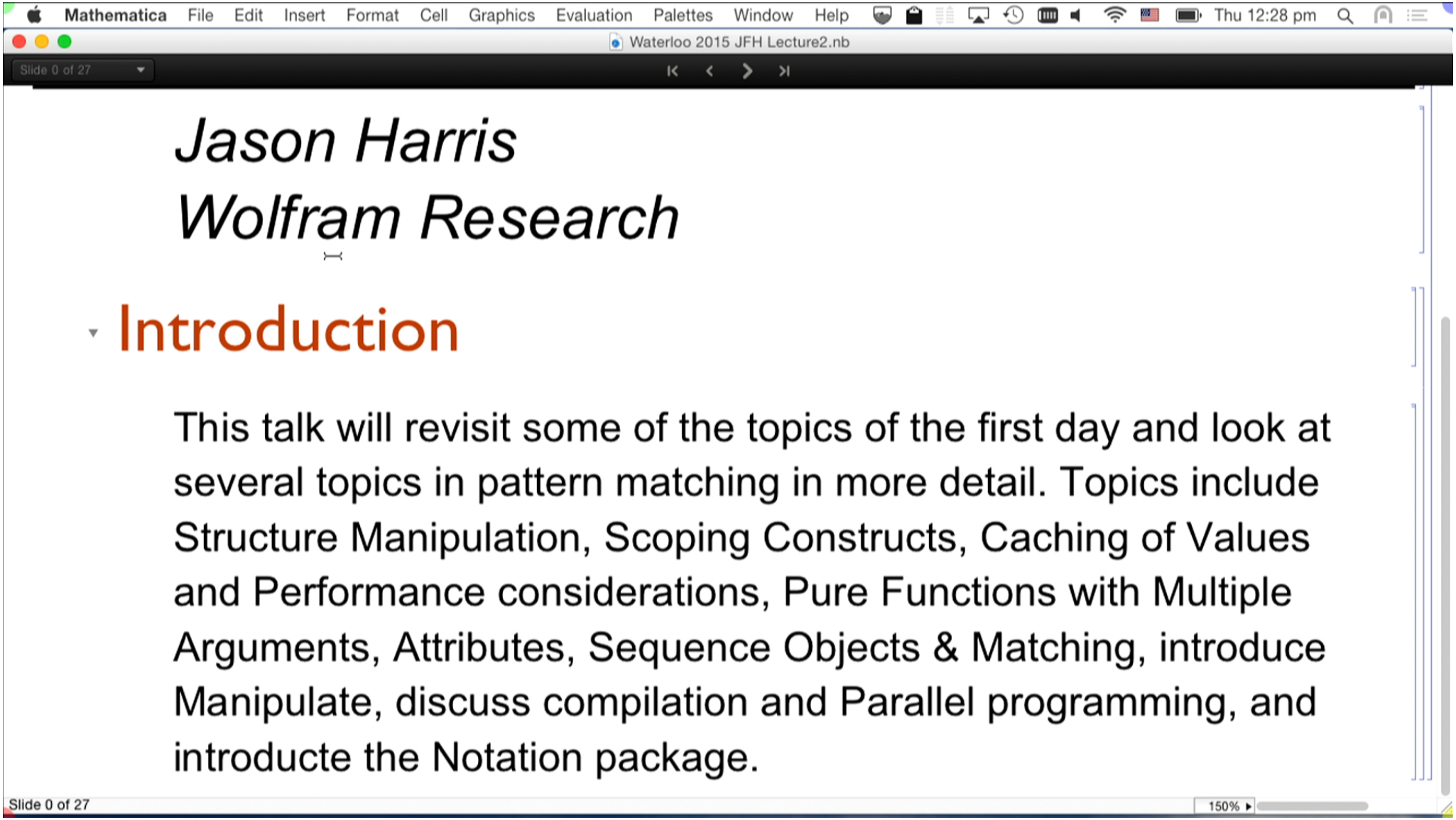
Title: TBA

Date: Aug 27, 2015 12:00 PM

URL: http://pirsa.org/15080051

Abstract: TBA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Waterloo 2015 JFH Lecture2.nb
Slide 0 of 27
Jason Harris
Wolfram Research
Introduction
This talk will revisit some of the topics of the first day and look at several topics in pattern matching in more detail. Topics include Structure Manipulation, Scoping Constructs, Caching of Values and Performance considerations, Pure Functions with Multiple Arguments, Attributes, Sequence Objects & Matching, introduce Manipulate, discuss compilation and Parallel programming, and introducte the Notation package.
Slide 0 of 27
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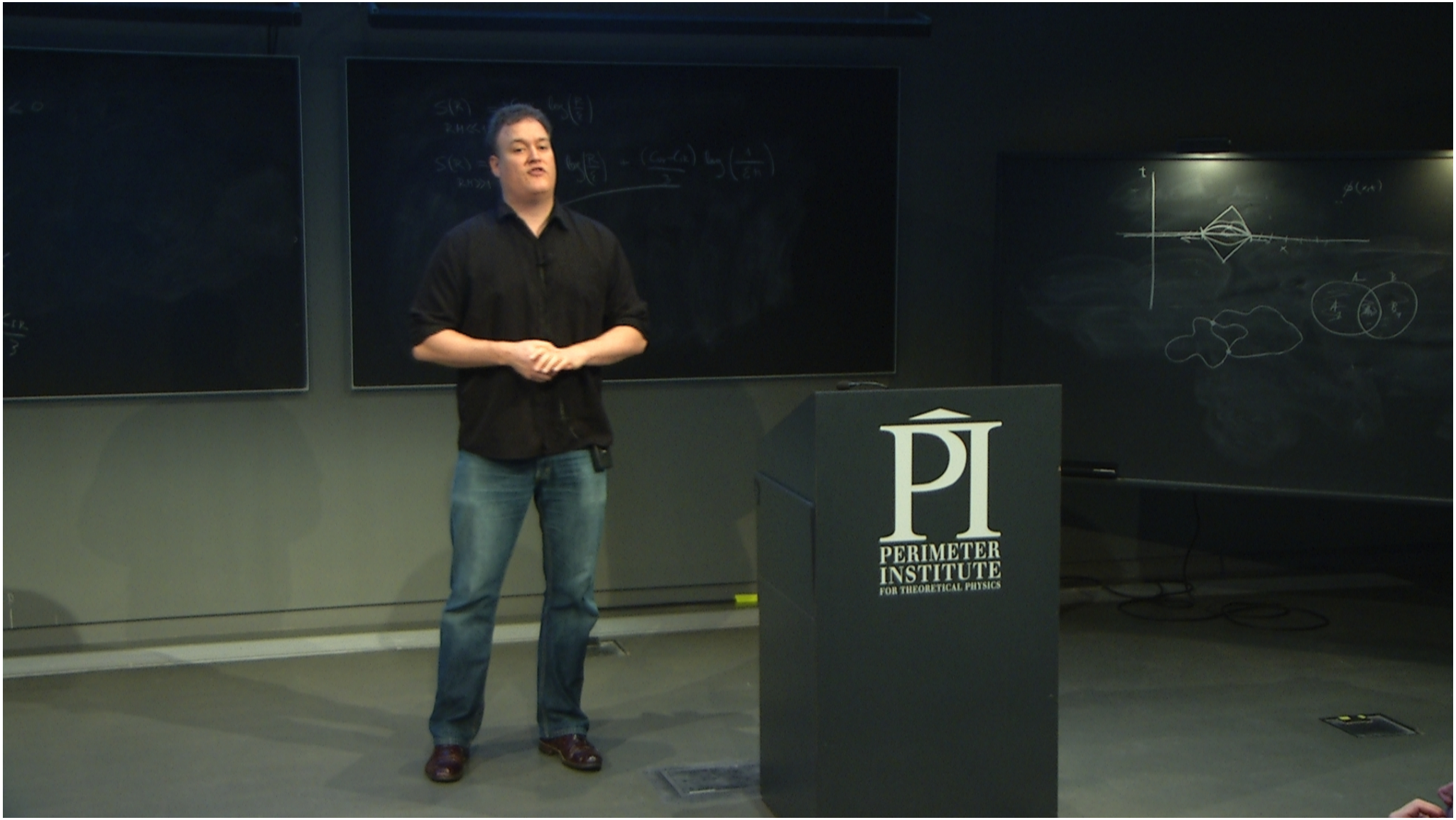
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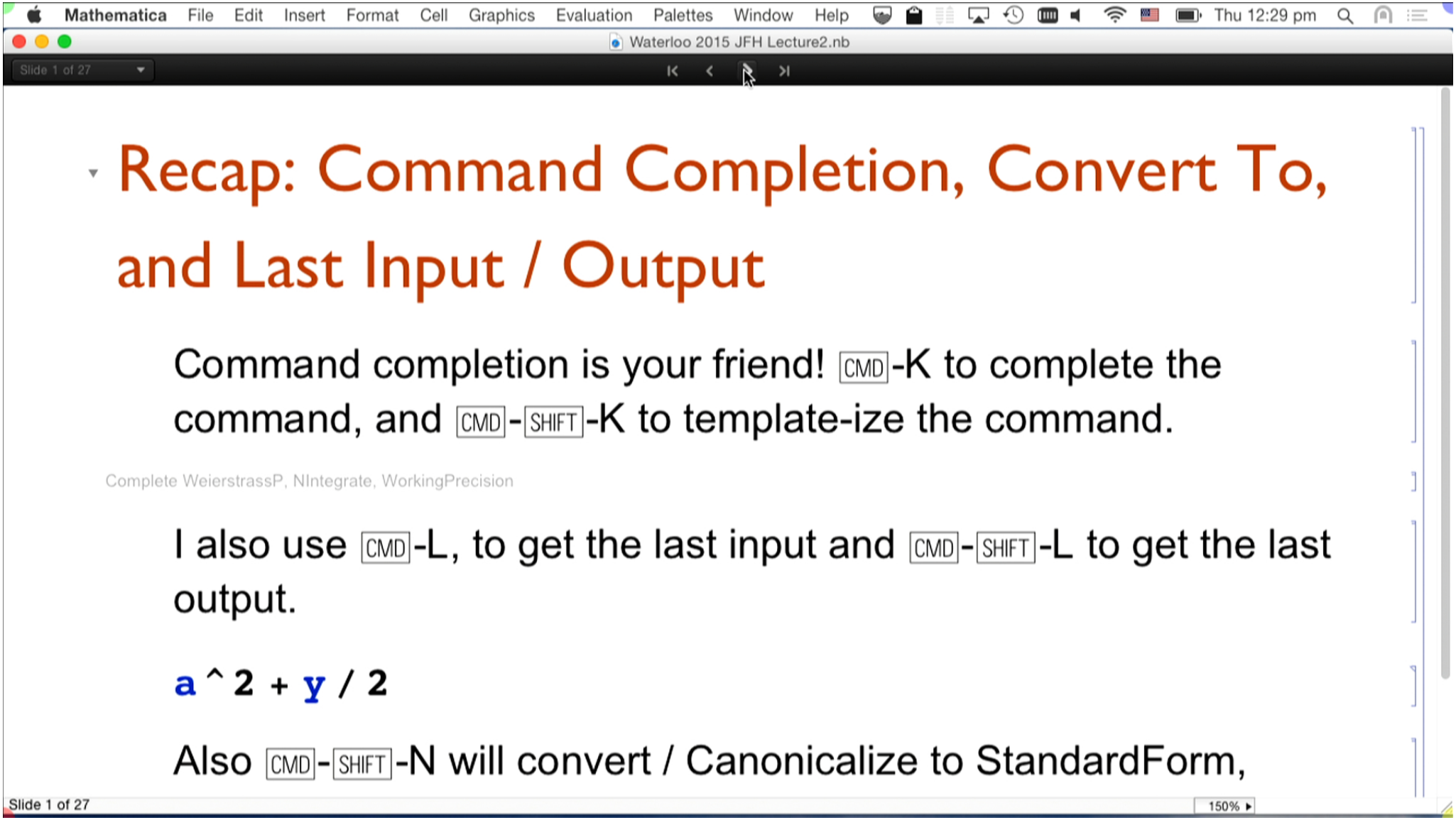
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Slide 1 of 27
Recap: Command Completion, Convert To, and Last Input / Output
Command completion is your friend! CMD-K to complete the command, and CMD-SHIFT-K to template-ize the command.
Complete WeierstrassP, NIntegrate, WorkingPrecision
I also use CMD-L, to get the last input and CMD-SHIFT-L to get the last output.
a ^ 2 + y / 2
Also CMD-SHIFT-N will convert / Canonicalize to StandardForm,
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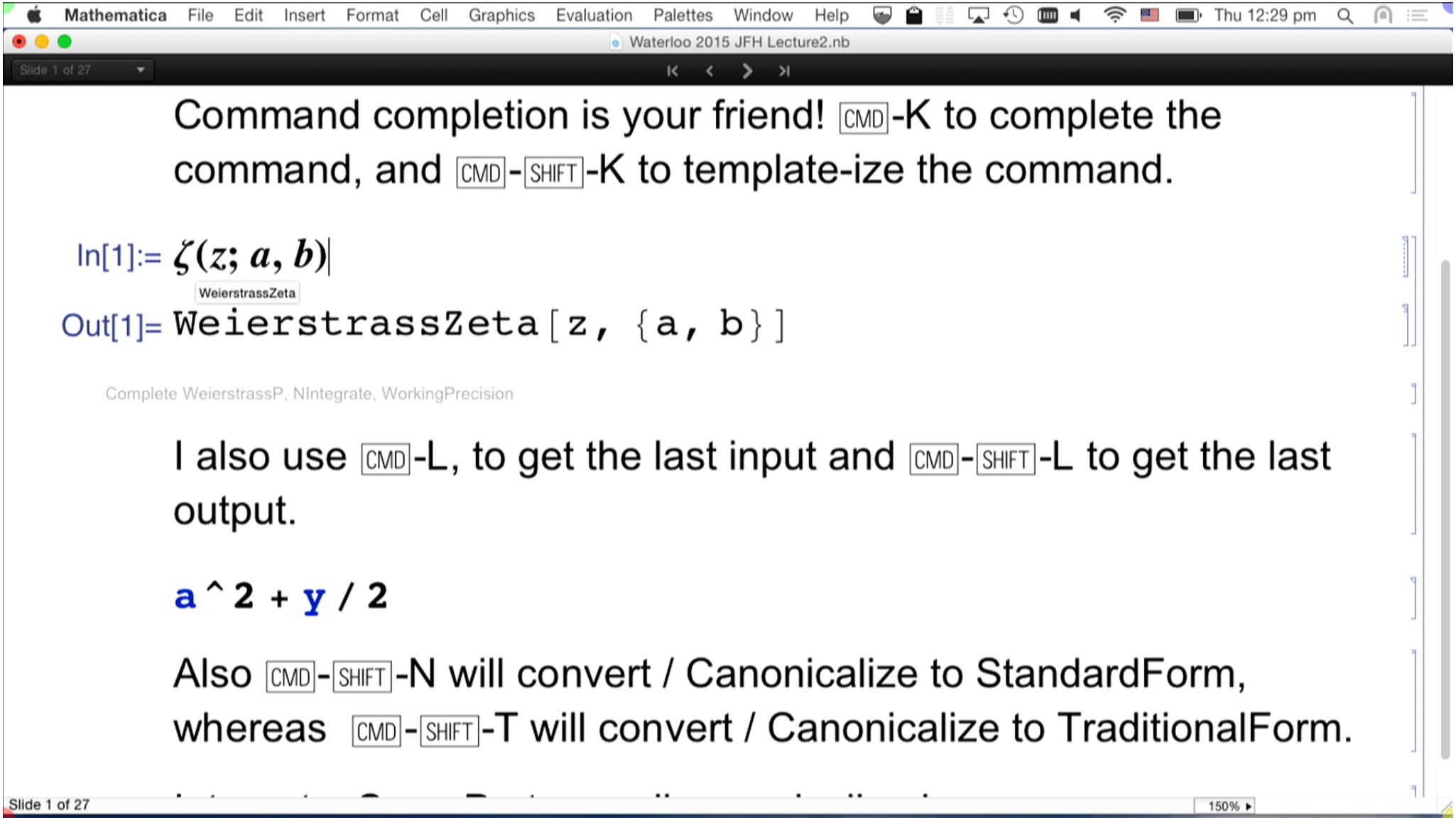
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Slide 1 of 27
Command completion is your friend! CMD-K to complete the command, and CMD-SHIFT-K to template-ize the command.
In[1]:= ζ(z; a, b)
WeierstrassZeta
Out[1]= WeierstrassZeta[z, {a, b}]
Complete WeierstrassP, NIntegrate, WorkingPrecision
I also use CMD-L, to get the last input and CMD-SHIFT-L to get the last output.
a ^ 2 + y / 2
Also CMD-SHIFT-N will convert / Canonicalize to StandardForm, whereas CMD-SHIFT-T will convert / Canonicalize to TraditionalForm.
Slide 1 of 27
