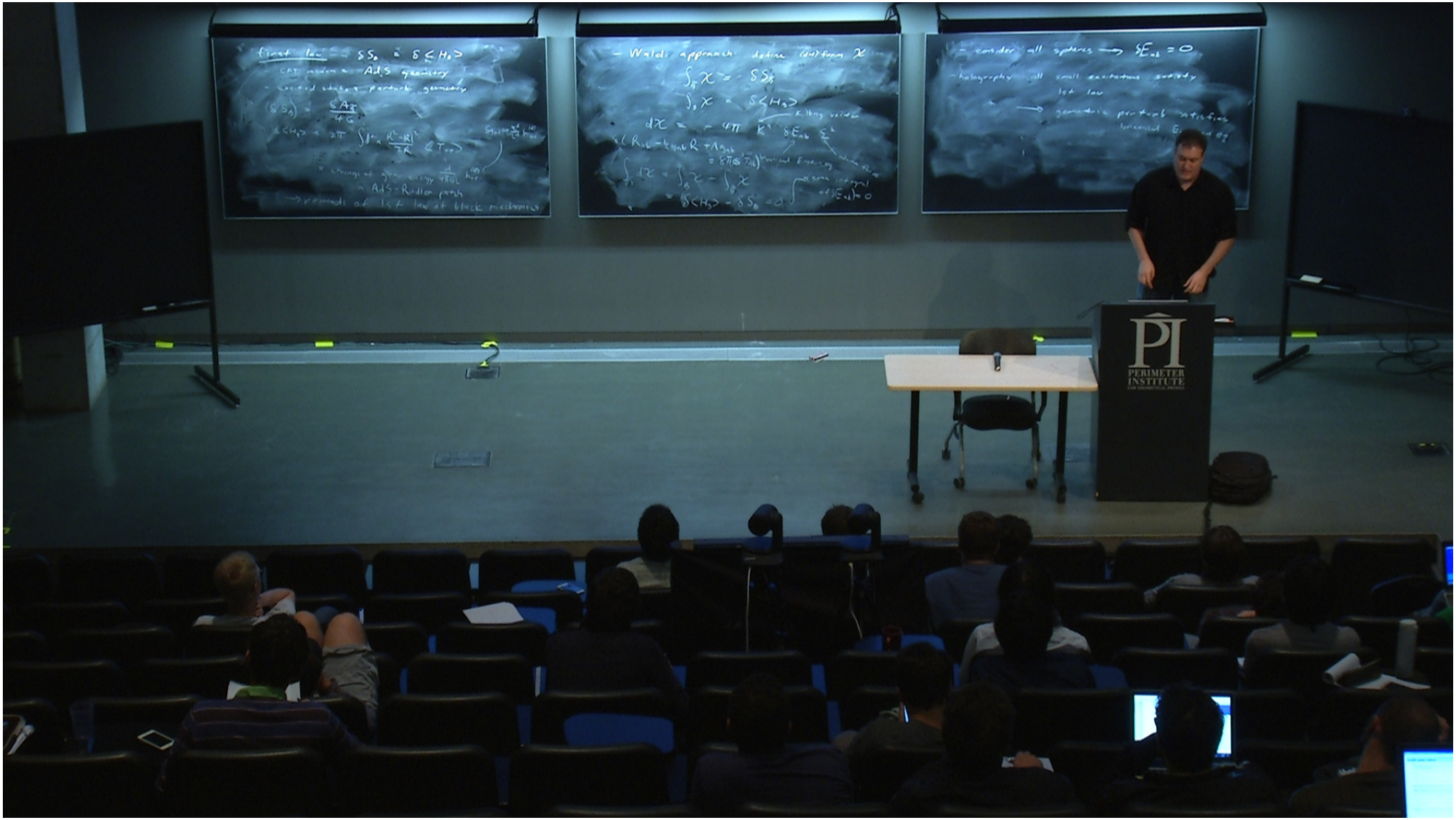
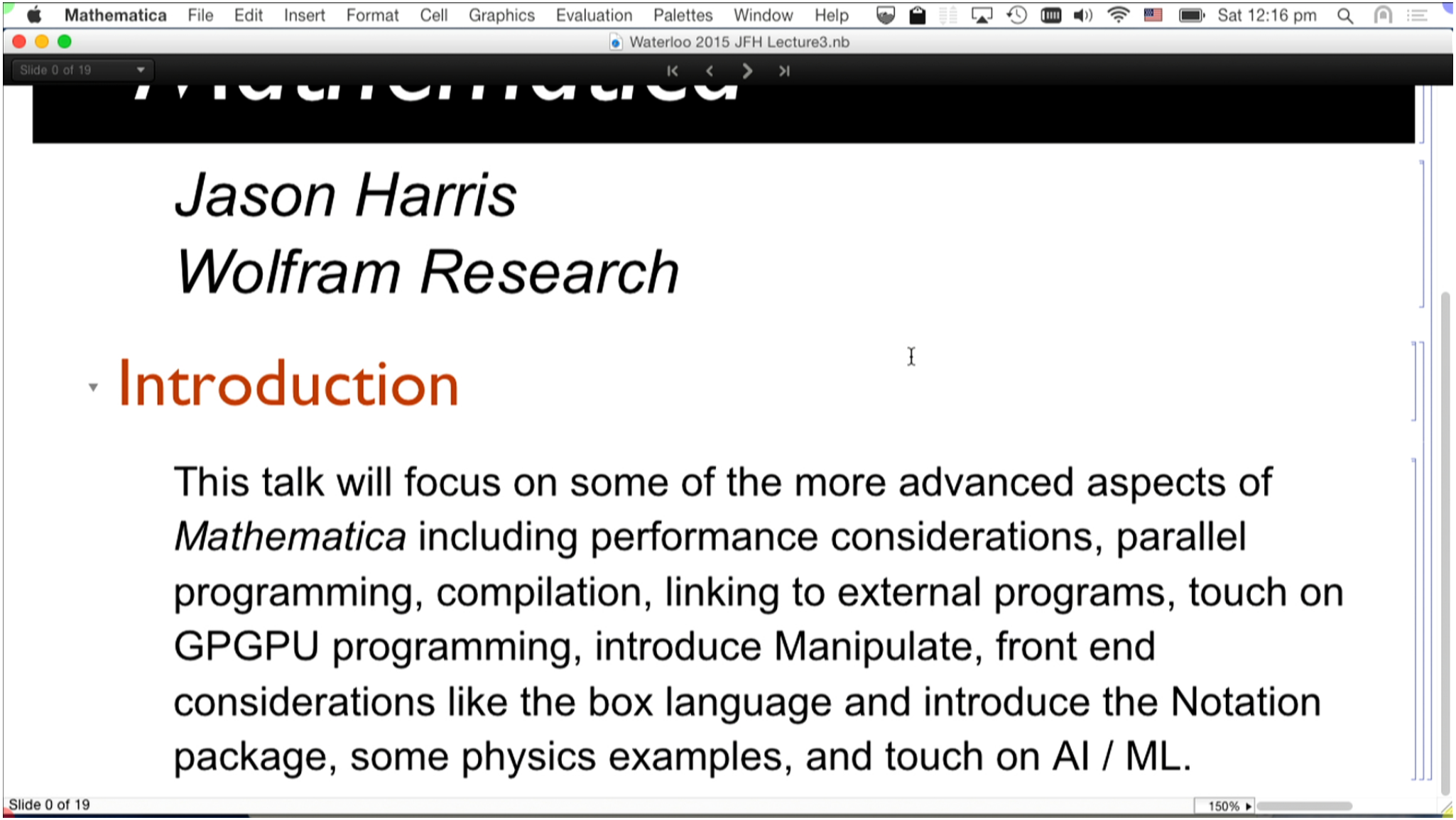
Title: TBA

