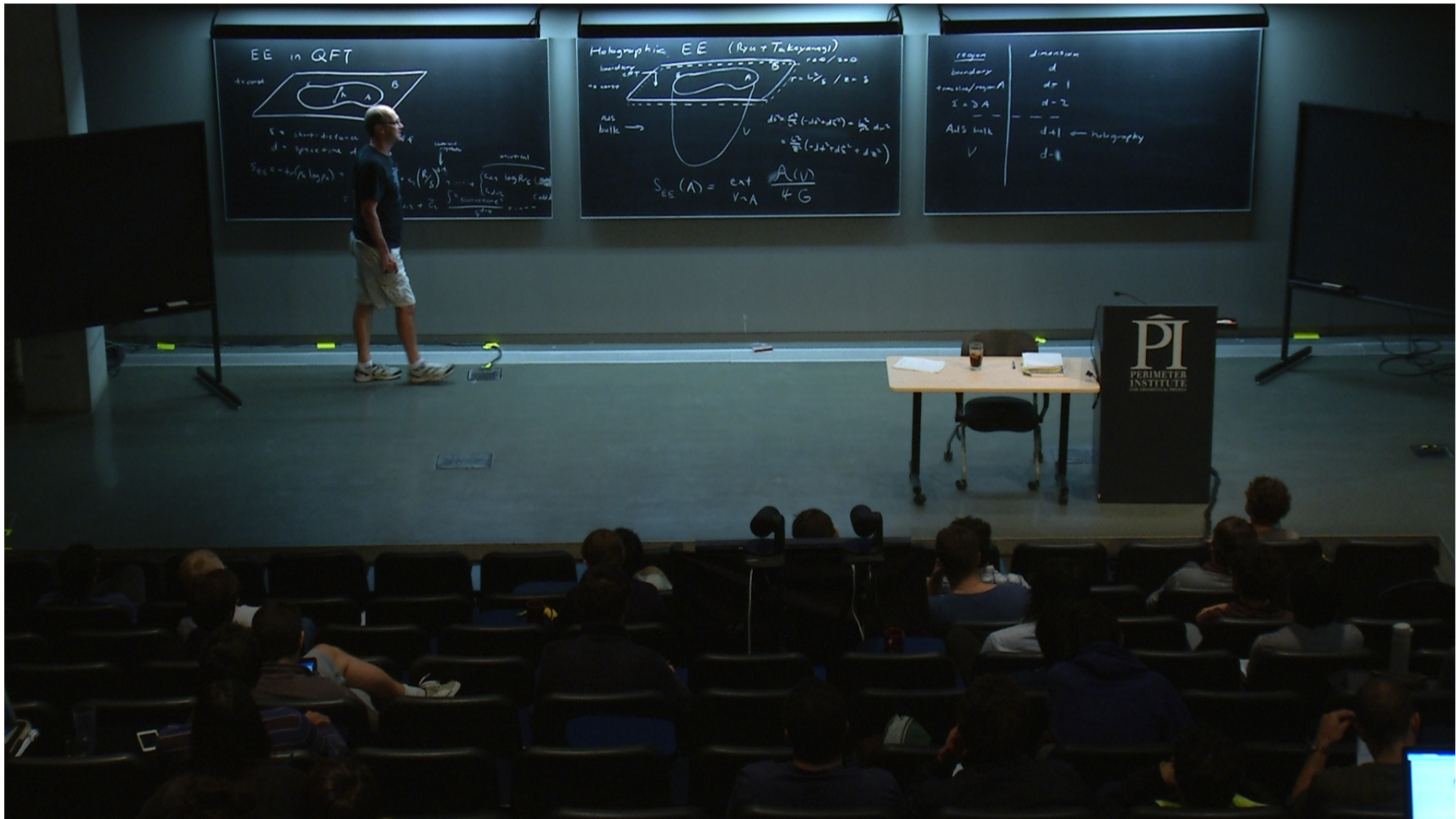


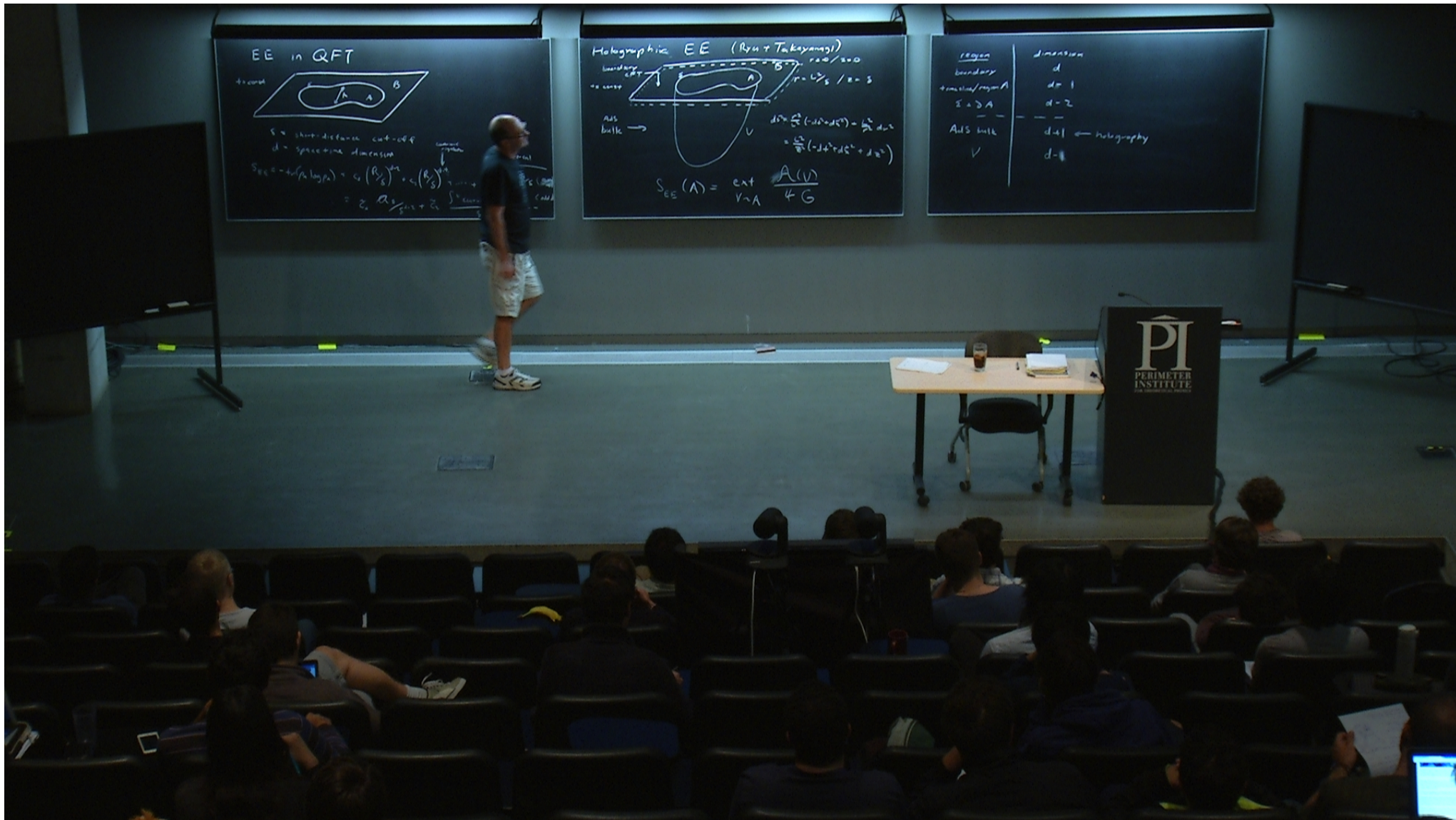
Title: Holographic entanglement entropy

Date: Aug 29, 2015 11:00 AM

URL: <http://pirsa.org/15080056>

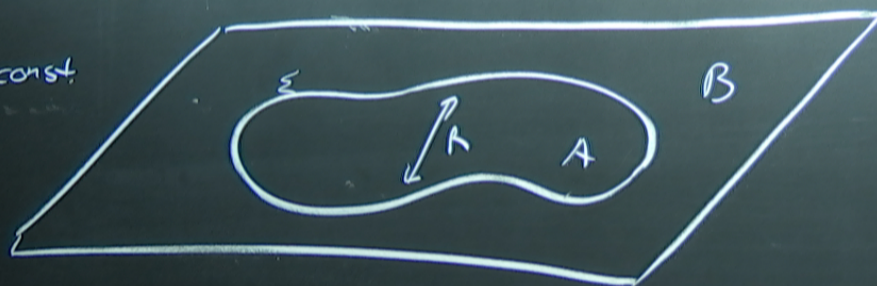
Abstract: TBA





EE in QFT

$t = \text{const}$



ϵ = short-distance cut-off

d = spacetime dimension

$$S_{EE} = -\text{tr}(p_A \log p_A) = c_0 \left(\frac{R}{\epsilon}\right)^{d-2} + c_2 \left(\frac{R}{\epsilon}\right)^{d-4} + \dots + \underbrace{\left\{ \begin{array}{l} c_{d-2} \log R/\epsilon \text{ (even } d) \\ c_{d-2} \text{ (odd } d) \end{array} \right\}}_{\text{universal}}$$

$$= \tilde{c}_0 \frac{A_\epsilon}{\epsilon^{d-2}} + \tilde{c}_2 \frac{\int \text{"curvature"}}{\epsilon^{d-4}} + \dots$$

