Title: Tensor network renormalization

Date: Aug 27, 2015 09:15 AM

URL: http://pirsa.org/15080049

Abstract: TBA

Summary MPS / MERA

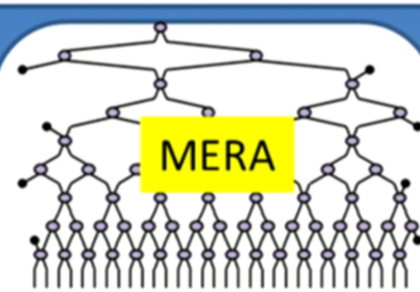
- Important aspects of a Tensor Network?
  - ➢ efficient representation and *efficient computation*
  - ➢ structural properties (correlations and entanglement)

MPS

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

$$S(A) \leq const$$

ground state
gapped Hamiltonian



$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx \frac{1}{L^p}$$

$$S(A) \approx \log L$$

ground state
gapless Hamiltonian


• Important aspects of a Tensor Network?
➢ efficient representation and *efficient computation*
➢ structural properties (correlations and entanglement)
MPS
$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$
$S(A) \leq const$
ground state
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MERA
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ground state
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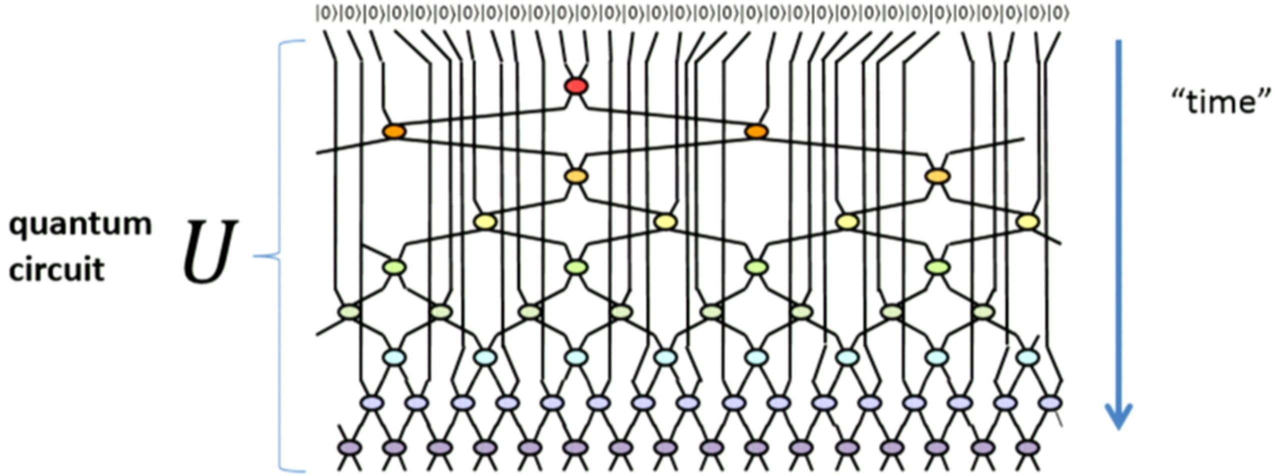
MERA as a quantum circuit



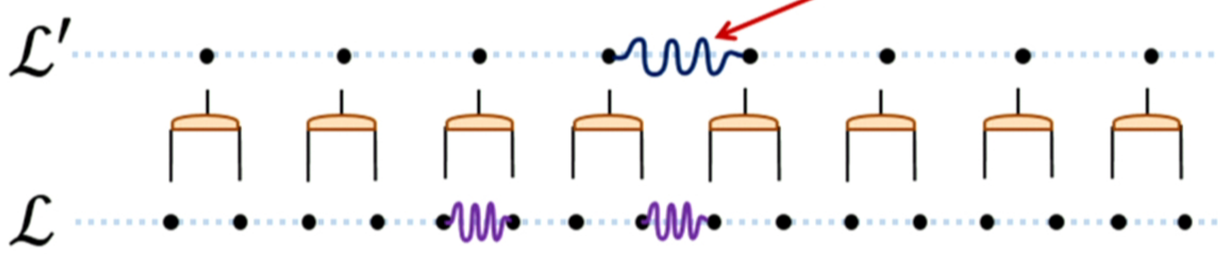
ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

Entanglement introduced by gates at different "times" (= length scales)

MERA as a (real space) Renormalizatin Group transformation

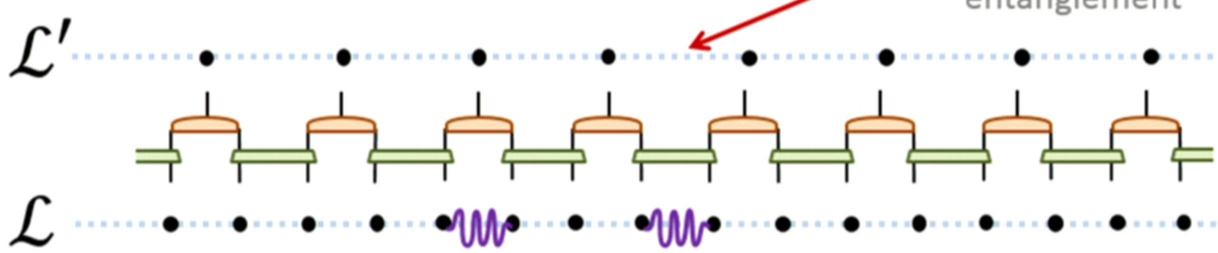
Kadanoff (1966) + White (1992)
blocking variational optimization

failure to remove *some* short-range entanglement !



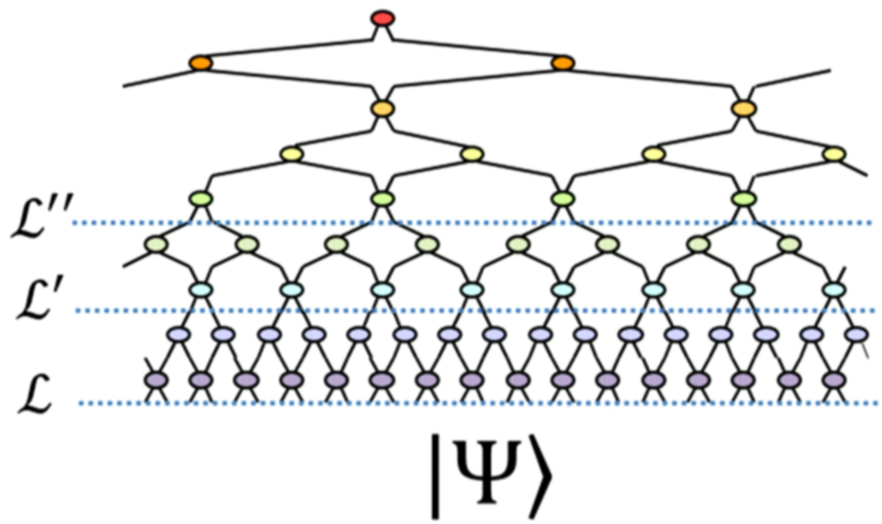
Entanglement renormalization (2005)

removal of *all* short-range entanglement



MERA as a (real space) Renormalizatin Group transformation

sequence of ground state wave-functions



$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

MERA defines an RG flow
in the space of wave-functions

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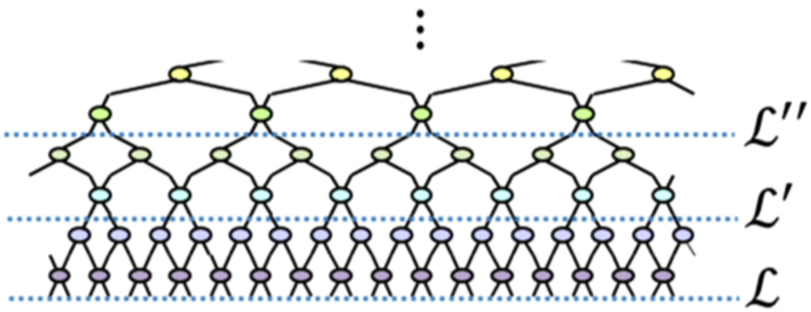
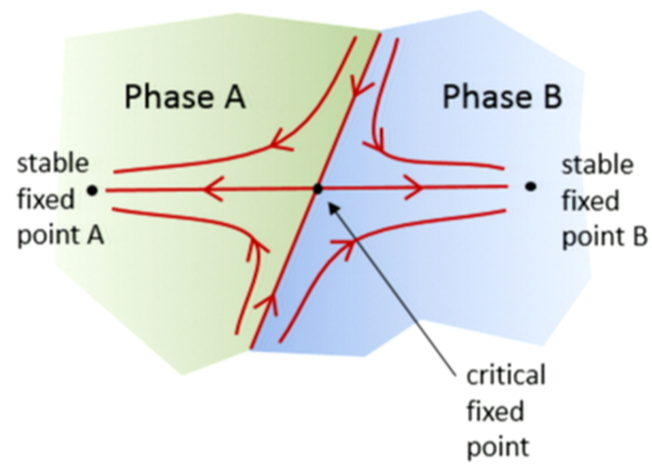
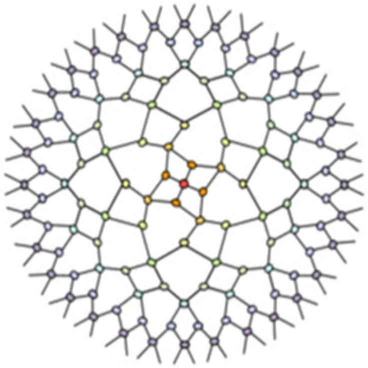


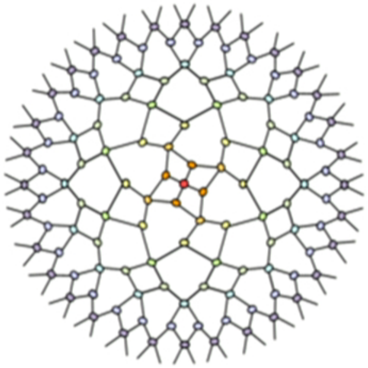


MERA and CFT



MERA

MERA and CFT



MERA

input

1D quantum Hamiltonian

- **on the lattice**
- **at a critical point**

output

Numerical determination of conformal data:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
  and conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

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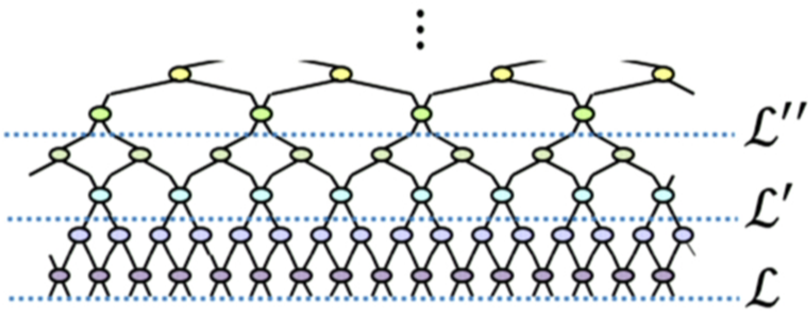
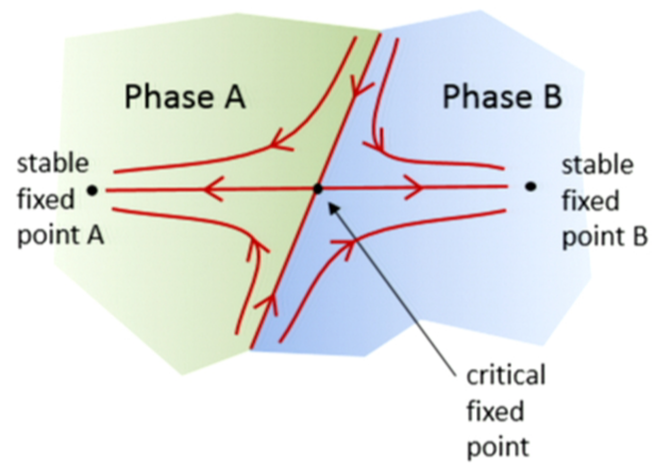
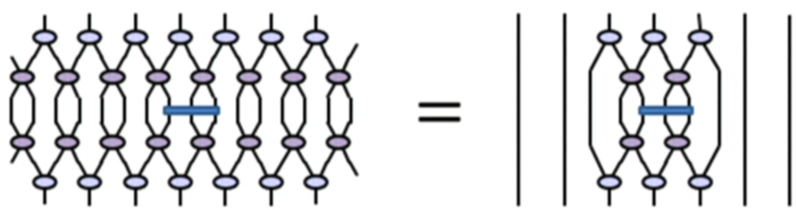
$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$





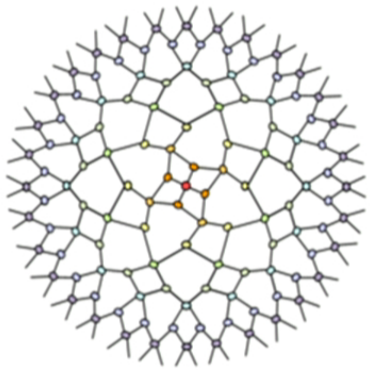
... and in the space of Hamiltonians

$$H \to H' \to H'' \to \cdots$$



local operators are mapped into local operators !

