Title: TBA

Date: Aug 28, 2015 12:00 PM

URL: http://pirsa.org/15080054

Abstract: TBA

ForLect1.nb * - Wolfram Mathematica 10.0

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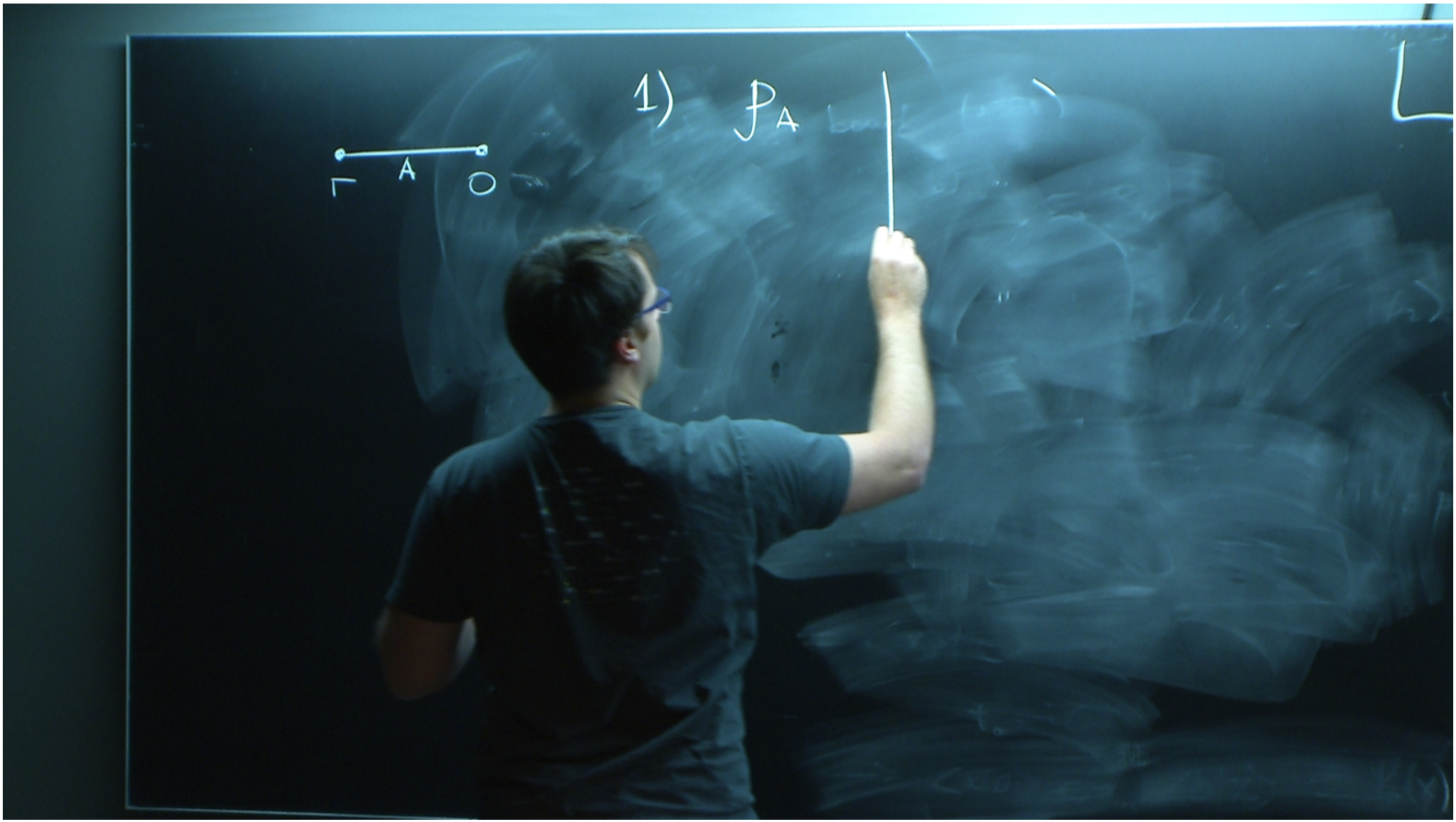
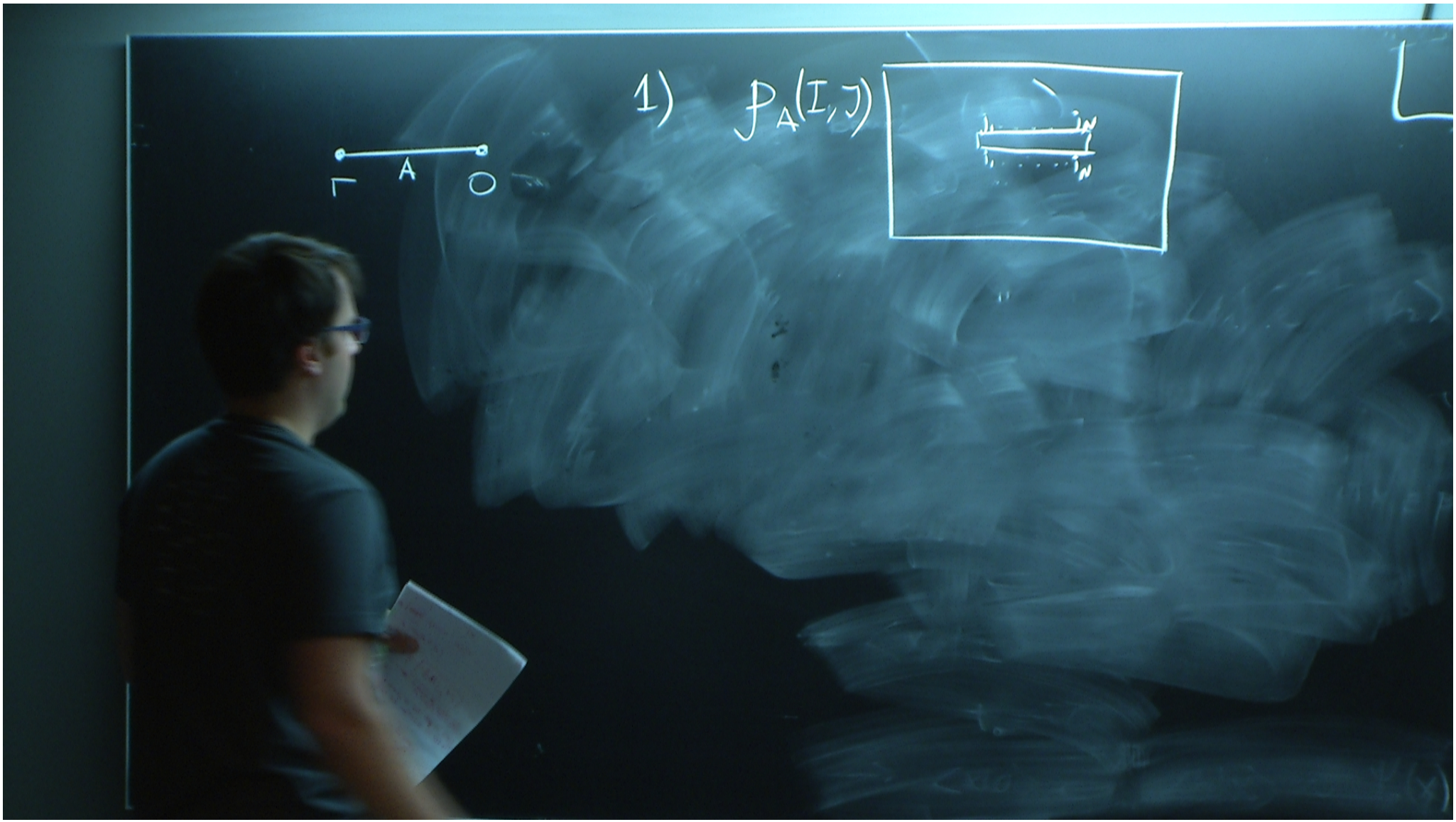
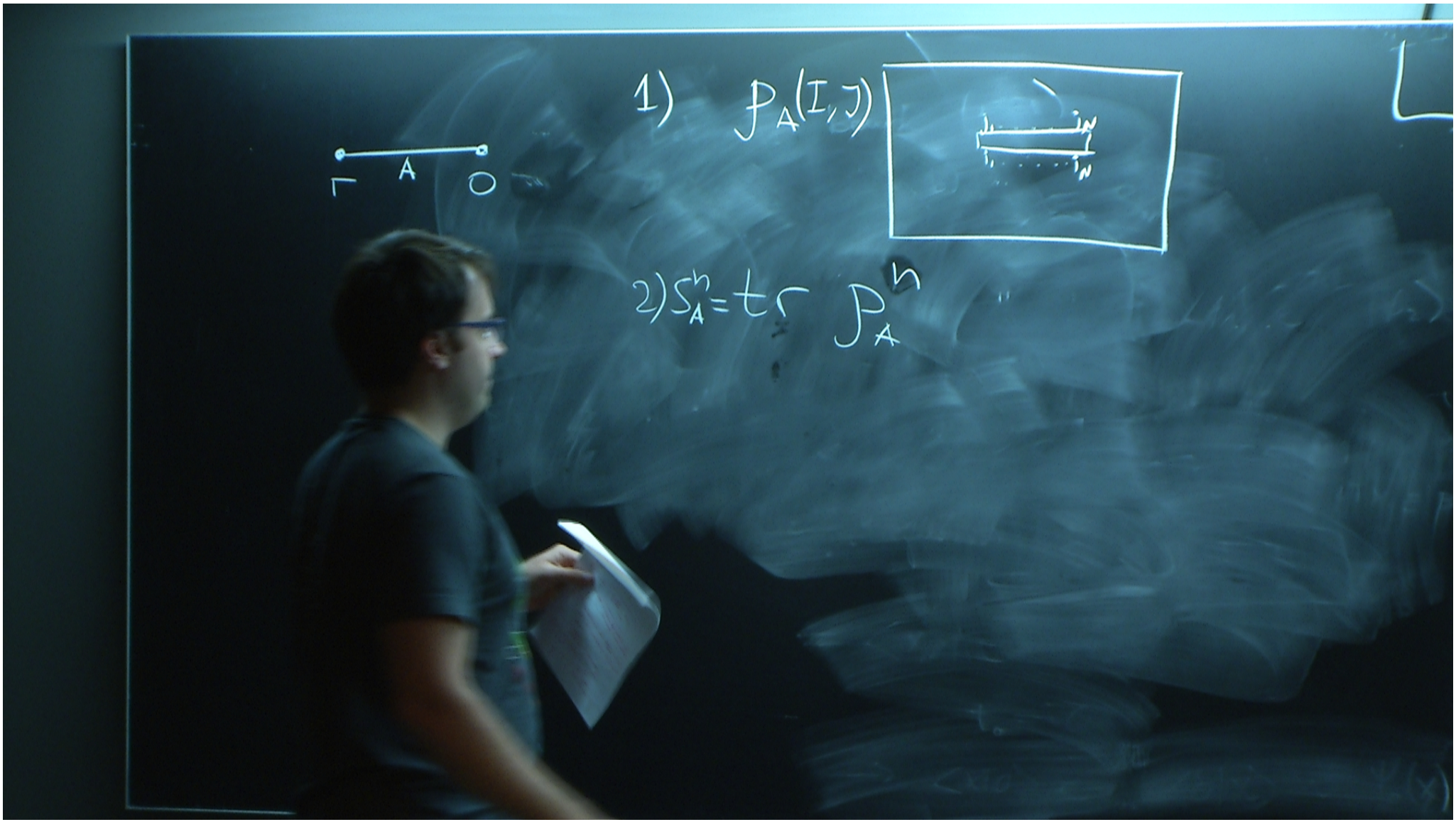
Poison resummation

