Title: Mathematica, tensor networks, MERA and entanglement

Date: Aug 28, 2015 09:15 AM

URL: http://pirsa.org/15080052

Abstract: TBA

OUTLINE

MERA, entanglement and AdS/MERA

- Tensor networks and contractions
- Entanglement on a slice of AdS?
- Goal: do MERA computations, and make AdS/MERA quantitative
  - →get simple MERA example + routines and do exercises

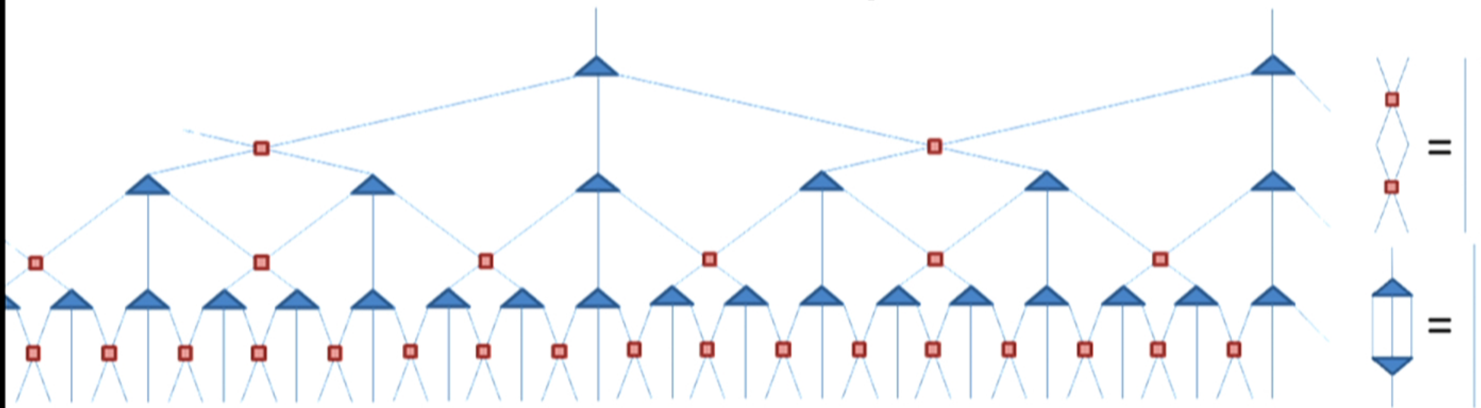
Frequently used ***Mathematica***

- Making and using packages
- NDSolve and/or spectral methods
- Transforming functions/coordinate transformations

MULTISCALE ENTANGLEMENT RENORMALISATION ANSATZ (MERA)

MPS correlations/entanglement requires larger χ

Choose different *ansatz* to incorporate RG flow:



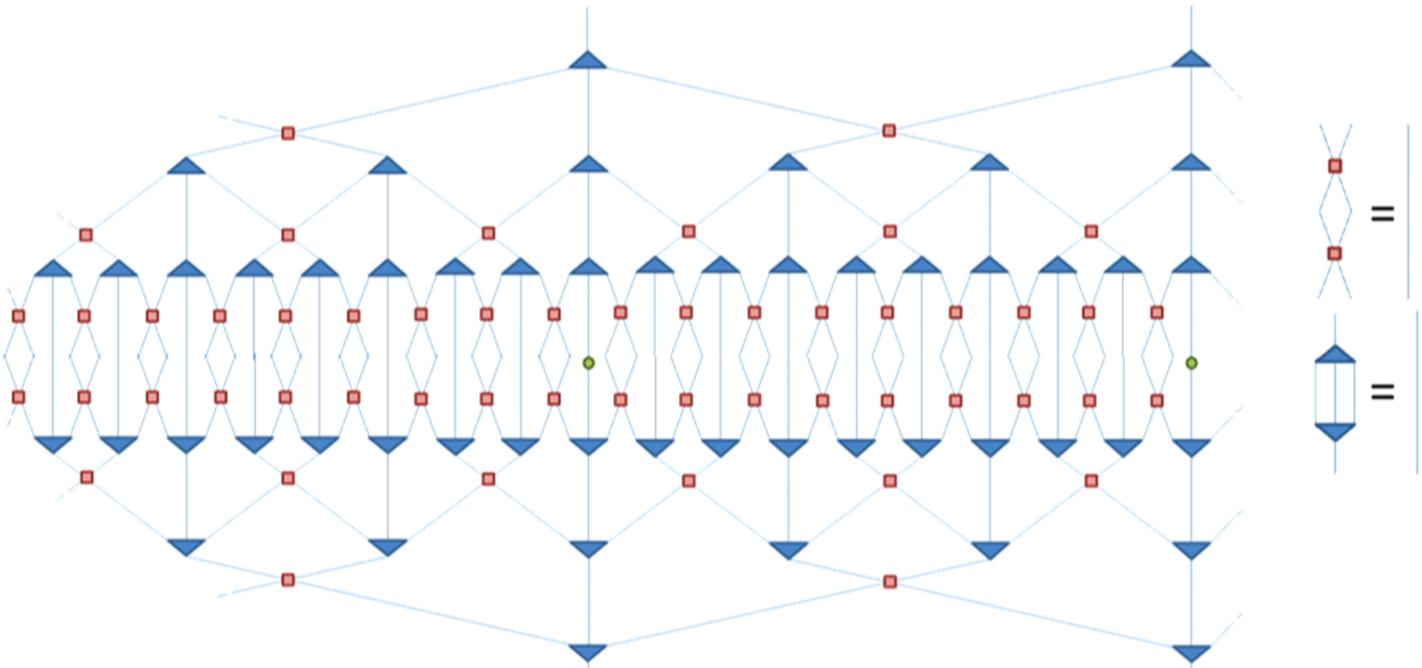
Disentanglers and coarse grainers (ternary)

Extra advantage: scale invariance is very natural!

G. Vidal, Entanglement Renormalization (2007)

EXAMPLE: CORRELATORS IN MERA

Choose operators at smart locations



Simplify ☺

R.N.C. Pfeifer, G. Evenbly and G. Vidal, Entanglement renormalization, scale invariance, and quantum criticality (2009)

ENTANGLEMENT ENTROPY

Reduced density matrix: $\rho_{red,L} = \mathrm{tr}(\rho)$

Obtain mixed state with probabilities: $p_\rho = \mathrm{eig}(\rho_{red,L})$

- **Has entropy:** $S_{EE} = -\sum_i p_{\rho,i} \log(p_{\rho,i}) = \frac{c}{3}\log(L) + \mathcal{O}(1)$
- **I.e. ground state → excited state!**

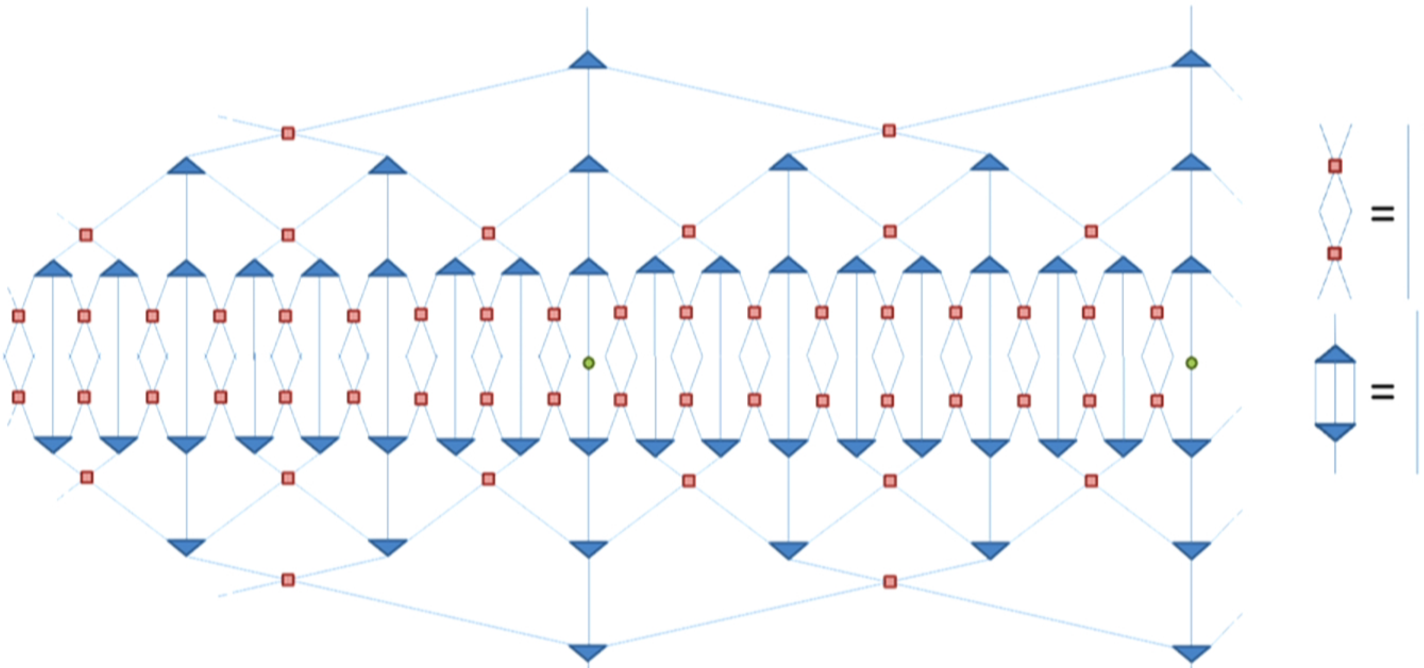
Ising model: $H_{\text{Ising}} = -\sum_r \left(\lambda\sigma_z^{[r]} + \sigma_x^{[r]}\sigma_x^{[r+1]}\right)$

- **Energy:** $e_0 = -\frac{2}{L\sin(\pi/2L)} \approx -\frac{4}{\pi} - \frac{\pi}{6L^2}$
- **Central charge 1/2**

P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory (2004)

