Title: The multi-scale entanglement renormalization ansatz

Date: Aug 26, 2015 11:00 AM

URL: http://pirsa.org/15080047

Abstract: TBA

Summary of matrix product state (MPS)

$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

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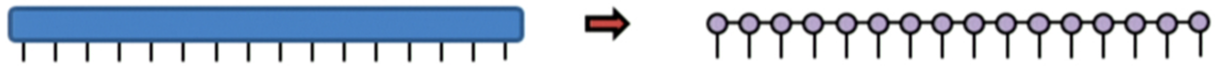

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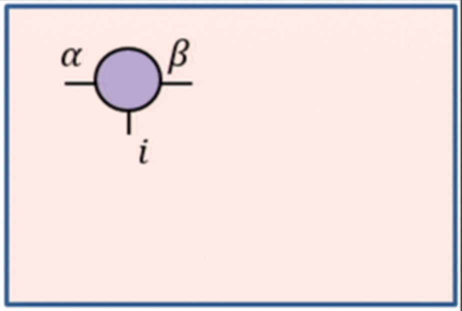

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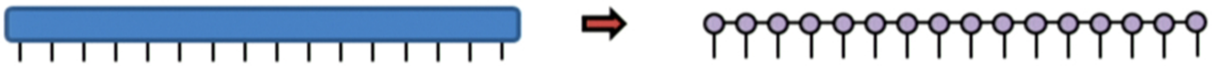

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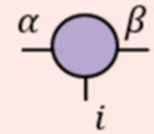

$|\alpha| = |\beta| = \chi$

$|i| = d$

$O(d\chi^2)$ parameters

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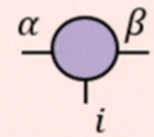

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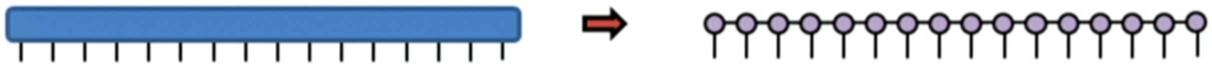

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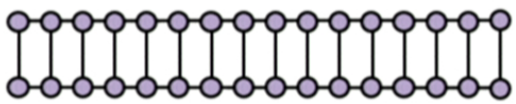
Efficient representation!

Efficient computation?

$\langle\Psi|\Psi\rangle =$



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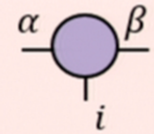

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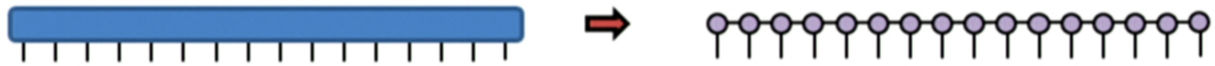

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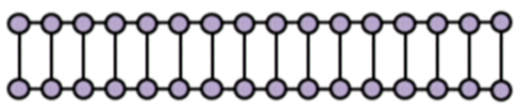

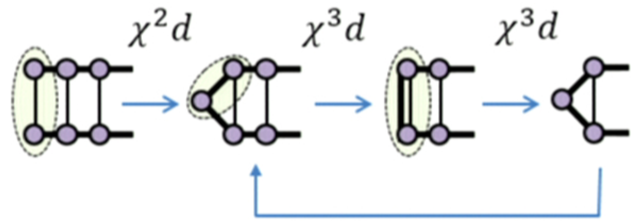

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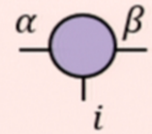

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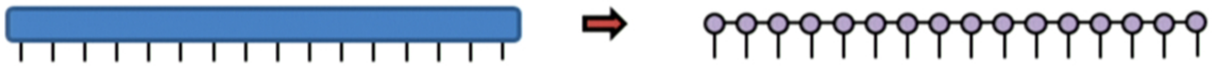

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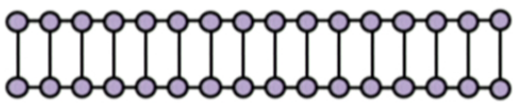
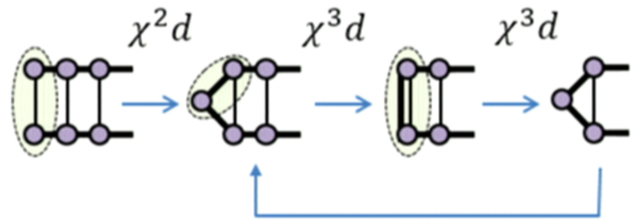
$\langle\Psi|\Psi\rangle =$ 



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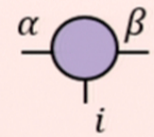

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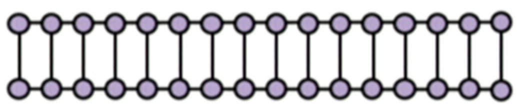

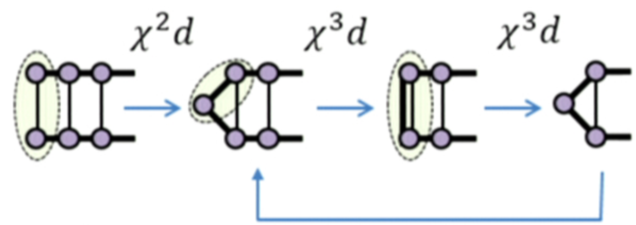

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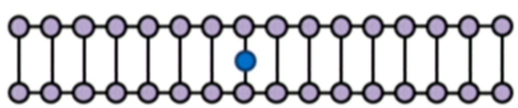

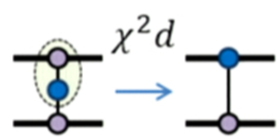

2.B Physics? Structural properties:

(a) entanglement entropy
(b) correlations

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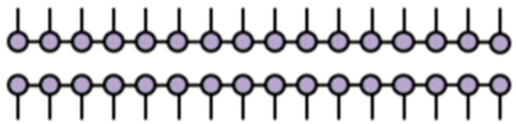
$|\Psi\rangle\langle\Psi| =$ 

