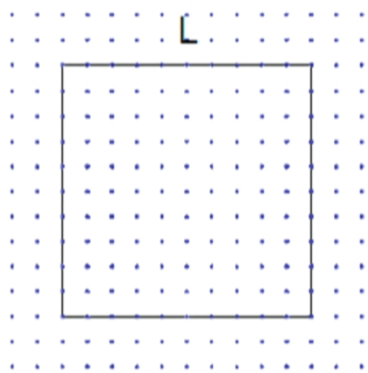
Title: TBA

Date: Aug 27, 2015 11:00 AM

URL: http://pirsa.org/15080050

Abstract: TBA

Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
Similar to phonons in a solid

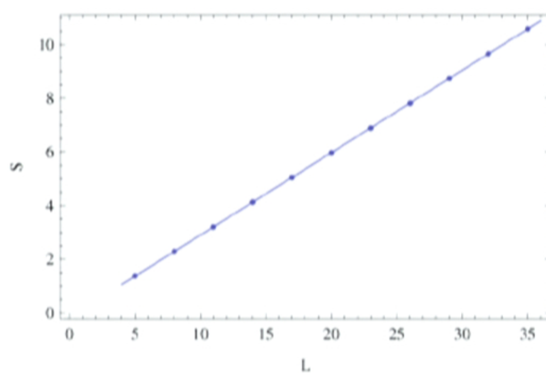


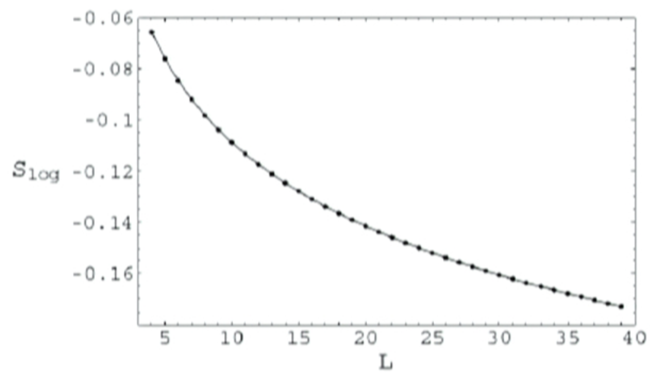
$$H=\frac{1}{2}\int d^2x\left(\dot{\phi}(x)^2+(\nabla\phi(x))^2\right)$$

$$\rightarrow H=\frac{1}{2}\sum_i \epsilon^2\left(\dot{\phi}_i{}^2+\sum_{j\sim i}\frac{(\phi_i-\phi_j)^2}{\epsilon^2}\right)$$

For interacting spin systems the Hilbert space dimension grows as 2^N

For coupled Harmonic oscillators we have only to diagonalize matrices of $N\times N$

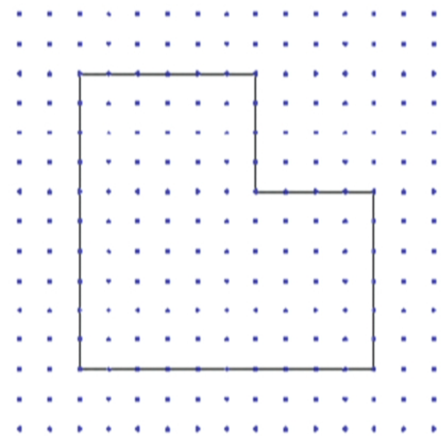




S=.075 (4 L/ε)-0.047 Log[L/ε] +const=.075 (perimeter/ε)-0.047 Log[L/ ε]+const

We have an «area» term and a logarithmic correction. These are divergent as ε ->0

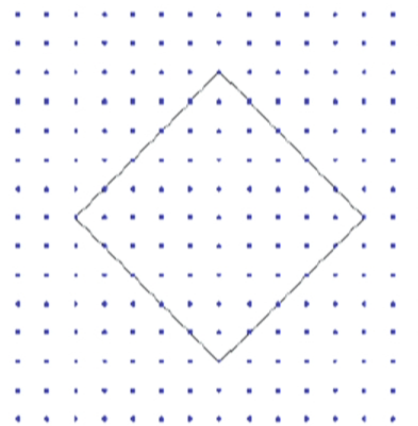
S=.075 (perimeter/ε)-(6/4) 0.047 Log[L/ε]+const



The same «area» term. A logarithmic coefficient growing
with the number of vertices.
(All vertices have the same angle S(A)=S(-A) for a global pure state)

In general:

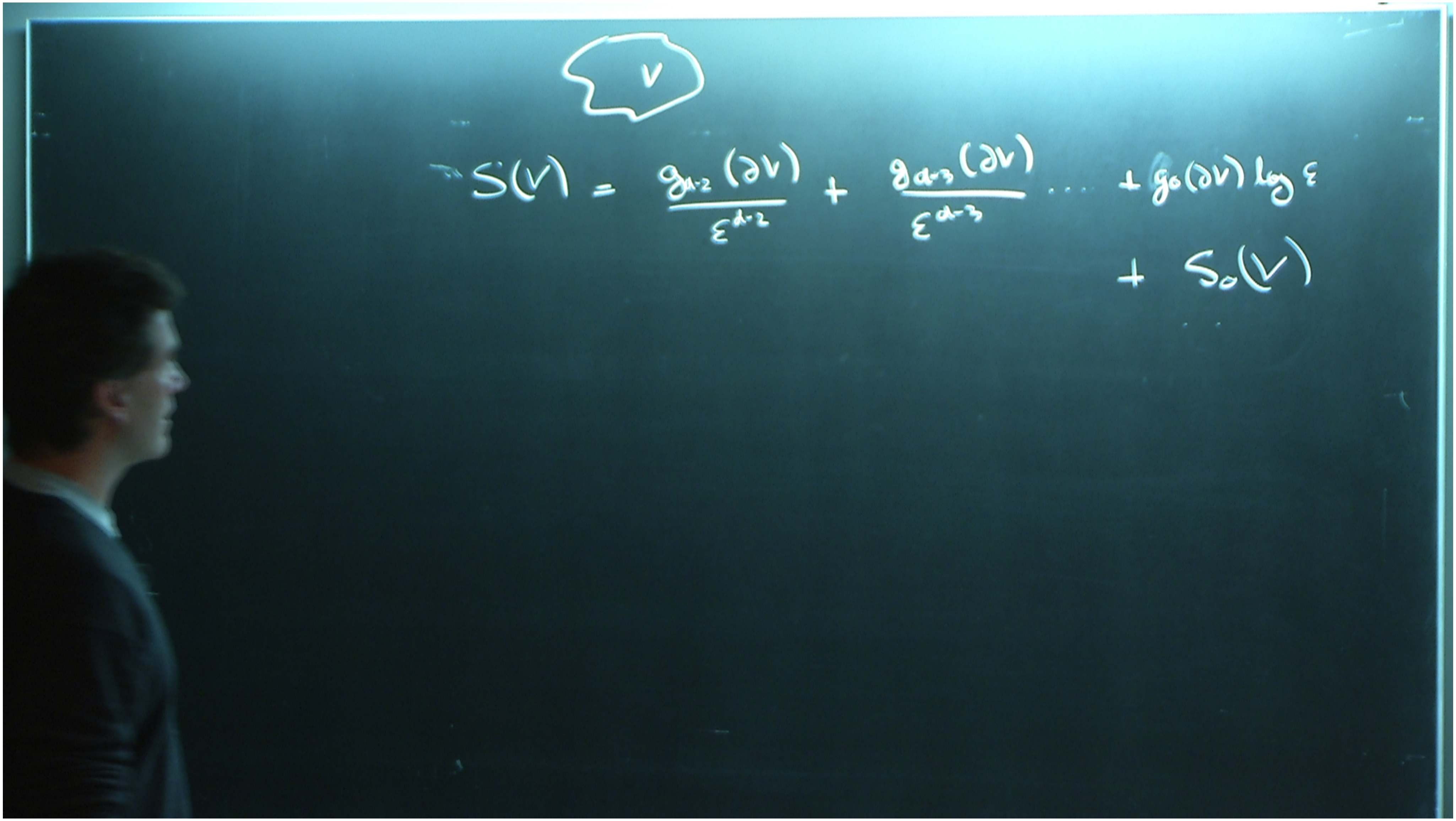
$$S(A) = c_1 \,(\mathbf{perimeter}/\epsilon) - \sum_{\mathbf{vertices}} c_{\log}(\theta)\log(R/\epsilon) + \mathbf{const}$$