covariant regulator
↓

<u>region</u>	dimension
boundary	d
time slice/region A	$d-1$
$\Sigma = \partial A$	$d-2$
— — — — —	
AdS bulk	$d+1 \leftarrow \text{holography}$
$\checkmark$	$d-1$

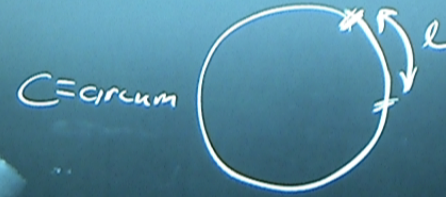
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<u>region</u>	dimension
boundary	d
timeslice/region A	$d-1$
$\Sigma = \partial A$ $= \underline{\partial V}$	$d-2$
AdS bulk	$d+1 \leftarrow \text{holography}$
$V \sim A$	$d-1 \leftarrow \text{codim } 2 \text{ surface}$

tests of HRE

① area law

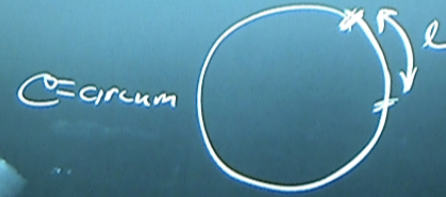
② $d=2$ CFT : $S = \frac{c}{3} \log \left(\frac{c}{\pi s} \sin \frac{\pi \ell}{c} \right)$



