Title: A new perspective on holographic entanglement

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Abstract: We will present a reformulation of the Ryu-Takayanagi holographic entanglement entropy formula which does not involve the areas of surfaces. The reformulation leads to a picture of entanglement entropy of boundary regions being carried by Planck-thickness "bit threads" in the bulk. We will argue that this picture resolves a number of conceptual difficulties surrounding the RT formula.

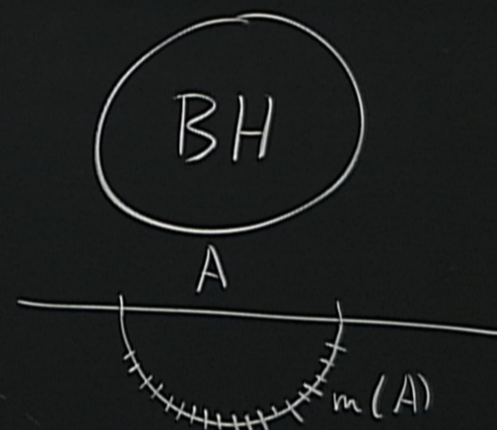
A new perspective on holographic entanglement

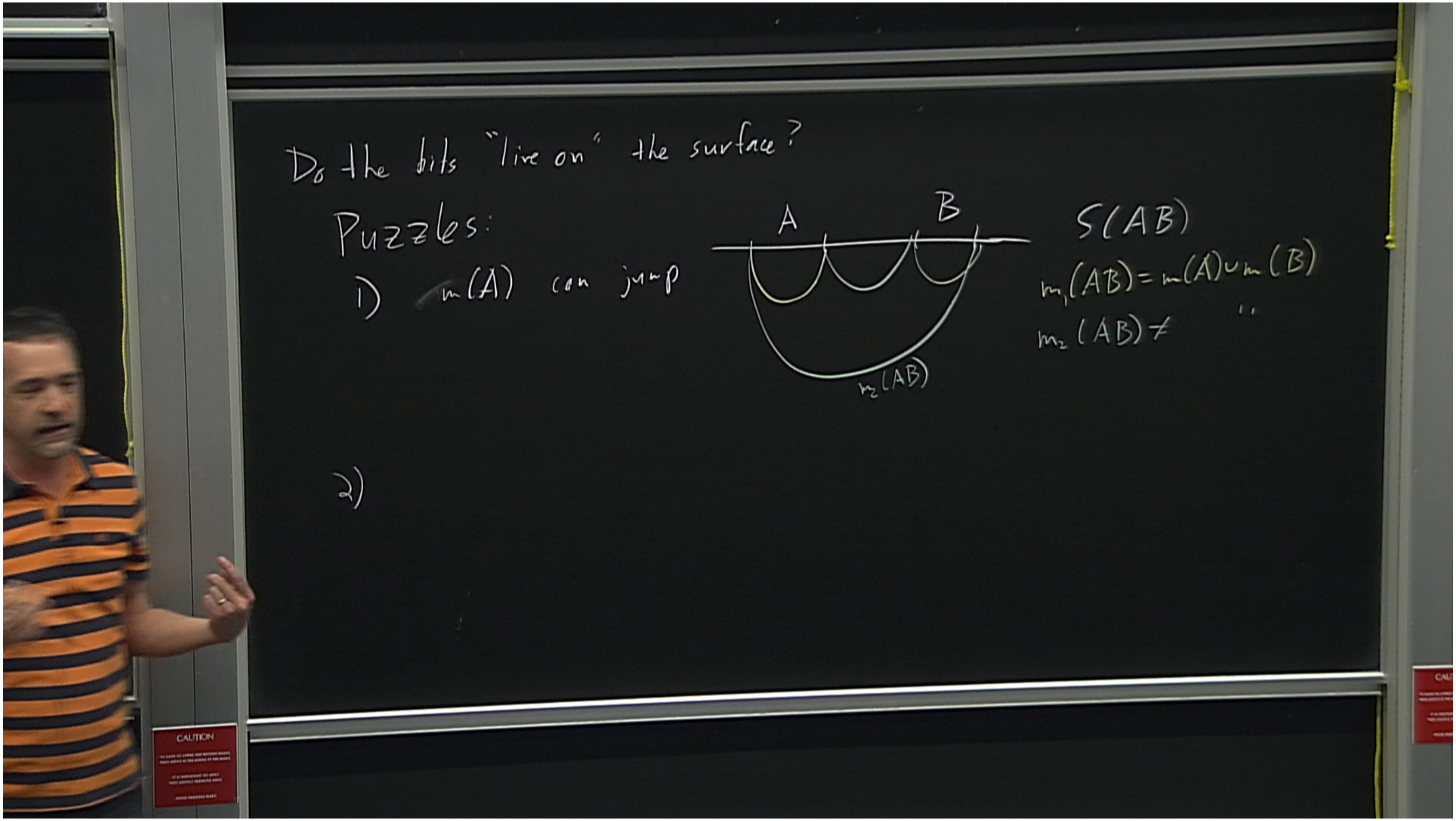
(w/ M. Freedman, to appear)