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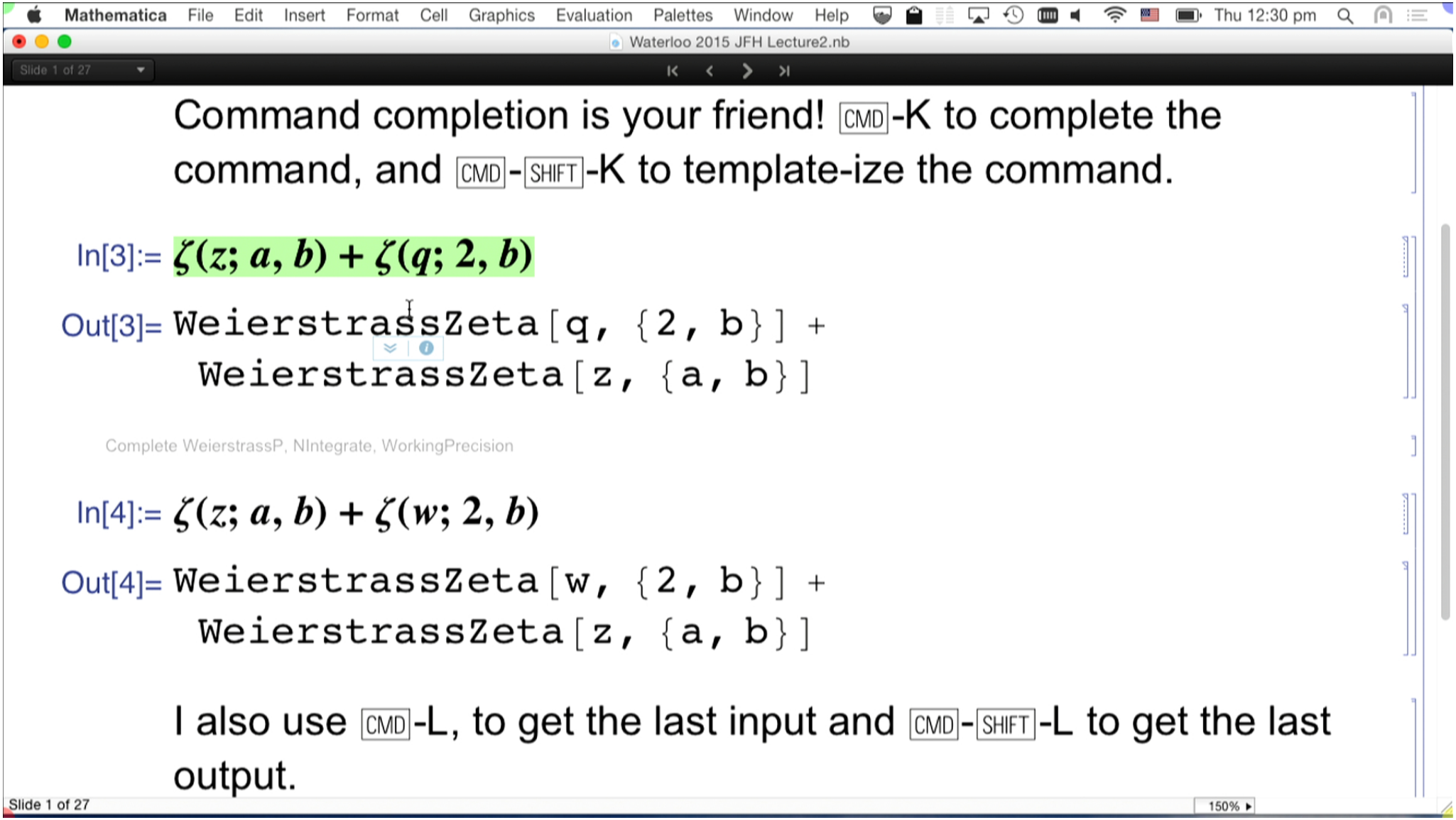
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Slide 1 of 27
Command completion is your friend! CMD-K to complete the command, and CMD-SHIFT-K to template-ize the command.
In[3]:= ζ(z; a, b) + ζ(q; 2, b)
Out[3]= WeierstrassZeta[q, {2, b}] +
WeierstrassZeta[z, {a, b}]
Complete WeierstrassP, NIntegrate, WorkingPrecision
In[4]:= ζ(z; a, b) + ζ(w; 2, b)
Out[4]= WeierstrassZeta[w, {2, b}] +
WeierstrassZeta[z, {a, b}]
I also use CMD-L, to get the last input and CMD-SHIFT-L to get the last output.
Slide 1 of 27
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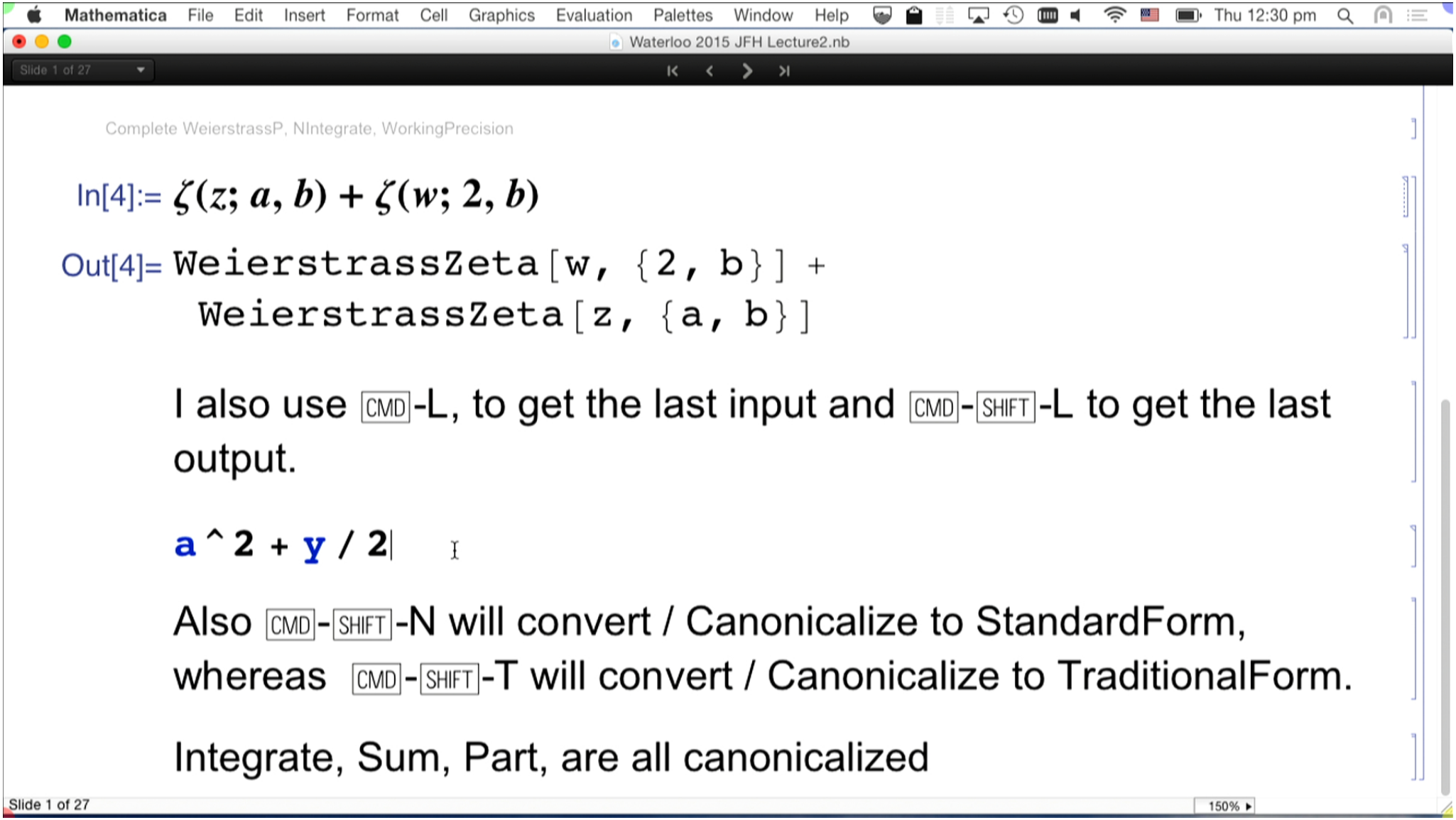
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Slide 1 of 27
Complete WeierstrassP, NIntegrate, WorkingPrecision
In[4]:= ζ(z; a, b) + ζ(w; 2, b)
Out[4]= WeierstrassZeta[w, {2, b}] +
WeierstrassZeta[z, {a, b}]
I also use CMD-L, to get the last input and CMD-SHIFT-L to get the last output.
a ^ 2 + y / 2
Also CMD-SHIFT-N will convert / Canonicalize to StandardForm, whereas CMD-SHIFT-T will convert / Canonicalize to TraditionalForm.
Integrate, Sum, Part, are all canonicalized
Slide 1 of 27
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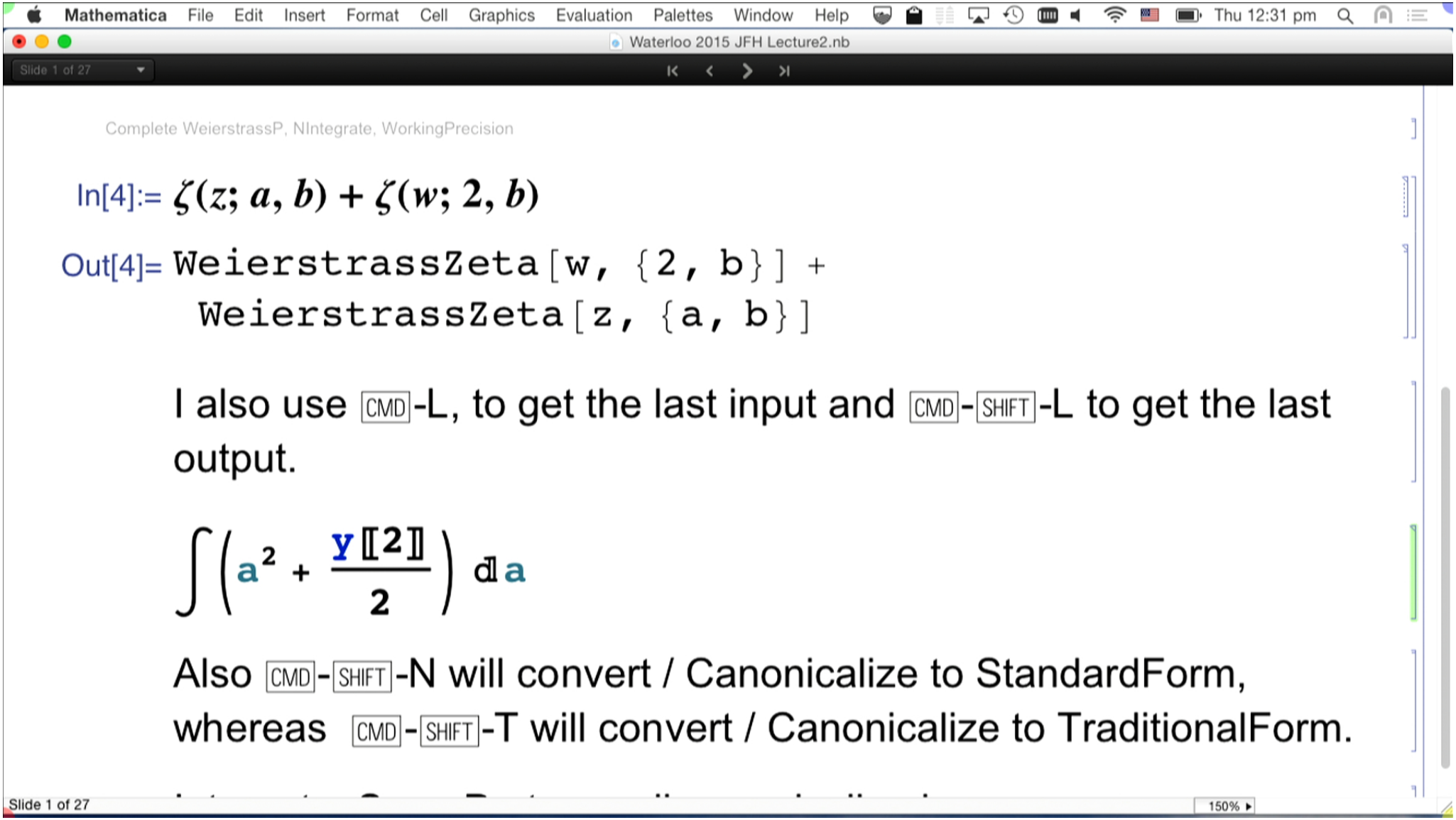
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Slide 1 of 27
Complete WeierstrassP, NIntegrate, WorkingPrecision
In[4]:= ζ(z; a, b) + ζ(w; 2, b)
Out[4]= WeierstrassZeta[w, {2, b}] +
WeierstrassZeta[z, {a, b}]
I also use CMD-L, to get the last input and CMD-SHIFT-L to get the last output.
∫ (a^2 + y[[2]]/2) da
Also CMD-SHIFT-N will convert / Canonicalize to StandardForm, whereas CMD-SHIFT-T will convert / Canonicalize to TraditionalForm.
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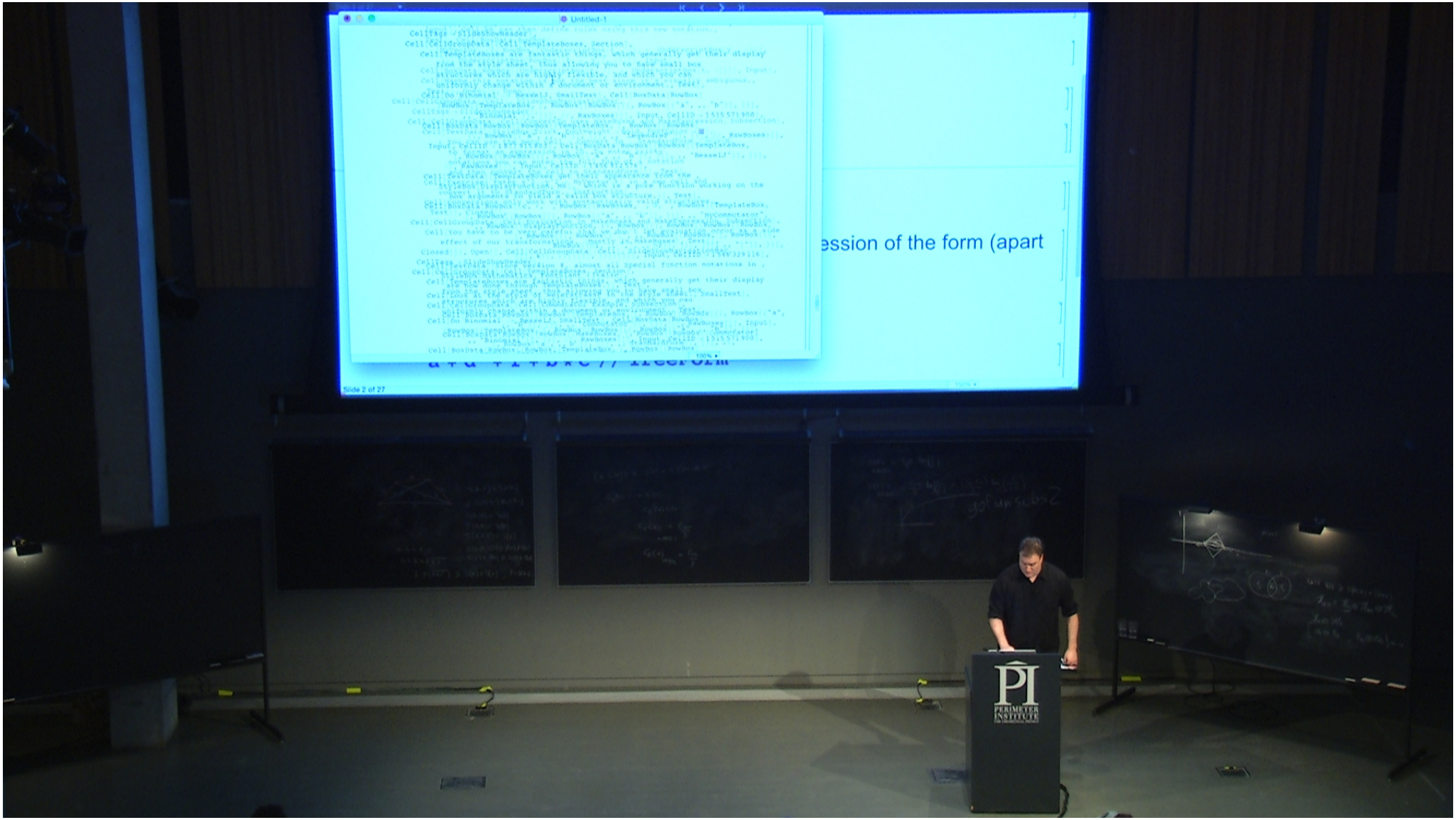
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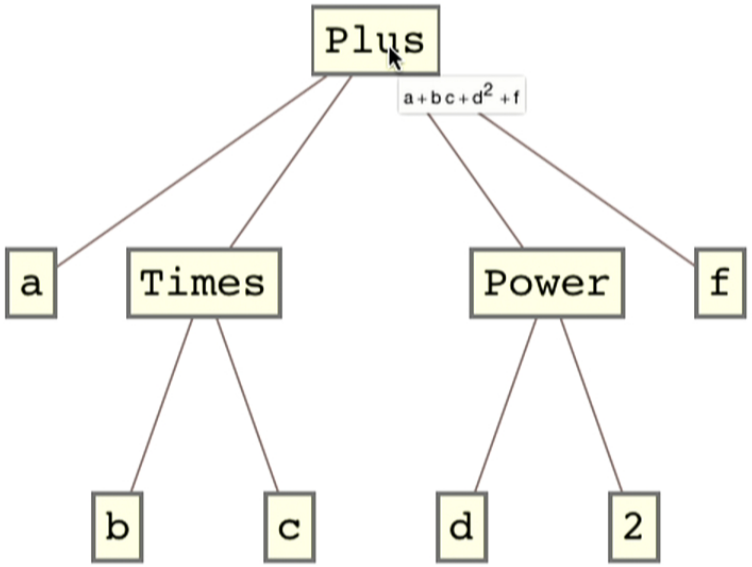
Slide 2 of 27

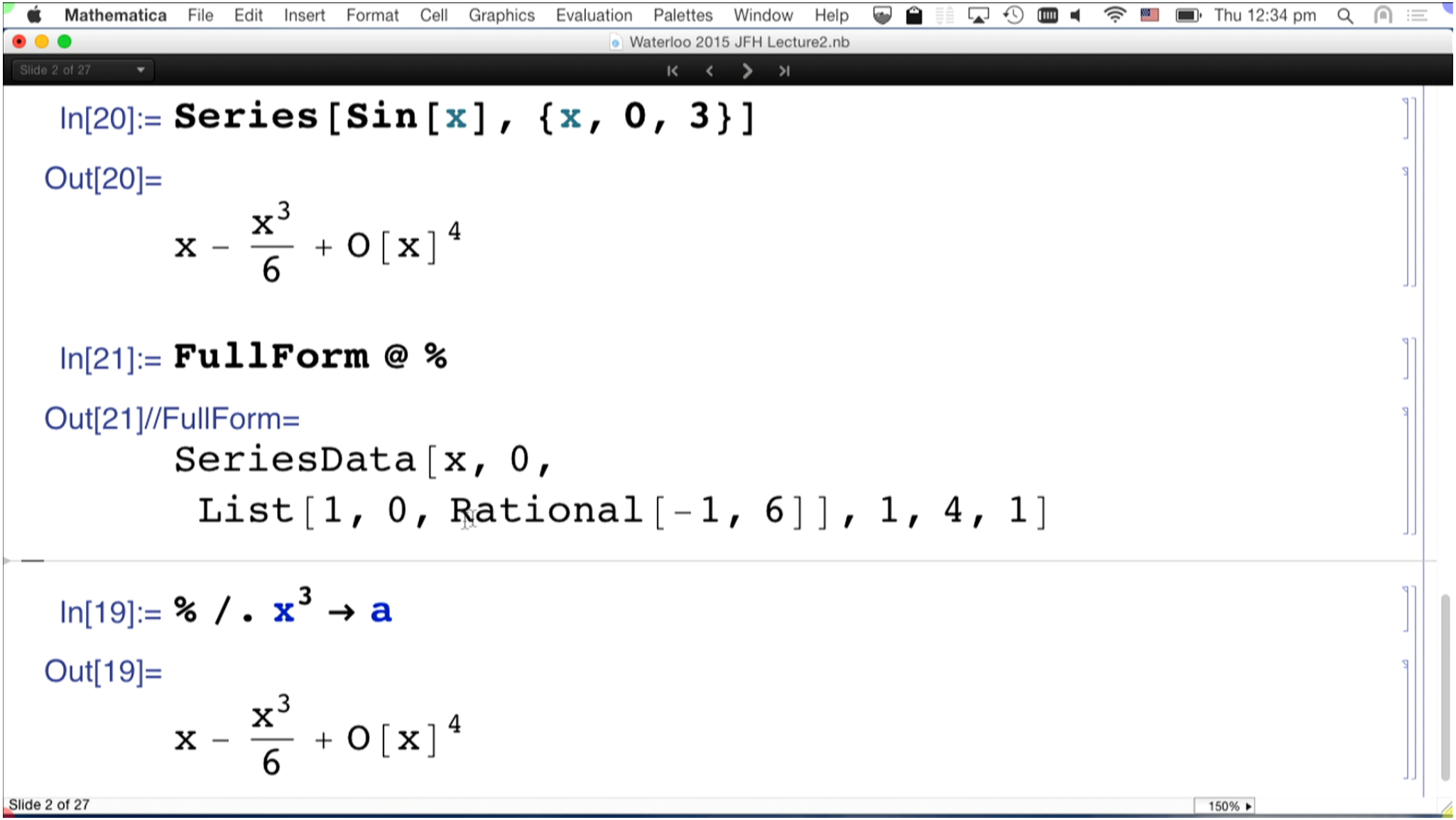
In[11]:= `a + d^2 + f + b * c // TreeForm`

