Title: TBA

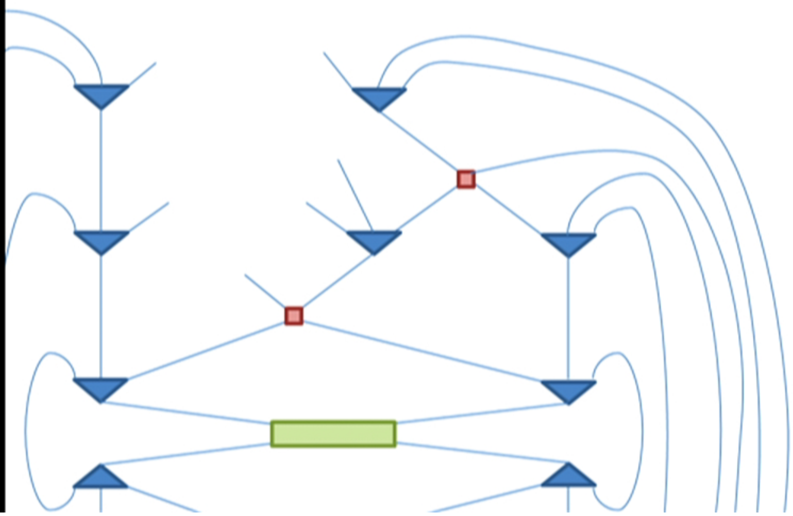
Date: Aug 26, 2015 12:00 PM

URL: http://pirsa.org/15080048

Abstract: TBA

MATHEMATICA, TENSOR NETWORKS, MERA AND ENTANGLEMENT

TOWARDS HOLOGRAPHY FROM THE FIELD THEORY SIDE



Wilke van der Schee

Mathematica School, PI, August 2015

OUTLINE

MERA, entanglement and AdS/MERA

- Tensor networks and contractions
- Entanglement on a slice of AdS?
- Goal: do MERA computations, and make AdS/MERA quantitative
  - →get simple MERA example + routines and do exercises

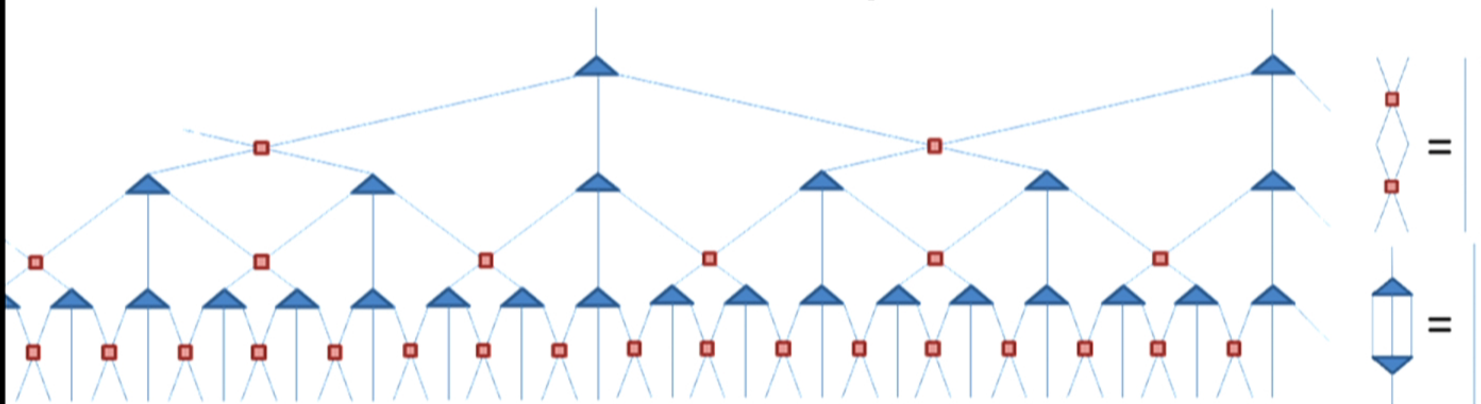
Frequently used *Mathematica*

- Making and using packages
- NDSolve and/or spectral methods
- Transforming functions/coordinate transformations

MULTISCALE ENTANGLEMENT RENORMALISATION ANSATZ (MERA)

MPS correlations/entanglement requires larger χ

Choose different *ansatz* to incorporate RG flow:



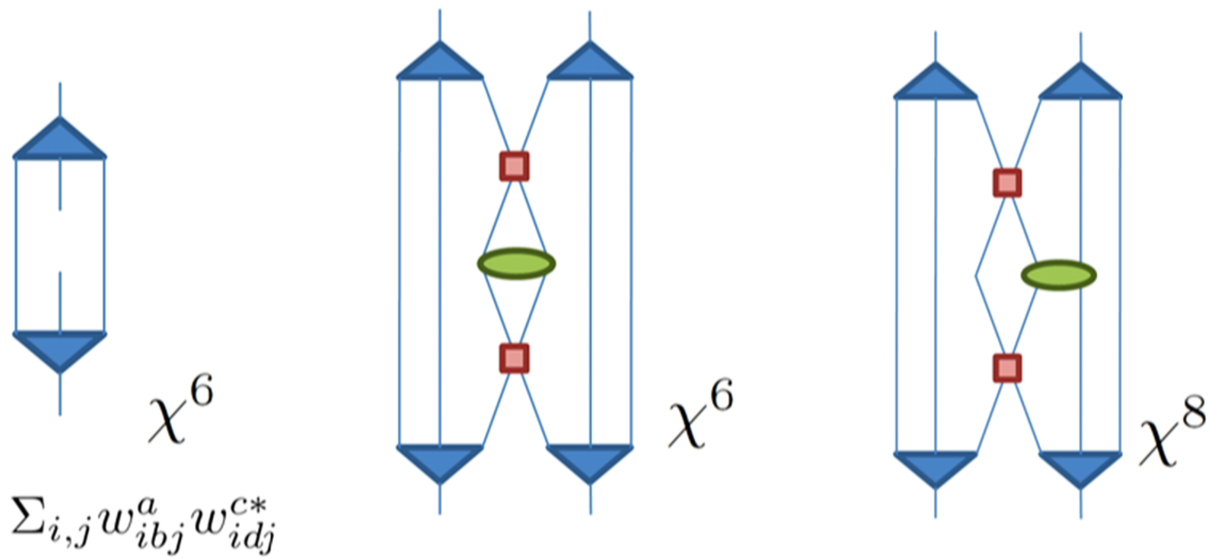
Disentanglers and coarse grainers (ternary)

Extra advantage: scale invariance is very natural!

G. Vidal, Entanglement Renormalization (2007)

FUN FACT: TENSOR CONTRACTIONS NP COMPLETE

Algorithm depends crucially on `efficiently contractible'

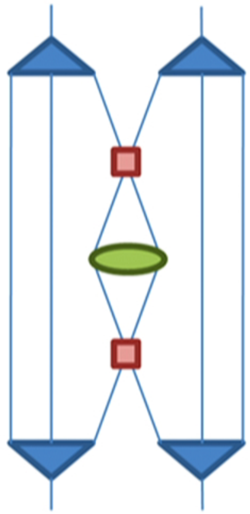


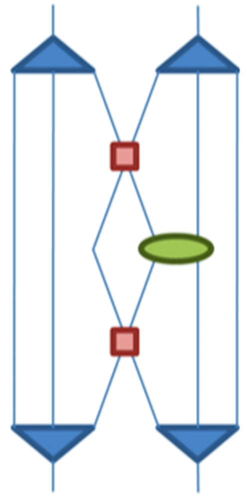
Much harder for 2 dimensions (i.e. χ^{16} or χ^{23})

R.N.C. Pfeifer, J. Haegeman and F. Verstraete, Faster identification of optimal contraction sequences for tensor networks (2013)

FUN FACT: TENSOR CONTRACTIONS NP COMPLETE

Algorithm depends crucially on `efficiently contractible'