EXAMPLE: CORRELATORS IN MERA

Choose operators at smart locations



Simplify ☺

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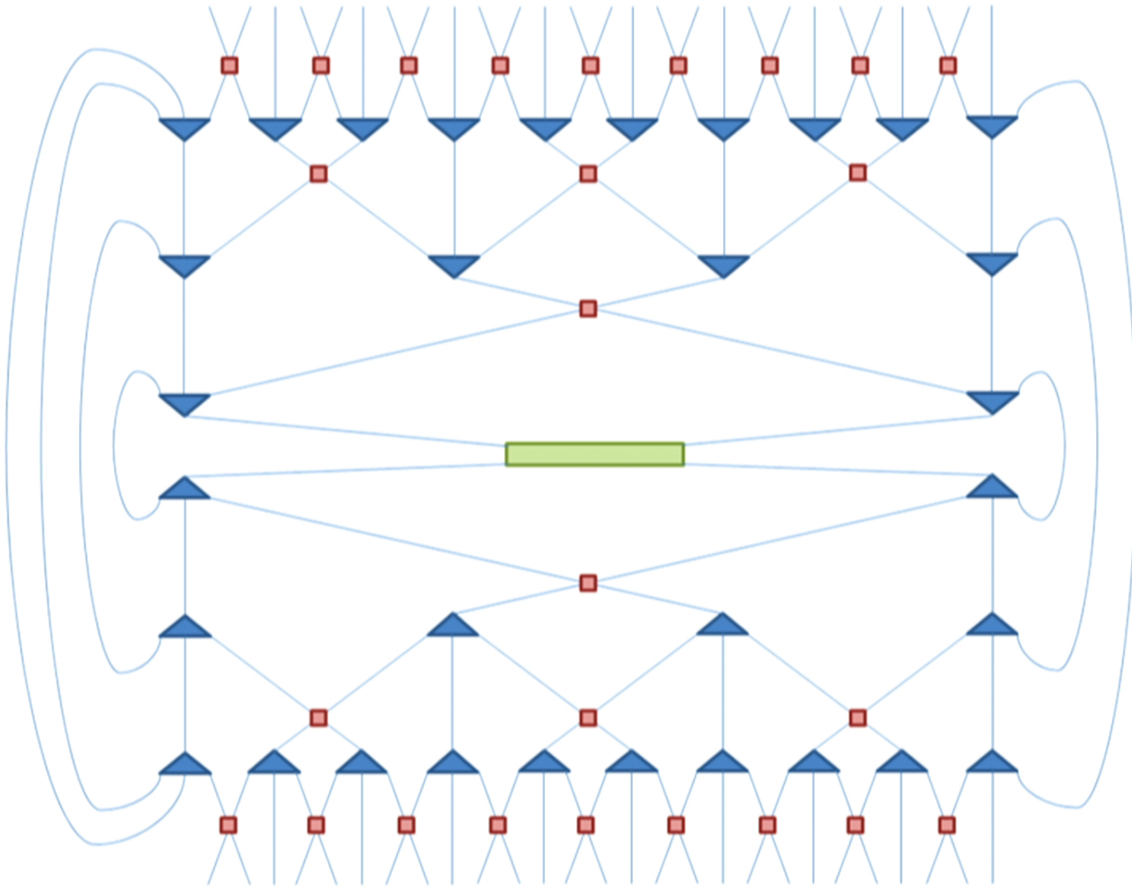
$\rho_{red,L} = \mathrm{tr}(\rho)$

Wilke van der Schee, MIT

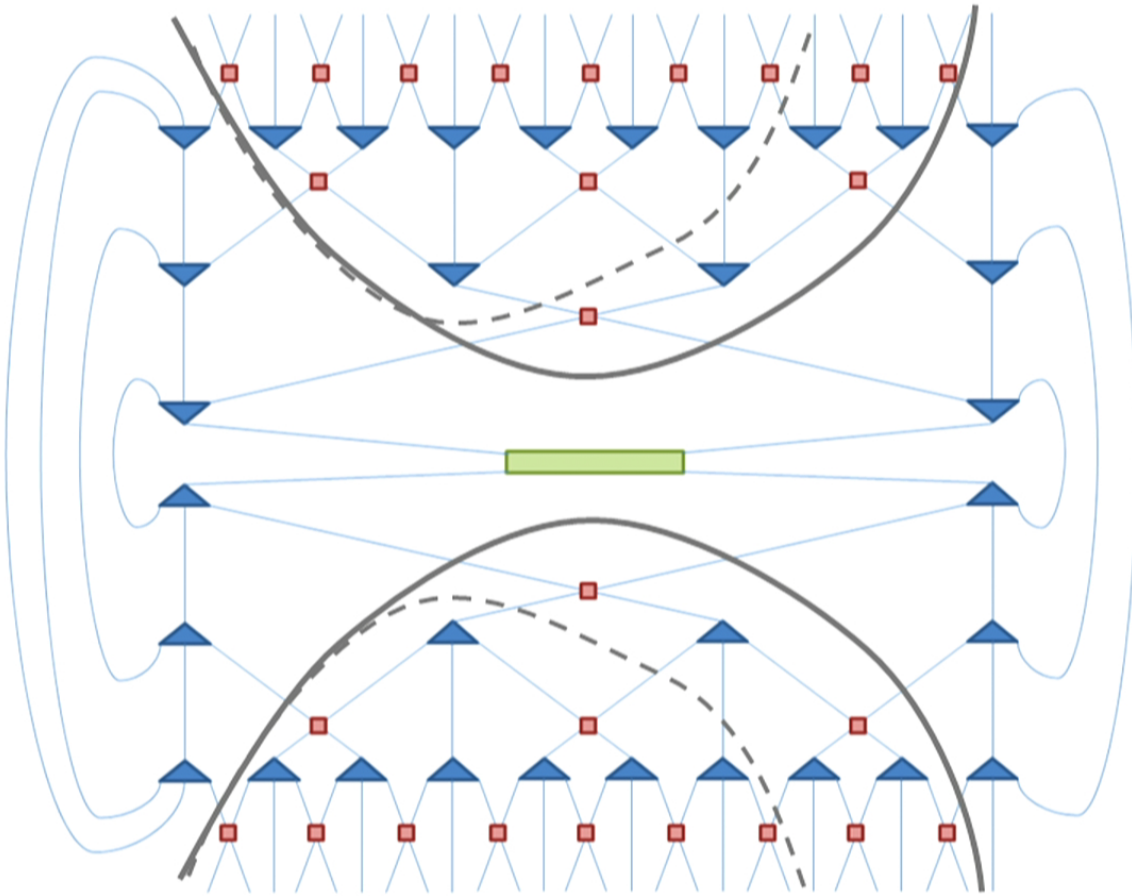
REDUCED DENSITY MATRIX IN MERA



$\rho_{red,L} = \mathrm{tr}(\rho)$

Wilke van der Schee, MIT

`GEODESIC' IN ADS SPACETIME (SWINGLE)

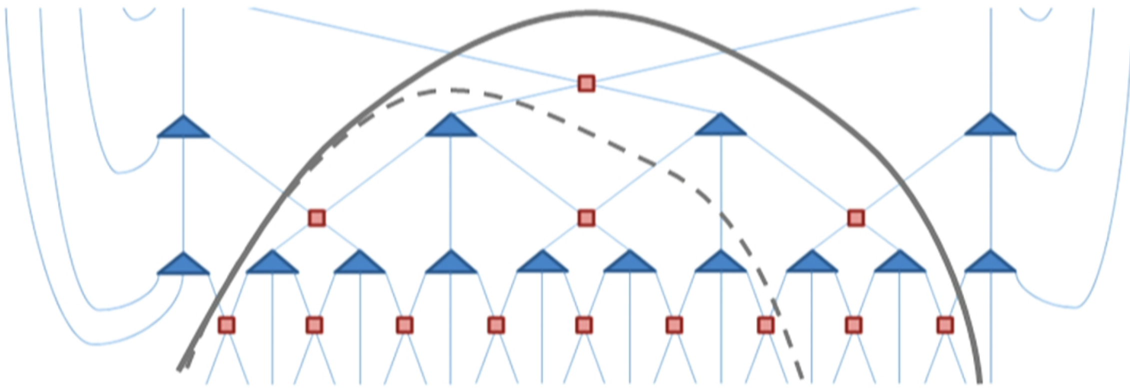


Brian Swingle, Entanglement Renormalization and Holography (2009)