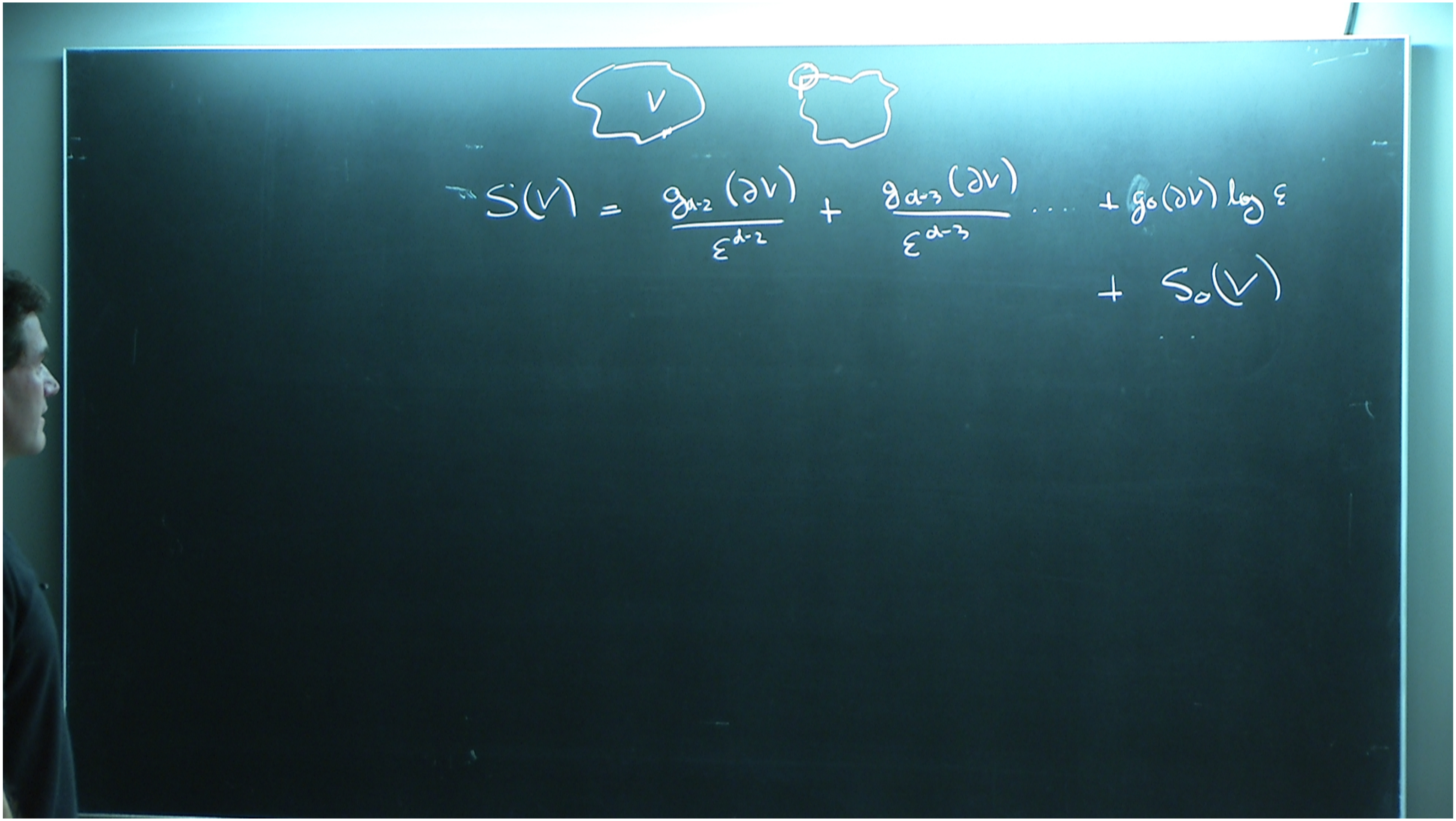


S=.085 (perimeter/ε)-0.047 Log[L/ε]+const

Bad: area term does not have the rotational
symmetry of the theory in the continuum limit

Good: the logarithmic term does not notice
the lattice


V
$S(V) = \frac{g_{d-2}(\partial V)}{\varepsilon^{d-2}} + \frac{g_{d-3}(\partial V)}{\varepsilon^{d-3}} \cdots + g_0(\partial V) \log \varepsilon$
$+ S_0(V)$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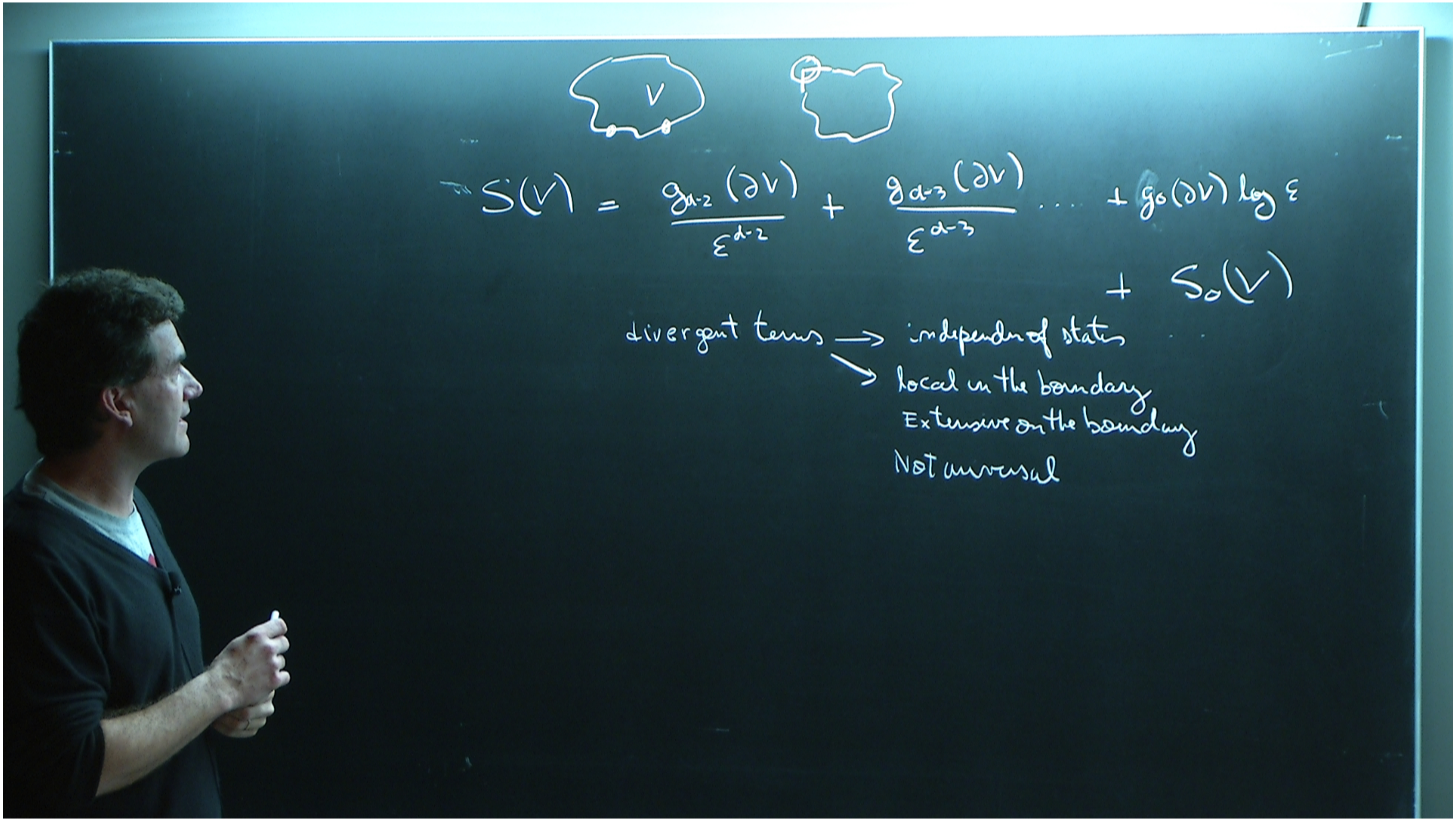
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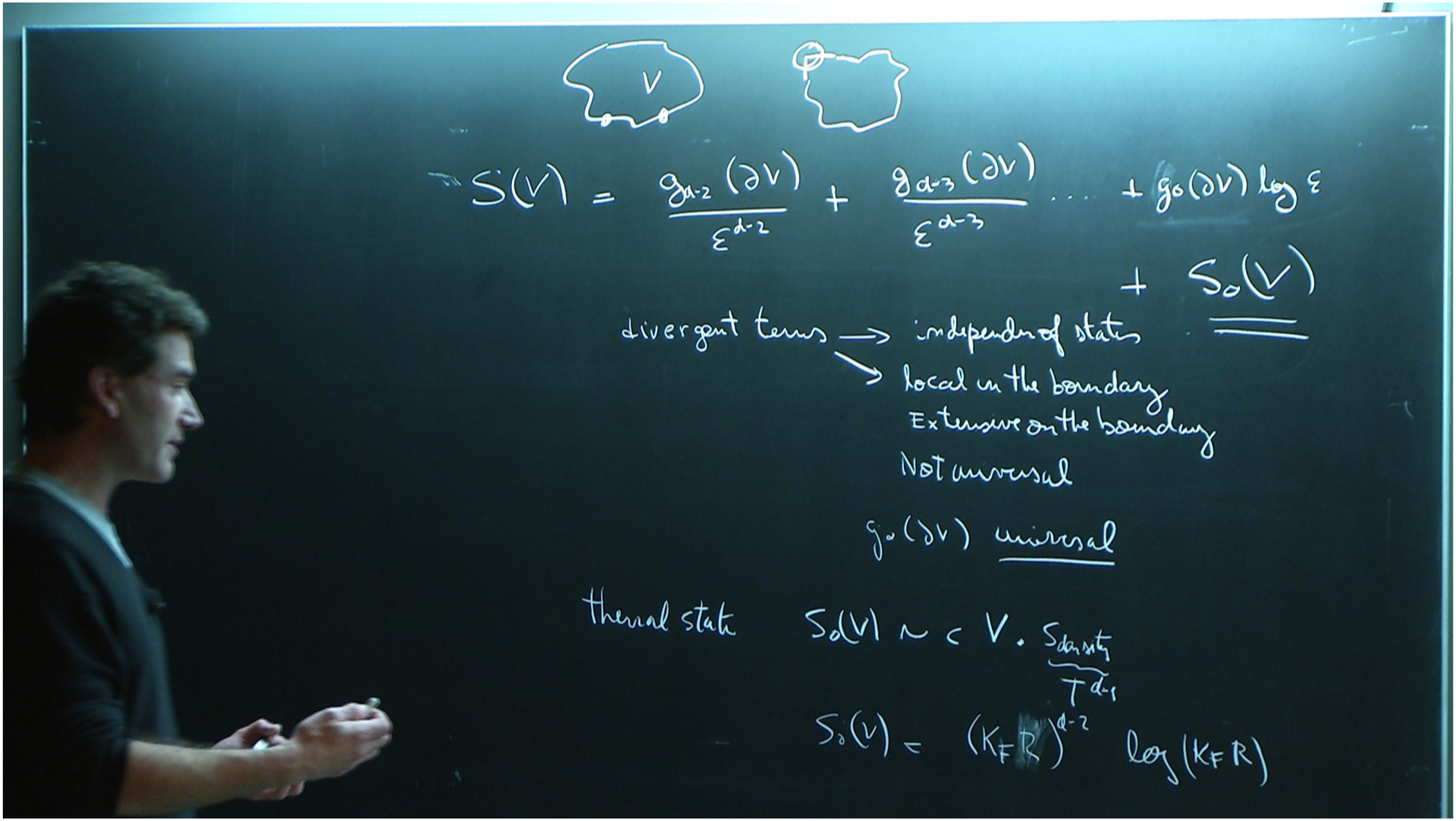
$+ S_0(V)$
divergent terms → independent of states
→ local in the boundary
Extensive on the boundary
Not universal