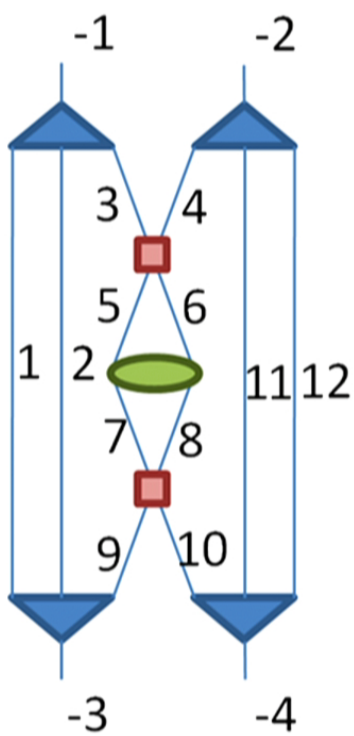


Much harder for 2 dimensions (i.e. χ^{16} or χ^{23})

R.N.C. Pfeifer, J. Haegeman and F. Verstraete, Faster identification of optimal contraction sequences for tensor networks (2013)

NCON FUNCTION

Idea: contract sequentially, contracting two tensors at a time:




```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
]
```


R. N. C. Pfeifer, G. Evenbly, S. Singh, G. Vidal, NCON: A tensor network contractor for MATLAB (2014)

presentation.nb * - Wolfram Mathematica 10.0 (New Kernel)

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Slide 2 of 5

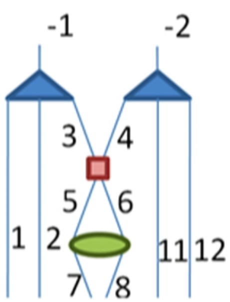
```
   legs1 = Drop[legs1, 2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
  (*rearranging legs according to -legs*)
  If[Length[legs1[[1]]] > 0, Transpose[tens1[[1]], -legs1[[1]]], tens1[[1]]]
 )
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

```
In[3]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
         {5, 6, 7, 8}}

Out[3]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}
```

```
nconp[{w, w*, w, w*, u, u*, operator},
 {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}]
```




```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
```

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R. N. C. Pfeifer, G. Evenbly, S. Singh, G. Vidal, NCON: A tensor network contractor for MATLAB (2014)

presentation.nb * - Wolfram Mathematica 10.0 (New Kernel)

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```
  While[Length[legs1] > 1, (*outer product, only at end*)
   tens1 = Drop[tens1, 2]~Join~{out[tens1[[1]], tens1[[2]]]};
   legs1 = Drop[legs1, 2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
  (*rearranging legs according to -legs*)
  If[Length[legs1[[1]]] > 0, Transpose[tens1[[1]], -legs1[[1]]], tens1[[1]]]
 )
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
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IdentityMatrix, change order, translational operator

```
In[3]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
   {5, 6, 7, 8}}

Out[3]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[4]:= Select[Flatten[legs], # > 0 &]

Out[4]= {1, 2, 3, 1, 2, 9, 4, 11, 12, 10, 11, 12, 3, 4, 5, 6, 7, 8, 9, 10, 5, 6, 7, 8}

nconp[{w, w*, w, w*, u, u*, operator},
 {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}]
```

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12:30 Wednesday 26-Aug-15

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Slide 2 of 5

```
  While[Length[legs1] > 1, (*outer product, only at end*)
   tens1 = Drop[tens1, 2]~Join~{out[tens1[[1]], tens1[[2]]]};
   legs1 = Drop[legs1, 2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
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  If[Length[legs1[[1]]] > 0, Transpose[tens1[[1]], -legs1[[1]]], tens1[[1]]]
 )
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

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In[3]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
   {5, 6, 7, 8}}

Out[3]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

nconp[{w, w*, w, w*, u, u*, operator},
 {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}]

  -1     -2
```

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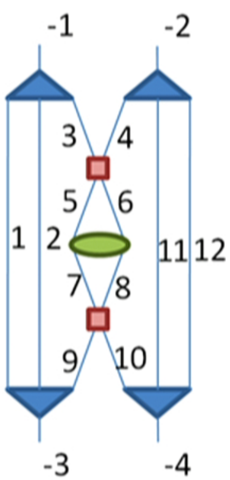
```
In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[7]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

        ncon[{w, w*, w, w*, u, u*, operator},
         {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}]
```




```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
]
```

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```
In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[7]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[10]:= ncon[{w, w*, w, w*, u, u*, operator},
          {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}]

Out[10]= {{{{1.67714×10^7, 1.63121×10^7, 1.72036×10^7, 1.61852×10^7, 1.6415×10^7, 1.58304×10^7},
    {1.63379×10^7, 1.58947×10^7, 1.67618×10^7, 1.57665×10^7, 1.59925×10^7, 1.54246×10^7},
    {1.7395×10^7, 1.69006×10^7, 1.7829×10^7, 1.67803×10^7, 1.70124×10^7, 1.63999×10^7},
    {1.64471×10^7, 1.59715×10^7, 1.68528×10^7, 1.58708×10^7, 1.60824×10^7, 1.54986×10^7},
    {1.6526×10^7, 1.60656×10^7, 1.69457×10^7, 1.59466×10^7, 1.617×10^7, 1.55911×10^7},
    {1.58288×10^7, 1.5378×10^7, 1.62247×10^7, 1.5277×10^7, 1.54836×10^7, 1.49246×10^7}},
   {{1.62487×10^7, 1.74267×10^7, 1.70935×10^7, 1.62246×10^7, 1.65623×10^7, 1.59648×10^7},
    {1.58286×10^7, 1.69816×10^7, 1.66546×10^7, 1.58049×10^7, 1.61366×10^7, 1.55561×10^7},
    {1.68533×10^7, 1.8053×10^7, 1.77162×10^7, 1.682×10^7, 1.71644×10^7, 1.65369×10^7},
    {1.59357×10^7, 1.70597×10^7, 1.67461×10^7, 1.59098×10^7, 1.62261×10^7, 1.56283×10^7},
    {1.60109×10^7, 1.71619×10^7, 1.68367×10^7, 1.59842×10^7, 1.63137×10^7, 1.5722×10^7},
```