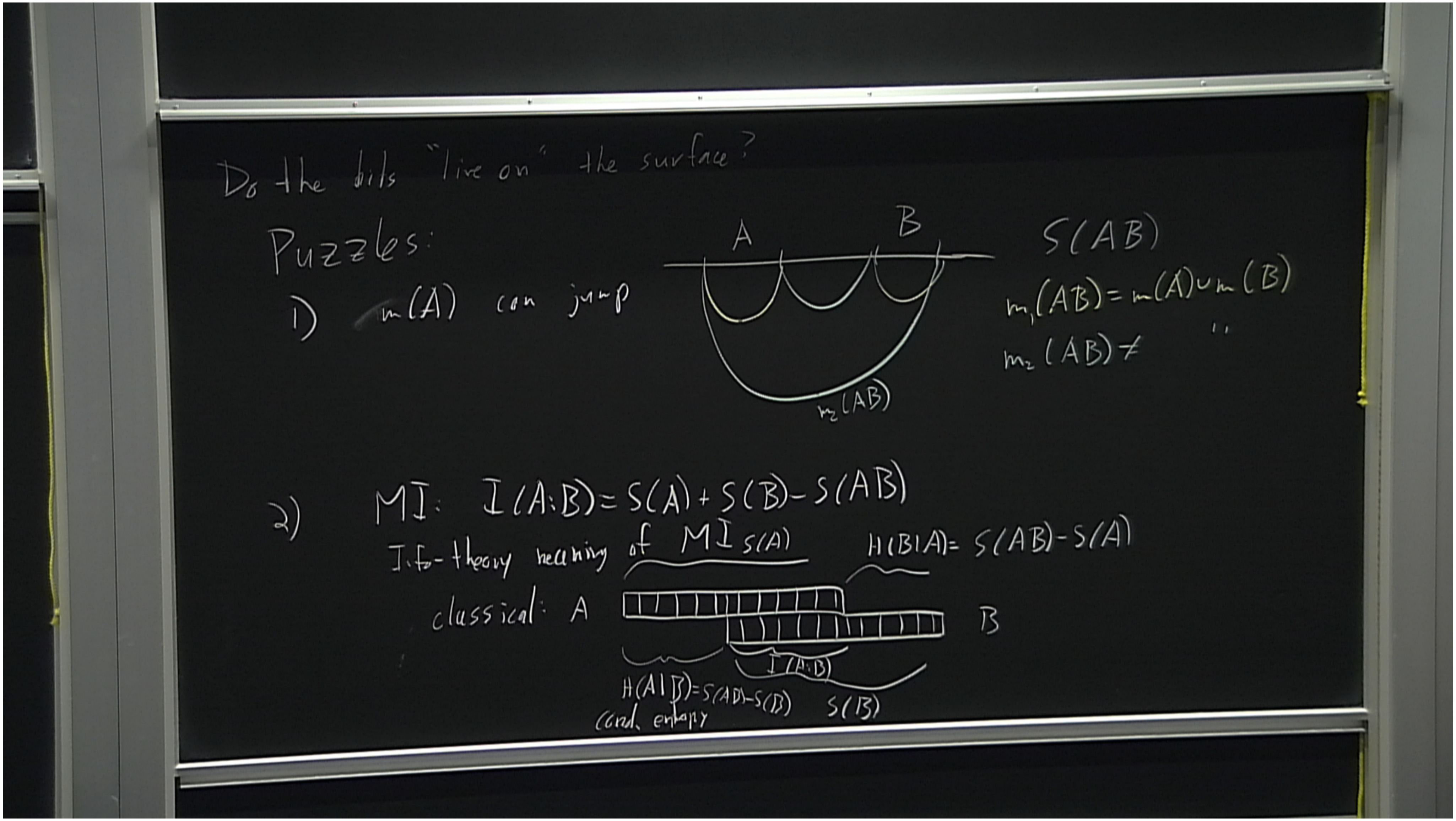
In gravity, area = entropy

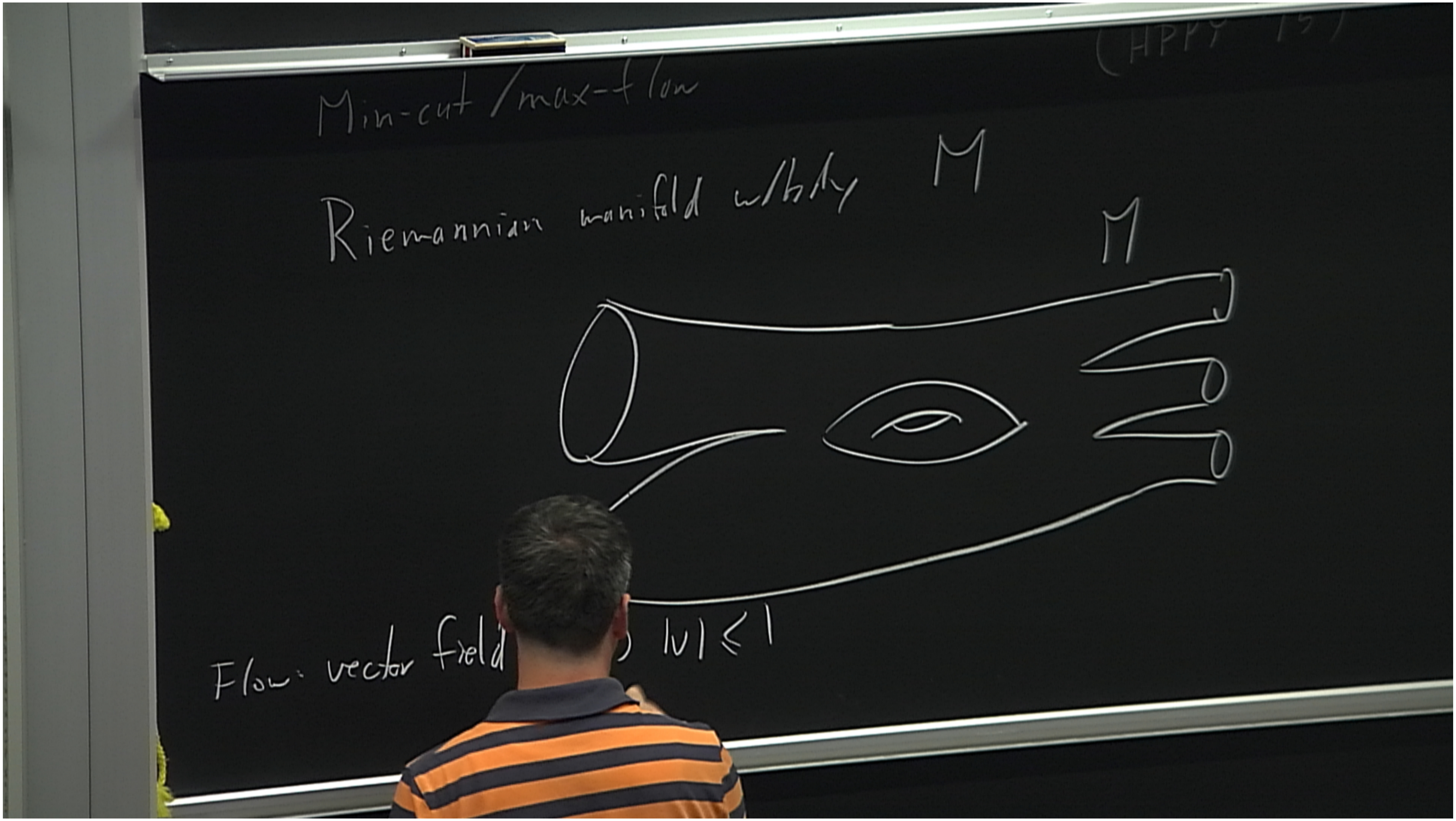
$$S_{BH} = \frac{1}{4G_N} \text{ area (horizon)}$$