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divergent terms → independent of states
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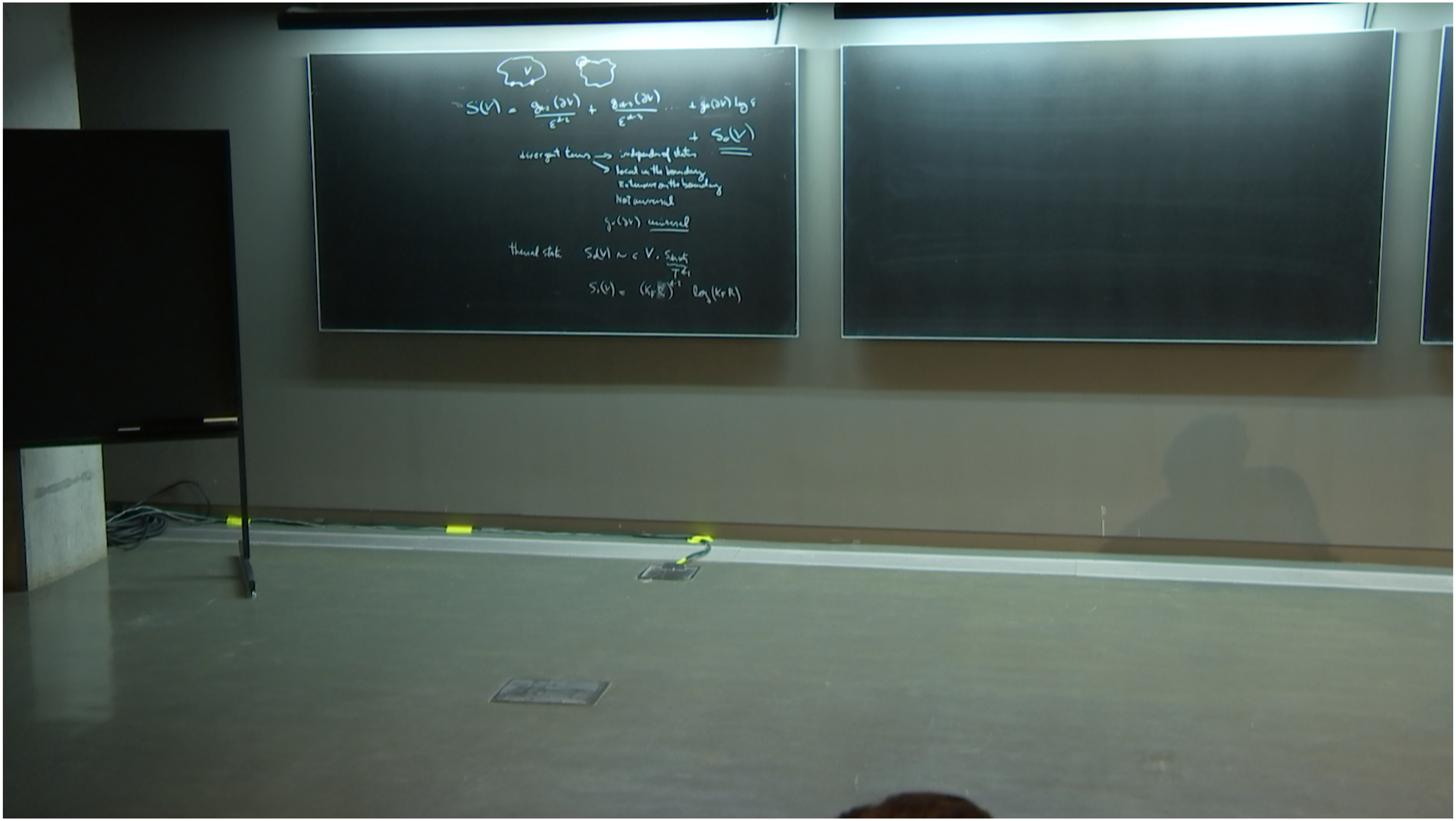
Extensive on the boundary
Not universal
$g_0(\partial V)$ universal
thermal state $S_0(V) \sim c\, V \cdot \frac{s_{density}}{T^{d-1}}$
$S_0(V) = (K_F R)^{d-2} \log(K_F R)$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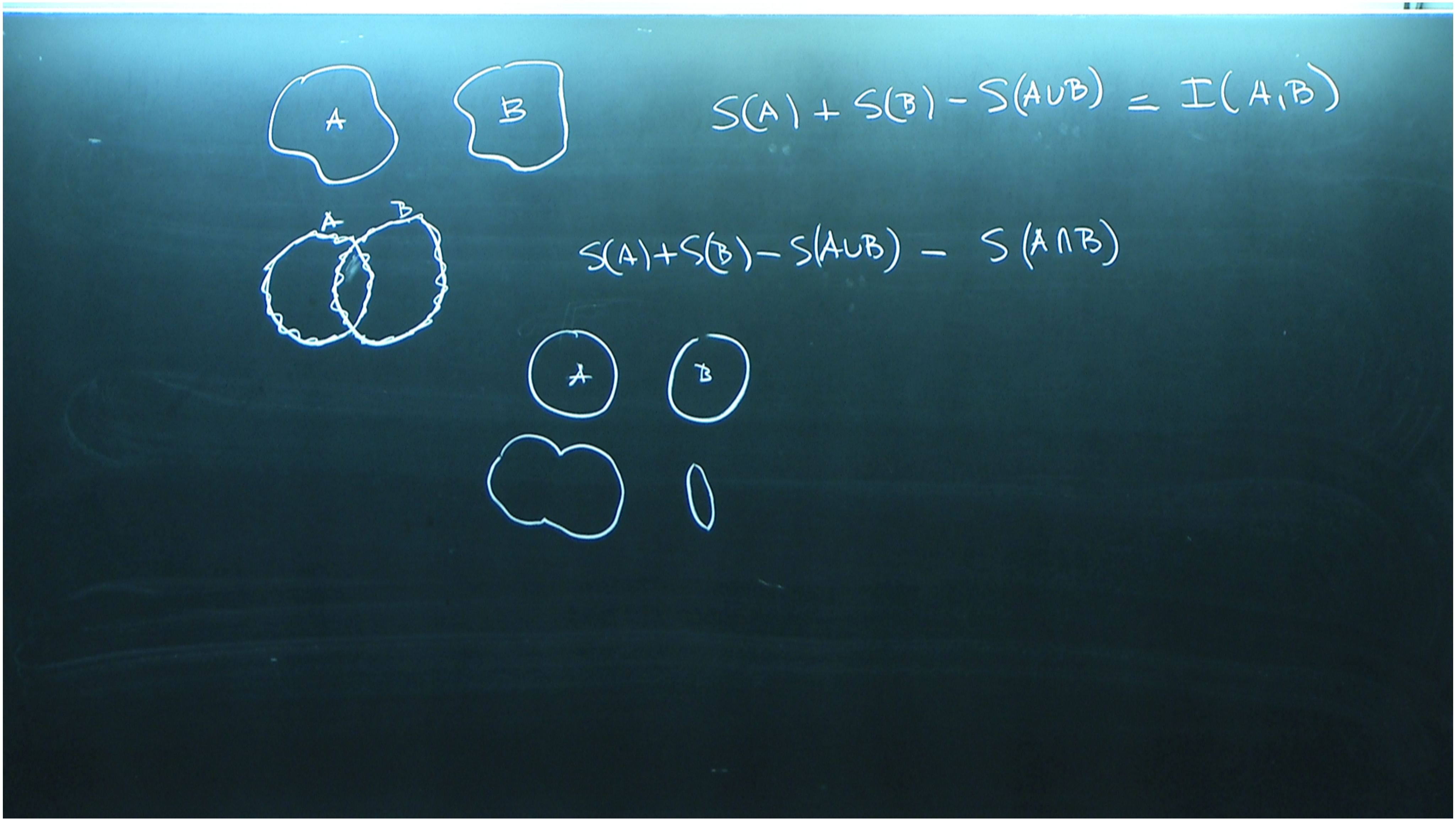
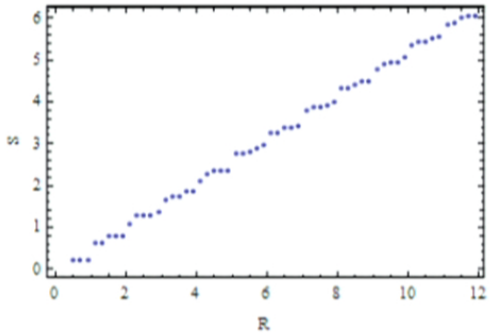
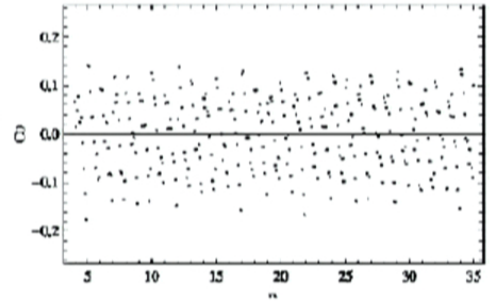
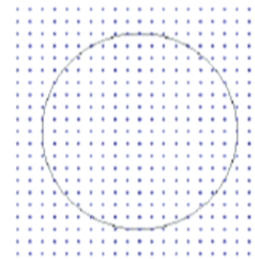