MERA and CFT



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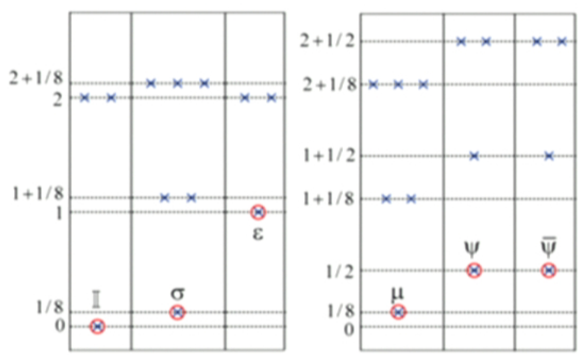
e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

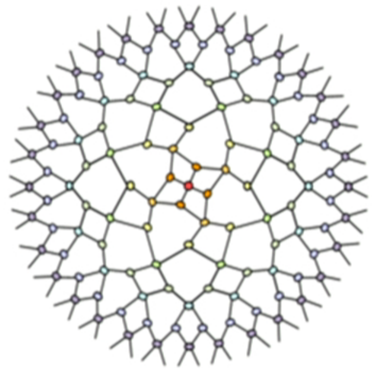
$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

$\Delta_{\bar{\psi}} \approx 0.500001$

MERA and CFT



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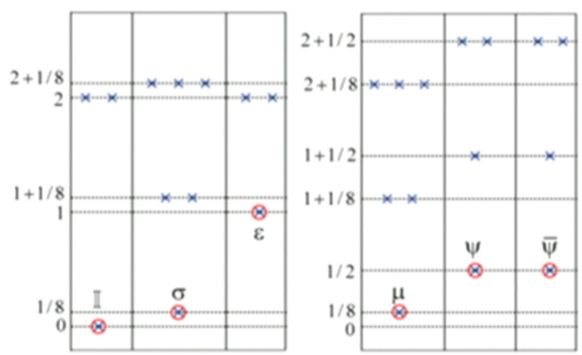

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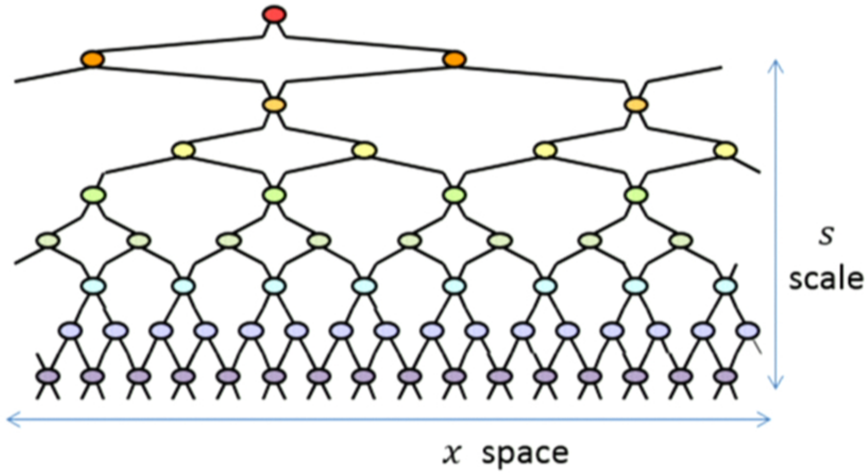
$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $\quad C_{\epsilon\mu\mu} = -\frac{1}{2}$

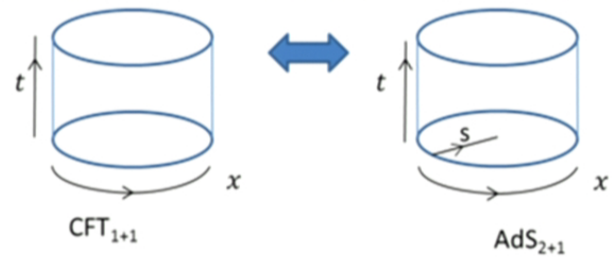
$C_{\epsilon\psi\bar{\psi}} = i$ $\quad C_{\epsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $\quad C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$

MERA and holography?





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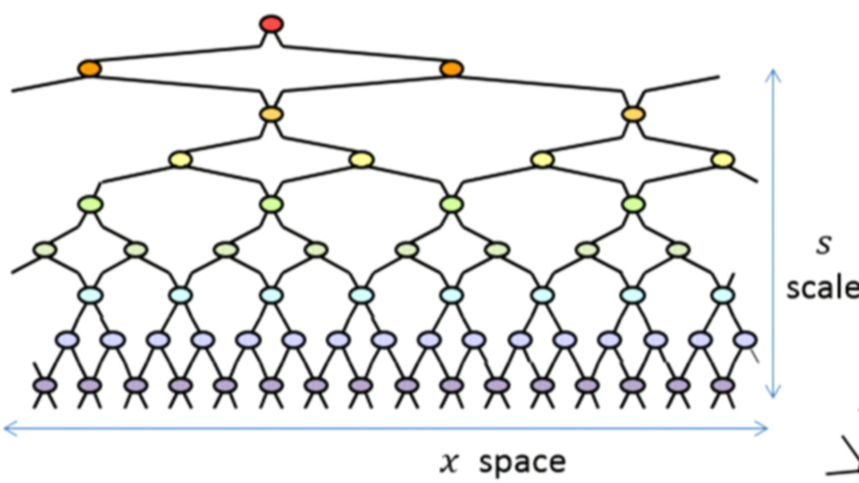

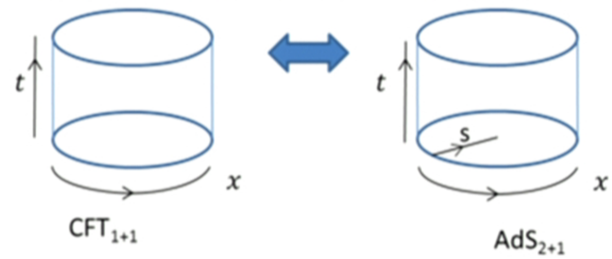

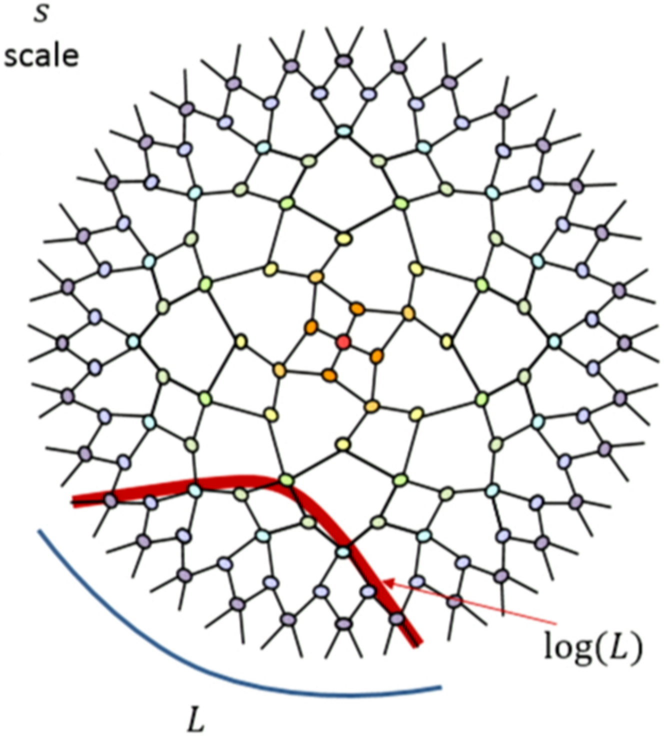

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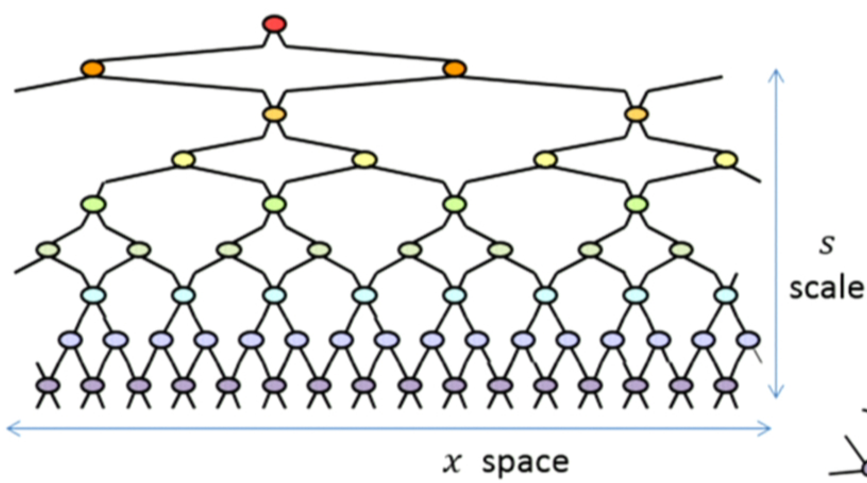
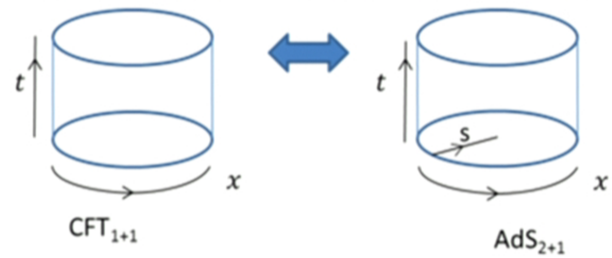
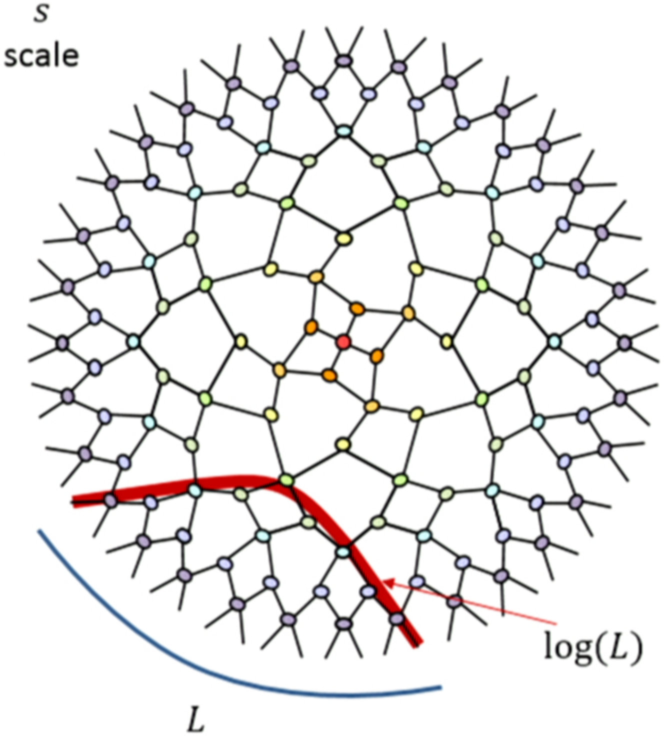




- entanglement entropy

$$S_L \approx \log{(L)}$$

parallel to area of minimal surface in Ryu-Takayanagi



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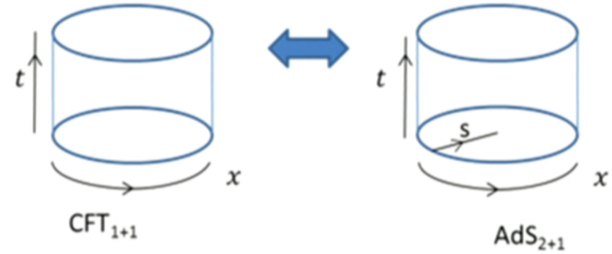
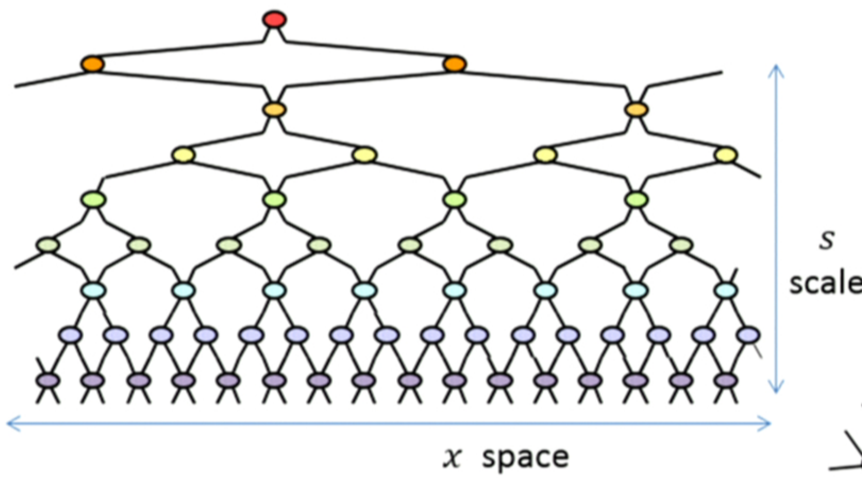




- entanglement entropy

$$S_L \approx \log(L)$$

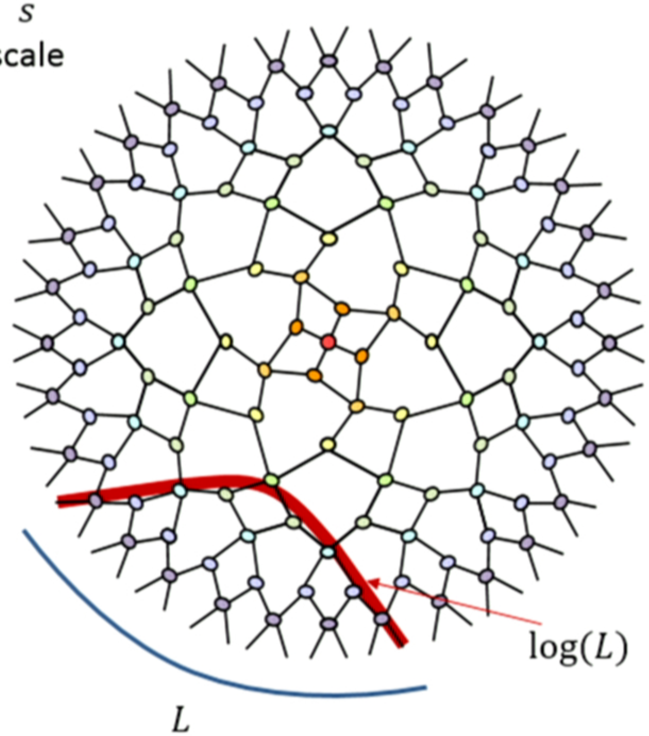
parallel to area of minimal surface in Ryu-Takayanagi

- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

geodesic distance $D \approx \log(L)$ as in a hyperbolic geometry

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



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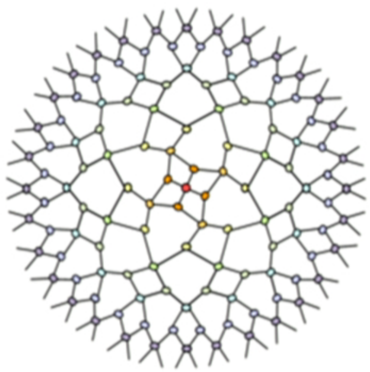

MERA
(2005)

MERA ↔ AdS/CFT

Swingle, 2009

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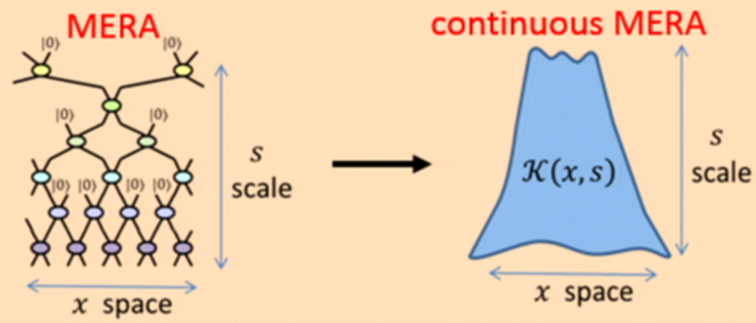

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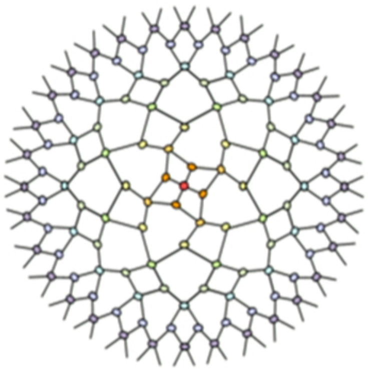

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thermal state + time evolution

as a Generalized Holography”
Miyaji, Takayanagi, 2015

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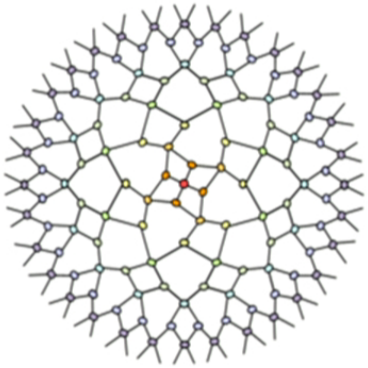

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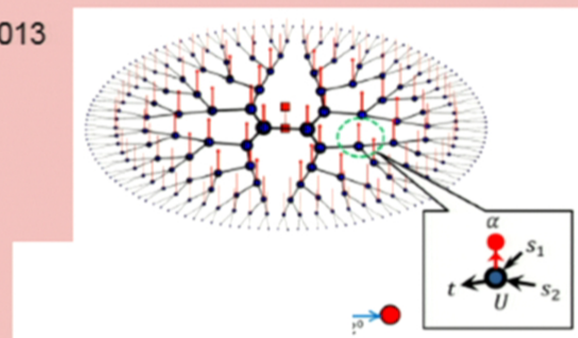

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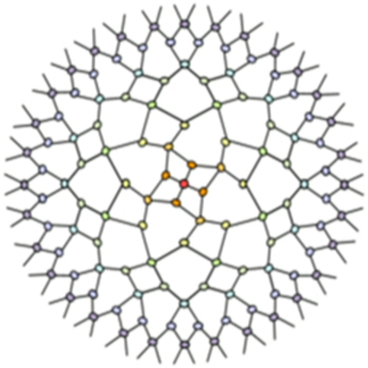

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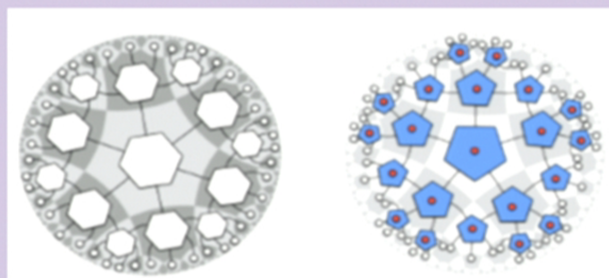
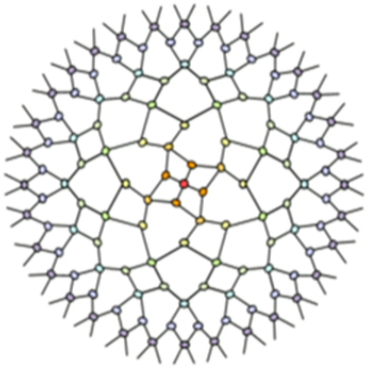


holographic hexagon state — holographic pentagon code

MERA and holography?



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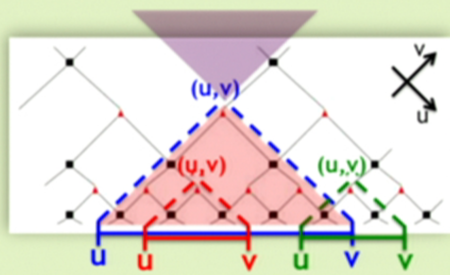
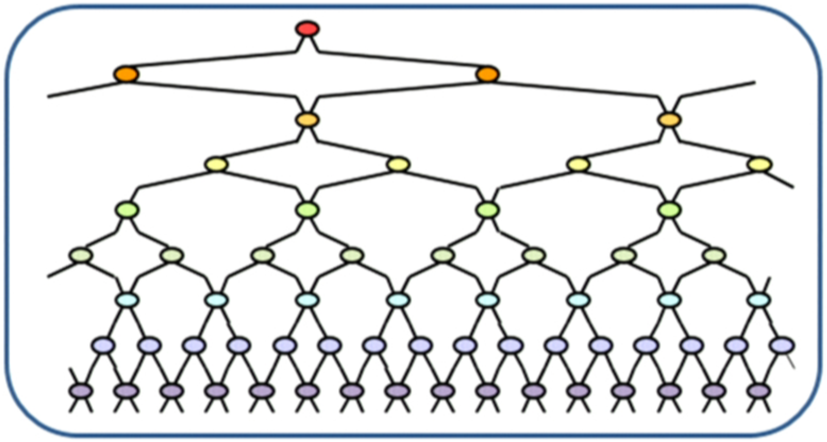
Czech, Lamprou, McCandlish, Sully, 2015



MERA = tensor network + isometric/unitary constraints



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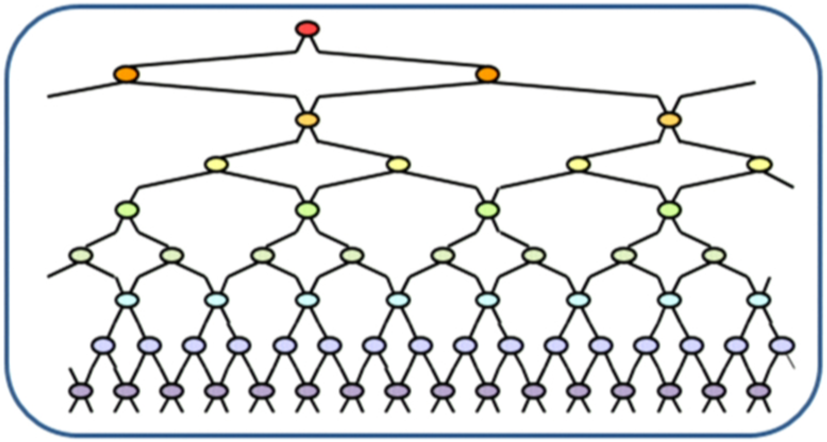

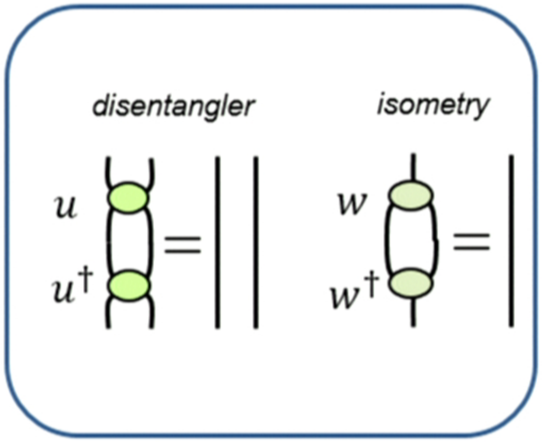

$\sim$ hyperbolic plane?

(Swingle 2009)

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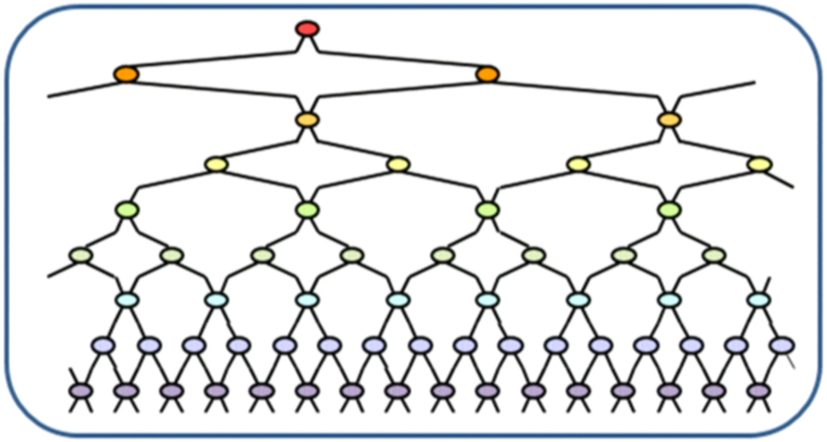

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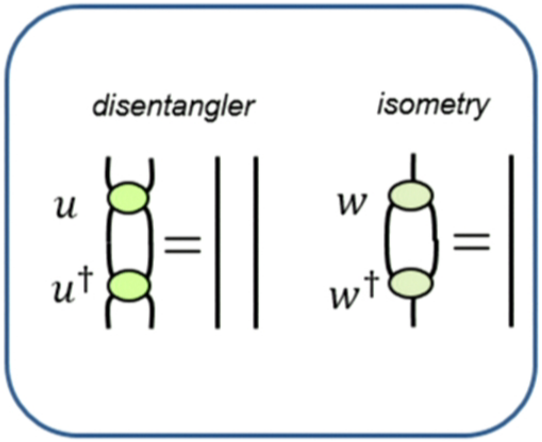


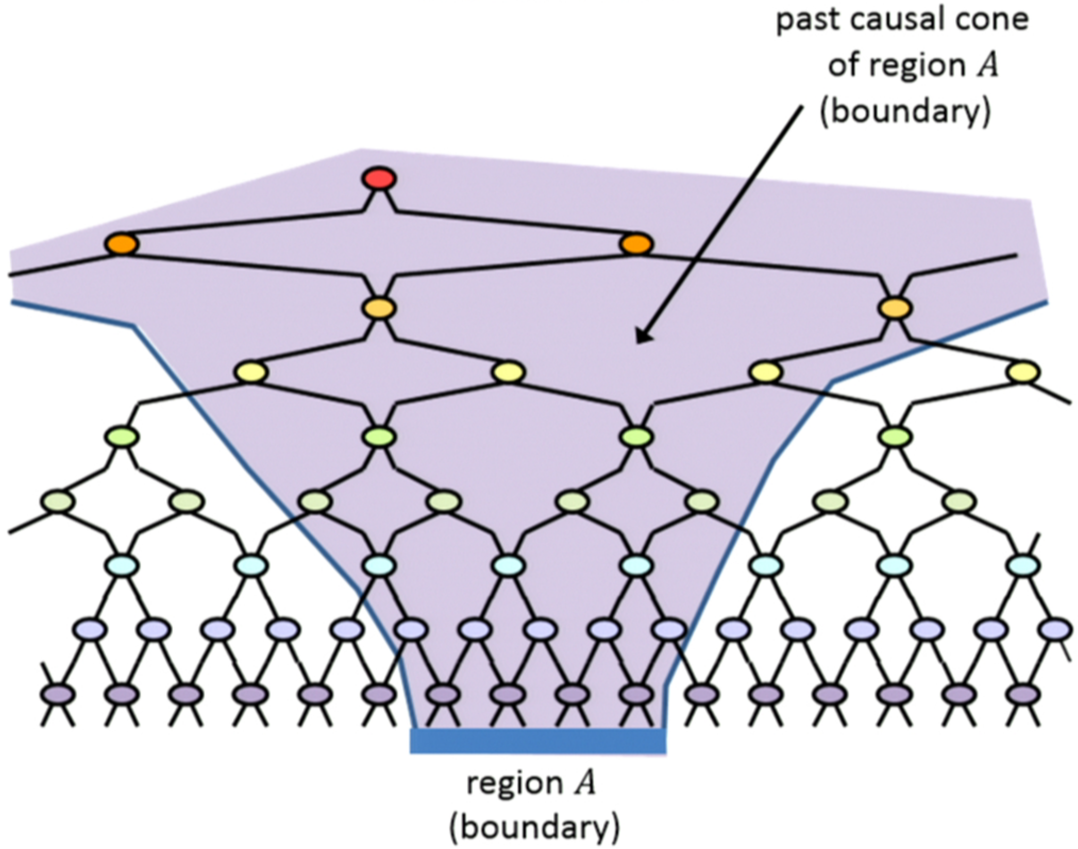
~ de Sitter space?