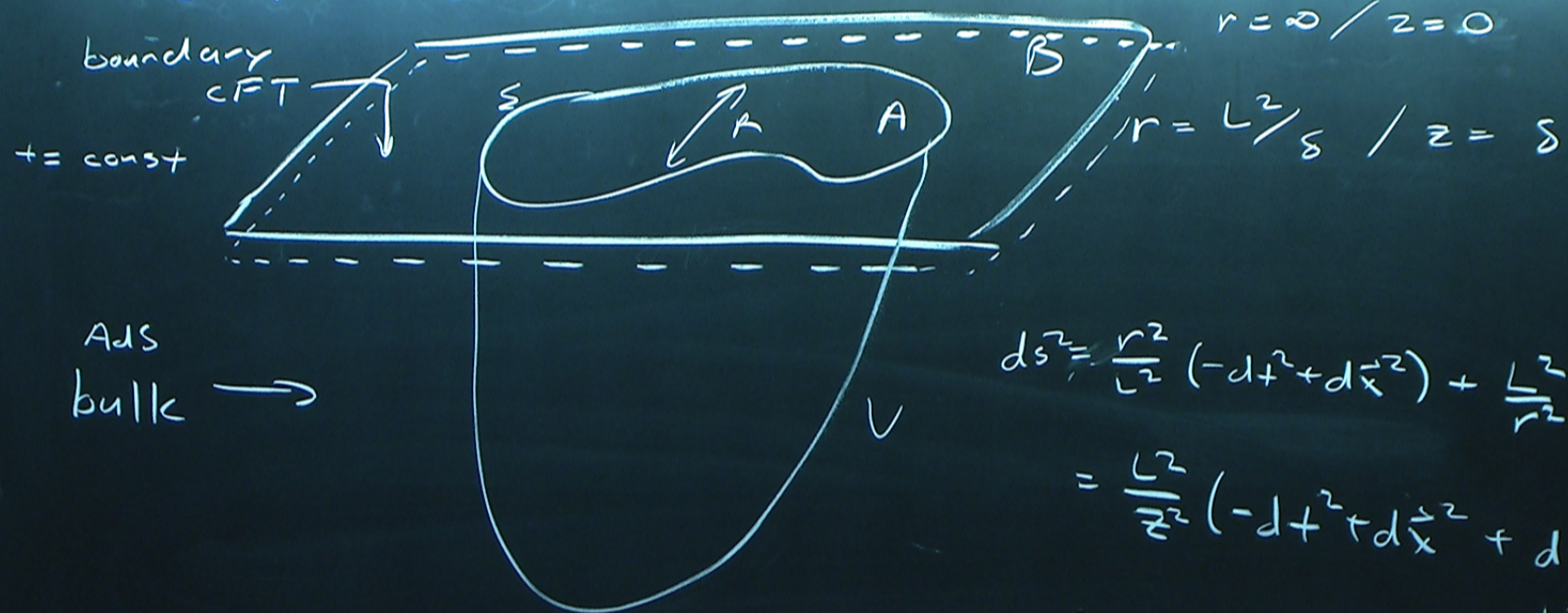
tests of HRE

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Holographic EE (Ryu + Takayanagi)



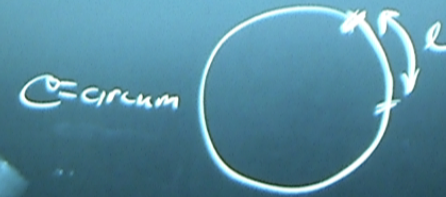
$$S_{EE}(A) = \text{ext}_{V \sim A} \frac{A(V)}{4G}$$

tests of HEE

① area law

② $d=2$ CFT : $S = \frac{c}{3} \log \left(\frac{c}{\pi s} \sin \frac{\pi \ell}{c} \right)$

③

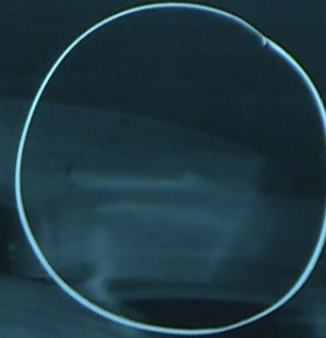
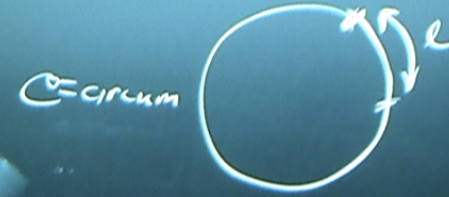


tests of HEE

① area law

② $d=2$ CFT: $S = \frac{c}{3} \log \left(\frac{c}{\pi s} \sin \frac{\pi \ell}{c} \right)$

③ $S(A) = S(\bar{A})$

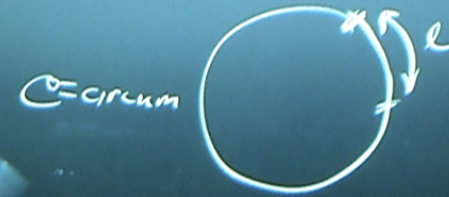


tests of HEE

① area law

② $d=2$ CFT: $S = \frac{c}{3} \log \left(\frac{c}{\pi s} \frac{\pi l}{c} \right)$