10/16

$\rho_{red,L} = \mathrm{tr}(\rho)$

LOG(L) SCALING

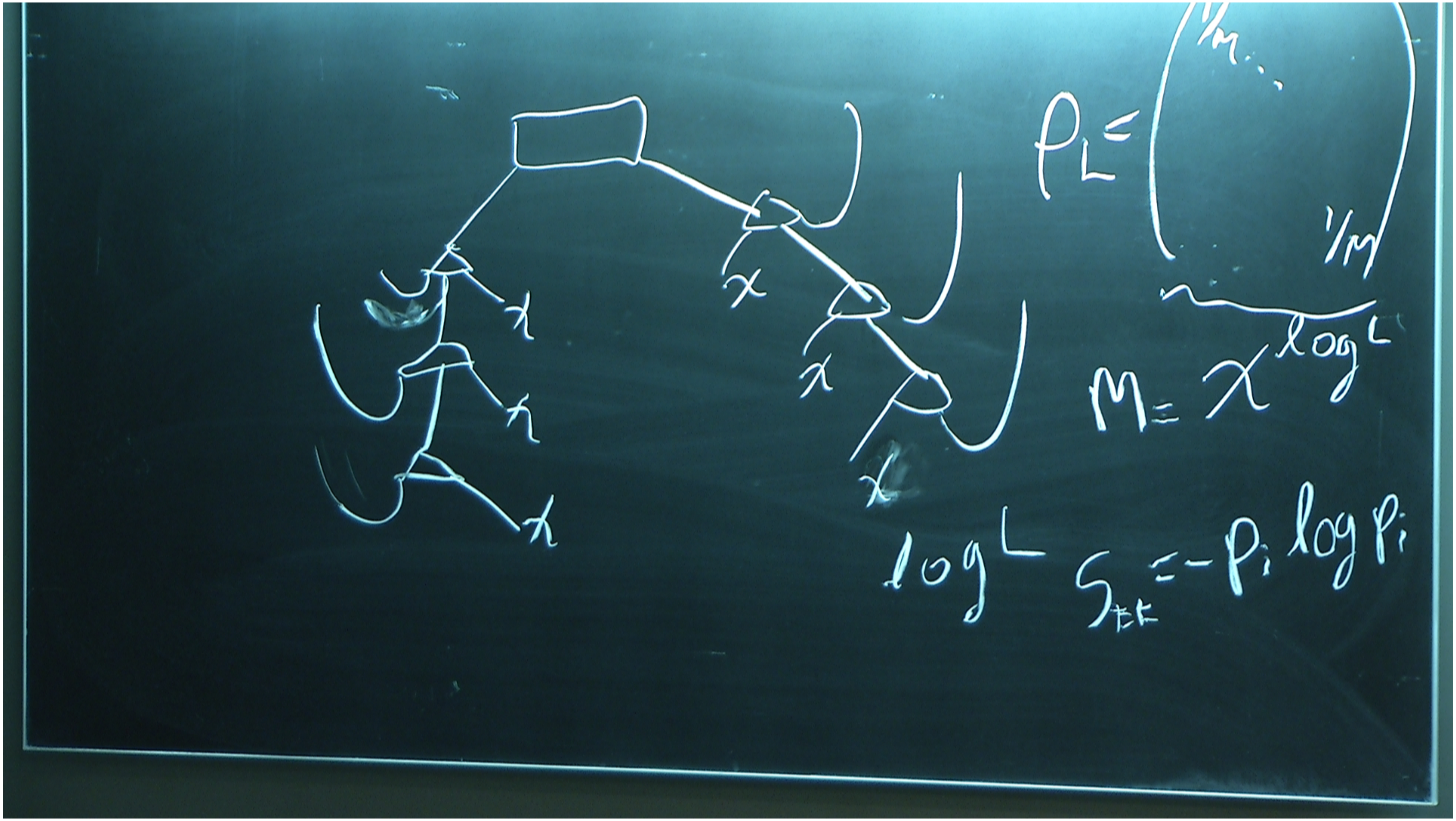


Important: MERA has $S_{EE} \lesssim \log(\chi)\log(L)$

Most local Hamiltonians obey this (but $S_{EE} \sim \sqrt{L}$ **possible)**

11/16

R. Movassagh, P.W. Shor, Power law violation of the area law in quantum spin chains (2014)

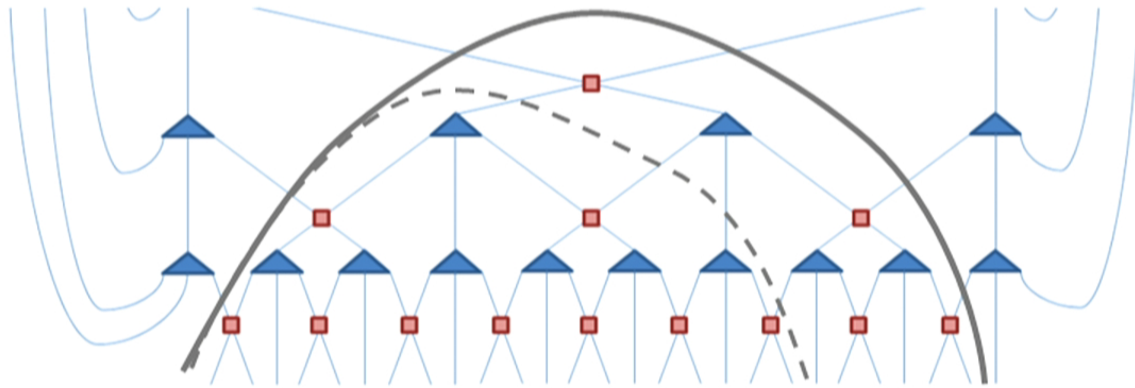


χ
χ
χ
χ
χ
χ
ρL =
1/M
1/M
M = χ log L
log L
SEE = −pi log pi

$\rho_{red,L} = \mathrm{tr}(\rho)$

Wilke van der Schee, MIT

LOG(L) SCALING



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11/16

R. Movassagh, P.W. Shor, Power law violation of the area law in quantum spin chains (2014)

GRAVITY AS AN EMERGENT FORCE, ADS/MERA

Field theory without gravity ⟷ string theory with gravity

Holographic: gravity has one extra dimension, RG scale

⟶ Propose connection between MERA and gravity

Caveat: gravity `emerges' only for specific field theories
(large N, strong coupling)

Brian Swingle, Entanglement Renormalization and Holography (2009)

GRAVITY AS AN EMERGENT FORCE, ADS/MERA

Field theory without gravity ↔ string theory with gravity

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→ Propose connection between MERA and gravity

Caveat: gravity `emerges' only for specific field theories
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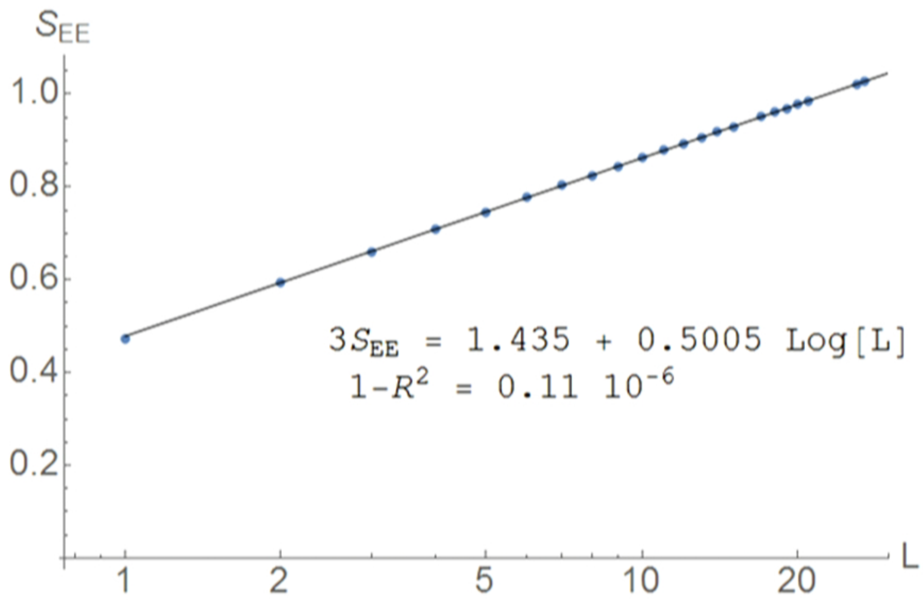
Brian Swingle, Entanglement Renormalization and Holography (2009)

$$S_{EE} = \frac{c}{3}\log(L) + \mathcal{O}(1)$$

Wilke van der Schee, MIT

EQUIVALENT REDUCED DENSITY MATRIX

Resulting entanglement entropies:

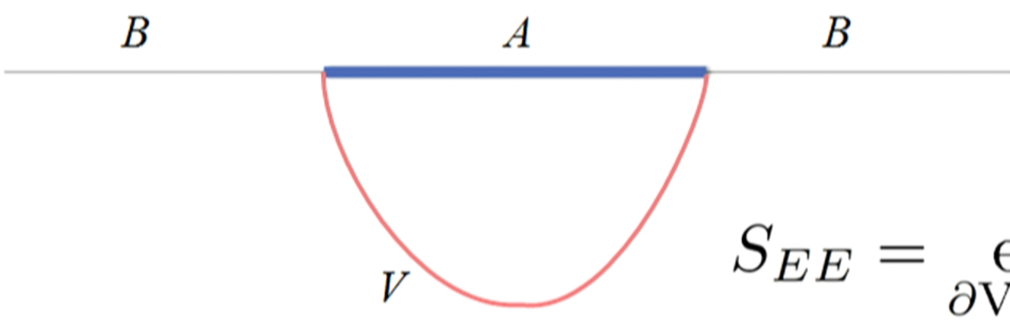


ENTANGLEMENT = GEOMETRY (PICTURE)

Entanglement: trace out part of space → mixed state (entropy!)

Remarkable statement (Ryu+Takayanagi):

entanglement entropy = area extremal surface in AdS



$$S_{EE} = \underset{\partial V = \partial A}{\text{ext}} \frac{A_V}{4G_N}$$

13/16

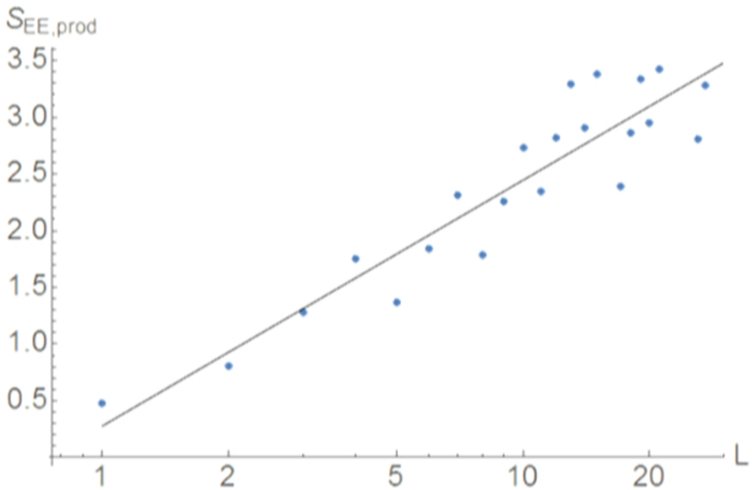
S. Ryu, T. Takayanagi, Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence (2006

QUESTION: WHAT GIVES A GEOMETRIC PICTURE?

Reduced density matrix gives a local `slice' in `AdS'

Hard to formalise: legs do not in general decouple

- Do they decouple with large *c*?
- If so, then entanglement entropy = sum over entropy/leg
- What about dual of Ising model?



$3S_{EE} = 0.8247 + 2.828 \text{ Log}[L]$
$R^2 = 0.69$

Slide 1 of 5

Packages

- **Almost always useful: different notebooks for different parameters/options, simulations versus analysing**
- **Easy to make: convert initialisation cells to package**
  Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options --> File Options --> Autogenerated package --> Manual

- **Either put parameters/options before loading package or after**

Example:

```
In[1]:= SetDirectory[NotebookDirectory[]];
<< MERA`

domodel[initIsing, 6]
```

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Hydrodynamics

Load and display

Standard values

Below are all standard values used. They are evaluated at the very last, so when using the notebook one can just change them and use ones own values; of particular interest are the gridsize (longitudinal), ngridpoints (AdS, longitudinal), the input function h, and the regulator energy density a4reg.