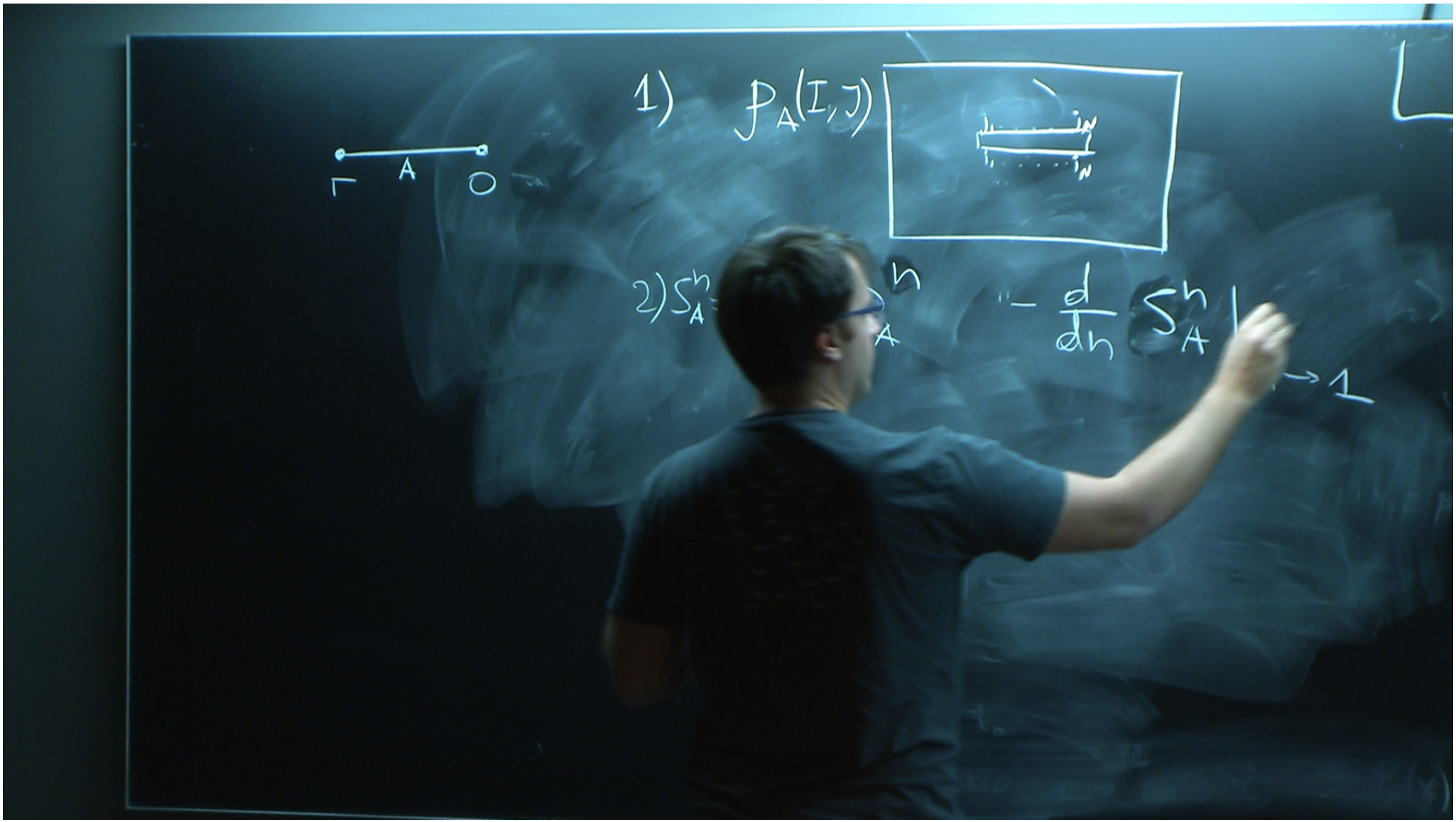
By Poles

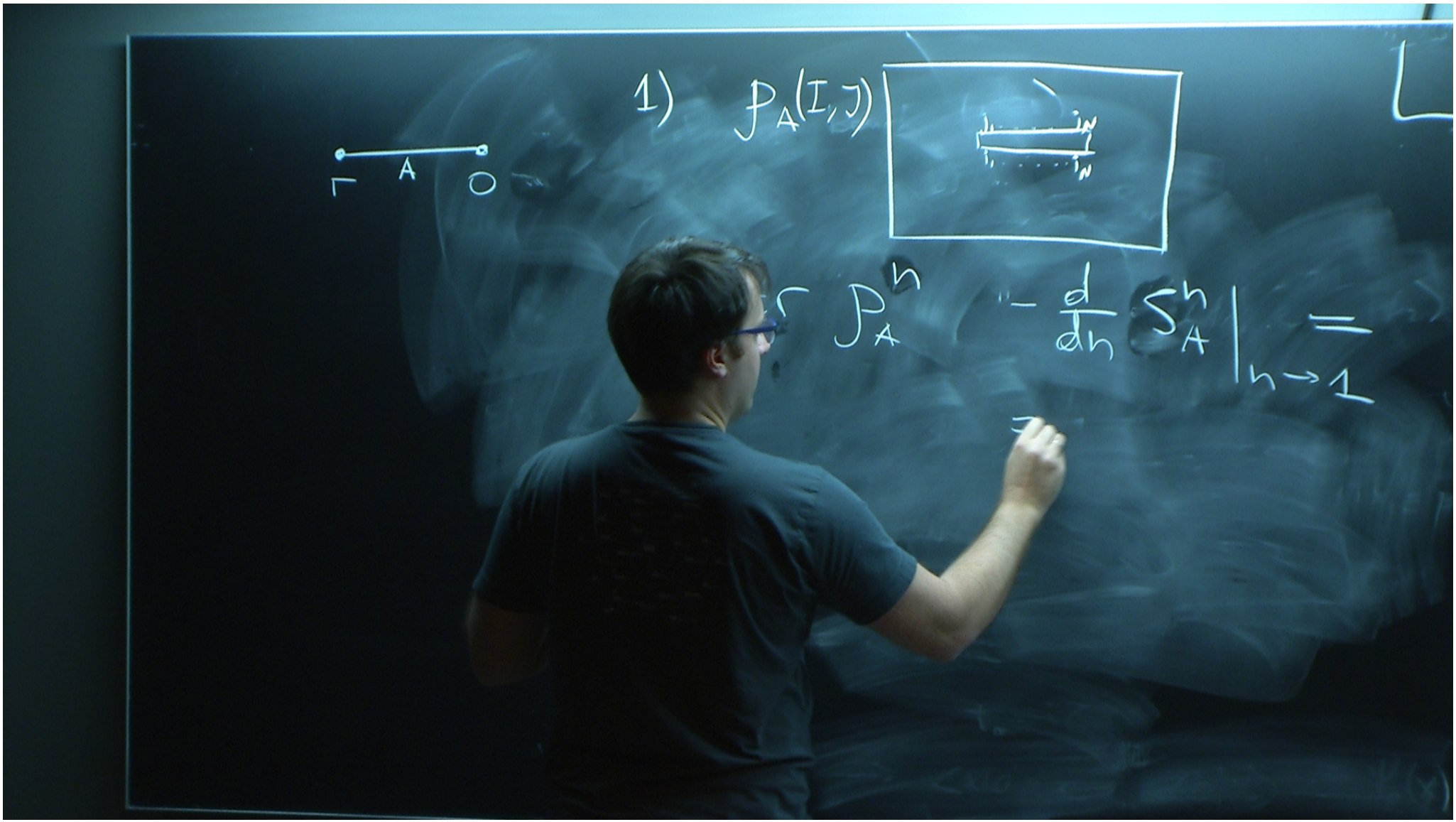

1)
ρ_A
A

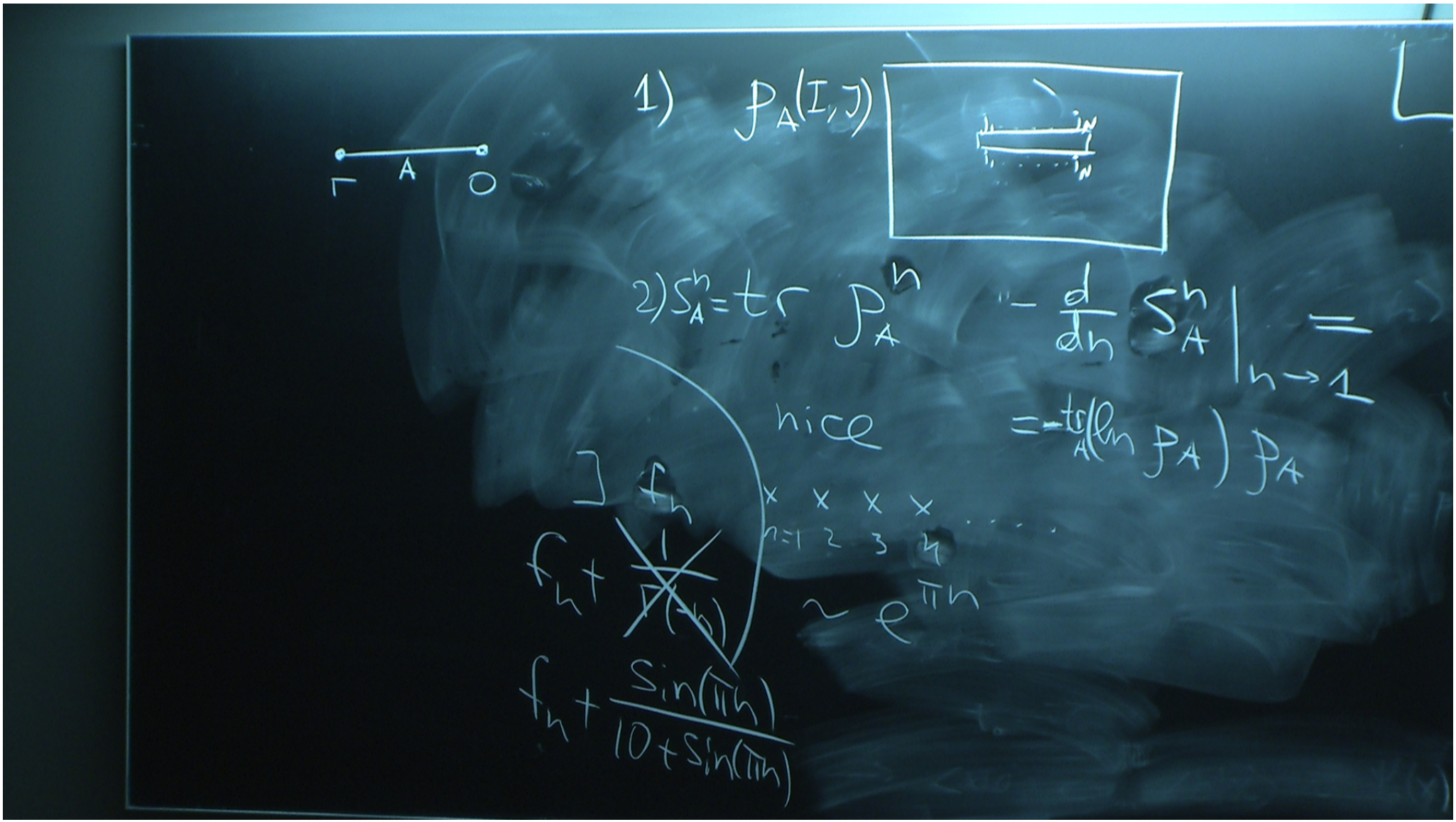

1) $\rho_A(I,J)$
A

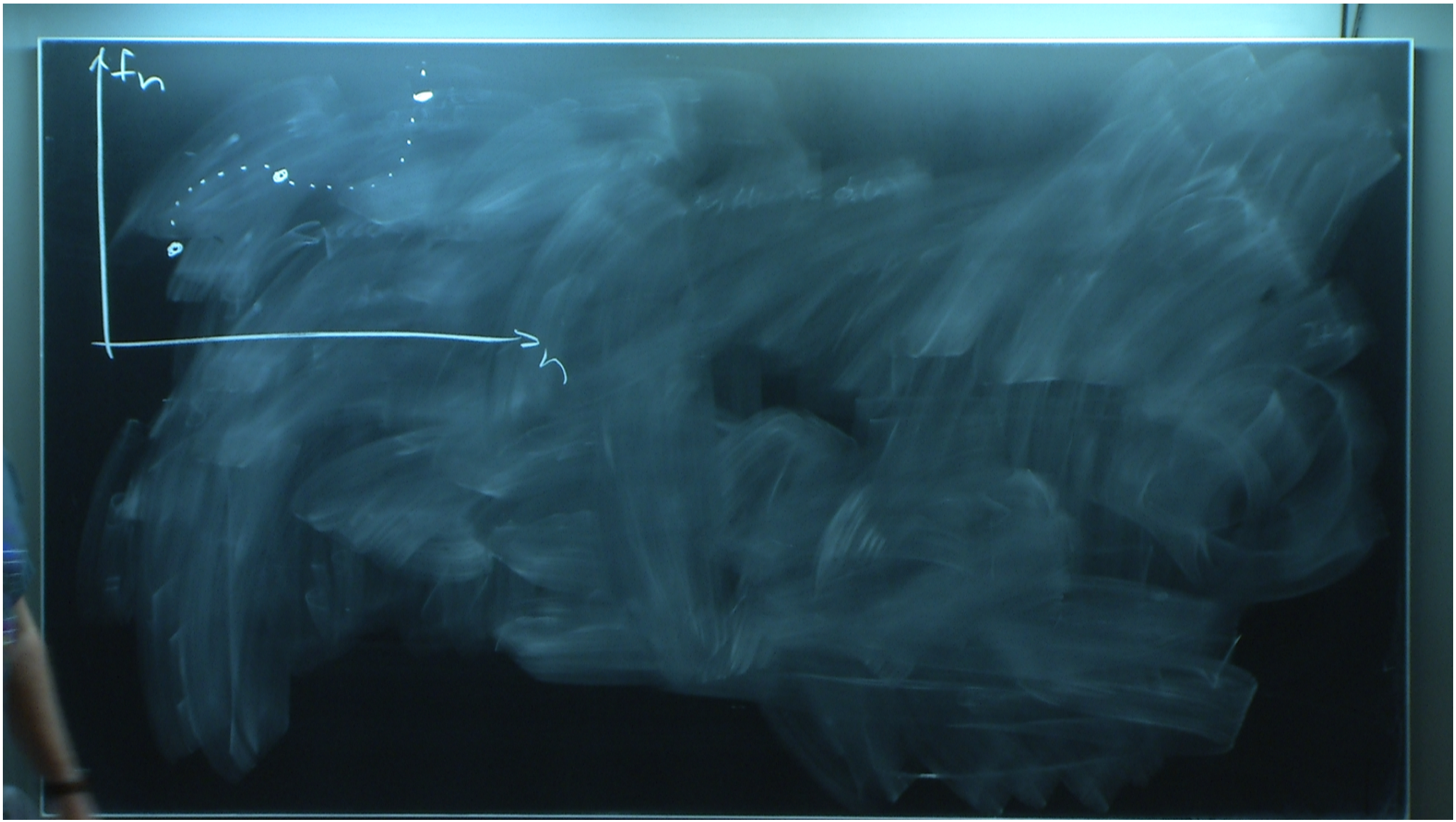
1) $\rho_A(I,J)$
A
2) tr ρ

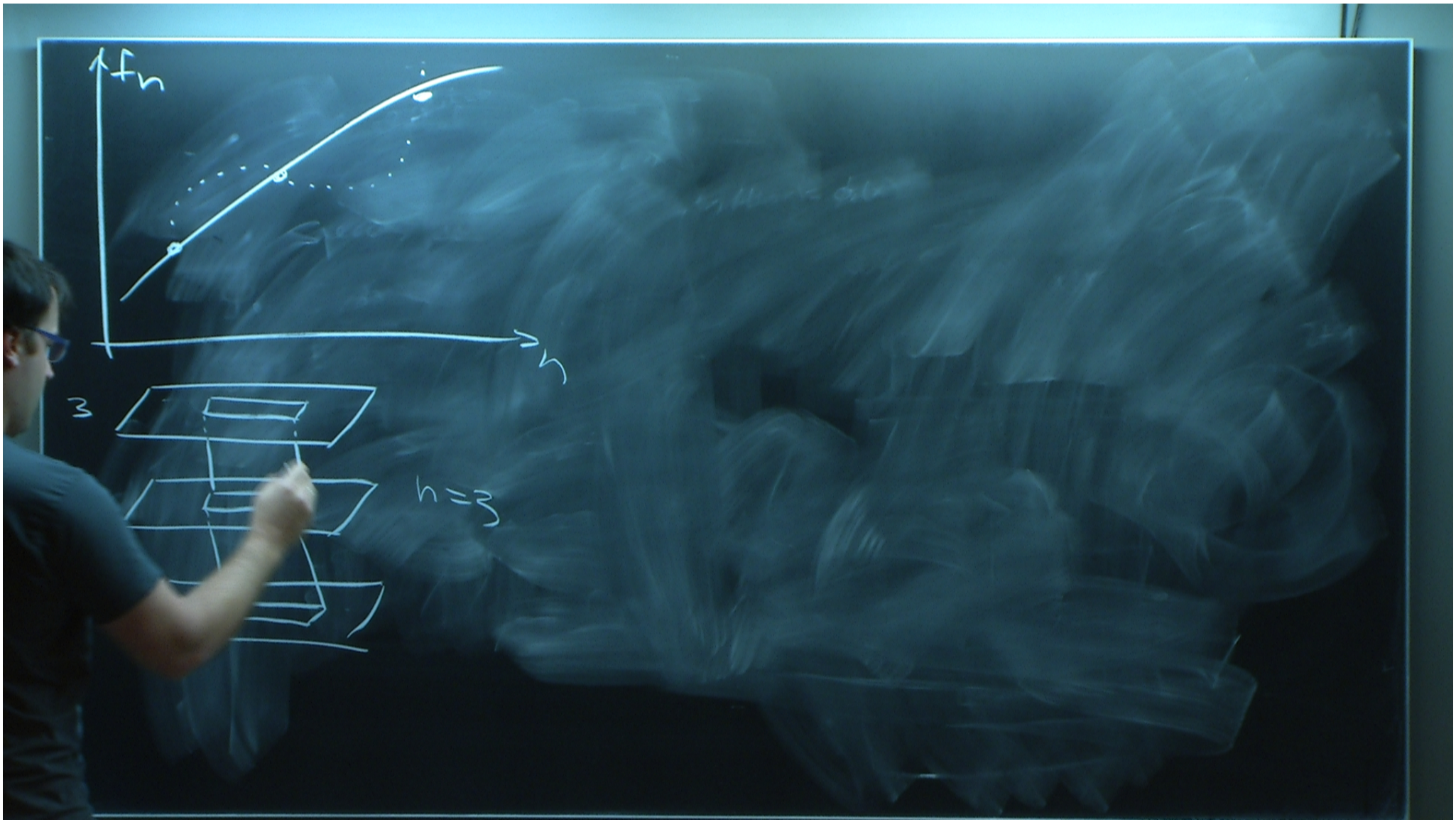

1) $\rho_A(I,J)$
A
2) $S_A^n = \mathrm{tr}\, \rho_A^n$

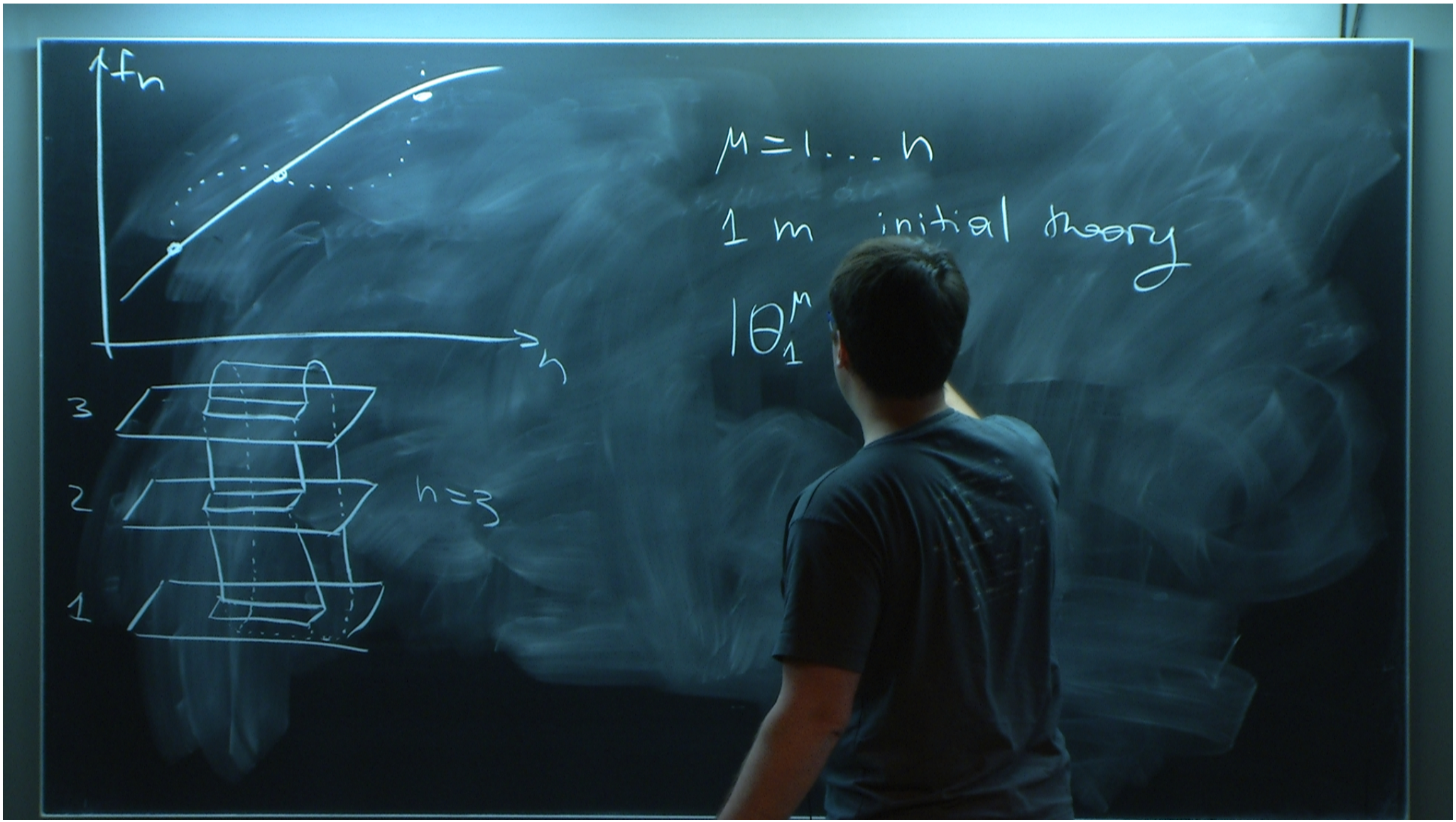

1) $\rho_A(I,J)$
A
2) S_A^h
$= -\frac{d}{dh} S_A^h \Big|$
$\to 1$

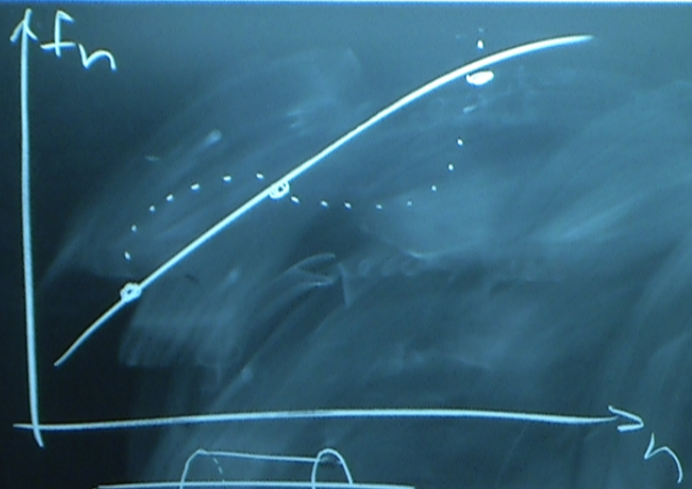

1) $\rho_A(I,J)$
A
ρ_A^n
$-\frac{d}{dn} S_A^n \Big|_{n\to 1} =$

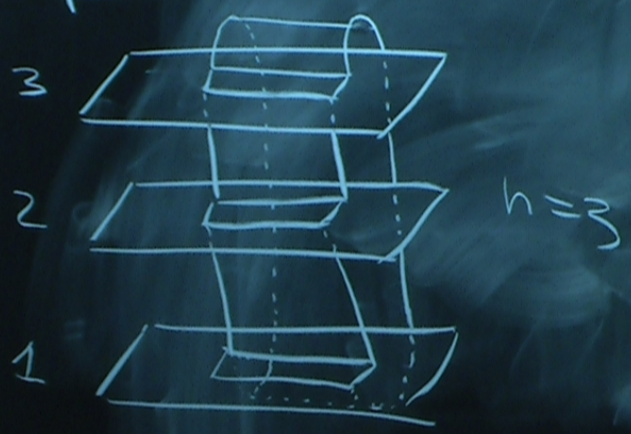

1) $\rho_A(I,J)$
A
2) $S_A^n = \mathrm{tr}\, \rho_A^n$
$= -\frac{d}{dn} S_A^n \Big|_{n\to 1} =$
$= -\mathrm{tr}_A(\ln \rho_A)\, \rho_A$
nice
f_n
$f_n + \frac{1}{P(n)}$
$\sim e^{\pi n}$
$f_n + \frac{\sin(\pi n)}{10 + \sin(\pi n)}$