A
B
S(A) + S(B) - S(A∪B) = I(A,B)
A
B
S(A)+S(B)-S(A∪B) - S(A∩B)
A
B

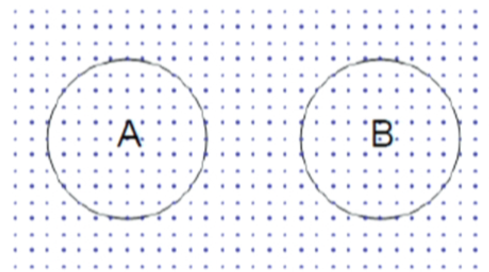
How to extract unambiguous information from the finite term?





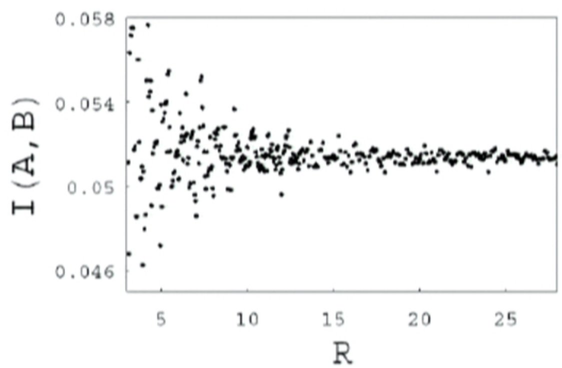


Circles in a square lattice (no log term): $S(R) = c_1\, R + c_0$



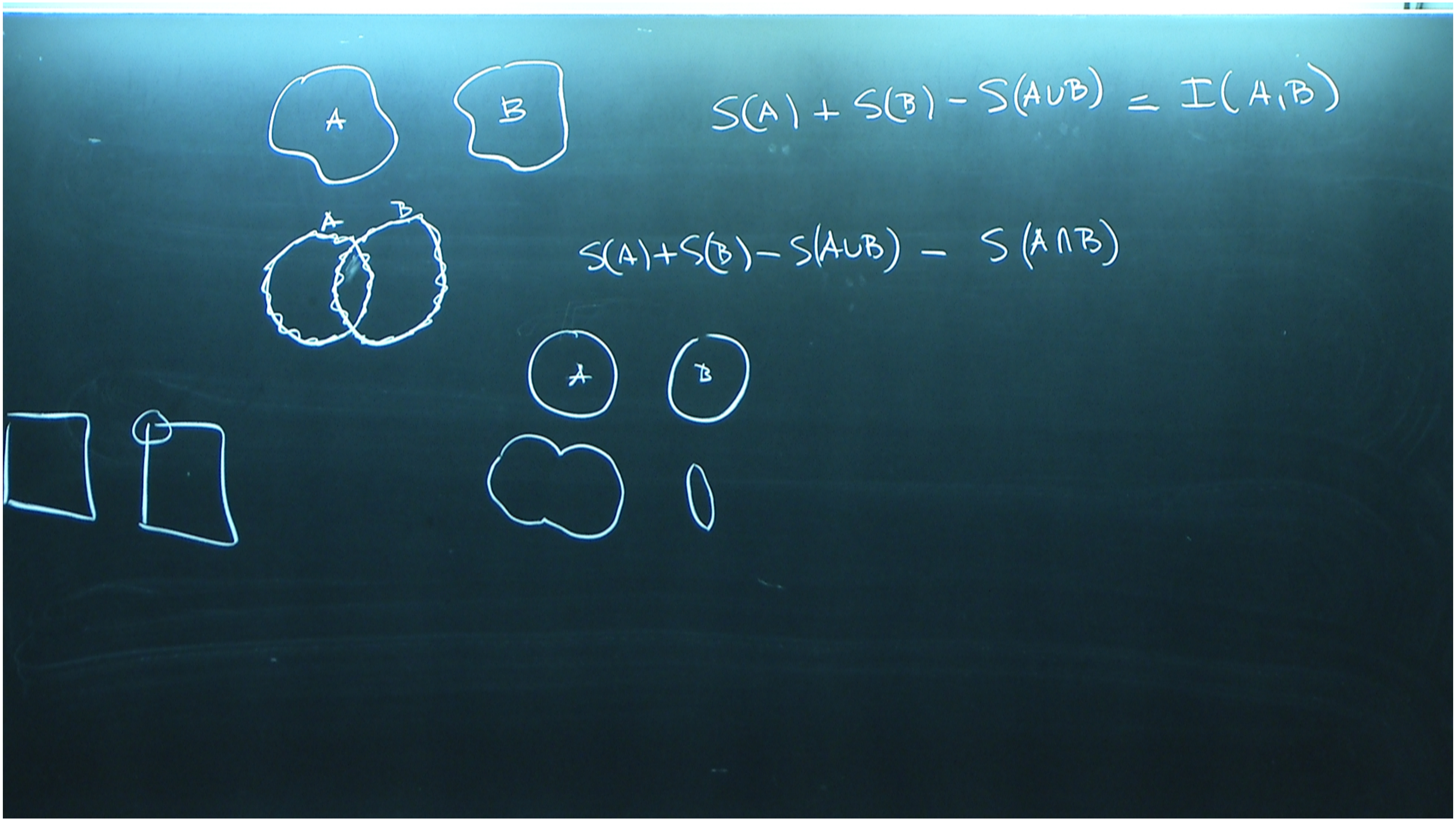
Mutual information $I(A,B) = S(A) + S(B) - S(A \cup B)$

The boundary divergences cancel out in the combination.



Very little large distance entanglement
Compare I(A,B)=0.05 with log(2)=0.69
Less than 1/10 bit for infinitely many degree of freedom!

