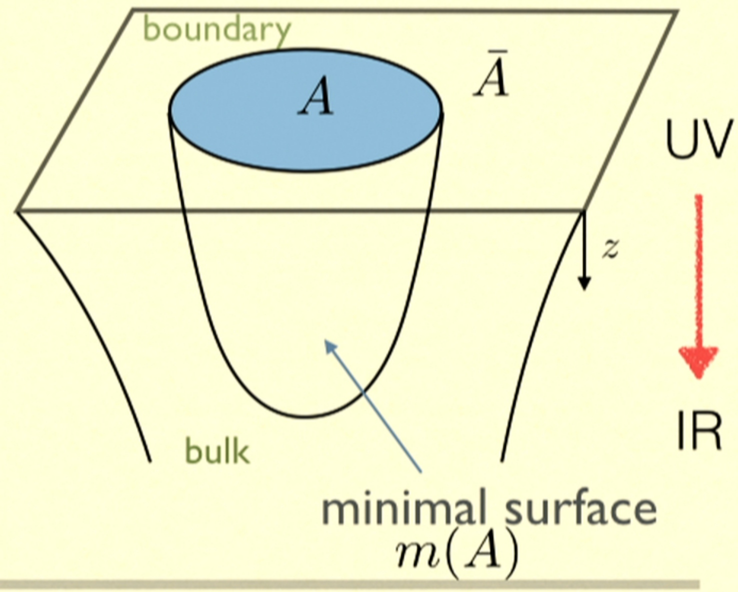
You all know this:

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$$

$$\rho_A = \text{tr}_{\bar{A}} \left|0\right\rangle \left\langle 0\right|$$

$$S_{EE}(\rho_A) = \frac{\text{Area}(m(A))}{4G_N}$$

Ryu, Takayanagi `06



BUT HOW GENERAL?

Expect: works for CFTs with classical gravity dual

- Large $N_{\rm dof}$ / very strong coupling
- $G_N \sim 1/N_{\rm dof}$ / local EFT for gravity in the bulk

Hartman et al.

- Hints it works beyond this: 2d CFTs, large-c and a low lying sparse spectrum (which does not preclude bulk = string theory)

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EXAMPLE I : DEFORMED CFTS

TF `14

- EE provides RG monotones (c-functions)
- c, F, a in 2,3,4 dimensions respectively
- Proven in 2d and 3d using EE and SSA

Casini, Huerta `04,`12

- e.g $F(R) = (R\partial_R - 1)S_{EE}(R)$ Liu, Mezei `12

$$F'(R) \leq 0$$

Equal to F_{UV}, F_{IR} fixed points

Important: ball shaped region

UV CFT (c_{UV})

↓ *energy scale*

IR CFT (c_{IR})

$$c_{UV} > c_{IR}$$

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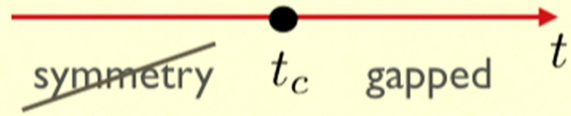
- Physically: $CFT_{UV} \sim$ quantum critical point - lattice system say in 2+1d



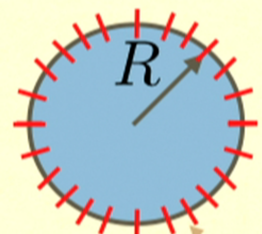
- Quantum fluctuations in ground state large; diverging correlation length ξ as $t \to t_c$

- Characterize fluctuations in g.s. wavefunction with EE

- $t = t_c$ $$S_{EE} = \frac{\text{Area}(\partial A)}{a} - F_{UV}$$

UV divergent

Interesting part



short ranged entanglement cut off by lattice scale a

- Derivatives remove divergence:

$$F(R) = F_{UV}$$

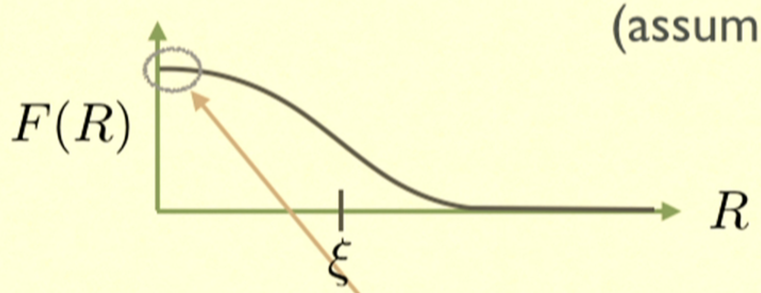
(or use Mutual Information)