(Beny 2011, Czech 2015)

Causal structure

essential for many MERA properties
and computational efficiency

Causal cone



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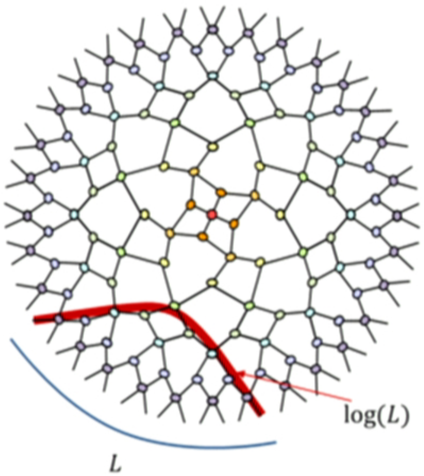
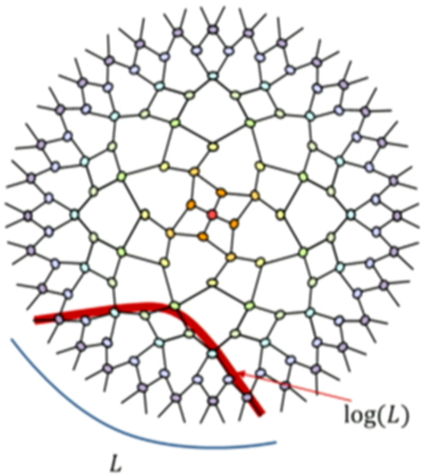


MERA = RG

Tensor network for ground state/Hilbert space of CFT, organized in extra dimension corresponding to scale



MERA represents a generic CFT *(no large N or strong interactions)*

e.g. for Ising model

MERA/CFT dictionary

boundary	bulk
state of CFT (or Hilbert space of CFT)	tensor network (?) (or unitary map, EHM)
scaling dimension Δ	mass $\sim\Delta$
entanglement entropy	"minimal connecting surface"
global on-site symmetry (e.g. Z_2)	local/gauge symmetry (e.g. Z_2)

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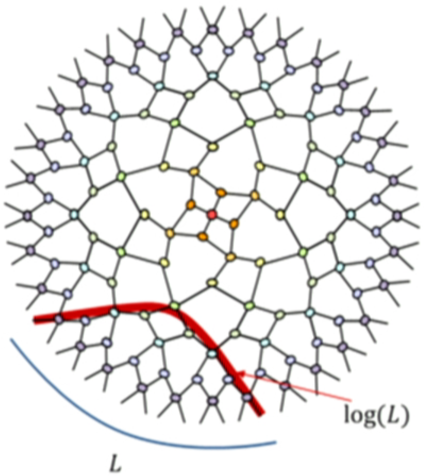

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MERA operates at scale of AdS radius
For smaller scale? → cMERA

Useful testing ground / nice drawings

MERA = RG

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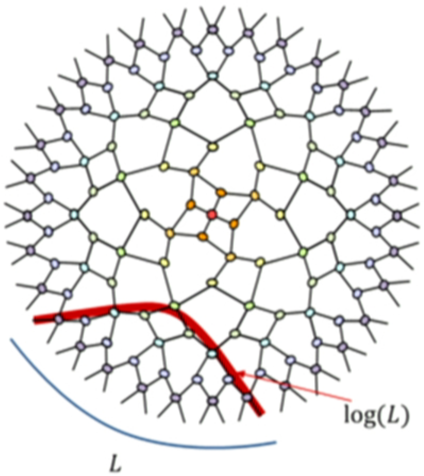

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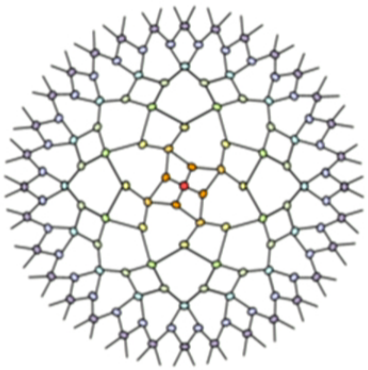
Generalized notion of *holographic* description?

MERA and holography?



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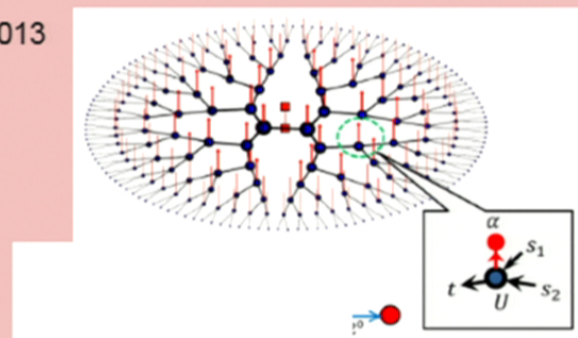
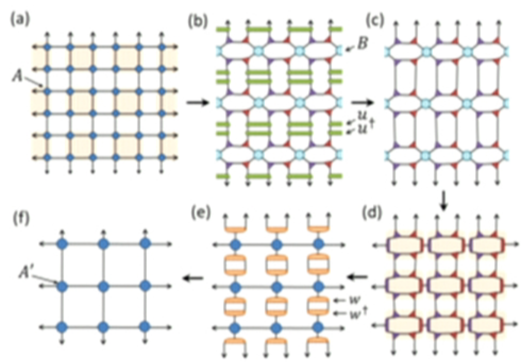
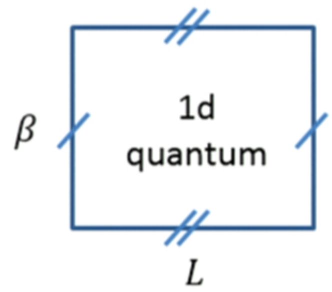


"Lecture 3"- Tensor network renormalization (TNR)



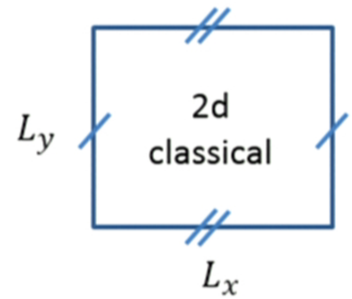
Euclidean path integral

$$Z(\lambda) = tr\; e^{-\beta H_q^{1d}}$$



Statistical partition function

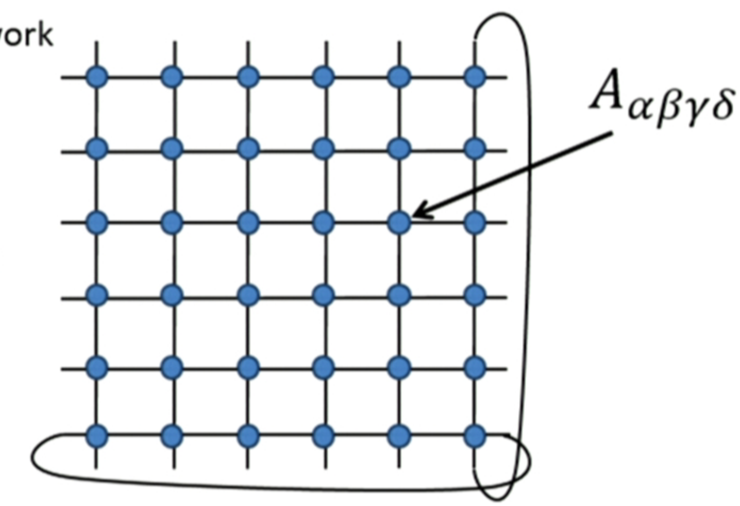
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$



$\sim$

as a tensor network

$$Z =$$



Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$

$$H_q^{1d} = \sum_i (\sigma_z^i + \sigma_x^i \sigma_x^{i+1})$$

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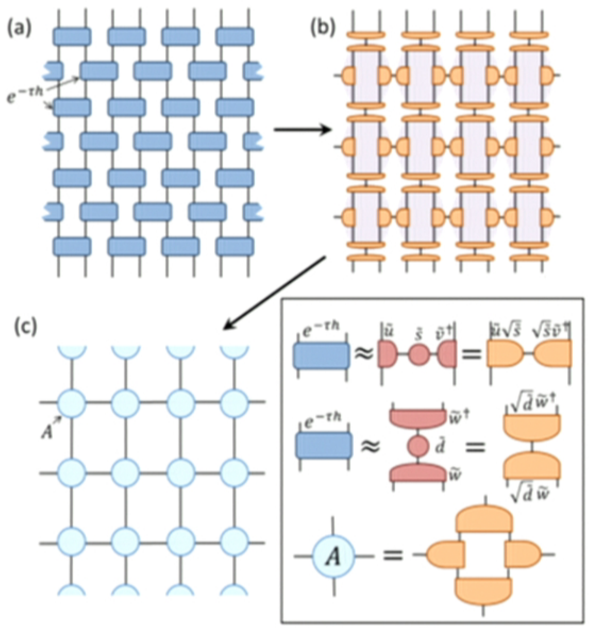

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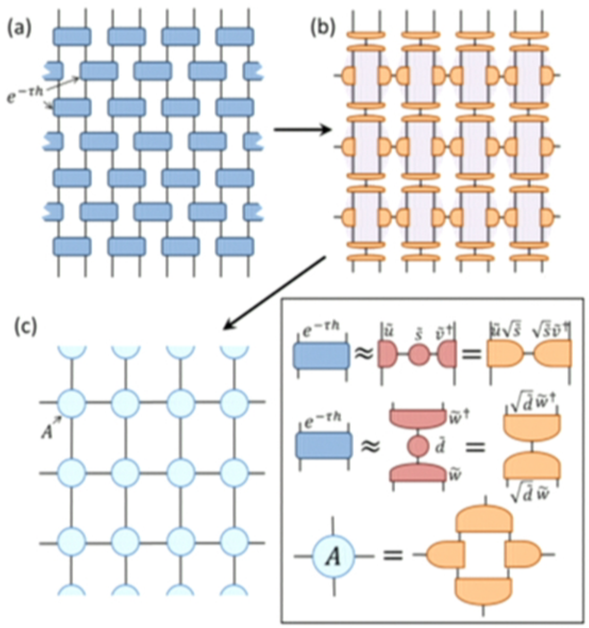

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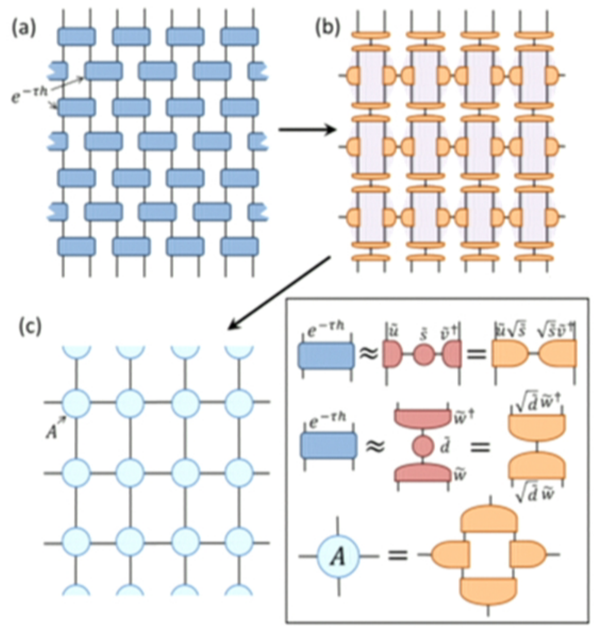

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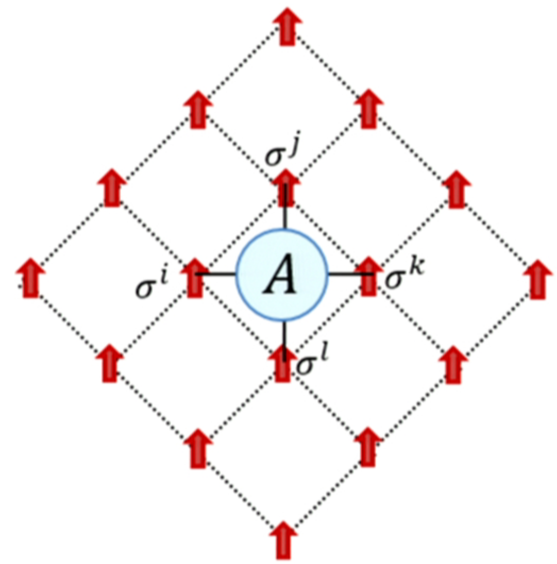

$$A_{ijkl} = e^{-(\sigma_i\sigma_j + \sigma_j\sigma_k + \sigma_k\sigma_l + \sigma_l\sigma_i)/T}$$

Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

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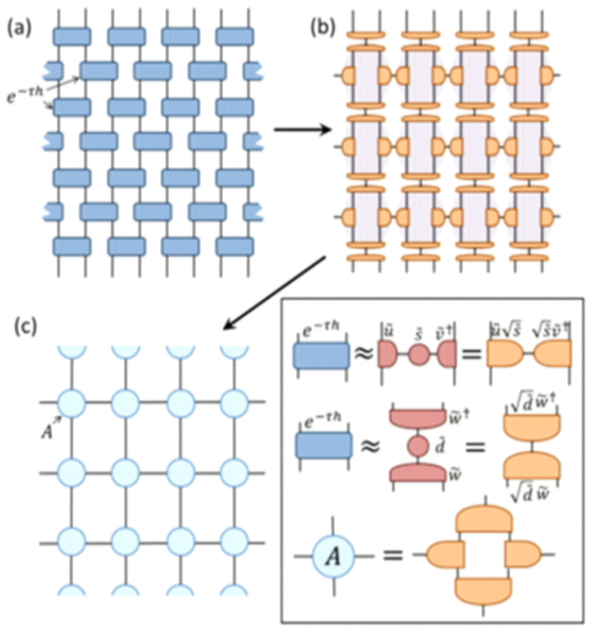