Date: Aug 29, 2015 12:00 PM

URL: http://pirsa.org/15080057

Abstract: TBA


PERIMETER INSTITUTE


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:16 pm
Waterloo 2015 JFH Lecture3.nb
Slide 0 of 19
Jason Harris
Wolfram Research
Introduction
This talk will focus on some of the more advanced aspects of Mathematica including performance considerations, parallel programming, compilation, linking to external programs, touch on GPGPU programming, introduce Manipulate, front end considerations like the box language and introduce the Notation package, some physics examples, and touch on AI / ML.
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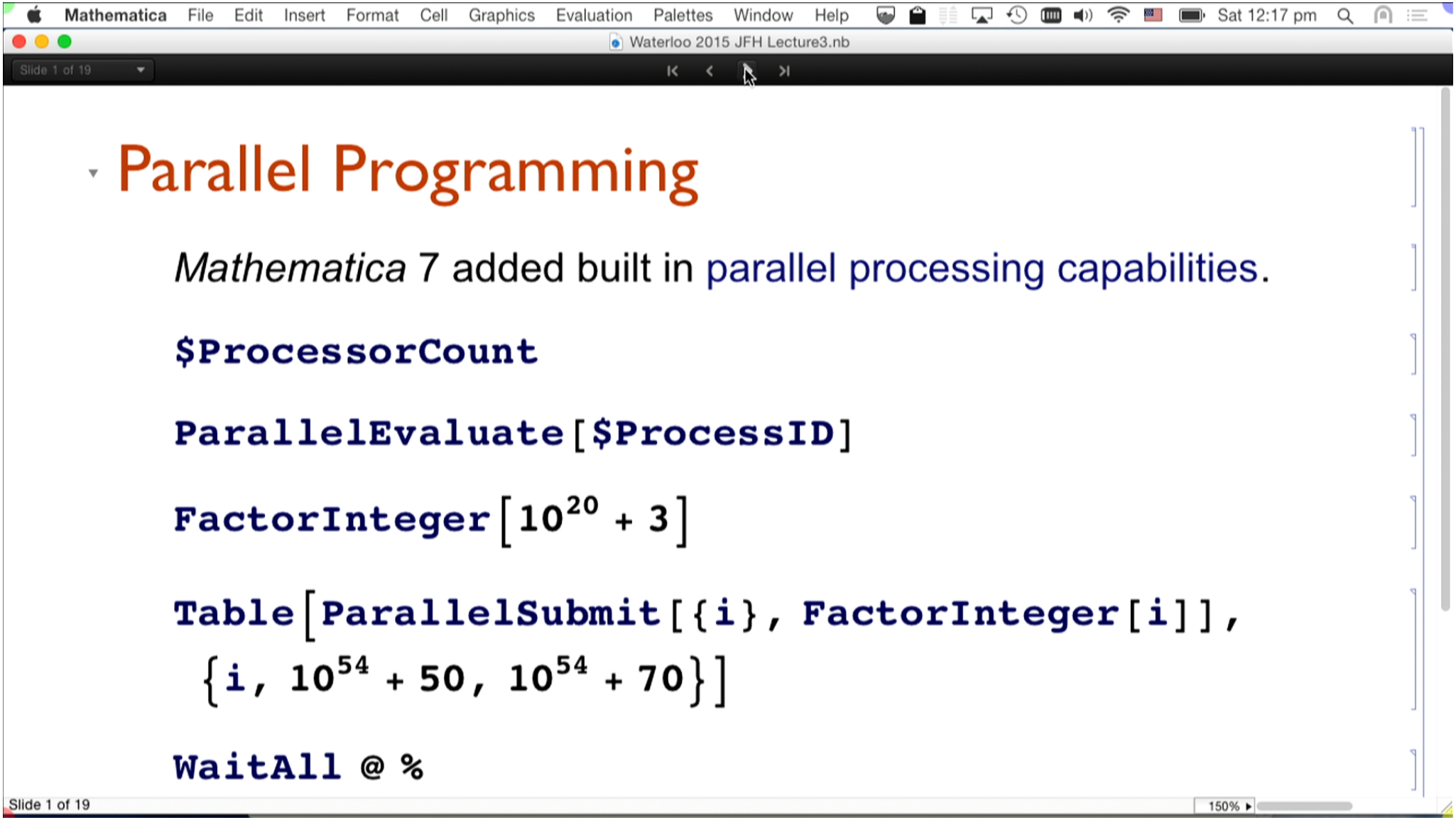
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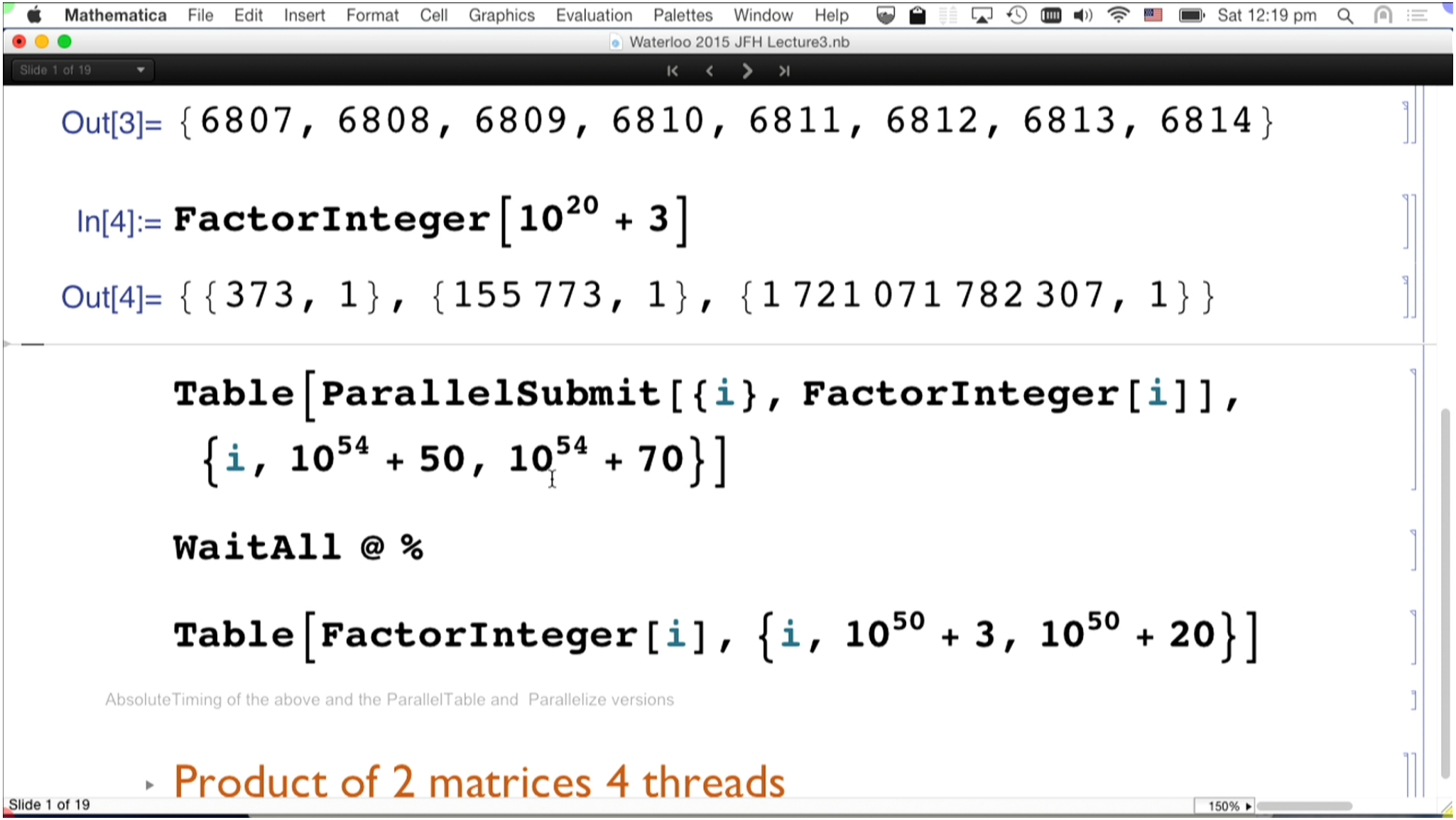
Slide 1 of 19
Parallel Programming
Mathematica 7 added built in parallel processing capabilities.
$ProcessorCount
ParallelEvaluate[$ProcessID]
FactorInteger[10^20 + 3]
Table[ParallelSubmit[{i}, FactorInteger[i]],
{i, 10^54 + 50, 10^54 + 70}]
WaitAll @ %
Slide 1 of 19
150%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Out[3]= {6807, 6808, 6809, 6810, 6811, 6812, 6813, 6814}
In[4]:= FactorInteger[10^20 + 3]
Out[4]= {{373, 1}, {155 773, 1}, {1 721 071 782 307, 1}}
Table[ParallelSubmit[{i}, FactorInteger[i]], {i, 10^54 + 50, 10^54 + 70}]
WaitAll @ %
Table[FactorInteger[i], {i, 10^50 + 3, 10^50 + 20}]
AbsoluteTiming of the above and the ParallelTable and Parallelize versions
Product of 2 matrices 4 threads
Slide 1 of 19
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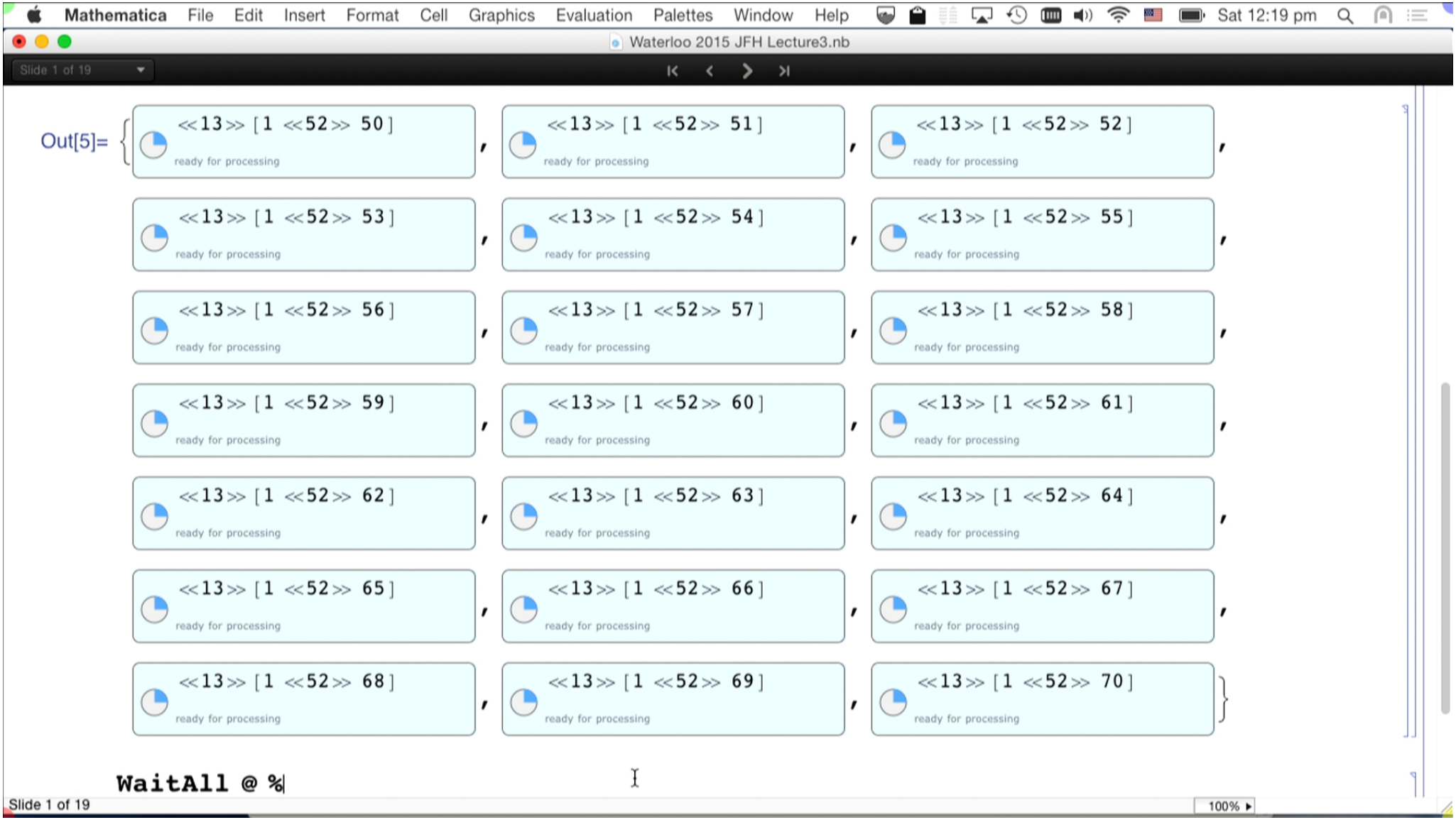
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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Out[5]=
<<13>> [1 <<52>> 50]
<<13>> [1 <<52>> 51]
<<13>> [1 <<52>> 52]
<<13>> [1 <<52>> 53]
<<13>> [1 <<52>> 54]
<<13>> [1 <<52>> 55]
<<13>> [1 <<52>> 56]
<<13>> [1 <<52>> 57]
<<13>> [1 <<52>> 58]
<<13>> [1 <<52>> 59]
<<13>> [1 <<52>> 60]
<<13>> [1 <<52>> 61]
<<13>> [1 <<52>> 62]
<<13>> [1 <<52>> 63]
<<13>> [1 <<52>> 64]
<<13>> [1 <<52>> 65]