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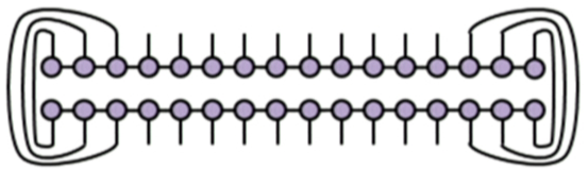
$\rho_A =$ 

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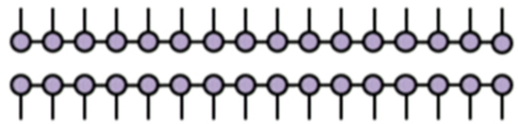

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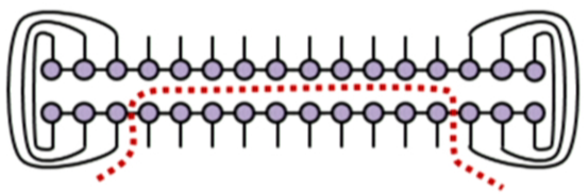

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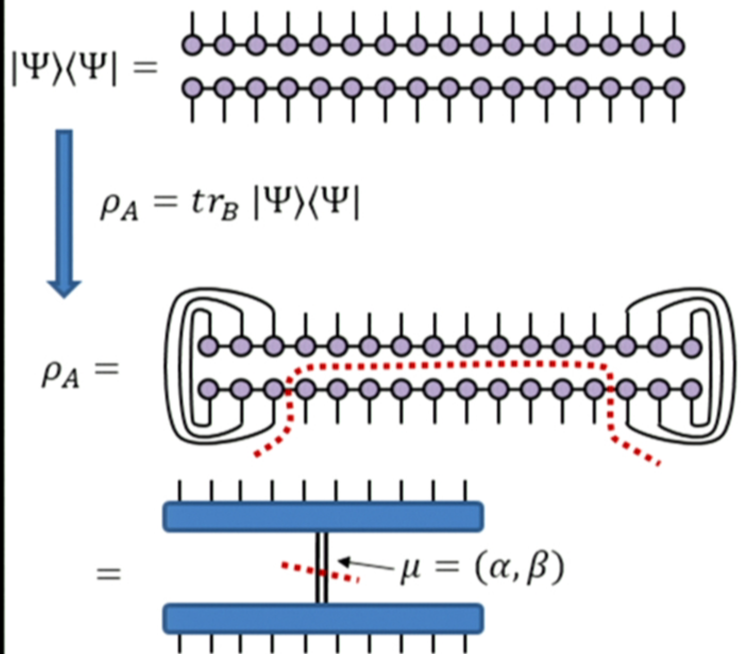

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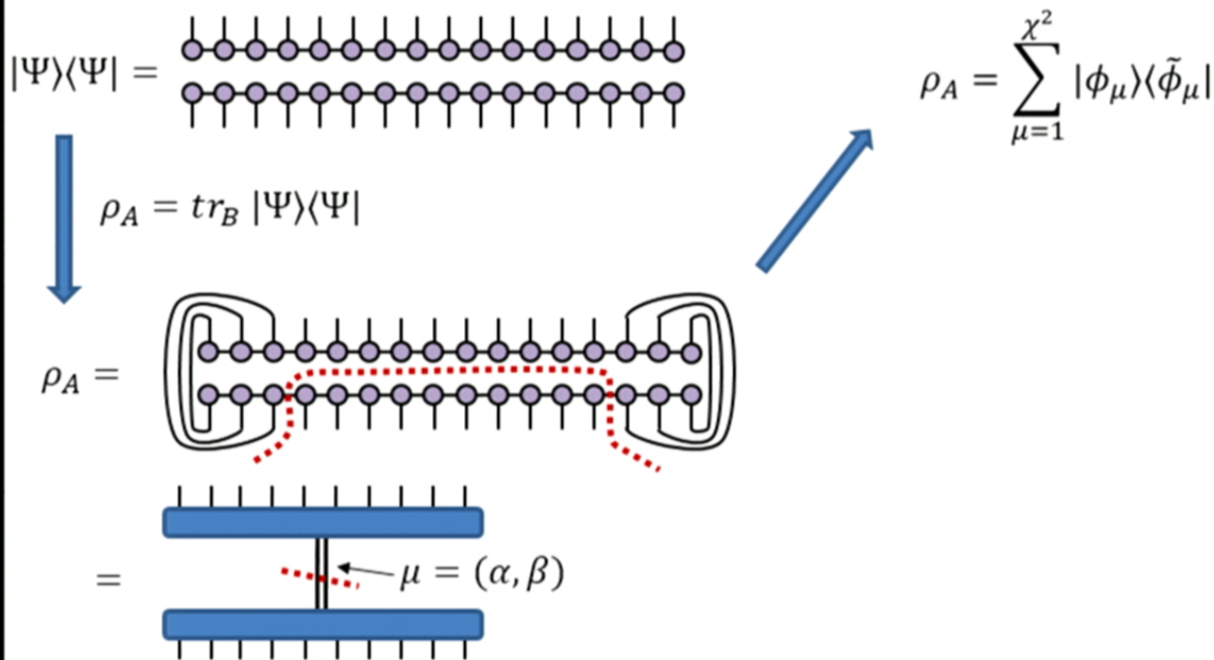

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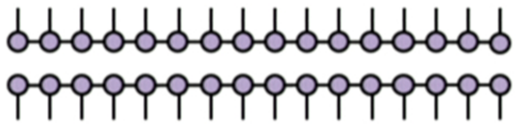

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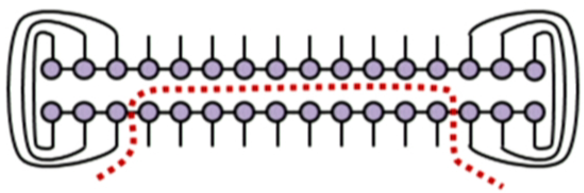