A lot of short distance entanglement:
I(A,B) diverges when A and B touch each other.
This reflects the locality of the theory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B
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A
B
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B

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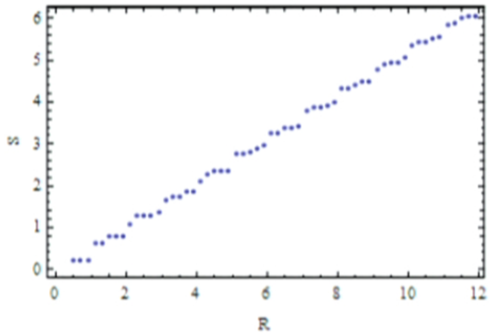
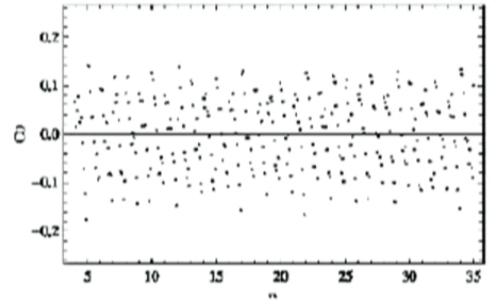
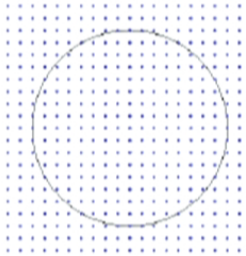
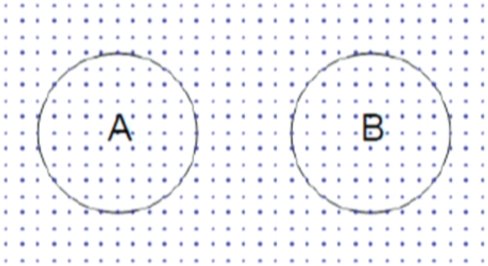






Circles in a square lattice (no log term): $S(R) = c_1 R + c_0$



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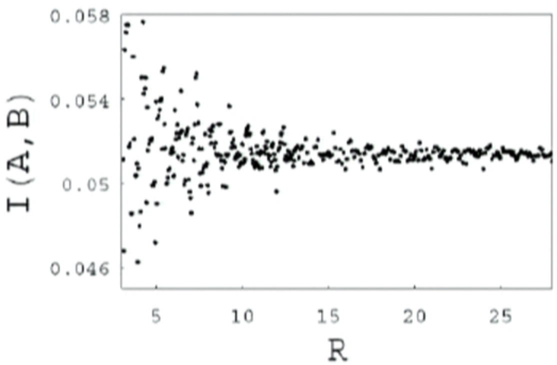

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Properties of entropy

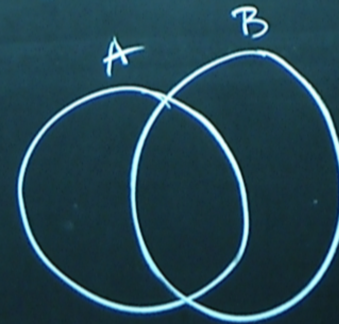
additivity $\quad \rho_A \otimes \rho_B \qquad S(AB) = S(A) + S(B)$

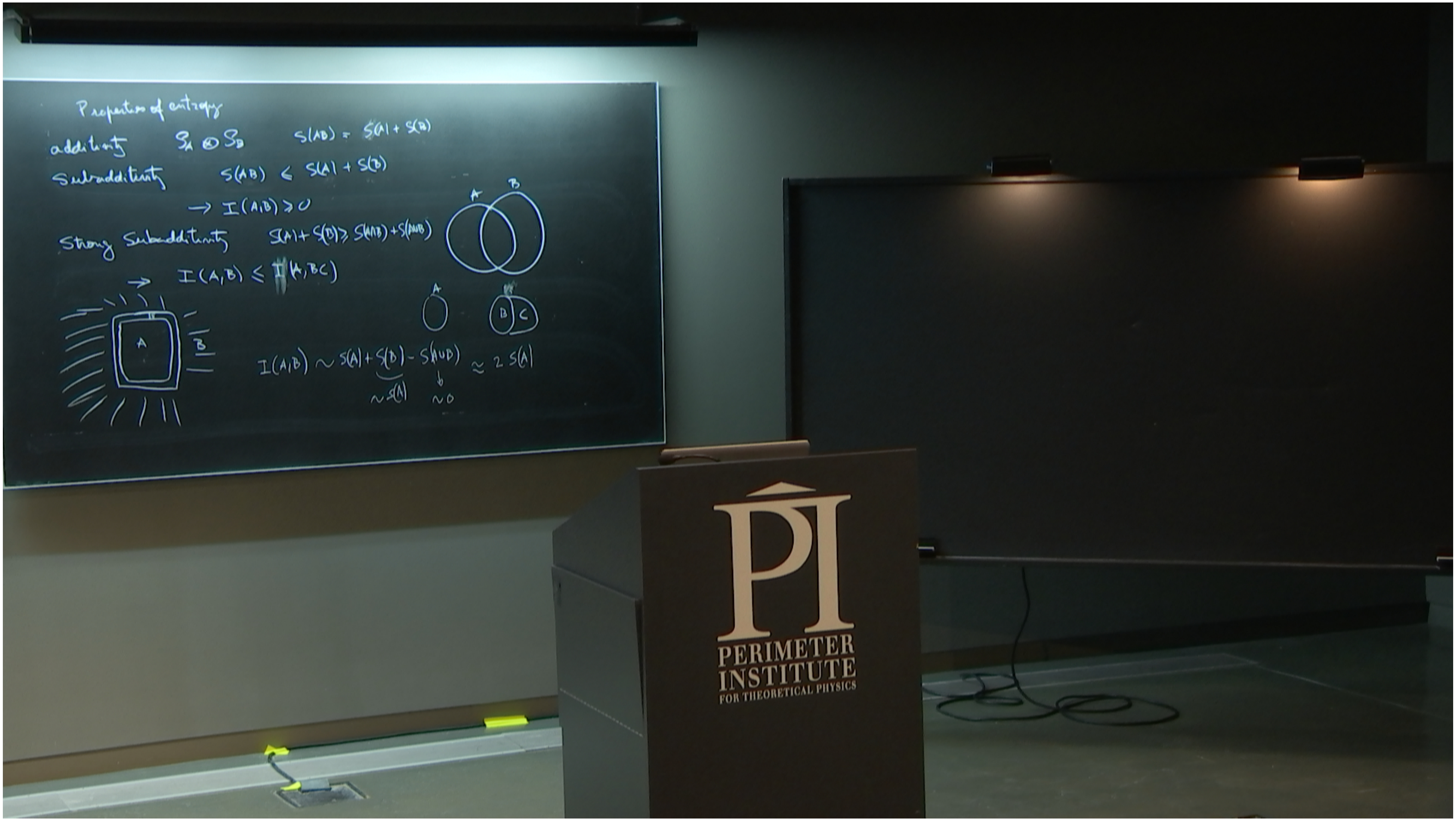
Subadditivity $\qquad S(AB) \leq S(A) + S(B)$

$\rightarrow I(A,B) \geq 0$

Strong Subadditivity $\qquad S(A) + S(B) \geq S(A \cap B) + S(A \cup B)$

$\rightarrow I(A,B) \leq S(A,BC)$




Properties of entropy
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Strong Subadditivity $S(A) + S(B) \geq S(A \cap B) + S(A \cup B)$
$\rightarrow I(A,B) \leq I(A,BC)$
A
B
C
$I(A,B) \sim S(A) + S(B) - S(A \cup B) \approx 2\,S(A)$
$\sim S(A)$
~ 0
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$$I(A,\varepsilon) \sim c\,\frac{\text{Area}}{\varepsilon^{d-2}} + \dots$$

A —— L —— B

$I(A,B) \sim \langle \phi(0)\phi(L)\rangle^2$ Cardy (2013)

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free Scalar field massless 2+1

free Maxwell field in 2+1

$$I \sim \langle \phi(0)\phi(L)\rangle^2 \sim \frac{1}{L^2}$$

$$I \sim \langle F_{\mu\nu}(0)\, F^{\mu\nu}(L)\rangle^2 \sim \frac{1}{L^6}$$

$$F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho}\,\partial^\rho\phi$$

$$E_i = \partial_i\phi$$

$$B = \partial_0\phi$$

ϕ

$$I_{gauge} \leq I_{scalar}$$

$$I(A,\varepsilon) \sim c\,\frac{\text{Area}}{\varepsilon^{d-2}} + \ldots$$

(A) L (B)

$$I(A,B) \sim c\,\boxed{\langle \phi(0)\phi(L)\rangle^2}$$

Cardy (2013)