Out[11]//TreeForm=



$\partial_x f[x]$

Slide 2 of 27 150%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Slide 2 of 27
In[20]:= Series[Sin[x], {x, 0, 3}]
Out[20]=
x - x^3/6 + O[x]^4
In[21]:= FullForm @ %
Out[21]//FullForm=
SeriesData[x, 0,
List[1, 0, Rational[-1, 6]], 1, 4, 1]
In[19]:= % /. x^3 → a
Out[19]=
x - x^3/6 + O[x]^4
Slide 2 of 27
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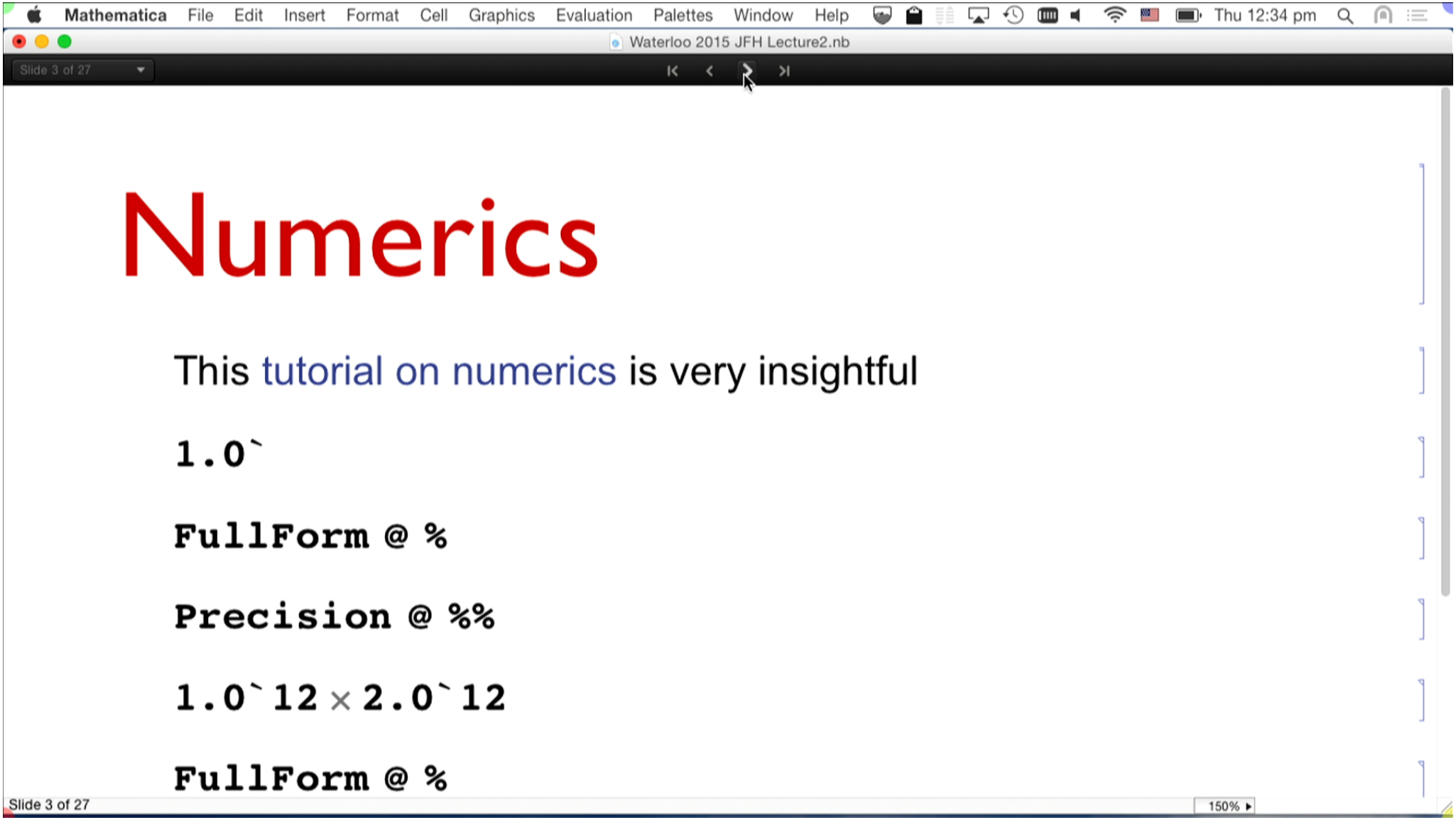
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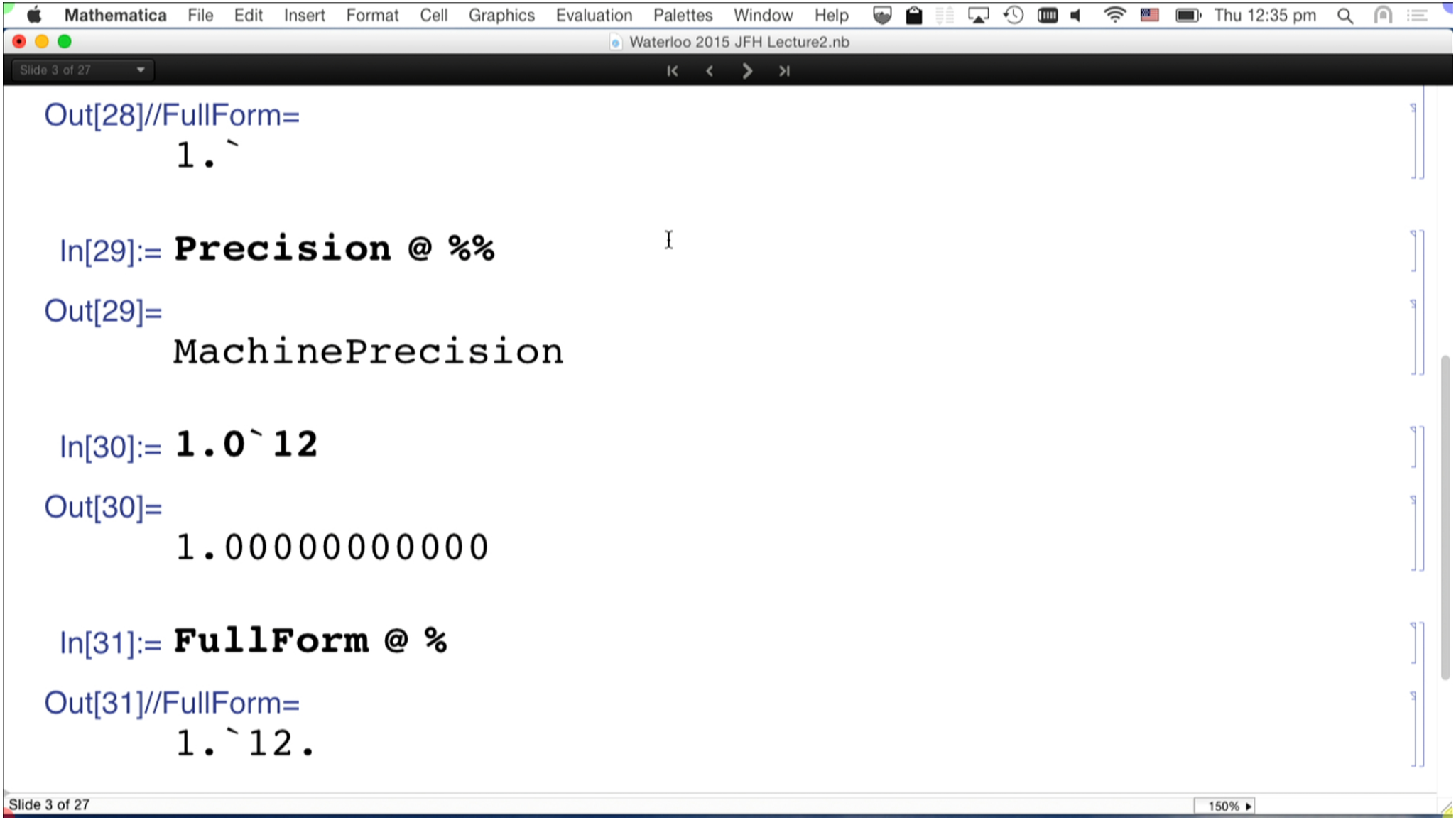
Slide 3 of 27
Numerics
This tutorial on numerics is very insightful
1.0`
FullForm @ %
Precision @ %%
1.0`12 × 2.0`12
FullForm @ %
Slide 3 of 27
150%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Slide 3 of 27
Out[28]//FullForm=
1.`
In[29]:= Precision @ %%
Out[29]=
MachinePrecision
In[30]:= 1.0`12
Out[30]=
1.00000000000
In[31]:= FullForm @ %
Out[31]//FullForm=
1.`12.
Slide 3 of 27
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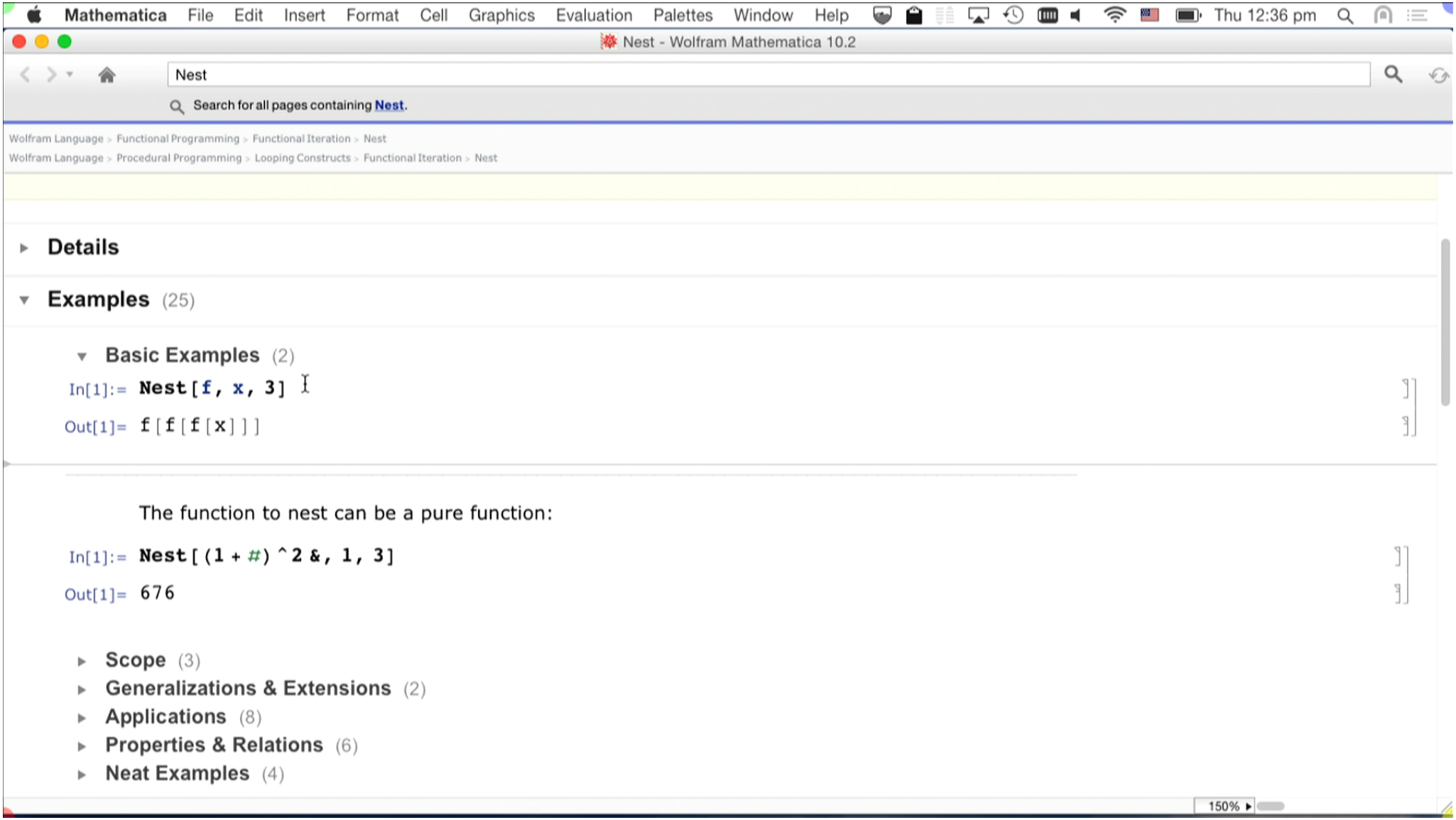
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Nest - Wolfram Mathematica 10.2
Nest
Search for all pages containing Nest.
Wolfram Language > Functional Programming > Functional Iteration > Nest
Wolfram Language > Procedural Programming > Looping Constructs > Functional Iteration > Nest
Details
Examples (25)
Basic Examples (2)
In[1]:= Nest[f, x, 3]
Out[1]= f[f[f[x]]]
The function to nest can be a pure function:
In[1]:= Nest[(1 + #)^2 &, 1, 3]
Out[1]= 676
Scope (3)
Generalizations & Extensions (2)
Applications (8)
Properties & Relations (6)
Neat Examples (4)
150%

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WorkingPrecision

Wolfram Language > Numerical Evaluation & Precision > Precision & Accuracy Control > WorkingPrecision

▶ **Details**

▼ **Examples** (9)

▼ Basic Examples (2)

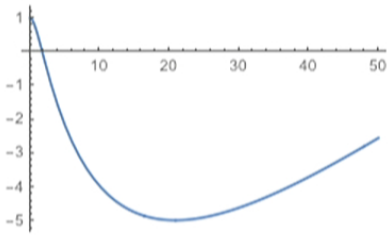
Find a root using 60-digit precision arithmetic:

```
In[1]:= FindRoot[x^2 - 2, {x, 1}, WorkingPrecision → 60]
Out[1]= {x → 1.41421356237309504880168872420969807856967187537694807317668}
```

Solve a differential equation using 24-digit precision arithmetic:

```
In[1]:= NDSolve[{x''[t] + x[t] / (1 + t^2) == 0, x[0] == 1, x'[0] == 0}, x, {t, 0, 50}, WorkingPrecision → 24]
Out[1]= {{x → InterpolatingFunction[ Domain: {{0, 50.000000000000000000000}}
                                     Output: scalar ]}}
```

```
In[2]:= Plot[First[x[t] /. %], {t, 0, 50}]
```



Out[2]=

▶ **Scope** (4)

▶ **Applications** (1)