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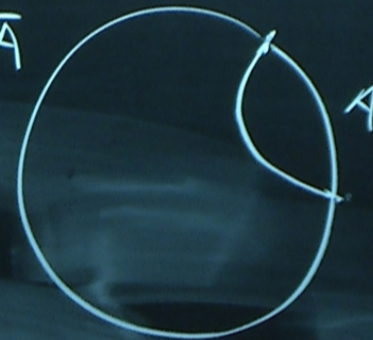
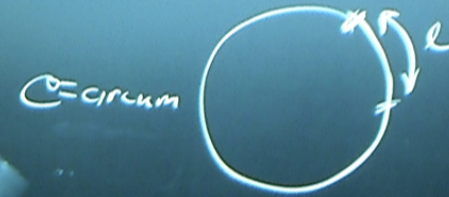


tests of HEE

① area law

$$d=2 \text{ CFT} : S = \frac{c}{3} \log \left(\frac{C}{\pi s} \sin \frac{\pi \ell}{C} \right)$$

$$S(A) = S(\bar{A})$$

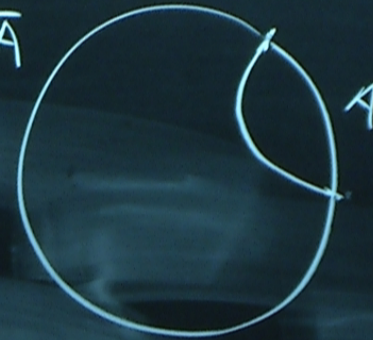
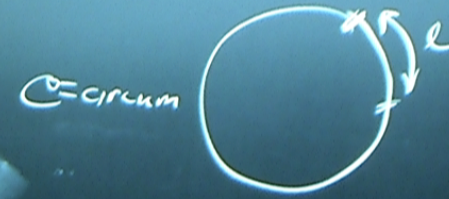


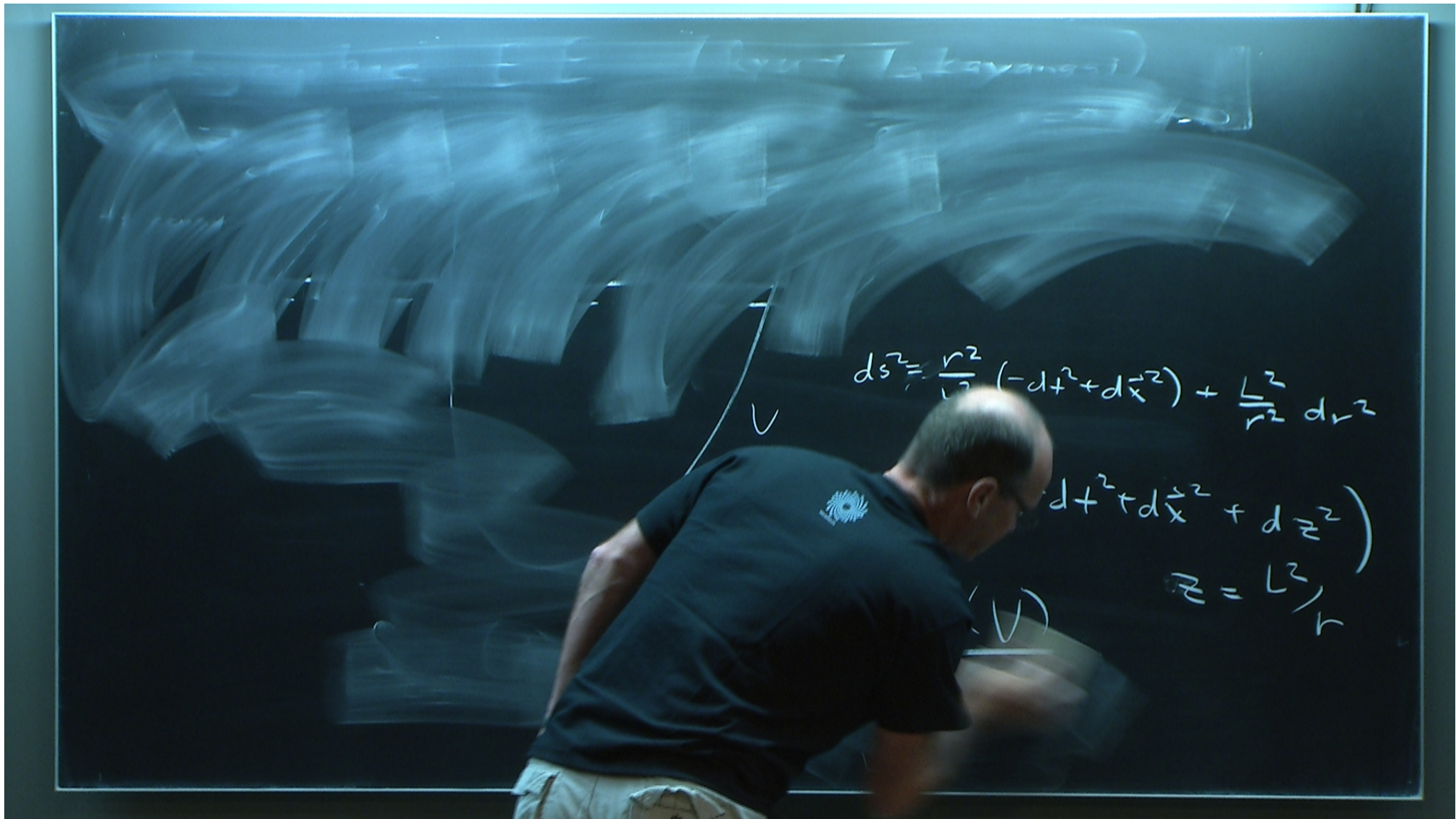
tests of HEE

① area law

② $d=2$ CFT: $S = \frac{c}{3} \log \left(\frac{c}{\pi s} \sin \frac{\pi \ell}{c} \right)$