f_n
n


f_n
n
3
$h=3$


f_n
n
$\mu = 1 \ldots n$
1 m initial theory
$|\theta_1^\mu$
3
2
1
$h=3$





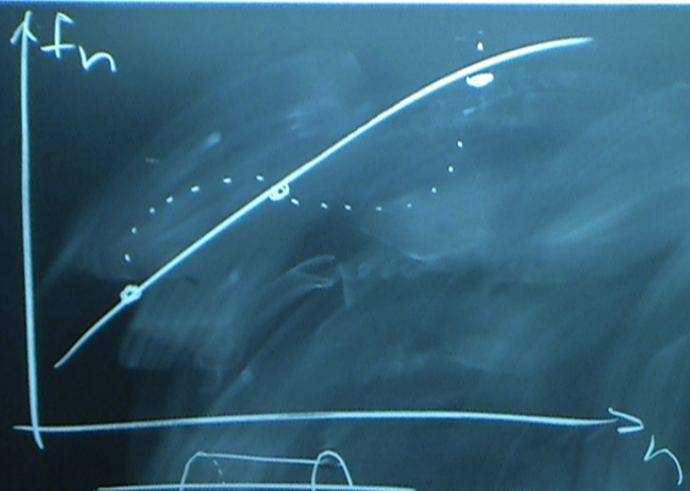
$\mu = 1 \ldots n$

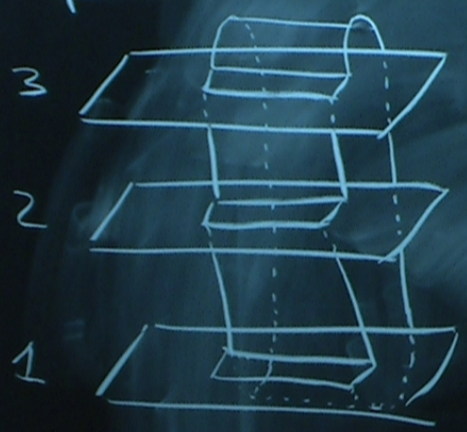
1 m initial theory

$$|\theta_1^{\mu_1}\, \theta_2^{\mu_2} \ldots \theta_k^{\mu_k}\rangle$$

$$E = m \operatorname{ch} \theta$$

$$P = m \operatorname{sh} \theta$$





$n=3$

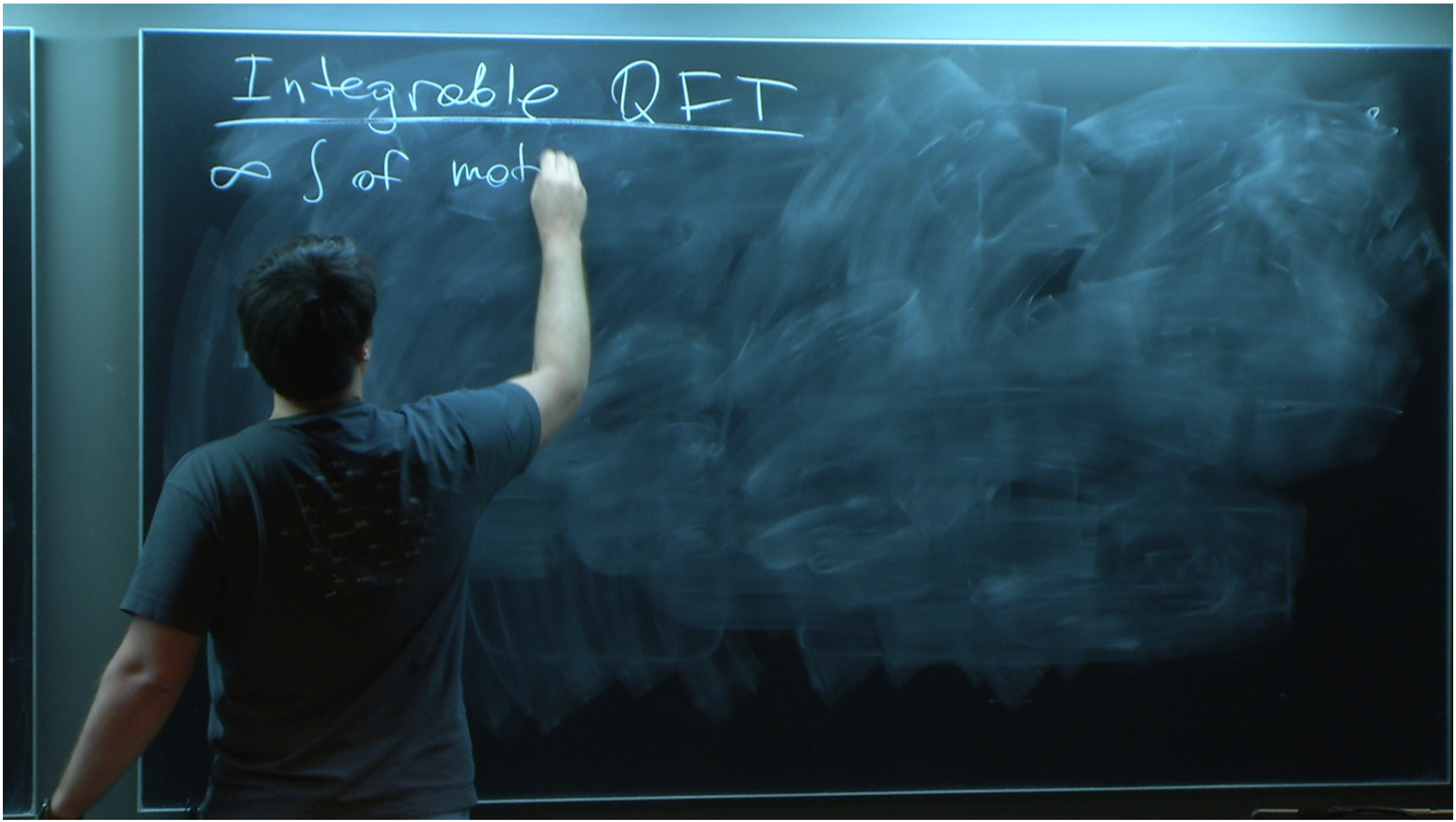
$\mu = 1 \ldots n$

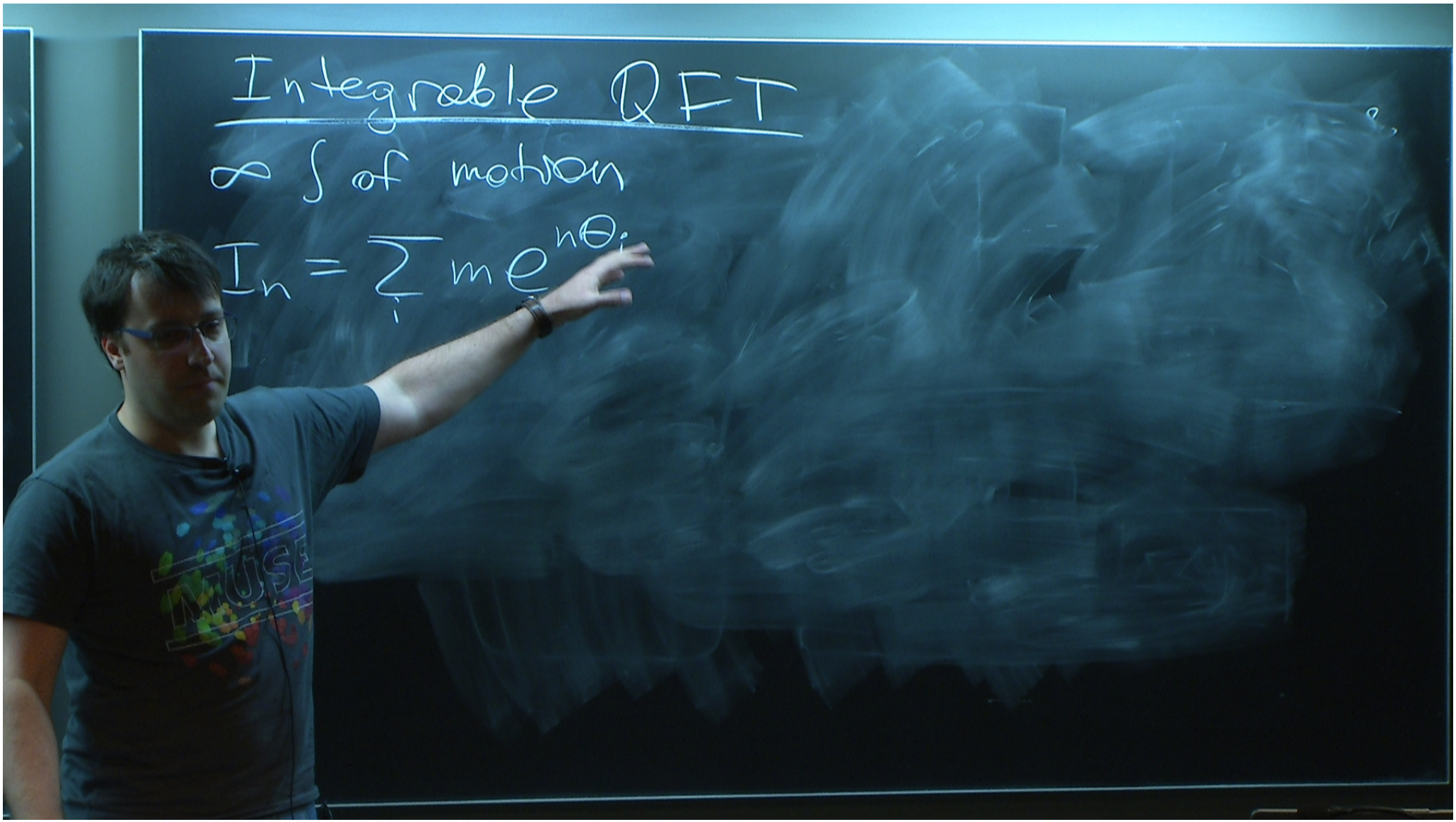
1 m initial theory

$$|\theta_1^{\mu_1}\, \theta_2^{\mu_2} \ldots \theta_k^{\mu_k}\rangle$$

$$E = m \operatorname{ch} \theta$$

$$P = m \operatorname{sh} \theta$$


Integrable QFT
∞ ∫ of mot


Integrable QFT
∞ ∫ of motion
$I_n = \sum_i m e^{n\theta_i}$

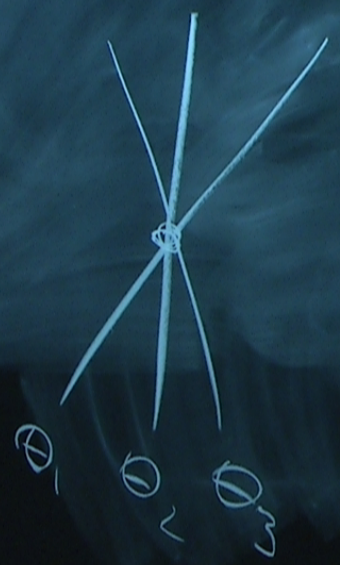