Slide 2 of 5

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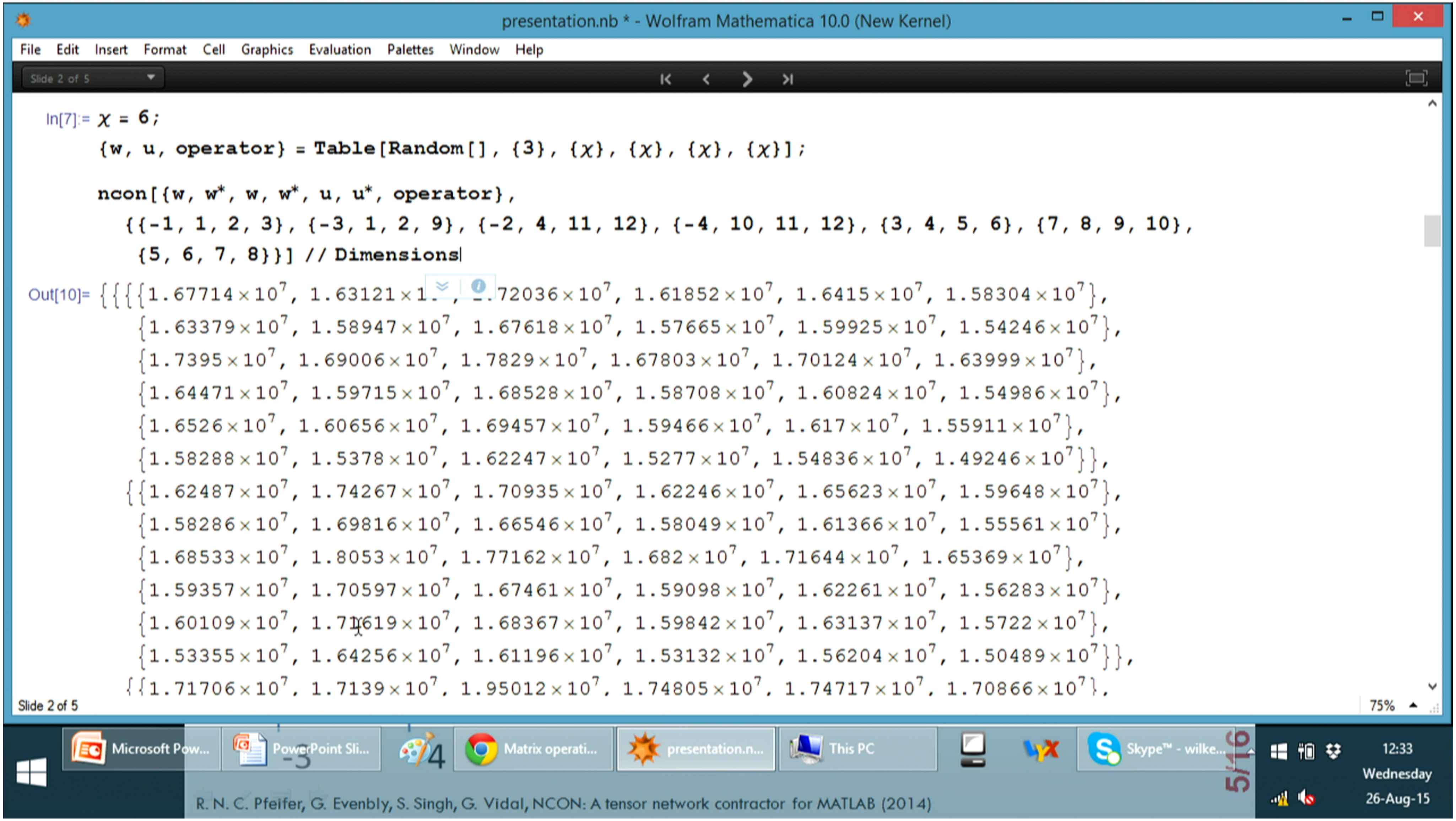
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File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Slide 2 of 5
In[7]:= χ = 6;
{w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];
ncon[{w, w*, w, w*, u, u*, operator},
{{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
{5, 6, 7, 8}}] // Dimensions
Out[10]= {{{{1.67714×10^7, 1.63121×1
.72036×10^7, 1.61852×10^7, 1.6415×10^7, 1.58304×10^7},
{1.63379×10^7, 1.58947×10^7, 1.67618×10^7, 1.57665×10^7, 1.59925×10^7, 1.54246×10^7},
{1.7395×10^7, 1.69006×10^7, 1.7829×10^7, 1.67803×10^7, 1.70124×10^7, 1.63999×10^7},
{1.64471×10^7, 1.59715×10^7, 1.68528×10^7, 1.58708×10^7, 1.60824×10^7, 1.54986×10^7},
{1.6526×10^7, 1.60656×10^7, 1.69457×10^7, 1.59466×10^7, 1.617×10^7, 1.55911×10^7},
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{{1.62487×10^7, 1.74267×10^7, 1.70935×10^7, 1.62246×10^7, 1.65623×10^7, 1.59648×10^7},
{1.58286×10^7, 1.69816×10^7, 1.66546×10^7, 1.58049×10^7, 1.61366×10^7, 1.55561×10^7},
{1.68533×10^7, 1.8053×10^7, 1.77162×10^7, 1.682×10^7, 1.71644×10^7, 1.65369×10^7},
{1.59357×10^7, 1.70597×10^7, 1.67461×10^7, 1.59098×10^7, 1.62261×10^7, 1.56283×10^7},
{1.60109×10^7, 1.71619×10^7, 1.68367×10^7, 1.59842×10^7, 1.63137×10^7, 1.5722×10^7},
{1.53355×10^7, 1.64256×10^7, 1.61196×10^7, 1.53132×10^7, 1.56204×10^7, 1.50489×10^7}},
{{1.71706×10^7, 1.7139×10^7, 1.95012×10^7, 1.74805×10^7, 1.74717×10^7, 1.70866×10^7},
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Wednesday
26-Aug-15
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presentation.nb * - Wolfram Mathematica 10.0 (New Kernel)

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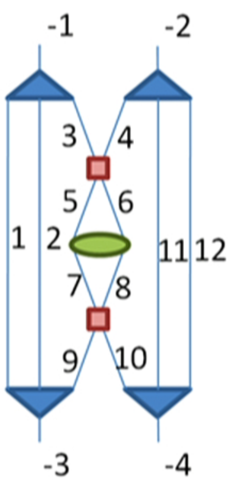
```
In[7]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[11]:= ncon[{w, w*, w, w*, u, u*, operator},
          {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
           {5, 6, 7, 8}}] // Dimensions

Out[11]= {6, 6, 6, 6}
```




```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
]
```

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presentation.nb * - Wolfram Mathematica 10.0 (New Kernel)

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```
  (List /@ Select[Range[ArrayDepth[a]], m ↦ ! MemberQ[Flatten[links[[ ;; ]]], m]]);
  Tr[Flatten[a, rearrange], Plus, 2]
 )

ncon[tensors_, legs_, sequence_] := (
  {tens1, legs1} = {tensors, legs};
  For[inc = 1, inc ≤ Length[sequence],
   {contr1, contr2} = Position[legs1, sequence[[inc]]][[ ;; , 1]];
   (*tensors to be contracted*)
   For[links = {}, (*look if we are to contract more than one leg*)
    Length[links] + inc ≤ Length[sequence] &&
     {contr1, contr2} == Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 1]],
    AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]]];

   nlegs = Select[legs1[[contr1]] ~Join~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
     ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #] &]; (*combine legs*)

   If[contr1 == contr2, (*trace, remove tensor and add traced*)
    tens1 = Drop[tens1, {contr2}] ~Join~ {trace[tens1[[contr1]], links, Length[tensors] > 1]};
    legs1 = Drop[legs1, {contr2}] ~Join~ {nlegs};
    , (*contract, remove tensor and add contracted*)
```

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presentation.nb * - Wolfram Mathematica 10.0 (New Kernel)

```
    (List /@ Select[Range[ArrayDepth[a]], m ↦ ! MemberQ[Flatten[links[[;; ]]], m]]);
    Tr[Flatten[a, rearrange], Plus, 2]
   )

In[12]:= legs

Out[12]= {{-1, 1, 1, 3}, {-3, 2, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[13]:= sequence = Range[12]

Out[13]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[14]:= inc = 1

Out[14]= 1

In[17]:= sequence[[inc]]

Out[17]= 1

In[16]:= Position[legs, sequence[[inc]]]

Out[16]= {{1, 2}, {1, 3}}

ncon[tensors_, legs_, sequence_] := (
  {tens1, legs1} = {tensors, legs};
  For[inc = 1, inc ≤ Length[sequence],
```

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```
In[13]:= sequence = Range[12]
Out[13]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[14]:= inc = 1
Out[14]= 1

In[17]:= sequence[[inc]]
Out[17]= 1

In[20]:= Position[legs, sequence[[inc]]][[ ;; , 1]]
Out[20]= {1, 2}

ncon[tensors_, legs_, sequence_] := (
  {tens1, legs1} = {tensors, legs};
  For[inc = 1, inc ≤ Length[sequence],
   {contr1, contr2} = Position[legs1, sequence[[inc]]][[ ;; , 1]];
   (*tensors to be contracted*)
   For[links = {}, (*look if we are to contract more than one leg*)
    Length[links] + inc ≤ Length[sequence] &&
     {contr1, contr2} == Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 1]],
    AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]];
```