③ $S(A) = S(\bar{A})$





1st Law of Entanglement

$$S_{EE} = - \text{tr}(\rho_A \log \rho_A)$$

new state: $\tilde{\rho}_A = \rho_A +$

$$\delta S = - \text{tr} \delta \rho \log \rho - \text{tr} \rho \log \delta \rho$$

1st Law of Entanglement

1st Law of Entanglement

$$S_{EE} = - \operatorname{tr}(\rho_A \log \rho_A)$$

new state: $\tilde{\rho}_A = \rho_A + \delta \rho$ $\operatorname{tr} \rho_A = 1 = \operatorname{tr} \tilde{\rho}_A$

$$\delta S = - \operatorname{tr} \delta \rho \log \rho - \operatorname{tr} (\rho \log \rho \delta \rho)$$

modular Hamiltonian:

$$\rho_A = \frac{e^{-H_A}}{Z} \quad \operatorname{tr} \delta \rho = 0$$

eg $A \rightarrow$ entire space

state $\rightarrow$ thermal state

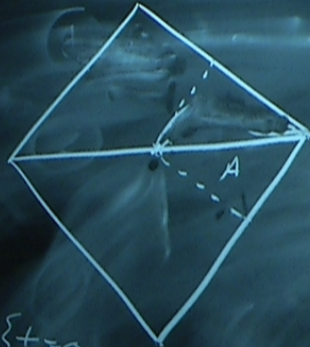
H

eg $A \rightarrow$ entire space

state $\rightarrow$ thermal state

$H_A = H/T \rightarrow$ 1st law of thermo.

Rindler



$$\Sigma = \{t=0, x=0\}$$

$$H_A = 2\pi \int d^2y dx \cdot x T_{++}$$

$$= 2\pi \int_A d\Sigma^\mu K^\nu$$

$$K^\mu$$

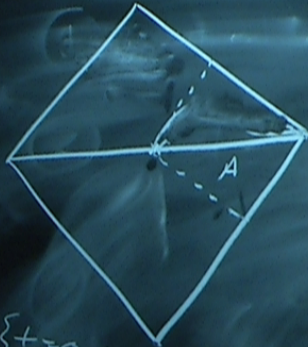
best Killing vector

→ any QFT

$$H_A =$$

large

Rindler



$\Sigma = \{t=0, x=0\}$

$$H_A = 2\pi \int d^2y dx \cdot x T_{++}$$

$$= 2\pi \int_A d\Sigma^\mu T_{\mu\nu} K^\nu$$

$K^\mu \partial_\mu = \text{Boost Killing vector}$

→ any QFT

$$= x \partial_t + t \partial_x$$

$$\nabla_{(a} K_{b)} = 0$$

$H_A = \text{conserved charge}$

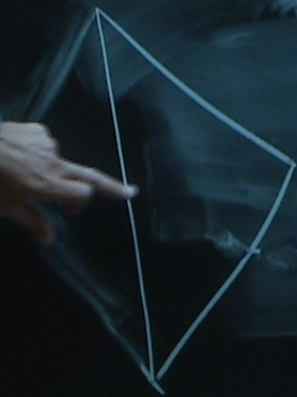
- For CFT : special conformal trans takes
 $\Sigma \rightarrow \tilde{\Sigma} = \{t=0, |\vec{x}|^2 = R\}$