<<13>> [1 <<52>> 66]
<<13>> [1 <<52>> 67]
<<13>> [1 <<52>> 68]
<<13>> [1 <<52>> 69]
<<13>> [1 <<52>> 70]
ready for processing
WaitAll @ %
Slide 1 of 19
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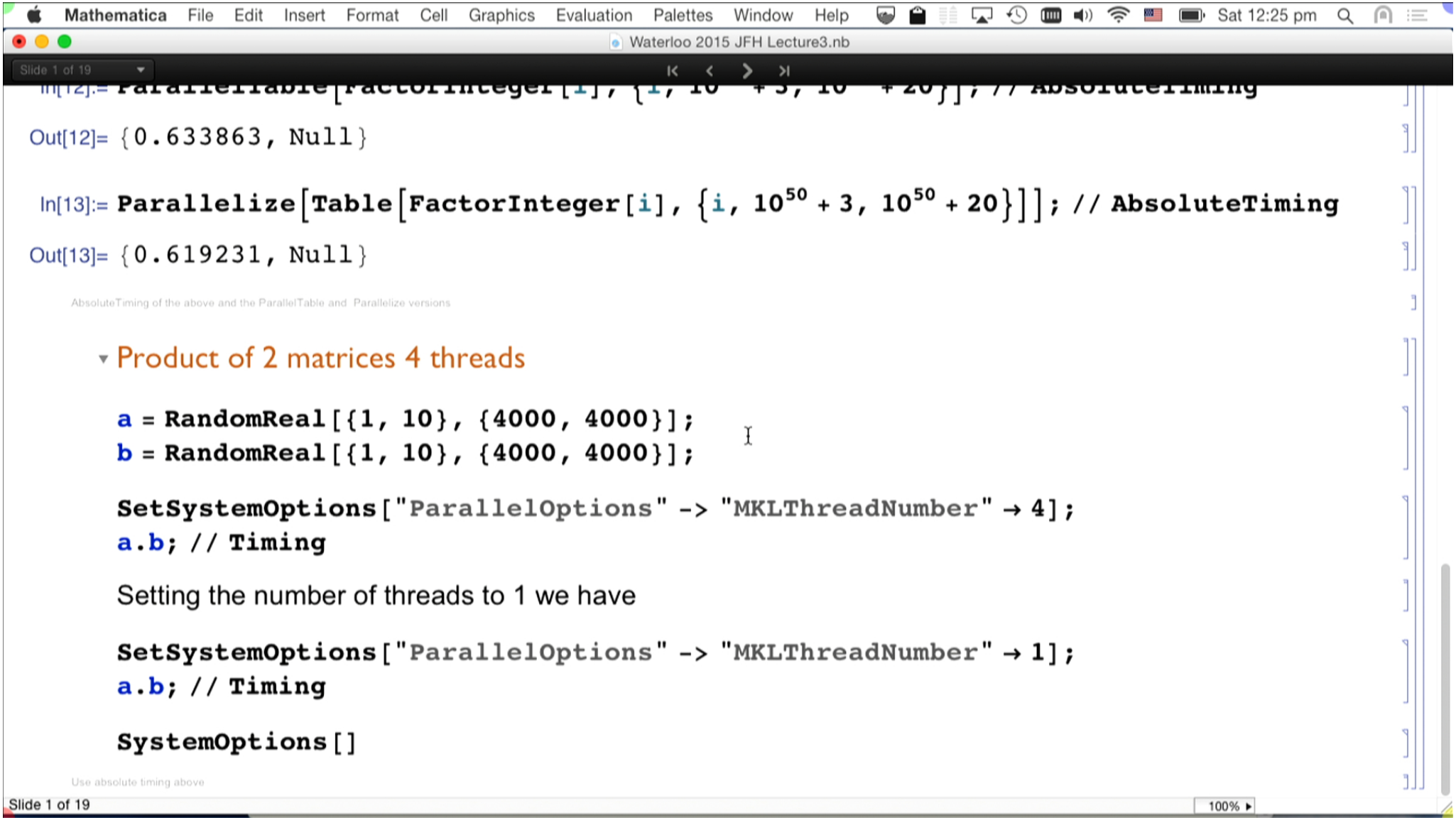
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Slide 1 of 19
Out[12]= {0.633863, Null}
In[13]:= Parallelize[Table[FactorInteger[i], {i, 10^50 + 3, 10^50 + 20}]]; // AbsoluteTiming
Out[13]= {0.619231, Null}
AbsoluteTiming of the above and the ParallelTable and Parallelize versions
Product of 2 matrices 4 threads
a = RandomReal[{1, 10}, {4000, 4000}];
b = RandomReal[{1, 10}, {4000, 4000}];
SetSystemOptions["ParallelOptions" -> "MKLThreadNumber" → 4];
a.b; // Timing
Setting the number of threads to 1 we have
SetSystemOptions["ParallelOptions" -> "MKLThreadNumber" → 1];
a.b; // Timing
SystemOptions[]
Use absolute timing above
Slide 1 of 19
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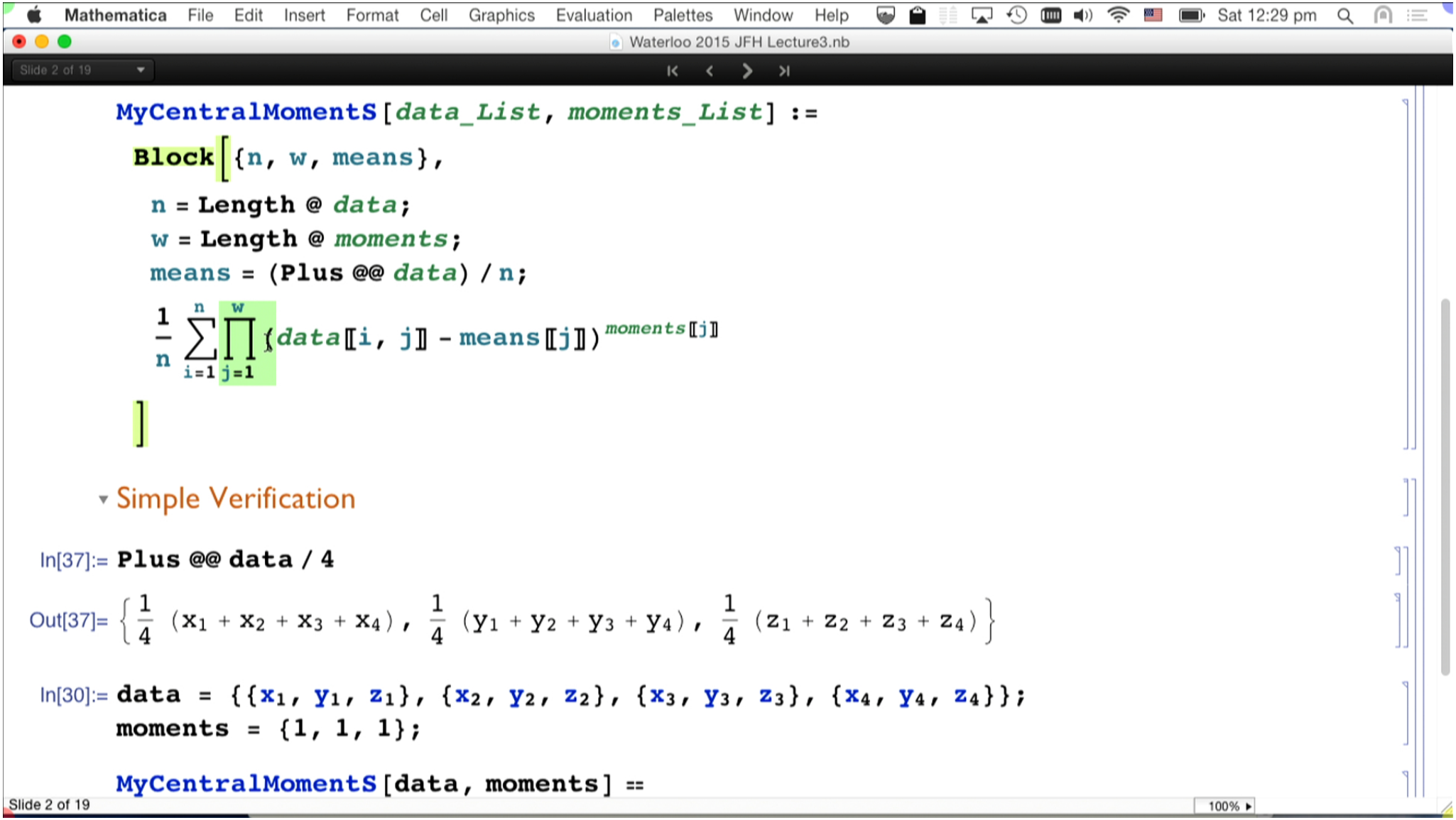
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:29 pm
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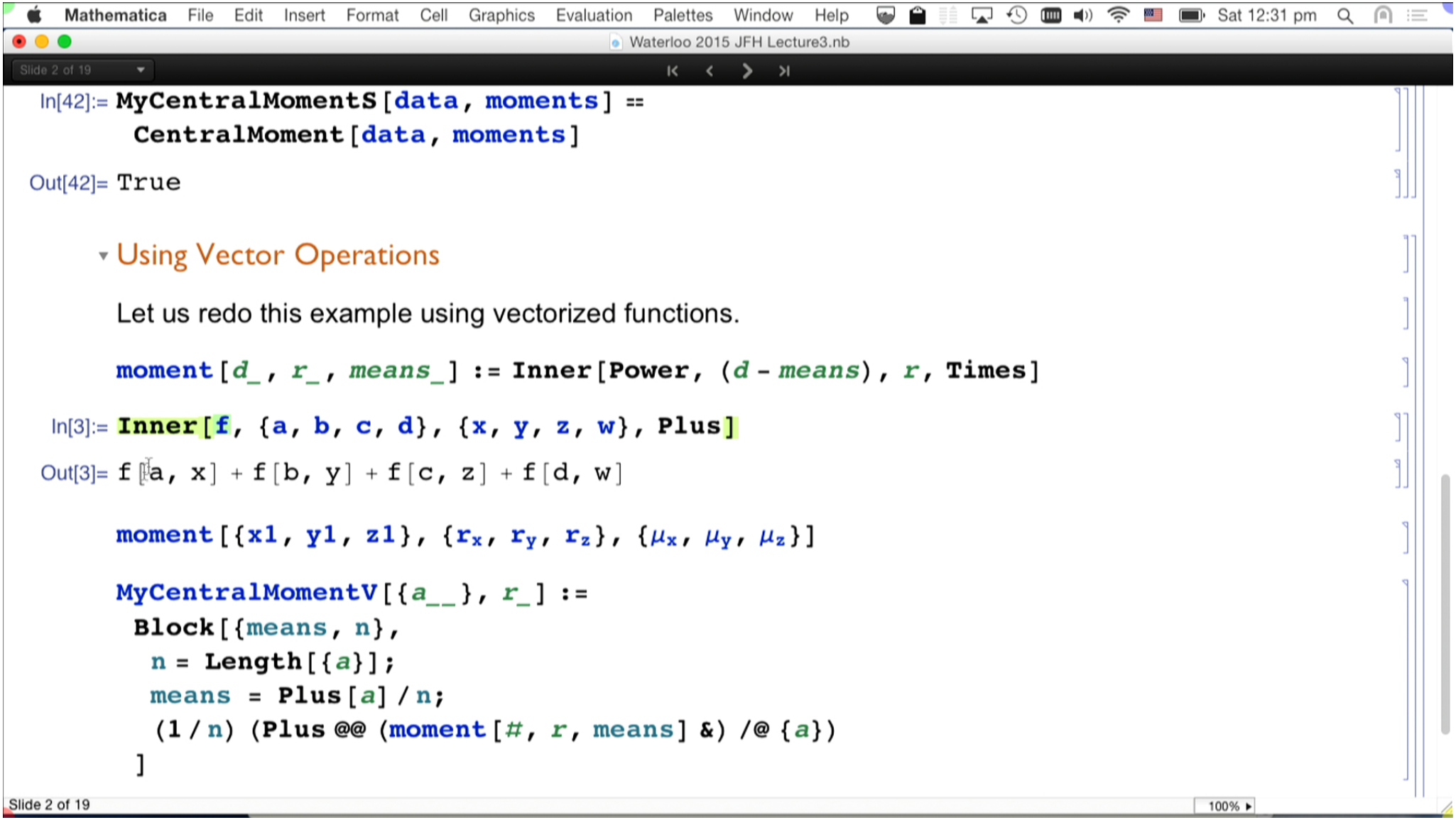
Slide 2 of 19
MyCentralMomentS[data_List, moments_List] :=
Block[{n, w, means},
n = Length @ data;
w = Length @ moments;
means = (Plus @@ data) / n;
1/n Σ_{i=1}^{n} Π_{j=1}^{w} (data[[i, j]] - means[[j]])^moments[[j]]
]
Simple Verification
In[37]:= Plus @@ data / 4
Out[37]= {1/4 (x1 + x2 + x3 + x4), 1/4 (y1 + y2 + y3 + y4), 1/4 (z1 + z2 + z3 + z4)}
In[30]:= data = {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}, {x4, y4, z4}};
moments = {1, 1, 1};
MyCentralMomentS[data, moments] ==
Slide 2 of 19
100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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In[42]:= MyCentralMomentS[data, moments] ==
CentralMoment[data, moments]
Out[42]= True
Using Vector Operations
Let us redo this example using vectorized functions.
moment[d_, r_, means_] := Inner[Power, (d - means), r, Times]
In[3]:= Inner[f, {a, b, c, d}, {x, y, z, w}, Plus]
Out[3]= f[a, x] + f[b, y] + f[c, z] + f[d, w]