Casini, Huerta, Myers, Yale `15

- $t > t_c$ gives a finite correlation length $a \ll R \sim \xi$

$F(R)$ now probes the crossover from UV to IR

(assuming Relativistic flow)



$\xi \sim (t - t_c)^{-\nu}$

- Leading correction:

$$F = F_{UV} - \mathcal{N}(R/\xi)^{2/\nu} + \dots$$

Have calculated this leading correction using conformal perturbation theory. Computation directly maps to classical GR problem in one higher dimension!

Entanglement Entropy = Area!

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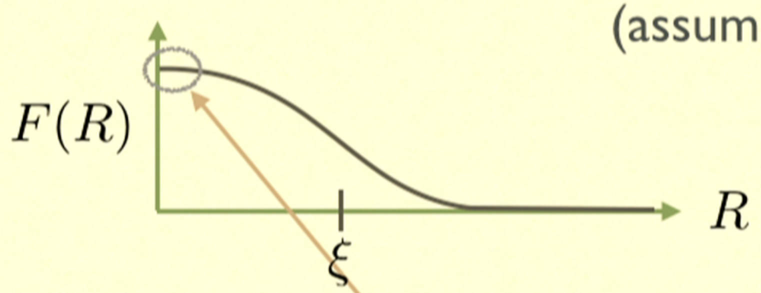

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PREVIEW OF RESULTS

- More precisely: $H = H_{CFT} + \lambda \int \mathcal{O}$ ← leading relevant scalar operator

$$[\mathcal{O}] = \Delta = d - 1/\nu$$

- Claim 1: Field theory calculation gives:

found previously for holographic theories
Liu, Mezei `12

$$F(R) = F_{UV} - \lambda^2 R^{6-2\Delta} 2\pi^2 \frac{(\Delta - 3)}{2\Delta - 7} + \dots$$

- Claim 2: Space dependent couplings can also be treated:

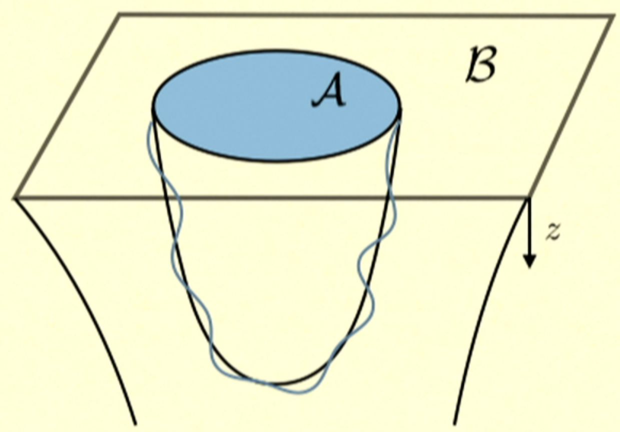
$$H = H_{CFT} + \int \lambda(x)\mathcal{O}(x)$$

- $\delta S_{EE}|_{\mathcal{O}(\lambda^2)} = \delta \mathrm{Area}(g)$ where the metric g is given by the following problem ...
- Solve Einstein's equations coupled to a free scalar field ϕ perturbatively about AdS in d+1 dimensions where