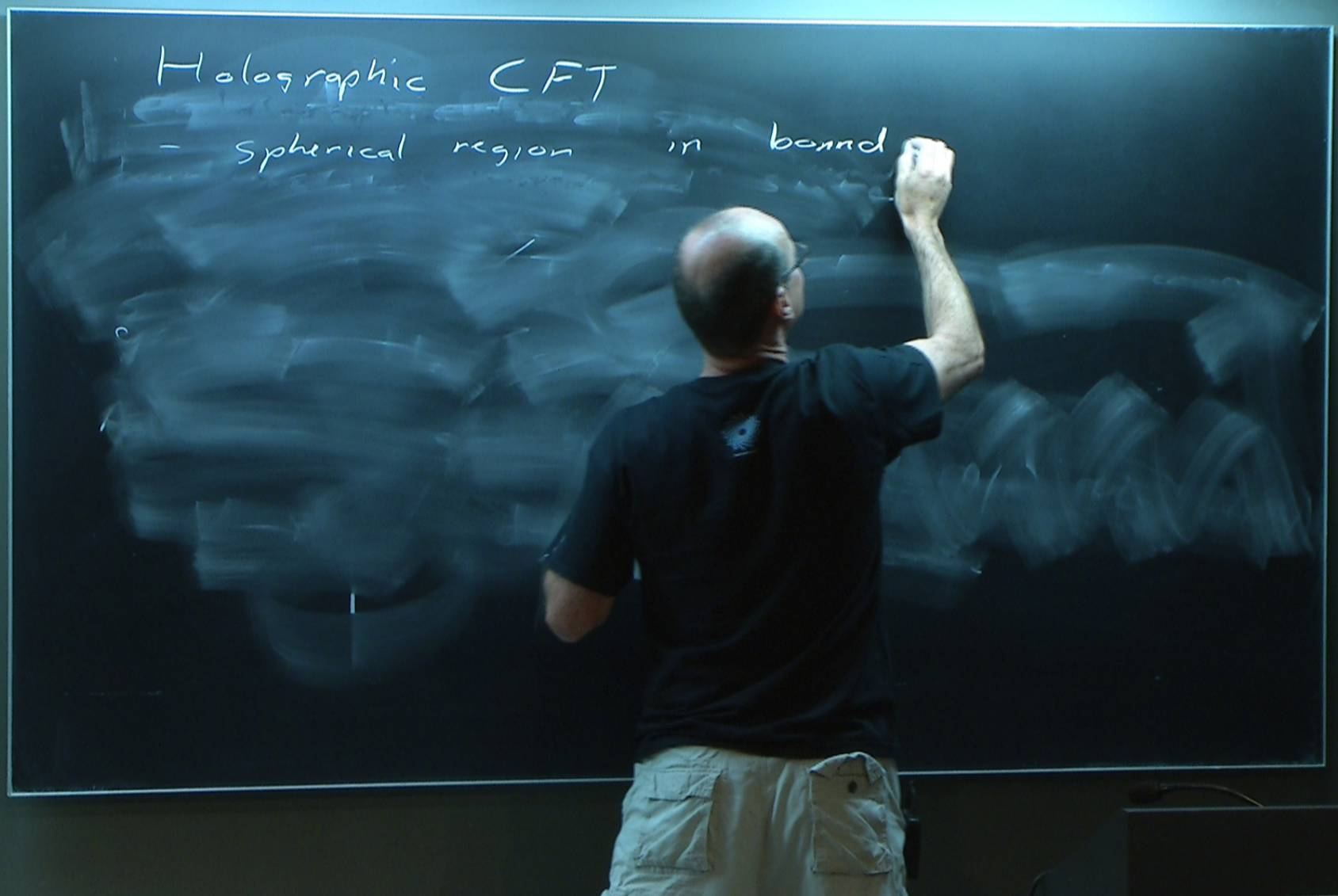
→ local modular Hamiltonian

$$H_B = 2\pi \int_B d\Sigma^\mu T_{\mu\nu} K^\nu$$

K^ν = conformal Killing vector

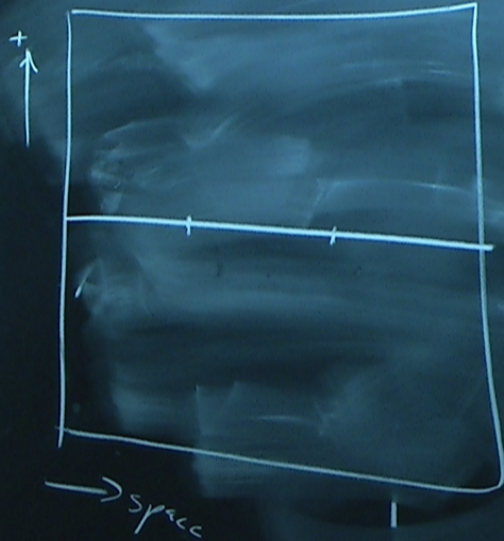
$$\nabla_\mu K_\nu = \frac{1}{d} g_{\mu\nu} \nabla \cdot K$$