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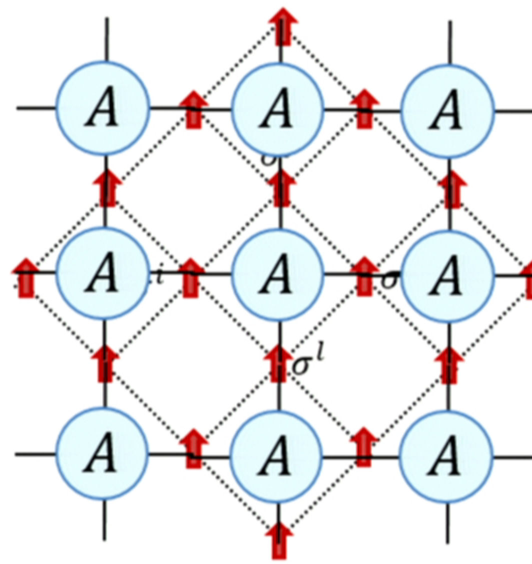
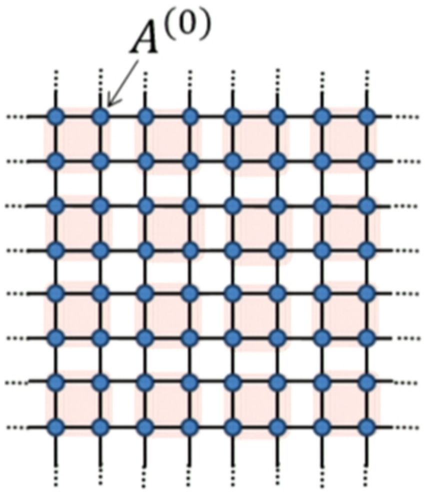


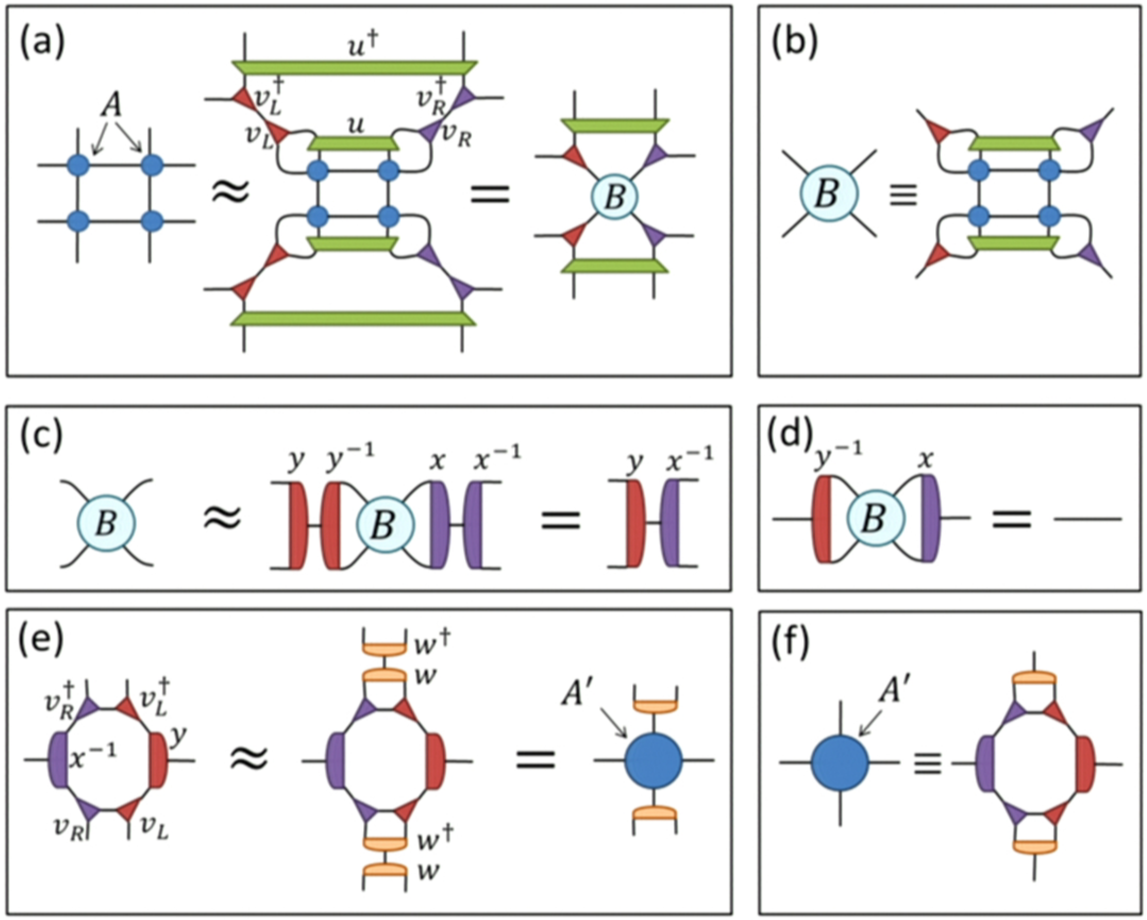
$$A_{ijkl} = e^{-(\sigma_i\sigma_j + \sigma_j\sigma_k + \sigma_k\sigma_l + \sigma_l\sigma_i)/T}$$

Goal: define an RG flow in the space of tensor networks



Tensor Network Renormalization (TNR)

[Evenbly, Vidal 14-15]



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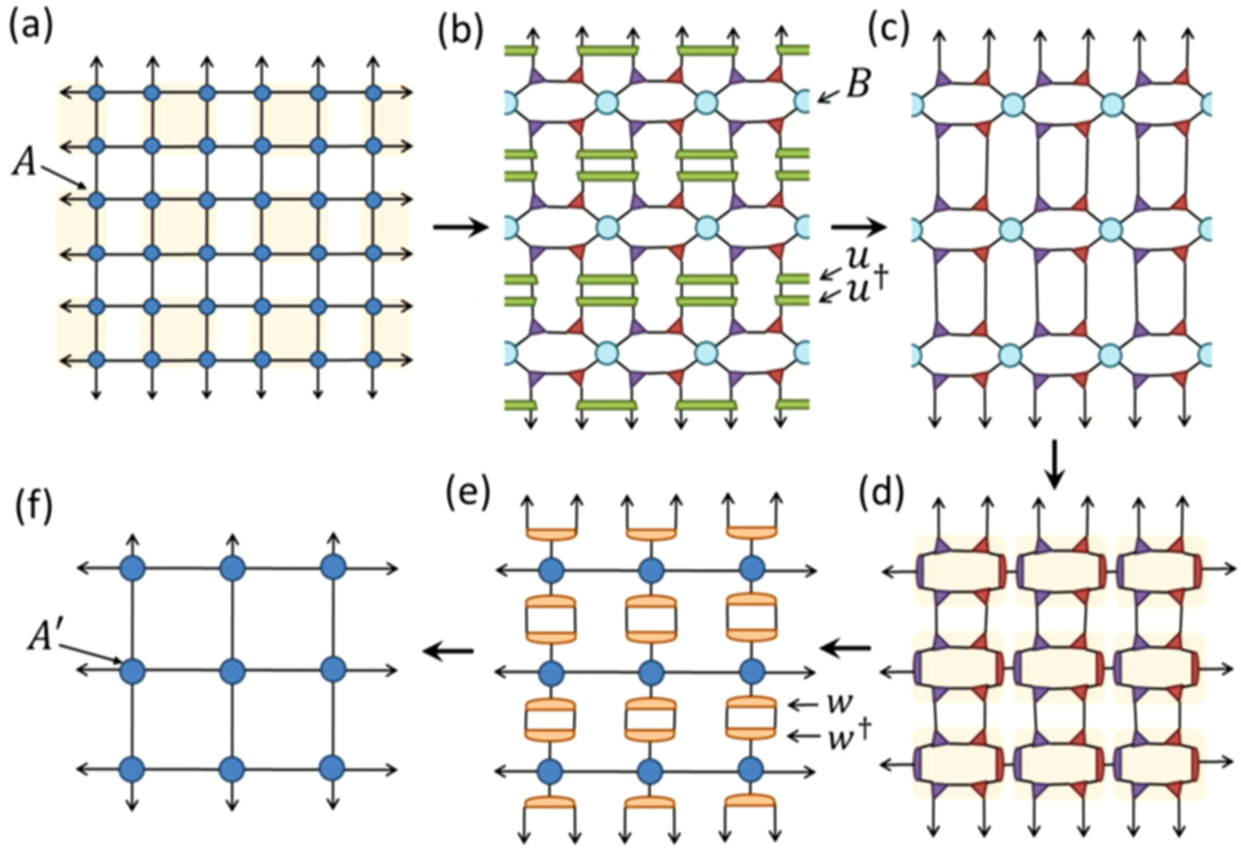
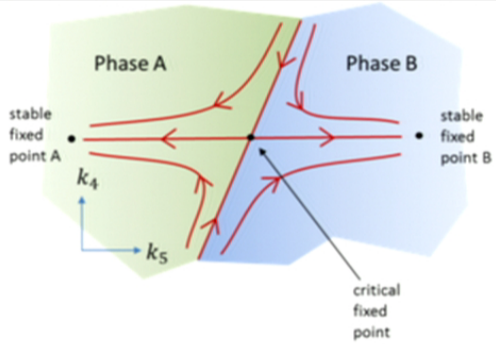
[Evenbly, Vidal 14-15]

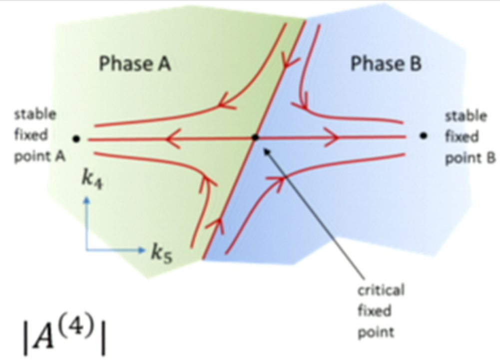


TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$

TNR -> proper RG flow
Example: 2D classical Ising

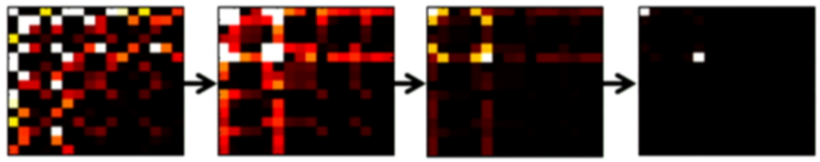


$$A \to A' \to A'' \to \cdots \to A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$

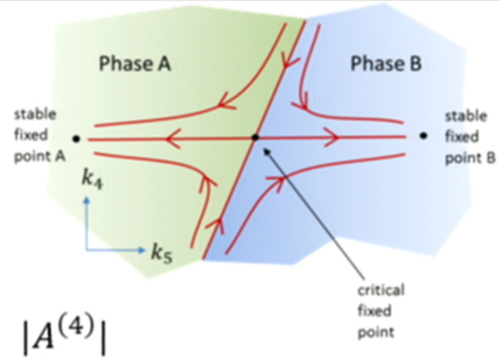
below critical

$T = 0.9\, T_c$



ordered (Z2) fixed point

TNR -> proper RG flow
Example: 2D classical Ising



$$A \to A' \to A'' \to \cdots \to A^{fp}$$

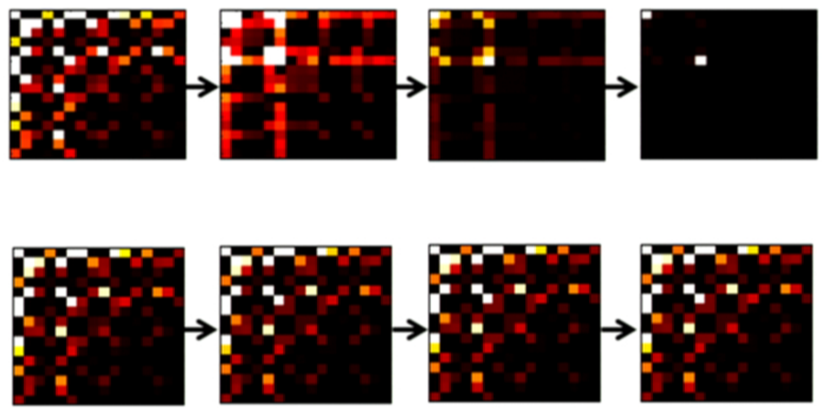
$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$

below critical

$T = 0.9\, T_c$

critical

$T = T_c$

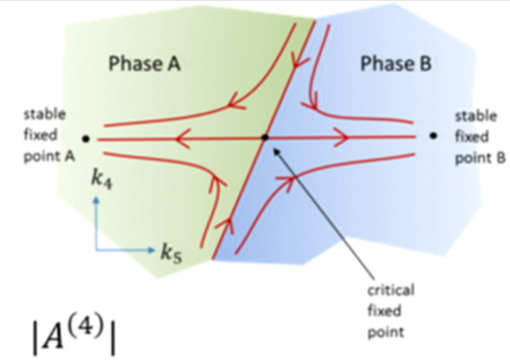


ordered (Z2) fixed point

critical fixed point

TNR -> proper RG flow

Example: 2D classical Ising



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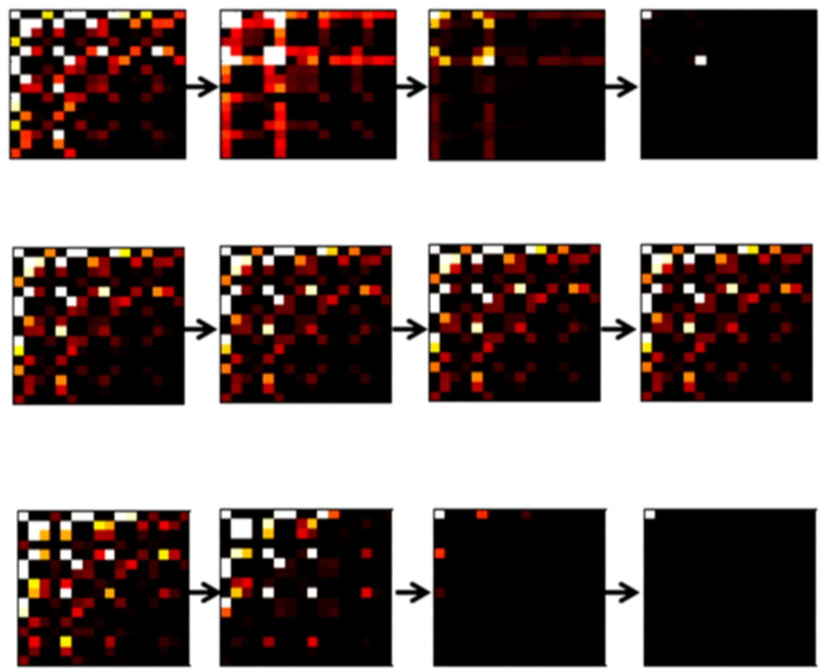