$$m_\phi^2 = \Delta(\Delta - d)$$

- Impose the following boundary conditions as $z \to 0$

$$\phi \to \lambda(x) z^{d-\Delta} + \dots$$
$$g \to g_{AdS} + \dots$$



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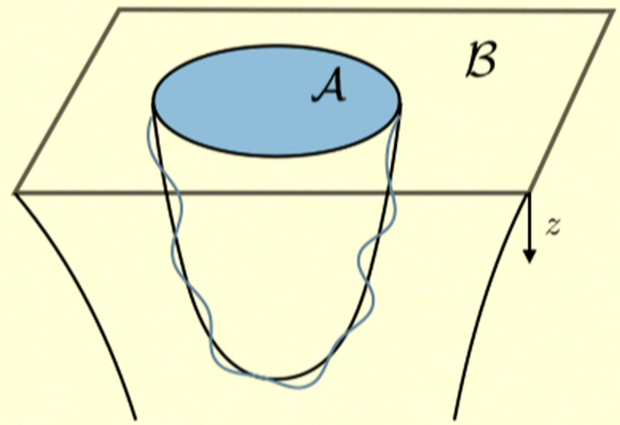


- Calculate the change in area of the perturbed minimal area surface due to $\delta g \sim \mathcal{O}(\lambda^2)$

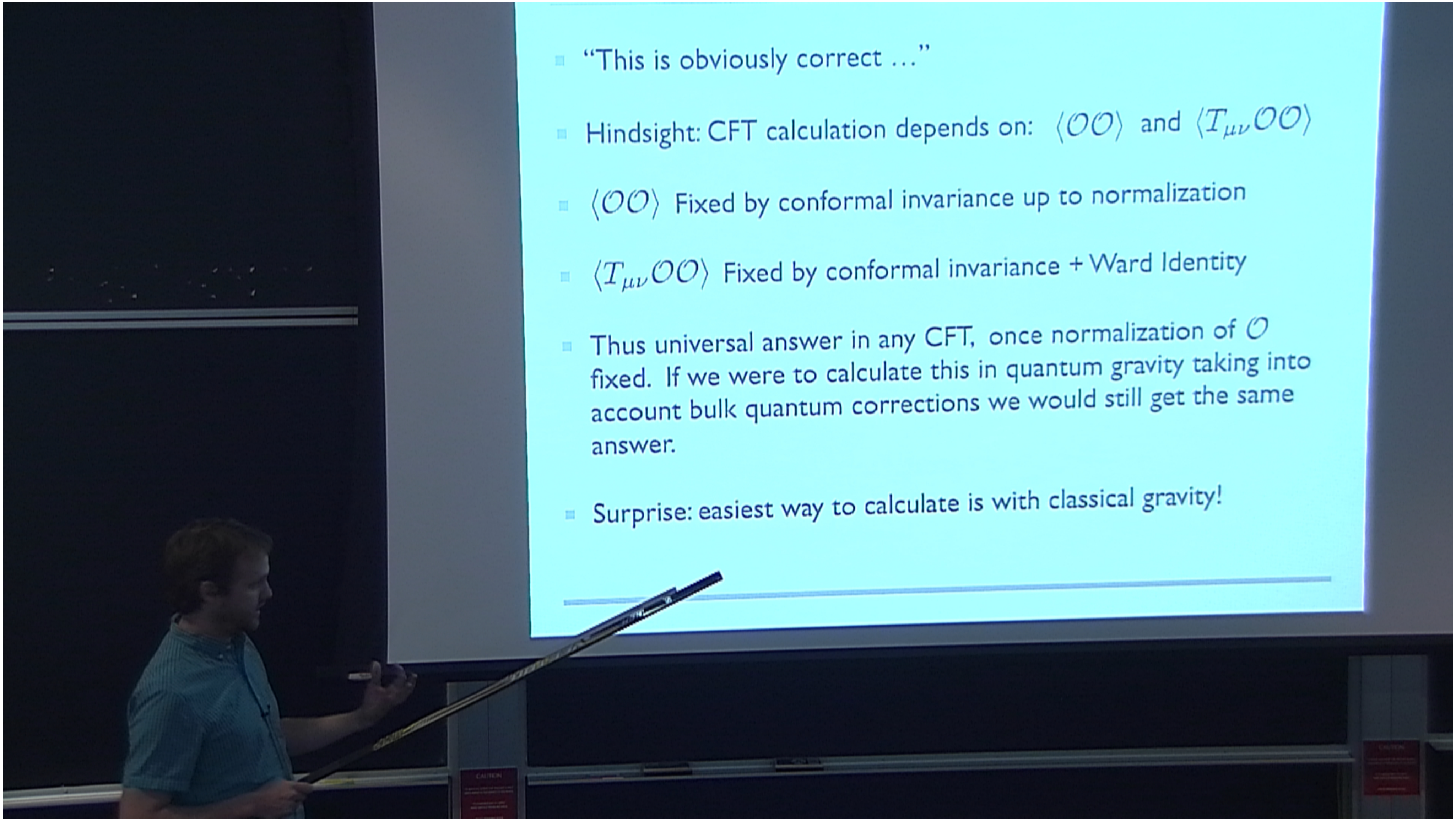
- Note that G_N cancels out of the final answer for δS_{EE}
- The only independent parameter is the normalization of the two point function of $\mathcal{O}$ which can be fixed in AdS by studying the two point function of ϕ
- Recognize: this as how you would calculate δS_{EE} in CFTs with a classical gravity dual