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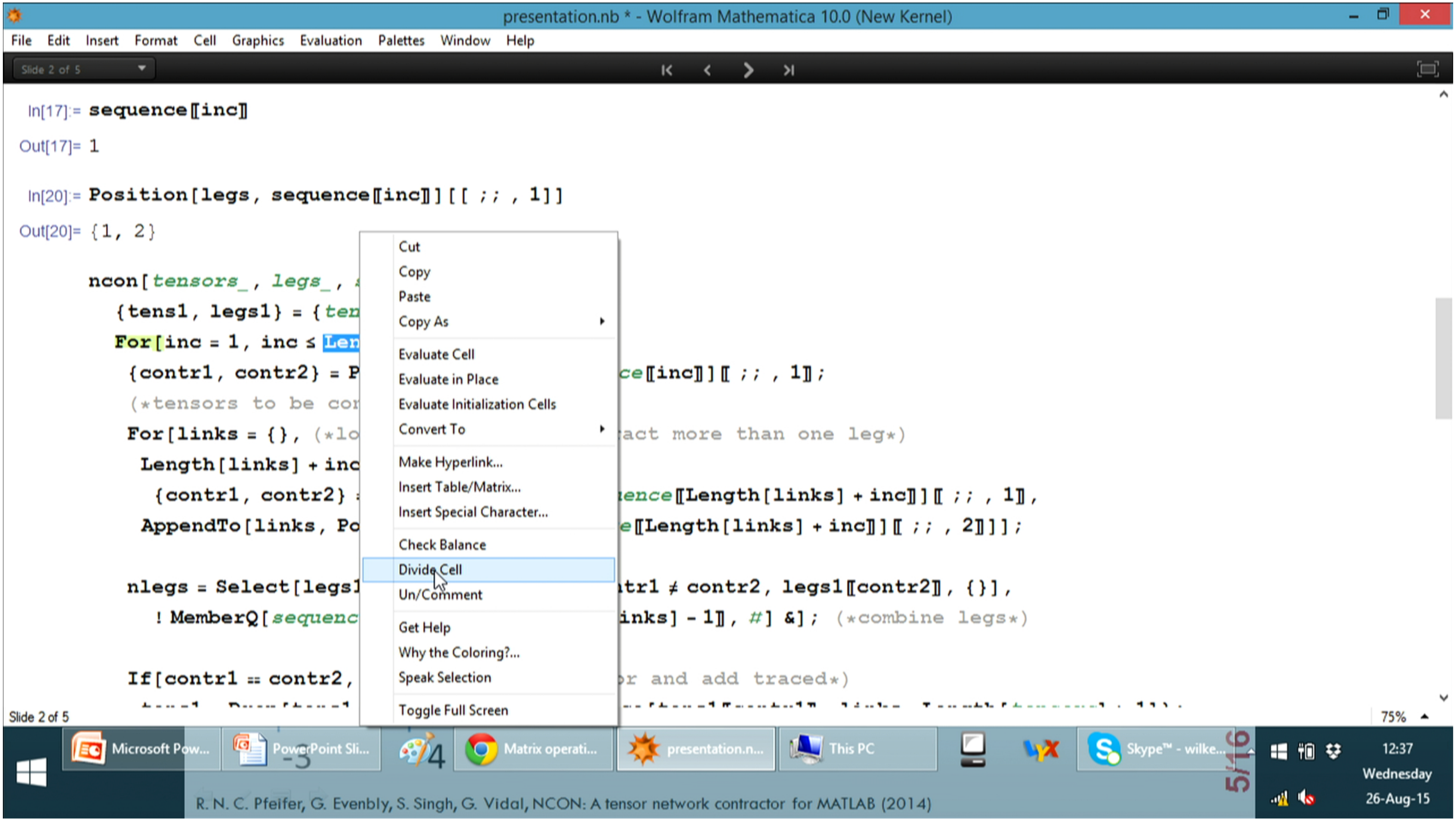

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Slide 2 of 5
In[17]:= sequence[[inc]]
Out[17]= 1
In[20]:= Position[legs, sequence[[inc]]][[;; , 1]]
Out[20]= {1, 2}
ncon[tensors_, legs_,
{tens1, legs1} = {ten
For[inc = 1, inc ≤ Len
{contr1, contr2} = P
ce[[inc]]][[;; , 1]];
(*tensors to be co
For[links = {}, (*lo
act more than one leg*)
Length[links] + inc
{contr1, contr2}
ence[[Length[links] + inc]]][[;; , 1]],
AppendTo[links, Po
e[[Length[links] + inc]]][[;; , 2]]]];
nlegs = Select[legs1
tr1 ≠ contr2, legs1[[contr2]], {}],
! MemberQ[sequenc
inks] - 1]], #] &]; (*combine legs*)
If[contr1 == contr2,
r and add traced*)
Cut
Copy
Paste
Copy As
Evaluate Cell
Evaluate in Place
Evaluate Initialization Cells
Convert To
Make Hyperlink...
Insert Table/Matrix...
Insert Special Character...
Check Balance
Divide Cell
Un/Comment
Get Help
Why the Coloring?...
Speak Selection
Toggle Full Screen
Slide 2 of 5
75%
Microsoft Pow...
PowerPoint Sli...
Matrix operati...
presentation.n...
This PC
Skype™ - wilke...
12:37
Wednesday
26-Aug-15
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```
In[17]:= sequence[[inc]]

Out[17]= 1

In[20]:= Position[legs, sequence[[inc]]][[ ;; , 1]]

Out[20]= {1, 2}

In[24]:= {contr1, contr2}

Out[24]= {1, 2}

In[21]:= ncon[tensors_, legs_, sequence_] := (
      {tens1, legs1} = {tensors, legs};
      For[inc = 1, inc ≤ (*Length[sequence]*)1,
       {contr1, contr2} = Position[legs1, sequence[[inc]]][[ ;; , 1]];
       (*tensors to be contracted*)
       For[links = {}, (*look if we are to contract more than one leg*)
        Length[links] + inc ≤ Length[sequence] &&
         {contr1, contr2} == Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 1]],
        AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]]];

       nlegs = Select[legs1[[contr1]] ~Join~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
         ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #1] &]; (*combine legs*)
```

Slide 2 of 5 75%

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12:38 Wednesday 26-Aug-15

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Slide 2 of 5

```
(*tensors to be contracted*)
For[links = {}, (*look if we are to contract more than one leg*)
 Length[links] + inc ≤ Length[sequence] &&
  {contr1, contr2} == Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 1]],
 AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]]];

nlegs = Select[legs1[[contr1]] ~Join~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
  ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #] &]; (*combine legs*)

If[contr1 == contr2, (*trace, remove tensor and add traced*)
 tens1 = Drop[tens1, {contr2}] ~Join~ {trace[tens1[[contr1]], links, Length[tensors] > 1]};
 legs1 = Drop[legs1, {contr2}] ~Join~ {nlegs};
 , (*contract, remove tensor and add contracted*)
 tens1 = Drop[Drop[tens1, {contr2}], {contr1}] ~Join~ {contract[tens1[[contr1]], tens1[[contr2]], links]};
 legs1 = Drop[Drop[legs1, {contr2}], {contr1}] ~Join~ {nlegs}];
inc += Length[links]];
(*While[Length[legs1]>1,(*outer product, only at end*)
tens1=Drop[tens1,2]~Join~{out[tens1[[1]],tens1[[2]]]};
legs1=Drop[legs1,2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
(*rearranging legs according to -legs*)
If[Length[legs1[[1]]]>0,Transpose[tens1[[1]],-legs1[[1]]],tens1[[1]]]
```

Slide 2 of 5 75%

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$$A_{\alpha\,\underline{\alpha_n}\,\beta} \quad B_{\beta\,\underline{\gamma_1}\,\gamma_2\,\gamma_3}$$

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Ncon