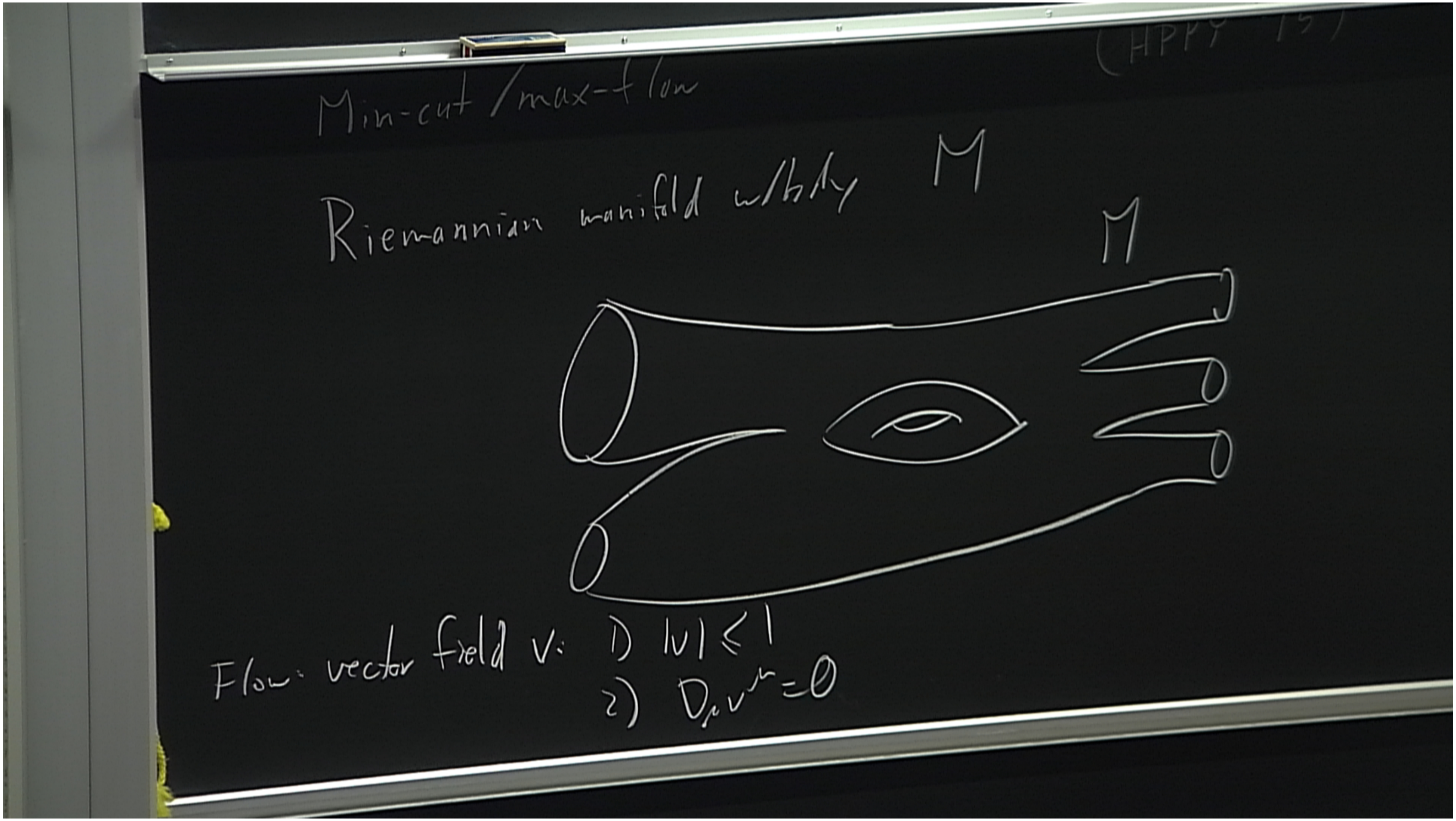
$$S(A) = \frac{1}{4G_N} \text{ area}(m(A))$$