Two reactions when I tell people this …

"This is obviously wrong; you've been watching too many disney movies with your daughter ..."

(complaint: "this is outside the regime of validity of AdS/CFT shouldn't there be strong quantum corrections if $N_{\rm dof}$ is small?")

- "This is obviously correct …"
- Hindsight: CFT calculation depends on: $\langle \mathcal{O}\mathcal{O} \rangle$ and $\langle T_{\mu\nu}\mathcal{O}\mathcal{O} \rangle$
- $\langle \mathcal{O}\mathcal{O} \rangle$ Fixed by conformal invariance up to normalization
- $\langle T_{\mu\nu}\mathcal{O}\mathcal{O} \rangle$ Fixed by conformal invariance + Ward Identity
- Thus universal answer in any CFT, once normalization of $\mathcal{O}$ fixed. If we were to calculate this in quantum gravity taking into account bulk quantum corrections we would still get the same answer.
- Surprise: easiest way to calculate is with classical gravity!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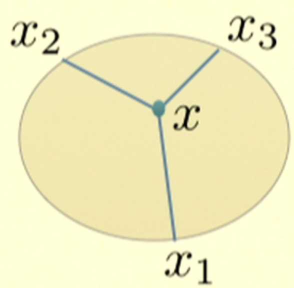
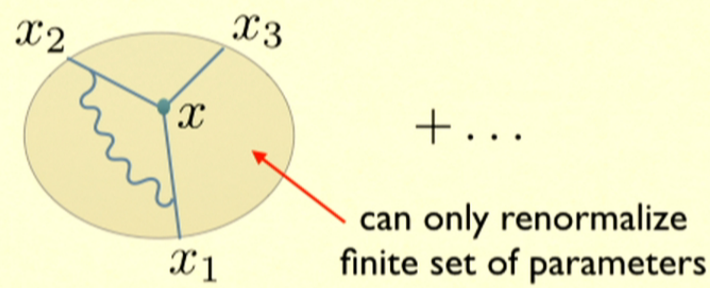
AN ANALOGY

- CFT 3 point functions fixed up to a finite set of parameters

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle = C(x_{12})^{-\Delta}(x_{13})^{-\Delta}(x_{23})^{-\Delta}$$

- Can also calculate this in AdS/CFT: (an early check of AdS/CFT)

$$= \int dx$$  $$+$$  $$+ \dots$$

- Here we have the reverse. AdS/CFT answer calculated first, only now checking it agrees with CFT calculation!