moment[{x1, y1, z1}, {rx, ry, rz}, {μx, μy, μz}]
MyCentralMomentV[{a__}, r_] :=
Block[{means, n},
n = Length[{a}];
means = Plus[a] / n;
(1 / n) (Plus @@ (moment[#, r, means] &) /@ {a})
]
Slide 2 of 19
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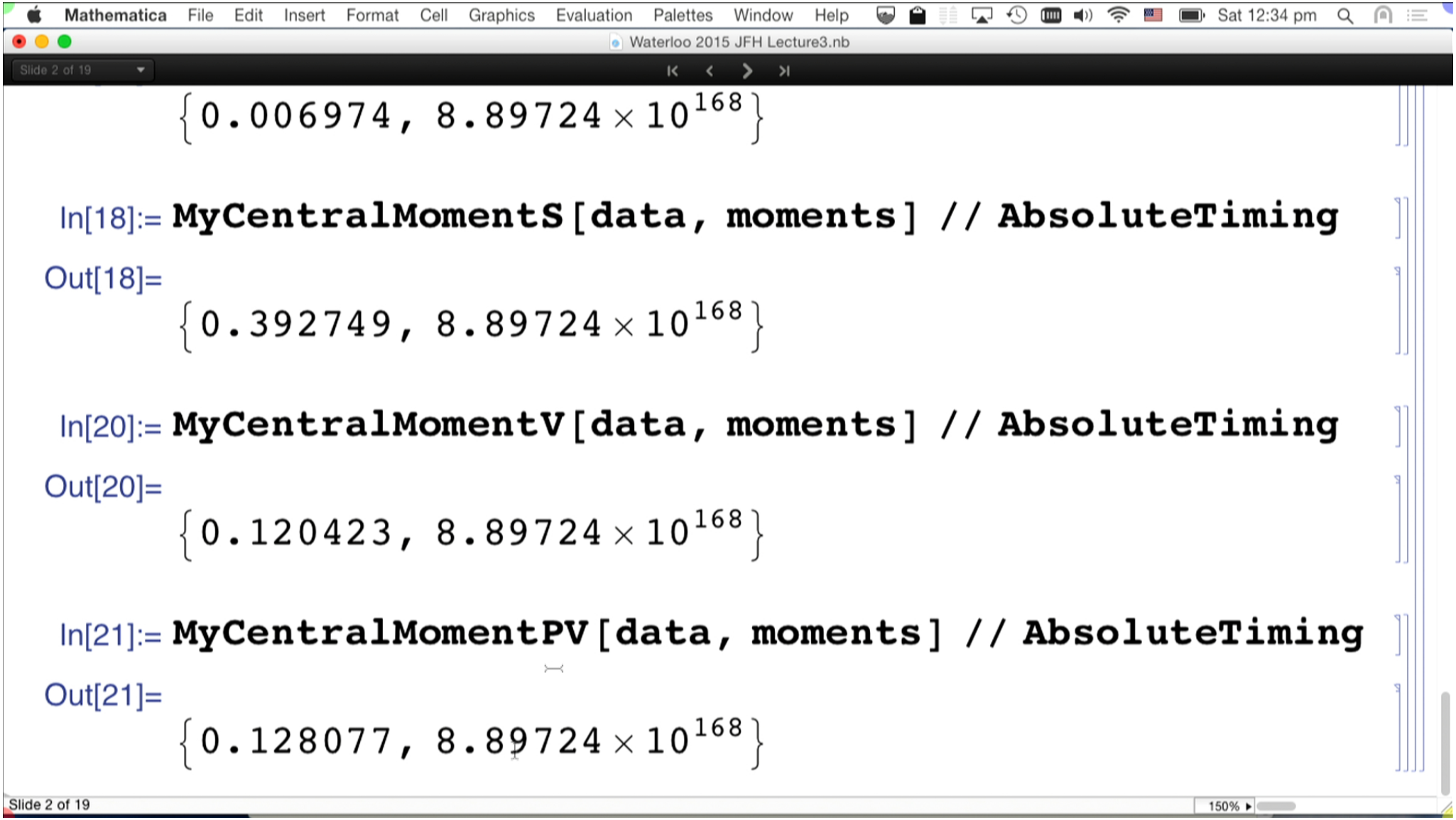
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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{0.006974, 8.89724 × 10^168}
In[18]:= MyCentralMomentS[data, moments] // AbsoluteTiming
Out[18]=
{0.392749, 8.89724 × 10^168}
In[20]:= MyCentralMomentV[data, moments] // AbsoluteTiming
Out[20]=
{0.120423, 8.89724 × 10^168}
In[21]:= MyCentralMomentPV[data, moments] // AbsoluteTiming
Out[21]=
{0.128077, 8.89724 × 10^168}
Slide 2 of 19
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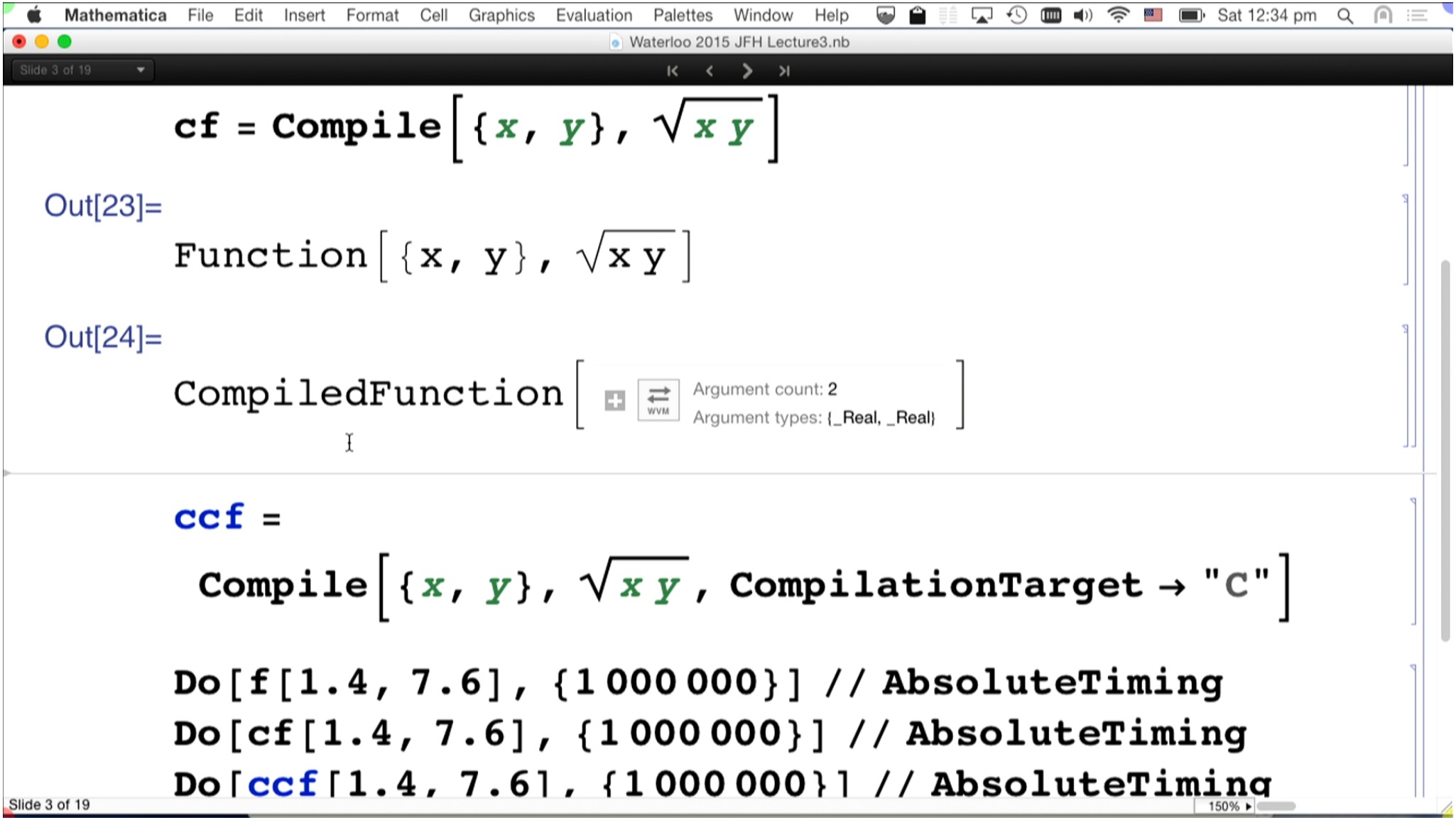
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cf = Compile[{x, y}, √(x y)]
Out[23]=
Function[{x, y}, √(x y)]
Out[24]=
CompiledFunction[Argument count: 2 Argument types: {_Real, _Real}]
ccf =
Compile[{x, y}, √(x y), CompilationTarget → "C"]
Do[f[1.4, 7.6], {1000000}] // AbsoluteTiming
Do[cf[1.4, 7.6], {1000000}] // AbsoluteTiming
Do[ccf[1.4, 7.6], {1000000}] // AbsoluteTiming
Slide 3 of 19
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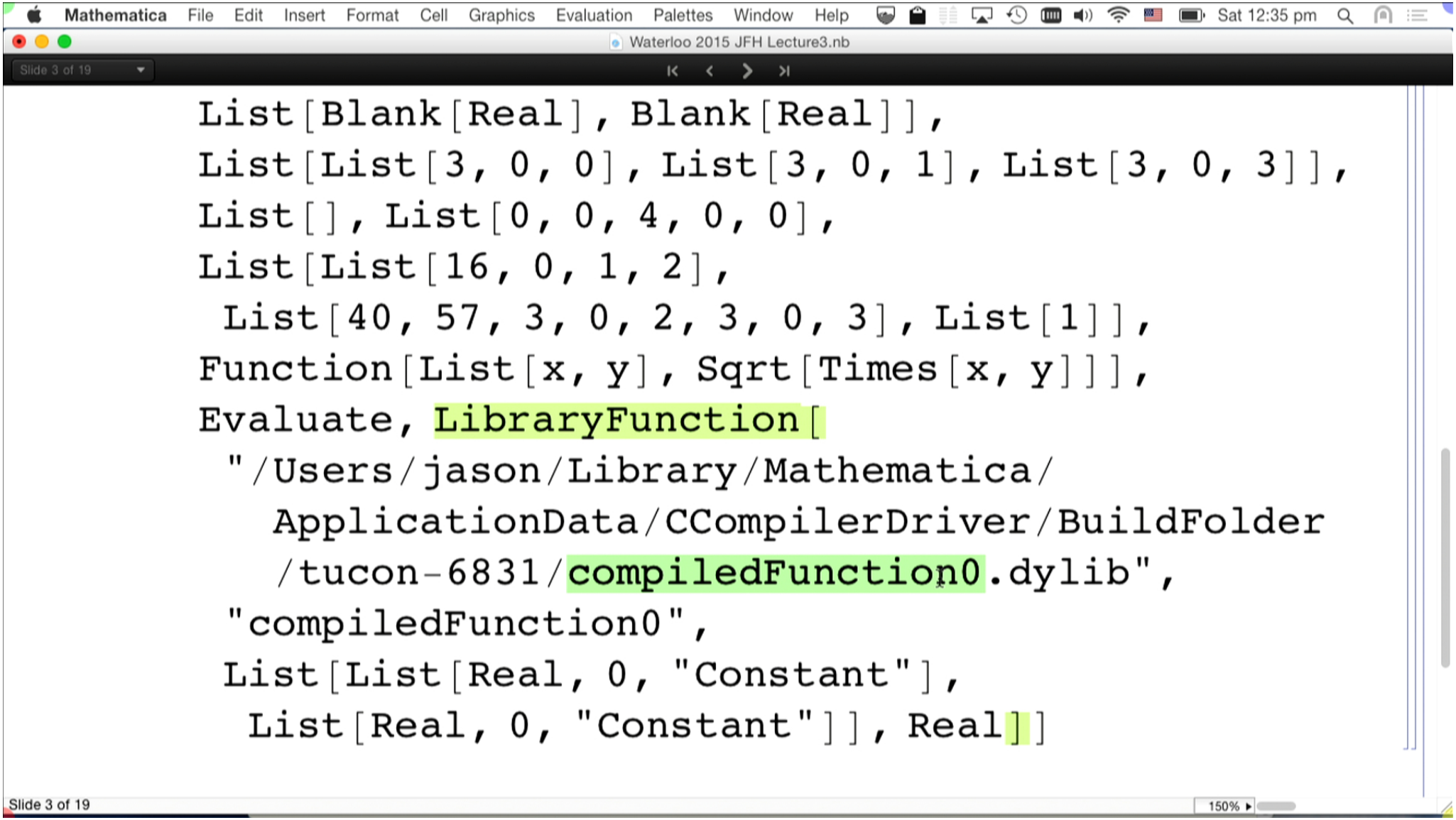
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Waterloo 2015 JFH Lecture3.nb
Slide 3 of 19
List[Blank[Real], Blank[Real]],
List[List[3, 0, 0], List[3, 0, 1], List[3, 0, 3]],
List[], List[0, 0, 4, 0, 0],
List[List[16, 0, 1, 2],
List[40, 57, 3, 0, 2, 3, 0, 3], List[1]],
Function[List[x, y], Sqrt[Times[x, y]]],
Evaluate, LibraryFunction[
"/Users/jason/Library/Mathematica/
ApplicationData/CCompilerDriver/BuildFolder
/tucon-6831/compiledFunction0.dylib",
"compiledFunction0",
List[List[Real, 0, "Constant"],
List[Real, 0, "Constant"]], Real]]
Slide 3 of 19
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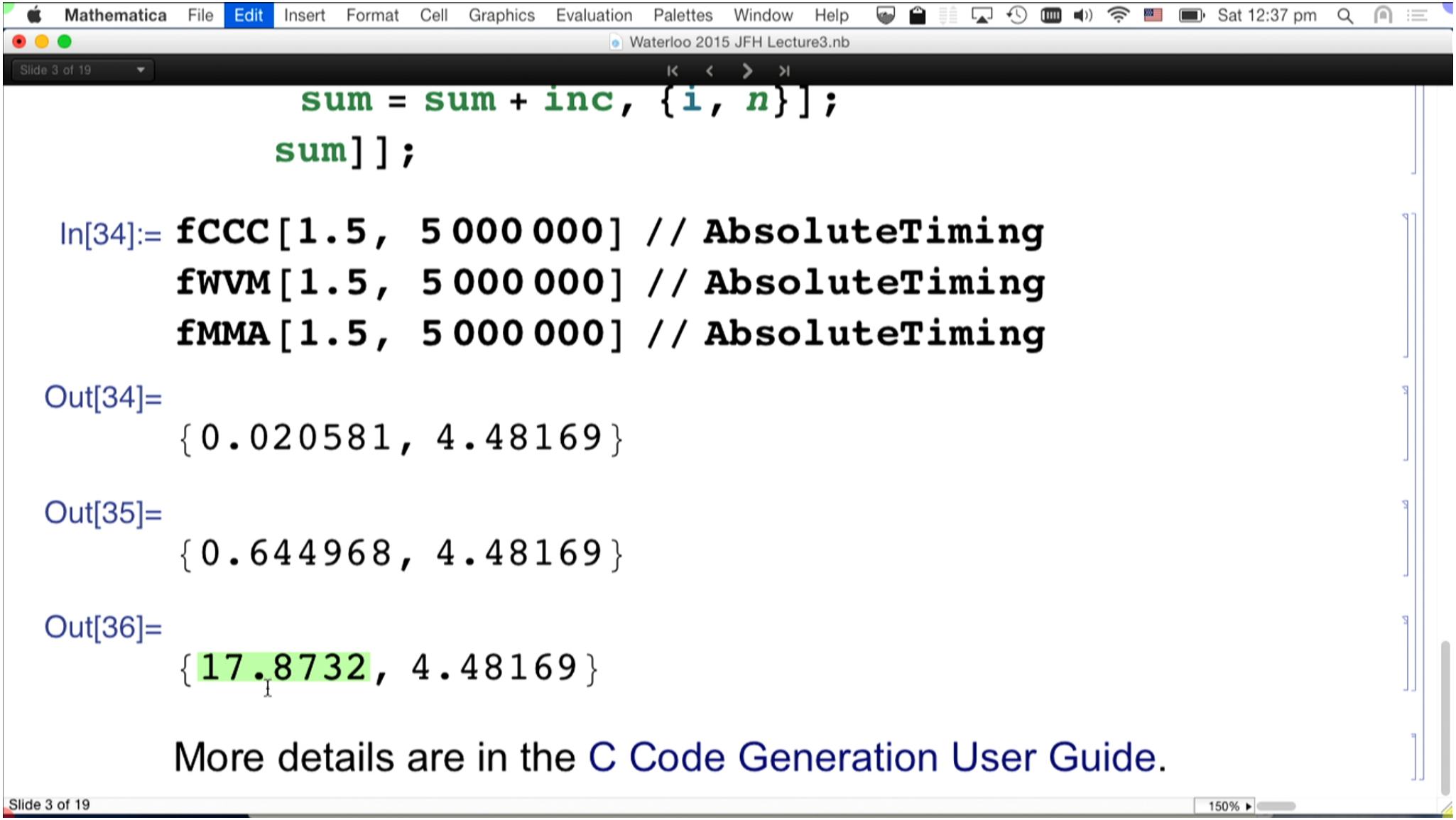
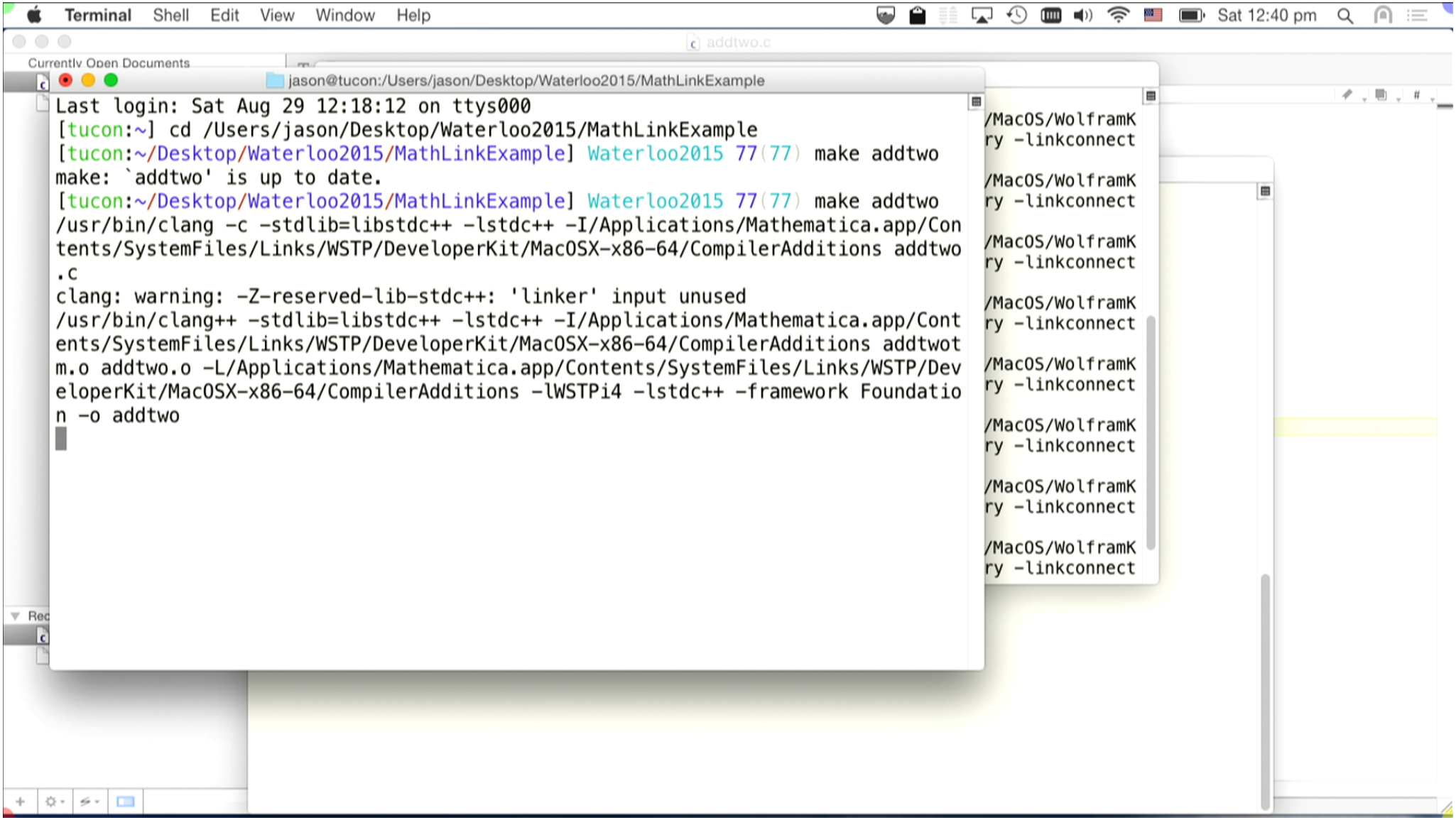
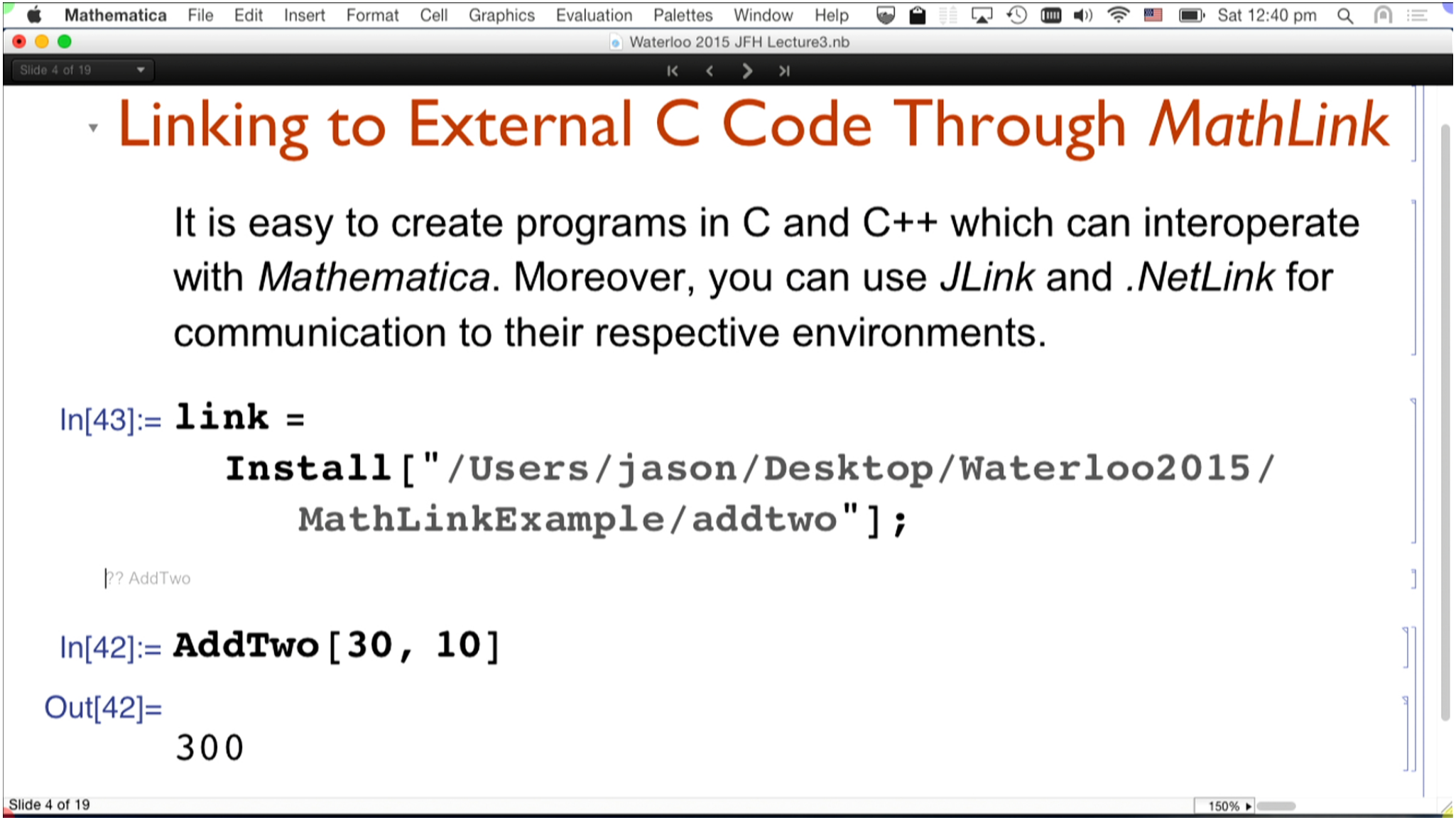