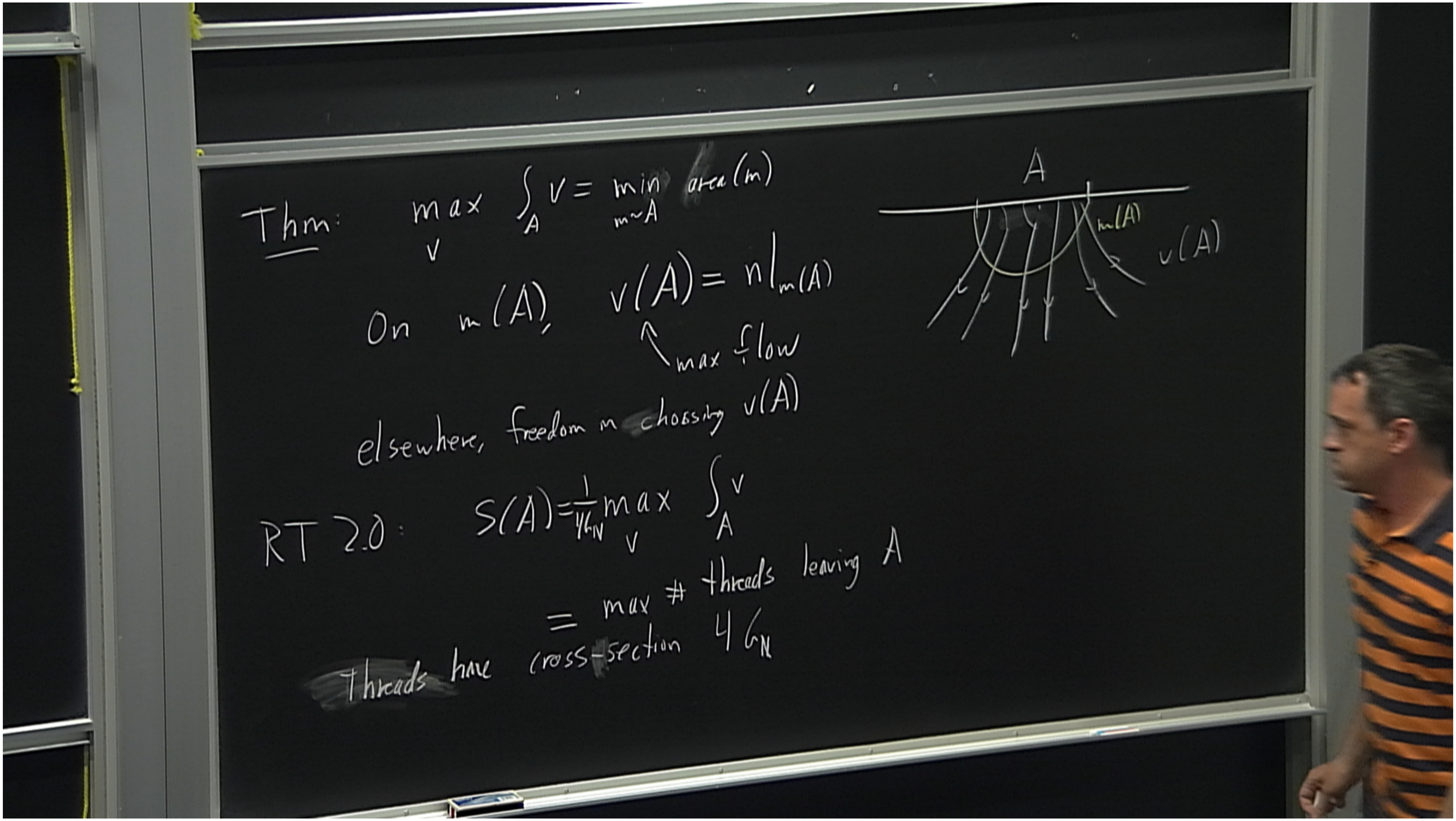


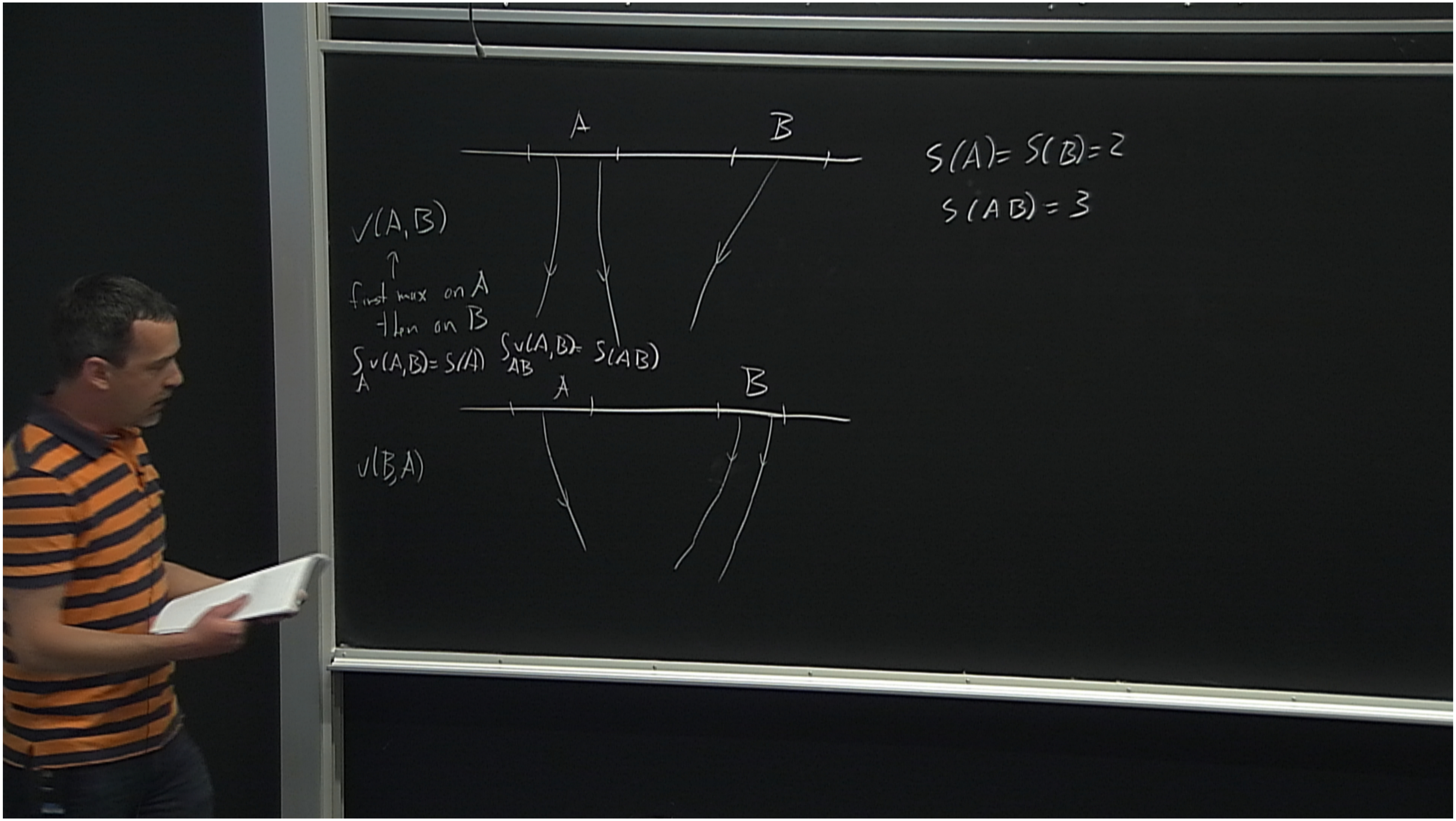

Do the bits "live on" the surface?
Puzzles:
1) m(A) can jump
A
B
S(AB)
m1(AB) = m(A) ∪ m(B)
m2(AB) ≠ "
m2(AB)
2)
CAUTION


Do the bits "live on" the surface?
Puzzles:
1) m(A) can jump
A
B
S(AB)
m₁(AB) = m(A) ∪ m(B)
m₂(AB) ≠ "
m₂(AB)
2) MI: I(A:B) = S(A) + S(B) − S(AB)
Info-theory meaning of MI
S(A)
H(B|A) = S(AB) − S(A)
classical: A
B
I(A:B)
H(A|B) = S(AB) − S(B)
cond. entropy
S(B)

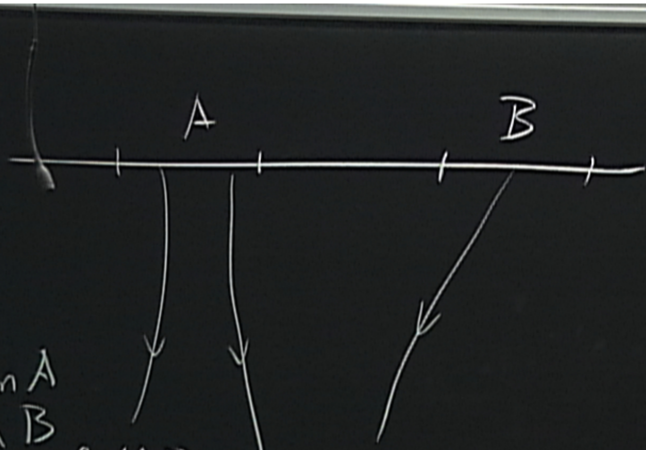

Min-cut / max-flow
Riemannian manifold w/bdy
M
M
Flow: vector field
|v| ≤ 1


Min-cut / max-flow
Riemannian manifold w/ bdy M
M
Flow: vector field v: 1) $|v| \leq 1$
2) $D_\mu v^\mu = 0$


Thm: $\max_{v} \int_A v = \min_{m \sim A} \text{area}(m)$
On $m(A)$, $v(A) = n|_{m(A)}$
max flow
elsewhere, freedom in choosing $v(A)$
RT 2.0: $S(A) = \frac{1}{4G_N} \max_{v} \int_A v$
= max # threads leaving A
Threads have cross-section $4G_N$
A
m(A)
v(A)