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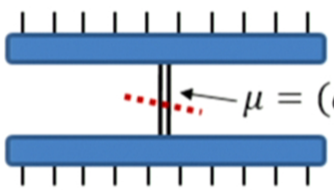

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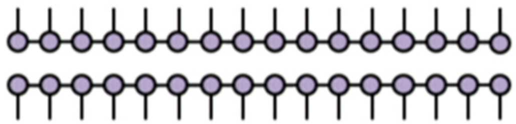
$|\Psi\rangle\langle\Psi| =$ 

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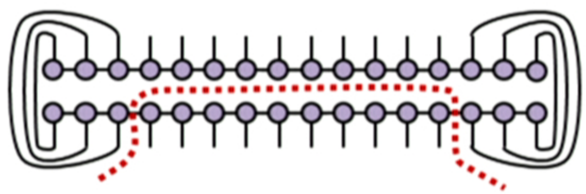
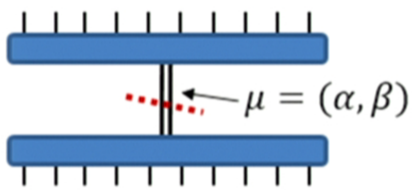
$\rho_A =$ 

$=$ 

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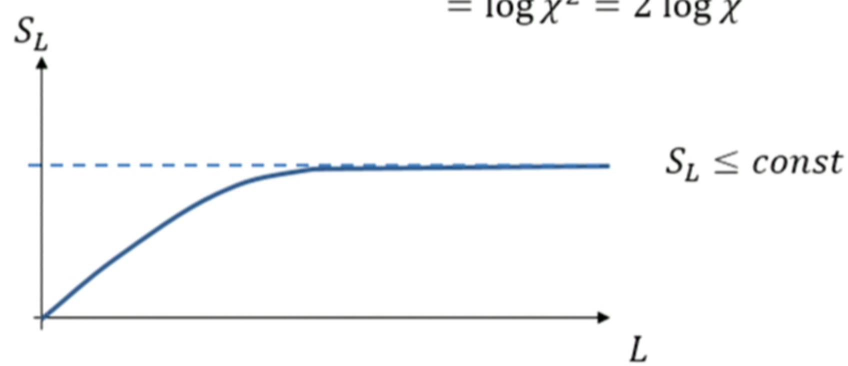

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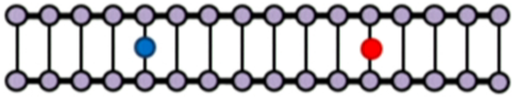
$$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle =$$ 

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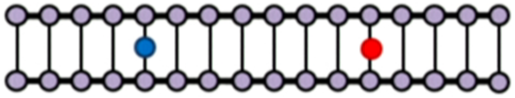
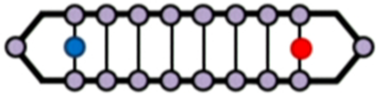
$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle =$  $=$ 

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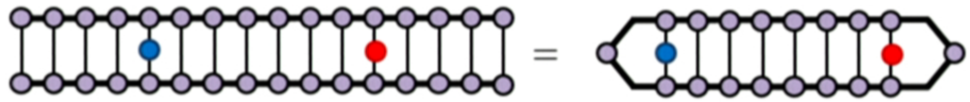

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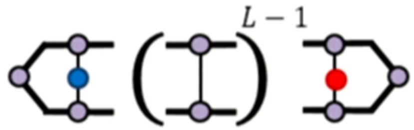

$$\approx a\,\lambda^{L} = ae^{-L/\xi}$$

$$\xi \equiv -\frac{1}{\log\lambda}$$

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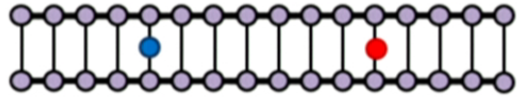
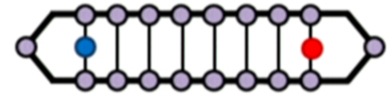
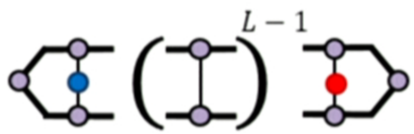
$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle =$  $=$ 

$=$ 

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Structural properties of MPS

correlations	$C(L) \approx e^{-L/\xi}$
entanglement	$S_L \leq 2\log\chi$

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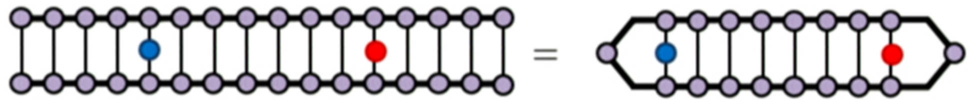
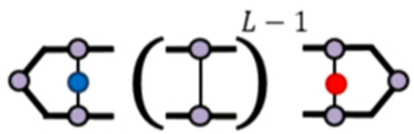


$=$  $\approx a\,\lambda^{L} = a e^{-L/\xi}$

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Structural properties of MPS

correlations	$C(L) \approx e^{-L/\xi}$
entanglement	$S_L \leq 2\log\chi$

match with
ground states of 1D
gapped Hamiltonians

MERA: definition

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

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Matrix product state
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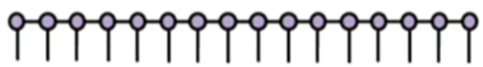

