

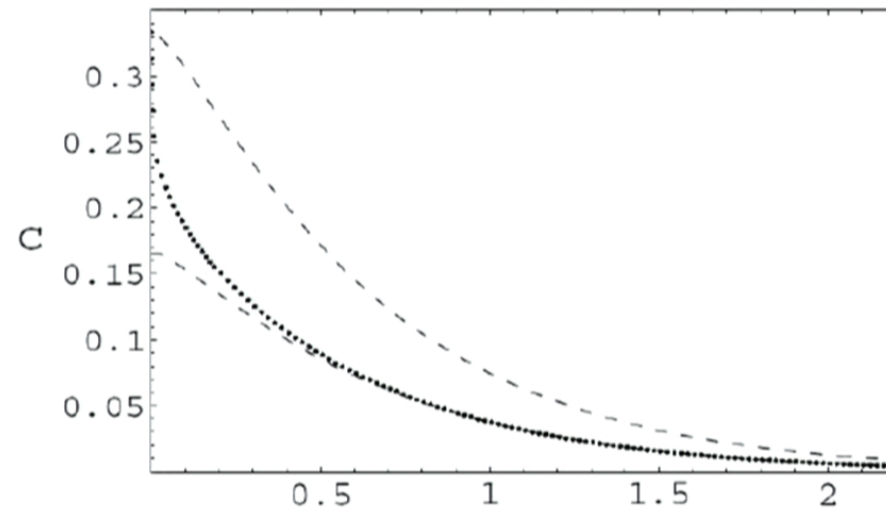
Title: TBA

Date: Aug 28, 2015 11:00 AM

URL: <http://pirsa.org/15080053>

Abstract: TBA

Entropic C-functions for free scalar and Dirac fields



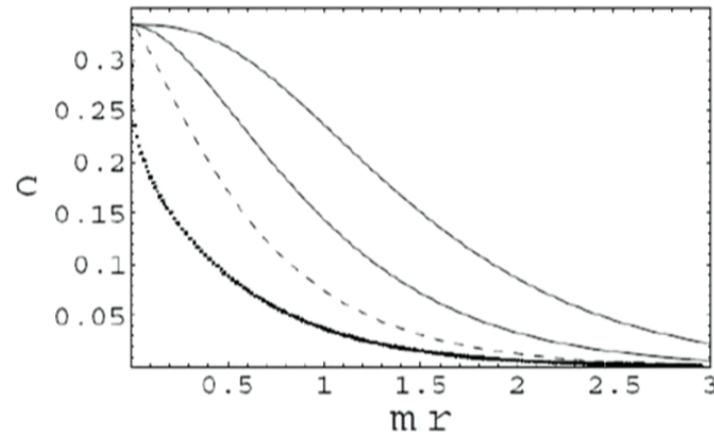


Figure 2. From top to bottom: one third of the Zamolodchikov c-functions for a real scalar and a Dirac field, and entropic c-functions for a Dirac (dashed curve) and a real scalar field (dotted curve).

Replica trick

$$S = -\lim_{n \rightarrow 0} \frac{1}{n} \log Z^n$$

Rényi $S_n = \frac{1}{1-n} \log Z^n$

$$\lim_{n \rightarrow 1} S_n = S$$

$$\langle x_1, 0 | x_0, t_0 \rangle = \int_{x_0, t_0}^{x_1, 0} \mathcal{D}x(t) e^{-iS[x]}$$