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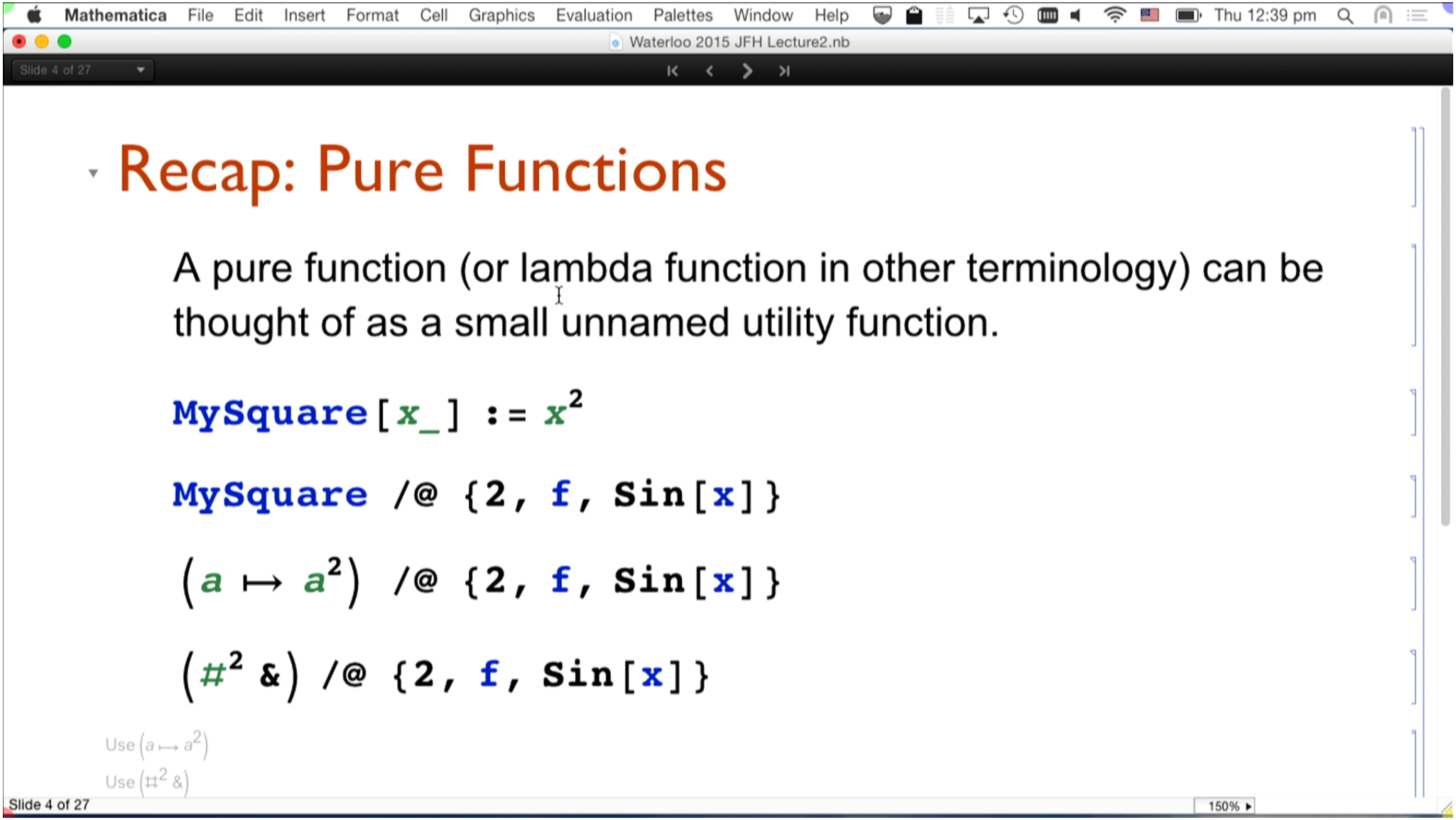
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Slide 4 of 27
Recap: Pure Functions
A pure function (or lambda function in other terminology) can be thought of as a small unnamed utility function.
MySquare[x_] := x^2
MySquare /@ {2, f, Sin[x]}
(a ↦ a^2) /@ {2, f, Sin[x]}
(#^2 &) /@ {2, f, Sin[x]}
Use (a ↦ a^2)
Use (#^2 &)
Slide 4 of 27
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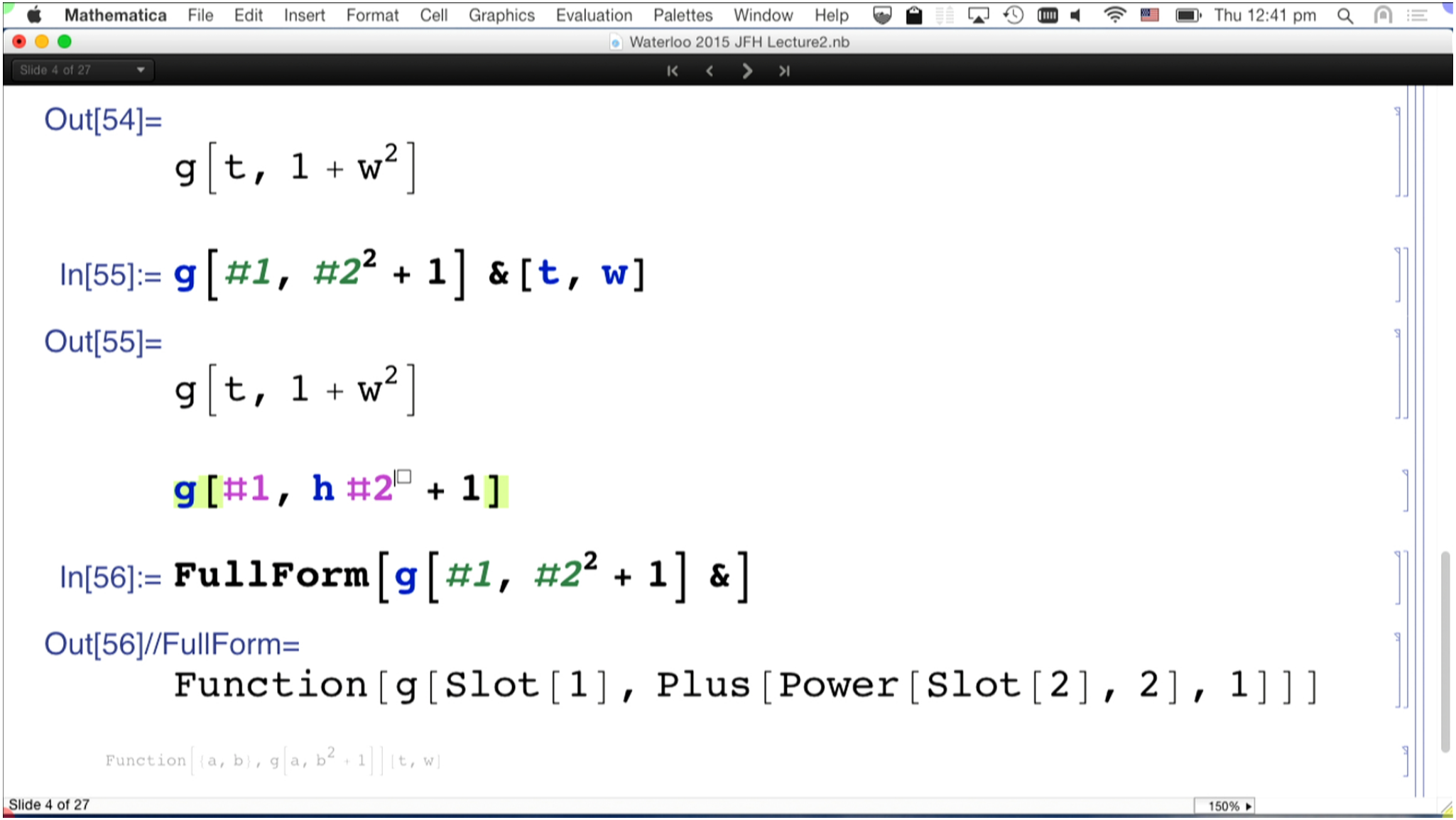
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Slide 4 of 27
Out[54]=
g[t, 1 + w^2]
In[55]:= g[#1, #2^2 + 1] &[t, w]
Out[55]=
g[t, 1 + w^2]
g[#1, h #2 + 1]
In[56]:= FullForm[g[#1, #2^2 + 1] &]
Out[56]//FullForm=
Function[g[Slot[1], Plus[Power[Slot[2], 2], 1]]]
Function[{a, b}, g[a, b^2 + 1]][t, w]
Slide 4 of 27
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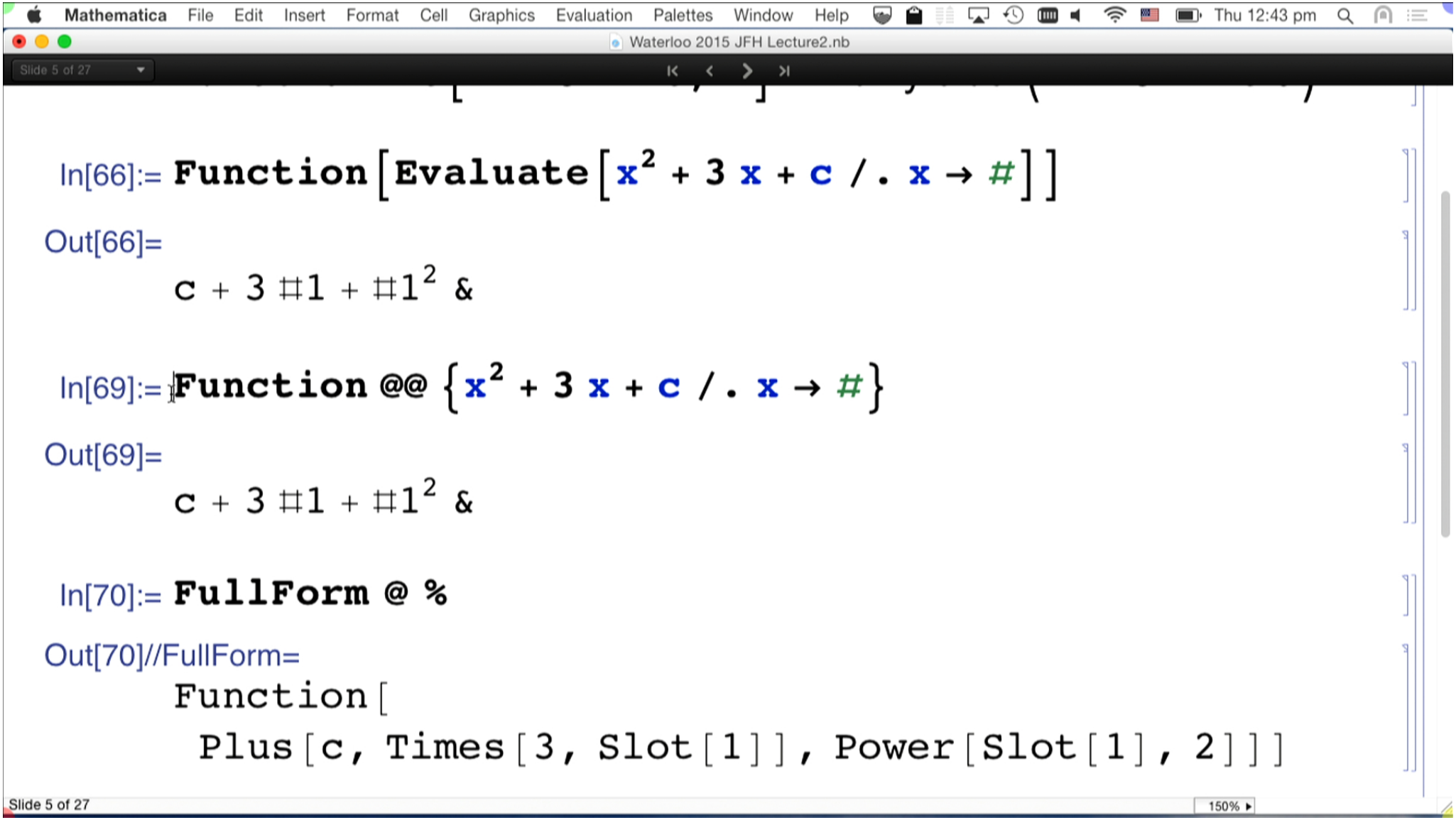
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Slide 5 of 27
In[66]:= Function[Evaluate[x^2 + 3 x + c /. x → #]]
Out[66]=
c + 3 #1 + #1^2 &
In[69]:= Function @@ {x^2 + 3 x + c /. x → #}
Out[69]=
c + 3 #1 + #1^2 &
In[70]:= FullForm @ %
Out[70]//FullForm=
Function[
Plus[c, Times[3, Slot[1]], Power[Slot[1], 2]]]
Slide 5 of 27
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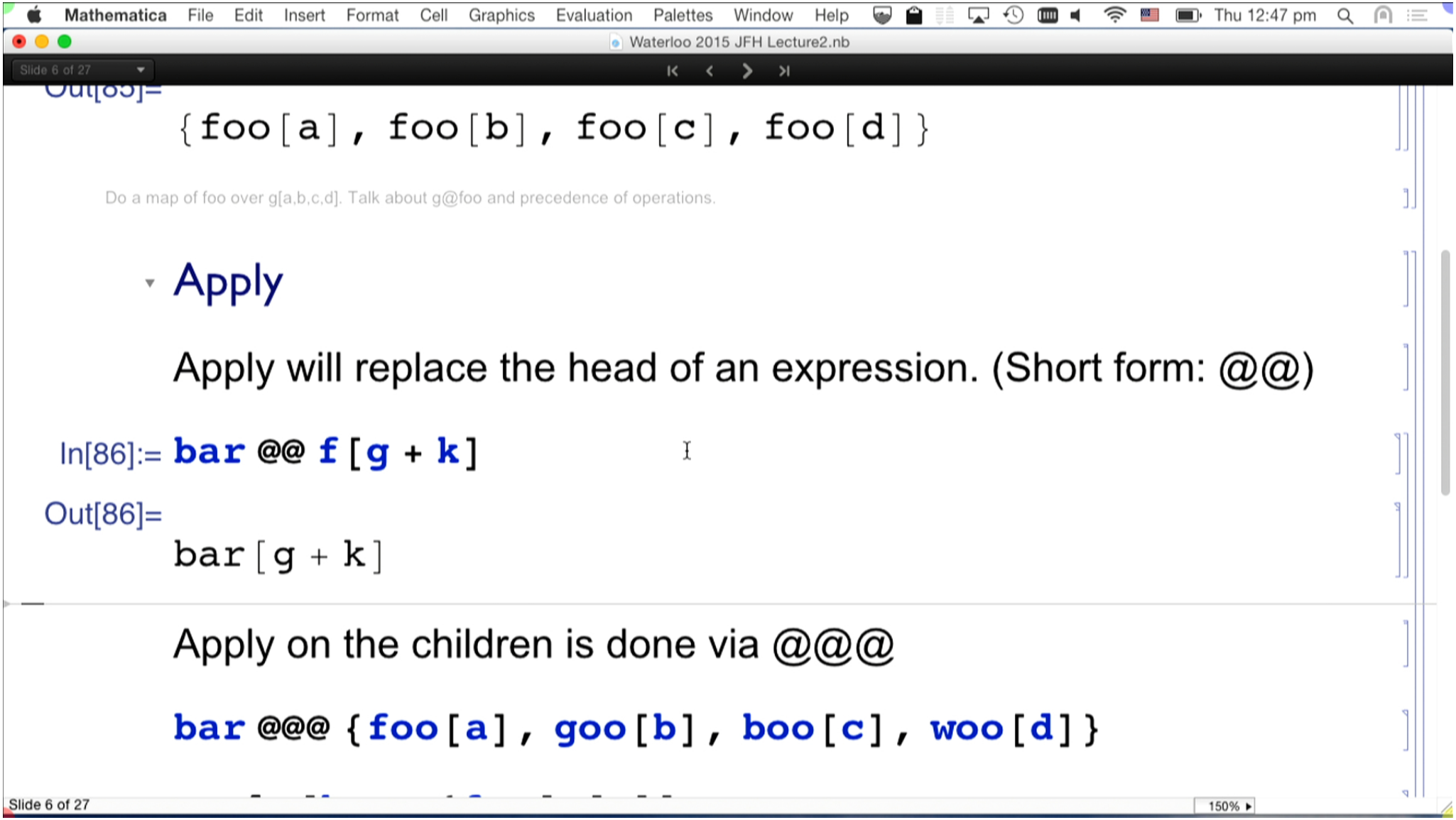
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Thu 12:47 pm
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Slide 6 of 27
Out[85]=
{foo[a], foo[b], foo[c], foo[d]}
Do a map of foo over g[a,b,c,d]. Talk about g@foo and precedence of operations.
Apply
Apply will replace the head of an expression. (Short form: @@)
In[86]:= bar @@ f[g + k]
Out[86]=
bar[g + k]
Apply on the children is done via @@@
bar @@@ {foo[a], goo[b], boo[c], woo[d]}
Slide 6 of 27
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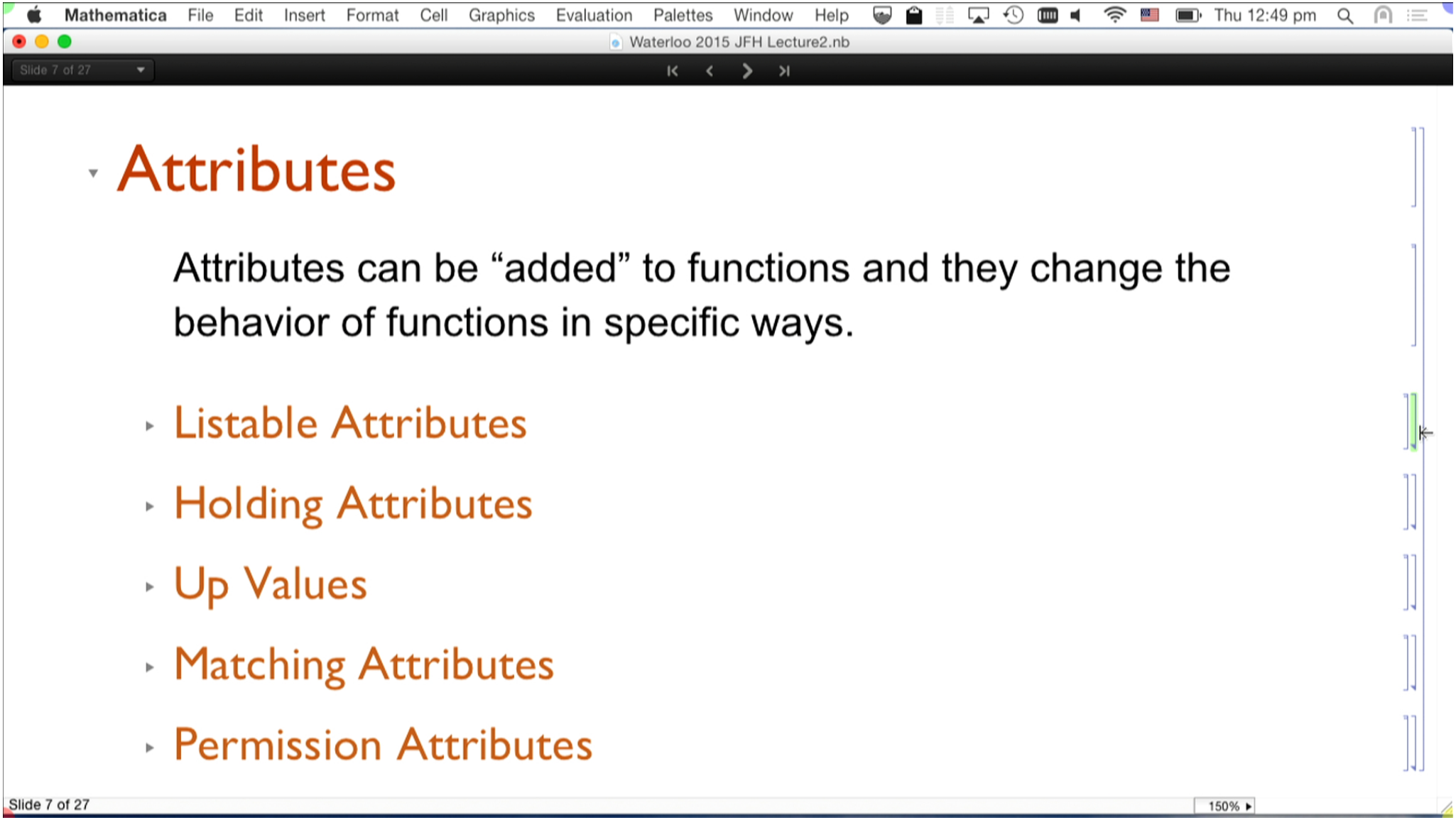
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Slide 7 of 27
Attributes
Attributes can be “added” to functions and they change the behavior of functions in specific ways.
Listable Attributes
Holding Attributes
Up Values
Matching Attributes
Permission Attributes
Slide 7 of 27
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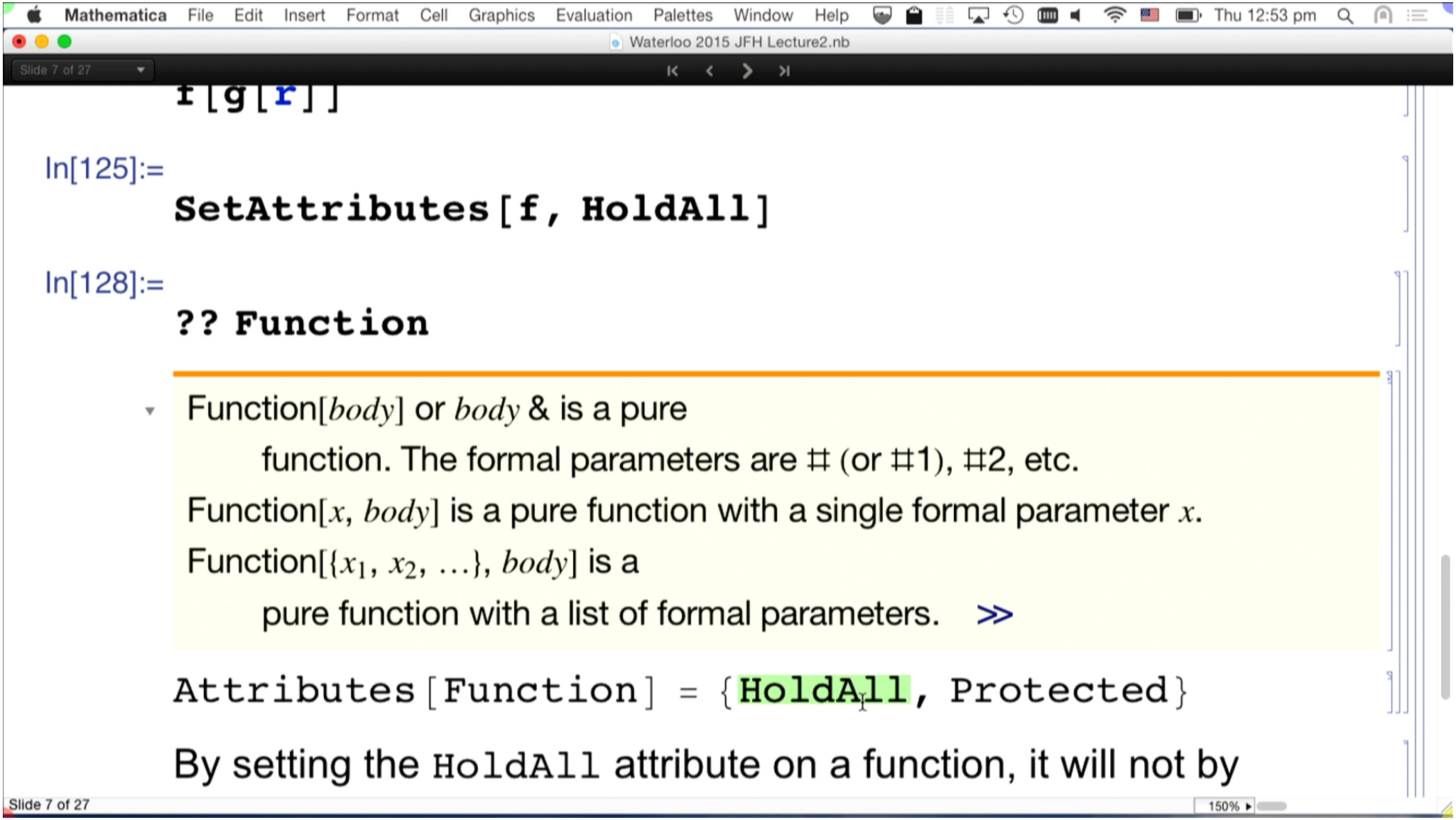
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Slide 7 of 27
f[g[r]]
In[125]:=
SetAttributes[f, HoldAll]
In[128]:=
?? Function
Function[body] or body & is a pure function. The formal parameters are # (or #1), #2, etc.
Function[x, body] is a pure function with a single formal parameter x.
Function[{x1, x2, ...}, body] is a pure function with a list of formal parameters. >>
Attributes[Function] = {HoldAll, Protected}
By setting the HoldAll attribute on a function, it will not by
Slide 7 of 27
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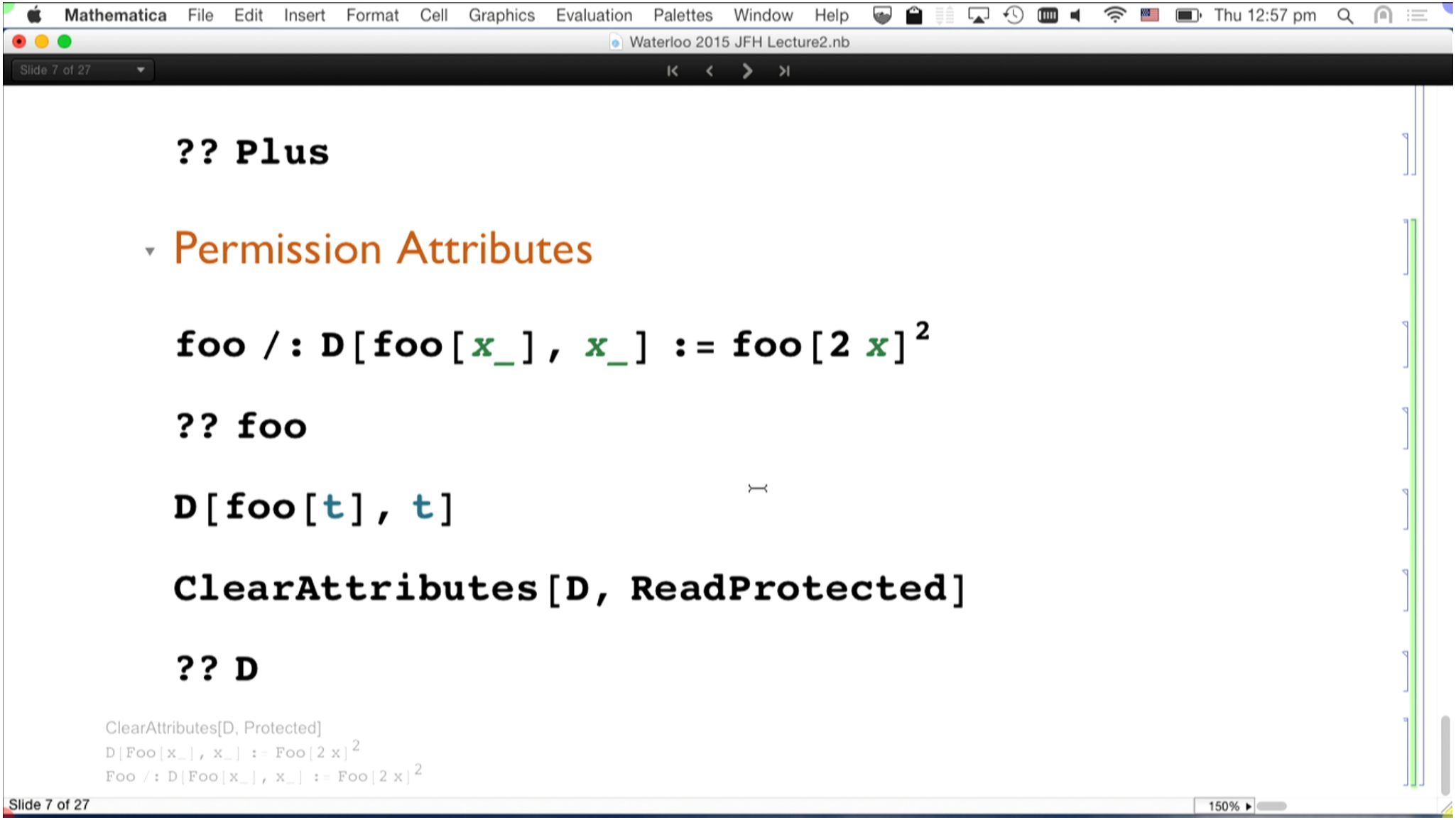
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Slide 7 of 27