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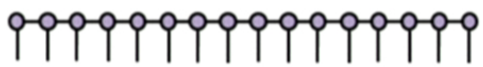
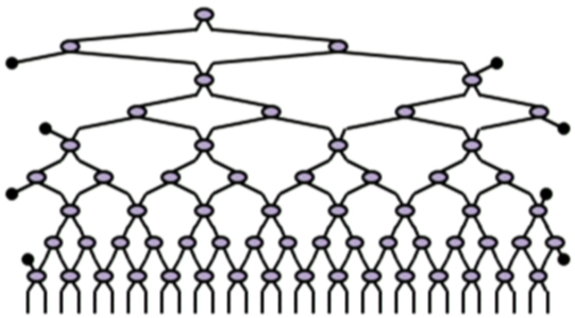
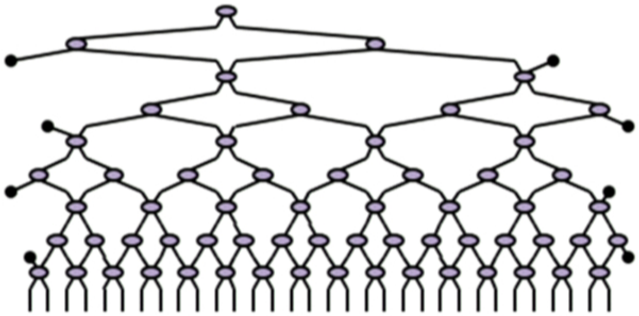


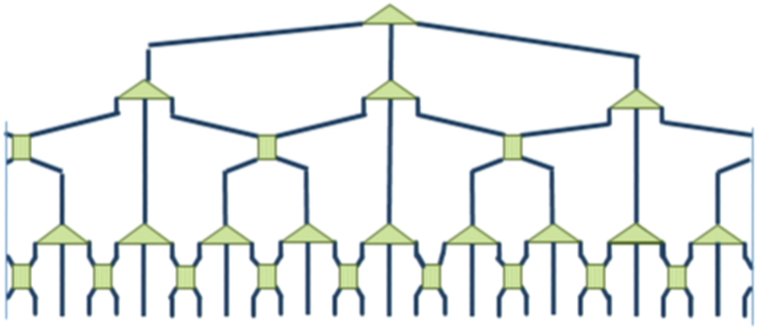
Multi-scale entanglement
renormalization ansatz
(MERA)



MERA



also MERA !



Efficiency

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

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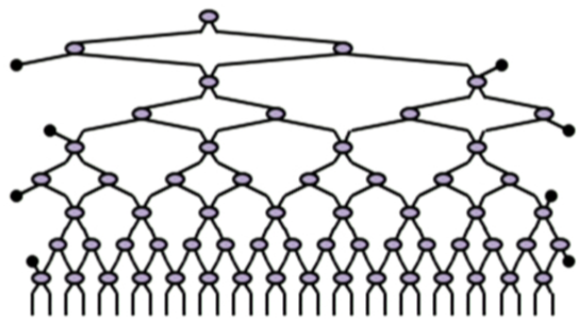

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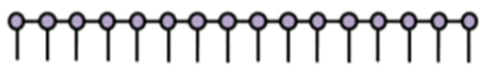


N spins

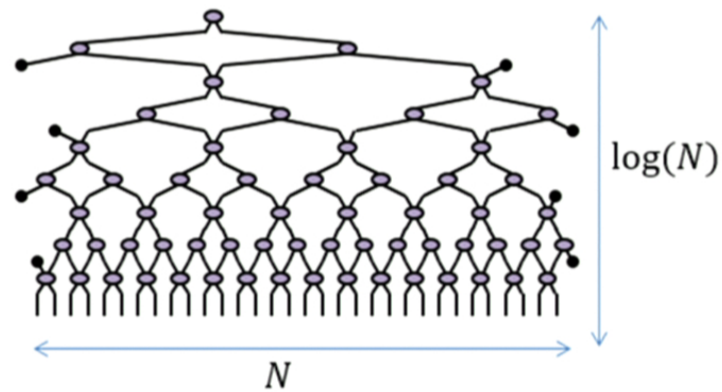
Efficiency

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers

$$N + \frac{N}{2} + \frac{N}{4} + \ldots = N\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) \leq 2N$$

Multi-scale entanglement renormalization ansatz (MERA)



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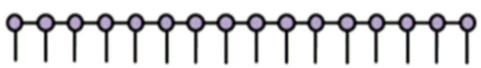


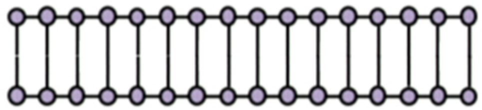
N spins $\Rightarrow$ N tensors
$\Rightarrow O(N)$ parameters

N spins $\Rightarrow N\log(N)$ tensors ?

efficiency

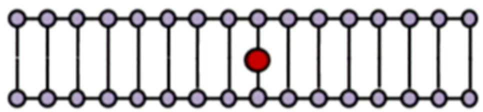
Matrix product state
(MPS)