$$= \langle x_1 | e^{iHt_0} | x_0 \rangle$$

$$t_0 = -i\tau$$

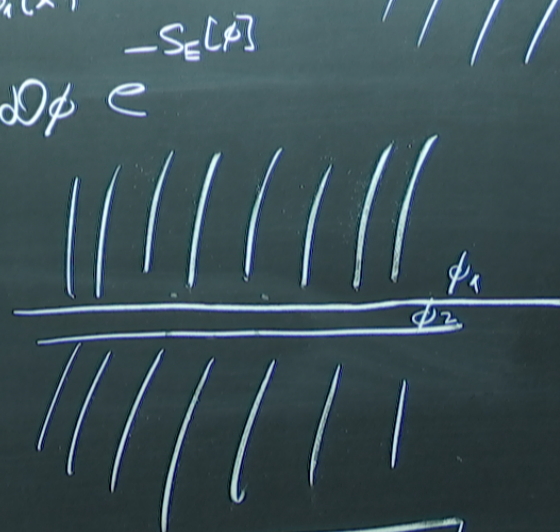
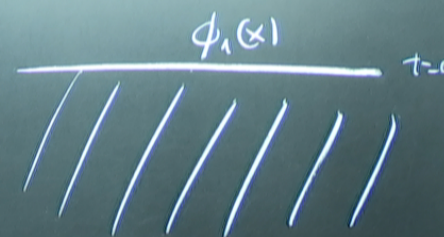
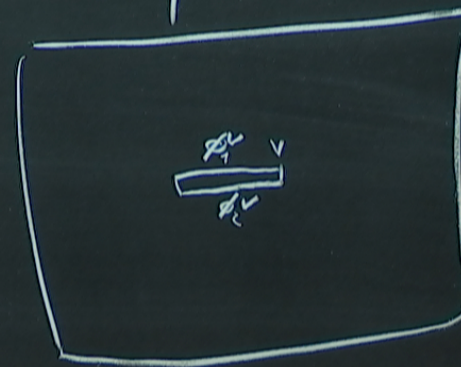
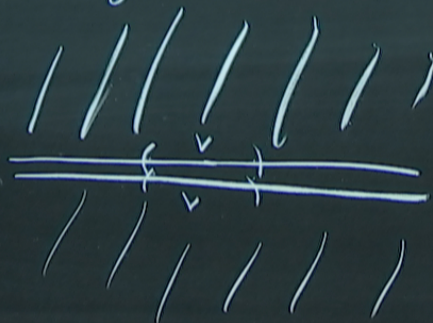
$$\rightarrow \langle x_1 | 0 \rangle e^{-E_0 \tau} \langle 0 | x_0 \rangle \sim \psi_0(x)$$

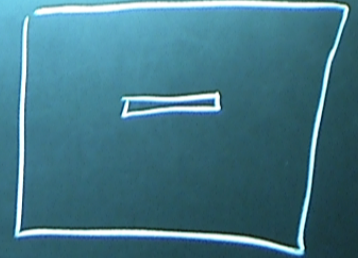
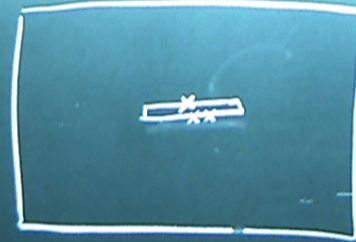
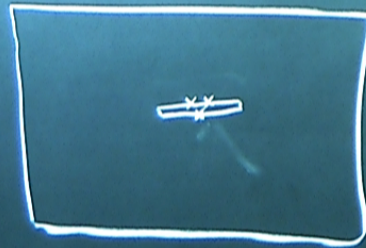
$$\psi_0(x) \sim \int_{-\infty}^{x, t=0} dx e^{-S_E[x]}$$

$$\Phi_0(\phi_1(x), t=0) = N^{-1/2} \int_{-\infty}^{t=0, \phi(t=0, x) = \phi_1(x)} d\phi e^{-S_E[\phi]}$$