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:37 pm
Waterloo 2015 JFH Lecture3.nb
Slide 3 of 19
sum = sum + inc, {i, n}];
sum]];
In[34]:= fCCC[1.5, 5 000 000] // AbsoluteTiming
fWVM[1.5, 5 000 000] // AbsoluteTiming
fMMA[1.5, 5 000 000] // AbsoluteTiming
Out[34]=
{0.020581, 4.48169}
Out[35]=
{0.644968, 4.48169}
Out[36]=
{17.8732, 4.48169}
More details are in the C Code Generation User Guide.
Slide 3 of 19
150%


Terminal Shell Edit View Window Help
Sat 12:40 pm
addtwo.c
Currently Open Documents
jason@tucon:/Users/jason/Desktop/Waterloo2015/MathLinkExample
Last login: Sat Aug 29 12:18:12 on ttys000
[tucon:~] cd /Users/jason/Desktop/Waterloo2015/MathLinkExample
[tucon:~/Desktop/Waterloo2015/MathLinkExample] Waterloo2015 77(77) make addtwo
make: `addtwo' is up to date.
[tucon:~/Desktop/Waterloo2015/MathLinkExample] Waterloo2015 77(77) make addtwo
/usr/bin/clang -c -stdlib=libstdc++ -lstdc++ -I/Applications/Mathematica.app/Contents/SystemFiles/Links/WSTP/DeveloperKit/MacOSX-x86-64/CompilerAdditions addtwo.c
clang: warning: -Z-reserved-lib-stdc++: 'linker' input unused
/usr/bin/clang++ -stdlib=libstdc++ -lstdc++ -I/Applications/Mathematica.app/Contents/SystemFiles/Links/WSTP/DeveloperKit/MacOSX-x86-64/CompilerAdditions addtwotm.o addtwo.o -L/Applications/Mathematica.app/Contents/SystemFiles/Links/WSTP/DeveloperKit/MacOSX-x86-64/CompilerAdditions -lWSTPi4 -lstdc++ -framework Foundation -o addtwo
/MacOS/WolframK
ry -linkconnect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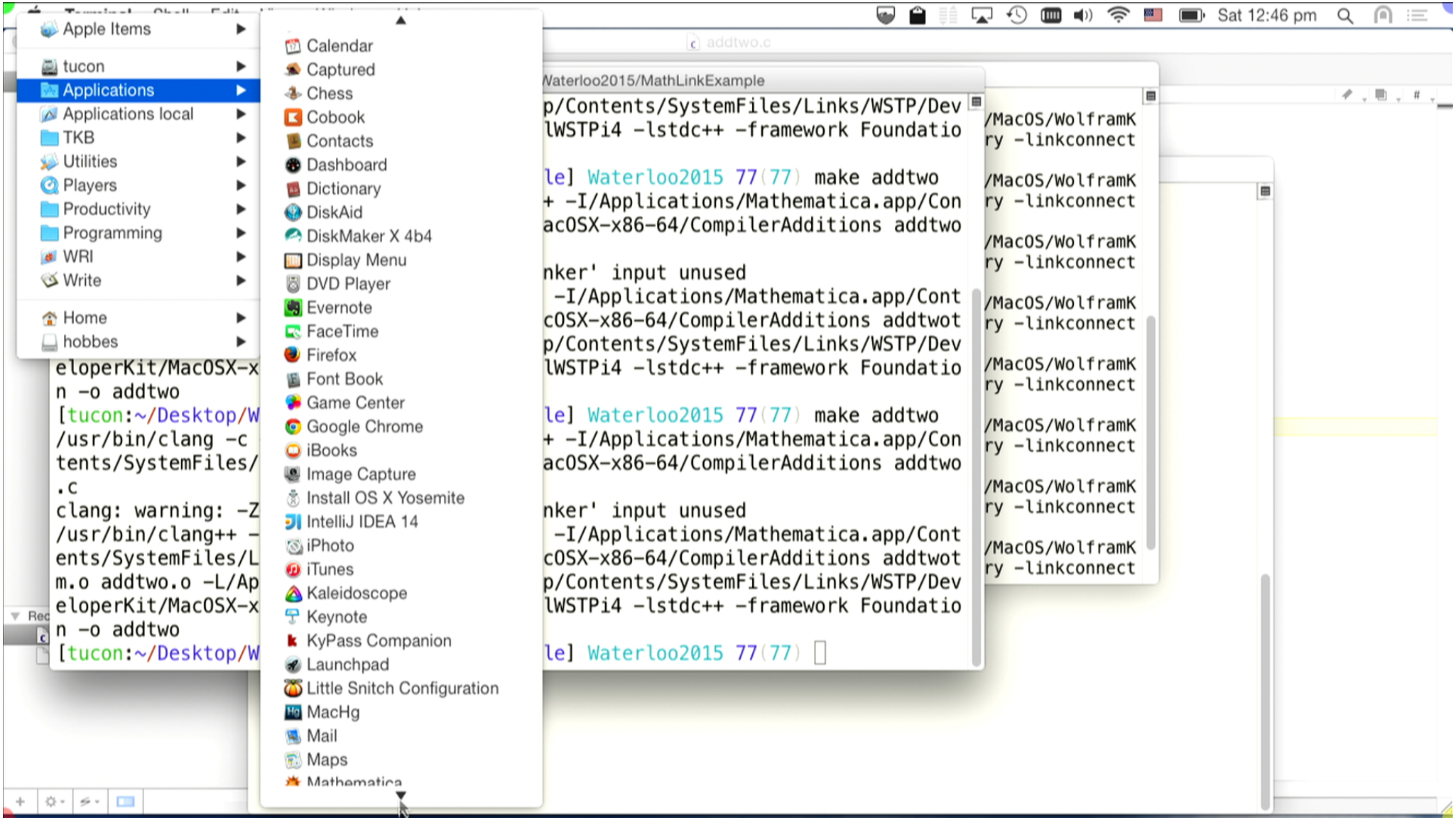
Linking to External C Code Through MathLink
It is easy to create programs in C and C++ which can interoperate with Mathematica. Moreover, you can use JLink and .NetLink for communication to their respective environments.
In[43]:= link =
Install["/Users/jason/Desktop/Waterloo2015/
MathLinkExample/addtwo"];
?? AddTwo
In[42]:= AddTwo[30, 10]
Out[42]=
300
Slide 4 of 19
150%


Apple Items
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Cobook
Contacts
Dashboard
Dictionary
DiskAid
DiskMaker X 4b4
Display Menu
DVD Player
Evernote
FaceTime
Firefox
Font Book
Game Center
Google Chrome
iBooks
Image Capture
Install OS X Yosemite
IntelliJ IDEA 14
iPhoto
iTunes
Kaleidoscope
Keynote
KyPass Companion
Launchpad
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Sat 12:46 pm
addtwo.c
Waterloo2015/MathLinkExample
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.c
clang: warning: -Z
/usr/bin/clang++ -
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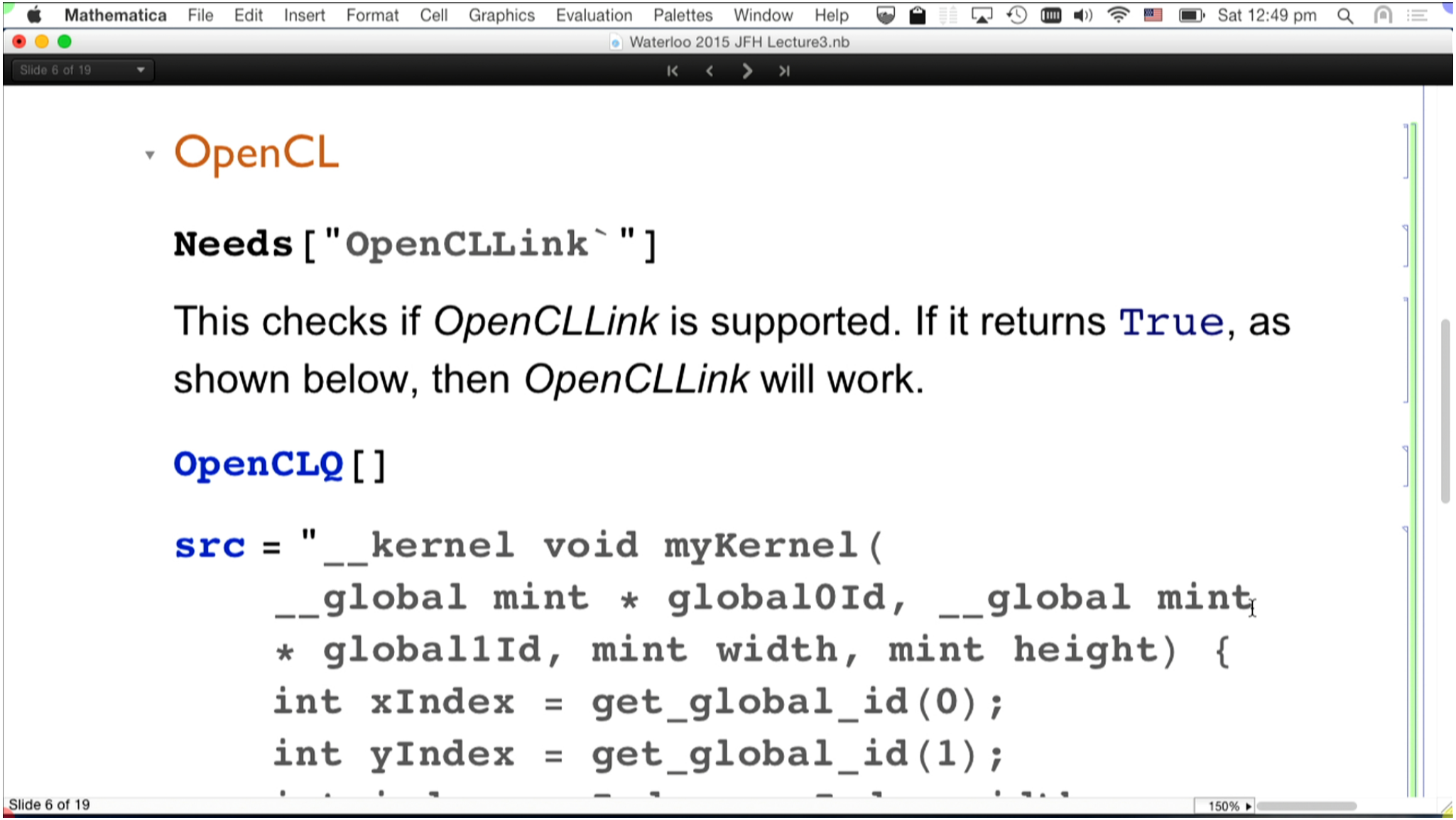
m.o addtwo.o -L/Ap
eloperKit/MacOSX-x
n -o addtwo
[tucon:~/Desktop/W