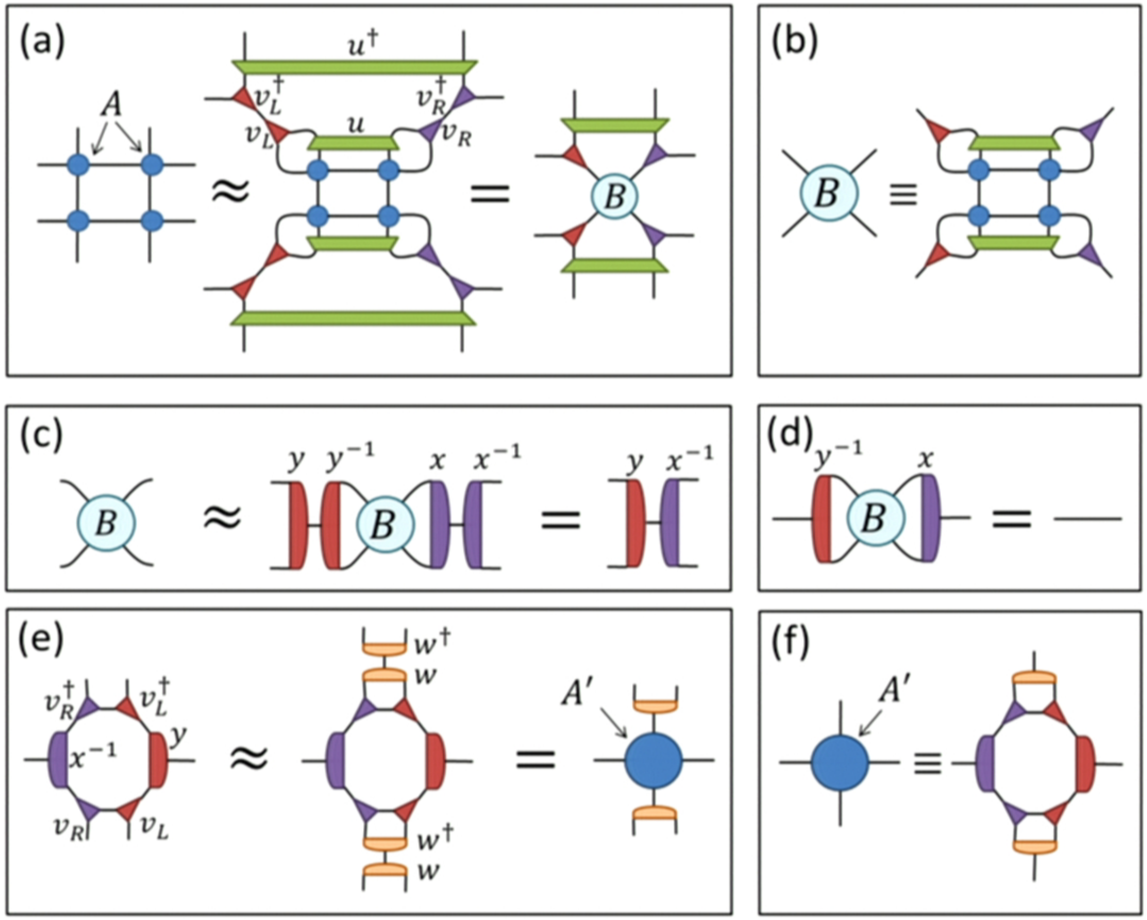
below critical $T = 0.9\,T_c$ → ordered (Z2) fixed point

critical $T = T_c$ → critical fixed point

above critical $T = 1.1\,T_c$ → disordered (trivial) fixed point

Tensor Network Renormalization (TNR)

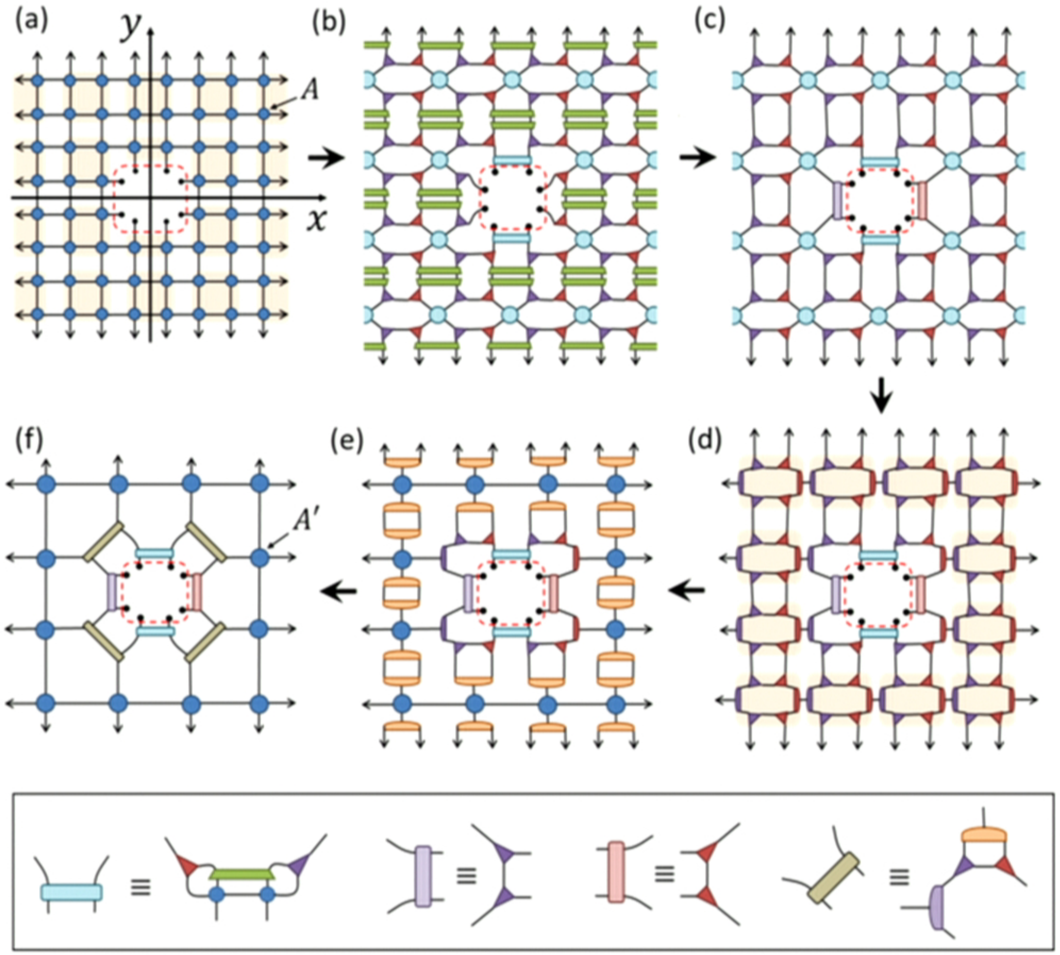
[Evenbly, Vidal 14-15]



local scale transformations

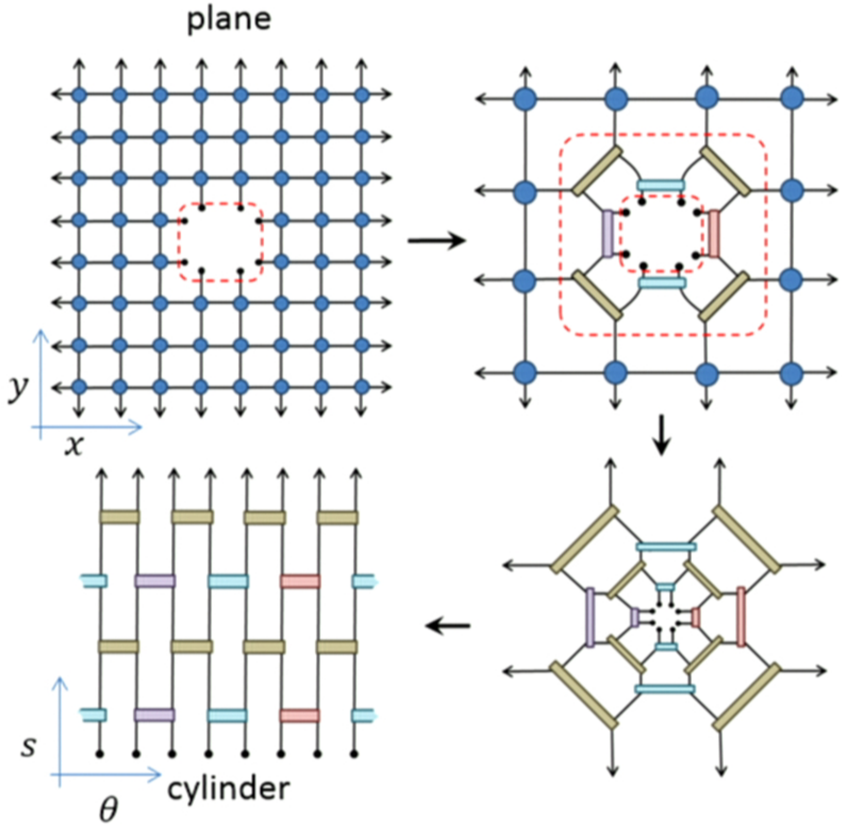
[Evenbly et al, in preparation]

example 1: Plane to cylinder



local scale transformations

[Evenbly et al, in preparation]

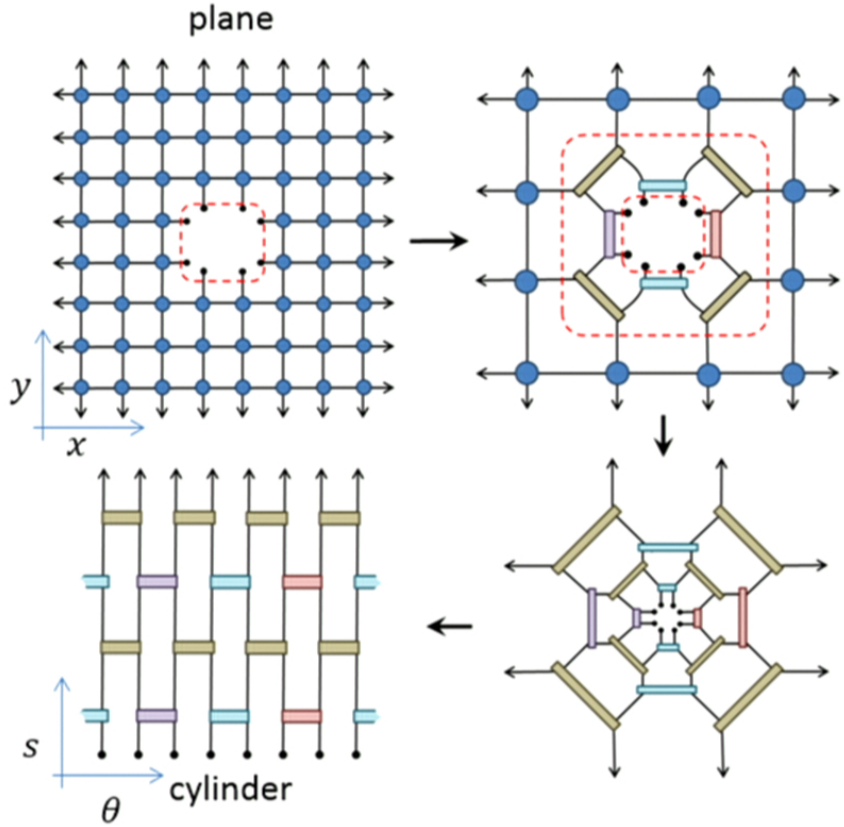


example 1: Plane to cylinder

(radial quantization in CFT)

local scale transformations

[Evenbly et al, in preparation]



example 1: Plane to cylinder

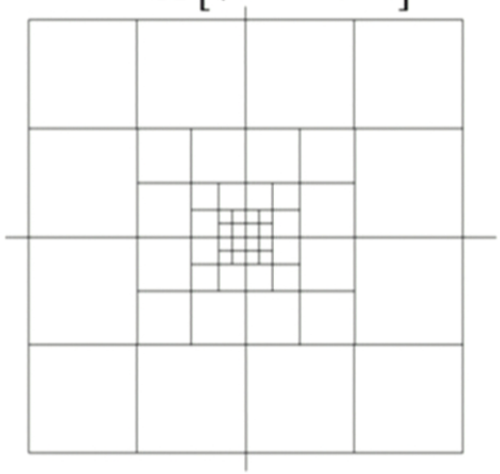
(radial quantization in CFT)