$$S_0[\phi_1, \phi_2] = \Phi_0^*(\phi_1) \Phi_0(\phi_2)$$

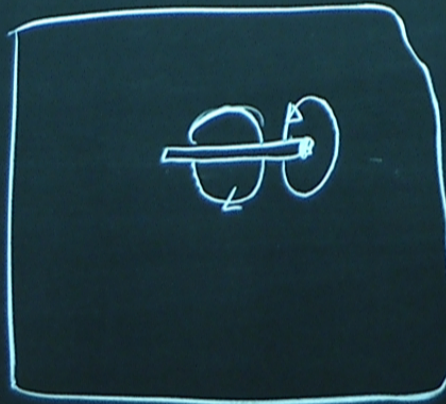
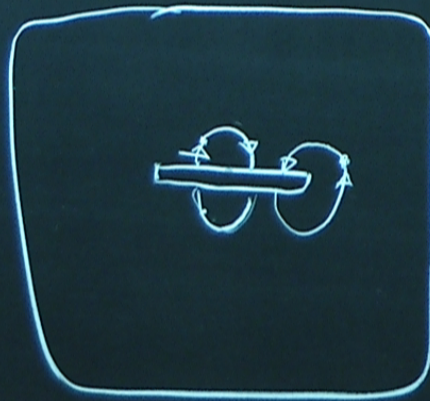
$$S_0^v[\phi_1^v, \phi_2^v] = \int d\phi_1^v = \phi_2^v e^{-S}$$





$$S^2(\phi_1^\vee, \phi_2^\vee) = \int \mathcal{D}\phi_3^\vee S_0^\vee(\phi_1^\vee, \phi_3^\vee) S_0^\vee(\phi_3^\vee, \phi_2^\vee)$$