Standard values for grid

```
gridsize = 10 π;
ngridpoints = {40, 500};
horloc = -1;
gridtypeset := {{"spectral", 2.0 / (1 + Cos[((-horloc - 1) π)/(ngridpoints[[1]] - 1)])}, {"fourier", gridsize}};
derivativeorder = {"spectral", 41};
periodicity = {False, True};
dt := Min[0.00007 45^2/ngridpoints[[1]]^2, 0.001];
```

Standard values for computing EF B

```
startzFG = 3 / 100;
rFGmax = 1 / 0.005^(1/4);
charge = 0;
orderRKiny = 6;
stepsizeFGy = 1 / 1000;
orderRKinz = 8;
stepsizeFGz = 1 / 500;
```

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CUDA

```
cuda = False;
usecuda := (
  Needs["CUDALink`"];
  cuda = True;)
CUDADot2[a_, b_] := (
  dima = Dimensions[a];
  dimb = Dimensions[b];
  If[Length[dima] > 2 || Length[dimb] > 2,
   ArrayReshape[CUDALink`CUDADot[Flatten[a, Length[dima] - 2] // N,
     Flatten /@ b // N], dima[[ ;; -2]] ~ Join ~ dimb[[2 ;;]]],
   CUDALink`CUDADot[a // N, b // N]]
 )

dot[a_, b_] := If[cuda && Dimensions[a][[-1]] < 2500, CUDADot2[a, b], a.b];

usecuda
```

```
  If[Length[legs1[[1]]] > 0, Transpose[tens1[[1]], -legs1[[1]]], tens1[[1]]]
 )
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

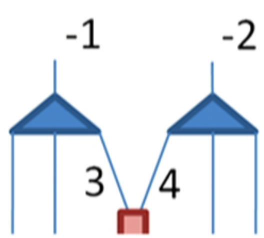
IdentityMatrix, change order, translational operator

```
In[3]:= χ = 18;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[5]:= AbsoluteTiming[
         nconp[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12},
            {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}},
           {5, 6, 7, 8, 1, 2, 3, 9, 11, 12, 4, 10}] // Dimensions]

        total cost is 6χ^6  total: 204 073 344  (old: 204 073 344)

Out[5]= {0.157110, {18, 18, 18, 18}}
```




```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
```

```
In[3]:= χ = 18;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[5]:= AbsoluteTiming[
         nconp[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12},
            {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}},
           {5, 6, 7, 8, 1, 2, 3, 9, 11, 12, 4, 10}] // Dimensions]

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Out[5]= {0.157110, {18, 18, 18, 18}}

In[5]:= AbsoluteTiming[
         nconp[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6},
            {7, 8, 9, 10}, {5, 6, 7, 8}}, {5, 6, 7, 8, 1, 2, 3, 9, 11, 12, 4, 10}] // Dimensions]
```

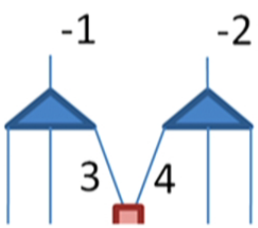


```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
```

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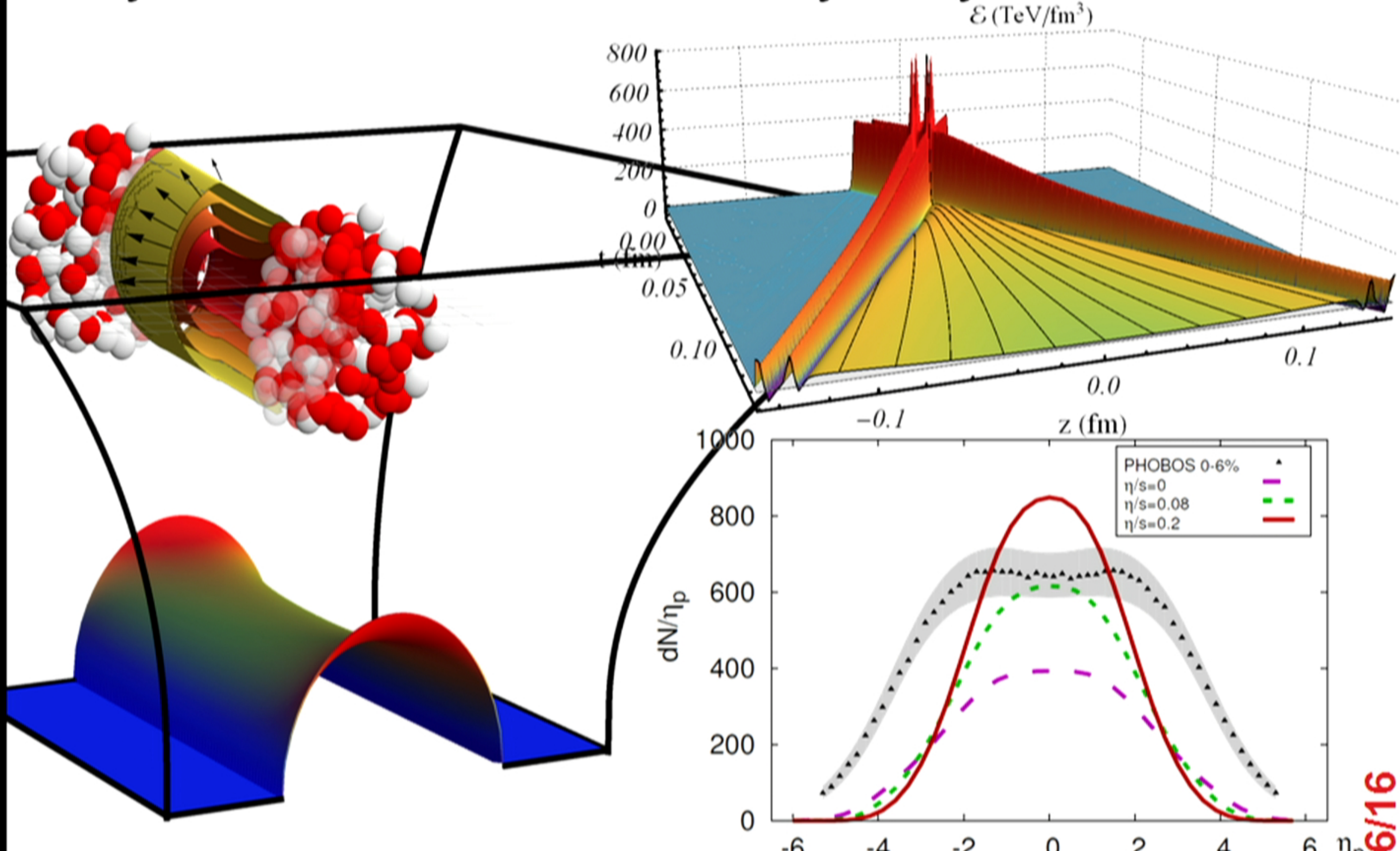
SOME OTHER WORK

Dynamics in Anti-de Sitter to study heavy ion collisions



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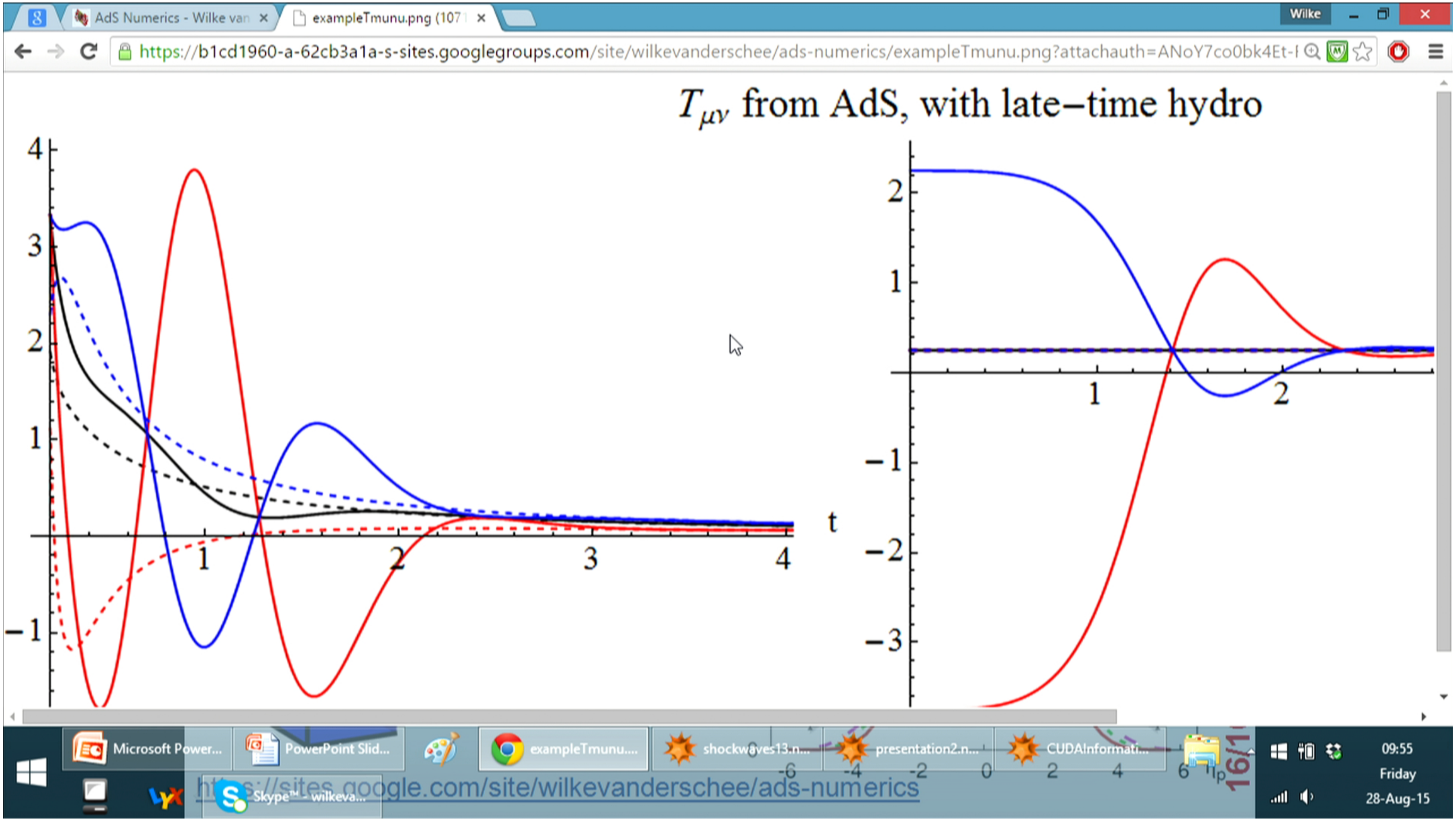
16/16



AdS Numerics - Wilke van
exampleTmunu.png (107
https://b1cd1960-a-62cb3a1a-s-sites.googlegroups.com/site/wilkevanderschee/ads-numerics/exampleTmunu.png?attachauth=ANoY7co0bk4Et-
$T_{\mu\nu}$ from AdS, with late−time hydro
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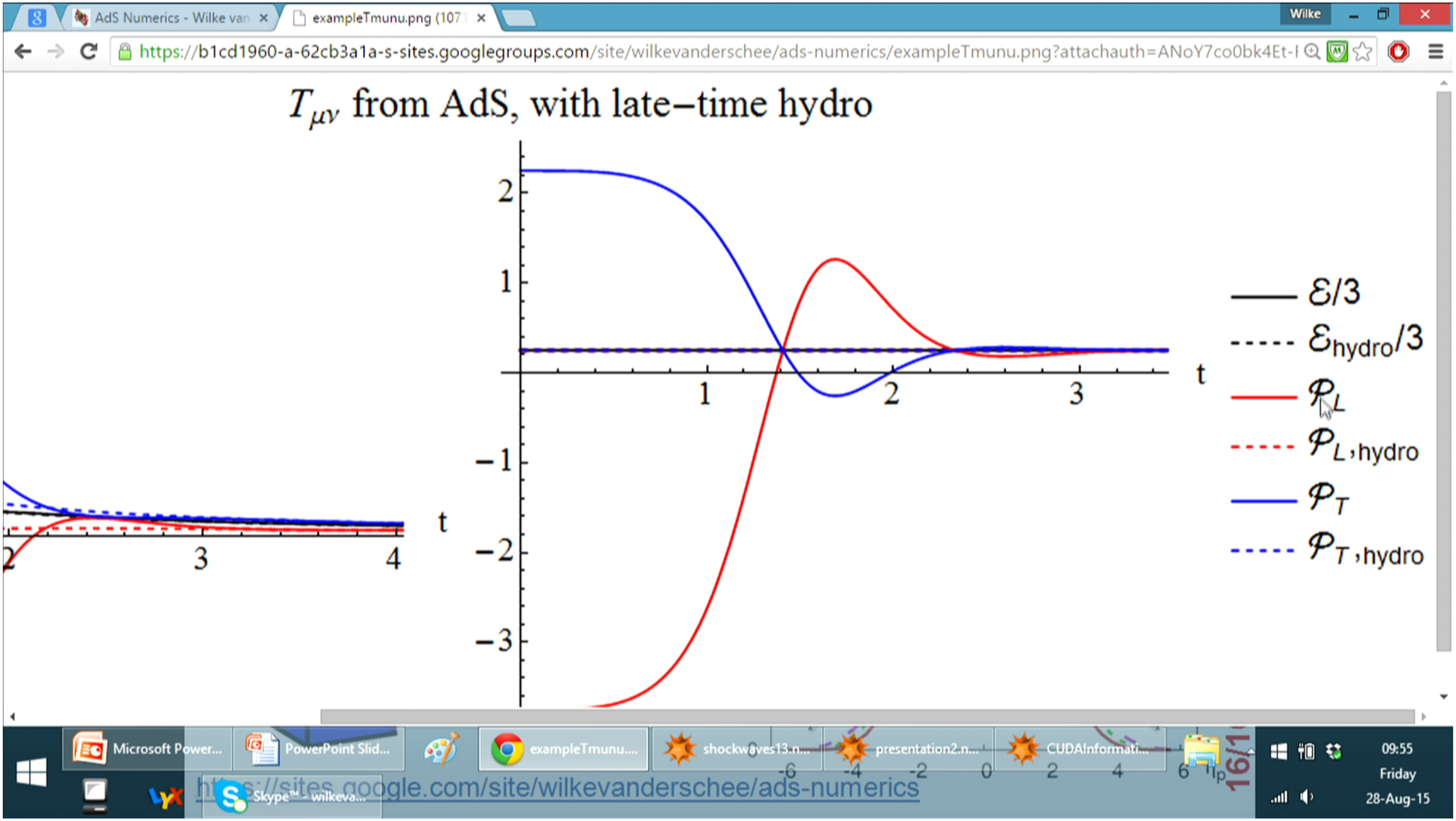

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$T_{\mu\nu}$ from AdS, with late−time hydro
t
$\mathcal{E}/3$
$\mathcal{E}_{hydro}/3$
$\mathcal{P}_L$
$\mathcal{P}_{L,hydro}$
$\mathcal{P}_T$
$\mathcal{P}_{T,hydro}$
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NDSolve and/or spectral methods

Solve linear ODE:

```
LDE = (2 + x) y''[x] + y'[x] - 20 x y[x];
LDEboundary = {y[-1] == 5, y[1] == -1};

dsolLDE = y /. DSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]
ndsolLDE = y /. NDSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]

pl = Plot[dsolLDE[x] // Re, {x, -1, 1.}]

initGrid[points_] := (

  nx = points;

  x = N[Table[-Cos[r π / ...], {r, 0, nx - 1}]];
```


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```
In[12]:= LDE = (2 + x) y''[x] + y'[x] - 20 x y[x];
         LDEboundary = {y[-1] == 5, y[1] == -1};

In[14]:= LDE
```

Out[14]= $-20\,x\,y[x] + y'[x] + (2 + x)\,y''[x]$

```
In[15]:= dsolLDE = y /. DSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]
         ndsolLDE = y /. NDSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]
```

Out[15]= $\text{Function}\Big[\{x\},\ \Big(e^{-2\sqrt{5}\,(1+x)}\Big(-\text{HypergeometricU}\Big[\frac{1}{2}-2\sqrt{5},\ 1,\ 4\sqrt{5}\,(2+x)\Big]$
$\Big(e^{4\sqrt{5}}\,\text{LaguerreL}\Big[-\frac{1}{2}+2\sqrt{5},\ 4\sqrt{5}\Big]+5\,\text{LaguerreL}\Big[-\frac{1}{2}+2\sqrt{5},\ 12\sqrt{5}\Big]\Big)+$
$\Big(e^{4\sqrt{5}}\,\text{HypergeometricU}\Big[\frac{1}{2}-2\sqrt{5},\ 1,\ 4\sqrt{5}\Big]+5\,\text{HypergeometricU}\Big[$
$\frac{1}{2}-2\sqrt{5},\ 1,\ 12\sqrt{5}\Big]\Big)\,\text{LaguerreL}\Big[-\frac{1}{2}+2\sqrt{5},\ 4\sqrt{5}\,(2+x)\Big]\Big)\Big)\Big/$
$\Big(\text{HypergeometricU}\Big[\frac{1}{2} - 2\sqrt{5},\ 1,\ 12\sqrt{5}\Big]\,\text{LaguerreL}\Big[-\frac{1}{2}+2\sqrt{5},\ 4\sqrt{5}\Big]$ ...

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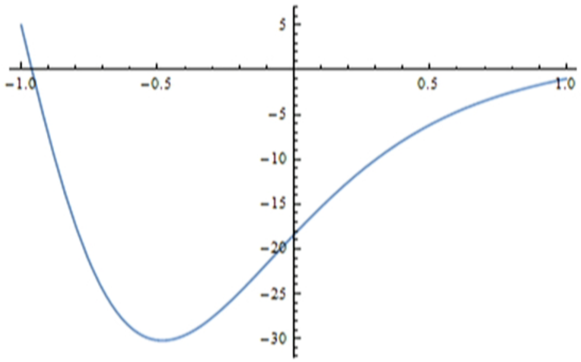
$$\frac{1}{2} - 2\sqrt{5},\ 1,\ 12\sqrt{5}\Big]\Big)\ \text{LaguerreL}\Big[-\frac{1}{2} + 2\sqrt{5},\ 4\sqrt{5}\ (2 + x)\Big]\Big)\Big)\Big/$$

$$\Big(\text{HypergeometricU}\Big[\frac{1}{2} - 2\sqrt{5},\ 1,\ 12\sqrt{5}\Big]\ \text{LaguerreL}\Big[-\frac{1}{2} + 2\sqrt{5},\ 4\sqrt{5}\Big] -$$

$$\text{HypergeometricU}\Big[\frac{1}{2} - 2\sqrt{5},\ 1,\ 4\sqrt{5}\Big]\ \text{LaguerreL}\Big[-\frac{1}{2} + 2\sqrt{5},\ 12\sqrt{5}\Big]\Big)\Big)$$

```
In[18]:= pl = Plot[dsolLDE[x] // Re, {x, -1, 1.}]
```