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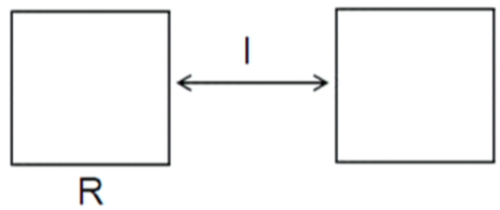
$$I_{gauge} \leq I_{scalar}$$

$$E_i = \partial_i\phi$$

ϕ

$$B = \partial_0\phi$$



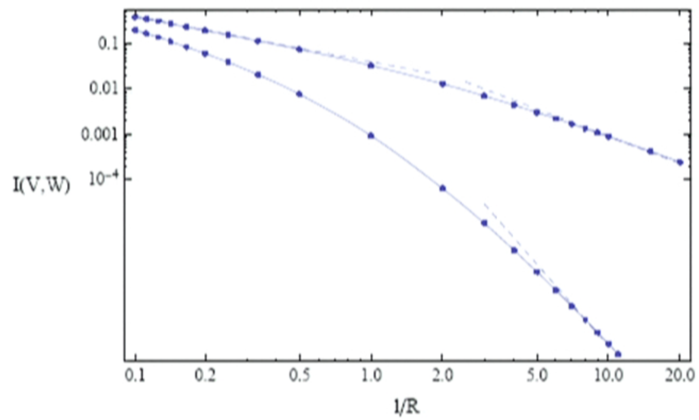
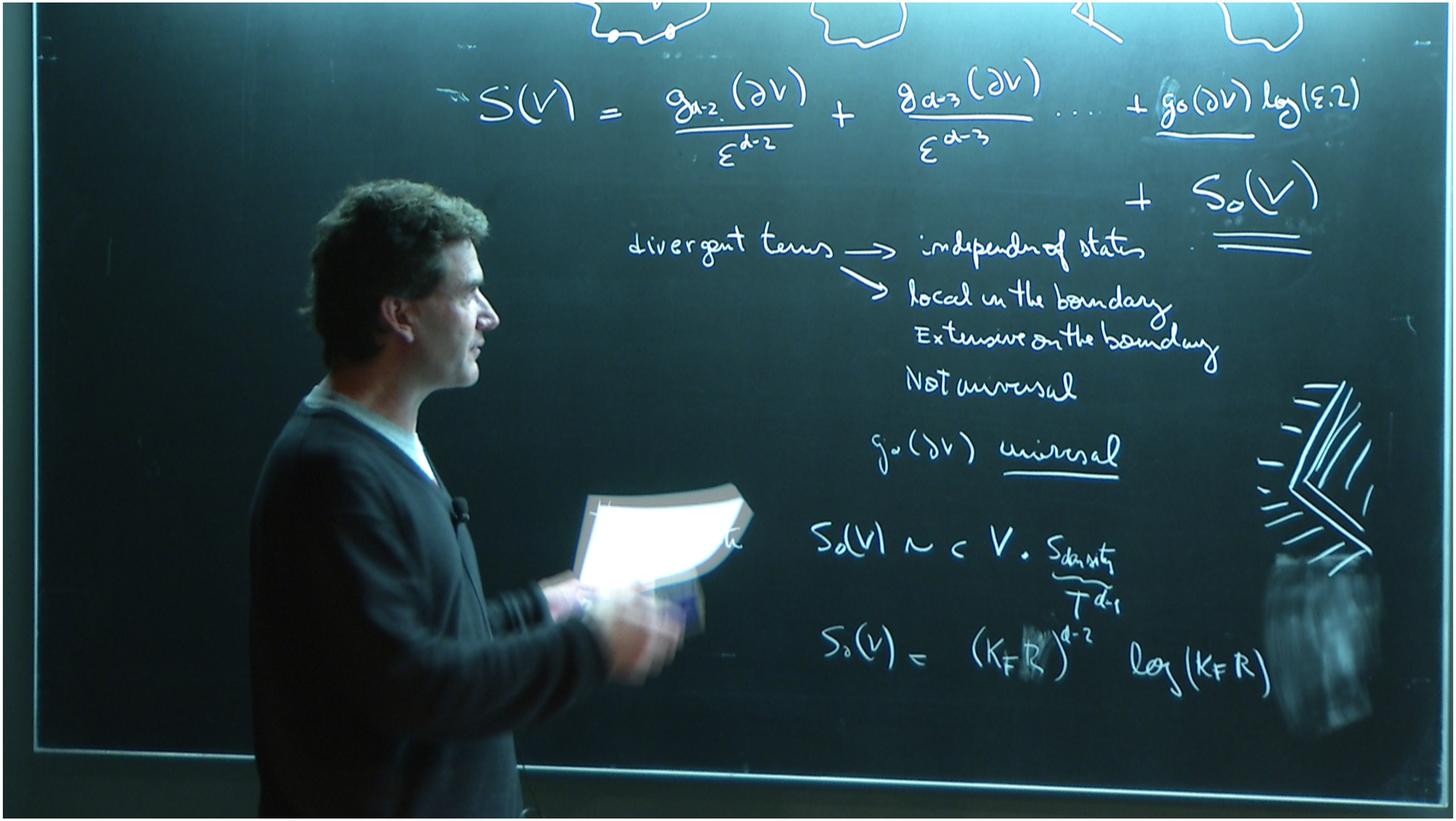