t_S^2



$2 \cdot (2\pi)$

Conical singularity

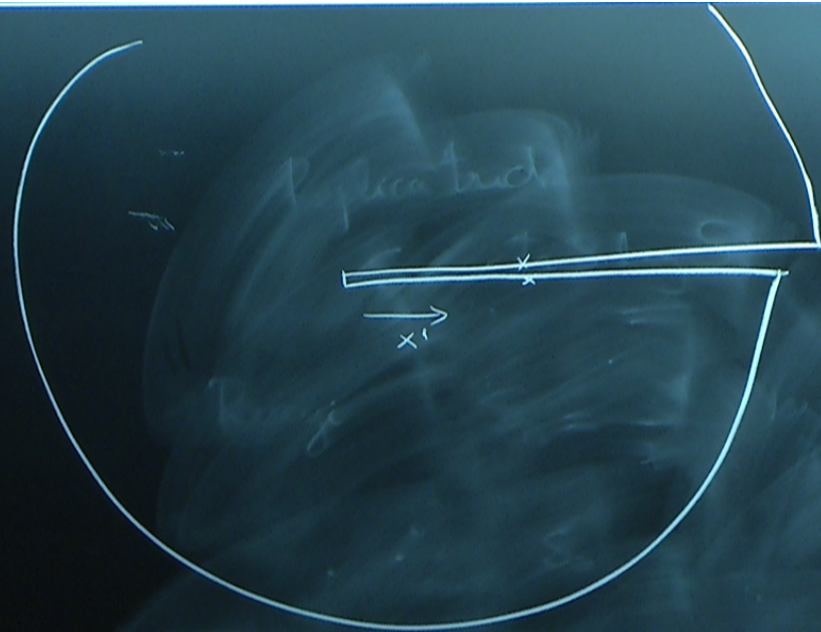
$m(2\pi)$

- Geometrical construction

- Geometrical only for integer m

$$h S^m = \frac{Z[m]}{Z[1]^m}$$

$$S_m = \frac{\log Z(m) - m \log Z[1]}{1 - m}$$



$$= S_v$$

$$S_v[\phi_1, \phi_2] = \langle \phi_1 | e^{-2\pi L} | \phi_2 \rangle$$

$$L = \int_{x>0} dx \, x_1 T_{00}$$

$$S_v = e^{-2\pi \int_{x>0} dx \, x_1 T_{00}}$$

Boost operator

Relativistic QFT, Vacuum, but for any QFT

$$S = \frac{e^{-K}}{\text{tr } e^{-K}}$$

K modular hamiltonian
entanglement hamiltonian

K is local

Relativistic QFT, Vacuum, but for any QFT

$$S = \frac{e^{-K}}{\text{tr } e^{-K}}$$

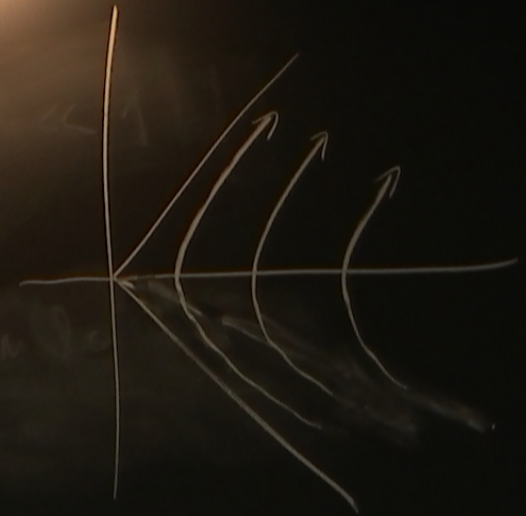
K Modular Hamiltonian
Entanglement Hamiltonian

K is local

$$\left(e^{i t K} \right)$$

→ energy density

→ Spheres in CFT



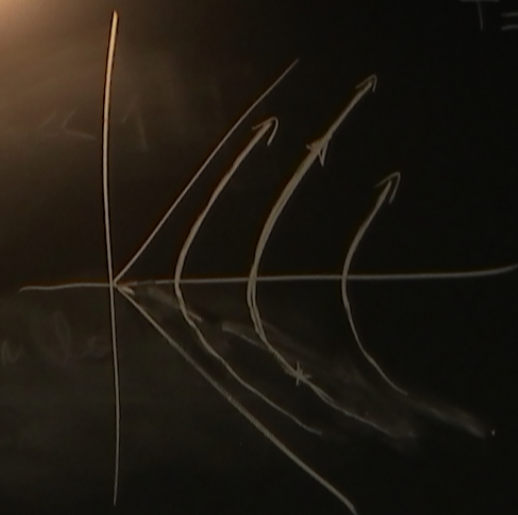
Relativistic QFT, Vacuum, but for any QFT

$$S = \frac{-K}{\text{tr } \bar{c}^K}$$

K modular hamiltonian
entanglement hamiltonian

K is local

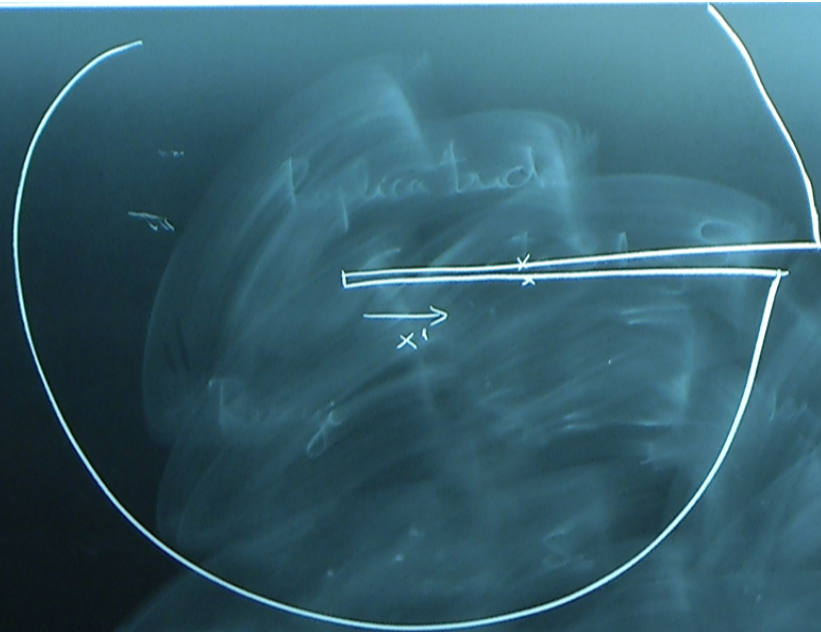
$$\left(\begin{matrix} i t K \\ e \end{matrix} \right)$$