```
In[29]:= rearrangea
         rearrangeb

Out[29]= {{1}, {4}, {2, 3}}

Out[30]= {{2, 3}, {1}, {4}}
```

```
contract[a_, b_, links_] := ((*contract takes 2 tensors and the links which have to be contracted;
  it then rearranges (with Flatten) the tensors in such a way that the summation can be
   done using an ordinary dot product, i.e. A.B sums the last index of A with the first index of B*)
  rearrangea = (List /@ Select[Range[ArrayDepth[a]], m ↦ ! MemberQ[links⟦ ;; , 1⟧, m]]) ~Join~
    {links⟦ ;; , 1⟧};
  rearrangeb = {links⟦ ;; , 2⟧} ~Join~
    (List /@ Select[Range[ArrayDepth[b]], m ↦ ! MemberQ[links⟦ ;; , 2⟧, m]]);
  Flatten[a, rearrangea].Flatten[b, rearrangeb]
 )
out[a_, b_] := contract[{a}, {b}, {{1, 1}}]

trace[a_, links_, print_: True] := (
```

Slide 2 of 5

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Slide 2 of 5

```
  AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]];

 nlegs = Select[legs1[[contr1]] ~ Join ~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
   ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #] &]; (*combine legs*)

 If[contr1 == contr2, (*trace, remove tensor and add traced*)
  tens1 = Drop[tens1, {contr2}] ~ Join ~ {trace[tens1[[contr1]], links, Length[tensors] > 1]};
  legs1 = Drop[legs1, {contr2}] ~ Join ~ {nlegs};
  , (*contract, remove tensor and add contracted*)
  tens1 = Drop[Drop[tens1, {contr2}], {contr1}] ~ Join ~ {contract[tens1[[contr1]], tens1[[contr2]], links]};
  legs1 = Drop[Drop[legs1, {contr2}], {contr1}] ~ Join ~ {nlegs}];
 inc += Length[links]];
(*While[Length[legs1]>1,(*outer product, only at end*)
 tens1=Drop[tens1,2]~Join~{out[tens1[[1]],tens1[[2]]]};
 legs1=Drop[legs1,2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
(*rearranging legs according to -legs*)
If[Length[legs1[[1]]]>0,Transpose[tens1[[1]],-legs1[[1]]],tens1[[1]]]
*))
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

Slide 2 of 5 75%

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Slide 2 of 5

```
   AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]];

  nlegs = Select[legs1[[contr1]] ~ Join ~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
    ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #] &]; (*combine legs*)

  If[contr1 == contr2, (*trace, remove tensor and add traced*)
   tens1 = Drop[tens1, {contr2}] ~ Join ~ {trace[tens1[[contr1]], links, Length[tensors] > 1]};
   legs1 = Drop[legs1, {contr2}] ~ Join ~ {nlegs};
   , (*contract, remove tensor and add contracted*)
   tens1 = Drop[Drop[tens1, {contr2}], {contr1}] ~ Join ~ {contract[tens1[[contr1]], tens1[[contr2]], links]};
   legs1 = Drop[Drop[legs1, {contr2}], {contr1}] ~ Join ~ {nlegs}];
  inc += Length[links]];
 (*While[Length[legs1]>1,(*outer product, only at end*)
  tens1=Drop[tens1,2]~Join~{out[tens1[[1]],tens1[[2]]]};
  legs1=Drop[legs1,2]~Join~{legs1[[1]]~Join~legs1[[2]]}];
 (*rearranging legs according to -legs*)
 If[Length[legs1[[1]]]>0,Transpose[tens1[[1]],-legs1[[1]]],tens1[[1]]]
 *))
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

Slide 2 of 5

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Slide 2 of 5

```
  AppendTo[links, Position[legs1, sequence[[Length[links] + inc]]][[ ;; , 2]]];

 nlegs = Select[legs1[[contr1]] ~ Join ~ If[contr1 ≠ contr2, legs1[[contr2]], {}],
    ! MemberQ[sequence[[inc ;; inc + Length[links] - 1]], #] &]; (*combine legs*)

 If[contr1 == contr2, (*trace, remove tensor and add traced*)
  tens1 = Drop[tens1, {contr2}] ~ Join ~ {trace[tens1[[contr1]], links, Length[tensors] > 1]};
  legs1 = Drop[legs1, {contr2}] ~ Join ~ {nlegs};
  , (*contract, remove tensor and add contracted*)
  tens1 = Drop[Drop[tens1, {contr2}], {contr1}] ~ Join ~ {contract[tens1[[contr1]], tens1[[contr2]], links]};
  legs1 = Drop[Drop[legs1, {contr2}], {contr1}] ~ Join ~ {nlegs}];
 inc += Length[links]];
While[Length[legs1] > 1, (*outer product, only at end*)
 tens1 = Drop[tens1, 2] ~ Join ~ {out[tens1[[1]], tens1[[2]]]};
 legs1 = Drop[legs1, 2] ~ Join ~ {legs1[[1]] ~ Join ~ legs1[[2]]}];
(*rearranging legs according to -legs*)
If[Length[legs1[[1]]] > 0, Transpose[tens1[[1]], -legs1[[1]]], tens1[[1]]]
)
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

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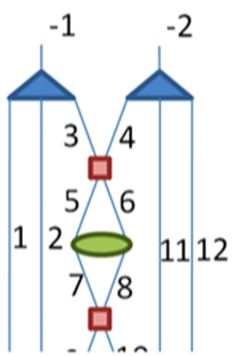
```
In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[7]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[36]:= ncon[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
            {5, 6, 7, 8}}] // Dimensions

No more memory available.
Mathematica kernel has shut down.
Try quitting other applications and then retry.
```




```
ncon[{w,w',w,w',u,u',o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
]
```

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Packages

- **Almost always useful: different notebooks for different parameters/options, simulations versus analysing**
- **Easy to make: convert initialisation cells to package**

  Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options -->

  File Options --> Autogenerated package --> Manual

- **Either put parameters/options before loading package or after**

Example:

```
In[1]:= SetDirectory[NotebookDirectory[]];
<< MERA`
```

```
domodel[initIsing, 6]
```

```
analysefile["Ising6.dat"]
```

Slide 1 of 5

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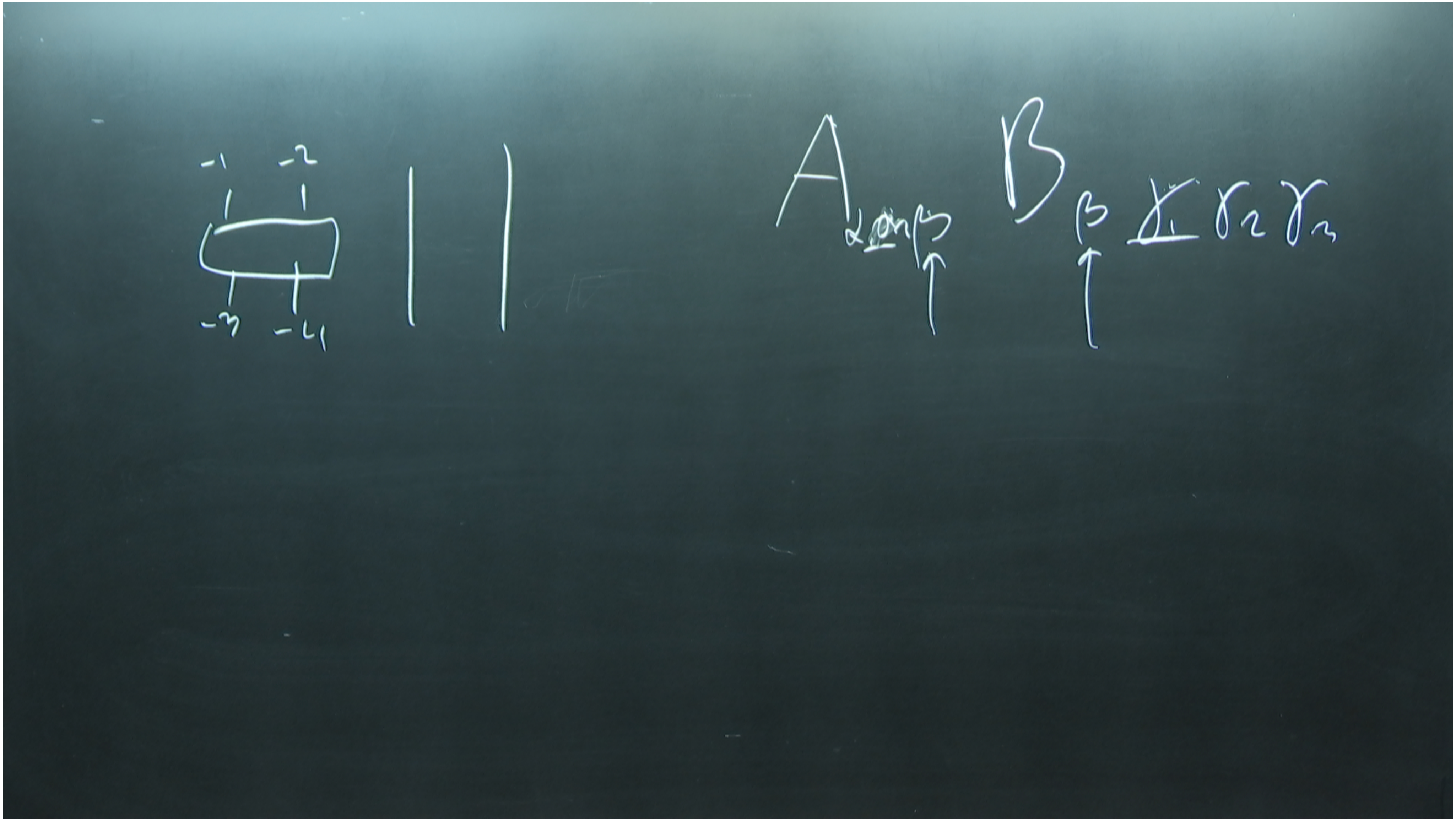
-1
-2
-3
-4