?? Plus
Permission Attributes
foo /: D[foo[x_], x_] := foo[2 x]^2
?? foo
D[foo[t], t]
ClearAttributes[D, ReadProtected]
?? D
ClearAttributes[D, Protected]
D[Foo[x_], x_] := Foo[2 x]^2
Foo /: D[Foo[x_], x_] := Foo[2 x]^2
Slide 7 of 27
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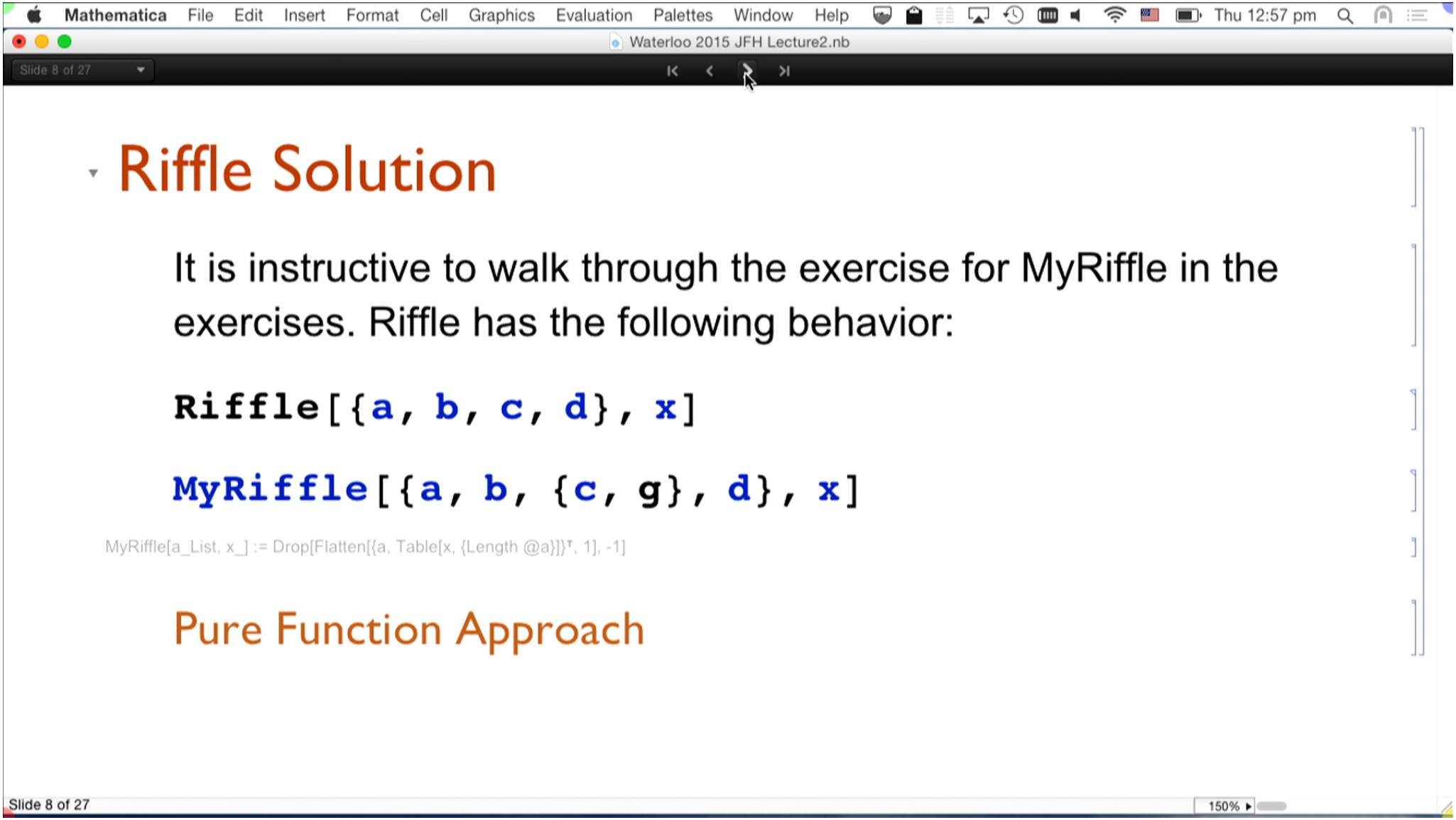
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Slide 8 of 27
Riffle Solution
It is instructive to walk through the exercise for MyRiffle in the exercises. Riffle has the following behavior:
Riffle[{a, b, c, d}, x]
MyRiffle[{a, b, {c, g}, d}, x]
MyRiffle[a_List, x_] := Drop[Flatten[{a, Table[x, {Length @a}]}ᵀ, 1], -1]
Pure Function Approach
Slide 8 of 27
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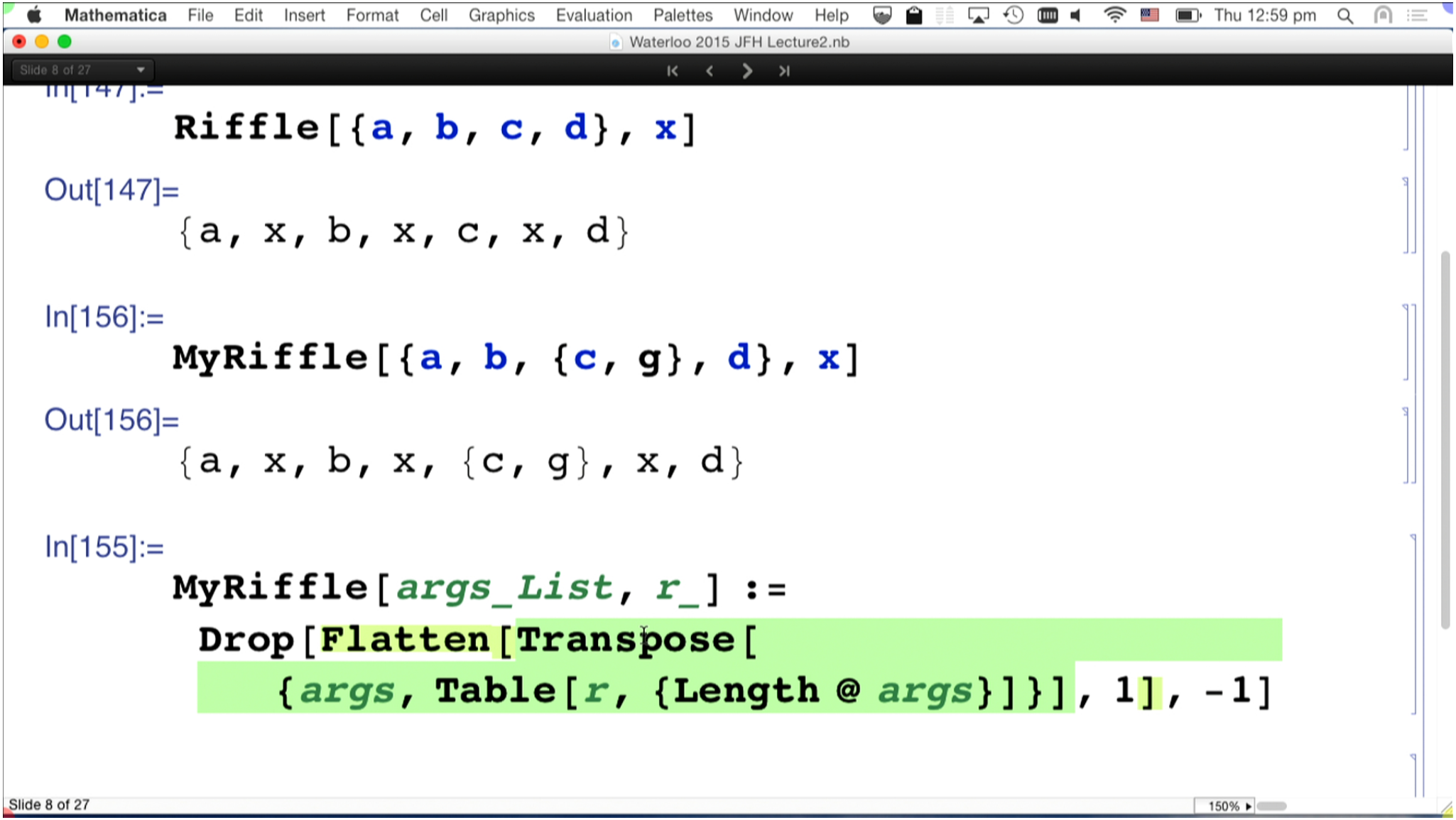
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Slide 8 of 27
In[147]:=
Riffle[{a, b, c, d}, x]
Out[147]=
{a, x, b, x, c, x, d}
In[156]:=
MyRiffle[{a, b, {c, g}, d}, x]
Out[156]=
{a, x, b, x, {c, g}, x, d}
In[155]:=
MyRiffle[args_List, r_] :=
Drop[Flatten[Transpose[
{args, Table[r, {Length @ args}]}], 1], -1]
Slide 8 of 27
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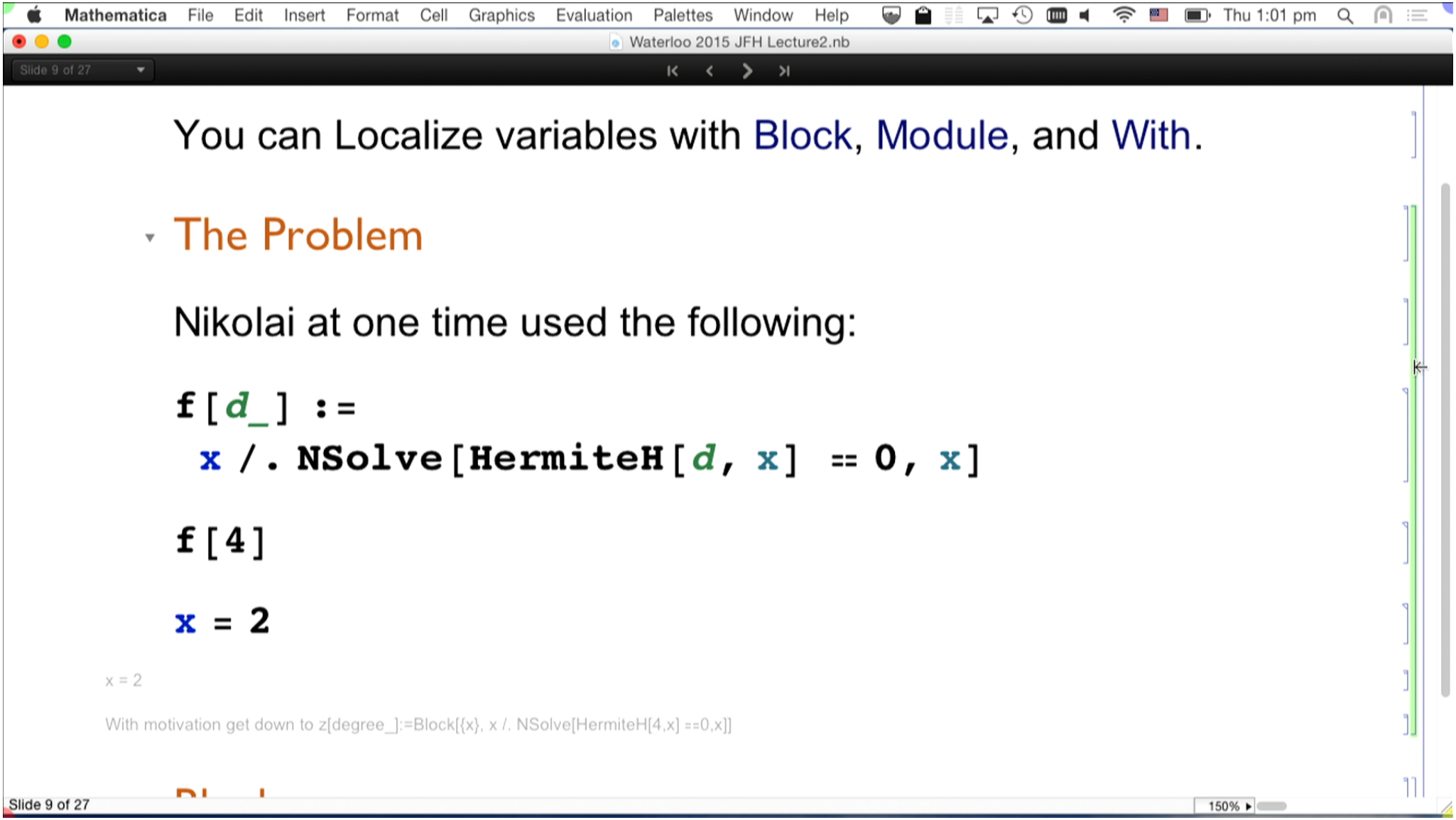
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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Slide 9 of 27
You can Localize variables with Block, Module, and With.
The Problem
Nikolai at one time used the following:
f[d_] :=
x /. NSolve[HermiteH[d, x] == 0, x]
f[4]
x = 2
x = 2
With motivation get down to z[degree_]:=Block[{x}, x /. NSolve[HermiteH[4,x] ==0,x]]
Slide 9 of 27
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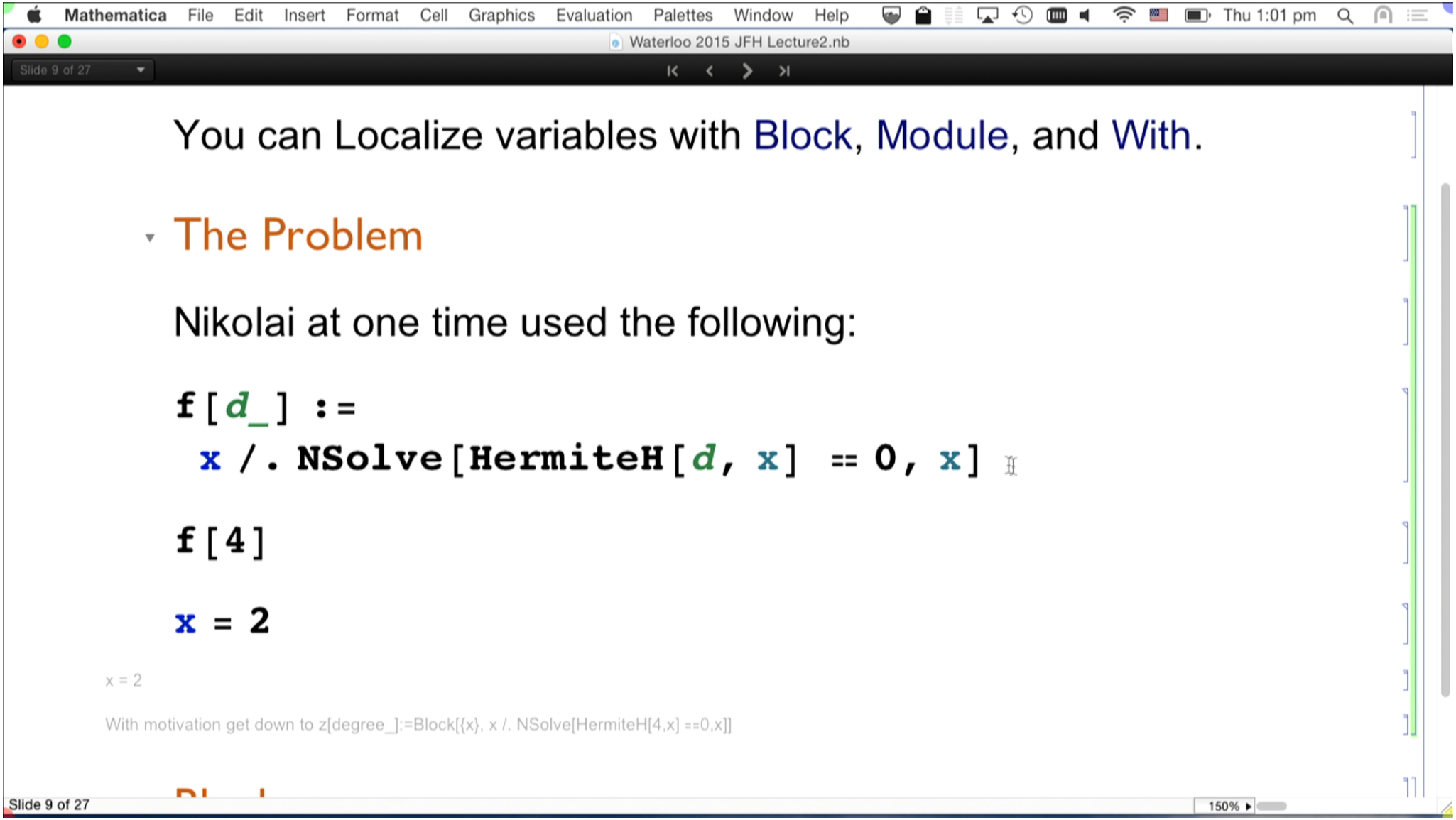
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Slide 9 of 27
You can Localize variables with Block, Module, and With.
The Problem
Nikolai at one time used the following:
f[d_] :=
x /. NSolve[HermiteH[d, x] == 0, x]
f[4]
x = 2
x = 2
With motivation get down to z[degree_]:=Block[{x}, x /. NSolve[HermiteH[4,x] ==0,x]]
Slide 9 of 27
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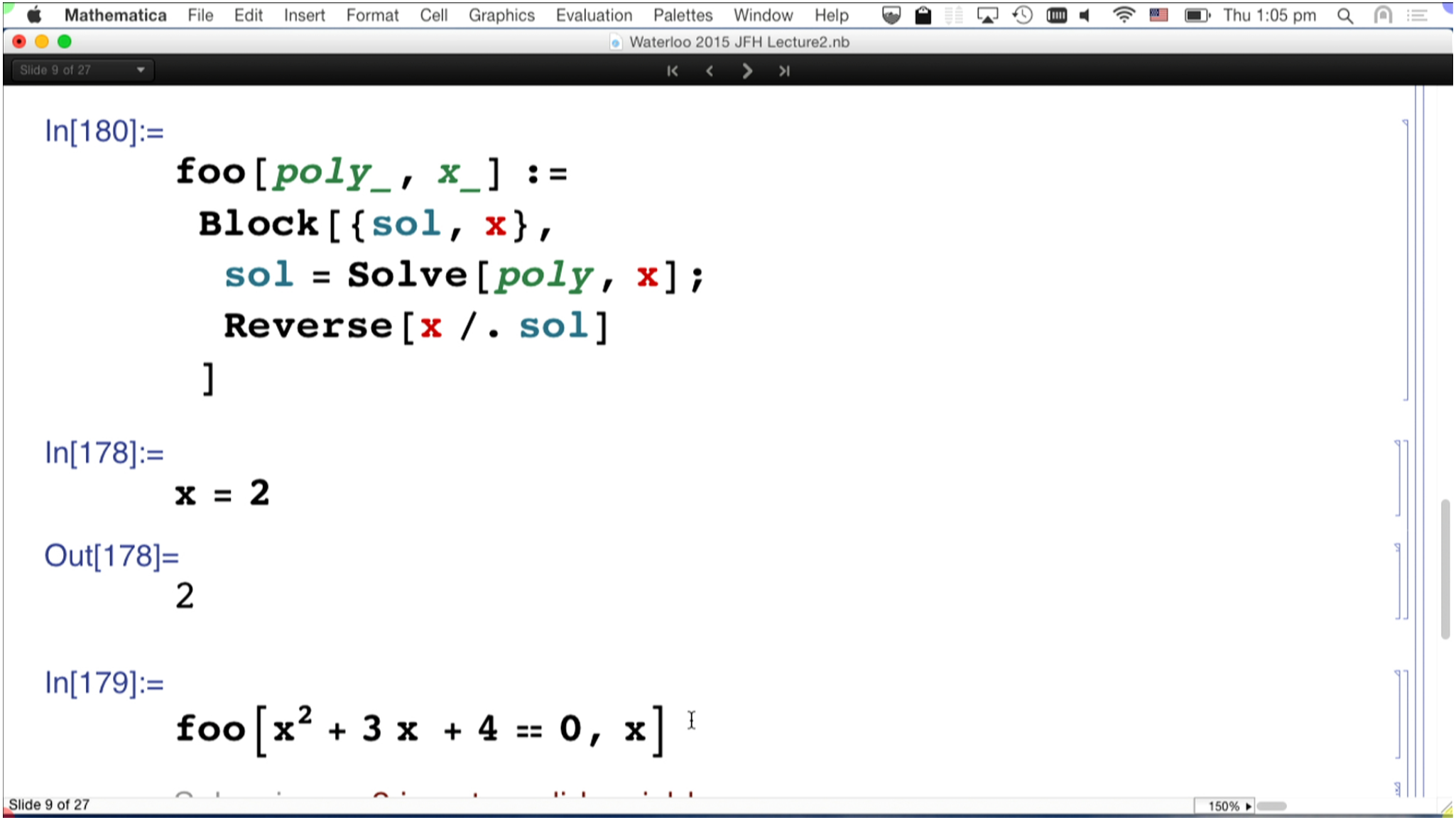
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Slide 9 of 27
In[180]:=
foo[poly_, x_] :=
Block[{sol, x},
sol = Solve[poly, x];
Reverse[x /. sol]
]
In[178]:=
x = 2
Out[178]=
2
In[179]:=
foo[x^2 + 3 x + 4 == 0, x]
Slide 9 of 27
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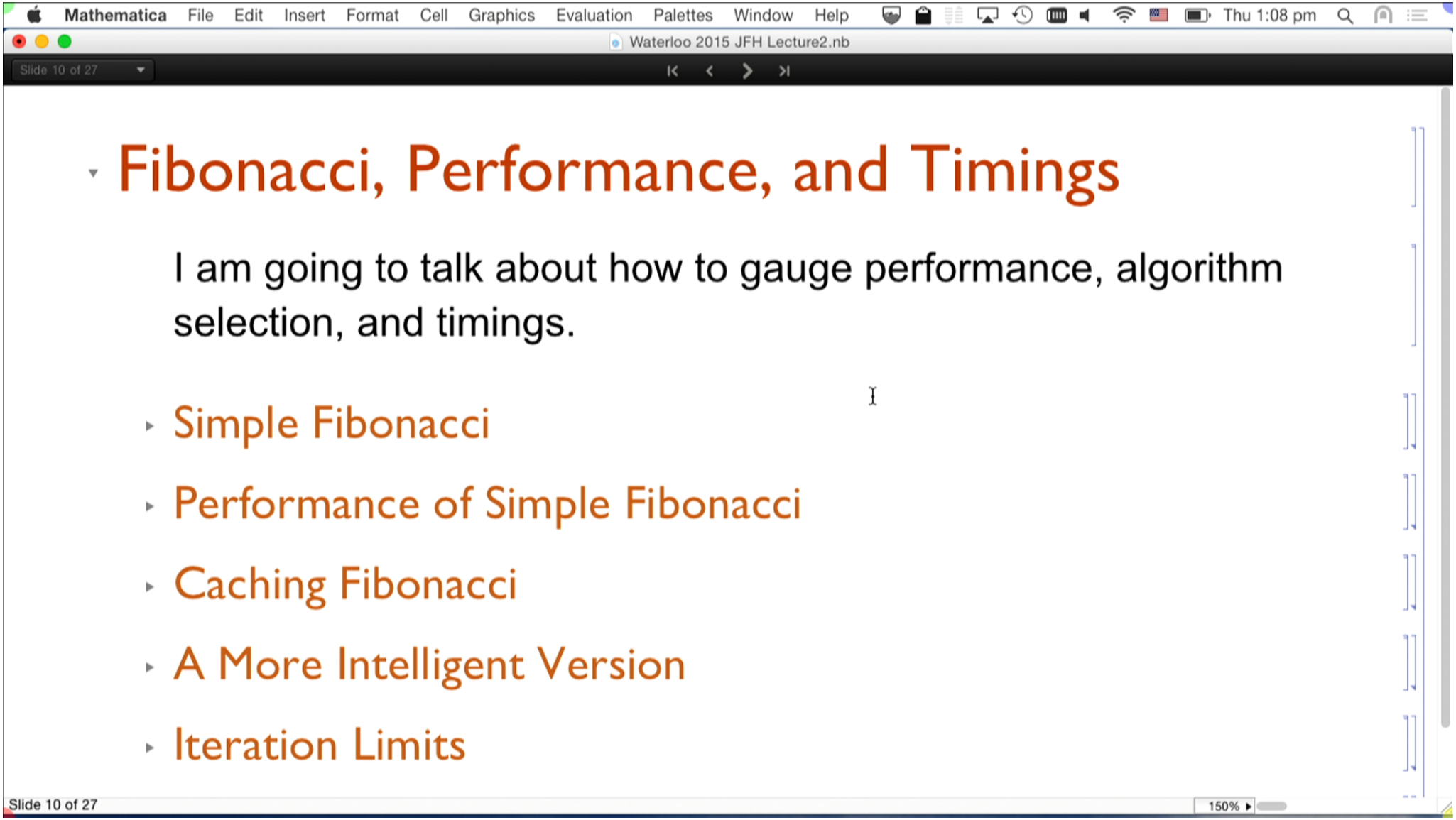
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Thu 1:08 pm
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Slide 10 of 27
Fibonacci, Performance, and Timings
I am going to talk about how to gauge performance, algorithm selection, and timings.
Simple Fibonacci
Performance of Simple Fibonacci
Caching Fibonacci
A More Intelligent Version
Iteration Limits
Slide 10 of 27
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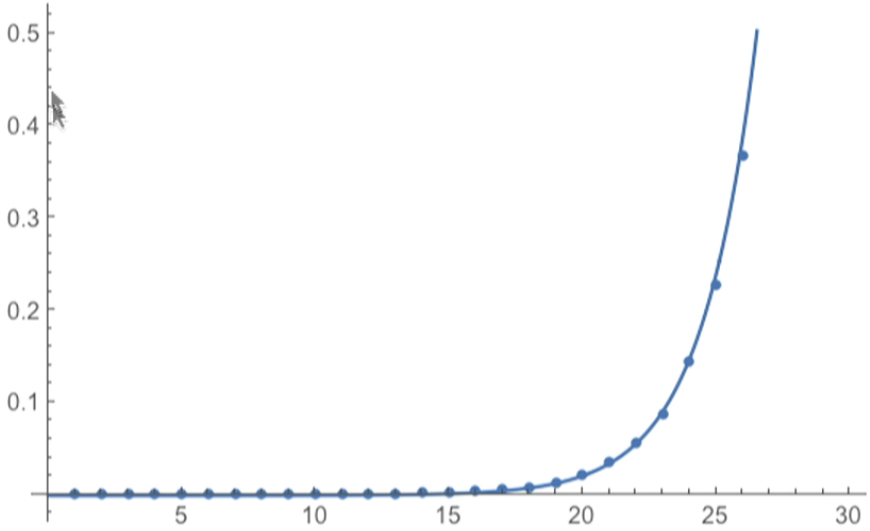
Slide 10 of 27