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l, as a function of l/R. The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V,W) \sim .0397R/l$ for both models, while for large distances we expect $I(V,W) \sim (l/R)^2$ for the scalar and $I(V,W) \sim (l/R)^6$ for the Maxwell field.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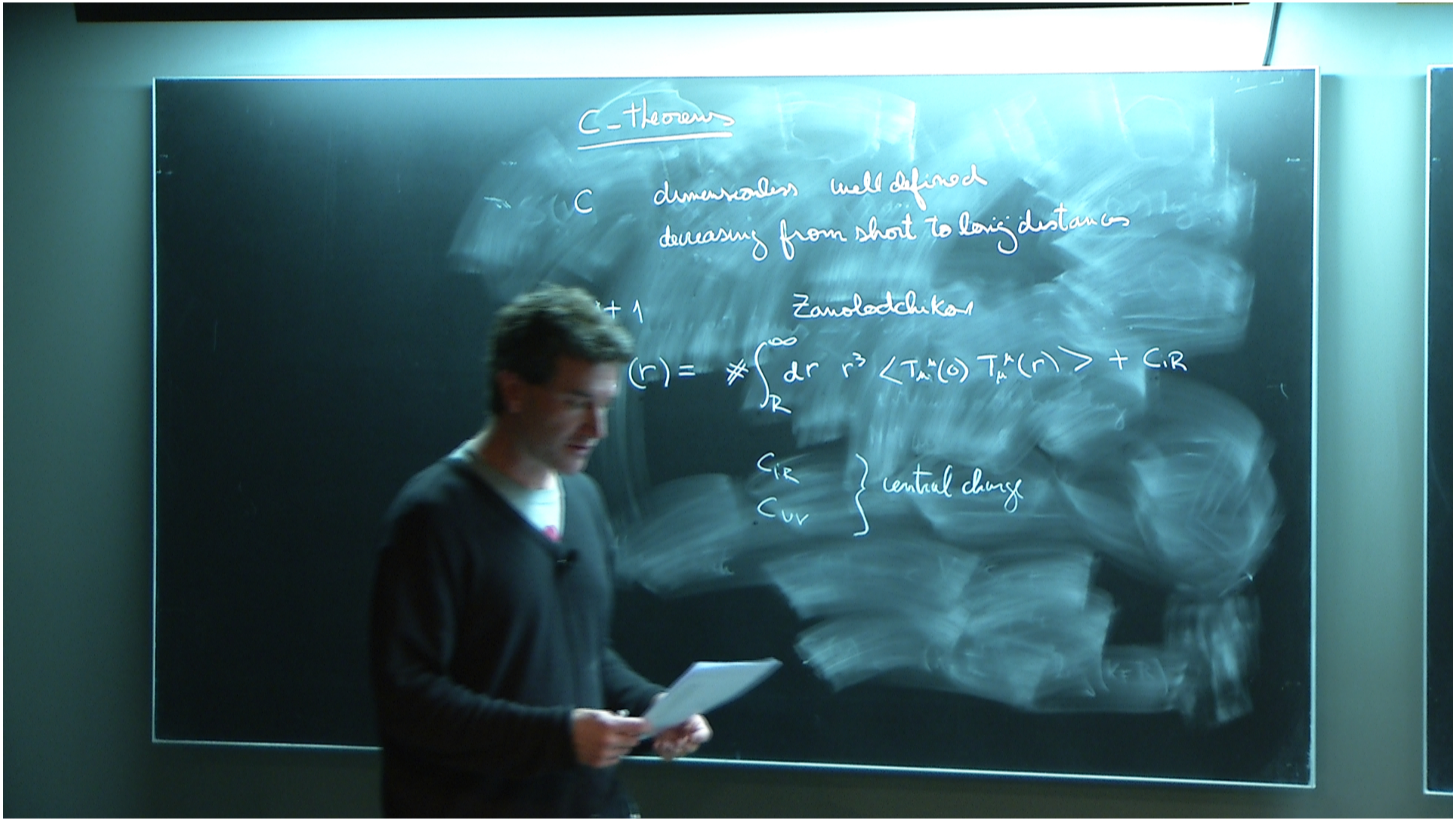
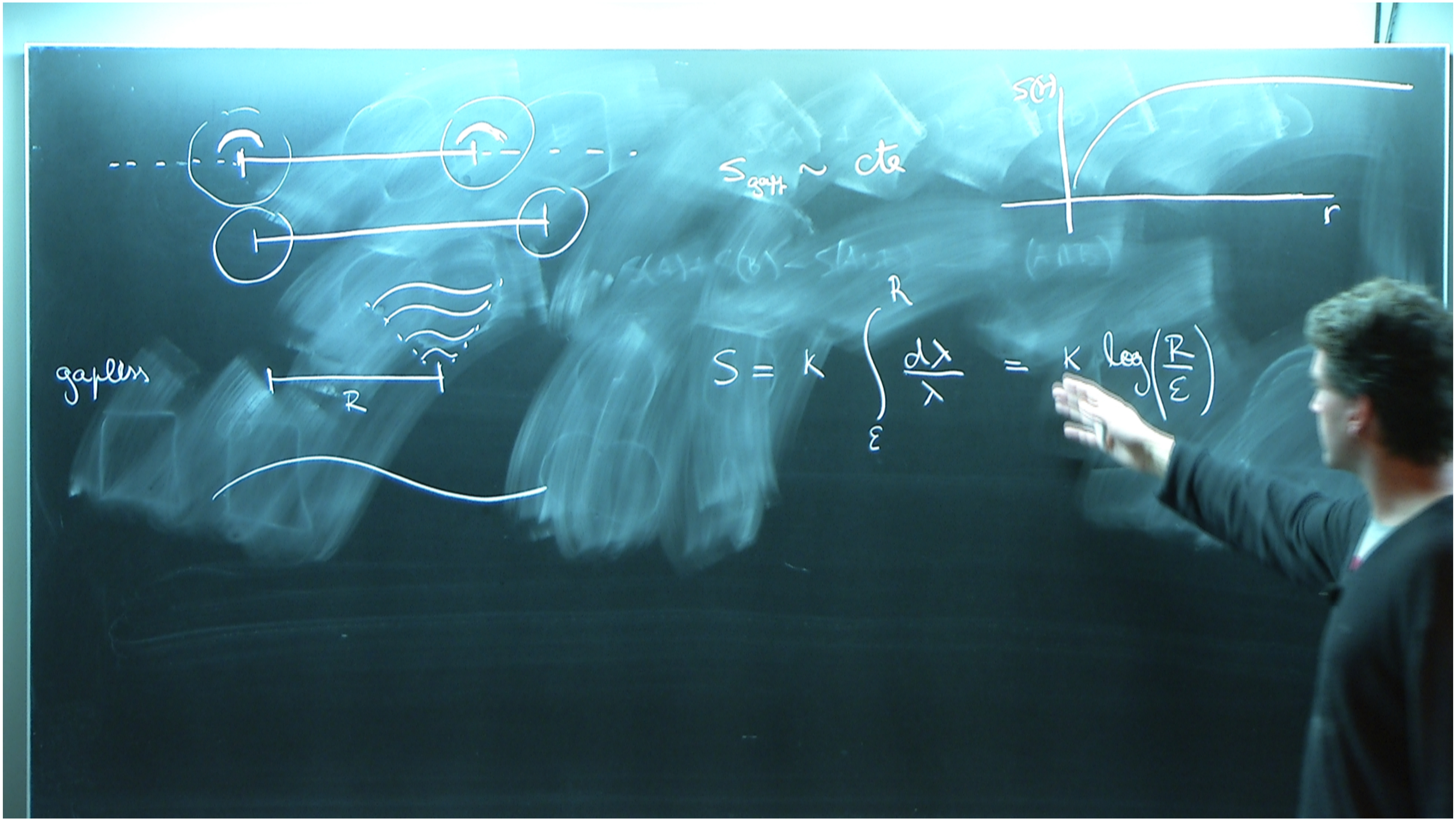
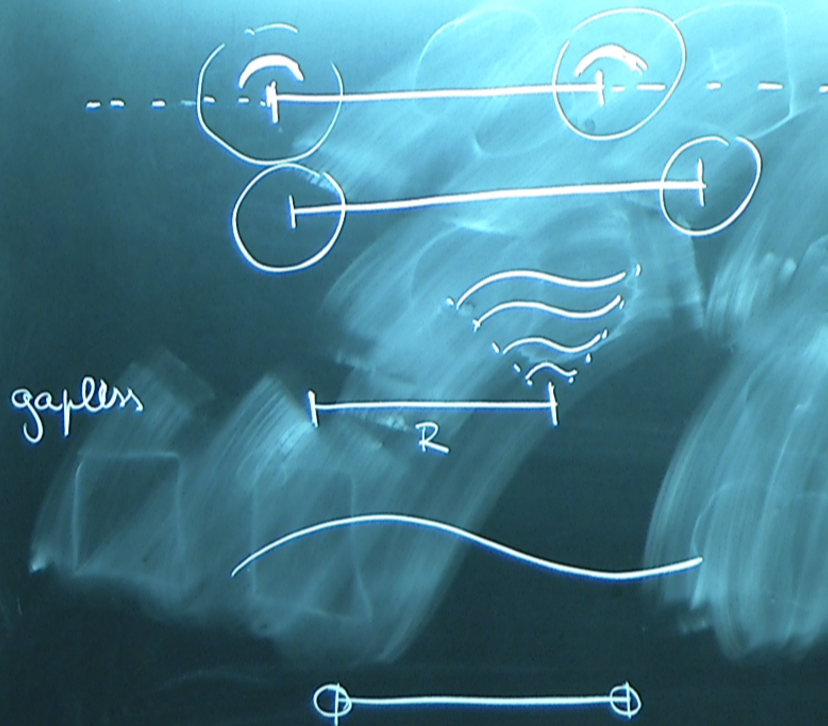
S(A) + S(B) - S(A∪B) = I(A,B)
S(A)+S(B)-S(A∪B) - S(A∩B)
A
B
bulk
ε

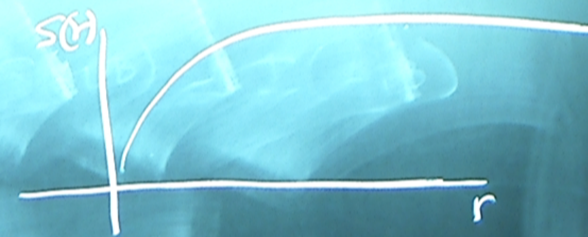

C_theorem
C dimensionless well defined
decreasing from short to long distances
Zamolodchikov
central charge



$S_{gapp} \sim cte$
$S(r)$
r
gapless
R
$S = K \int_{\varepsilon}^{R} \frac{d\lambda}{\lambda} = K \log\left(\frac{R}{\varepsilon}\right)$