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Slide 2 of 5

```
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

```
ncon[{Identity

In[18]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
   {5, 6, 7, 8}}

Out[18]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[3]:= χ = 6;
  {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[6]:= slow = nconp[{w, w*, w, w*, u, u*, operator},
    {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}];

  total cost is 1χ^6 + 2χ^7 + 2χ^9 + 1χ^10  total: 81228096  (old: 81228096)

In[10]:= fast = nconp[{w, w*, w, w*, u, u*, operator},
    {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}},
    {1, 2, 5, 6, 7, 8, 3, 9, 11, 12, 4, 10}];

  total cost is 6χ^6  total: 279936  (old: 279936)
```

Slide 2 of 5 75%

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Slide 2 of 5

```
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

```
In[13]:= ncon[{IdentityMatrix[2], IdentityMatrix[2]}, {{-1, -3}, {-2, -4}}]

Out[13]= {{{{1, 0}, {0, 0}}, {{0, 1}, {0, 0}}}, {{{0, 0}, {1, 0}}, {{0, 0}, {0, 1}}}}

In[18]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
           {5, 6, 7, 8}}

Out[18]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[3]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];

In[6]:= slow = nconp[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}];

        total cost is 1χ^6 + 2χ^7 + 2χ^9 + 1χ^10  total: 81228096  (old: 81228096)

In[10]:= fast = nconp[{w, w*, w, w*, u, u*, operator},
           {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}},
```

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Slide 2 of 5

```
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

```
In[13]:= ncon[{IdentityMatrix[2], IdentityMatrix[2]}, {{-1, -3}, {-2, -4}}]

Out[13]= {{{{1, 0}, {0, 0}}, {{0, 1}, {0, 0}}}, {{{0, 0}, {1, 0}}, {{0, 0}, {0, 1}}}}

In[15]:= ncon[{IdentityMatrix[2], IdentityMatrix[2]}, {{-1, -2}, {-3, -4}}]
         ncon[{%}, {-1, -3, -2, -4}]

Out[15]= {{{{1, 0}, {0, 1}}, {{0, 0}, {0, 0}}}, {{{0, 0}, {0, 0}}, {{1, 0}, {0, 1}}}}

error: length legs != length tensors
error: tensor 1 should have 4 indices.
Part::partw : Part 2 of {{2, 2, 2, 2}} does not exist. >>
Part::partw : Part 2 of {{2, 2, 2, 2}} does not exist. >>
error: tensor 2 should have 2 indices.
Part::partw : Part 3 of {{2, 2, 2, 2}} does not exist. >>
General::stop : Further output of Part::partw will be suppressed during this calculation. >>
error: tensor 3 should have 2 indices.
error: tensor 4 should have 2 indices.

In[18]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
```

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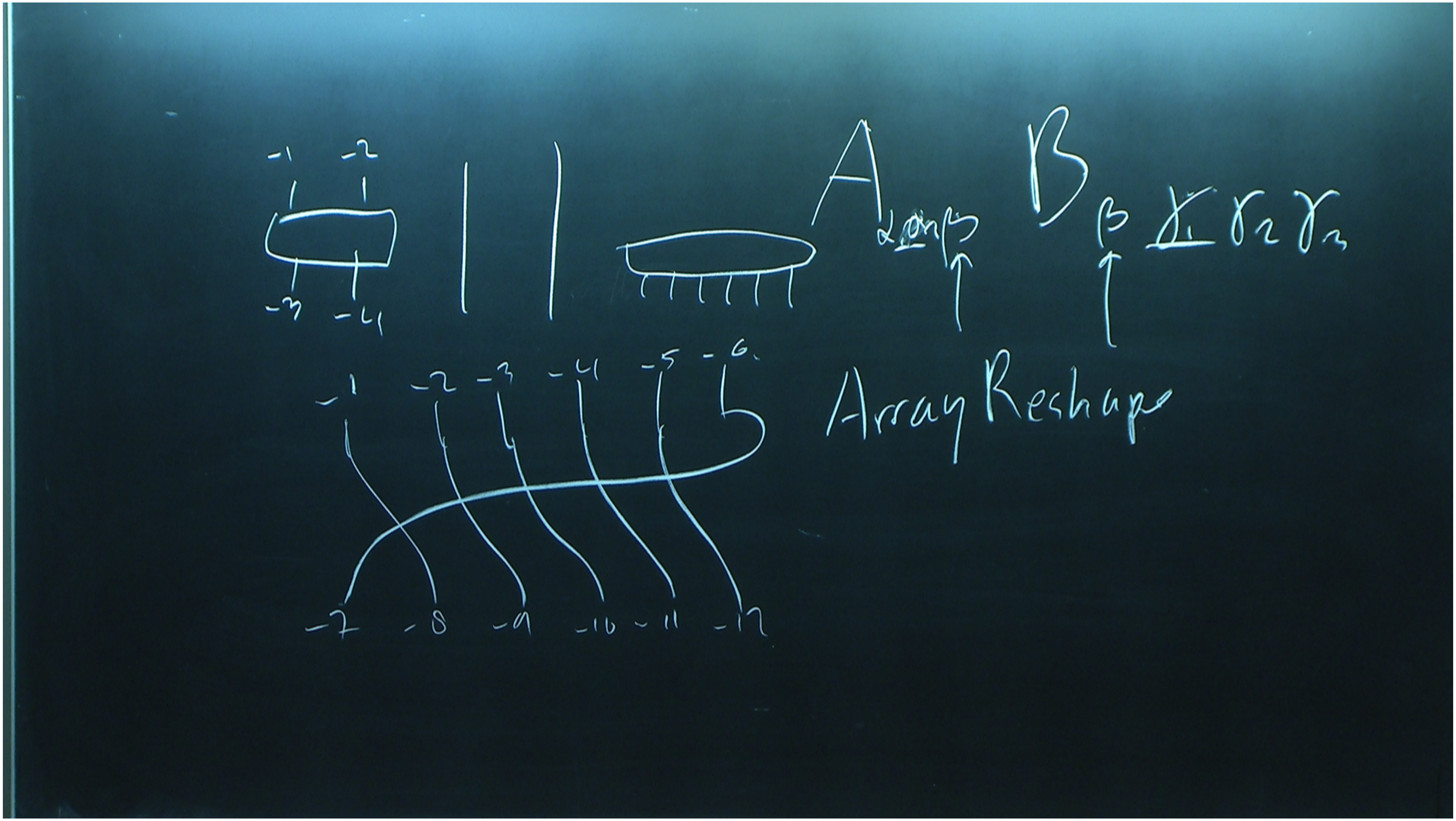
```
ncon[tensors_, legs_] := ncon[tensors, legs, Union[Select[Flatten[legs], # > 0 &]]]
```

IdentityMatrix, change order, translational operator