Out[18]=




```
initGrid[points_] := (
```

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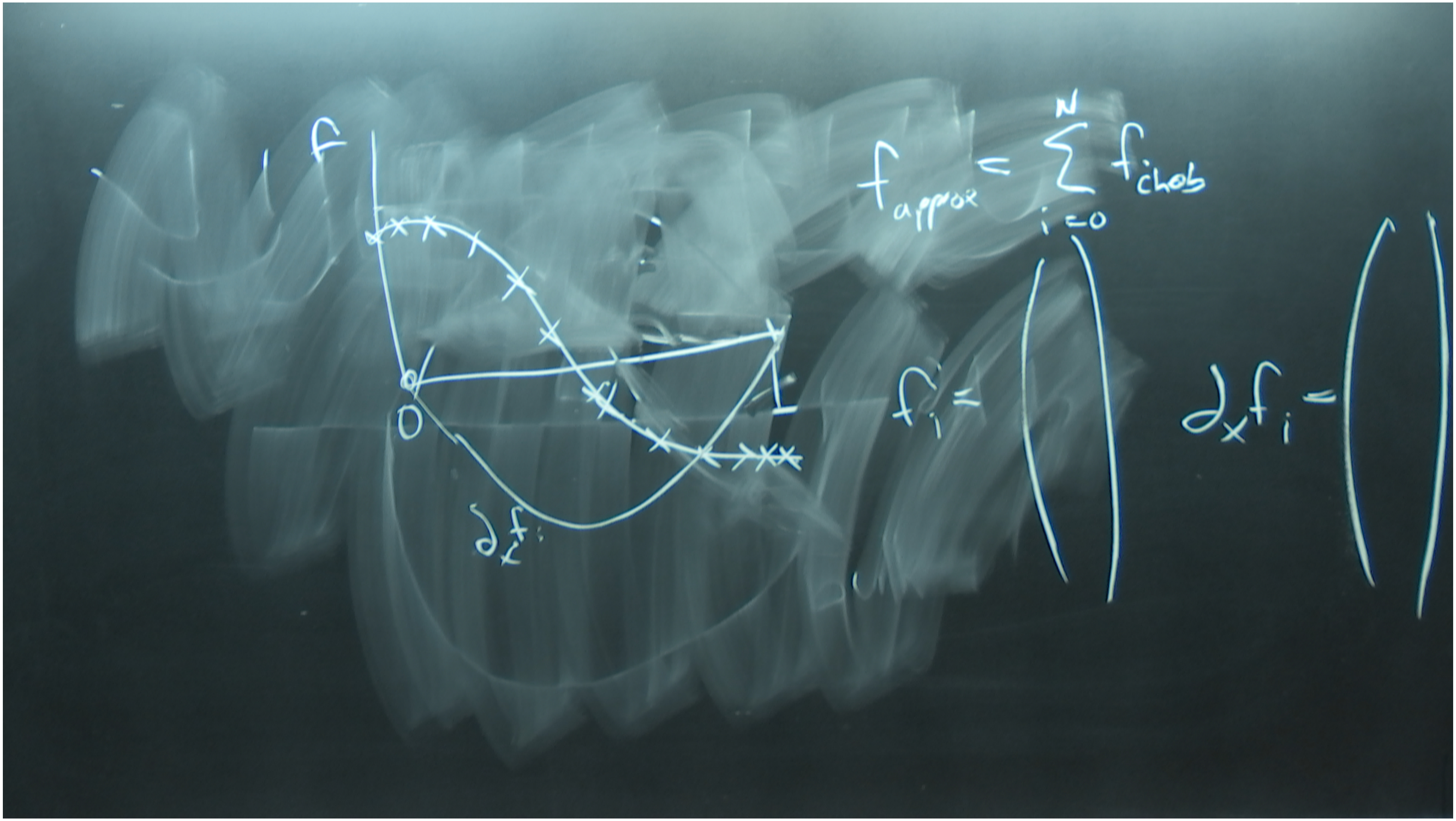
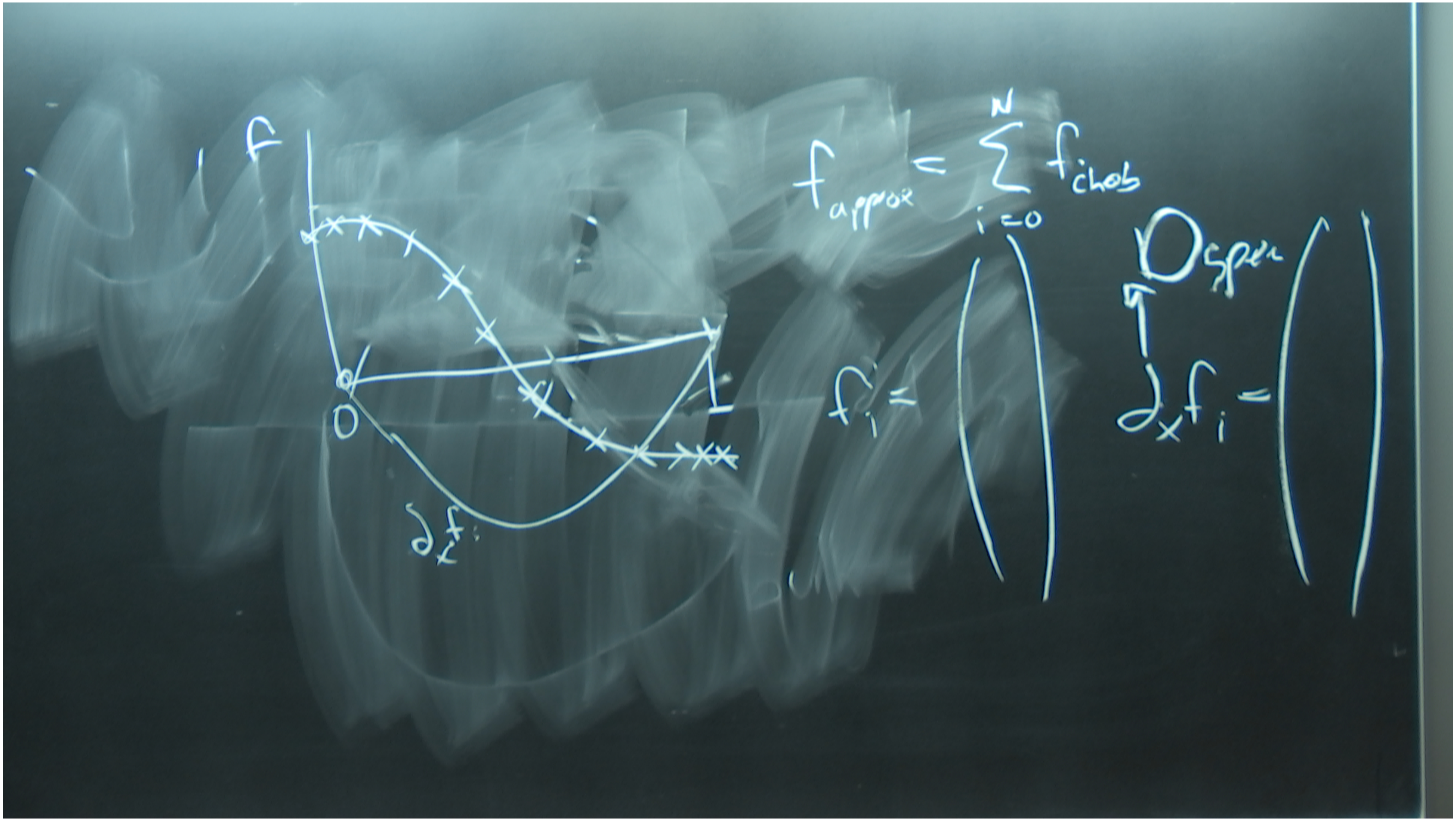

f
0
L
$f_{approx} = \sum_{i=0}^{N} f_{i\,cheb}$
$f_i =$
$\partial_x f_i =$
$\partial_x f$:


f
0
L
$f_{approx} = \sum_{i=0}^{N} f_{cheb}$
$f_i =$
O_{spec}
$\partial_x f_i =$
$\partial_x f_i$

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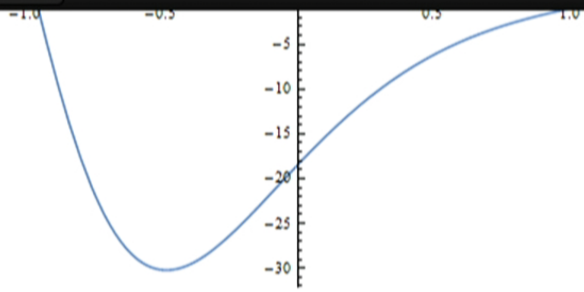
Out[18]=




```
initGrid[points_] := (
  nx = points;
  x = N[Table[-Cos[(r π)/(nx - 1)], {r, 0, nx - 1}]];
  DSpec = NDSolve`FiniteDifferenceDerivative[1, x, DifferenceOrder → points - 1][
    "DifferentiationMatrix"];
  D2Spec = DSpec.DSpec;
  x)
```

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```
In[19]:= initGrid[points_] := (
    nx = points;
    x = N[Table[-Cos[(r π)/(nx - 1)], {r, 0, nx - 1}]];
    DSpec = NDSolve`FiniteDifferenceDerivative[1, x, DifferenceOrder → points - 1][
      "DifferentiationMatrix"];
    D2Spec = DSpec.DSpec;
    x)

eqm = DiagonalMatrix[D[LDE, y''[x]]].D2Spec +
   DiagonalMatrix[D[LDE, y'[x]]].DSpec + DiagonalMatrix[D[LDE, y[x]]];
source = LDE /. y → (0 &);

LinearSolve[eqm, -source]
```

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```
x = N[Table[-Cos[ ... / (nx - 1)], {r, 0, nx - 1}]];
DSpec = NDSolve`FiniteDifferenceDerivative[1, x, DifferenceOrder → points - 1][
  "DifferentiationMatrix"];
D2Spec = DSpec.DSpec;
x)
```

```
In[20]:= initGrid[25]

Out[20]= {-1., -0.991445, -0.965926, -0.92388, -0.866025, -0.793353, -0.707107, -0.608761,
  -0.5, -0.382683, -0.258819, -0.130526, 0., 0.130526, 0.258819, 0.382683,
  0.5, 0.608761, 0.707107, 0.793353, 0.866025, 0.92388, 0.965926, 0.991445, 1.}
```

```
eqm = DiagonalMatrix[D[LDE, y''[x]]].D2Spec +
   DiagonalMatrix[D[LDE, y'[x]]].DSpec + DiagonalMatrix[D[LDE, y[x]]];
source = LDE /. y → (0 &);
```

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NDSolve and/or spectral methods

Solve linear ODE:

```
In[12]:= LDE = (2 + x) y''[x] + y'[x] - 20 x y[x];
         LDEboundary = {y[-1] == 5, y[1] == -1};

In[14]:= LDE

Out[14]= -20 x y[x] + y'[x] + (2 + x) y''[x]

In[15]:= dsolLDE = y /. DSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]
         ndsolLDE = y /. NDSolve[{LDE == 0, LDEboundary} // Flatten, y, {x, -1, 1}][[1]]
```

Out[15]= Function$\left[\{x\}, \left(e^{-2\sqrt{5}\,(1+x)}\left(-\text{HypergeometricU}\left[\frac{1}{2}-2\sqrt{5},\,1,\,4\sqrt{5}\,(2+x)\right]\right.\right.\right.$

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```
Out[21]= {1., 1.00856, 1.03407, 1.07612, 1.13397, 1.20665, 1.29289, 1.39124,
  1.5, 1.61732, 1.74118, 1.86947, 2., 2.13053, 2.25882, 2.38268, 2.5,
  2.60876, 2.70711, 2.79335, 2.86603, 2.92388, 2.96593, 2.99144, 3.}