$\langle\Psi|\Psi\rangle$



cost $O(N)$

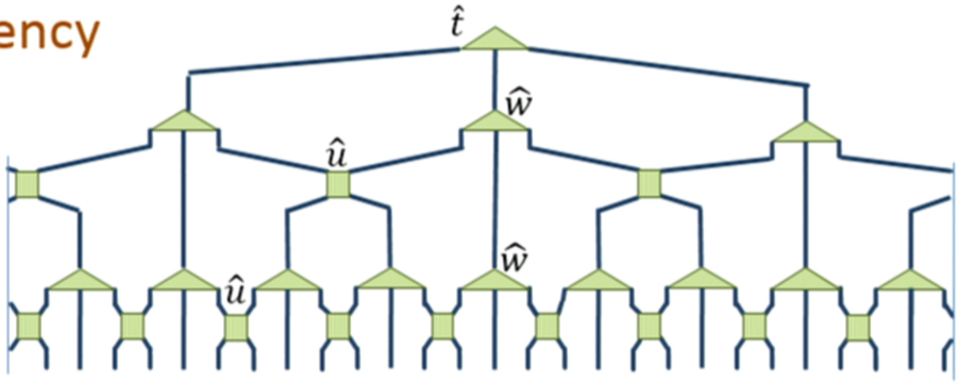
$\langle\Psi|\hat{o}|\Psi\rangle$



cost $O(N)$

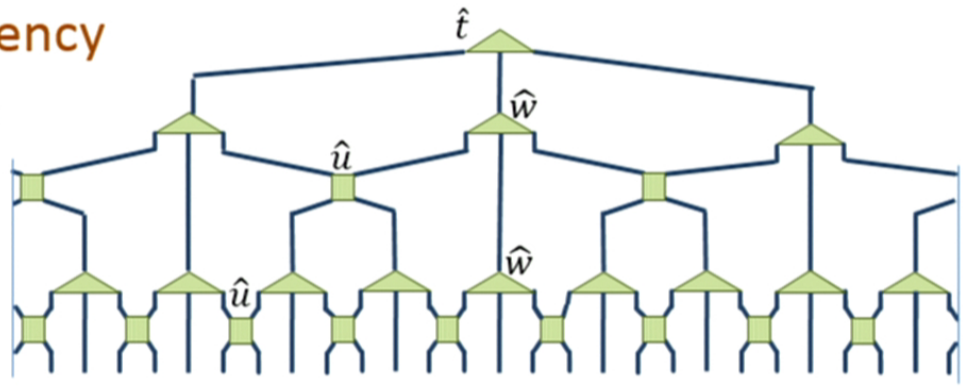
efficiency

$|\Psi\rangle$

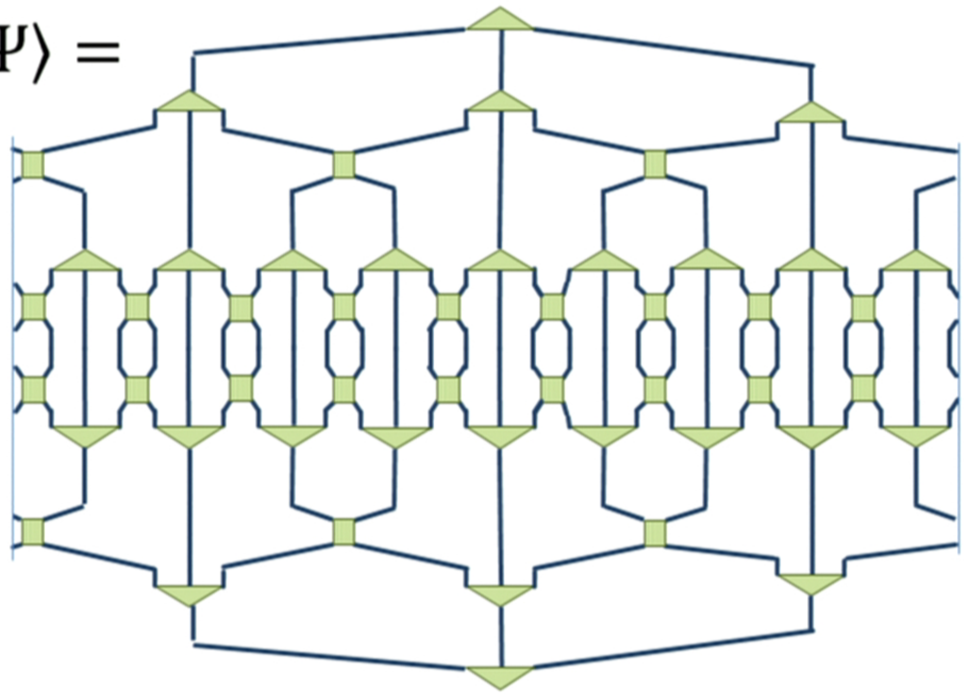


efficiency

$|\Psi\rangle$



$\langle\Psi|\Psi\rangle =$



efficiency

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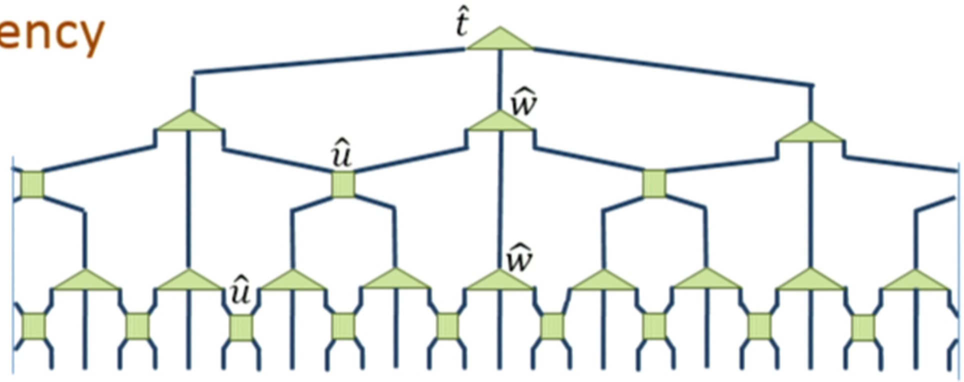
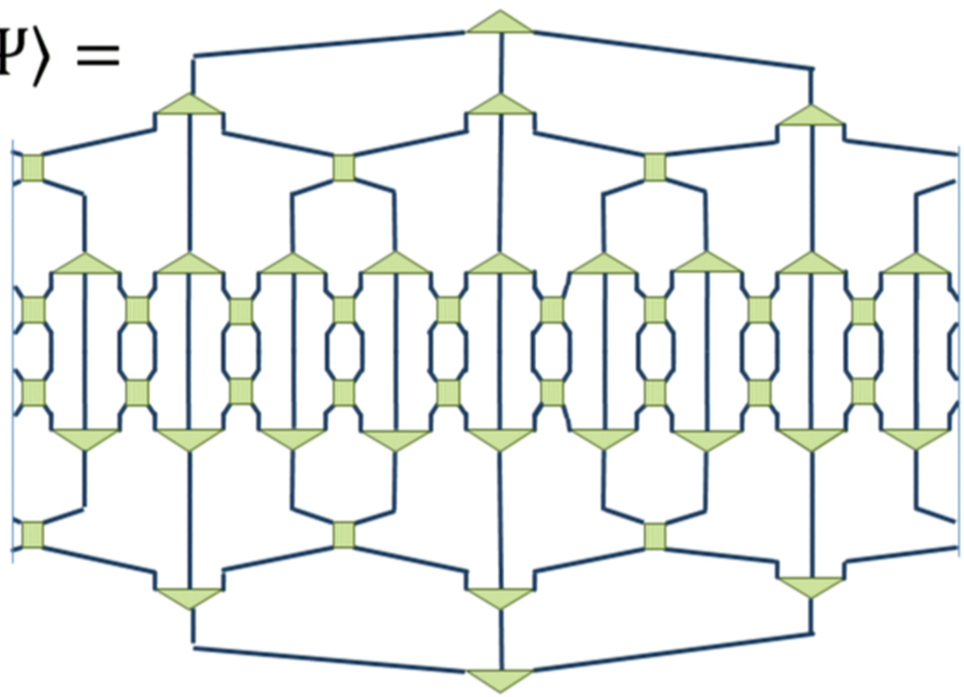
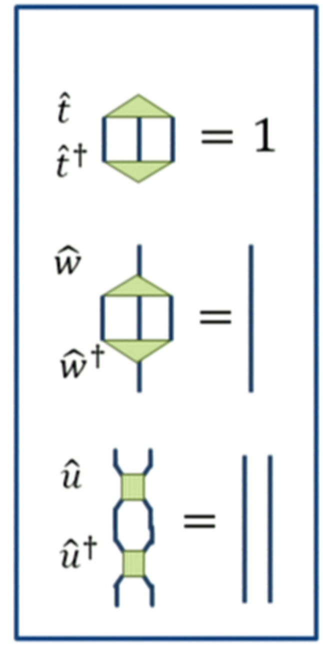