Integrable QFT

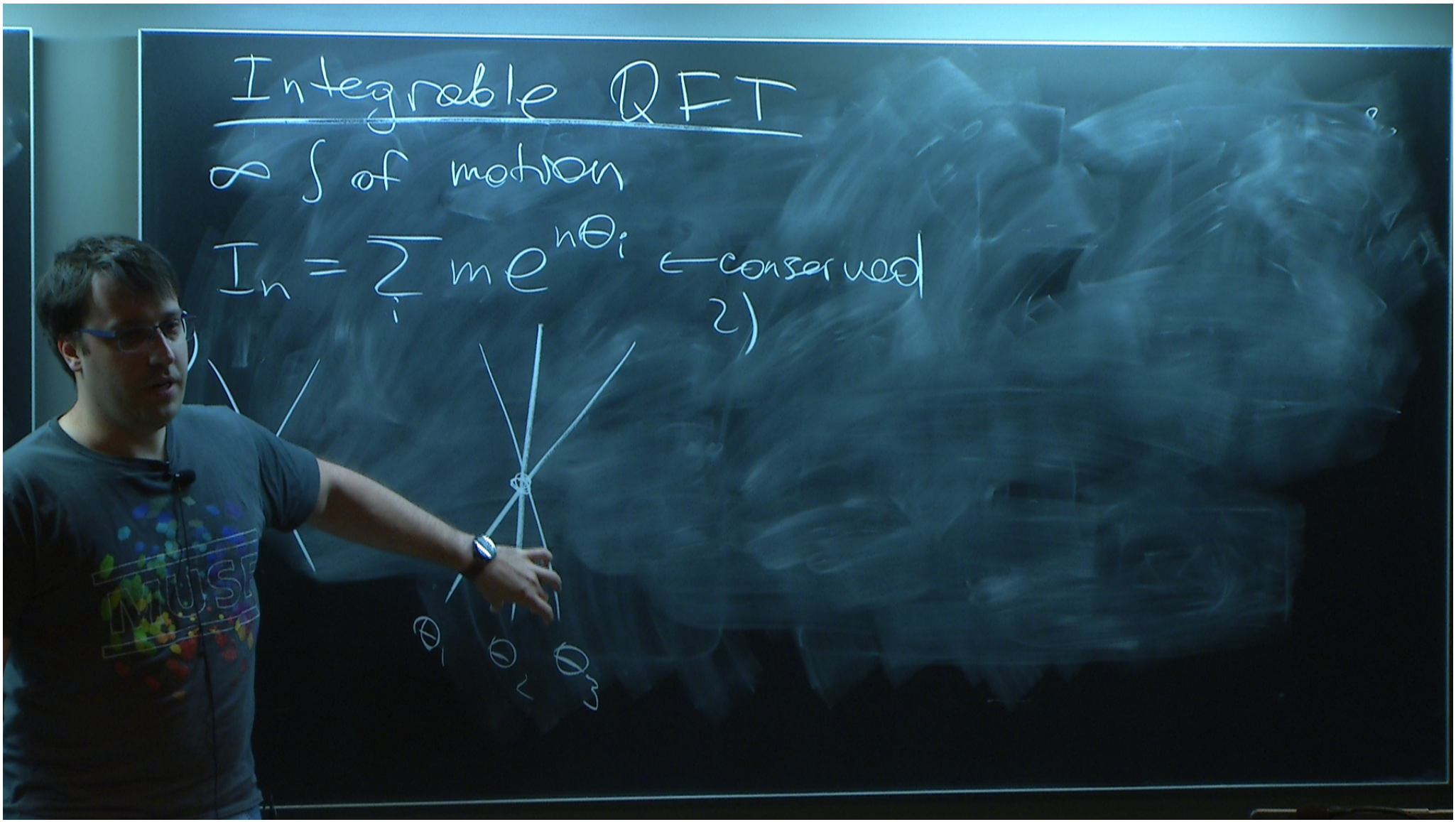
∞ ∫ of motion

$$I_n = \sum_i m e^{n\theta_i} \leftarrow \text{conserved}$$

1)

2)



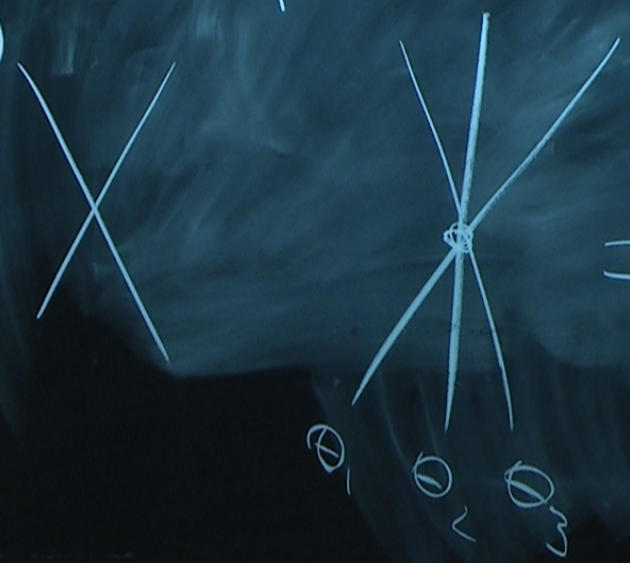

Integrable QFT
∞ ∫ of motion
$I_n = \sum_i m e^{n\theta_i}$ ← conserved
2)
θ_1 θ_2 θ_3

Integrable QFT

∞ ∫ of motion

$$I_n = \sum_i m e^{n\theta_i} \leftarrow \text{conserved}$$

1)



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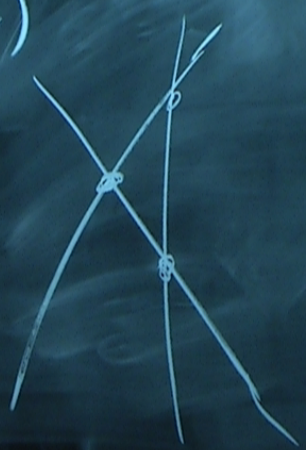
2)

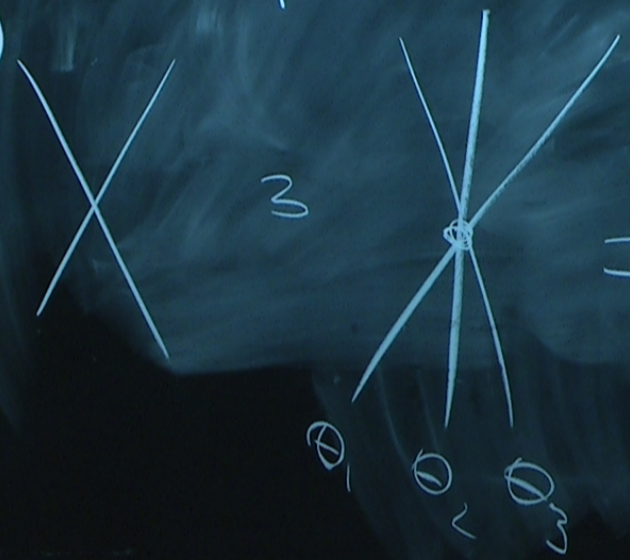


Integrable QFT

∞ ∫ of motion

$I_n = \sum_i m e^{n\theta_i}$ ← conserved

1)



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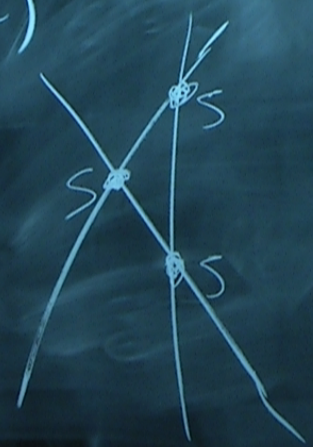
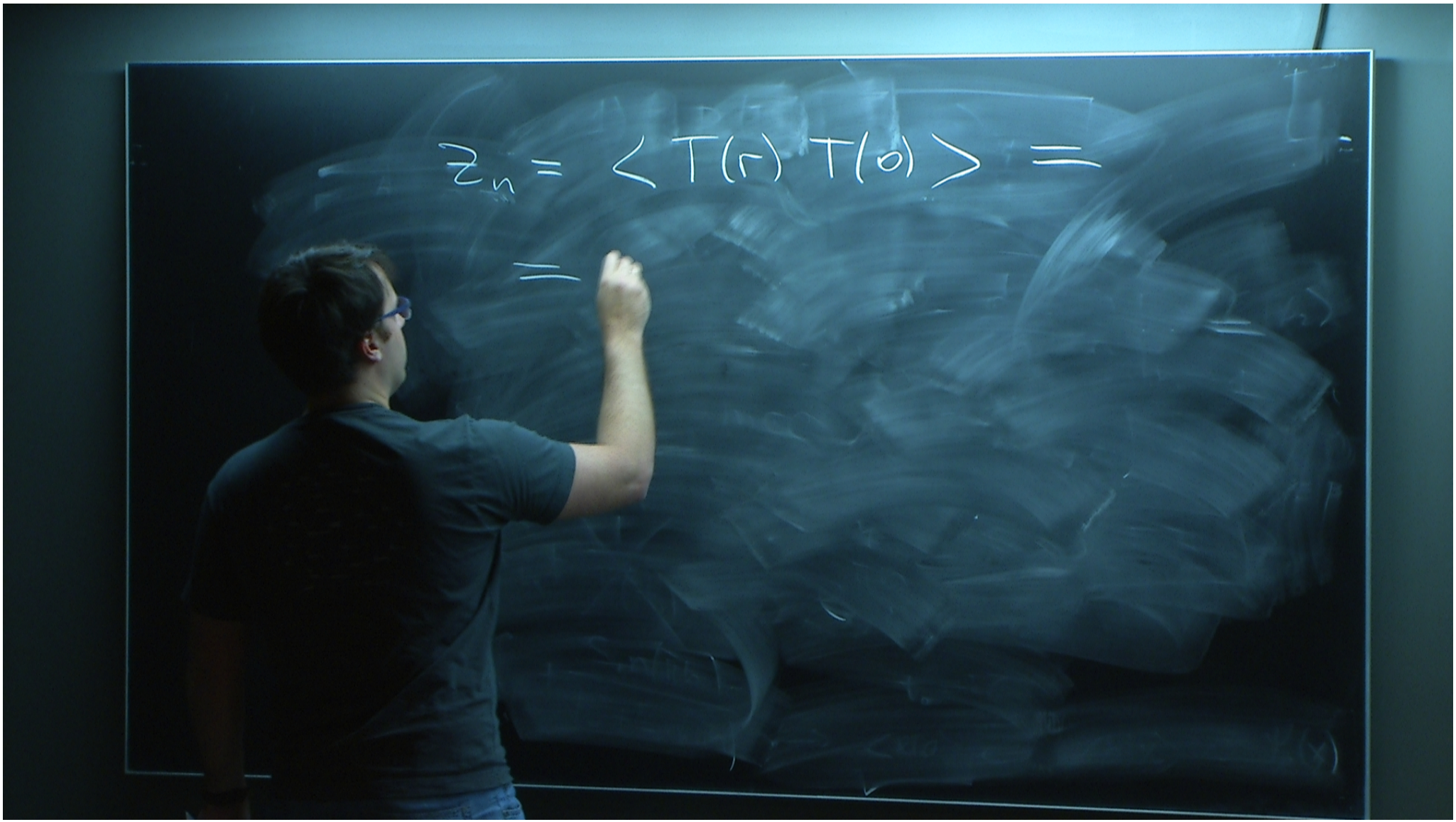
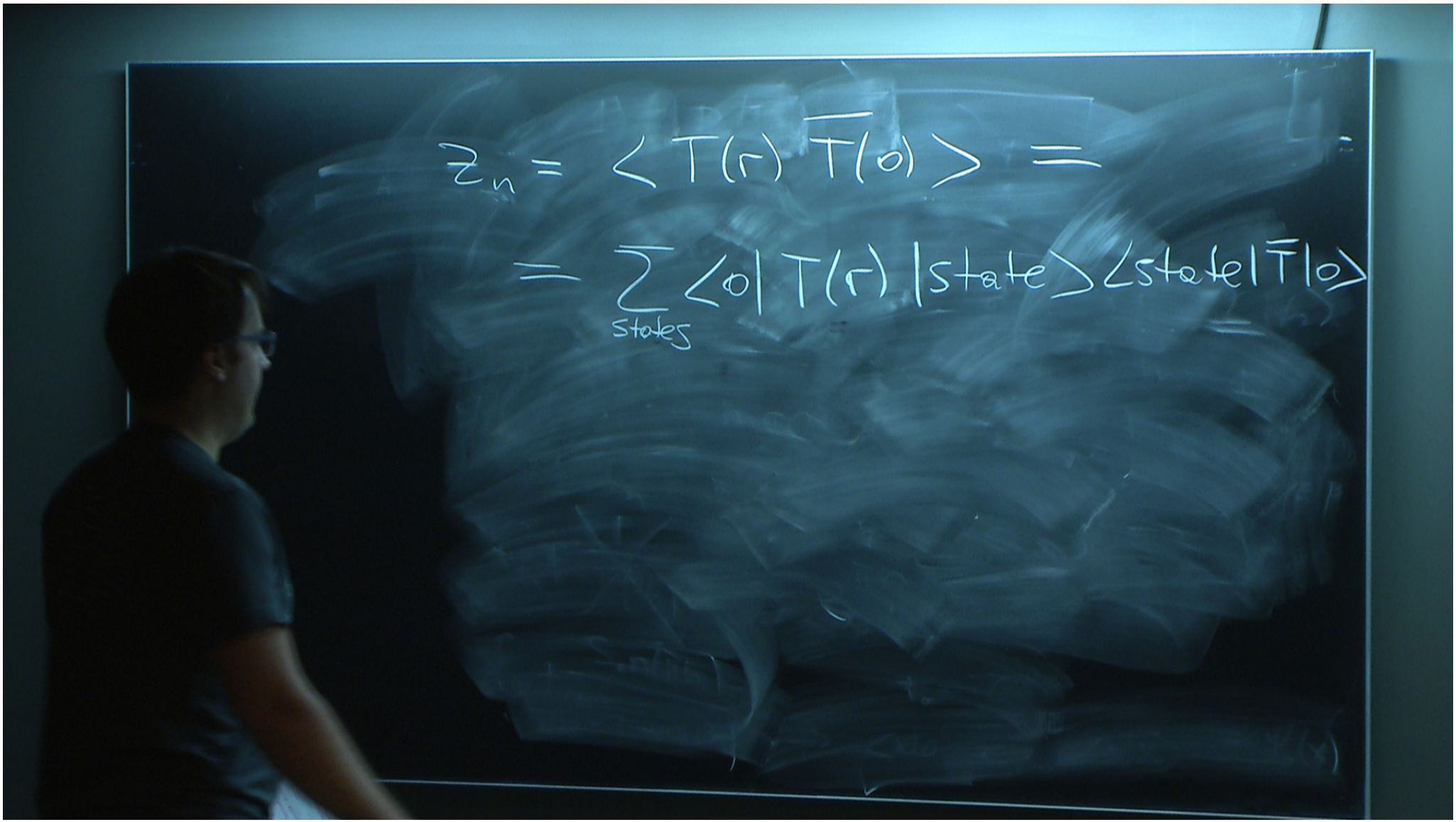
2)