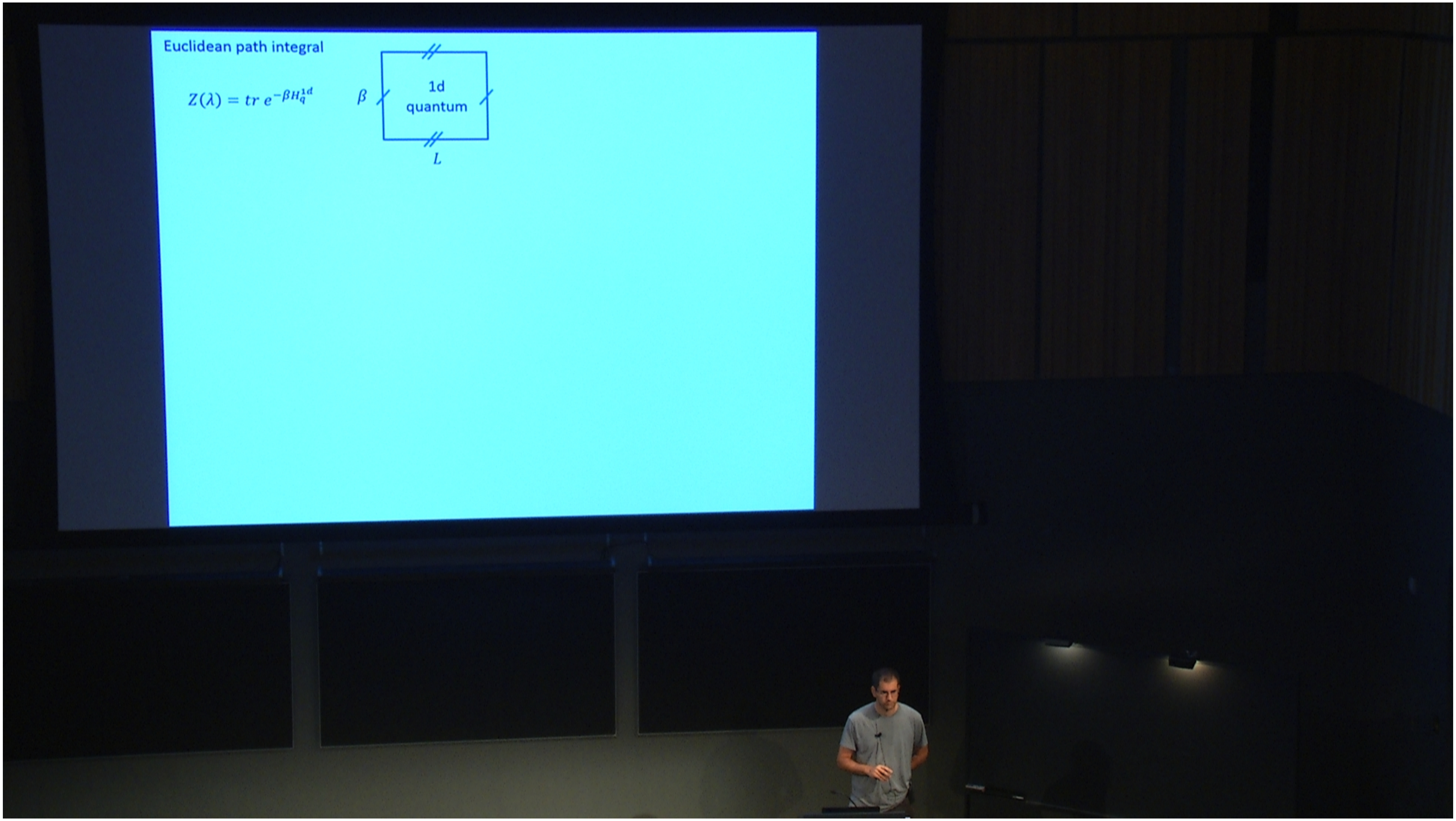
$z \equiv x + iy$

$$z = 2^w$$

$w \equiv s + i\theta$

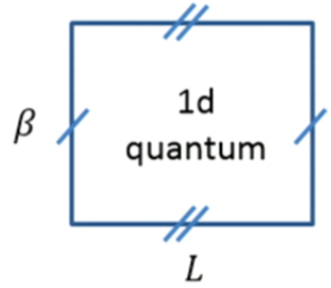
$s \equiv \log_2\left[\sqrt{(x^2+y^2)}\right]$




Euclidean path integral
$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$
β
1d
quantum
L

Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



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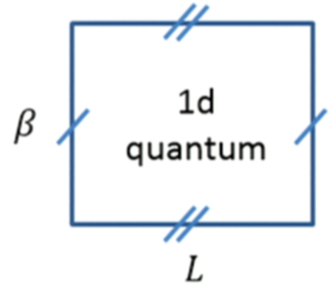
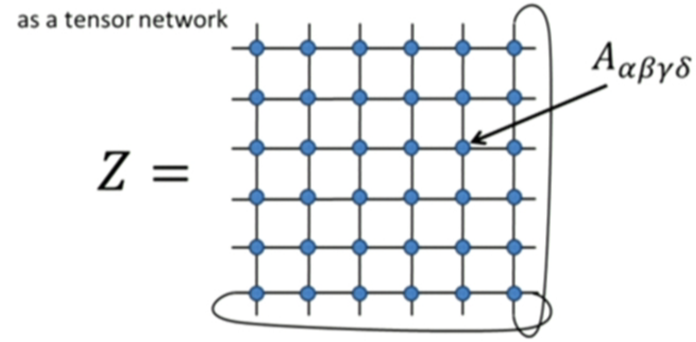
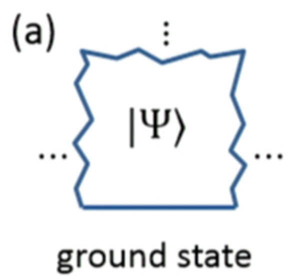


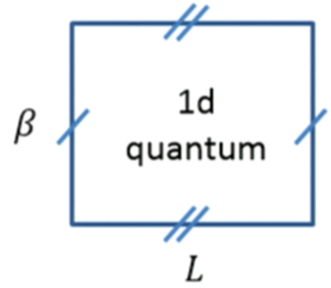


Euclidean time evolution on different geometries

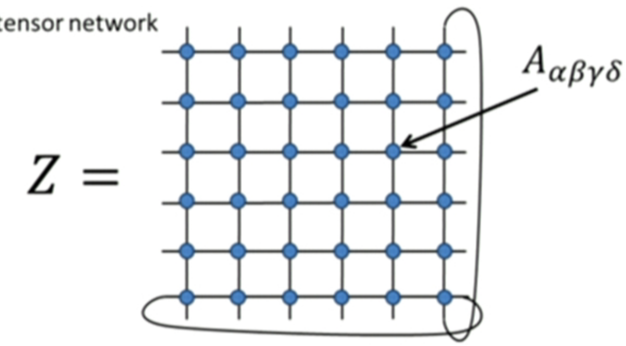


Euclidean path integral

$$Z(\lambda) = tr\ e^{-\beta H_q^{1d}}$$



as a tensor network



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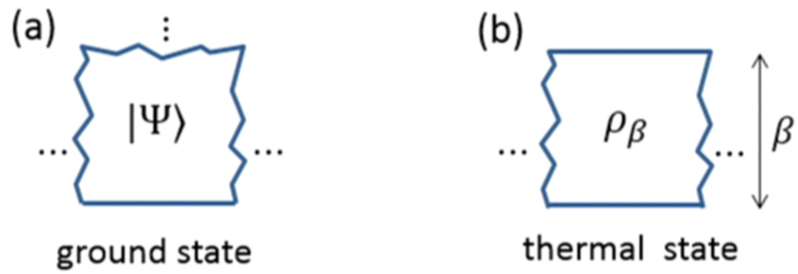

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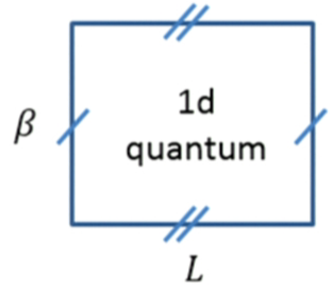
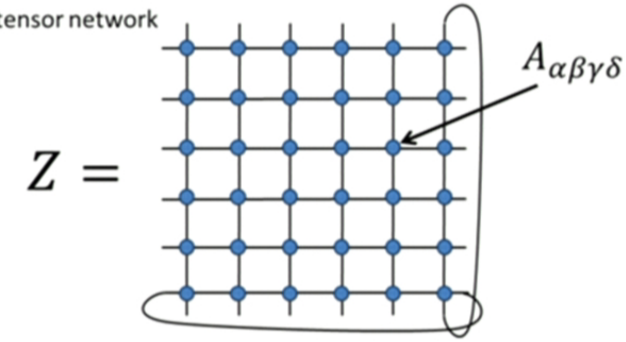


as a tensor network



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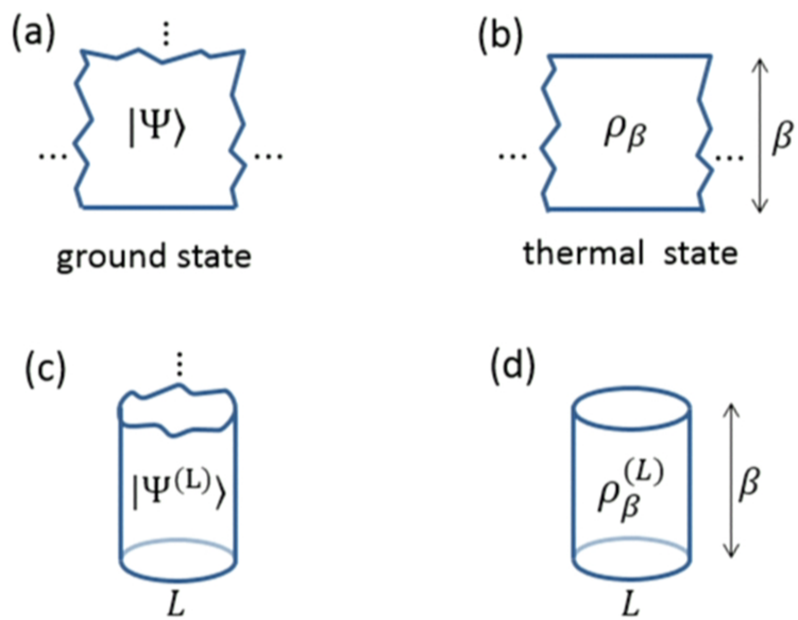


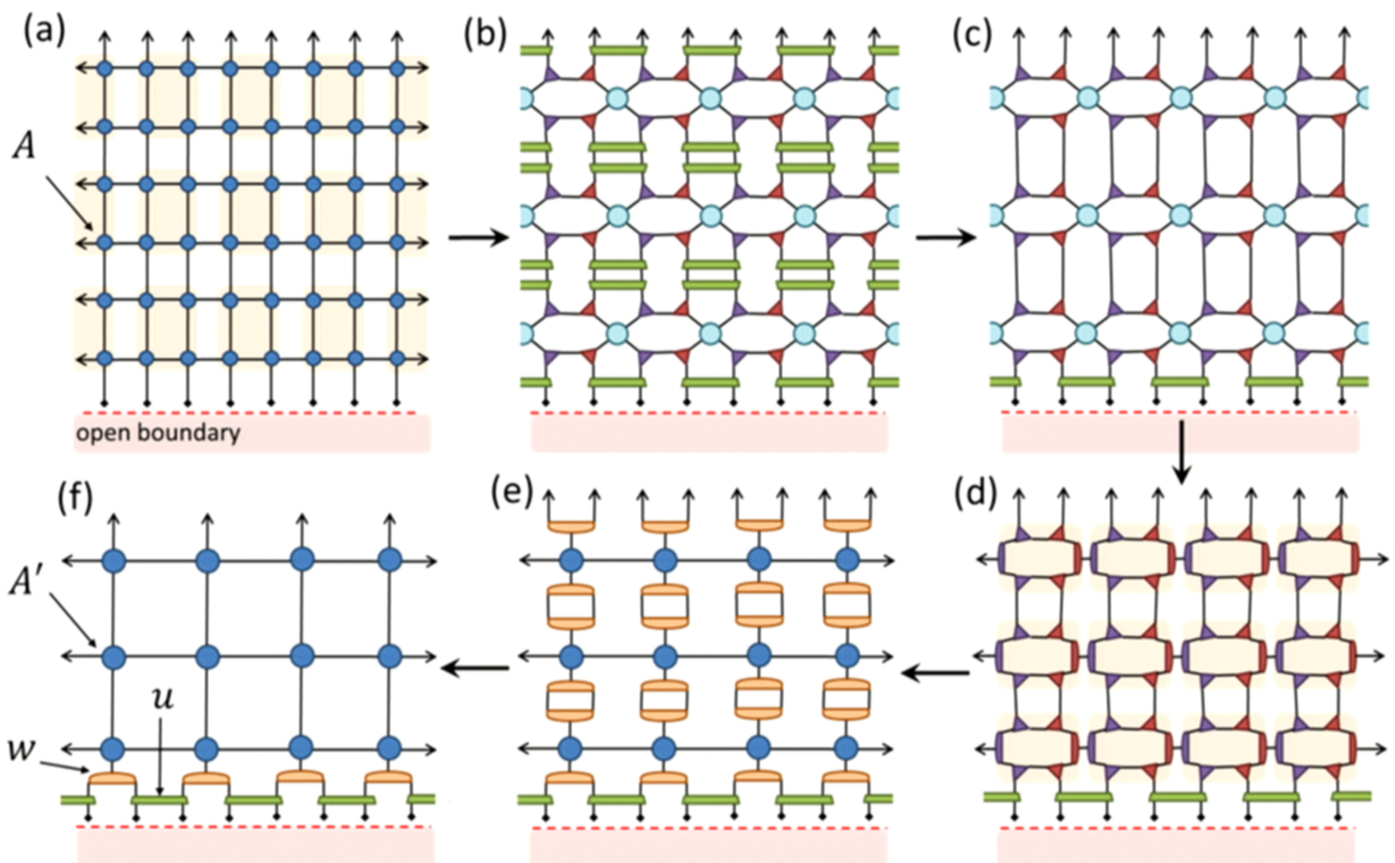
local scale transformations

example 2:

[Evenbly, Vidal, 15]