A
B
S(A) = S(B) = 2
S(AB) = 3
v(A,B)
first max on A
then on B
∫_A v(A,B) = S(A)
∫_AB v(A,B) = S(AB)
A
B
v(B,A)

Ex 1

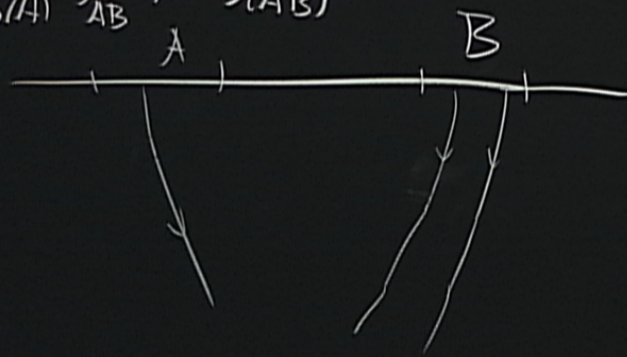


$v(A,B)$

$\uparrow$

first max on A
then on B

$\int_A v(A,B) = S(A)$ $\quad \int_{AB} v(A,B) = S(AB)$



$v(B,A)$

$S(A) = S(B) = 2$

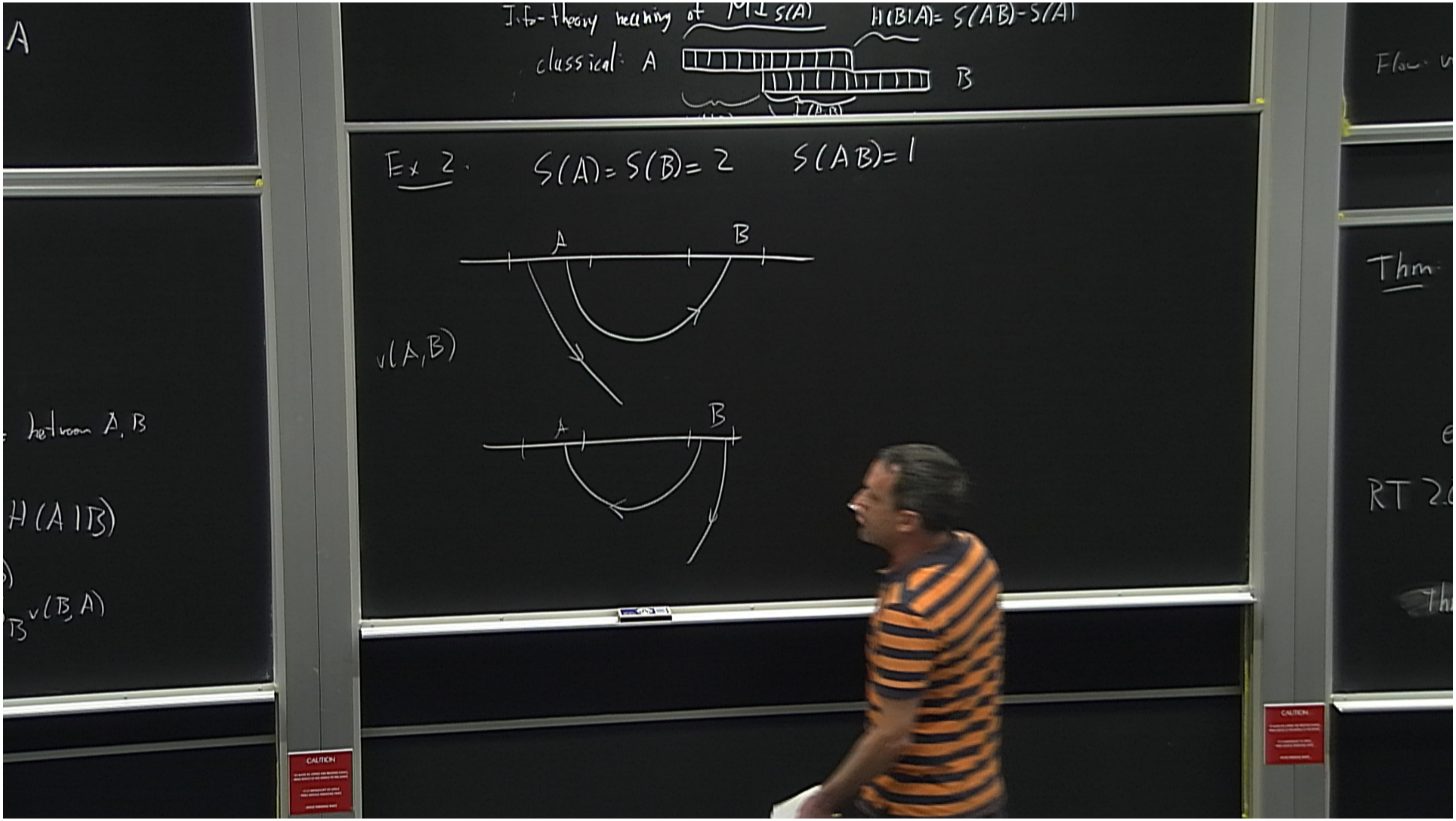
$S(AB) = 3$

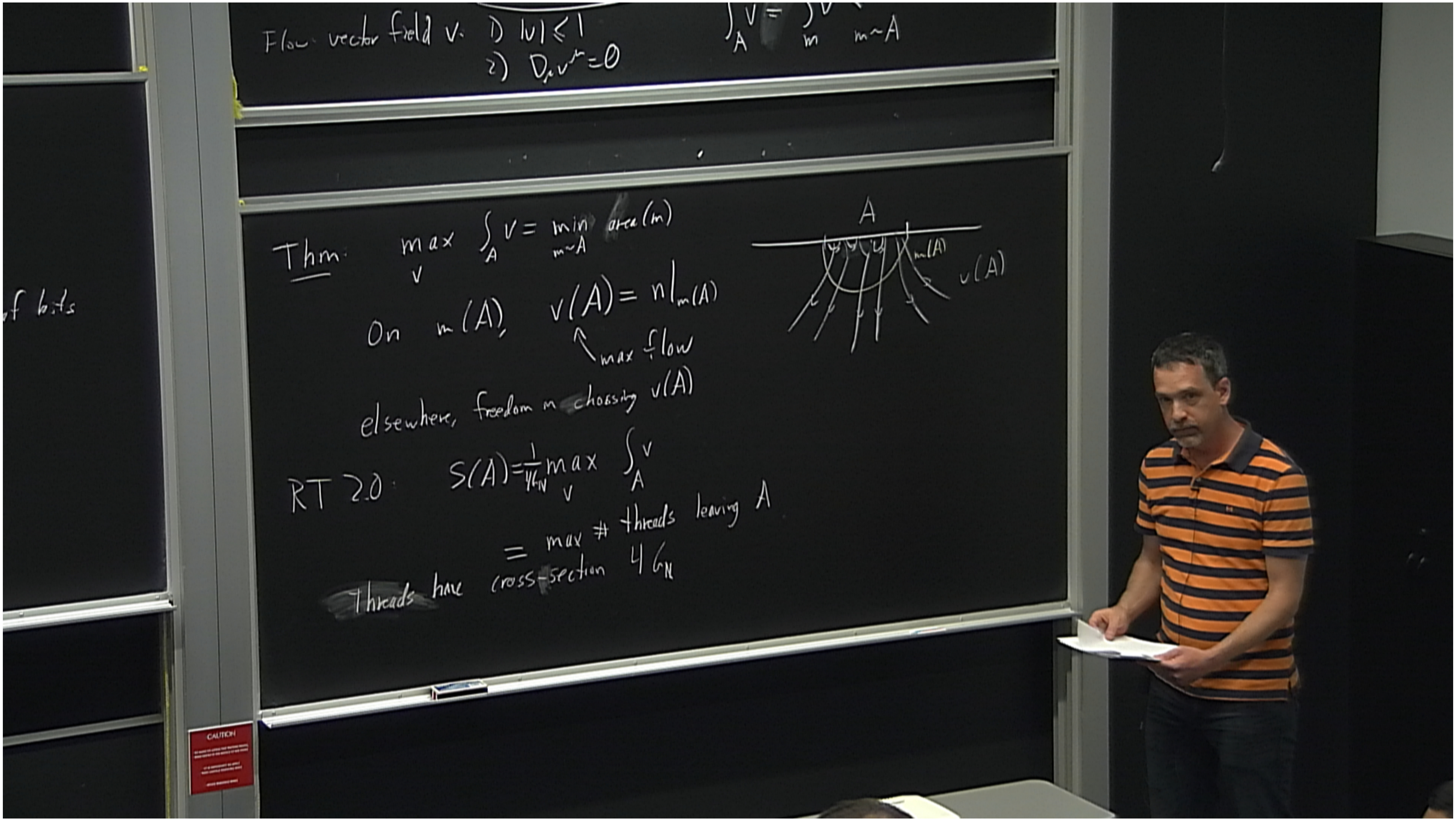
threads free to move between A, B

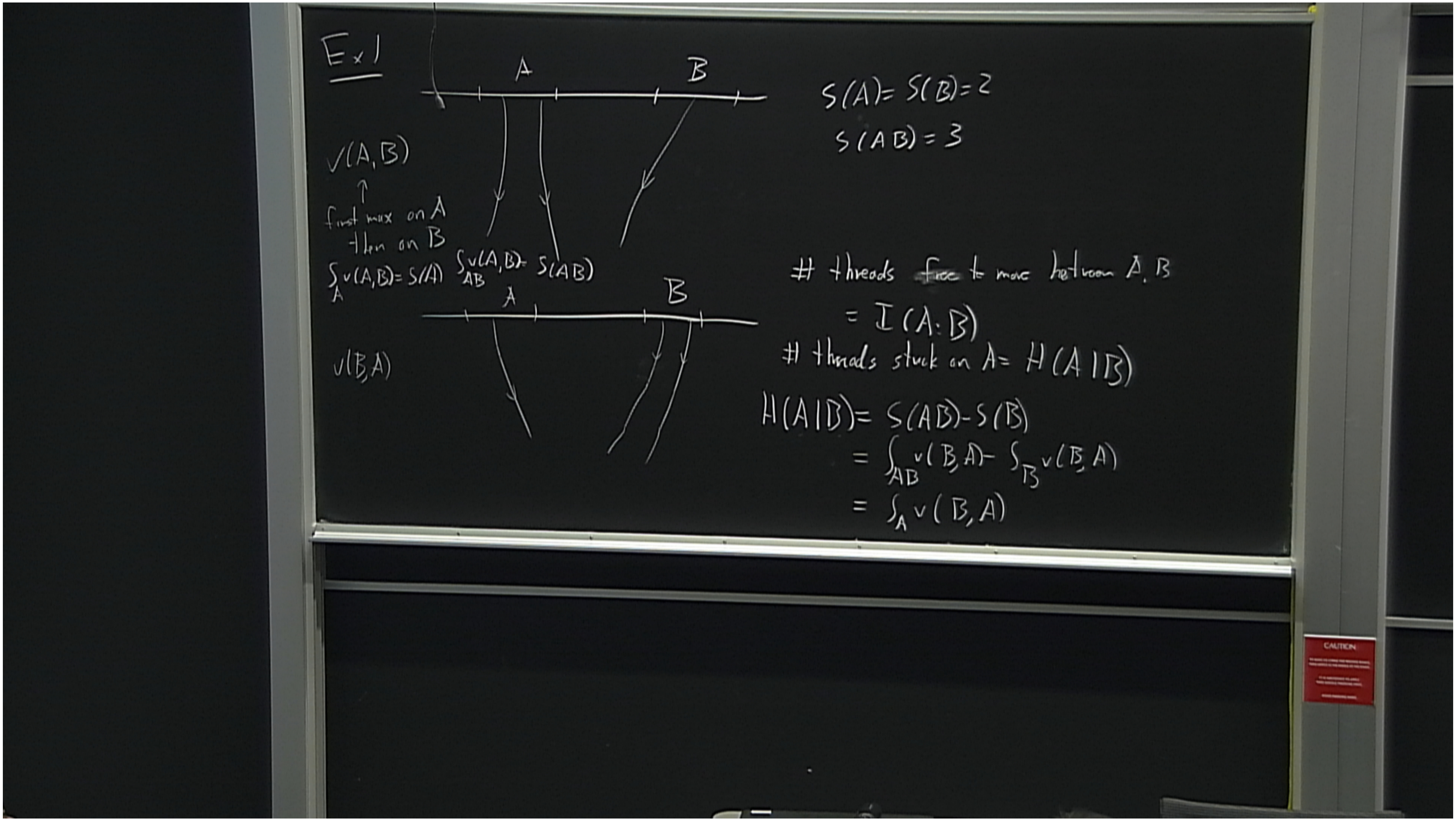