$S(\theta_1 - \theta_2)$

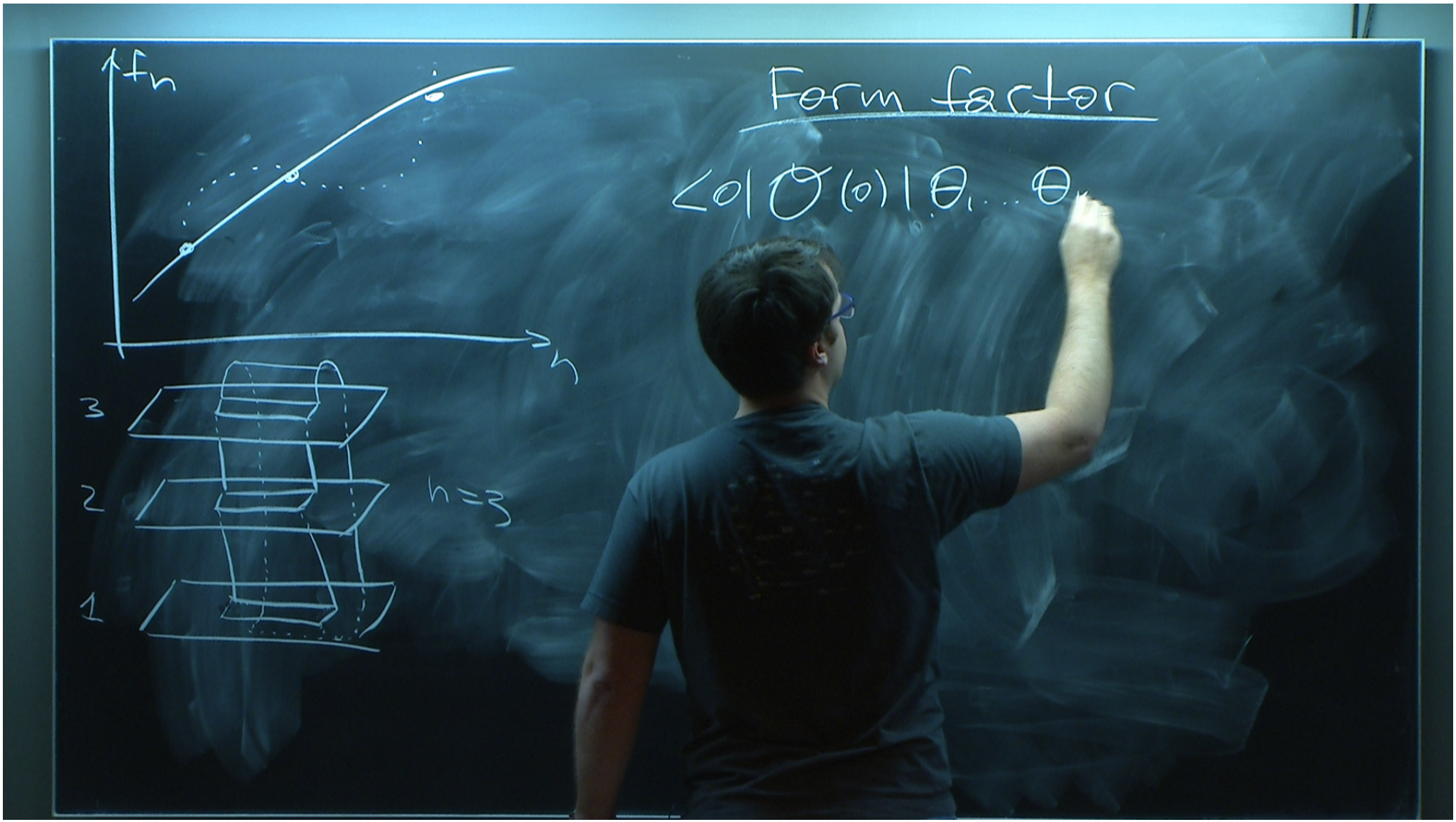

$z_n = \langle T(r)\, T(0) \rangle =$
$=$

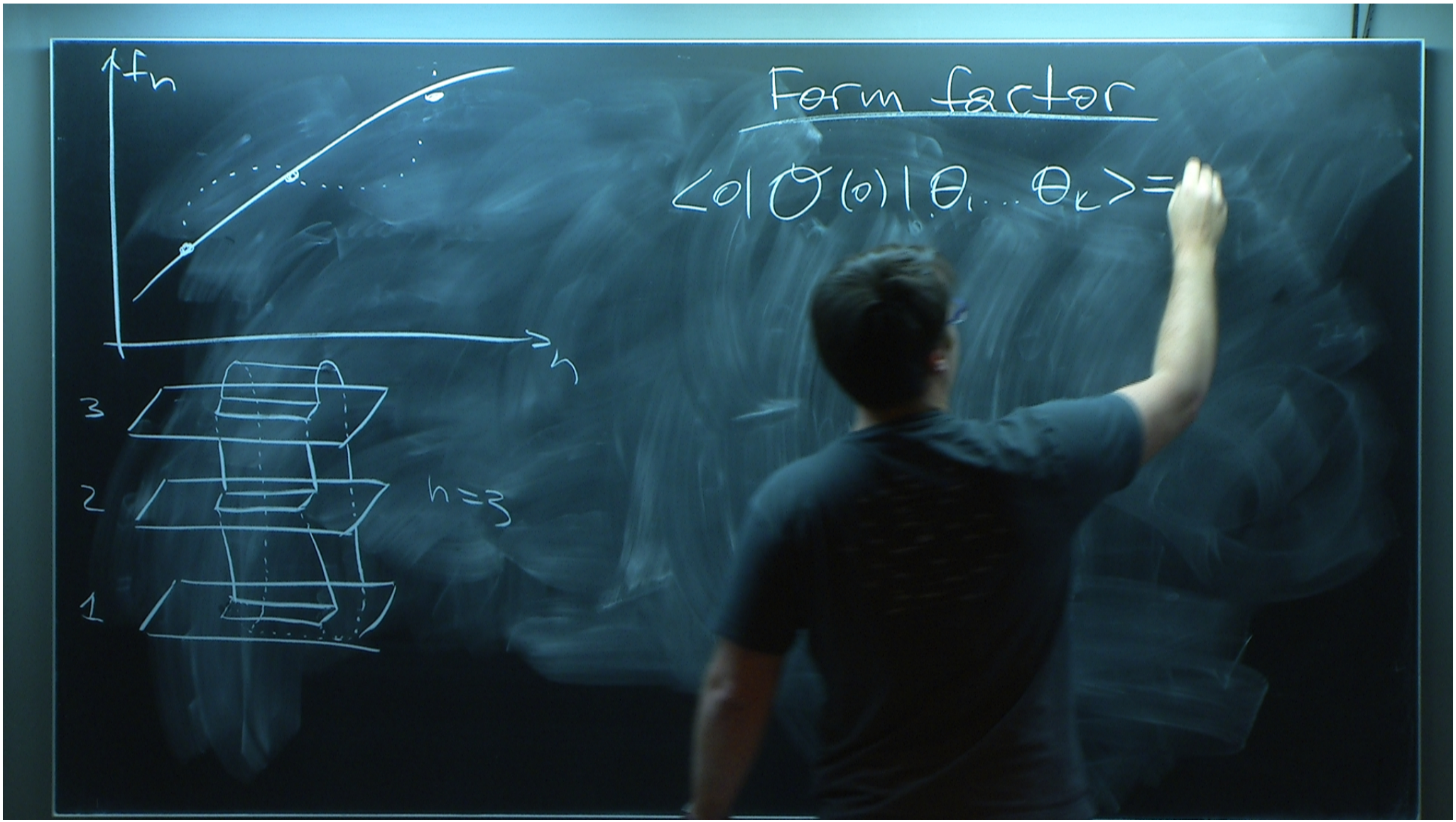

$z_n = \langle T(r)\,\bar{T}(0) \rangle =$
$= \sum_{states} \langle 0 | T(r) | state \rangle \langle state | \bar{T} | 0 \rangle$

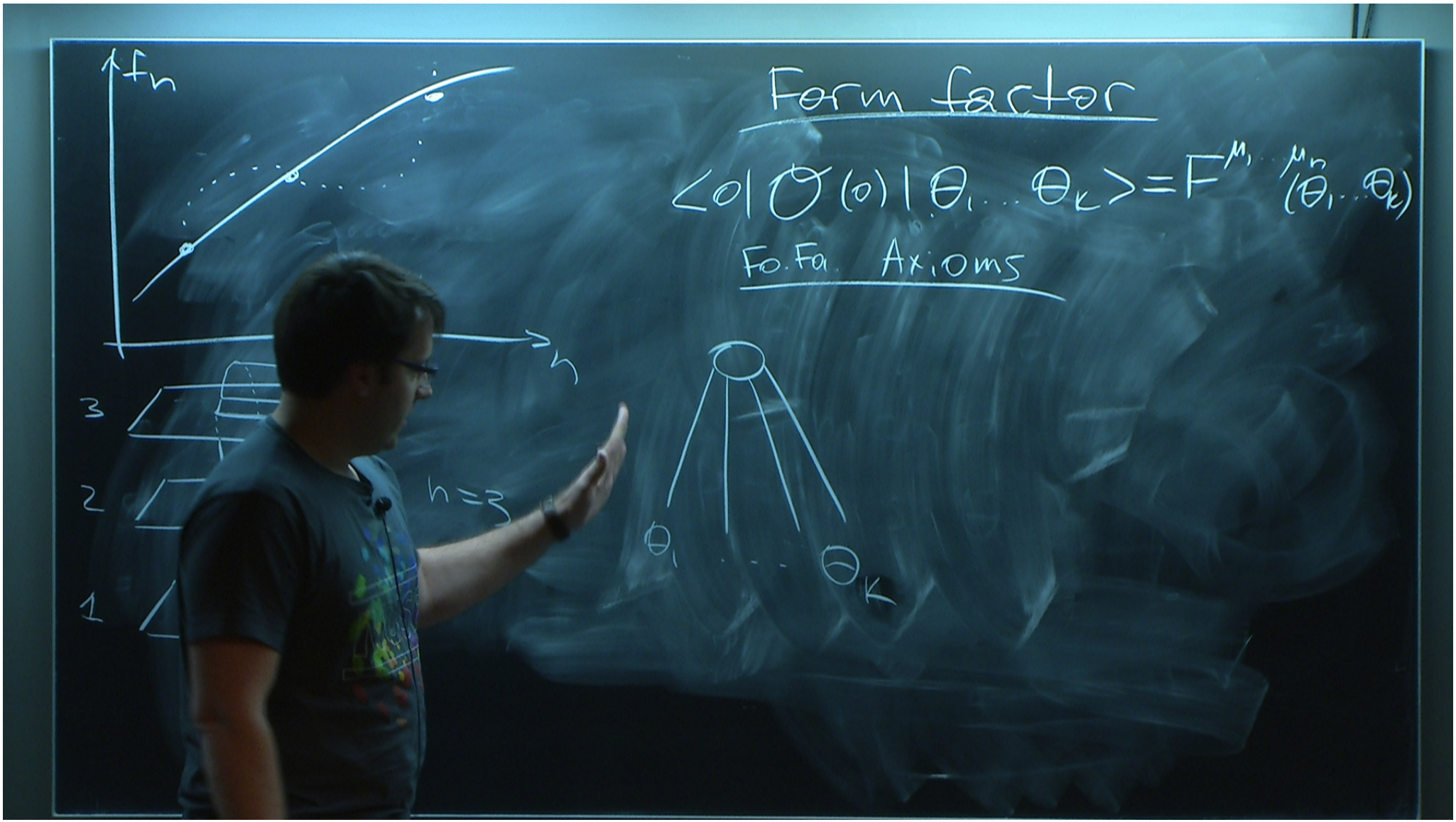
$$Z_n = \langle T(r)\,\bar{T}(0)\rangle =$$

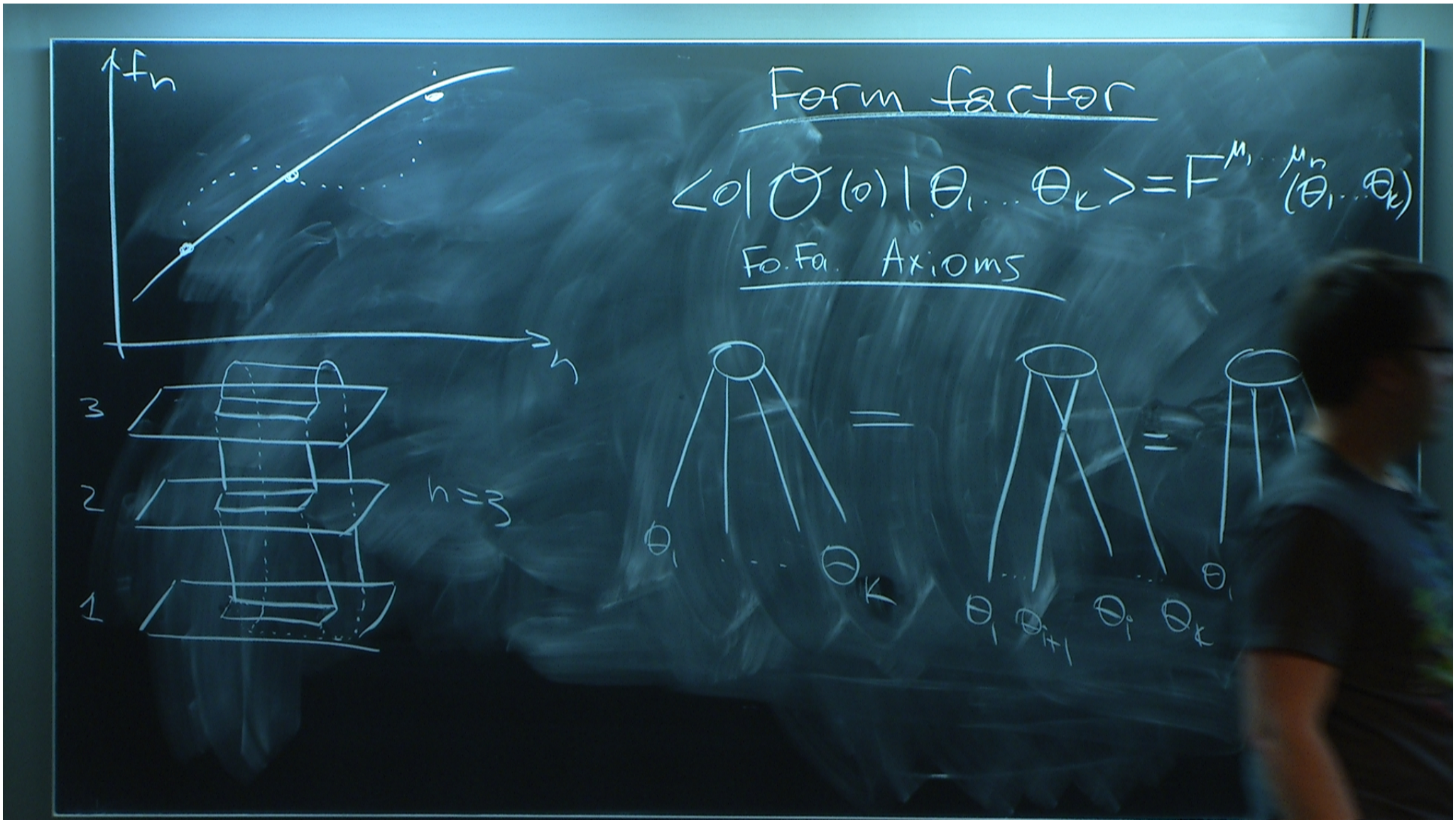
$$= \sum_{\text{states}} \langle 0|\,T(r)\,|\text{state}\rangle\langle \text{state}|\,\bar{T}\,|0\rangle$$

$$|\text{state}\rangle = |0\rangle,\ |\theta\rangle,\ |\theta_1, \theta_2 \ldots\rangle$$