Upper half plane to hyperbolic plane

$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$



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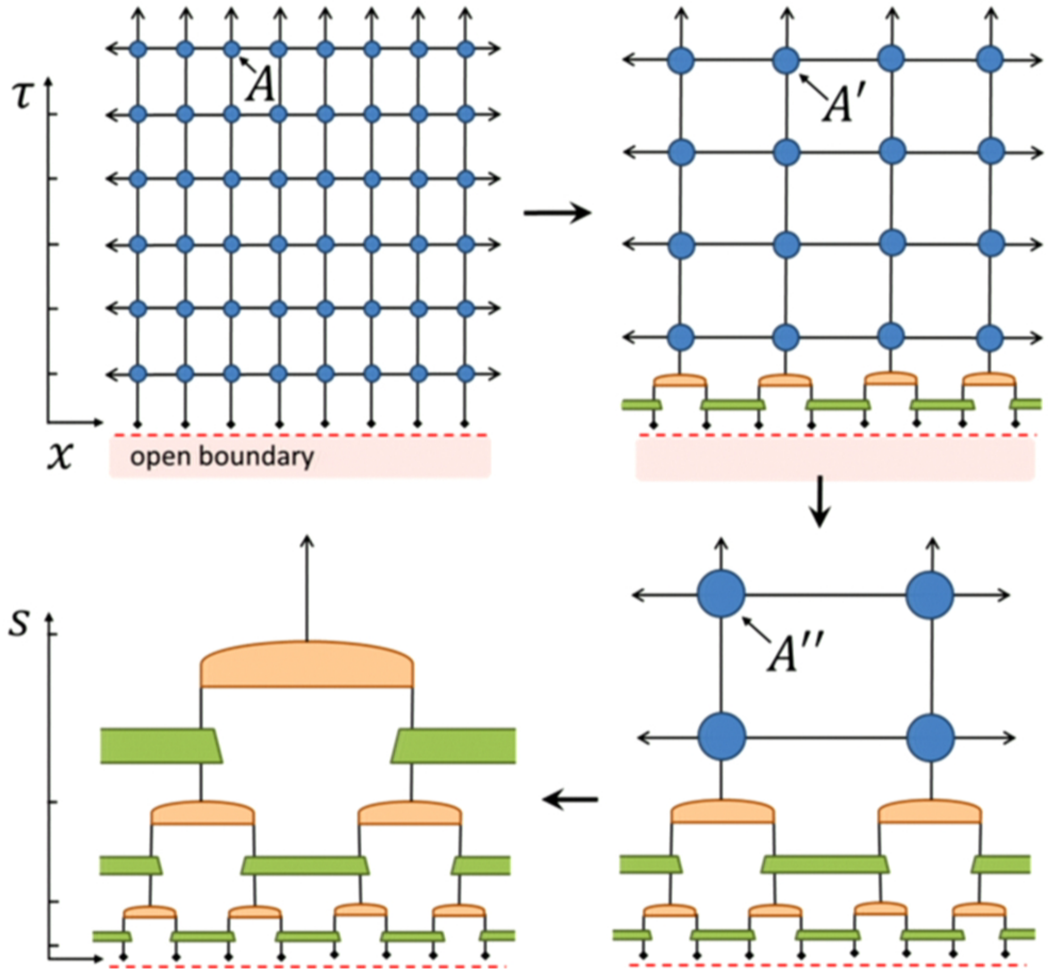
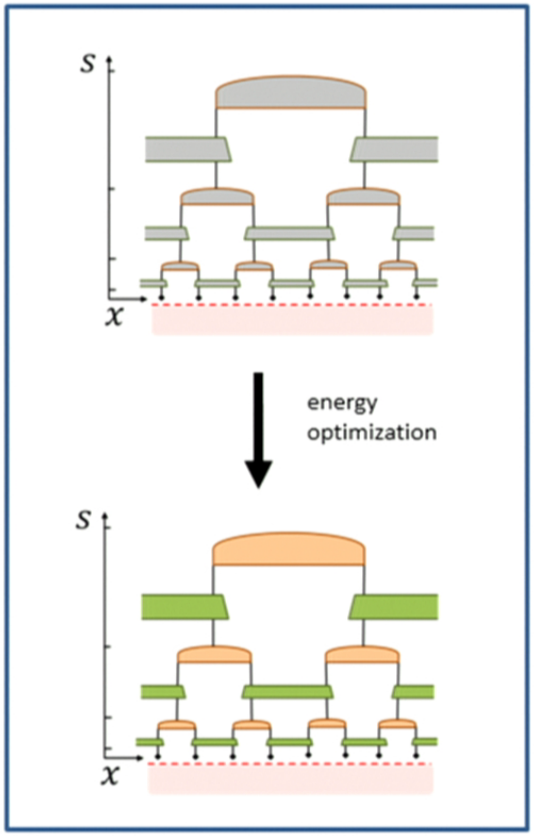


MERA = variational ansatz



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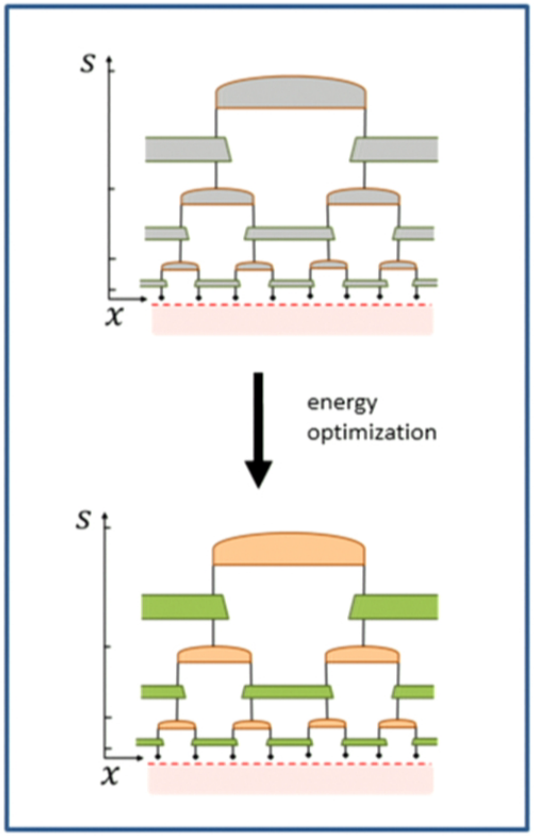
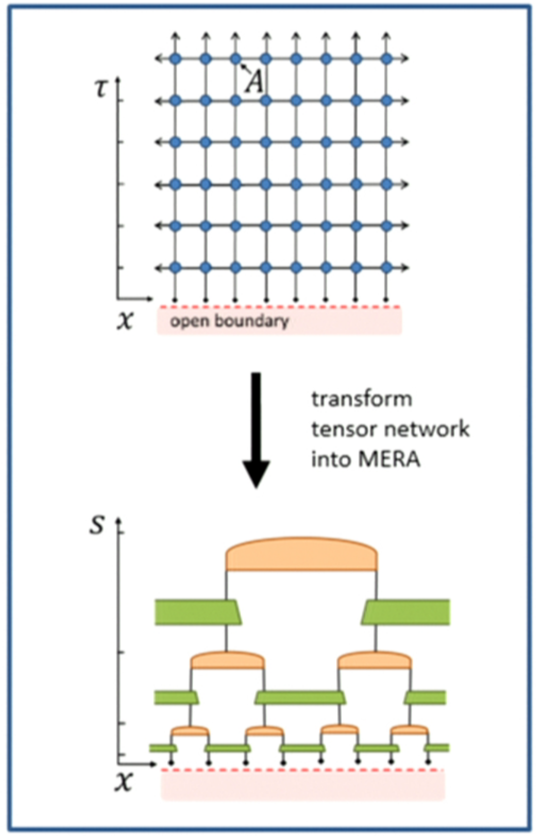
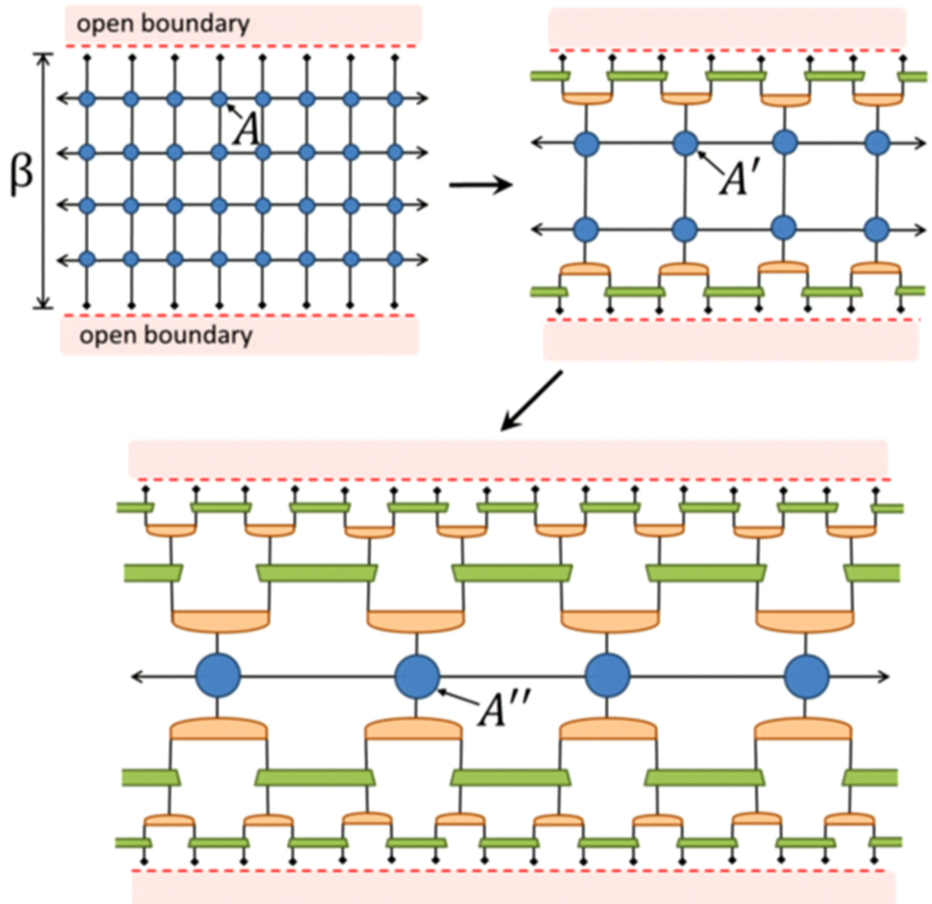


MERA = by-product of TNR



MERA for a thermal state (or black hole in holography)

$\rho_\beta \sim e^{-\beta H}$



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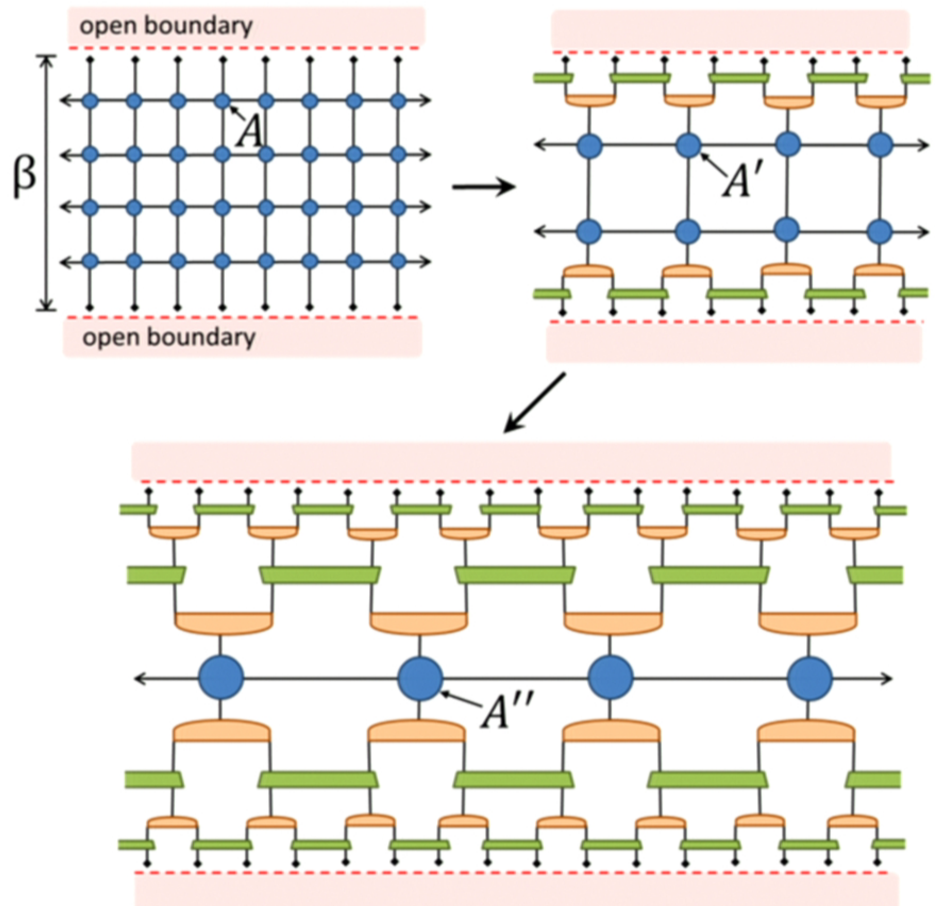


Summary: three lectures on Tensor Networks

Lecture 1

Lecture 2

Lecture 3

Summary: three lectures on Tensor Networks

Lecture 1

Tensor networks and many-body entanglement

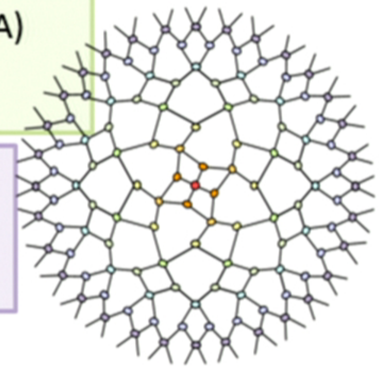
$$S_L^{gapped} = const.$$

$$S_L^{gapless} = \log L$$

Matrix product state (MPS)

Lecture 2

Multi-scale entanglement renormalization ansatz (MERA)



Lecture 3

MERA and holography

Tensor network renormalization (TNR)