```
In[13]:= ncon[{IdentityMatrix[2], IdentityMatrix[2]}, {{-1, -3}, {-2, -4}}]

Out[13]= {{{{1, 0}, {0, 0}}, {{0, 1}, {0, 0}}}, {{{0, 0}, {1, 0}}, {{0, 0}, {0, 1}}}}

In[17]:= ncon[{IdentityMatrix[2], IdentityMatrix[2]}, {{-1, -2}, {-3, -4}}]
         ncon[{%}, {{-1, -3, -2, -4}}]

Out[17]= {{{{1, 0}, {0, 1}}, {{0, 0}, {0, 0}}}, {{{0, 0}, {0, 0}}, {{1, 0}, {0, 1}}}}

Out[18]= {{{{1, 0}, {0, 0}}, {{0, 1}, {0, 0}}}, {{{0, 0}, {1, 0}}, {{0, 0}, {0, 1}}}}

In[18]:= legs = {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10},
           {5, 6, 7, 8}}

Out[18]= {{-1, 1, 2, 3}, {-3, 1, 2, 9}, {-2, 4, 11, 12}, {-4, 10, 11, 12}, {3, 4, 5, 6}, {7, 8, 9, 10}, {5, 6, 7, 8}}

In[5]:= Select[Flatten[legs], # > 0 &] // Union

Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

In[3]:= χ = 6;
        {w, u, operator} = Table[Random[], {3}, {χ}, {χ}, {χ}, {χ}];
```

Slide 2 of 5

R. N. C. Pfeifer, G. Evenbly, S. Singh, G. Vidal, NCON: A tensor network contractor for MATLAB (2014)


Array Reshape

ENTANGLEMENT ENTROPY

Reduced density matrix: $\rho_{red,L} = \mathrm{tr}(\rho)$

Obtain mixed state with probabilities: $p_\rho = \mathrm{eig}(\rho_{red,L})$

- **Has entropy:** $S_{EE} = -\sum_i p_{\rho,i} \log(p_{\rho,i}) = \frac{c}{3}\log(L) + \mathcal{O}(1)$
- **I.e. ground state → excited state!**

Ising model: $H_{\text{Ising}} = -\sum_r \left(\lambda\sigma_z^{[r]} + \sigma_x^{[r]}\sigma_x^{[r+1]}\right)$

- **Energy:** $e_0 = -\frac{2}{L\sin(\pi/2L)} \approx -\frac{4}{\pi} - \frac{\pi}{6L^2}$
- **Central charge 1/2**

P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory (2004)

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Packages

- **Almost always useful: different notebooks for different parameters/options, simulations versus analysing**
- **Easy to make: convert initialisation cells to package**

  Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options -->

  File Options --> Autogenerated package --> Manual

- **Either put parameters/options before loading package or after**

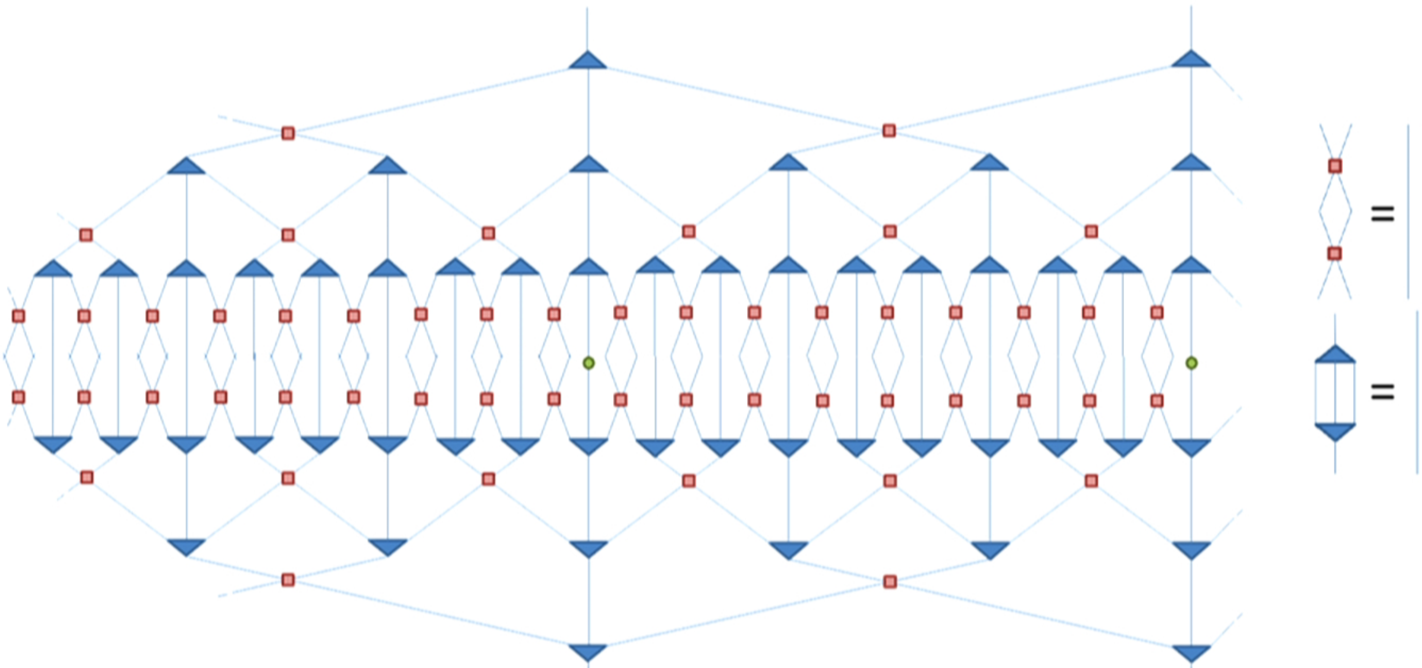
Example:

```
In[19]:= SetDirectory[NotebookDirectory[]];
<< MERA`

domodel[initIsing, 6]