$= I(A:B)$

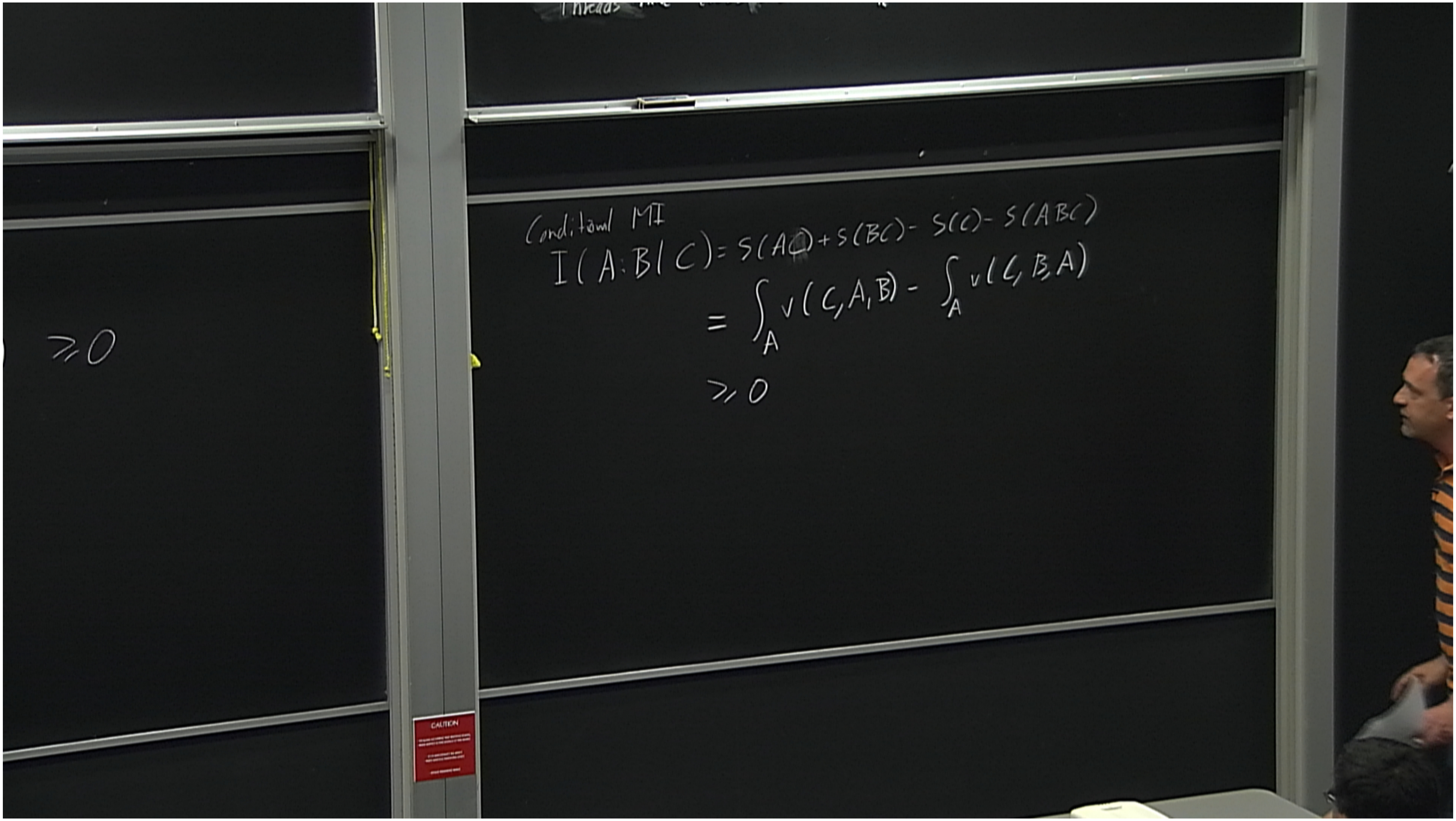
threads stuck on $A = H(A|B)$

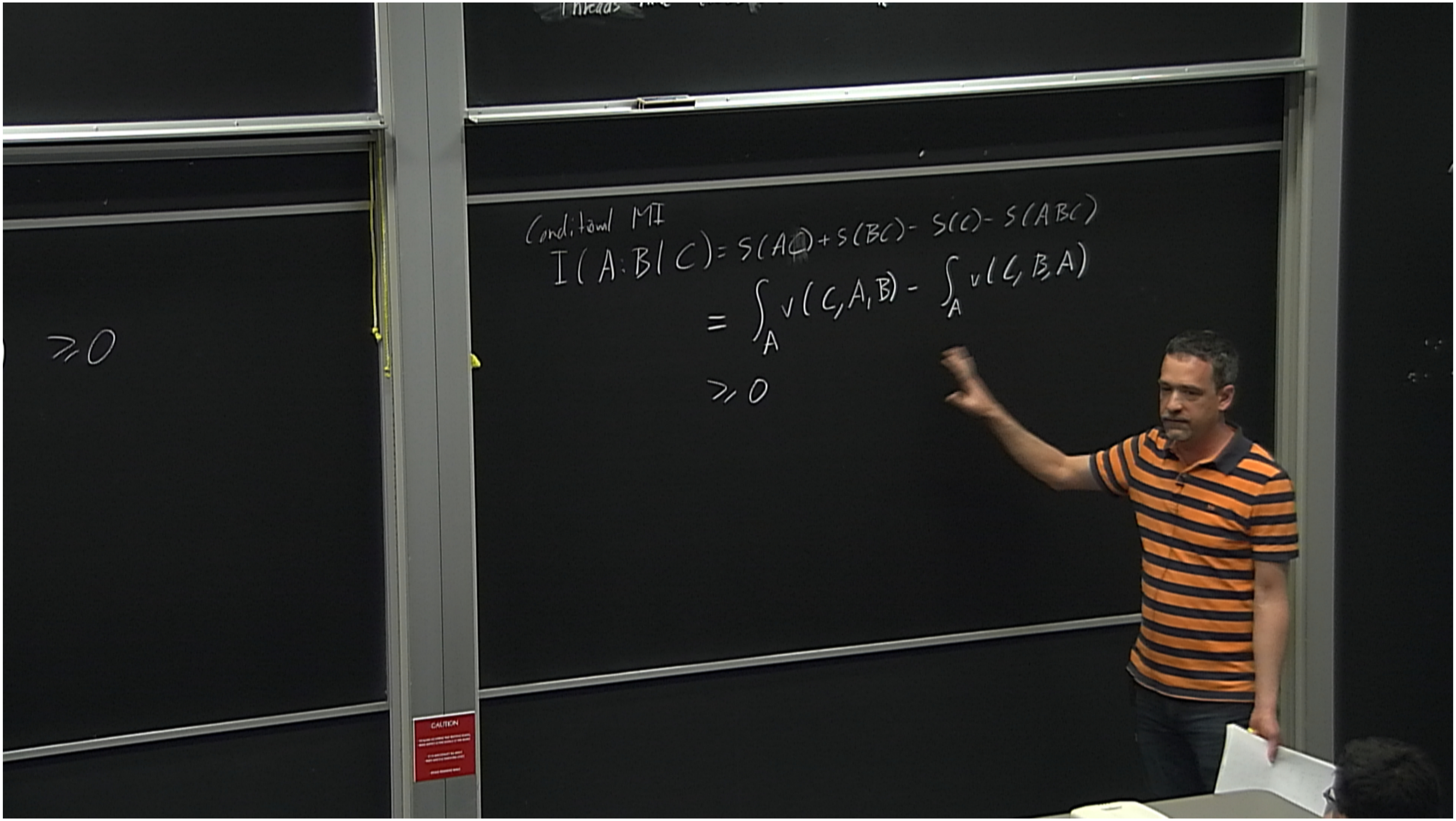
$$H(A|B) = S(AB) - S(B)$$
$$= \int_{AB} v(B,A) - \int_B v(B,A)$$
$$= \int_A v(B,A)$$