In[218]:=

```
Show[p1, p2]
```

Out[218]=




```
theFit /. x → 500
```

So our naive method would of course take much much longer

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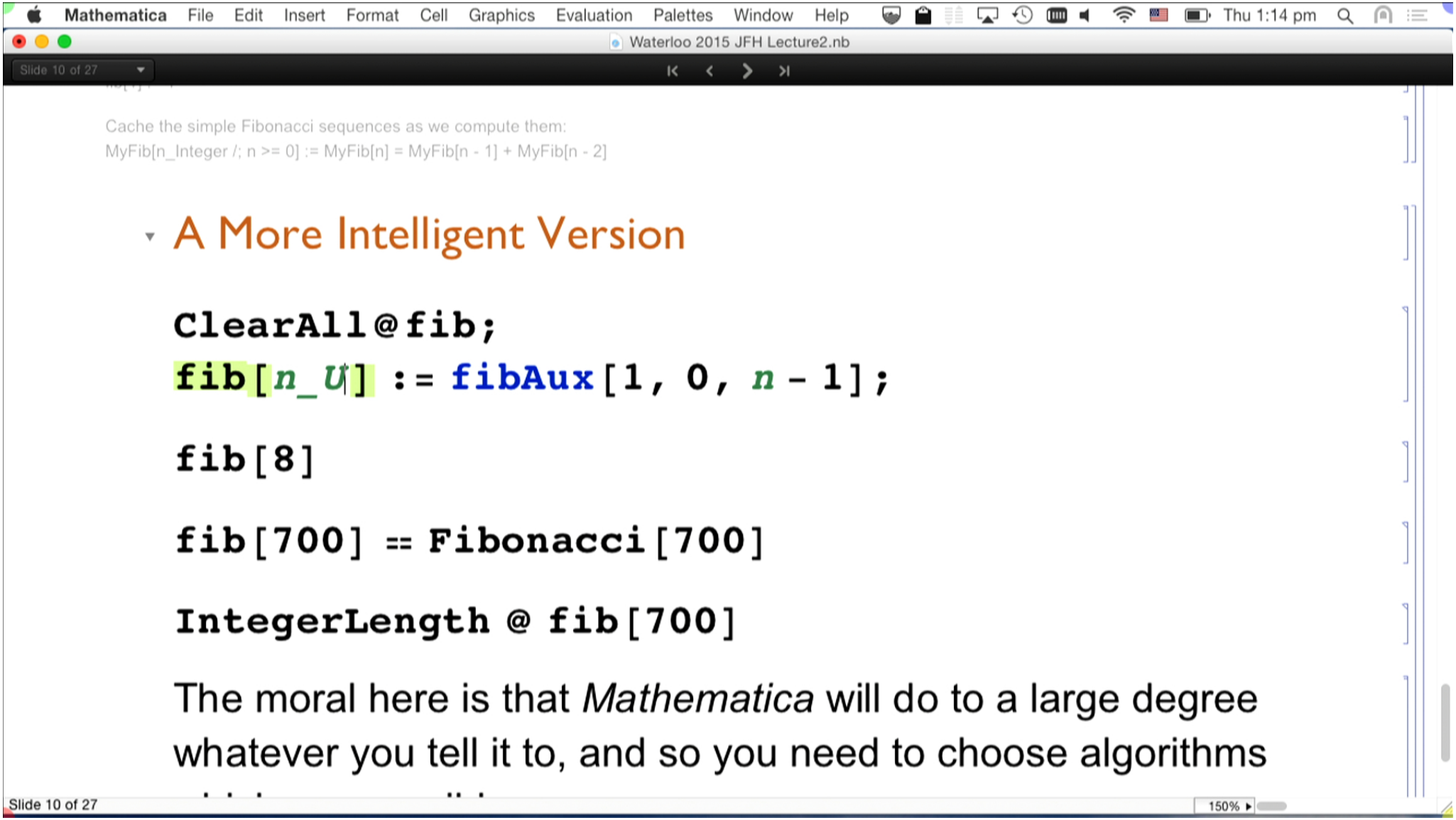