In[24]:= DSpec // Normal // MatrixForm;

  DiagonalMatrix[D[LDE, y''[x]]].D2Spec

In[25]:= eqm = DiagonalMatrix[D[LDE, y''[x]]].D2Spec +
    DiagonalMatrix[D[LDE, y'[x]]].DSpec + DiagonalMatrix[D[LDE, y[x]]];
  source = LDE /. y → (0 &);

In[30]:= LinearSolve[eqm, -source];

  LinearSolve::luc : Result for LinearSolve of badly conditioned matrix {≪1≫} may contain significant numerical errors. ≫

  eqm[[1, ;;]] = Table[If[i == 1, 1, 0], {i, nx}];
  eqm[[-1, ;;]] = Table[If[i == nx, 1, 0], {i, nx}];
```

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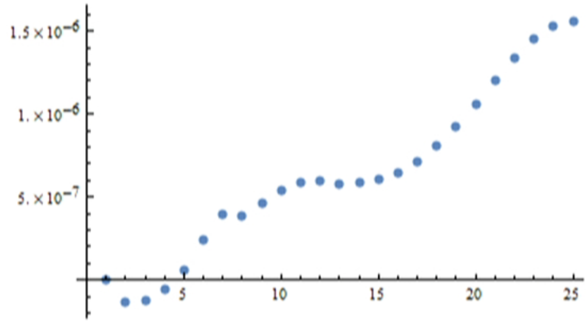
```
-24.1764, -28.4537, -30.1825, -29.3739, -26.5942, -22.6775,
-18.4271, -14.4284, -11.0015, -8.24794, -6.13099, -4.54953,
-3.388, -2.54316, -1.93452, -1.50526, -1.21881, -1.05392, -1.}
```

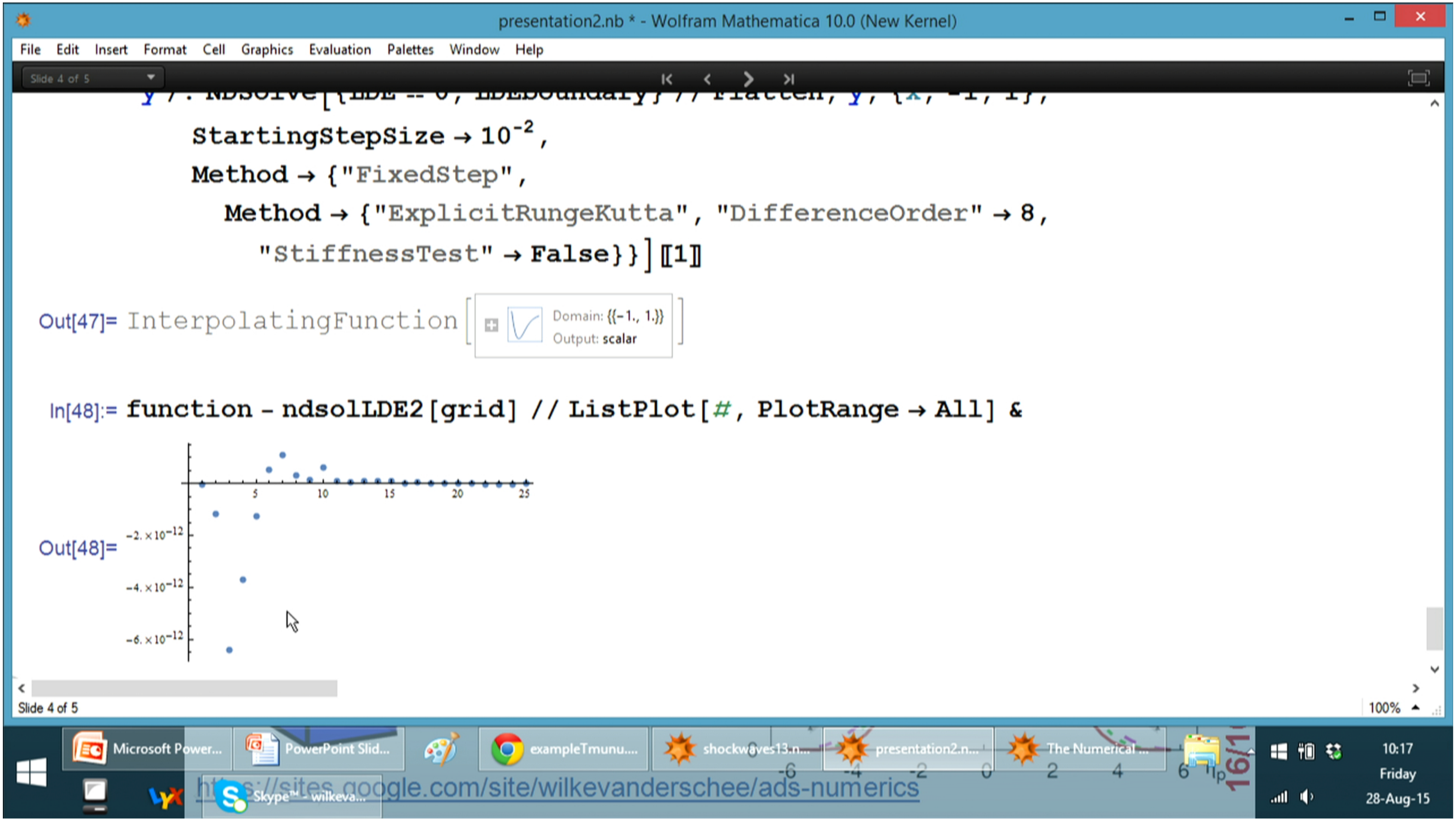
In[44]:=
```
function - ndsolLDE[x] // ListPlot
```

Out[44]=




```
grid = x;
x =.

ndsolLDE2 =
```



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StartingStepSize → 10^-2,
Method → {"FixedStep",
Method → {"ExplicitRungeKutta", "DifferenceOrder" → 8,
"StiffnessTest" → False}}] [[1]]
Out[47]= InterpolatingFunction[Domain: {{-1., 1.}} Output: scalar]
In[48]:= function - ndsolLDE2[grid] // ListPlot[#, PlotRange → All] &
Out[48]=
5 10 15 20 25
-2. × 10^-12
-4. × 10^-12
-6. × 10^-12
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10:17
Friday
28-Aug-15

Transforming functions/coordinate transformation

metric =

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -A[t,r] & 0 & 0 & 0 \\ 0 & 0 & S[t,r]^2\,\mathrm{Exp}[-2\,B[t,r]] & 0 & 0 \\ 0 & 0 & 0 & S[t,r]^2\,\mathrm{Exp}[B[t,r]] & 0 \\ 0 & 0 & 0 & 0 & S[t,r]^2\,\mathrm{Exp}[B[t,r]] \end{pmatrix}$$

```
coord = {r, t, x1, x2, x3};

SetDirectory[NotebookDirectory[]];
<< "diffgeo.m";
```

SetDelayed::write : Tag Laplacian in Laplacian[scalar_?scalarQ] is Protected. >>

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```
Method →
 {"FixedStep",
  Method → {"ExplicitRungeKutta", "DifferenceOrder" → 8,
    "StiffnessTest" → False}}]
```

For a time dependent problem:

```
NDSolve[ ...., StartingStepSize → 10^-2,
 Method → {"MethodOfLines",
   "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 50,
     "MaxPoints" → 50, "DifferenceOrder" → 4}}]
```

- **One could compute entanglement entropy (1506.02658)**

```
u = .

σ = {λ, λ2, λ3};
```

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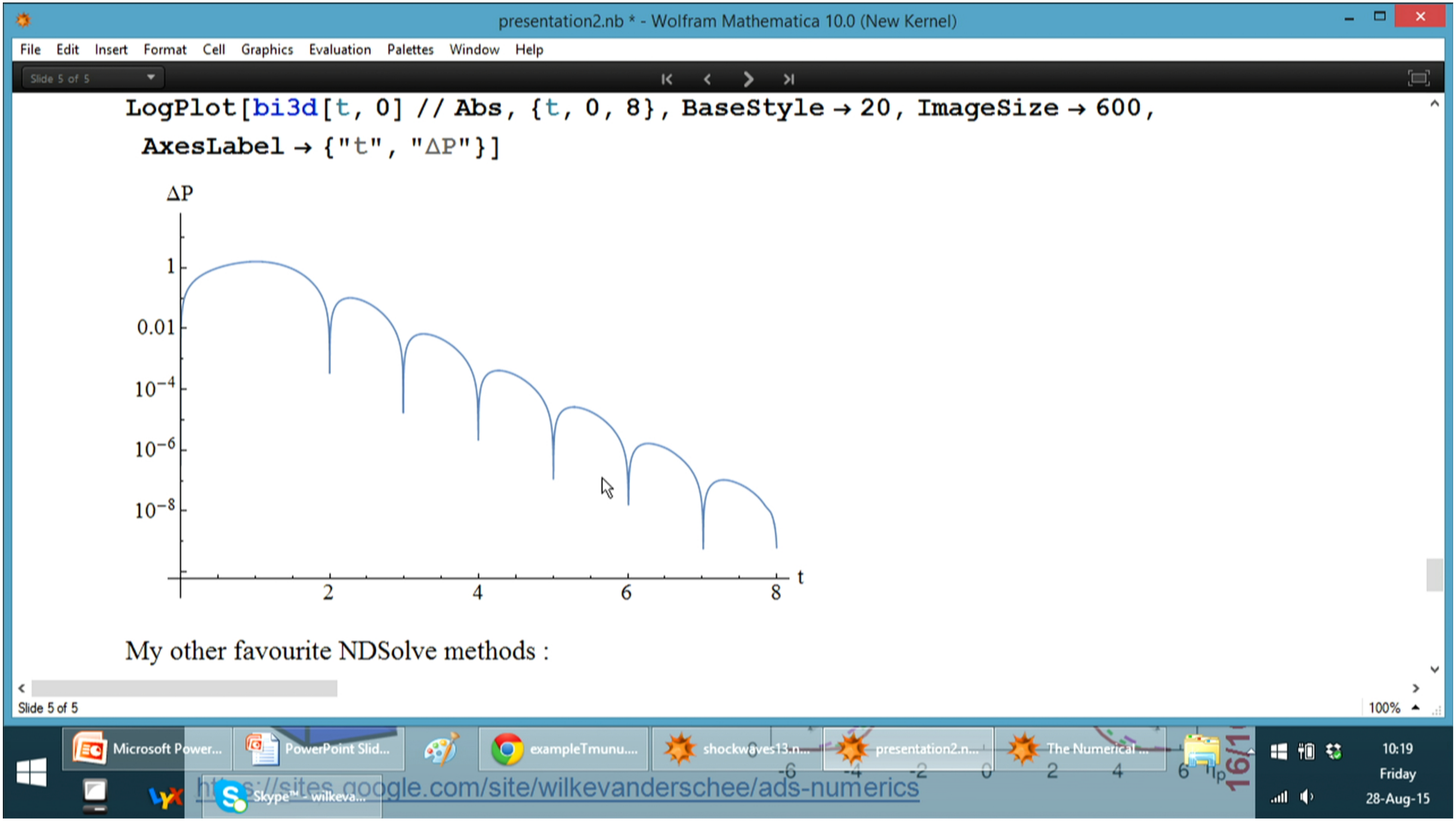
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LogPlot[bi3d[t, 0] // Abs, {t, 0, 8}, BaseStyle → 20, ImageSize → 600,
AxesLabel → {"t", "ΔP"}]
ΔP
1
0.01
10^-4
10^-6
10^-8
2
4
6
8
t
My other favourite NDSolve methods :
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$$\frac{}{2\,S[t,r]}\left(A[t,r]\left(3A^{(0,1)}[t,r]\,S^{(0,1)}[t,r]+S[t,r]\left(-8+A^{(0,2)}[t,r]\right)\right)-3\left(S^{(0,1)}[t,r]\,A^{(1,0)}[t,r]+S[t,r]\,B^{(1,0)}[t,r]^2-A^{(0,1)}[t,r]\,S^{(1,0)}[t,r]+2\,S^{(2,0)}[t,r]\right)\right)\Big\}$$