EXAMPLE II: ENTANGLEMENT DENSITY

Preliminary/Work in progress with
Leigh, Parrikar, Balakrishnan

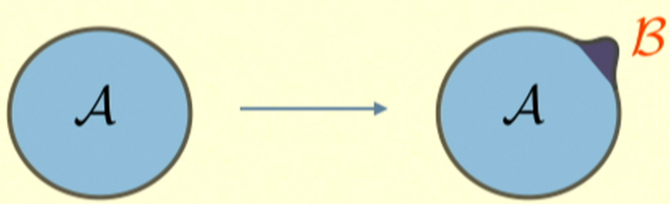
- Would like to study universality extended to stress tensor defs:

$$S_{CFT} \to S_{CFT} + \int h_{\mu\nu} T^{\mu\nu}$$ at second order in $\mathcal{O}(h^2)$

Expect fixed by $\underbrace{\langle TT \rangle \ \langle TTT \rangle}$

3 parameters

- Easier problem: deform shape of the ball

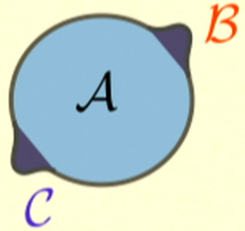


pure diffeomorphism:

$$h_{\mu\nu} = \partial_\mu \xi_\nu^{\mathcal{B}} + \partial_\nu \xi_\mu^{\mathcal{B}}$$

Bhattacharya, Hubeny, Rangamani, Takayanagi `14

- Define second order variation, "entanglement density":



$$S(\mathcal{AC}) - S(\mathcal{ABC}) - S(\mathcal{A}) + S(\mathcal{AB})$$
$$= \int_{\partial A} \delta r^{\mathcal{B}}(x_1) \int_{\partial A} \delta r^{\mathcal{C}}(x_2) n(x_1, x_2)$$

- Again can prove this is universal in any CFT (conformal pert. theory)

$$n(\vec{x}_1, \vec{x}_2) = \frac{2\pi^2 C_T}{(d+1)} \frac{1}{(\vec{x}_1 - \vec{x}_2)^{2(d-1)}}$$

Found in holography first: Nozaki, Numasawa, Prudenziati, Takayanagi `13
also: Mezei '14

- where C_T is fixed by: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle = C_T \frac{I_{\mu\nu;\alpha\beta}(x_1 - x_2)}{(x_{12})^{2d}}$

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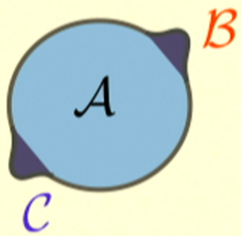

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DIFFERENT CFT UNIVERSALITY

- These results should be contrasted with:

  in even dimensions the trace anomaly leads to: Solodukhin '08

$$S_{EE} \sim \frac{|\partial \mathrm{A}|}{\epsilon^2} + \ln(\epsilon/L) \int_{\partial A} \left[a R^{(2)} + c \left(2 K_{\alpha\beta} K^{\alpha\beta} - K^2 \right) \right]$$

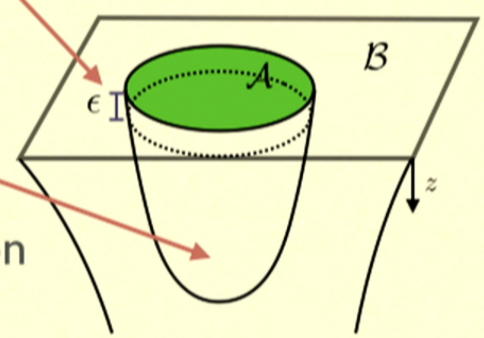
d=4

$$+ \text{const.}$$

- Main difference; sensitivity to the bulk:

  $F(R)$ and entanglement density etc. probe the bulk metric and fields

- Constant/non-local term in UV expansion of S_{EE}



AN ASIDE - A DIFFERENT APPROACH:

- Many recent papers have observed such universality
  Mezei `14, Bueno, Myers,Witczak-Krempa `15, Haehl `15 …

- study large set of higher derivative gravitational theories
- calculate EE via various generalizations to RT   Dong, Camps
- always find the same answer - claim victory!
- But: quantum corrections? not quite a proof

- Hybrid approach: show CFT calculations only depends on discrete set of data
- Match that data to a gravitational theory
- Use this gravitational theory to calculate EE (easier)

SKETCH OF CALCULATION

- Important tool: $\rho_A = \mathrm{tr}_{\bar{A}} \left|0\right\rangle \left\langle 0\right| \equiv \exp(-H_A)$