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:14 pm
Waterloo 2015 JFH Lecture2.nb
Slide 10 of 27
Cache the simple Fibonacci sequences as we compute them:
MyFib[n_Integer /; n >= 0] := MyFib[n] = MyFib[n - 1] + MyFib[n - 2]
A More Intelligent Version
ClearAll@fib;
fib[n_U] := fibAux[1, 0, n - 1];
fib[8]
fib[700] == Fibonacci[700]
IntegerLength @ fib[700]
The moral here is that Mathematica will do to a large degree whatever you tell it to, and so you need to choose algorithms
Slide 10 of 27
150%

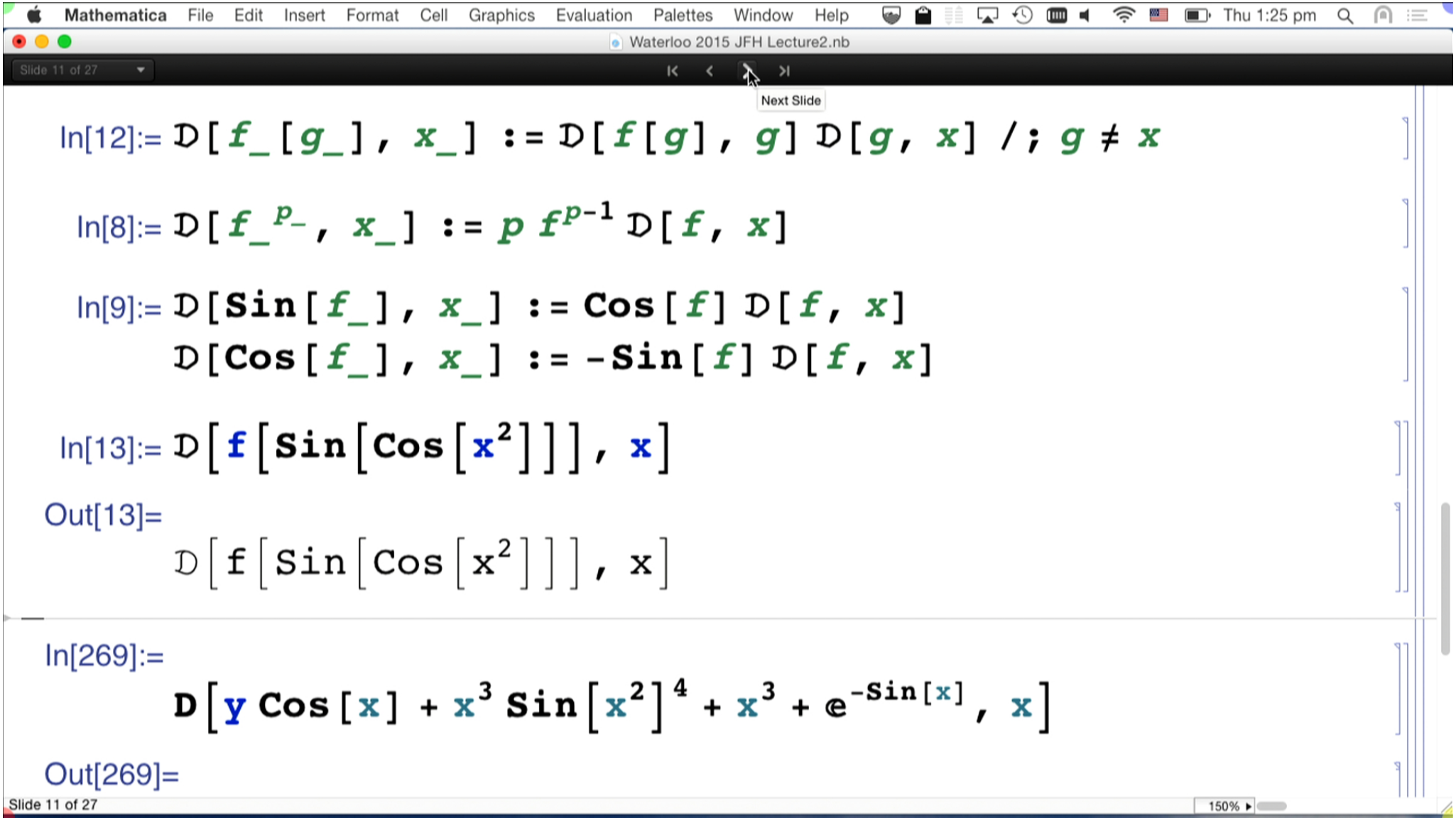
Implementing Differentation

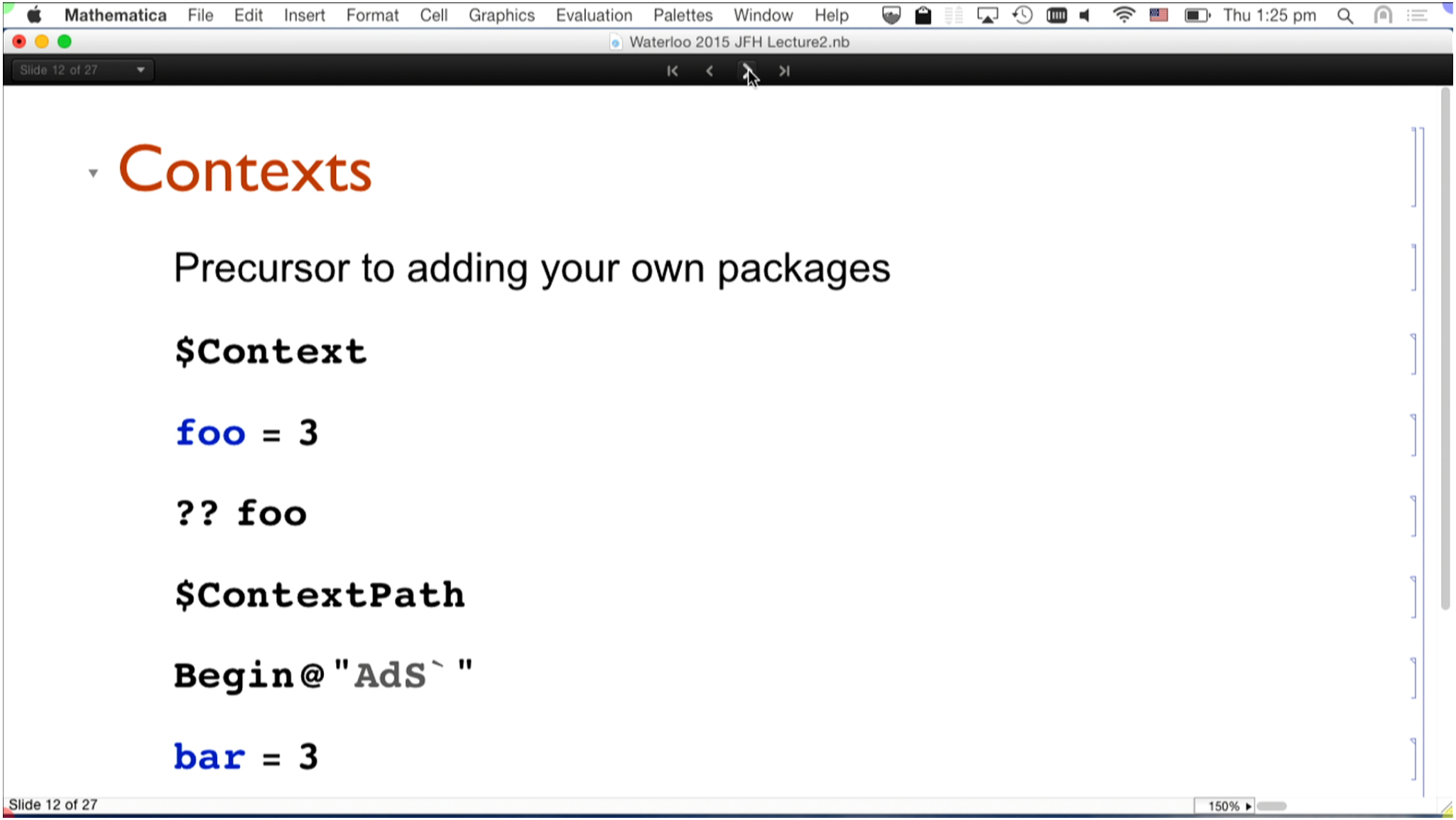
Let us now put much of the previous pattern matching together to implement a simple form of differentiation.

$$\partial_x \left(2\,\text{Sin}[x]^3\, e^{2x}\right)$$

We need: linearity, products, free, x wrt x, exp, log, chain, power, sin, cos

Steps


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:25 pm
Waterloo 2015 JFH Lecture2.nb
Slide 11 of 27
Next Slide
In[12]:= D[f_[g_], x_] := D[f[g], g] D[g, x] /; g ≠ x
In[8]:= D[f_^p_, x_] := p f^(p-1) D[f, x]
In[9]:= D[Sin[f_], x_] := Cos[f] D[f, x]
D[Cos[f_], x_] := -Sin[f] D[f, x]
In[13]:= D[f[Sin[Cos[x^2]]], x]
Out[13]=
D[f[Sin[Cos[x^2]]], x]
In[269]:=
D[y Cos[x] + x^3 Sin[x^2]^4 + x^3 + e^(-Sin[x]), x]
Out[269]=
Slide 11 of 27
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:25 pm
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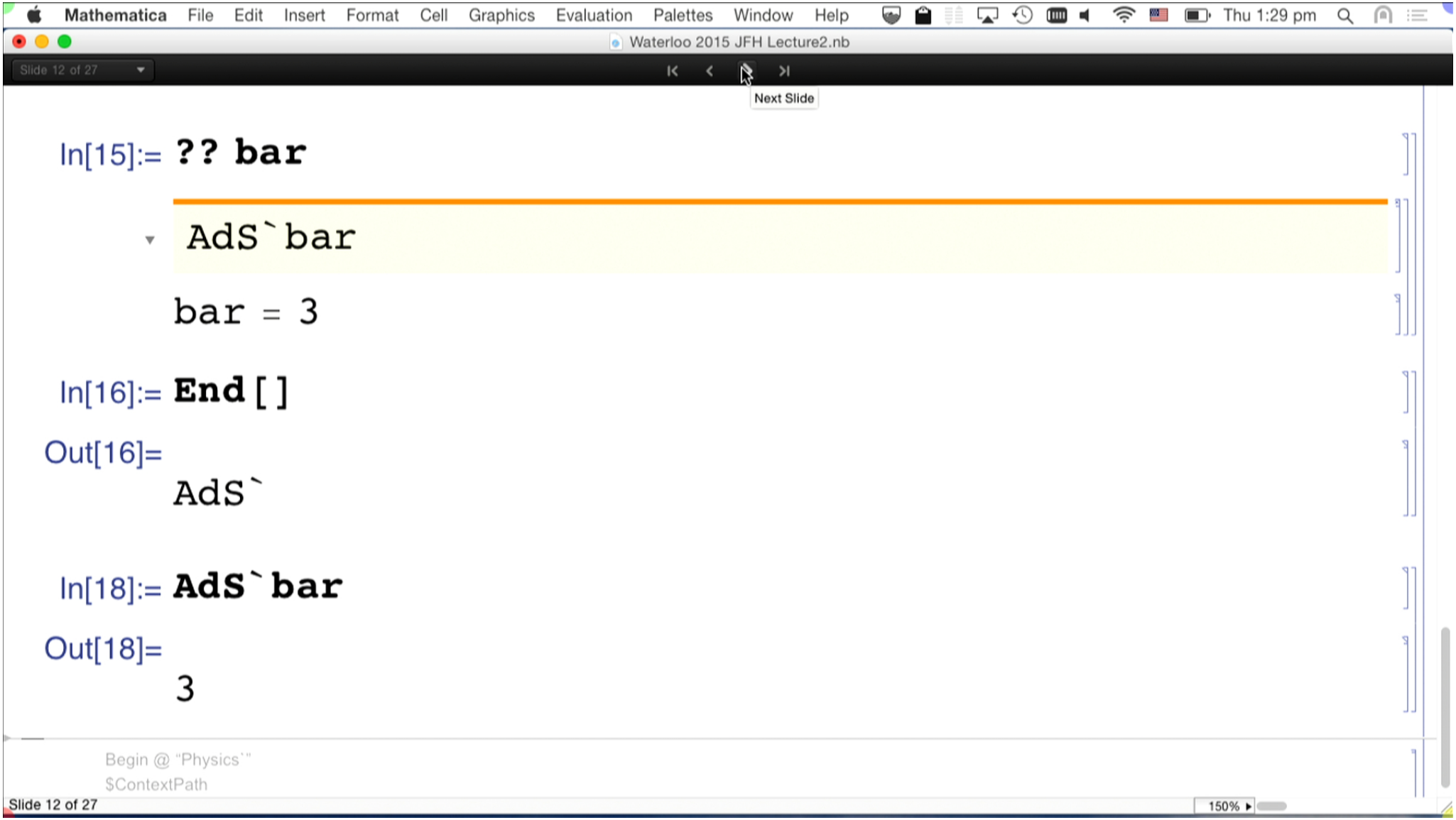
Slide 12 of 27
Contexts
Precursor to adding your own packages
$Context
foo = 3
?? foo
$ContextPath
Begin@"AdS`"
bar = 3
Slide 12 of 27
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:29 pm
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Slide 12 of 27
Next Slide
In[15]:= ?? bar
AdS`bar
bar = 3
In[16]:= End[]
Out[16]=
AdS`
In[18]:= AdS`bar
Out[18]=
3
Begin @ "Physics`"
$ContextPath
Slide 12 of 27
150%

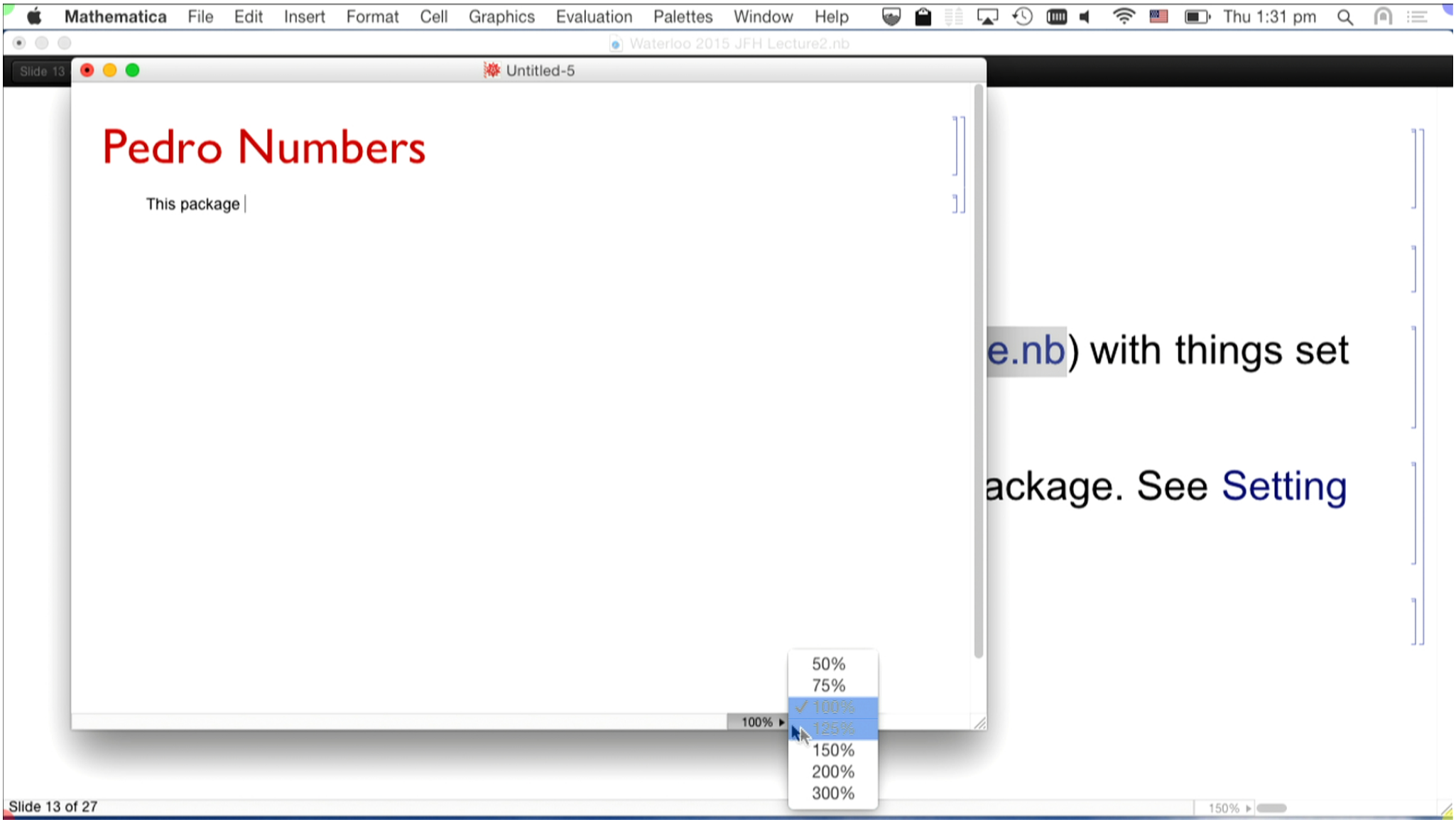
Programming In The Large.

Structure your notebooks for readability!!!

Here is an example notebook (NotationSource.nb) with things set up to program in the large...

If you repeatedly use functionality, create a package. See Setting Up *Mathematica* Packages

Consider Wolfram Workbench.