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:49 pm
Waterloo 2015 JFH Lecture3.nb
Slide 6 of 19
OpenCL
Needs["OpenCLLink`"]
This checks if OpenCLLink is supported. If it returns True, as shown below, then OpenCLLink will work.
OpenCLQ[]
src = "__kernel void myKernel(
__global mint * global0Id, __global mint
* global1Id, mint width, mint height) {
int xIndex = get_global_id(0);
int yIndex = get_global_id(1);
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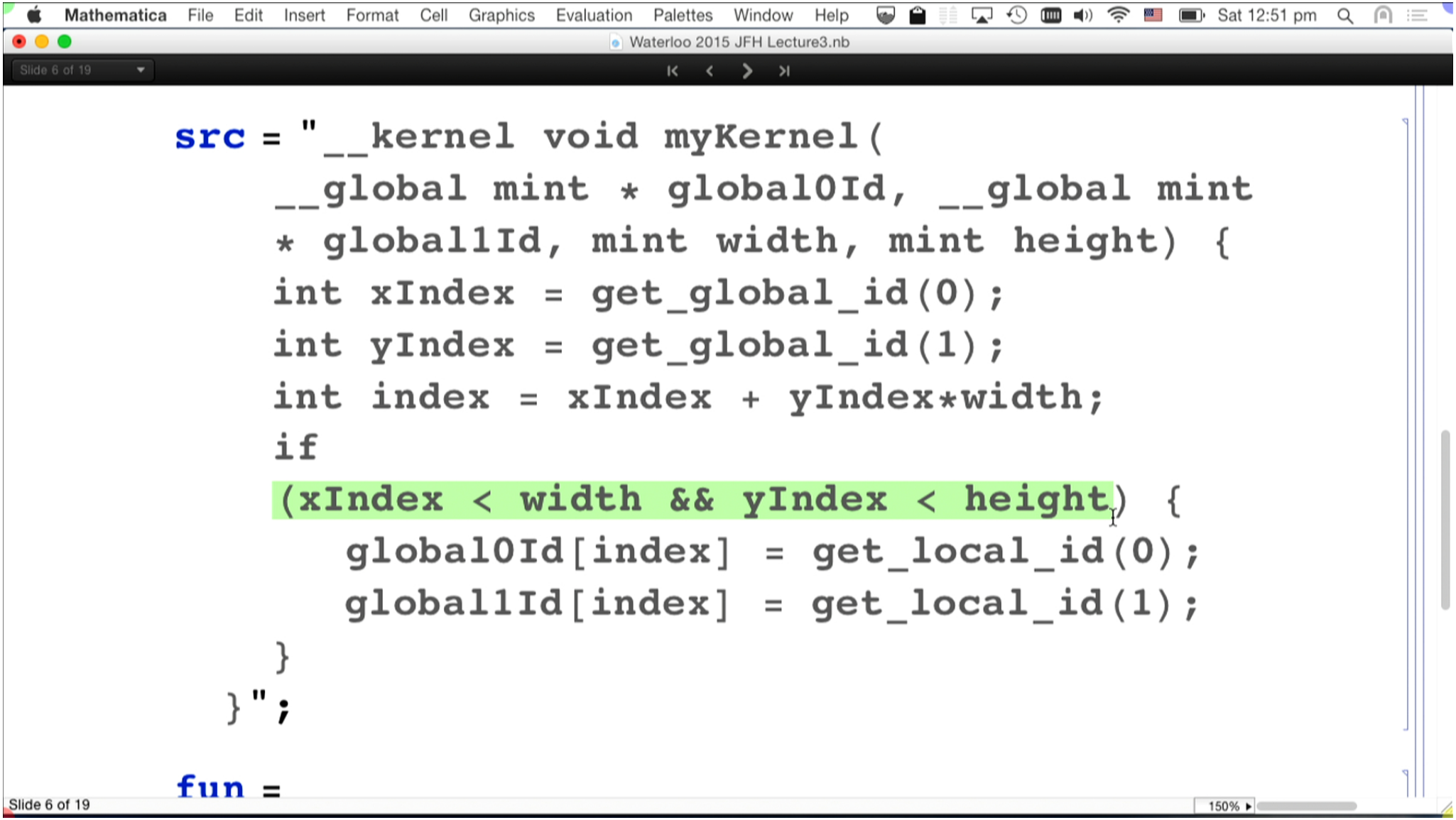
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src = "__kernel void myKernel(
__global mint * global0Id, __global mint
* global1Id, mint width, mint height) {
int xIndex = get_global_id(0);
int yIndex = get_global_id(1);
int index = xIndex + yIndex*width;
if
(xIndex < width && yIndex < height) {
global0Id[index] = get_local_id(0);
global1Id[index] = get_local_id(1);
}
}";
fun =
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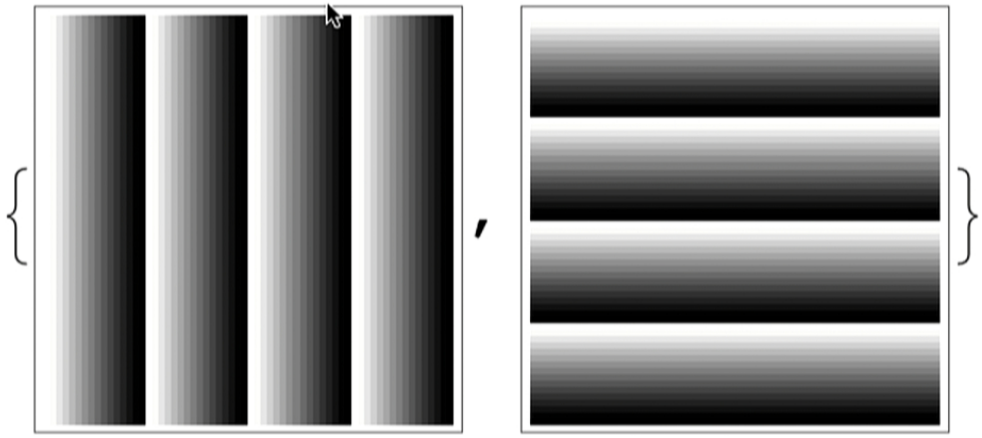
This launches the function.

In[9]:=
```
res = fun[global0Id, global1Id, width, height];
```

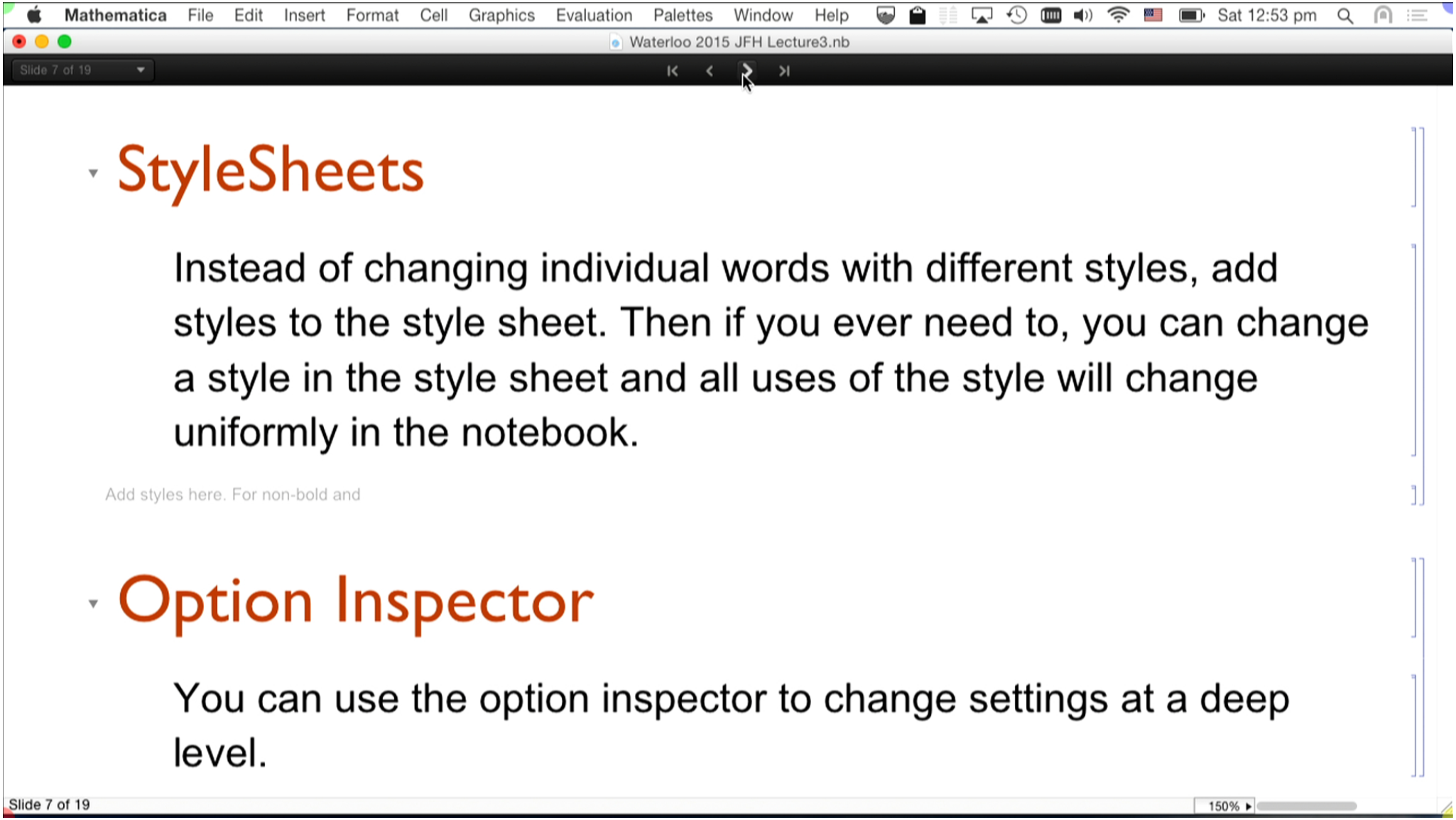
This visualizes the results.

In[10]:=
```
ArrayPlot /@ res
```

Out[10]=



Slide 6 of 19 150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:53 pm
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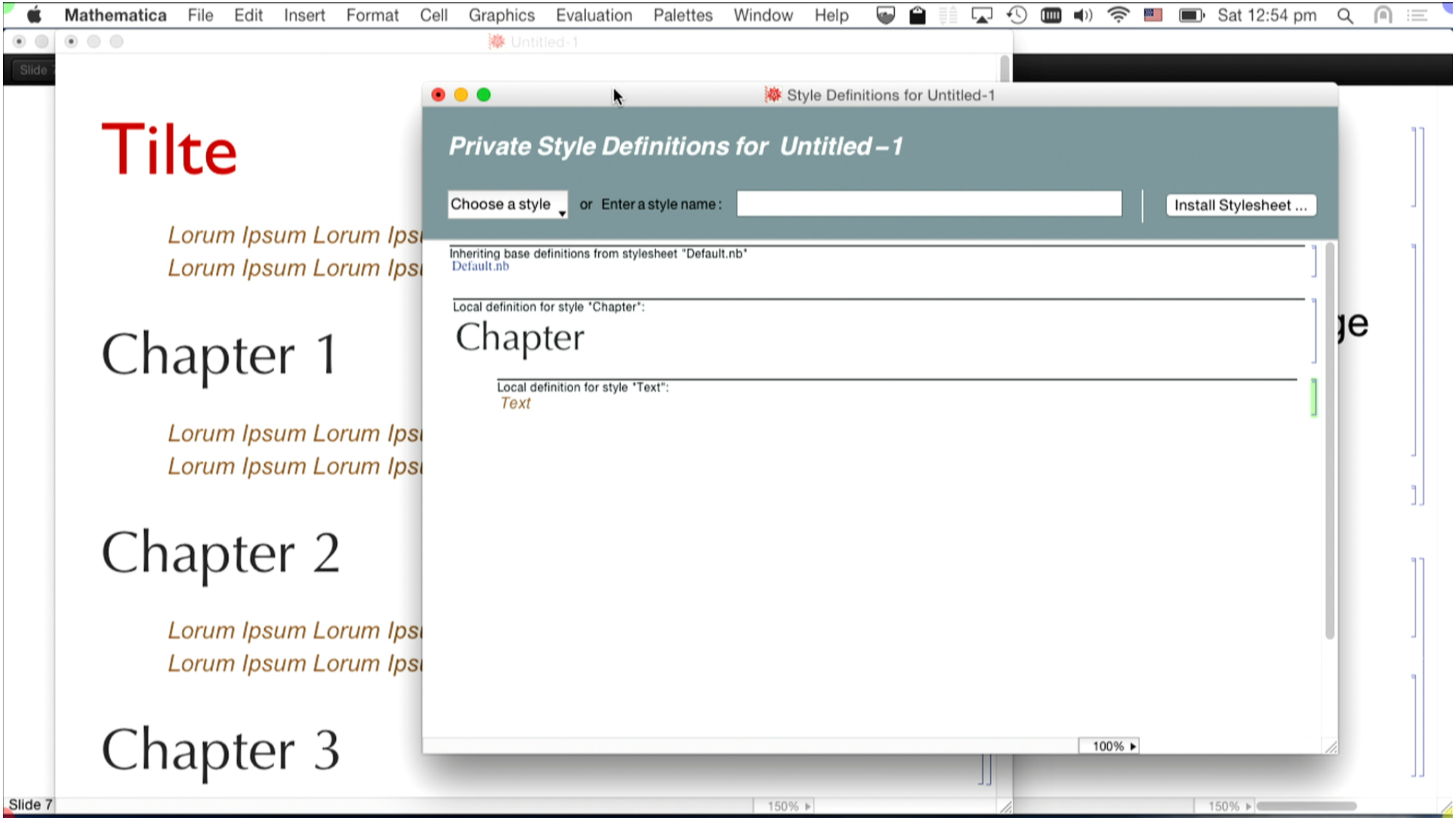
Slide 7 of 19
StyleSheets
Instead of changing individual words with different styles, add styles to the style sheet. Then if you ever need to, you can change a style in the style sheet and all uses of the style will change uniformly in the notebook.
Add styles here. For non-bold and
Option Inspector
You can use the option inspector to change settings at a deep level.
Slide 7 of 19
150%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Untitled-1
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Chapter 2
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Chapter 3
Slide 7
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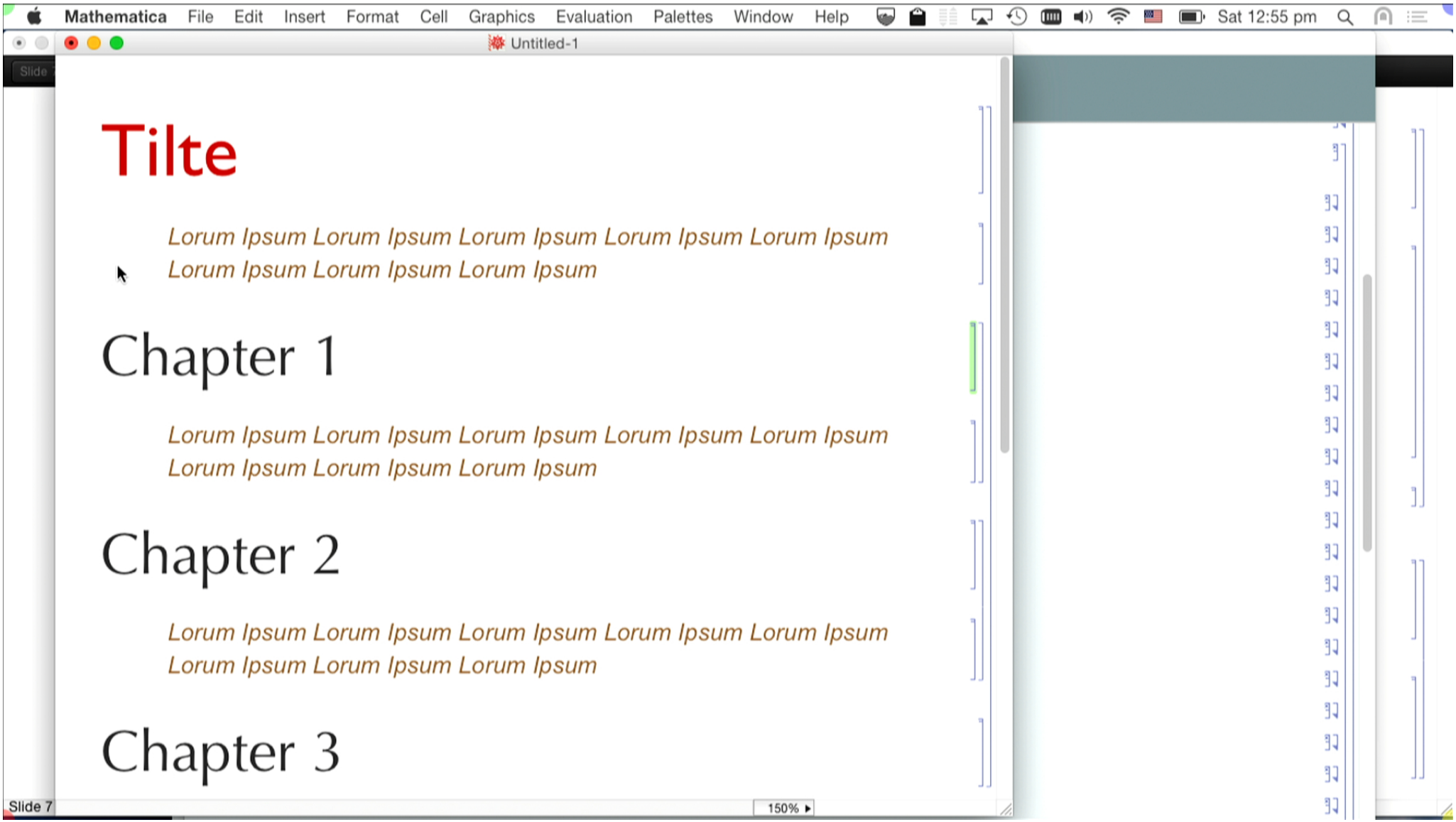
Style Definitions for Untitled-1
Private Style Definitions for Untitled–1
Choose a style
or Enter a style name:
Install Stylesheet ...
Inheriting base definitions from stylesheet "Default.nb"
Default.nb
Local definition for style "Chapter":
Chapter
Local definition for style "Text":
Text
100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 12:55 pm
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Chapter 1
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Chapter 2
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Chapter 3
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Manipulate