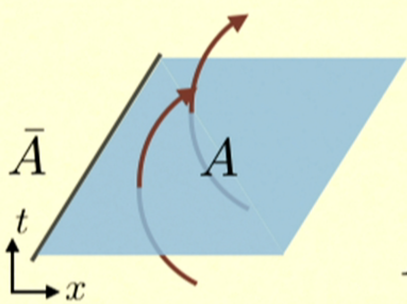
$$H_A = 2\pi \int_A dx \frac{(R^2 - |x|^2)}{2R} T_{tt} = \int_A d\Sigma^\nu \xi^\mu T_{\mu\nu}$$

CFT ($\lambda = 0$) &
ball shaped region

Modular/Entanglement Hamiltonian

Casini, Huerta, Myers `11

- Generlization of Unruh effect in Rindler space:



Rindler observers are uniformly accelerated in x direction. Only have acces to ρ_A

$$\rho_A = \exp(-K_x)$$

K_x is the generator of boosts in QFT

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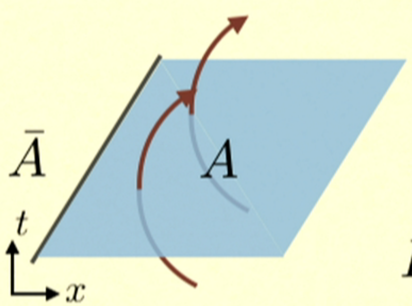

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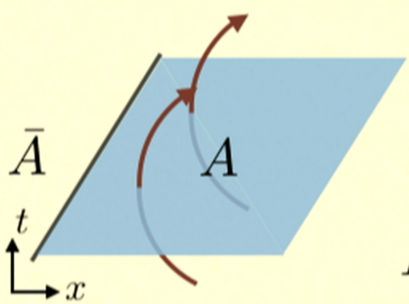

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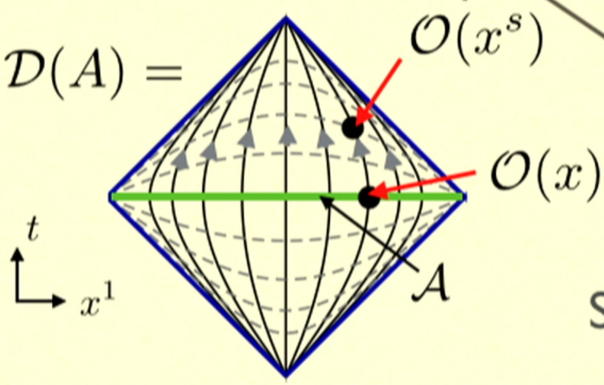
- H_A is "conformal boost" generator (symmetry of CFT vacuum)
- Consider finite "conformal boost" acting on a local CFT operator in A:

rapidity factor: $\eta = e^s$

$$e^{iH_A s}\mathcal{O}(x)e^{-iH_A s} \; \left(= \rho_A^{-is}\mathcal{O}(x)\rho_A^{is}\right)$$

- Claim: this is a local operator at a different point in: $\mathcal{D}(A)$

$\mathcal{D}(A) =$



Call this point: x^s

$\propto \mathcal{O}(x^s)$

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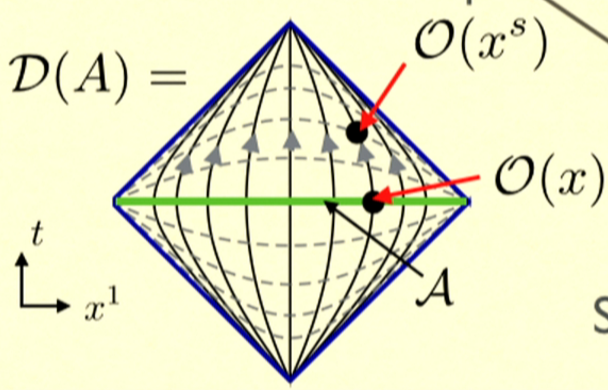

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$$H \to H_{CFT} + \lambda \int \mathcal{O} \longrightarrow \rho_A = \rho_0 + \delta^1\rho + \delta^2\rho + \ldots$$