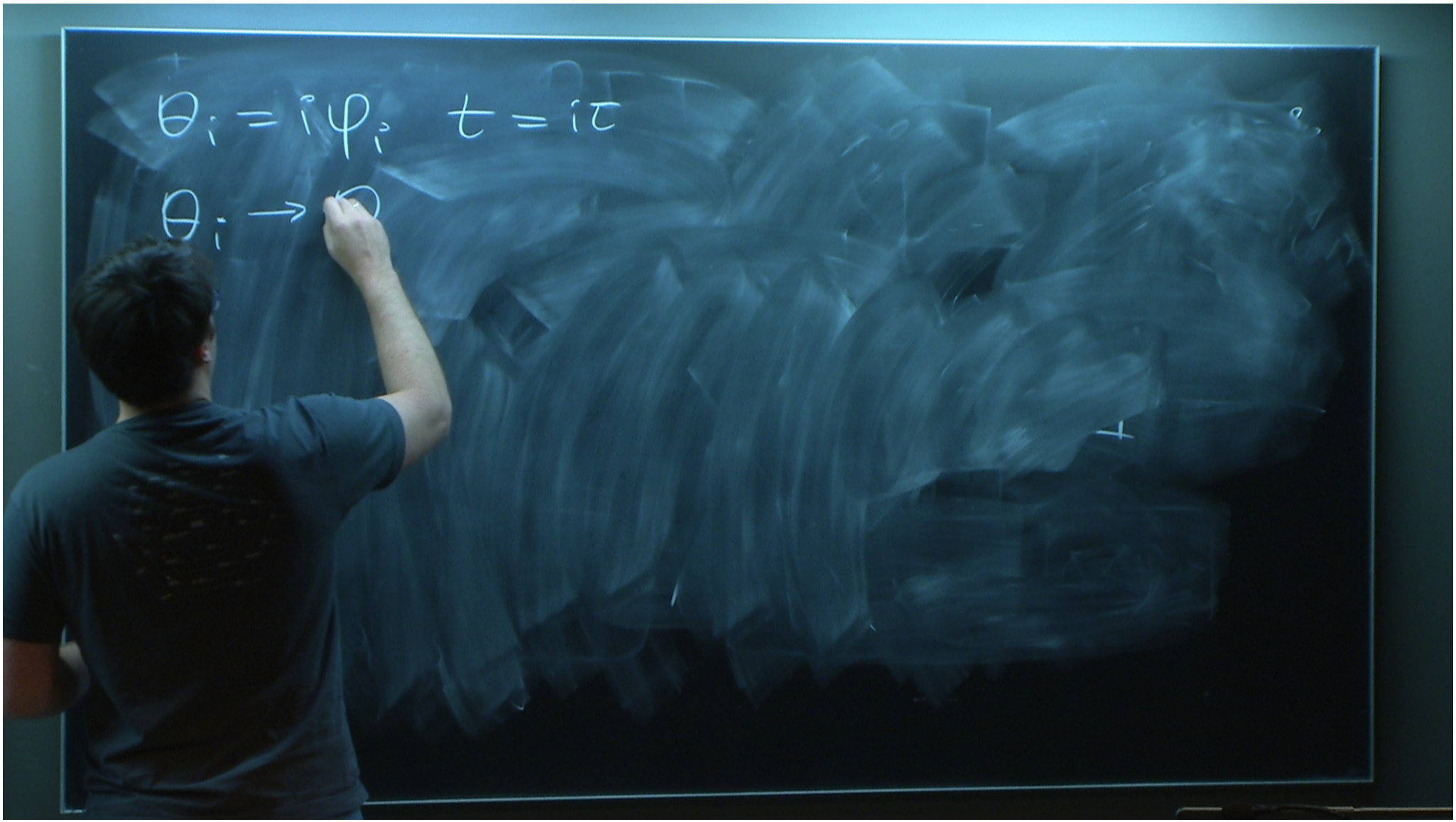

f_n
n
Form factor
$\langle 0 | \mathcal{O}(0) | \theta_1 \ldots \theta_n$
3
2
1
$h=3$


f_n
n
3
2
1
$n=3$
Form factor
$\langle 0|\mathcal{O}(0)|\theta_1 \ldots \theta_k\rangle =$


Form factor
$\langle 0 | \mathcal{O}(0) | \theta_1 \ldots \theta_k \rangle = F^{\mu_1 \ldots \mu_n}(\theta_1 \ldots \theta_k)$
Fo.Fa. Axioms
f_n
n
3
2
1
$n=3$
θ_1
θ_k


Form factor
$\langle 0 | \mathcal{O}(0) | \theta_1 \dots \theta_k \rangle = F^{\mu_1 \dots \mu_n}(\theta_1 \dots \theta_k)$
Fo.Fa. Axioms
f_n
n
3
2
1
$n=3$
θ_1
θ_k
θ_1 θ_{i+1} θ_i θ_k

$$\theta_i = i\varphi_i \quad t = i\tau$$


$\theta_i = i\varphi_i \quad t = i\tau$
$\theta_i \to$

$$\theta_i = i\varphi_i \quad t = i\tau$$

$$\theta_i \to \theta_i + 2\pi i$$

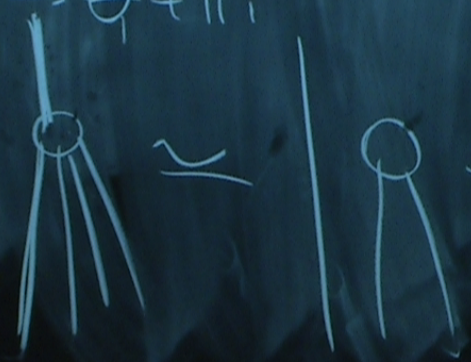
$$F^{\mu_1 \dots \mu_k}(\theta_1 + 2\pi i, \dots, \theta_k) = F^{\mu_2 \dots \mu_k (\mu_1 + 1)}(\theta_2 \dots \theta_k \theta_1)$$

$$\theta_i = i\varphi_i \qquad t = i\tau$$

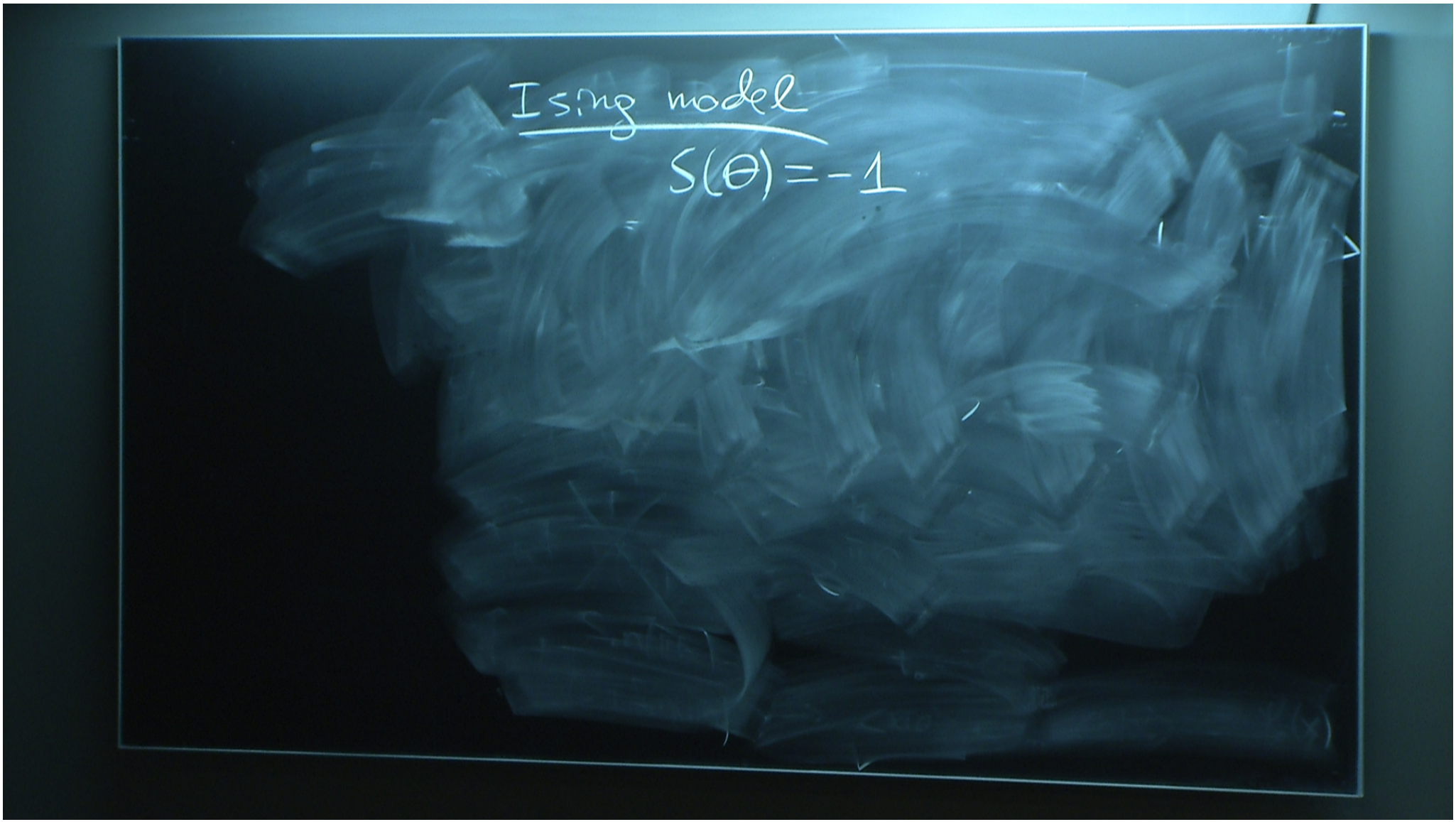
$$\theta_i \to \theta_i + 2\pi i$$

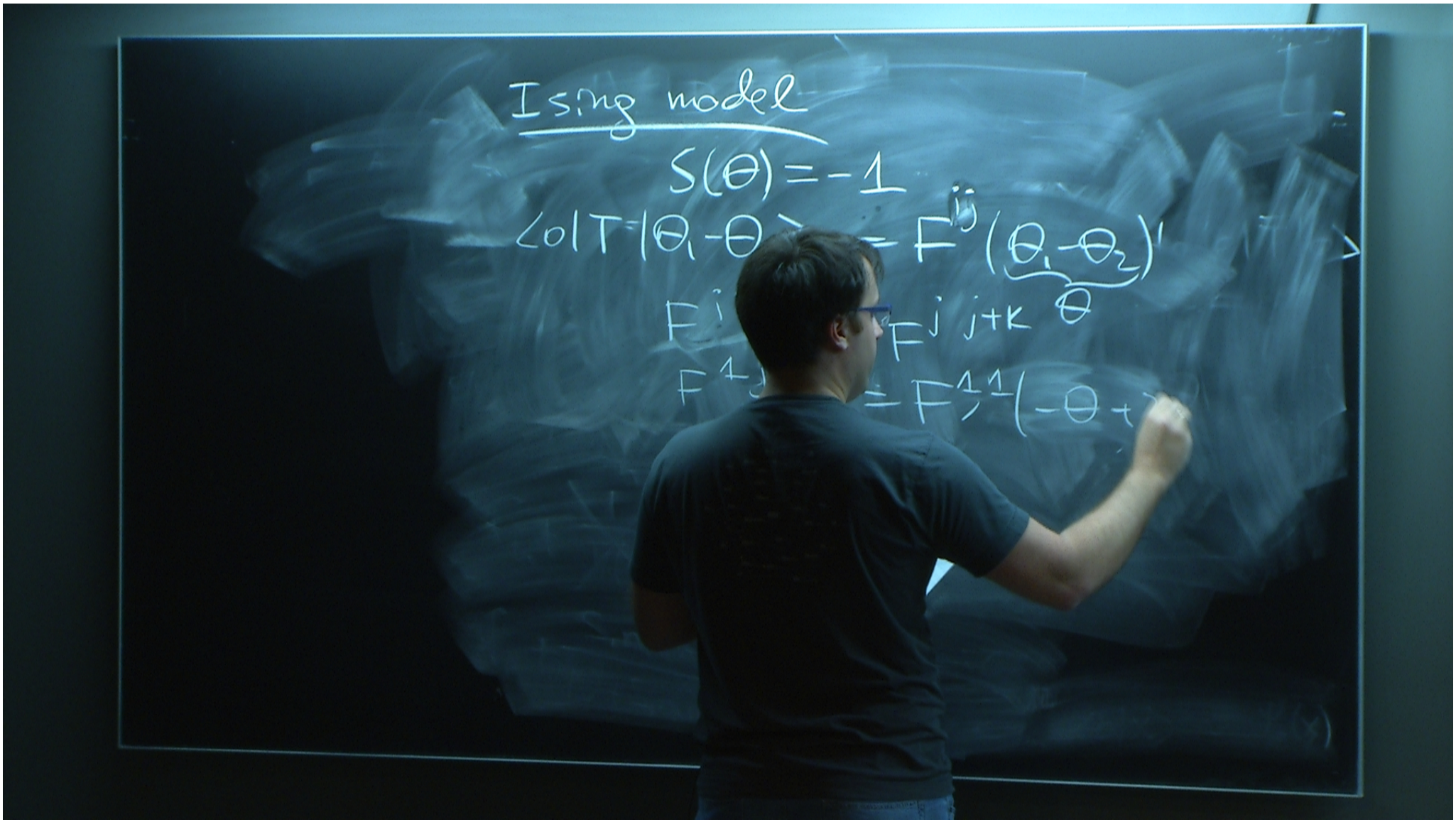
- $$F^{\mu_1 \dots \mu_k}(\theta_1 + 2\pi i, \dots, \theta_k) = F^{\mu_2 \dots \mu_k (\mu_1 + 1)}(\theta_2 \dots \theta_k \theta_1)$$

- $$\theta_1 \to \theta_1 + \pi i \qquad -i \operatorname*{Res}_{\bar\theta_0 = \theta_0} F^{\mu\mu\mu_1 \dots \mu_k}(\bar\theta_0 + i\pi, \theta_0, \theta_1 \dots \theta_k) = F^{\mu_1 \dots \mu_k}(\theta_1 \dots \theta_k)$$



$$\frac{1}{\theta_1 - \theta_2}$$


Ising model
$S(\theta) = -1$


Ising model
S(θ) = -1

Ising model

$$S(\theta) = -1$$

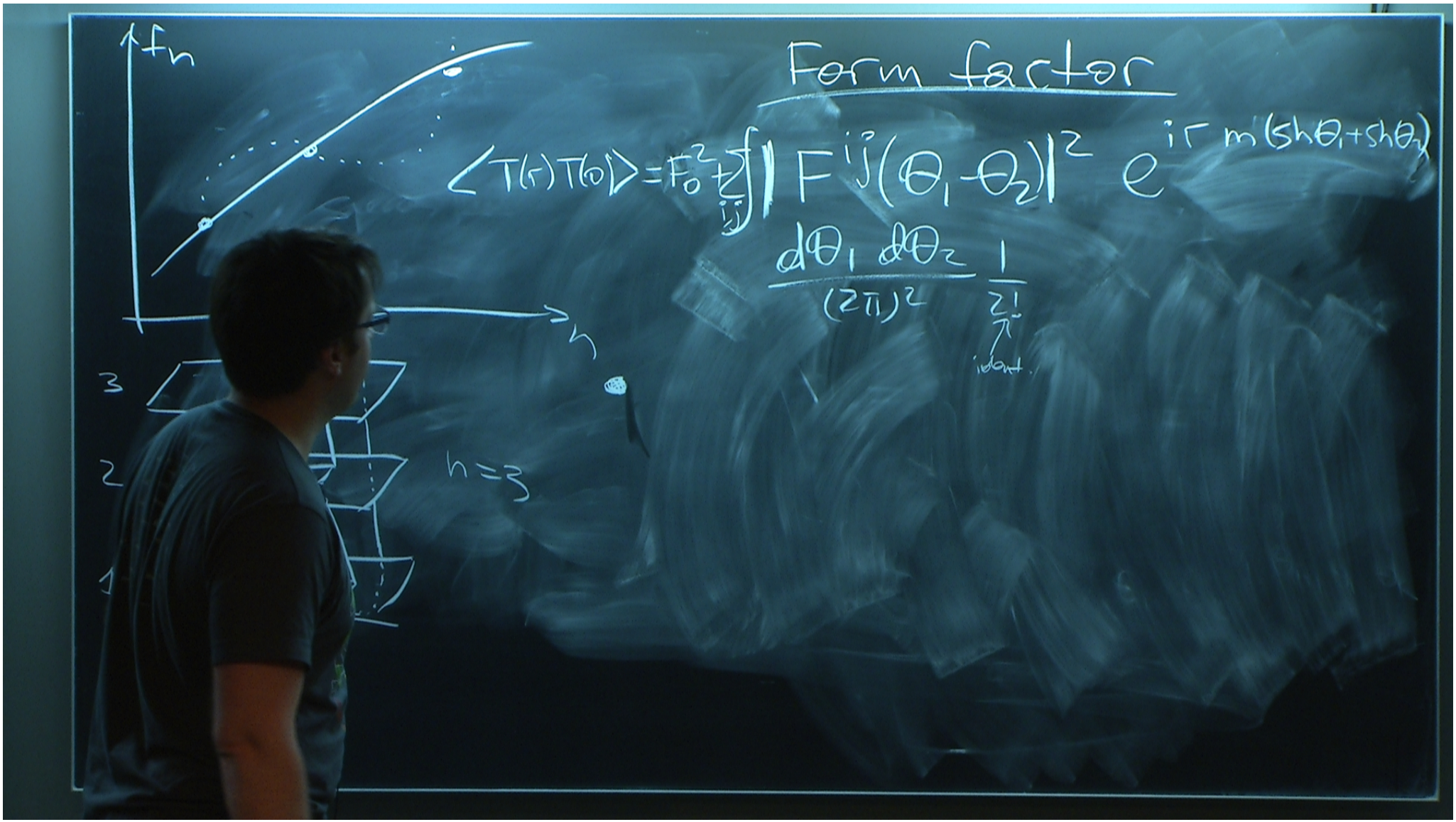
$$\langle 0|T|\theta_1 - \theta_2\rangle = F^{ij}(\underbrace{\theta_1 - \theta_2}_{\theta})$$