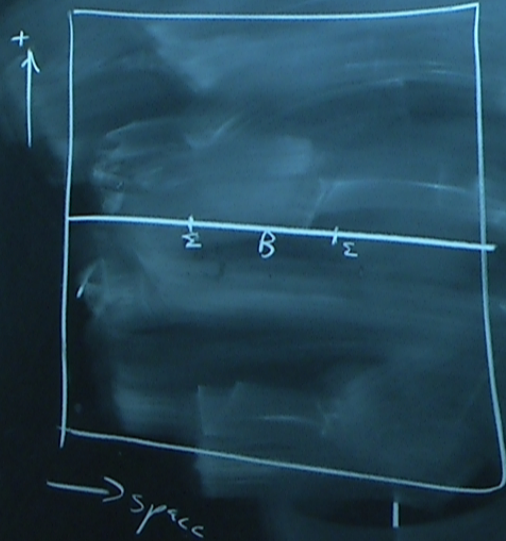
Holographic CFT

- Spherical region in boundary CFT



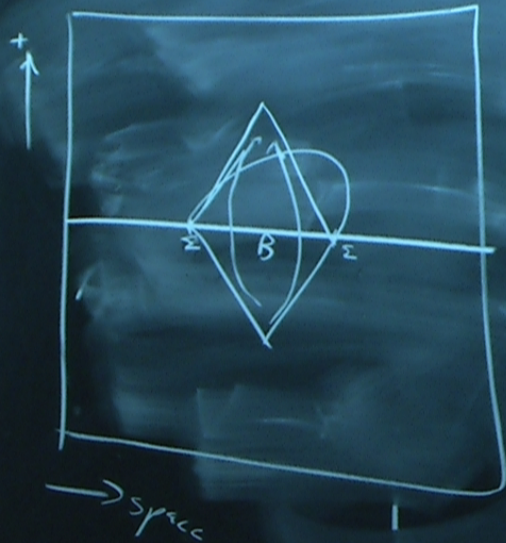
Holographic CFT

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Holographic CFT

- Spherical region in boundary CFT

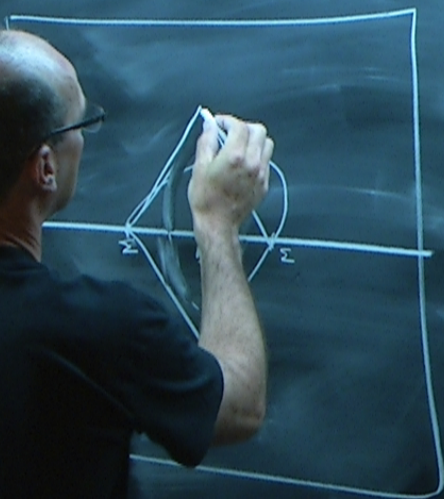


- RT surface:

$$l^2 + \bar{z}^2 = R^2$$

Holographic CFT

- spherical region in boundary CFT



- RT surface:

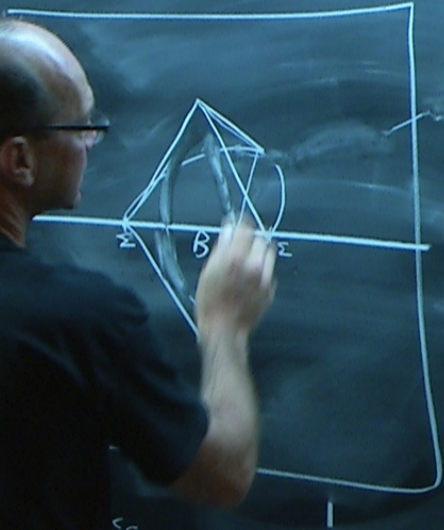
$$|\vec{x}|^2 + \bar{z}^2 = R^2$$

- boundary conformal KV

extends to a Killing vector
in bulk

Holographic CFT

- spherical region in boundary CFT



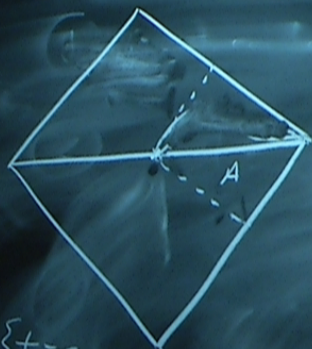
- RT surface:

$$|\vec{x}|^2 + \vec{z}^2 = R^2$$

- boundary conformal KV
extends to a Killing vector

→ domain of dependence in bulk
is delineated by Killing horizon
in bulk

Rindler



$\Sigma = \{t=0, x=0\}$

$$H_A = 2\pi \int d^2y dx \cdot x T_{++}$$

$$= 2\pi \int_A d\Sigma^\mu T_{\mu\nu} K^\nu$$

$K^\mu \partial_\mu = \text{Boost Killing vector}$

→ any QFT

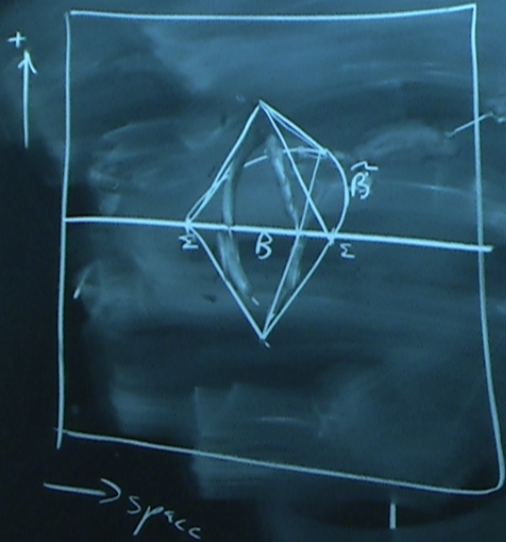
$$= x \partial_t + t \partial_x$$

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Holographic CFT

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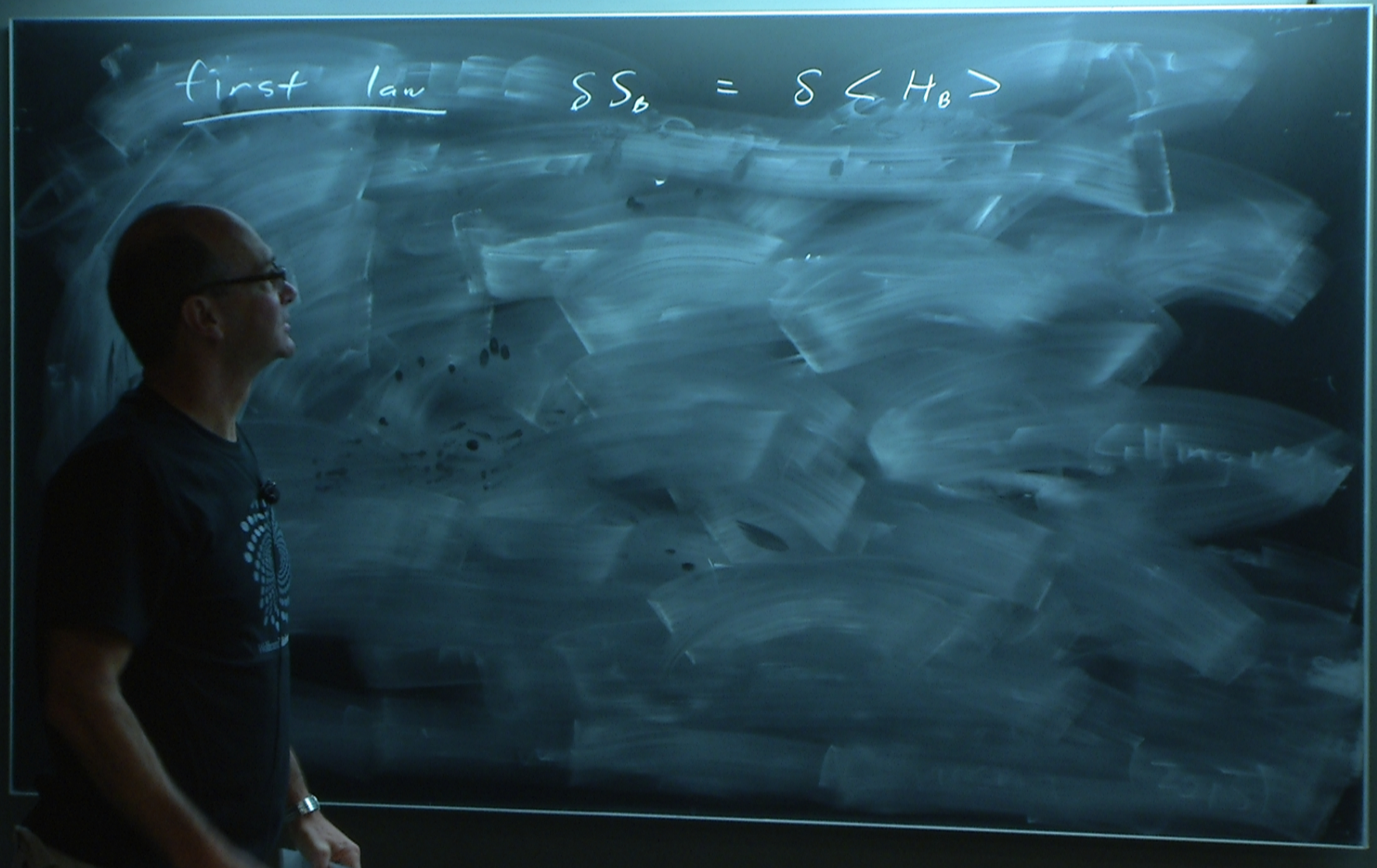


- RT surface: $\vec{B}$

$$|\vec{x}|^2 + \vec{z}^2 = R^2$$

- boundary conformal KV
extends to a Killing vector

→ domain of dependence in bulk
is delineated by Killing horizon in bulk



first law $\delta S_B = \delta \langle H_B \rangle$

- CFT vacuum = AdS geometry

- excited state = perturb geometry

$\delta S_B =$

first law $\delta S_B = \delta \langle H_B \rangle$

- CFT vacuum $\approx$ AdS geometry
- excited state $\approx$ perturb. geometry

$$(\delta S_B) = \frac{\delta A_B}{4G}$$

$$\delta \langle H_B \rangle = 2\pi \int d^{d+1}x \frac{R^2 - |\vec{k}|^2}{2R} \langle T_{++} \rangle$$

$$\delta g_{\mu\nu} \rightarrow \frac{2}{L^d} h_{\mu\nu}^{(d)}$$

= change of grav.
in AdS

→ reminds of

- Wald approach: define $(d-1)$ form

- Wald approach: define $(d-1)$ form χ

$$\int_{\tilde{B}} \chi = \delta S_{\tilde{B}}$$

$$\int_B \chi = \delta \langle H_B \rangle$$

$$d\chi =$$

-4

K^a

Killing vector

$\delta \bar{E}_{ab} \Sigma^b$

volume form

- Wald approach: define $(d-1)$ form χ

$$\int_{\tilde{B}} \chi = \delta S_{\tilde{B}}$$

$$\int_B \chi = \delta \langle H_B \rangle$$

$$d\chi = 4\pi$$

Killing vector

$$K^a \delta \bar{E}_{ab} \Sigma^b$$

linearized Einstein eq

volume form

- Wald approach: define $(d-1)$ form χ

$$\int_{\tilde{B}} \chi = \delta S_{\tilde{B}}$$

$$\int_B \chi = \delta \langle H_B \rangle$$

$$d\chi = -4\pi K^a \delta \bar{E}_{ab} \Sigma^b$$

Killing vector

linearized Einstein eq

volume form

$$\int_Y d\chi = \int_B \chi - \int_{\tilde{B}} \chi$$

$$= \delta \langle H_B \rangle - \delta S_{\tilde{B}} = 0$$

(some integral of $\bar{E}_{ab}$) = 0

- consider all spheres $\rightarrow \delta E_{ab} = 0$

- consider all spheres $\rightarrow \delta E_{ab} = 0$
- holography: all small excitations satisfy 1st law
 - $\rightarrow$ geometric perturb satisfies linearized Einstein eq.