Not modular Hamiltonian! $\propto \lambda$ $\propto \lambda^2$

- Construct $\delta\rho$ using a Euclidean path integral

Rosenhaus, Smolkin `14

$$\longrightarrow S_{EE} = -\mathrm{tr}\rho_A \ln \rho_A$$

- At second order in λ there are two terms:

$$\delta S_{EE} = \delta \langle H_A \rangle - S(\rho_A || \rho_0)$$

Entanglement first law

Relative entropy
Always vanishes on first order changes
$\mathcal{O}(\delta^1\rho)^2$

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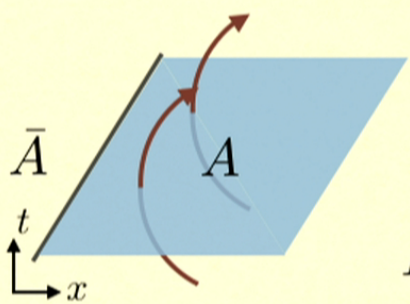

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here I differ from
Rosenhaus, Smolkin `14

- Relative entropy term most interesting:

$$S(\rho_A||\rho_0) \equiv \mathrm{tr}\rho_A(\ln \rho_A - \ln \rho_0)$$

Difficult to expand $\ln(\rho_0 + \delta\rho)$ because $[\rho_0, \delta\rho] \neq 0$

- Baker-Campbell-Hausdorff formula has ∞ number of terms each term naively contains a CFT n-point function for n large

⟶ doom universality?

Casini, Heurta

- Trick is to re-write: $-\ln \rho_A = \int_0^\infty d\beta \left(\frac{1}{\beta + \rho_A} - \frac{1}{\beta + 1} \right)$

$$\longrightarrow S(\rho||\rho_0) = \int_0^\infty d\beta\beta\mathrm{tr}\frac{1}{(\beta+\rho_0)^2}\delta\rho\frac{1}{\beta+\rho_0}\delta\rho$$

$$= \int_{-\infty}^{\infty} dsK(s)\mathrm{tr}\left(\rho_0^{is}\delta\rho\rho_0^{-is-1}\delta\rho\right)$$

Last line: insert complete set of entanglement eigenstates then fourier transform

- Relative entropy term most interesting:

here I differ from Rosenhaus, Smolkin `14

$$S(\rho_A||\rho_0) \equiv \mathrm{tr}\rho_A(\ln\rho_A - \ln\rho_0)$$

Difficult to expand $\ln(\rho_0 + \delta\rho)$ because $[\rho_0, \delta\rho] \neq 0$

- Baker-Campbell-Hausdorff formula has ∞ number of terms each term naively contains a CFT n-point function for n large

$\longrightarrow$ doom universality?

Casini, Heurta

- Trick is to re-write: $-\ln\rho_A = \int_0^\infty d\beta \left(\frac{1}{\beta+\rho_A} - \frac{1}{\beta+1}\right)$

$$\longrightarrow S(\rho||\rho_0) = \int_0^\infty d\beta\beta\mathrm{tr}\frac{1}{(\beta+\rho_0)^2}\delta\rho\frac{1}{\beta+\rho_0}\delta\rho$$

$$= \int_{-\infty}^\infty dsK(s)\mathrm{tr}\left(\rho_0^{is}\delta\rho\rho_0^{-is-1}\delta\rho\right)$$

Last line: insert complete set of entanglement eigenstates then fourier transform

$$S(\rho_A||\rho_0) = \int_{-\infty}^{\infty} ds K(s) \mathrm{tr}\left(\rho_0^{is}\delta\rho\rho_0^{-is-1}\delta\rho\right)$$

- Now Euclidean path integral gives for the change in the state:

Rosenhaus, Smolkin `14

$$\delta\rho = \rho_0 \int_{\mathcal{M}} \lambda \mathcal{O}(x)$$

$\mathcal{M}$ ← full Euclidean manifold

- See appearance of modular flow: $\mathcal{O}(x) \to \mathcal{O}(x^s)$

$$= \int ds \int \int_{\mathcal{M}} (\ldots) \langle \mathcal{O}(x_1^s)\mathcal{O}(x_2)\rangle$$

- This is enough to claim victory (c.f. Hybrid approach)
- But why stop now??? Here comes the fun …

Lets do some integrals!