→ energy density

→ spheres in CFT

→ Unruh temperature



$$= S_v$$

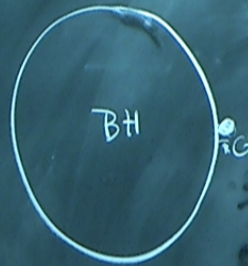
$$S_v[\phi_1, \phi_2] = \langle \phi_1 | e^{-2\pi L} | \phi_2 \rangle$$

$$L = \int_{x>0} dx \, x_1 \, T_{00}$$

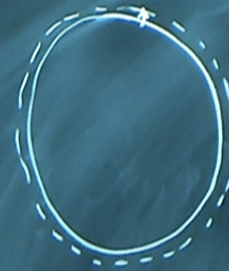
$$S_v = e^{-2\pi \int_{x>0} dx \, \underbrace{x_1 T_{00}}_{\text{Boost operator}}} \quad 2\pi x_1 \equiv \beta$$

Bekenstein bound (1981)

a)



b)



$$S_B = \frac{A}{4G}$$

$$S_{BH} + S_G \leq S_{BH}^{\text{!}} \rightarrow \text{GSL}$$

Entropy equation

$$S_G \leq 2\pi E \cdot R$$

- E, S global, R ? $x, t \rightarrow 0$
- Boundary condition? NO
- Species problem ✓
- S divergent ✓
- $E < 0$? ✓

$$S_0 \rightarrow S_V - S_V^0$$

$$2\pi ER \rightarrow 2\pi \int d^{d-1}x \, x^1 (\langle T_{00} \rangle_0 - \langle T_{00} \rangle_0)$$

$$2\pi \int d^{d-1}x \, x^1 (\langle T_{00} \rangle_0 - \langle T_{00} \rangle_0) \geq S_V - S_V^0$$

quantum Beh. bound

Relative entropy

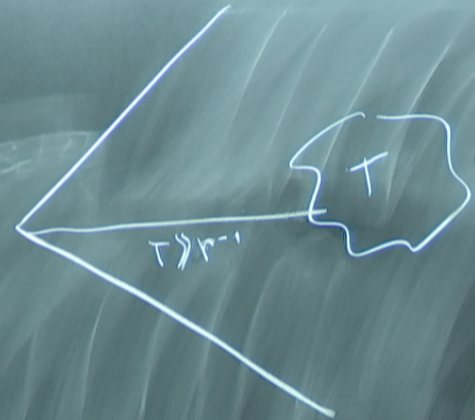
$$S(S_1 \| S_0) = -\ln(S_1 \log S_1 - S_1 \log S_0) \geq 0$$

$$S_0 = \frac{e^{-K}}{\ln e^K}$$

$$S(S_1 \| S_0) = \Delta \langle K \rangle - \Delta S = (\langle K \rangle_1 - \langle K \rangle_0) - (S_1 - S_0) \geq 0$$

$$K_0 = 2\pi \int d^{d-1}x \, T_{00}$$

- E, S global, R ? $x, t=0$
- Boundary condition? NO
- Species problem ✓
- S divergent ✓
- $E < 0$? ✓



$$S_0 \rightarrow S_V - S_V^0$$

$$2\pi ER \rightarrow 2\pi \int d^{d-1}x \, x^1 (\langle T_{00} \rangle_0 - \langle T_{00} \rangle)$$

$$2\pi \int d^{d-1}x \, x^1 (\langle T_{00} \rangle_0 - \langle T_{00} \rangle) \geq S_V - S_V^0$$

quantum Beh. bound ✓

Relative entropy

$$S(S_1 \| S_0) = -k \ln(S_1 \log S_1 - S_1 \log S_0) \geq 0$$

$$S_0 = \frac{e^{-k}}{\ln e^{-k}}$$

$$S(S_1 \| S_0) = \Delta \langle K \rangle - \Delta S = (\langle K \rangle_1 - \langle K \rangle_0) - (S_1 - S_0) \geq 0$$

$$K_0 = 2\pi \int d^3x \, x^1 T_{00}$$

measure of distinguishability

$$\sim e^{-m S(S_1 \| S_0)}$$