$\langle\Psi|\Psi\rangle =$



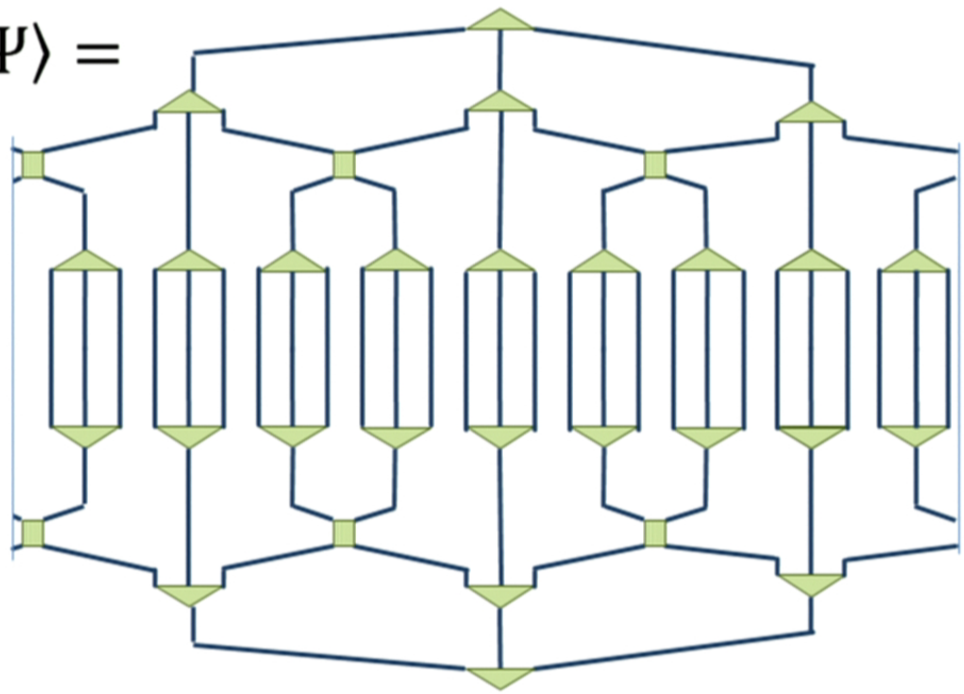
isometric tensors!

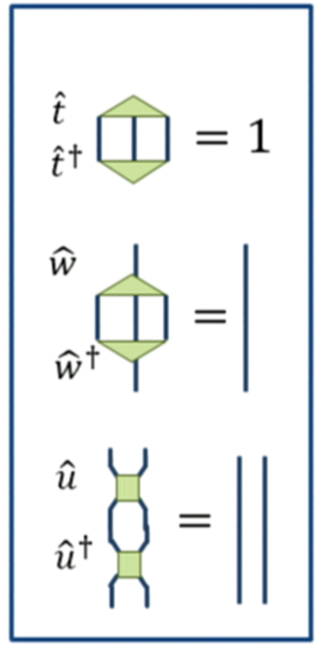


efficiency

isometric tensors!