Compounding Interest

Let us start with the extremely simple example. Consider the value of a quantity of money A continually compounding under an interest rate r for time t, which is given by $A(1+r)^t$.

```
A = 300;
r = 0.09;

Plot[A (1 + r)^t, {t, 0, 7}, PlotRange → {0, 600}]
```

Example $A(1+r)^t$ with {A,0,200}, {r,0,0.3}

```
Manipulate
```

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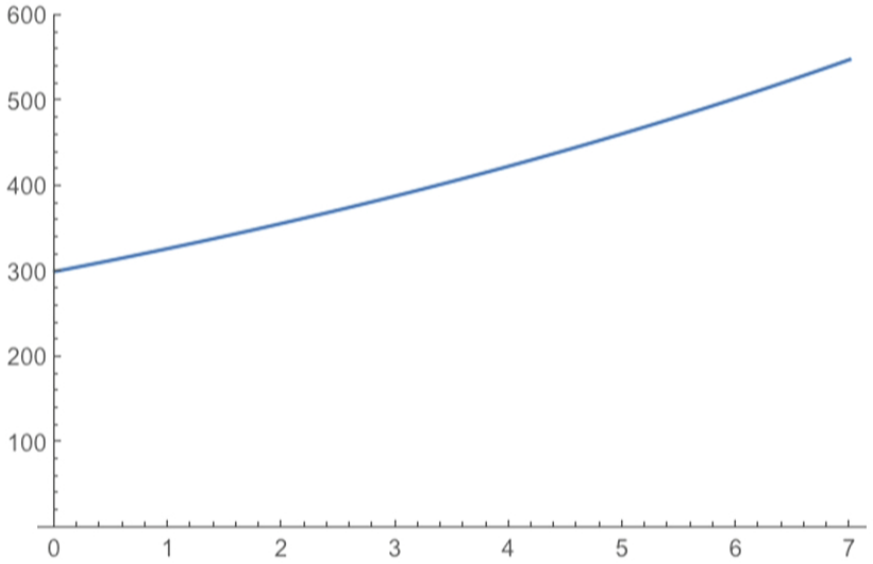
```
In[19]:= A = 300;
         r = 0.09;

In[21]:= Plot[A (1 + r)^t, {t, 0, 7}, PlotRange → {0, 600}]
```

Out[21]=



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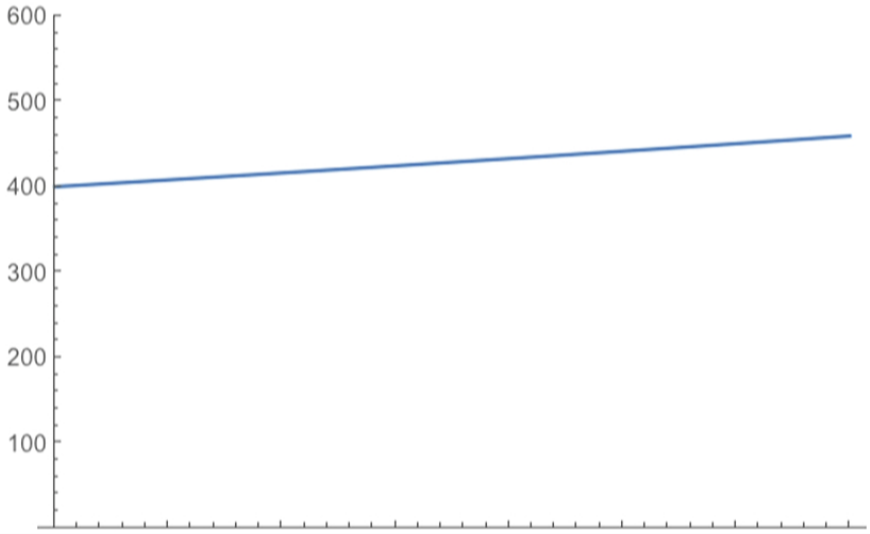
interest rate r for time t, which is given by $A(1+r)^t$.

In[28]:=
```
A = 400;
r = 0.02;
```

In[30]:=
```
Plot[A (1 + r)^t, {t, 0, 7}, PlotRange → {0, 600}]
```

Out[30]=



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interest rate r for time t, which is given by $A(1+r)^t$.

In[28]:=
```
A = 400;
r = 0.02;

Manipulate[Plot[A (1 + r)^t, {t, 0, 7},
  PlotRange → {0, 600}], {r, 0, 0.25}, {
```

Out[30]=

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TemplateBoxes

TemplateBoxes are fantastic things, which generally get their display from the style sheet, thus allowing you to have small box structures which are highly flexible, and which you can uniformly change within a document or environment.