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:31 pm
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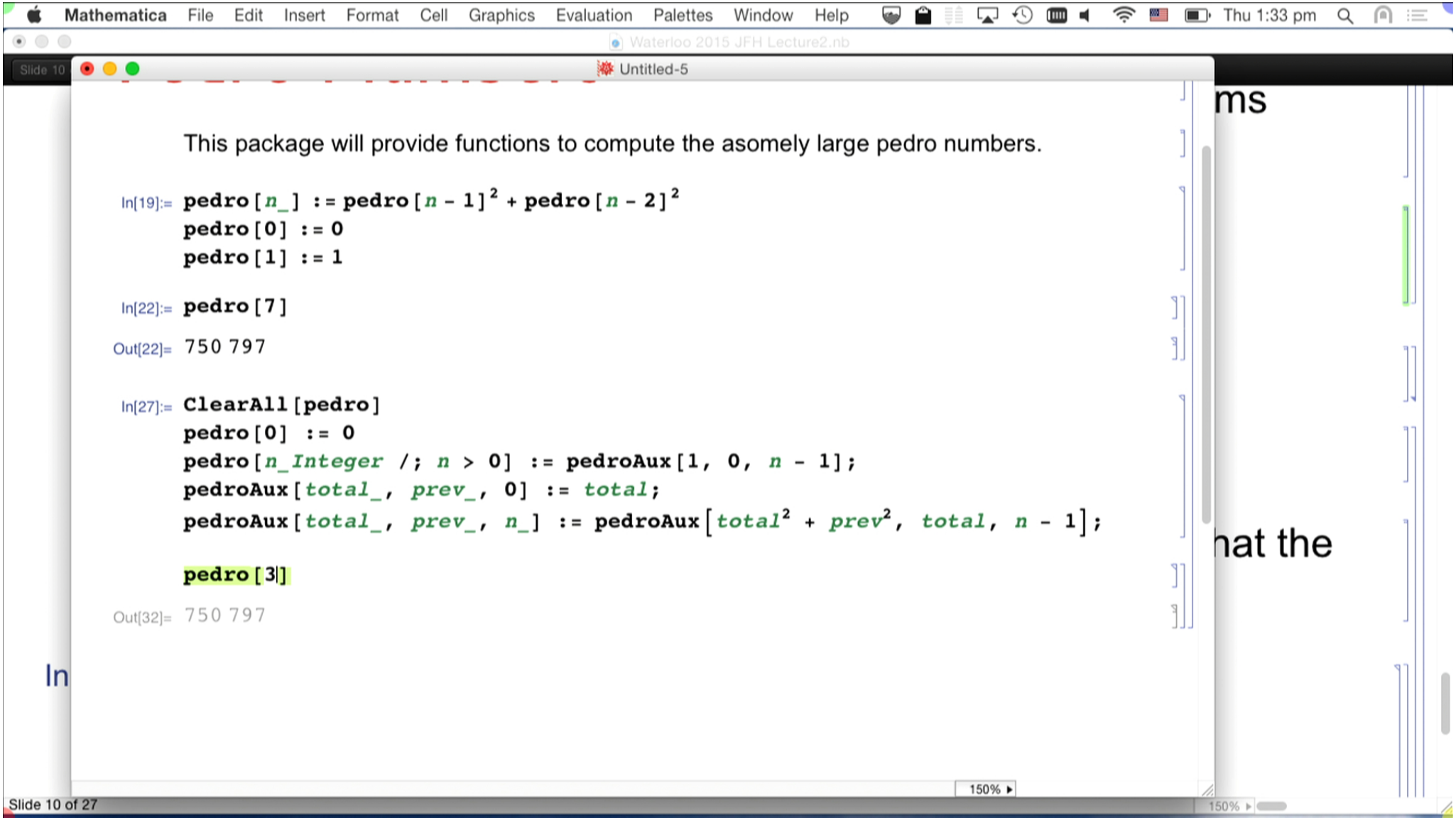
Slide 13
Untitled-5
Pedro Numbers
This package
e.nb) with things set
ackage. See Setting
50%
75%
100%
125%
150%
200%
300%
100%
Slide 13 of 27
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:33 pm
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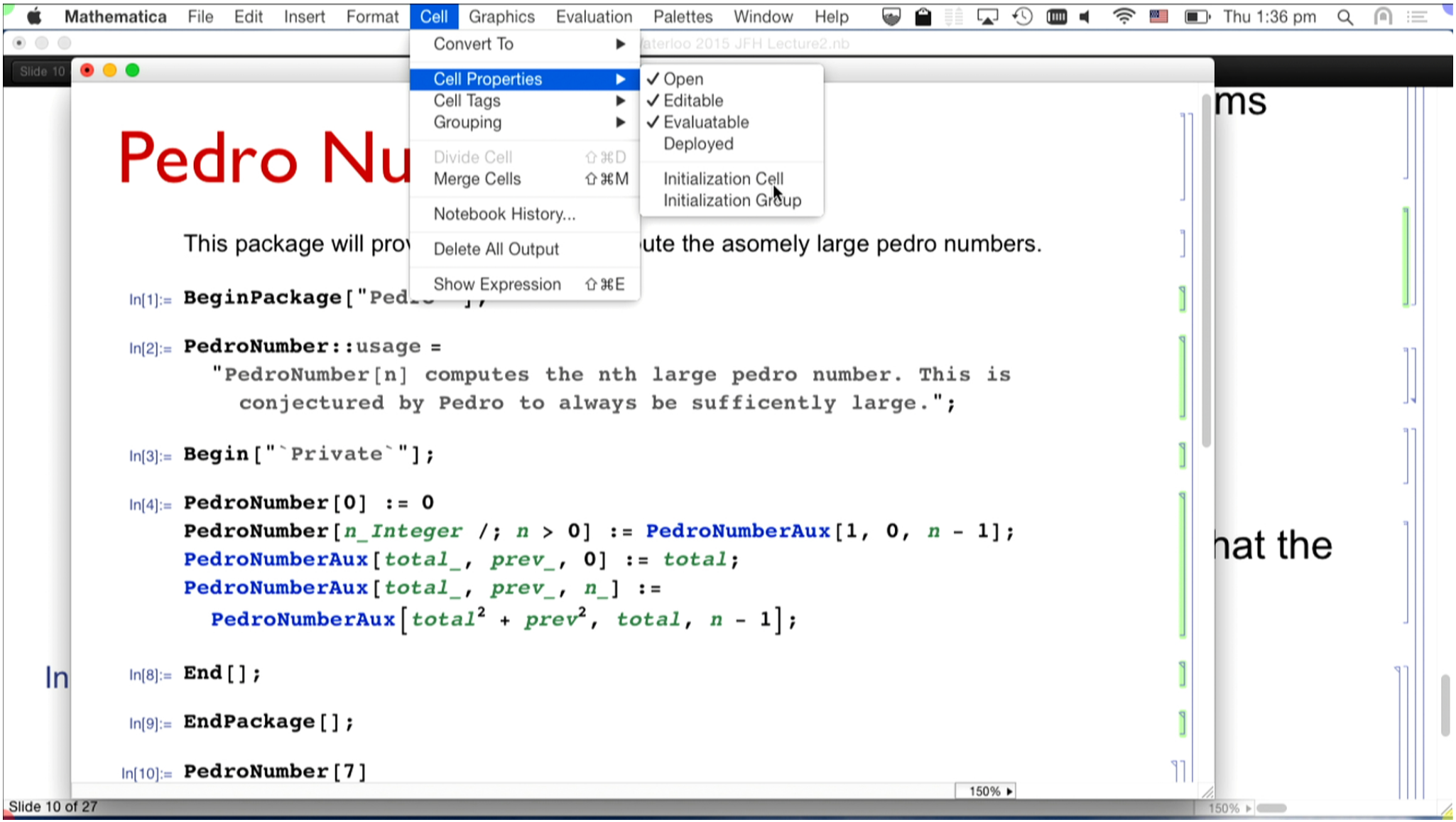
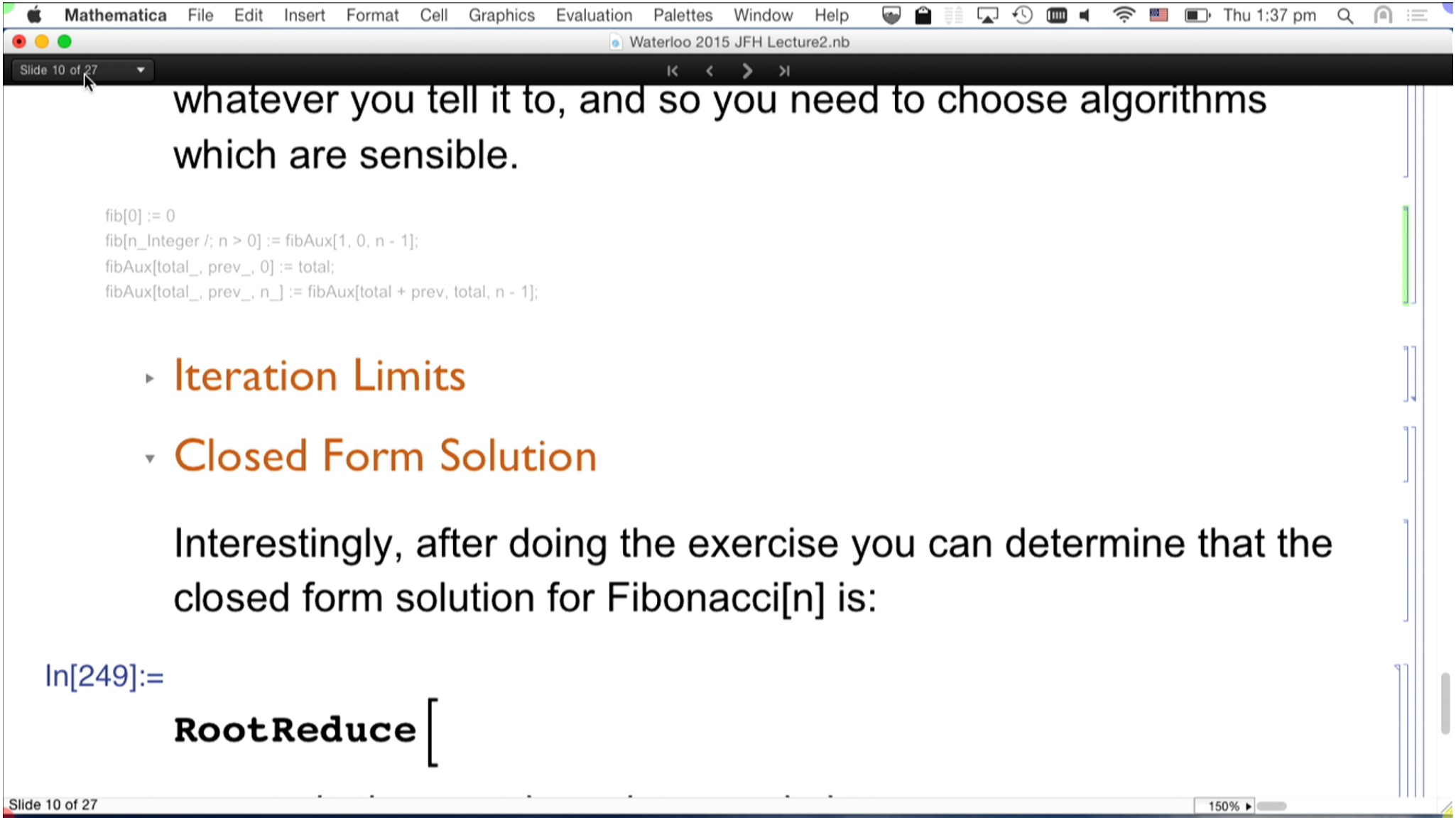
Slide 10
Untitled-5
This package will provide functions to compute the asomely large pedro numbers.
In[19]:= pedro[n_] := pedro[n - 1]^2 + pedro[n - 2]^2
pedro[0] := 0
pedro[1] := 1
In[22]:= pedro[7]
Out[22]= 750 797
In[27]:= ClearAll[pedro]
pedro[0] := 0
pedro[n_Integer /; n > 0] := pedroAux[1, 0, n - 1];
pedroAux[total_, prev_, 0] := total;
pedroAux[total_, prev_, n_] := pedroAux[total^2 + prev^2, total, n - 1];
pedro[3]
Out[32]= 750 797
150%
Slide 10 of 27
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:36 pm
Convert To
Cell Properties
Cell Tags
Grouping
Divide Cell ⇧⌘D
Merge Cells ⇧⌘M
Notebook History...
Delete All Output
Show Expression ⇧⌘E
Open
Editable
Evaluatable
Deployed
Initialization Cell
Initialization Group
Pedro Nu
This package will prov ute the asomely large pedro numbers.
In[1]:= BeginPackage["Ped
In[2]:= PedroNumber::usage =
"PedroNumber[n] computes the nth large pedro number. This is
conjectured by Pedro to always be sufficently large.";
In[3]:= Begin["`Private`"];
In[4]:= PedroNumber[0] := 0
PedroNumber[n_Integer /; n > 0] := PedroNumberAux[1, 0, n - 1];
PedroNumberAux[total_, prev_, 0] := total;
PedroNumberAux[total_, prev_, n_] :=
PedroNumberAux[total^2 + prev^2, total, n - 1];
In[8]:= End[];
In[9]:= EndPackage[];
In[10]:= PedroNumber[7]
150%
Slide 10 of 27
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 1:37 pm
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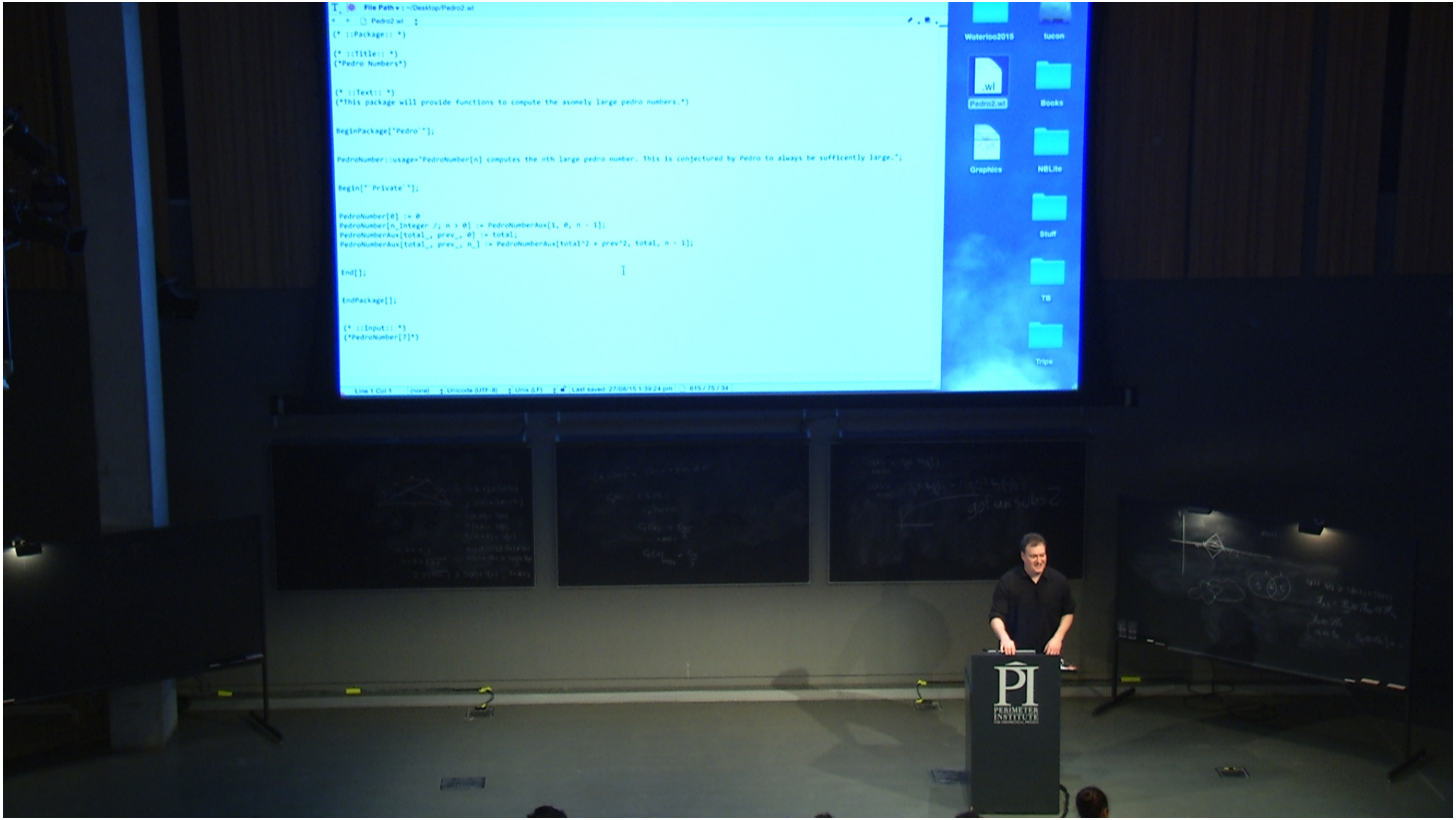
Slide 10 of 27
whatever you tell it to, and so you need to choose algorithms which are sensible.
fib[0] := 0
fib[n_Integer /; n > 0] := fibAux[1, 0, n - 1];
fibAux[total_, prev_, 0] := total;
fibAux[total_, prev_, n_] := fibAux[total + prev, total, n - 1];
Iteration Limits
Closed Form Solution
Interestingly, after doing the exercise you can determine that the closed form solution for Fibonacci[n] is:
In[249]:=
RootReduce [
Slide 10 of 27
150%

Summary:

- Choose your algorithims wisely
- For matching, check the FullForm of expressions
- Attributes effect the Evaluation and Pattern matching of expressions
- You can exectue things in parallel
- You can compile Code to the WVM, or to a C target
- You can link to external C libraries


File Path : ~/Desktop/Pedro2.wl
Pedro2.wl
(* ::Package:: *)
(* ::Title:: *)
(*Pedro Numbers*)
(* ::Text:: *)
(*This package will provide functions to compute the asomely large pedro numbers.*)
BeginPackage["Pedro`"];
PedroNumber::usage="PedroNumber[n] computes the nth large pedro number. This is conjectured by Pedro to always be sufficently large.";
Begin["`Private`"];
PedroNumber[0] := 0
PedroNumber[n_Integer /; n > 0] := PedroNumberAux[1, 0, n - 1];
PedroNumberAux[total_, prev_, 0] := total;
PedroNumberAux[total_, prev_, n_] := PedroNumberAux[total^2 + prev^2, total, n - 1];
End[];
EndPackage[];
(* ::Input:: *)
(*PedroNumber[7]*)
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tucon
Pedro2.wl
Books
Graphics
NBLite
Stuff
TB
Trips
PERIMETER INSTITUTE