$$S_{gapp.} \sim \boxed{cte}$$

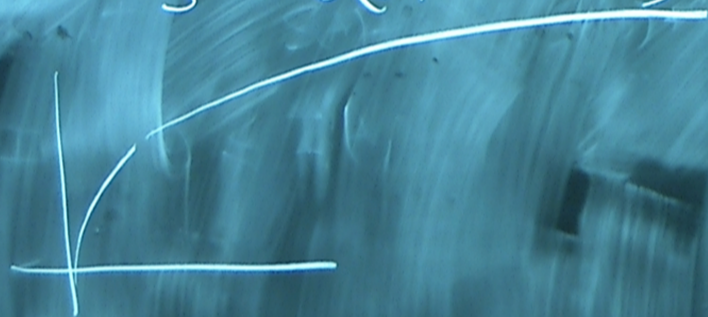


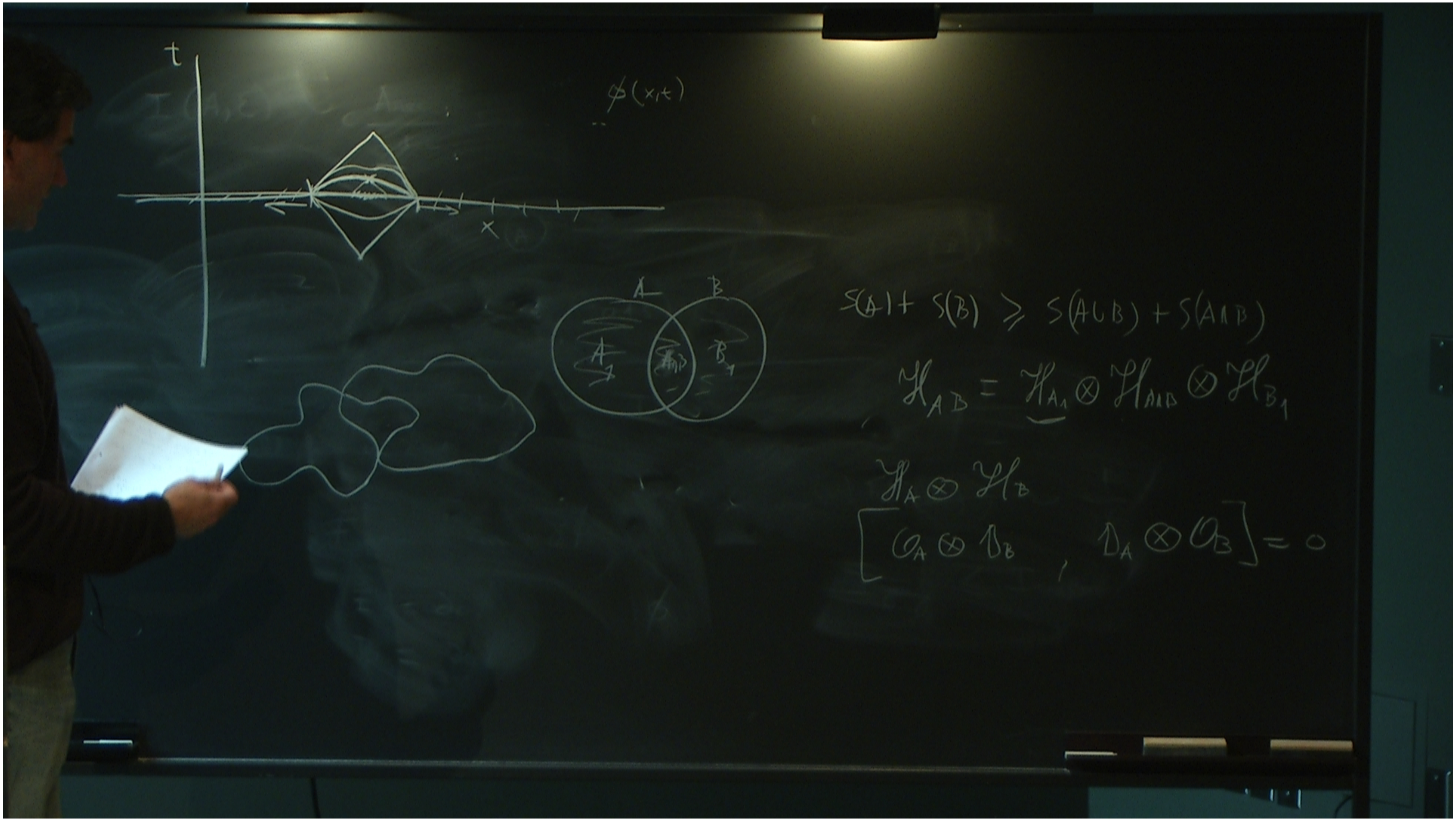
$$S = K \int_{\varepsilon}^{R} \frac{d\lambda}{\lambda} = K \log\left(\frac{R}{\varepsilon}\right) = \frac{c}{3} \log\left(\frac{R}{\varepsilon}\right)$$

$$\underset{RM \gg 1}{S(R)} \simeq \frac{c}{3} \log\left(\frac{1}{\varepsilon\, m}\right) + cte$$

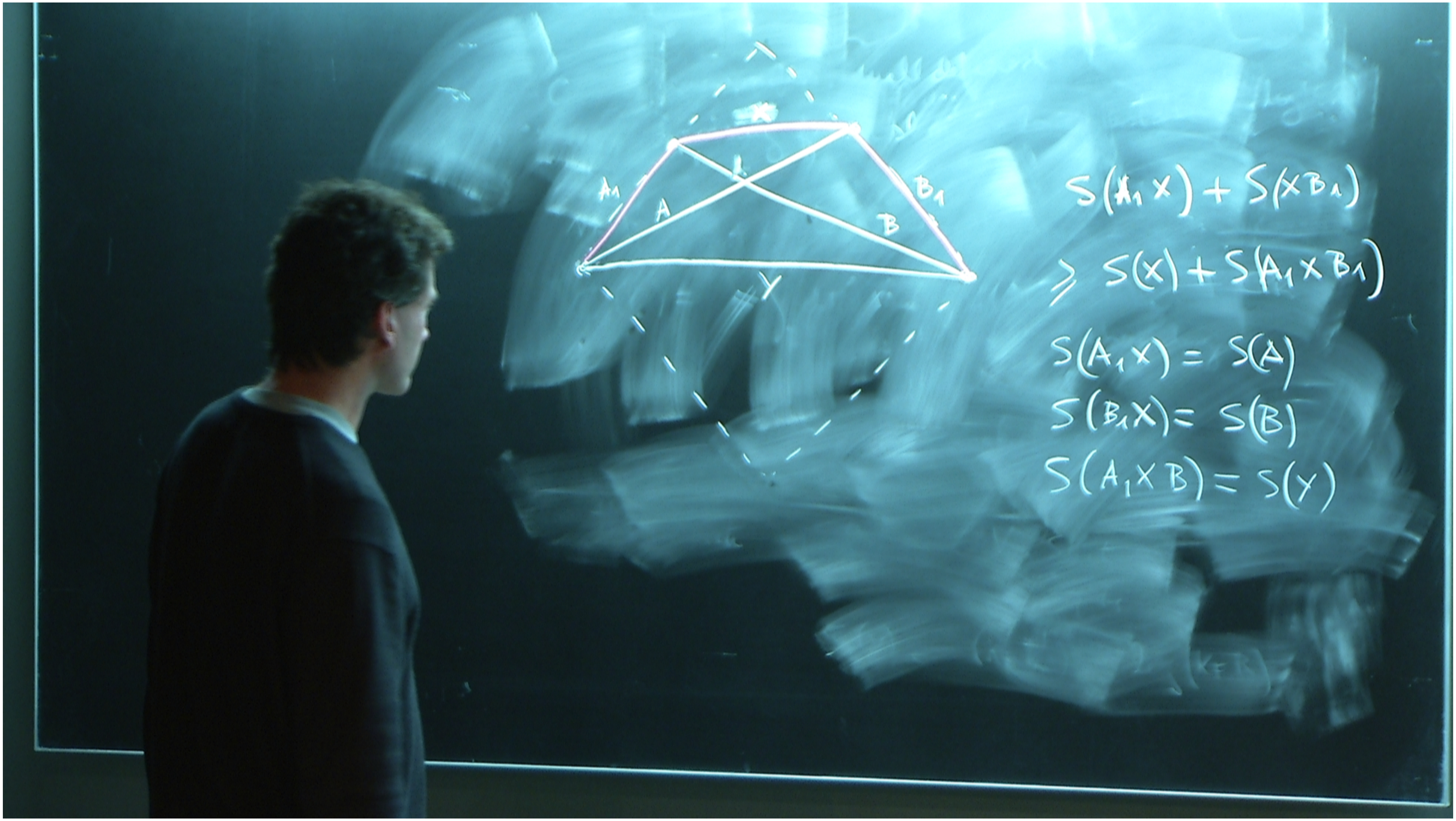
$$S(R) \underset{RM \ll 1}{=} \frac{c_{UV}}{3} \log\left(\frac{R}{\varepsilon}\right)$$

$$S(R) \underset{RM \gg 1}{=} \frac{c_{IR}}{3} \log\left(\frac{R}{\varepsilon}\right) + \frac{(c_{UV} - c_{IR})}{3} \log\left(\frac{1}{\varepsilon M}\right)$$





t
φ(x,t)
x
A
B
S(A)+S(B) ≥ S(A∪B)+S(A∩B)
H_AB = H_A1 ⊗ H_A∩B ⊗ H_B1
H_A ⊗ H_B
[O_A ⊗ 1_B , 1_A ⊗ O_B] = 0



A1
A
B1
B
X
Y
S(A1 X) + S(X B1)
≥ S(X) + S(A1 X B1)
S(A1 X) = S(A)
S(B1 X) = S(B)
S(A1 X B) = S(Y)


S(A1 X) + S(X B1)
≥ S(X) + S(A1 X B1)
S(A1 X) = S(A)
S(B1 X) = S(B)
S(A1 X B) = S(Y)
S(A) + S(B) ≥ S(X) + S(Y)
S(a) + S(b) ≥ S(x) + S(y)
A1
A
a
b
B
B1
Y

$$(x S'(x))' = S'(x) + x S''(x) \leq 0$$

$$C_E(x) = x S'(x)$$

$$C_E'(x) < 0$$

$$C_E(x) = \frac{C_{UV}}{3}$$

$$xM \ll 1$$

$$C_E(x)\Big|_{xM \gg 1} = \frac{C_{IR}}{3}$$