Do Binomial -> BesselJ

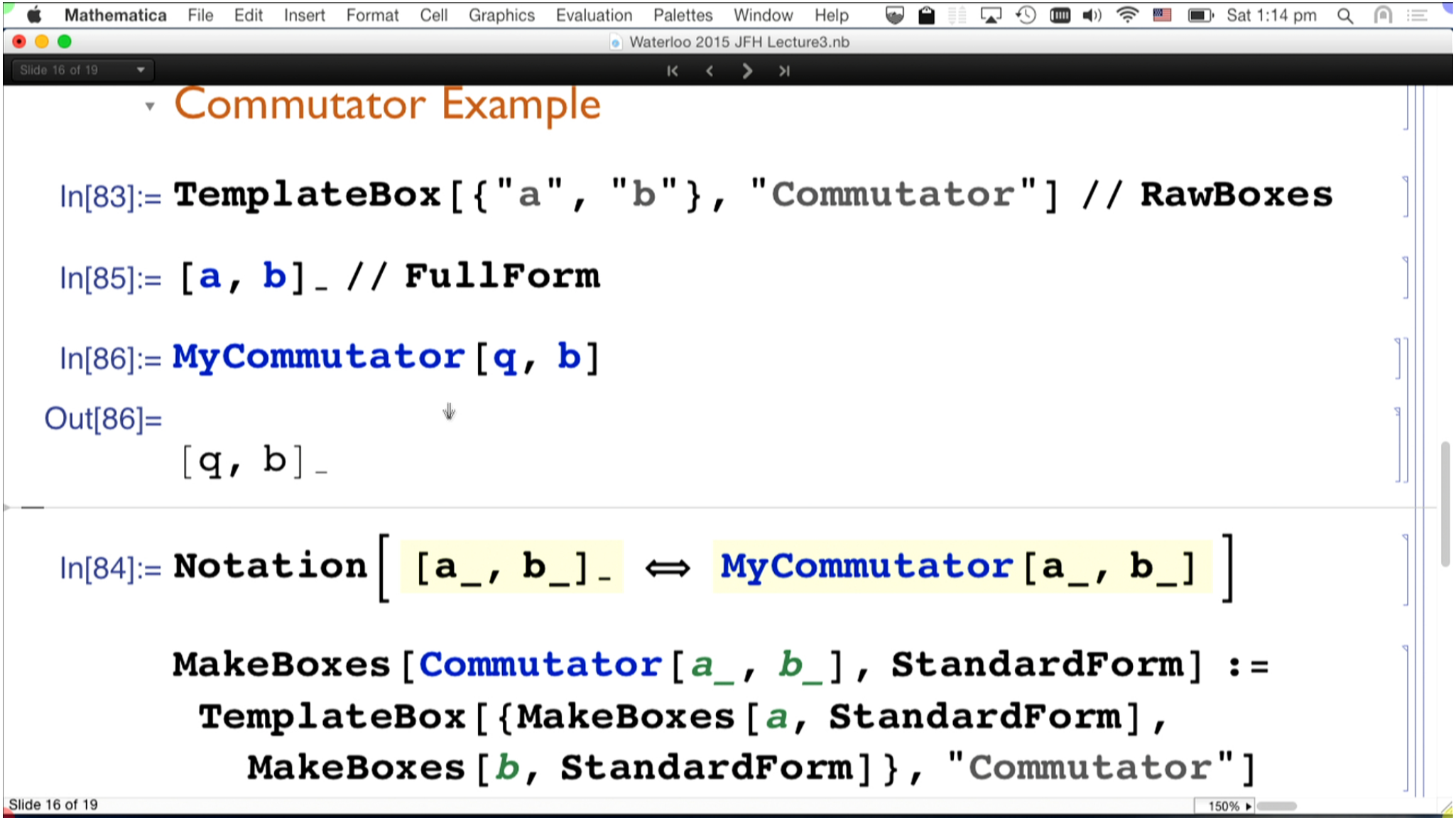
$P_n(x)$

LegendreP

```
TemplateBox[{"a", "b"}, "Binomial"] // RawBoxes

TemplateBox[{"a", "b"}, "LegendreP"] // RawBoxes

TemplateBox[{"a", "b"}, "BesselJ"] // RawBoxes
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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Commutator Example
In[83]:= TemplateBox[{"a", "b"}, "Commutator"] // RawBoxes
In[85]:= [a, b]_ // FullForm
In[86]:= MyCommutator[q, b]
Out[86]=
[q, b]_
In[84]:= Notation[[a_, b_]_ ⟺ MyCommutator[a_, b_]]
MakeBoxes[Commutator[a_, b_], StandardForm] :=
TemplateBox[{MakeBoxes[a, StandardForm],
MakeBoxes[b, StandardForm]}, "Commutator"]
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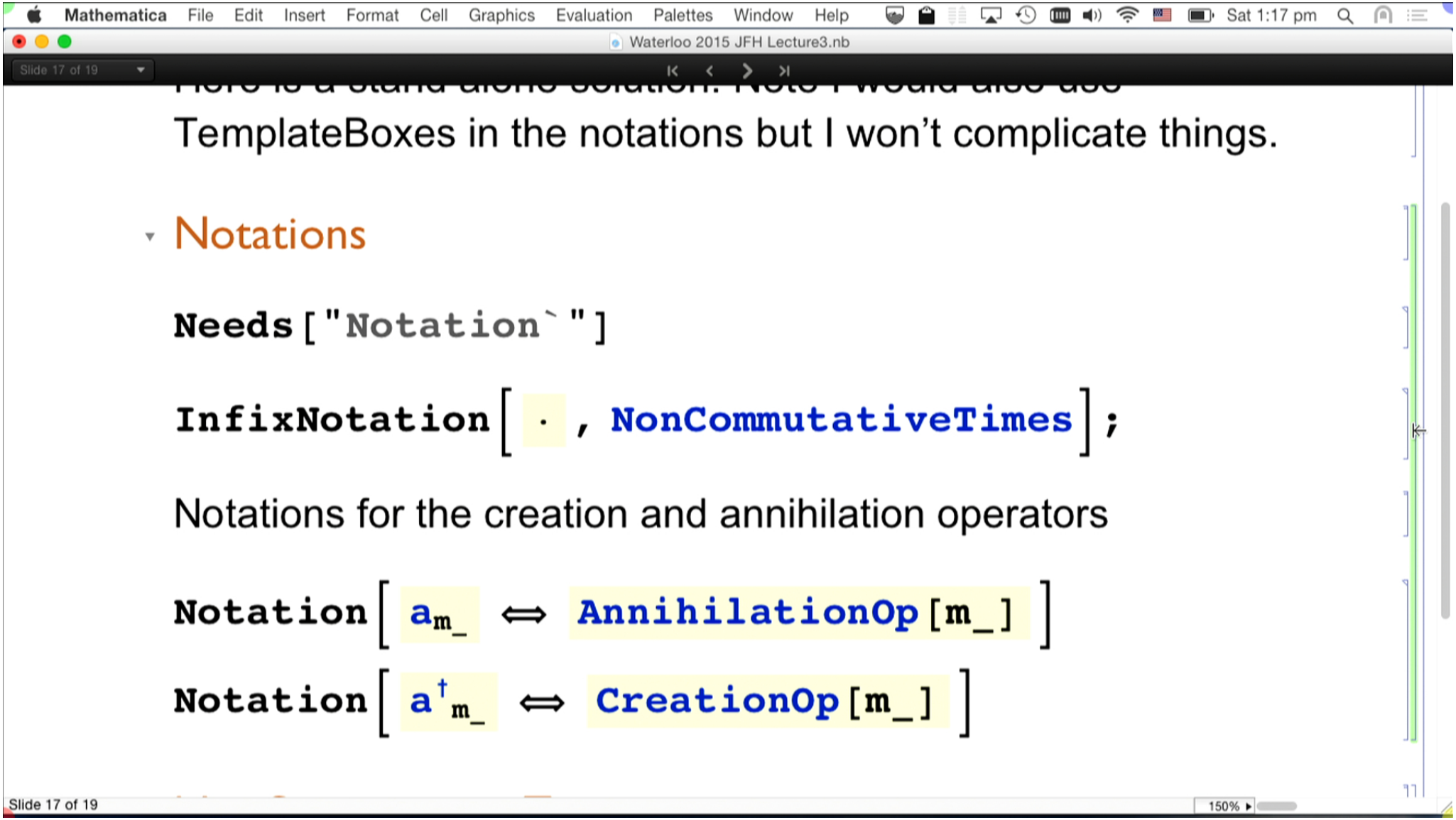
150%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TemplateBoxes in the notations but I won't complicate things.
Notations
Needs["Notation`"]
InfixNotation[· , NonCommutativeTimes];
Notations for the creation and annihilation operators
Notation[a_m_ ⟺ AnnihilationOp[m_]]
Notation[a†_m_ ⟺ CreationOp[m_]]
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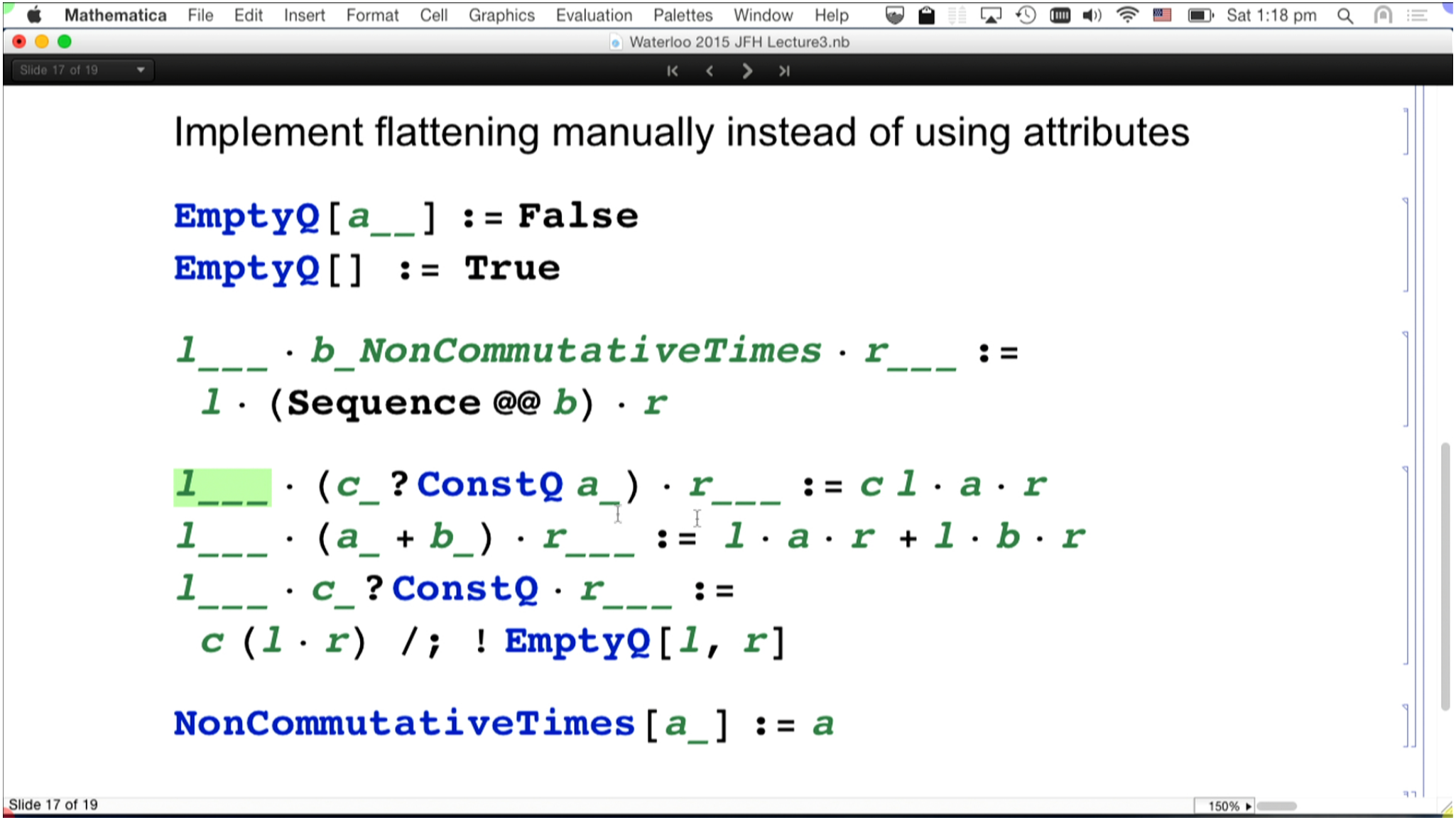
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Sat 1:18 pm
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Implement flattening manually instead of using attributes
EmptyQ[a__] := False
EmptyQ[] := True
l___ · b_NonCommutativeTimes · r___ :=
l · (Sequence @@ b) · r
l___ · (c_ ?ConstQ a_) · r___ := c l · a · r
l___ · (a_ + b_) · r___ := l · a · r + l · b · r
l___ · c_ ?ConstQ · r___ :=
c (l · r) /; ! EmptyQ[l, r]
NonCommutativeTimes[a_] := a
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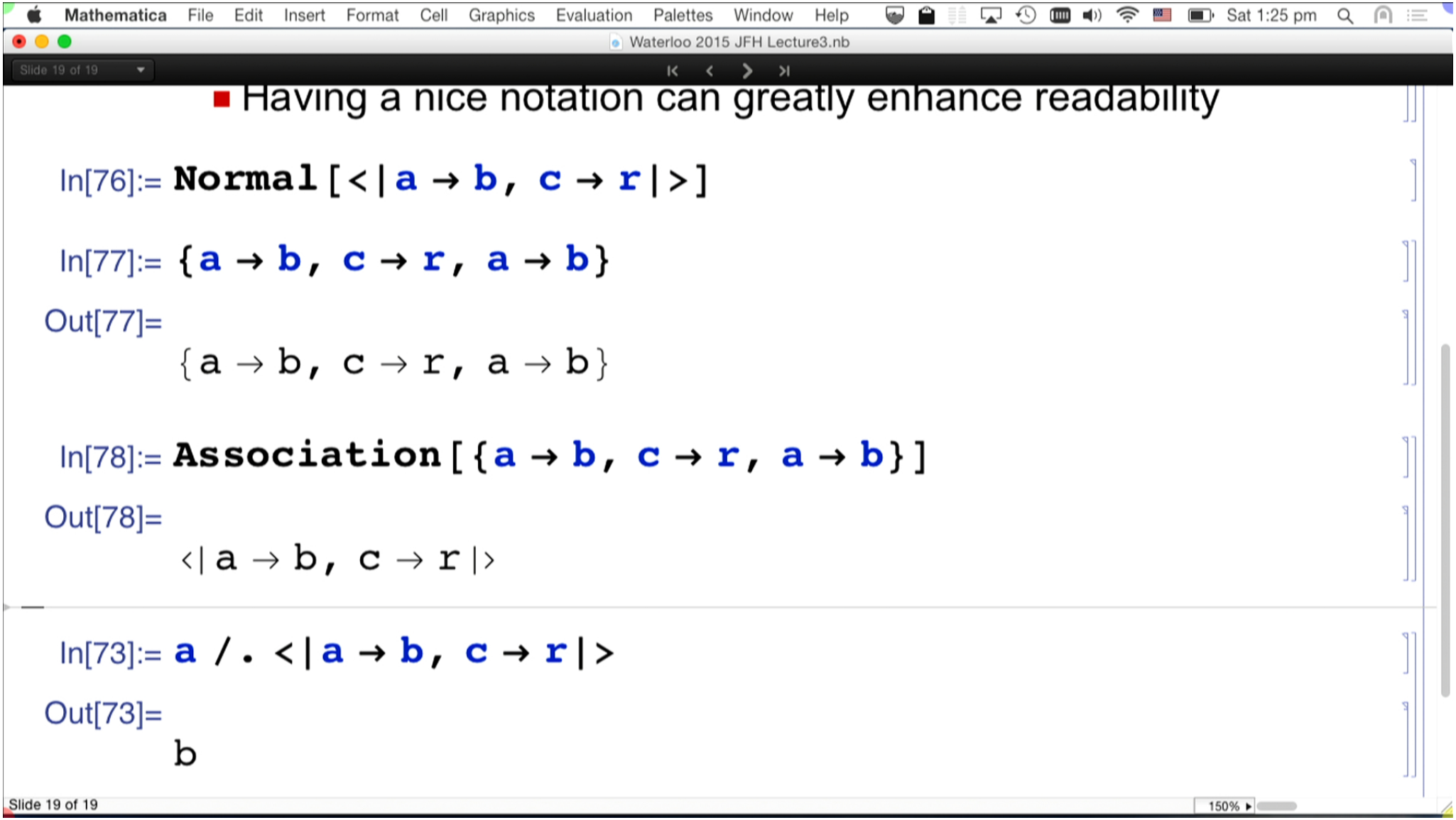
Summary:

- You can execute things in parallel
- You can compile Code to the WVM, or to a C target
- You can link to external C libraries
- Everything internally in the Front End is a box
- Most advanced typesetting is done through TemplateBoxes
- Having a nice notation can greatly enhance readability

```
<|a → b, c -
```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Sat 1:25 pm
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Having a nice notation can greatly enhance readability
In[76]:= Normal[<|a → b, c → r|>]
In[77]:= {a → b, c → r, a → b}
Out[77]=
{a → b, c → r, a → b}
In[78]:= Association[{a → b, c → r, a → b}]
Out[78]=
<|a → b, c → r|>
In[73]:= a /. <|a → b, c → r|>
Out[73]=
b
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150%

Digit Classification

Train a digit recognizer on some examples from the MNIST database of handwritten digits:

```
digitData = ExampleData[{"MachineLearning", "MNIST"},
    "Data"];
DumpSave["LocalDigitData.mx", digitData];

DumpGet["LocalDigitData.mx"]

RandomSample[digitData, 50]

mc = Classify[RandomSample[digitData, 200],
   Method → "LogisticRegression"]

rr = RandomSample[digitData, 50]
```

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Clasify.nb

Digit Classification

Train a digit recognizer on some examples from the MNIST database of handwritten digits:

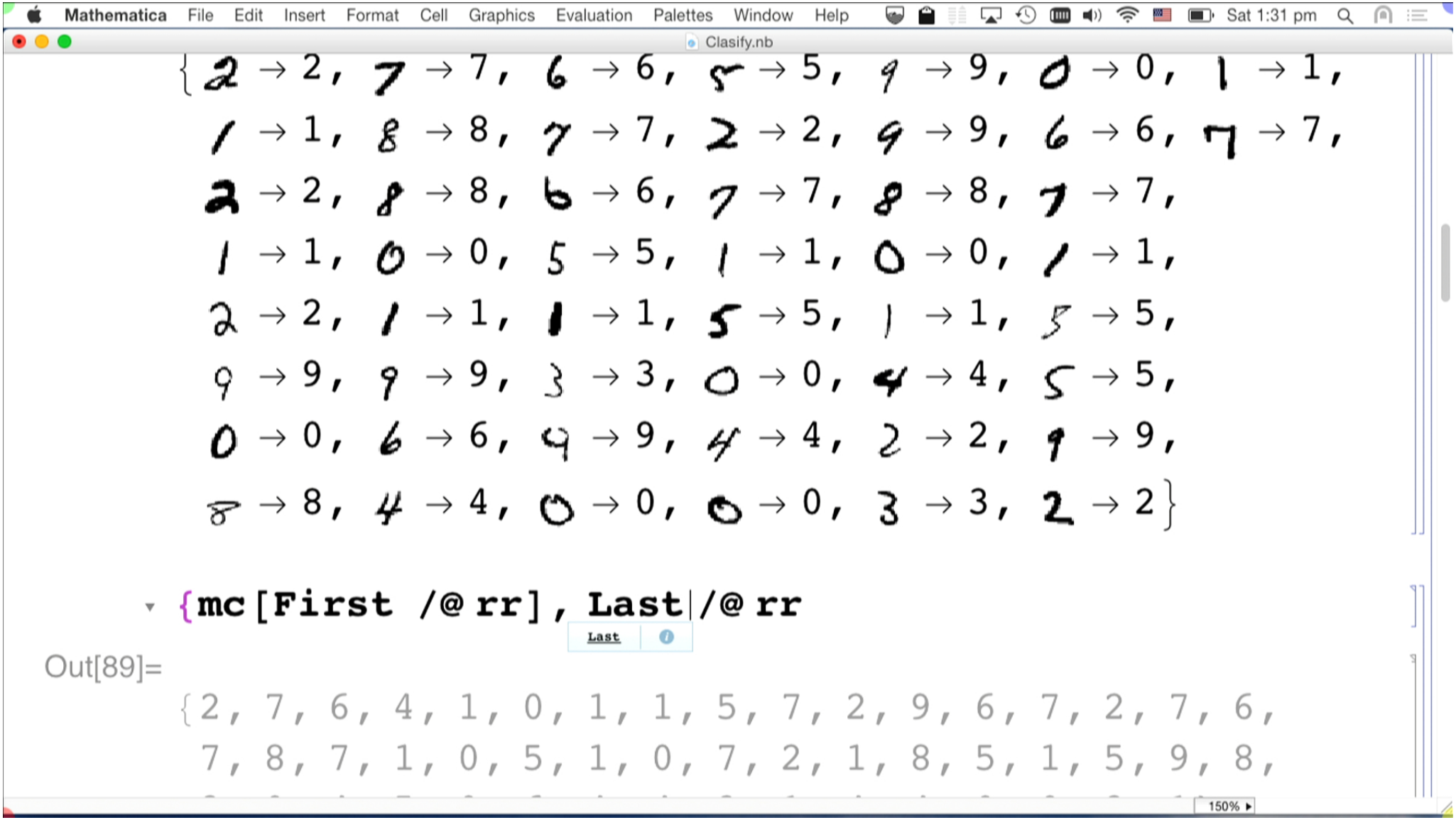
```
digitData =
  ExampleData[{"MachineLearning", "MNIST"},
   "Data"];
DumpSave["LocalDigitData.mx", digitData];

DumpGet["LocalDigitData.mx"]

RandomSample[digitData, 50]

mc = Classify[RandomSample[digitData, 200],
  Method → "LogisticRegression"]
```

150%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Clasify.nb
{2 → 2, 7 → 7, 6 → 6, 5 → 5, 9 → 9, 0 → 0, 1 → 1,
1 → 1, 8 → 8, 7 → 7, 2 → 2, 9 → 9, 6 → 6, 7 → 7,
2 → 2, 8 → 8, 6 → 6, 7 → 7, 8 → 8, 7 → 7,
1 → 1, 0 → 0, 5 → 5, 1 → 1, 0 → 0, 1 → 1,
2 → 2, 1 → 1, 1 → 1, 5 → 5, 1 → 1, 5 → 5,
9 → 9, 9 → 9, 3 → 3, 0 → 0, 4 → 4, 5 → 5,
0 → 0, 6 → 6, 9 → 9, 4 → 4, 2 → 2, 9 → 9,
8 → 8, 4 → 4, 0 → 0, 0 → 0, 3 → 3, 2 → 2}
{mc[First /@ rr], Last/@ rr
Last
Out[89]=
{2, 7, 6, 4, 1, 0, 1, 1, 5, 7, 2, 9, 6, 7, 2, 7, 6,
7, 8, 7, 1, 0, 5, 1, 0, 7, 2, 1, 8, 5, 1, 5, 9, 8,
150%