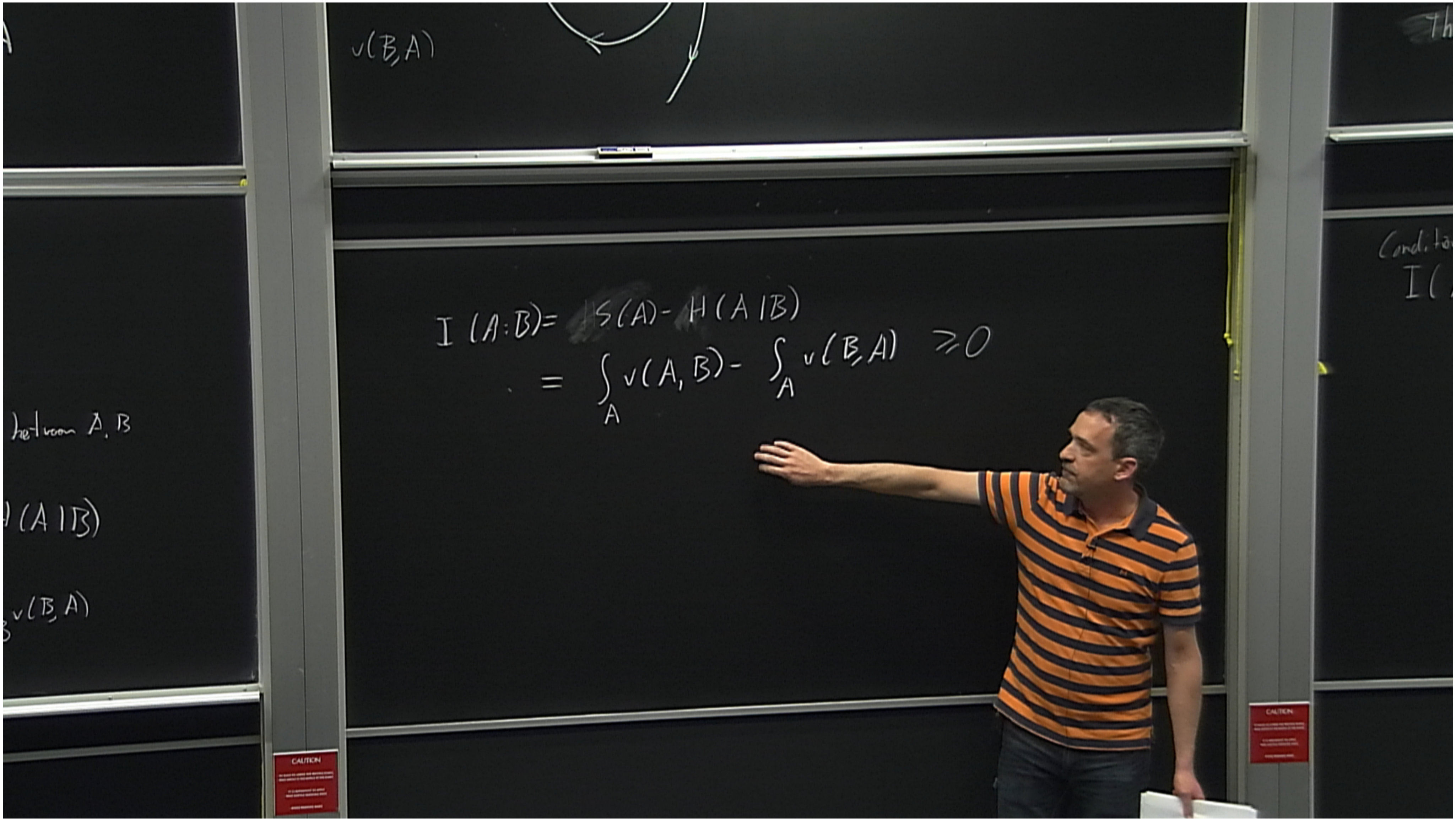

H(B|A)= S(AB)-S(A)
classical: A
B
Ex 2.
S(A)=S(B)=2
S(AB)=1
A
B
v(A,B)
between A, B
H(A|B)
v(B,A)
Thm
RT


Flow: vector field v: 1) |v| ≤ 1
2) ∇_μ v^μ = 0
∫_A v = ∫_m
m ~ A
Thm: max_v ∫_A v = min_{m~A} area(m)
On m(A), v(A) = n|_{m(A)}
max flow
elsewhere, freedom in choosing v(A)
RT 2.0: S(A) = 1/(4G_N) max_v ∫_A v
= max # threads leaving A
Threads have cross-section 4G_N
A
m(A)
v(A)
CAUTION


Ex 1
A
B
S(A) = S(B) = 2
S(AB) = 3
v(A, B)
first max on A
then on B
∫_A v(A,B) = S(A)
∫_AB v(A,B) = S(AB)
A
B
threads free to move between A, B
= I(A:B)
# threads stuck on A = H(A|B)
v(B,A)
H(A|B) = S(AB) - S(B)
= ∫_AB v(B,A) - ∫_B v(B,A)
= ∫_A v(B,A)
CAUTION


≥ 0
Conditional MI
I(A:B|C) = S(AC) + S(BC) - S(C) - S(ABC)
= ∫A v(C,A,B) - ∫A v(C,B,A)
≥ 0


Conditional MI
I(A:B|C) = S(AC) + S(BC) - S(C) - S(ABC)
= ∫A v(C,A,B) - ∫A v(C,B,A)
≥ 0
≥ 0
CAUTION


v(B,A)
I (A:B) = S(A) - H(A|B)
= ∫_A v(A,B) - ∫_A v(B,A) ≥ 0
between A, B
(A|B)
v(B,A)

Questions

1) Tripartite info $I_3(A:B:C)$

Superadditivity

~~Monogamy~~ of MI

$I_3 \leq 0$

2) What kinds of entanglement + correlation can be represented by threads?

3) Higher-curvature corrections?

4) Quantum "