analysefile["Ising6.dat"]
```

Slide 1 of 5

EXAMPLE: CORRELATORS IN MERA

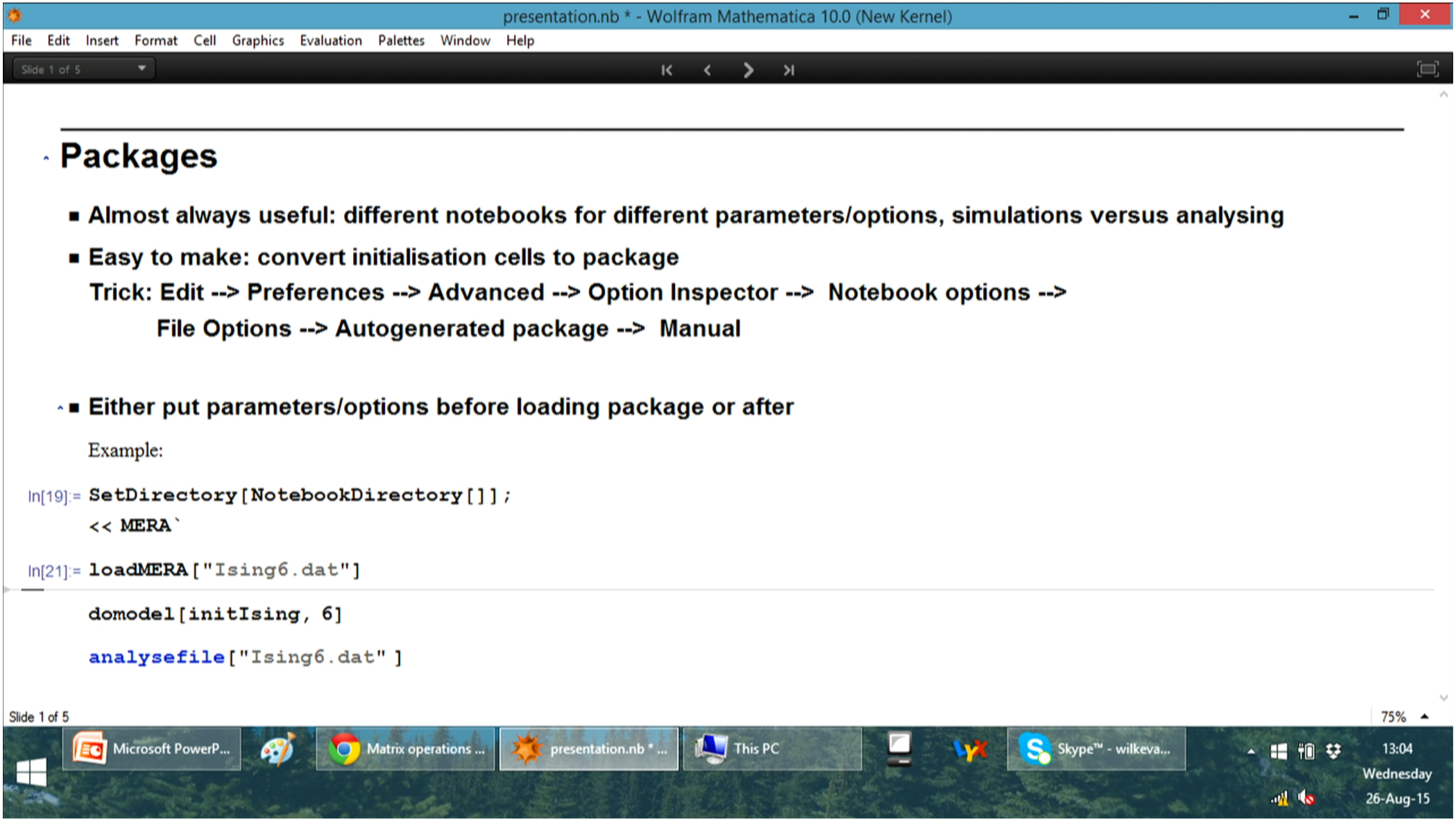
Choose operators at smart locations



Simplify ☺

6/16

R.N.C. Pfeifer, G. Evenbly and G. Vidal, Entanglement renormalization, scale invariance, and quantum criticality (2009)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Packages
Almost always useful: different notebooks for different parameters/options, simulations versus analysing
Easy to make: convert initialisation cells to package
Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options -->
File Options --> Autogenerated package --> Manual
Either put parameters/options before loading package or after
Example:
In[19]:= SetDirectory[NotebookDirectory[]];
<< MERA`
In[21]:= loadMERA["Ising6.dat"]
domodel[initIsing, 6]
analysefile["Ising6.dat"]
Slide 1 of 5
75%
Microsoft PowerP...
Matrix operations ...
presentation.nb * ...
This PC
Skype™ - wilkeva...
13:04
Wednesday
26-Aug-15

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Packages

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  Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options -->

  File Options --> Autogenerated package --> Manual

- **Either put parameters/options before loading package or after**

Example:

```
In[19]:= SetDirectory[NotebookDirectory[]];
         << MERA`

In[21]:= loadMERA["Ising6.dat"]

In[22]:= Nlayers

Out[22]= 6
```

```
domodel[initIsing, 6]
```

Slide 1 of 5 75%

- **Easy to make: convert initialisation cells to package**

  Trick: Edit --> Preferences --> Advanced --> Option Inspector --> Notebook options -->

  File Options --> Autogenerated package --> Manual

- **Either put parameters/options before loading package or after**

  Example:

```
In[19]:= SetDirectory[NotebookDirectory[]];
         << MERA`

In[21]:= loadMERA["Ising6.dat"]

In[22]:= Nlayers

Out[22]= 6

In[23]:= w // Length

Out[23]= 6

In[24]:= w[[1]] // Length

Out[24]= 1
```

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```
Out[23]= 6

In[24]:= w[[1]] // Length

Out[24]= 1

In[25]:= w[[1, 1]]

Out[25]= {{{{-0.636605 + 0.355596 i, -0.0195527 + 0.029982 i}, {0.191257 - 0.105473 i, -0.0972201 - 0.215114 i}},
   {{-0.0862657 - 0.000894288 i, 0.195033 + 0.255812 i}, {0.0159311 + 0.497034 i, -0.0678153 + 0.0275647 i}}},
  {{{-0.00221638 - 0.0110813 i, -0.755618 + 0.328789 i}, {-0.159952 + 0.0958883 i, -0.0304076 - 0.0425354 i}},
   {{0.510069 - 0.110394 i, 0.0453588 + 0.0383305 i}, {0.0575752 + 0.0604461 i, -0.00311667 + 0.0208724 i}}},
  {{{-0.14525 + 0.594785 i, 0.0534487 - 0.0349045 i}, {-0.243585 + 0.173231 i, 0.331089 + 0.346293 i}},
   {{0.123643 - 0.0882619 i, -0.215403 - 0.0315381 i}, {-0.454609 - 0.10925 i, 0.0924616 - 0.0674626 i}}},
  {{{0.0601131 - 0.0534626 i, 0.168413 - 0.2237 i}, {-0.0835755 - 0.636832 i, 0.021022 - 0.108673 i}},
   {{0.364829 - 0.545759 i, -0.0670552 + 0.0388807 i}, {-0.0527447 + 0.0892406 i, 0.165888 - 0.122906 i}}},
  {{{-0.101707 + 0.144917 i, -0.0730914 + 0.0015973 i}, {0.00856382 + 0.00994589 i, -0.0450318 - 0.64262 i}},
   {{-0.0661826 - 0.0188993 i, -0.177712 - 0.684117 i}, {-0.171995 - 0.113953 i, -0.0352731 - 0.00123301 i}}},
  {{{0.0664468 - 0.0912909 i, 0.17568 + 0.387098 i}, {-0.535549 - 0.324394 i, 0.207598 - 0.190993 i}},
   {{-0.0340752 + 0.421054 i, -0.0186992 + 0.0992451 i}, {-0.131632 + 0.185301 i, 0.120939 + 0.282819 i}}}}

domodel[initIsing, 6]

analysefile["Ising6.dat"]
```

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Either put parameters/options before loading package or after

Example:

```
In[19]:= SetDirectory[NotebookDirectory[]];
         << MERA`

In[21]:= loadMERA["Ising6.dat"]

In[22]:= Nlayers

Out[22]= 6

In[23]:= w // Length

Out[23]= 6

In[24]:= w[[1]] // Length

Out[24]= 1

In[26]:= w[[1, 1]] // Dimensions

Out[26]= {6, 2, 2, 2}

domodel[initIsing, 6]

analysefile["Ising6.dat" ]
```

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```
In[21]:= loadMERA["Ising6.dat"]

In[22]:= Nlayers

Out[22]= 6

In[23]:= w // Length

Out[23]= 6

In[24]:= w[[1]] // Length

Out[24]= 1

In[26]:= w[[1, 1]] // Dimensions

Out[26]= {6, 2, 2, 2}

In[27]:= u // Length

Out[27]= 6

In[28]:= dens // Length

Out[28]= 7

domodel[initIsing, 6]

analysefile["Ising6.dat"]
```

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