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Lets do some integrals!

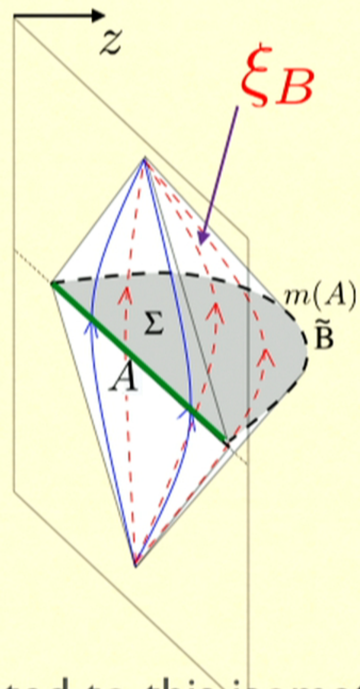
- Actually I failed to do these integrals directly.
- Can rewrite these integrals suggestively as:

$$S(\rho_A||\rho_0) = \langle H_\Sigma^{\text{bulk}}(\phi) \rangle$$

- Becomes a bulk AdS_{d+1} quantity
- Associate to "conformal boost" isometry on boundary - a boost isometry in the bulk

→ AdS Rindler wedge between

$\mathcal{D}(A); m(A)$



- $H_\Sigma^{\text{bulk}}(\phi)$ is the charge/Hamiltonian associated to this isometry for a free scalar field theory in the bulk

- Final form: $\delta S_{EE} = \langle H_A \rangle - \int_\Sigma d\Sigma^\mu T^B_{\mu\nu}(\phi)\xi^\nu_B$
- Any metric fluctuation h in AdS satisfies:

Wald, Iyer:
TF, Guica, Hartman, Myers
van Raamsdonk

$$\delta A(h) = \delta E_A(h) - \int d\Sigma^\mu \xi^\nu_B \delta(G_{\mu\nu} + \Lambda g_{\mu\nu})$$

Change in the area of $m(A)$

Falloff of h at boundary

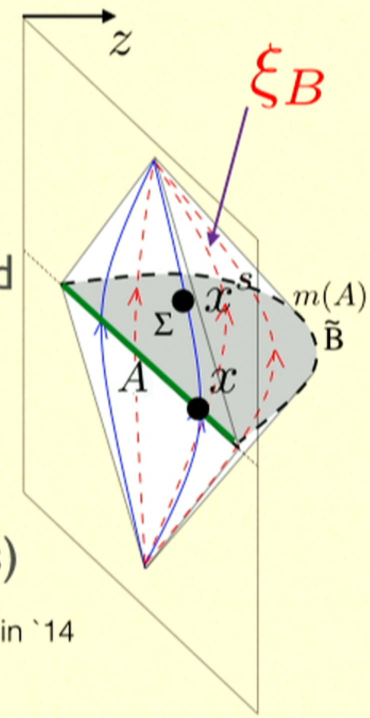
LHS of Einstein's equations

- Compare terms: if we demand that h satisfies Einstein's equations coupled to $T^B_{\mu\nu}(\phi)$ then: $\delta S_{EE} = \delta A(h)$
- So we can encode the CFT EE in this metric!

- Some intuition for this calculation:

  The s integral: $\int ds K(s) \mathcal{O}(x_s)$ is like a smearing function that allows us to reach into the bulk and construct the dual bulk field

- Essential to include this effect to derive these universal CFT results (maybe not so essential to see the $\ln \epsilon$ UV divergent terms)



Rosenhaus, Smolkin `14

CONCLUSSIONS

- Derived universal contributions to EE for CFTs
- Always: various small deformations of ball shaped EE
- Only depends on CFT 2 & 3 point functions, thus fixed by a finite set of CFT parameters
- Agrees with holographic theories in a non-trivial way
- Most natural derivation involves: Entropy = Area

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