Do a linearized approximations (look up black brane, or solve by near boundary analysis)

```
funcs = {A, B, S};

linear =
 (f ↦
    (f → (ToExpression[ToString[f] <> "0"][#1, #2] +
         ϵ ToExpression["δ" <> ToString[f]][#1, #2] &))) /@ funcs
D[A[t, r], t] /. linear

{A → (ToExpression[ToString[A] <> 0][#1, #2] +
```

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Transforming functions/coordinate transformation

```
In[53]:= metric =
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -A[t,r] & 0 & 0 & 0 \\ 0 & 0 & S[t,r]^2\,\mathrm{Exp}[-2\,B[t,r]] & 0 & 0 \\ 0 & 0 & 0 & S[t,r]^2\,\mathrm{Exp}[B[t,r]] & 0 \\ 0 & 0 & 0 & 0 & S[t,r]^2\,\mathrm{Exp}[B[t,r]] \end{pmatrix}$$

```
coord = {r, t, x1, x2, x3};

In[55]:= SetDirectory[NotebookDirectory[]];
<< "diffgeo.m";

SetDelayed::write : Tag Laplacian in Laplacian[scalar_?scalarQ] is Protected. >>
```

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■ Change coordinates to 1/r and B to B/z^4, which makes B regular at the boundary:

In[75]:= `EE1[[4]]`

Out[75]= $$\frac{1}{r}\left(\left(-1+5\,r^4\right)\delta B^{(0,1)}[t,r]+r\left(\left(-1+r^4\right)\delta B^{(0,2)}[t,r]+r\left(3\,\delta B^{(1,0)}[t,r]+2\,r\,\delta B^{(1,1)}[t,r]\right)\right)\right)$$

In[72]:= `replrtou = {f_^(a_,b_)[t, r] → D[f^(a,0)[t, 1/r], {r, b}], f_[t, r] → f[t, u], r → 1/u}`

Out[72]= {f_^(a_,b_)[t, r] → ∂{r,b} f^(a,0)[t, 1/r], ⋯ 1 ⋯, r → {{⋯ 1 ⋯}, {⋯ 1 ⋯}, ⋯ 14 ⋯, {{⋯ 1 ⋯}, ⋯ 16 ⋯, {{2.87775, 2.05096, 116.219, ⋯ 13 ⋯, 5.38225, 2.15256}, ⋯ 17 ⋯}}, {{⋯ 1 ⋯}, ⋯ 17 ⋯}}}

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Out[75]= $\frac{1}{r}\left(\left(-1+5\,r^4\right)\,\delta B^{(0,1)}[t,\,r]+ r\left(\left(-1+r^4\right)\,\delta B^{(0,2)}[t,\,r]+r\left(3\,\delta B^{(1,0)}[t,\,r]+2\,r\,\delta B^{(1,1)}[t,\,r]\right)\right)\right)$

u == 1 / r

In[76]:= D[f[t, 1 / r], t]

Out[76]= $f^{(1,0)}\left[t,\,\frac{1}{r}\right]$

In[77]:= $\delta B^{(0,1)}[t,\,r]$ /. $f\_^{(a\_,b\_)}[t,\,r]$ -> $D\left[f^{(a,0)}\left[t,\,\frac{1}{r}\right],\,\{r,\,b\}\right]$

Out[77]= $-\frac{\delta B^{(0,1)}\left[t,\,\frac{1}{r}\right]}{r^2}$

In[72]:= replrtou = $\left\{f\_^{(a\_,b\_)}[t,\,r]\to D\left[f^{(a,0)}\left[t,\,\frac{1}{r}\right],\,\{r,\,b\}\right],\,f\_[t,\,r]\to f[t,\,u],\,r\to 1/u\right\}$

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Out[76]= $f^{(1,0)}\left[t, \frac{1}{r}\right]$

In[78]:= u =.

In[81]:= $\delta B^{(0,5)}[t, r]$ /. $f\_^{(a\_,b\_)}[t, r]$ -> $D\left[f^{(a,0)}\left[t, \frac{1}{r}\right], \{r, b\}\right]$

% /. r → 1 / u

Out[81]= $$-\frac{120\,\delta B^{(0,1)}\left[t, \frac{1}{r}\right]}{r^6} - \frac{240\,\delta B^{(0,2)}\left[t, \frac{1}{r}\right]}{r^7} - \frac{120\,\delta B^{(0,3)}\left[t, \frac{1}{r}\right]}{r^8} - \frac{20\,\delta B^{(0,4)}\left[t, \frac{1}{r}\right]}{r^9} - \frac{\delta B^{(0,5)}\left[t, \frac{1}{r}\right]}{r^{10}}$$

Out[82]= $$-120\,u^6\,\delta B^{(0,1)}[t, u] - 240\,u^7\,\delta B^{(0,2)}[t, u] - 120\,u^8\,\delta B^{(0,3)}[t, u] - 20\,u^9\,\delta B^{(0,4)}[t, u] - u^{10}\,\delta B^{(0,5)}[t, u]$$

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Out[82]= $-120\,u^6\,\delta B^{(0,1)}[t,u] - 240\,u^7\,\delta B^{(0,2)}[t,u] - 120\,u^8\,\delta B^{(0,3)}[t,u] - 20\,u^9\,\delta B^{(0,4)}[t,u] - u^{10}\,\delta B^{(0,5)}[t,u]$

In[83]:= **replrtou** = $\{f\_^{(a\_,b\_)}[t,r] \to D[f^{(a,0)}[t,\frac{1}{r}],\{r,b\}],\ f\_[t,r] \to f[t,u],\ r \to 1/u\}$

Out[83]= $\{f\_^{(a\_,b\_)}[t,r] \to \partial_{\{r,b\}} f^{(a,0)}[t,\frac{1}{r}],\ f\_[t,r] \to f[t,u],\ r \to \frac{1}{u}\}$

EE1reg = $-\frac{1}{u^3}$ **EE1[[4]] //. replrtou /. δB → (δBreg[#1, #2] #2⁴ &) // FullSimplify**

$16\,u^3\,\delta Breg[t,u] + (-5 + 9\,u^4)\,\delta Breg^{(0,1)}[t,u] + u\,(-1 + u^4)\,\delta Breg^{(0,2)}[t,u] + 5\,\delta Breg^{(1,0)}[t,u] + 2\,u\,\delta Breg^{(1,1)}[t,u]$

- **Now write time derivative equation numerically, as Bc $\partial_z(\partial_t B)$+Cc($\partial_t B$) + Sc = 0, which is easy to solve spectrally**

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```
1.726, 1.79609, 1.85686, 1.90758, 1.94765, 1.97662, 1.99414, 2.}

dBregdt[Bnum_List] :=
 LinearSolve[DiagonalMatrix[Bc].DSpec + DiagonalMatrix[Cc + 0 u], -Snum[Bnum]]
```

- **Time stepping with NDSolve and/or ones favorite method (Runge-Kutta, Adams-Bashforth, implicit/explicit etc)**

```
Timing[
 ndsol = NDSolve[{D[b[t], t] == dBregdt[b[t]], b[0] == Bnum}, b, {t, 0, 8}][[1]];]
bi = b /. ndsol

{0.140625, Null}

InterpolatingFunction[ Domain: {{0., 8.}} Output dimensions: {30} ]

tgrid = bi["Grid"][[ ;; , 1]]
```

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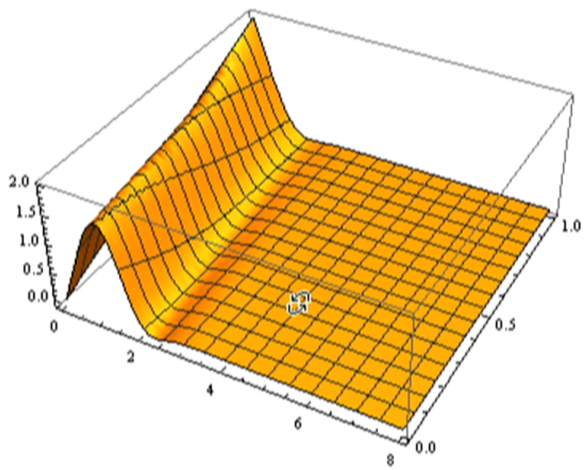
```
Plot3D[bi3d[x, y], {x, 0., 8.}, {y, 0., 1.}, PlotRange → All]
```



- **One clearly sees the lowest QNM:**

```
LogPlot[bi3d[t, 0] // Abs, {t, 0, 8}, BaseStyle → 20, ImageSize → 600,
 AxesLabel → {"t", "ΔP"}]
```

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