$$\langle \Psi | \Psi \rangle =$$





efficiency

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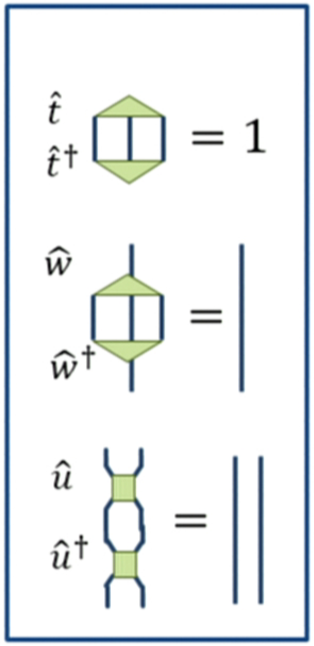
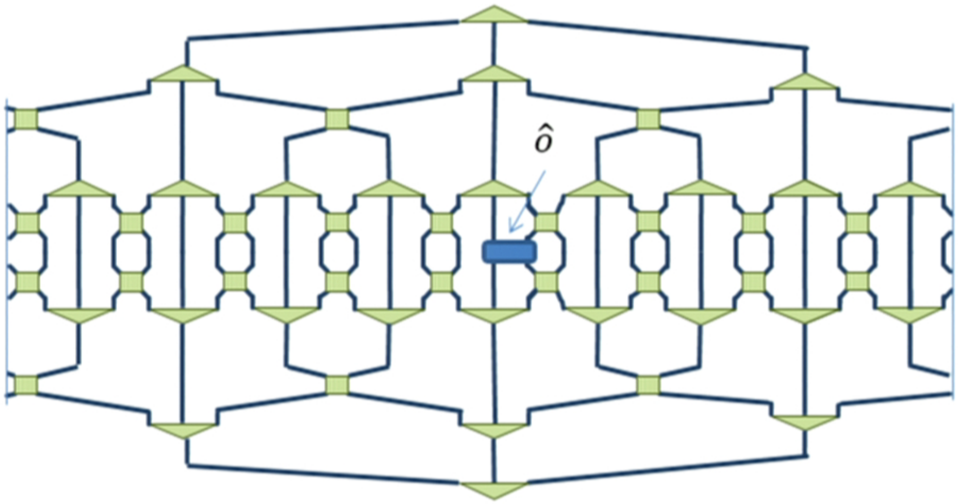
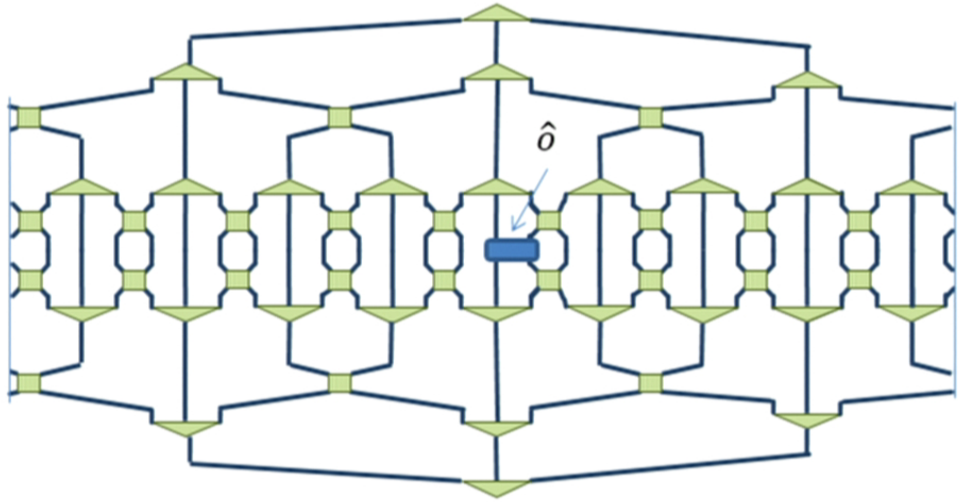


$$\langle\Psi|\Psi\rangle = 1$$

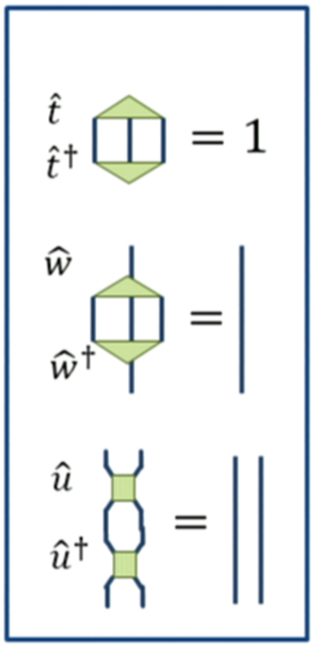
$$\langle\Psi|\hat{o}|\Psi\rangle =$$



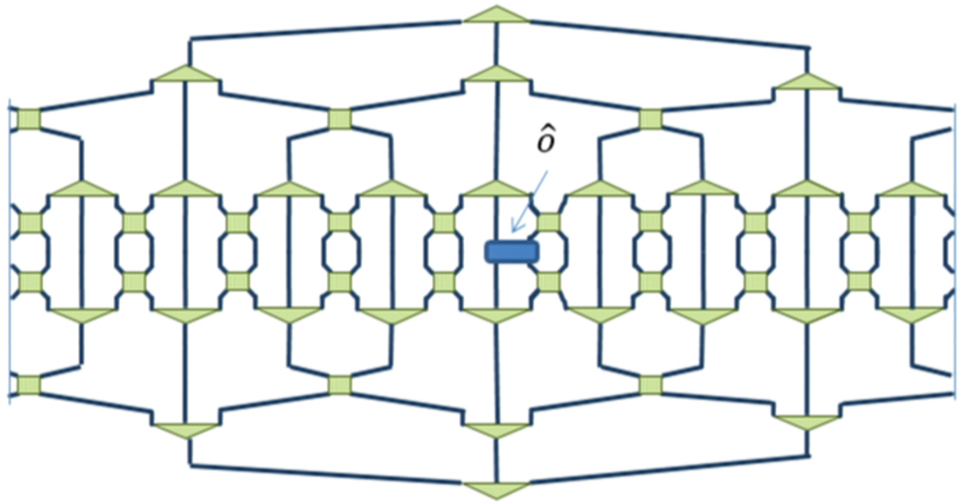
$$\langle \Psi | \hat{o} | \Psi \rangle =$$



isometric tensors!



$$\langle \Psi | \hat{o} | \Psi \rangle =$$



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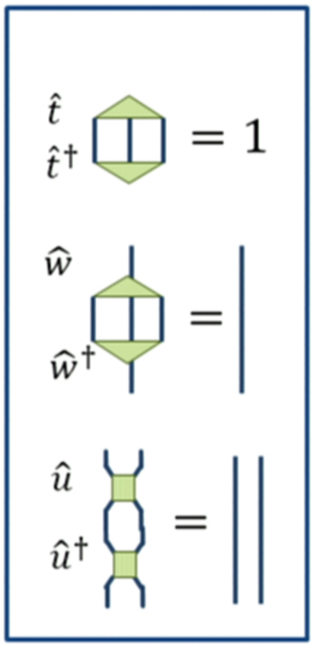