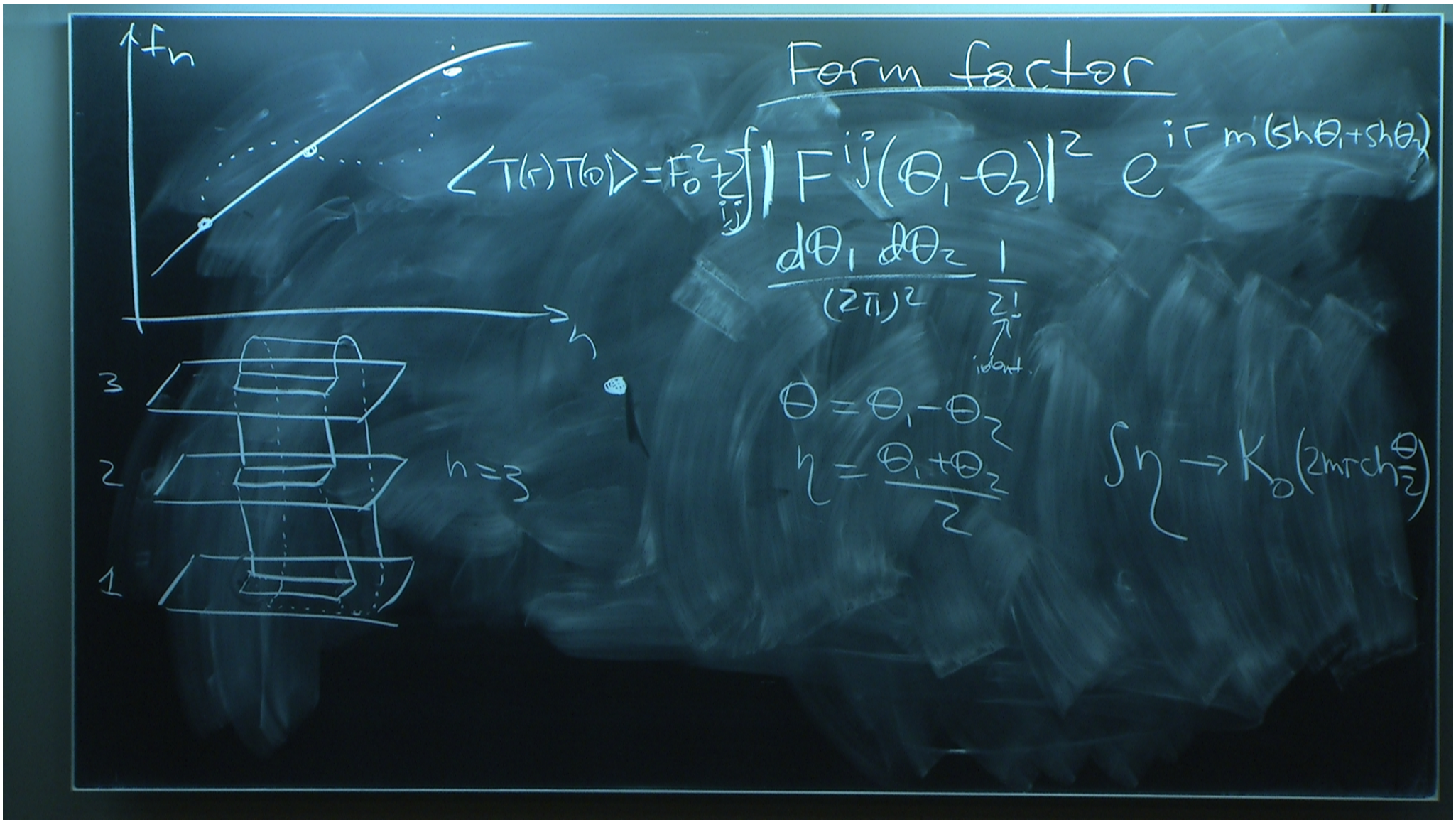
$$F^{i\,i+k} = F^{j\,j+k}$$

$$F^{1j}(\theta) \underset{j>1}{=} F^{11}(-\theta + 2\pi(j-1))$$

$$F^{11}(\theta) = \frac{F_0}{n} \frac{\cos\left(\frac{\pi}{2n}\right) \operatorname{sh}\left(\frac{\theta}{2n}\right)}{\operatorname{sh}\left(\frac{\pi - i\theta}{2n}\right) \operatorname{sh}\left(\frac{\pi + i\theta}{2n}\right)}$$

$F_0 = \langle 0|T|0\rangle$


Form factor
$\langle T(r)T(0)\rangle = F_0^2 + \sum_{ij} \int |F^{ij}(\theta_1-\theta_2)|^2 e^{irm(\mathrm{sh}\theta_1+\mathrm{sh}\theta_2)} \frac{d\theta_1\, d\theta_2}{(2\pi)^2} \frac{1}{2!}$
f_n
n
3
2
1
$n=3$


f_n
n
3
2
1
$n=3$
Form factor
$\langle T(r)T(0)\rangle = F_0^2 + \sum_{ij}\int |F^{ij}(\theta_1-\theta_2)|^2 \, e^{irm(\mathrm{sh}\theta_1+\mathrm{sh}\theta_2)} \frac{d\theta_1\, d\theta_2}{(2\pi)^2}\frac{1}{2!}$
identt.
$\theta = \theta_1 - \theta_2$
$\eta = \frac{\theta_1+\theta_2}{2}$
$\int \eta \to K_0\left(2mr\,\mathrm{ch}\frac{\theta}{2}\right)$

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Poison resummation

$$ts = \left(4\,\mathrm{Cos}\left[\frac{\pi}{2\,n}\right]^2 \left(\mathrm{Cos}\left[\frac{2\,\pi\,(j-1)}{n}\right] - \mathrm{Cosh}\left[\frac{\theta}{n}\right]\right)\right) \Big/ \left(n\left(\mathrm{Cos}\left[\frac{\pi\,(3-2\,j)}{n}\right] - \mathrm{Cosh}\left[\frac{\theta}{n}\right]\right)\left(\mathrm{Cosh}\left[\frac{\theta}{n}\right] - \mathrm{Cos}\left[\frac{\pi - 2\,\pi\,j}{n}\right]\right)\right);$$

By Poles

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Poison resummation

In[1]:=

```
ts = (4 Cos[π/(2 n)]^2 (Cos[(2 π (j - 1))/n] - Cosh[θ/n]))/
   (n (Cos[(π (3 - 2 j))/n] - Cosh[θ/n]) (Cosh[θ/n] - Cos[(π - 2 π j)/n]));
```

In[6]:=

```
sum[n0_] := Sum[ts /. θ → 1/2 /. n → n0, {j, n0}]
```

```
||
```

In[5]:=

```
sum[2]
```

Out[5]=

$$\frac{4\cos^2\left(\frac{\pi}{2n}\right)\left(1-\cosh\left(\frac{1}{2n}\right)\right)}{n\left(\cos\left(\frac{\pi}{n}\right)-\cosh\left(\frac{1}{2n}\right)\right)\left(\cosh\left(\frac{1}{2n}\right)-\cos\left(\frac{\pi}{n}\right)\right)}+\frac{4\cos^2\left(\frac{\pi}{2n}\right)\left(\cos\left(\frac{2\pi}{n}\right)-\cosh\left(\frac{1}{2n}\right)\right)}{n\left(\cos\left(\frac{\pi}{n}\right)-\cosh\left(\frac{1}{2n}\right)\right)\left(\cosh\left(\frac{1}{2n}\right)-\cos\left(\frac{3\pi}{n}\right)\right)}$$

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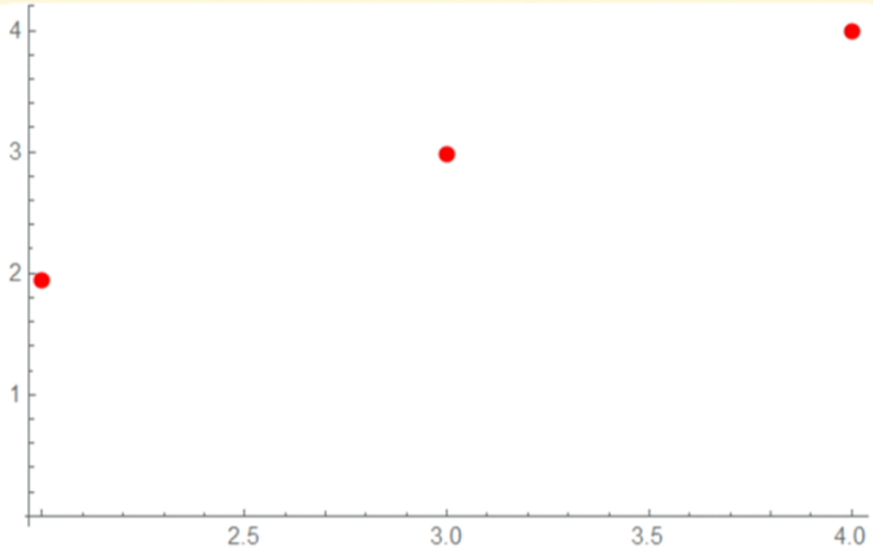
In[6]:=
```
sum[n0_] := Sum[ts /. θ → 1 / 2 /. n → n0, {j, n0}]
```

In[12]:=
```
p1 = ListPlot[Table[{n, sum[n]}, {n, 2, 4}], PlotStyle → Red]
```

Out[12]=



By Poles

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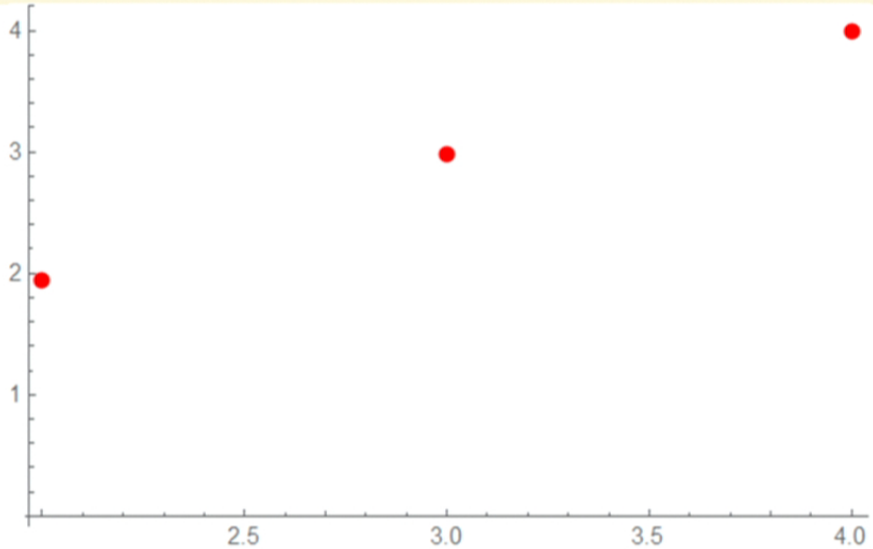
In[6]:=
```
sum[n0_] := Sum[ts /. θ → 1 / 2 /. n → n0, {j, n0}]
```

In[12]:=
```
p1 = ListPlot[Table[{n, sum[n]}, {n, 2, 4}], PlotStyle → Red]
```

Out[12]=



By Poles

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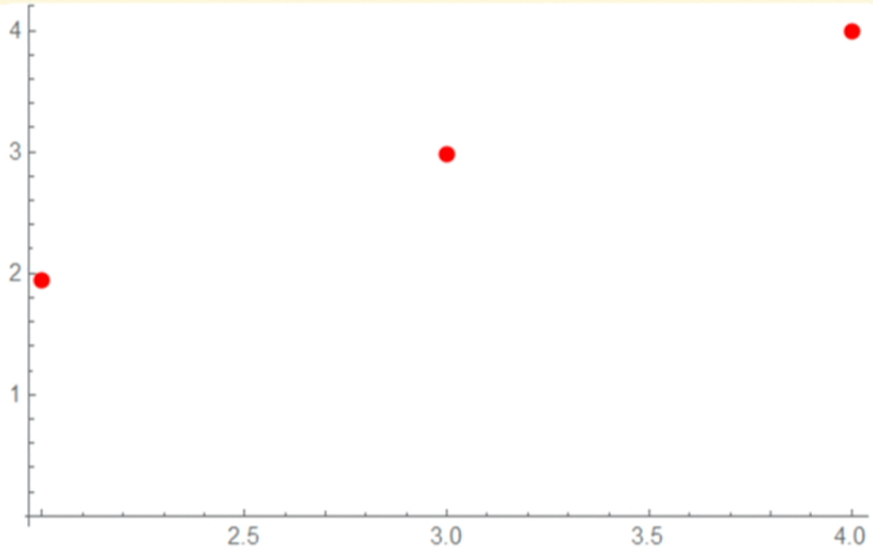
In[6]:=
```
sum[n0_] := Sum[ts /. θ → 1 / 2 /. n → n0, {j, n0}]
```

In[12]:=
```
p1 = ListPlot[Table[{n, sum[n]}, {n, 2, 4}], PlotStyle → Red]
```

Out[12]=



By Poles

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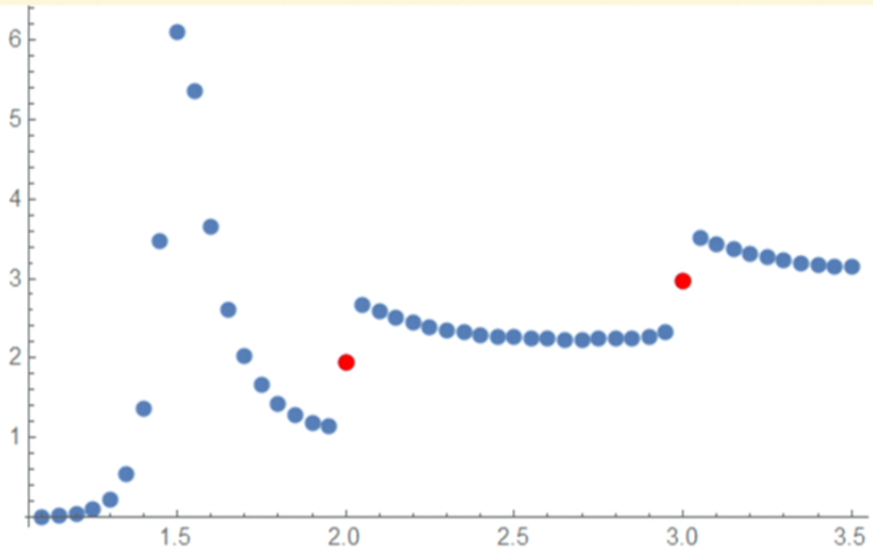
In[30]:=

```
Show[p2, p1]
```

Out[30]=



In[19]:=

```
sum[2] // N
```

Out[19]=

1.93909

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In[31]:=

```
ts
```

Out[31]=

$$\frac{4\cos^2\left(\frac{\pi}{2n}\right)\left(\cos\left(\frac{2\pi(j-1)}{n}\right)-\cosh\left(\frac{\theta}{n}\right)\right)}{n\left(\cos\left(\frac{\pi(3-2j)}{n}\right)-\cosh\left(\frac{\theta}{n}\right)\right)\left(\cosh\left(\frac{\theta}{n}\right)-\cos\left(\frac{\pi-2\pi j}{n}\right)\right)}$$

```
tsSeries[ts, {j, (-(i θ) - 2 π l n + π)/(2 π), -1}]
```

```
(*representation by poles*)
sing = 4 Sin[π/(2 n)] Cos[π/(2 n)]^2 Csc[π/n] Sin[π/(2 n) + i θ/n] Csc[π/n + i θ/n] Csch[θ/n]
    (1/(θ - (2 π I (j - 1) + I π + 2 π I n l)) + 1/(θ + (2 π I (j - 1) + 2 π I n - I π + 2 π I n l))) +
   4 Sin[π/(2 n)] Cos[π/(2 n)]^2 Csc[π/n] Sin[π/(2 n) - i θ/n] Csc[π/n - i θ/n] Csch[θ/n]
    (1/(θ + (2 π I (j - 1) + I π + 2 π I n l)) + 1/(θ - (2 π I (j - 1) + 2 π I n - I π + 2 π I n l)));
```

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In[35]:=
```
Sum[sing, {l, -∞, ∞}] / ts // FullSimplify
```

Out[35]= 1

In[37]:=
```
Sum[sing, {j, n}]
```

Out[37]=

$$\frac{1}{\pi} i \left(\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-\right.$$
$$\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-$$
$$\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)+\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)$$
$$\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+2n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)+\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-$$
$$\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+2n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)$$
$$\left.\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\psi^{(0)}\left(l n+n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)$$

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n

In[35]:=
```
Sum[sing, {l, -∞, ∞}] / ts // FullSimplify
```

Out[35]= 1

In[38]:=
```
Collect[Sum[sing, {j, n}], 1/Sin[_], Simplify]
```

Out[38]=

$$\frac{i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(\psi^{(0)}\left(l\,n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)+\psi^{(0)}\left((l+2)\,n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)\right)}{\pi}+$$

$$\frac{i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(-\psi^{(0)}\left((l+2)\,n-\frac{i\theta+\pi}{2\pi}\right)+\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)}{\pi}$$

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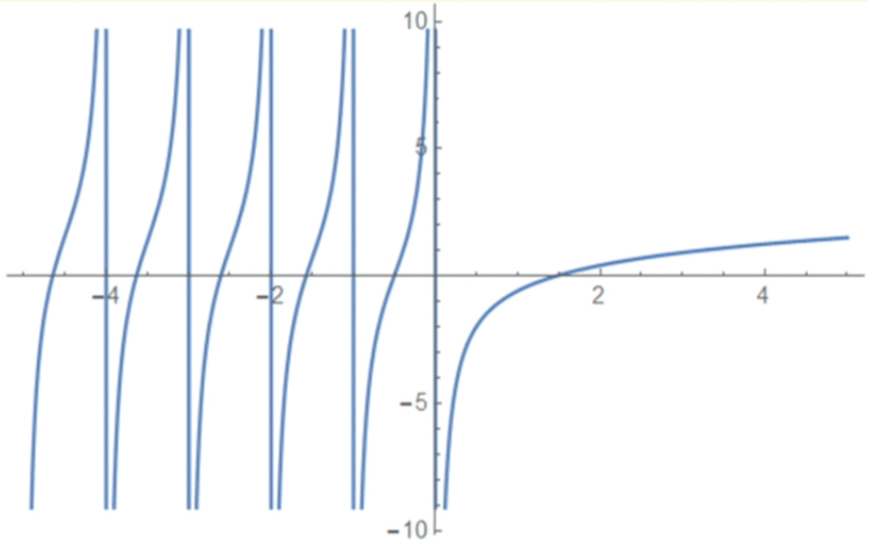
n

$$\frac{i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(-\psi^{(0)}\left((l+2)n-\frac{i\theta+\pi}{2\pi}\right)+\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)}{\pi}$$

In[39]:=

```
Plot[PolyGamma[0, x], {x, -5, 5}]
```

Out[39]=



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$$\frac{i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(-\psi^{(0)}\left((l+2)n-\frac{i\theta+\pi}{2\pi}\right)+\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)}{\pi}$$

In[44]:=

```
summed2 = Collect[Sum[sing /. j → n - j + 1, {j, n}], 1/Sin[_], Simplify]
```

Out[44]=

$$\frac{1}{\pi}\,i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(-\psi^{(0)}\left(\frac{i\theta+\pi}{2\pi}-(l+1)n\right)-\psi^{(0)}\left(-(l+1)n-\frac{i\theta}{2\pi}+\frac{3}{2}\right)+\psi^{(0)}\left(-(l+2)n-\frac{i\theta}{2\pi}+\frac{3}{2}\right)+\psi^{(0)}\left(-l\,n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)+$$
$$\frac{1}{\pi}\,i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(\psi^{(0)}\left(\frac{\pi-i\theta}{2\pi}-(l+1)n\right)-\psi^{(0)}\left(-l\,n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\psi^{(0)}\left(-(l+1)n+\frac{i\theta}{2\pi}+\frac{3}{2}\right)-\psi^{(0)}\left(-(l+2)n+\frac{i\theta}{2\pi}+\frac{3}{2}\right)\right)$$

```
p3 = Table[{n, Sum[summed1, {l, 0, 20}] +|}]
```

In[39]:=

```
Plot[PolyGamma[0, x], {x, -5, 5}]
```

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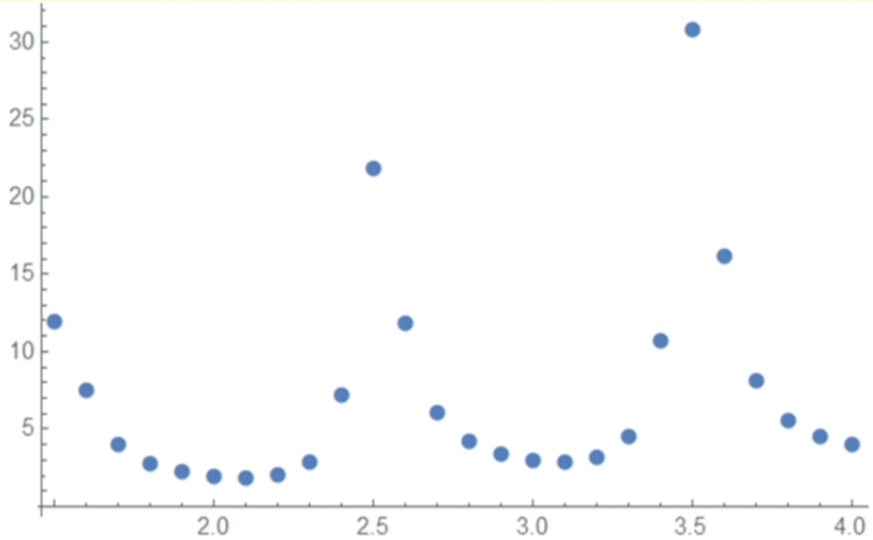
$$\left(\psi^{(0)}\left(\frac{\pi - i\,\theta}{2\,\pi} - (l+1)\,n\right) - \psi^{(0)}\left(-l\,n - \frac{i\,\theta}{2\,\pi} + \frac{1}{2}\right) + \psi^{(0)}\left(-(l+1)\,n + \frac{i\,\theta}{2\,\pi} + \frac{3}{2}\right) - \psi^{(0)}\left(-(l+2)\,n + \frac{i\,\theta}{2\,\pi} + \frac{3}{2}\right)\right)$$

In[47]:=

```
p3 = Table[{n, Sum[summed1, {l, 0, 20}] + Sum[summed2, {l, -20, -1}] /. θ → 1 / 2} // N,
   {n, 1 + 1 / 2, 4, 1 / 10}] // ListPlot
```

Out[47]=



In[39]:=

```
Plot[PolyGamma[0, x], {x, -5, 5}]
```

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In[51]:=

```
(*representation by poles*)
sing = 4 Sin[π/(2 n)] Cos[π/(2 n)]^2 Csc[π/n] Sin[π/(2 n) + i θ/n] Csc[π/n + i θ/n] Csch[θ/n]
    (1/(θ - (2 π I (j - 1) + I π + 2 π I n l)) + 1/(θ + (2 π I (j - 1) - I π + 2 π I n l))) +
   4 Sin[π/(2 n)] Cos[π/(2 n)]^2 Csc[π/n] Sin[π/(2 n) - i θ/n] Csc[π/n - i θ/n] Csch[θ/n]
    (1/(θ + (2 π I (j - 1) + I π + 2 π I n l)) + 1/(θ - (2 π I (j - 1) - I π + 2 π I n l)));
```

In[52]:=

```
Sum[sing, {l, -∞, ∞}] / ts // FullSimplify
```

Out[35]= 1

In[48]:=

```
summed1 = Collect[Sum[sing, {j, n}], 1/Sin[_], Simplify]
```

Out[48]=

$$\frac{i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi-2i\theta}{2n}\right)\csc\left(\frac{\pi-i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(\psi^{(0)}\left(l\,n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}+\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)+\psi^{(0)}\left((l+2)\,n+\frac{i\theta}{2\pi}-\frac{1}{2}\right)\right)}{\pi}+$$

$$i\cos\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi+2i\theta}{2n}\right)\csc\left(\frac{\pi+i\theta}{n}\right)\operatorname{csch}\left(\frac{\theta}{n}\right)\left(-\psi^{(0)}\left((l+2)\,n-\frac{i\theta+\pi}{2\pi}\right)+\psi^{(0)}\left(l\,n+n-\frac{i\theta}{2\pi}-\frac{1}{2}\right)-\psi^{(0)}\left(l\,n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)+\psi^{(0)}\left(l\,n+n+\frac{i\theta}{2\pi}+\frac{1}{2}\right)\right)$$

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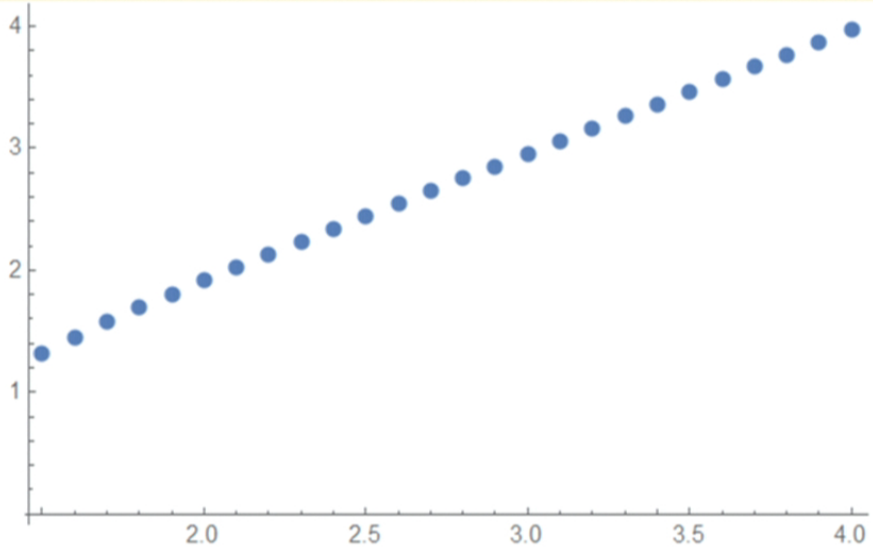
$$\psi^{(0)}\left(\frac{\cdots}{2\pi}-(l+1)\,n\right)+\psi^{(0)}\left(-l\,n-\frac{\cdots}{2\pi}+\frac{\cdots}{2}\right)-\psi^{(0)}\left(-l\,n+\frac{\cdots}{2\pi}+\frac{\cdots}{2}\right)\Bigg)\left(-\cosh\left(\frac{\cdots}{n}\right)+2\cos\left(\frac{\cdots}{n}\right)-1\right)\Bigg)$$

In[55]:=

```
p3 = Table[{n, Sum[summed1, {l, 0, 20}] + Sum[summed2, {l, -20, -1}] /. θ → 1 / 2} // N,
   {n, 1 + 1 / 2, 4, 1 / 10}] // ListPlot
```

Out[55]=



In[56]:=

```
Show[p3, p1]
```

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4.

In[65]:=

```
Series[summed1, {n, 1, 1}]
```

Out[65]=

$$(n-1)\left(\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(-i\theta+2\pi l-\pi)(-i\theta+2\pi l+\pi)}+\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(i\theta+2\pi l-\pi)(i\theta+2\pi l+\pi)}\right)+O\left((n-1)^2\right)$$

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4.

In[66]:=

```
Series[summed1, {n, 1, 1}]
```

Out[66]=

$$(n-1)\left(\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(-i\theta+2\pi l-\pi)(-i\theta+2\pi l+\pi)}+\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(i\theta+2\pi l-\pi)(i\theta+2\pi l+\pi)}\right)+O\left((n-1)^2\right)$$

In[67]:=

```
Series[summed1, {θ, I π (n - 1), -1}]
```

Out[67]=

$$-\frac{1}{\theta-i(\pi(n-1))}\,i\,n\left(\frac{2n\sin\left(\frac{\pi}{n}\right)\left(\cot\left(\frac{\pi}{2n}\right)+\cot\left(\frac{\pi(n-1)}{n}\right)\right)}{\pi(2ln+n-2)(2ln+3n-2)}+\frac{1}{\pi}\left(2\cos\left(\frac{\pi}{n}\right)-\cos\left(\frac{2\pi(n-1)}{n}\right)-1\right)\cos^2\left(\frac{\pi}{2n}\right)\csc\left(\frac{\pi(n-2)}{n}\right)\csc\left(\frac{\pi(n-1)}{n}\right)\right.$$

$$\left.\left(-\psi^{(0)}\left(\frac{1}{2}(2ln-n)\right)+\psi^{(0)}\left(\frac{1}{2}(2ln+n-2)\right)+\psi^{(0)}\left(\frac{1}{2}(2ln+n)\right)-\psi^{(0)}\left(\frac{1}{2}(2ln+3n-2)\right)\right)\right)+O\left((\theta-i(\pi(n-1)))^0\right)$$

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4.

In[66]:=

```
Series[summed1, {n, 1, 1}]
```

Out[66]=

$$(n-1)\left(\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(-i\theta+2\pi l-\pi)(-i\theta+2\pi l+\pi)}+\frac{2\pi^2\coth(\theta)\,\mathrm{csch}(\theta)}{(i\theta+2\pi l-\pi)(i\theta+2\pi l+\pi)}\right)+O\left((n-1)^2\right)$$

In[67]:=

```
Series[summed1, {θ, I π (n - 1), -1}]
```

Out[67]=

$$-\frac{1}{\theta-i(\pi(n-1))}\,i\,n\left(\frac{2n\sin\left(\frac{\pi}{n}\right)\left(\cot\left(\frac{\pi}{2n}\right)+\cot\left(\frac{\pi(n-1)}{n}\right)\right)}{\pi(2ln+n-2)(2ln+3n-2)}+\frac{1}{\pi}\left(2\cos\left(\frac{\pi}{n}\right)-\cos\left(\frac{2\pi(n-1)}{n}\right)-1\right)\cos^2\left(\frac{\pi}{2n}\right)\csc\left(\frac{\pi(n-2)}{n}\right)\csc\left(\frac{\pi(n-1)}{n}\right)\right.$$

$$\left.\left(-\psi^{(0)}\left(\frac{1}{2}(2ln-n)\right)+\psi^{(0)}\left(\frac{1}{2}(2ln+n-2)\right)+\psi^{(0)}\left(\frac{1}{2}(2ln+n)\right)-\psi^{(0)}\left(\frac{1}{2}(2ln+3n-2)\right)\right)\right)+O\left((\theta-i(\pi(n-1)))^0\right)$$

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Nikolay. Challenging Problems [Good Luck]

- Plot the density distribution of the first 1000 zeros of zeta function at $1/2+it$ for real t.
- Reproduce a fractal picture from https://en.wikipedia.org/wiki/Newton_fractal
- Do a function which will randomly replace pluses by minuses in your notebook
- Using Mathematica retrieve the list of all participants from http://perimeterinstitute.ca/conferences/mathematica-summer-school-2015 and randomly pick 5 people.
- Reproduce the lindep function of http://wayback.cecm.sfu.ca/projects/EZFace/ using `LatticeReduce`.

  In particular you should get

  `Lindep[35.42067282289080185433479921459357753664865663590 4,Zeta[3],Zeta[5]]`

  to give `{-1,1,33}`

  i.e. your function should decode that a number is a simple combination of zeta functions.

  In the website this runs as

  lindep([35.42067282289080185433479921459357753664865663590 4,z(3),z(5)])