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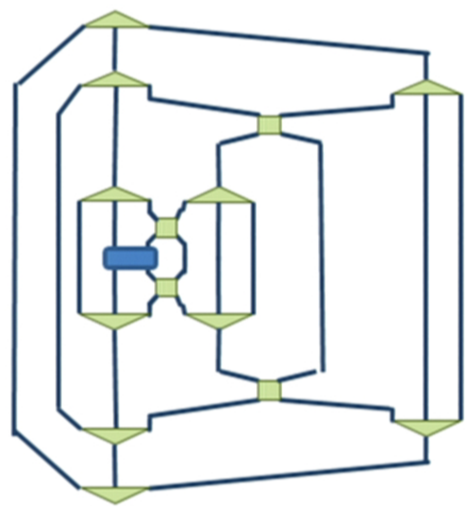

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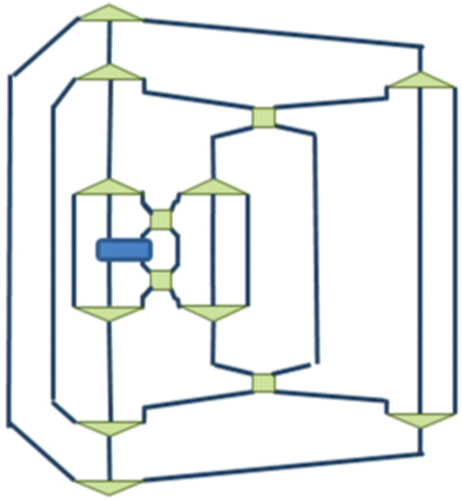
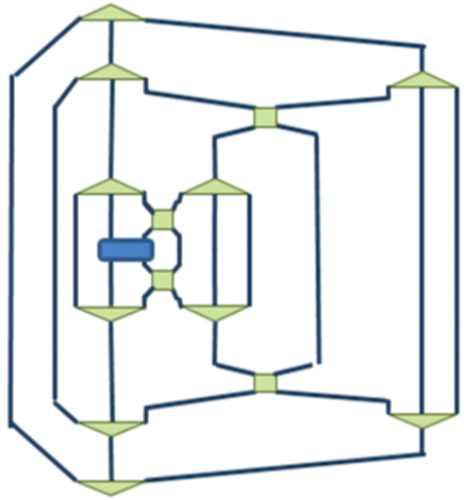
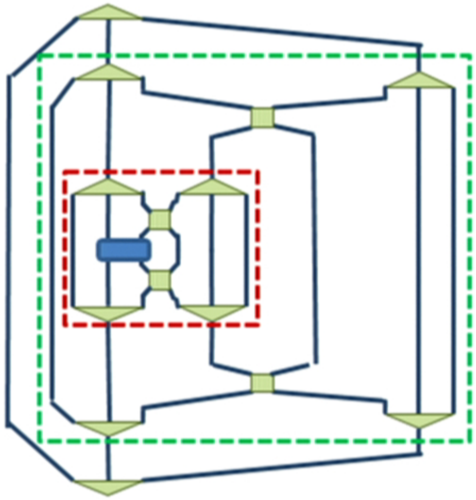
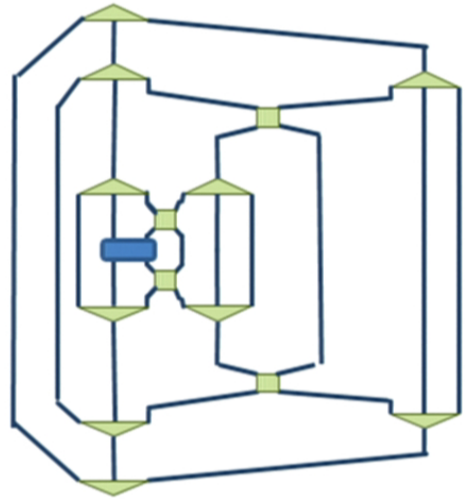


$$\langle \Psi | \hat{o} | \Psi \rangle =$$

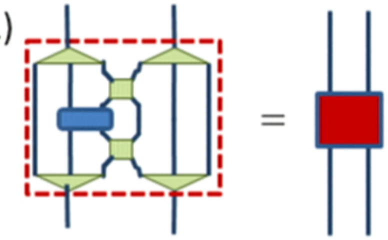




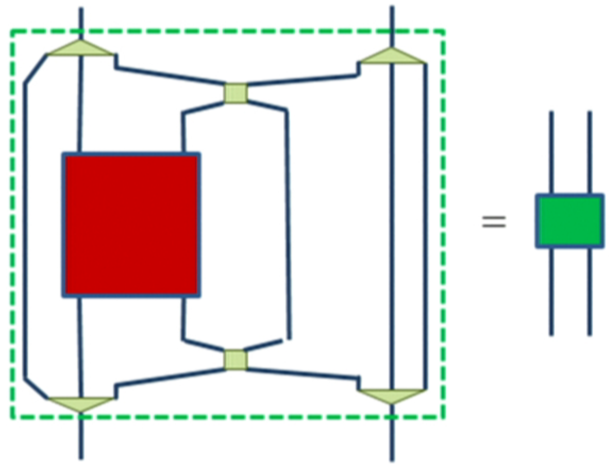
$$\langle\Psi|\hat{o}|\Psi\rangle =$$

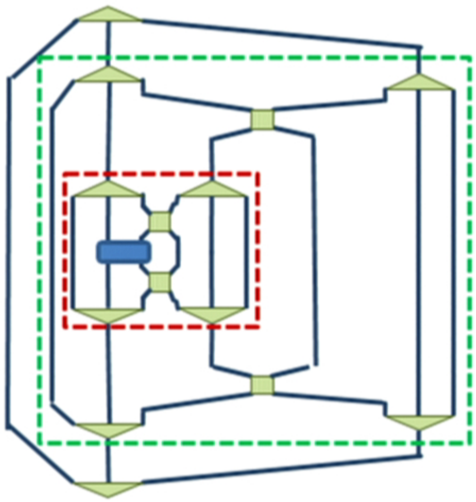


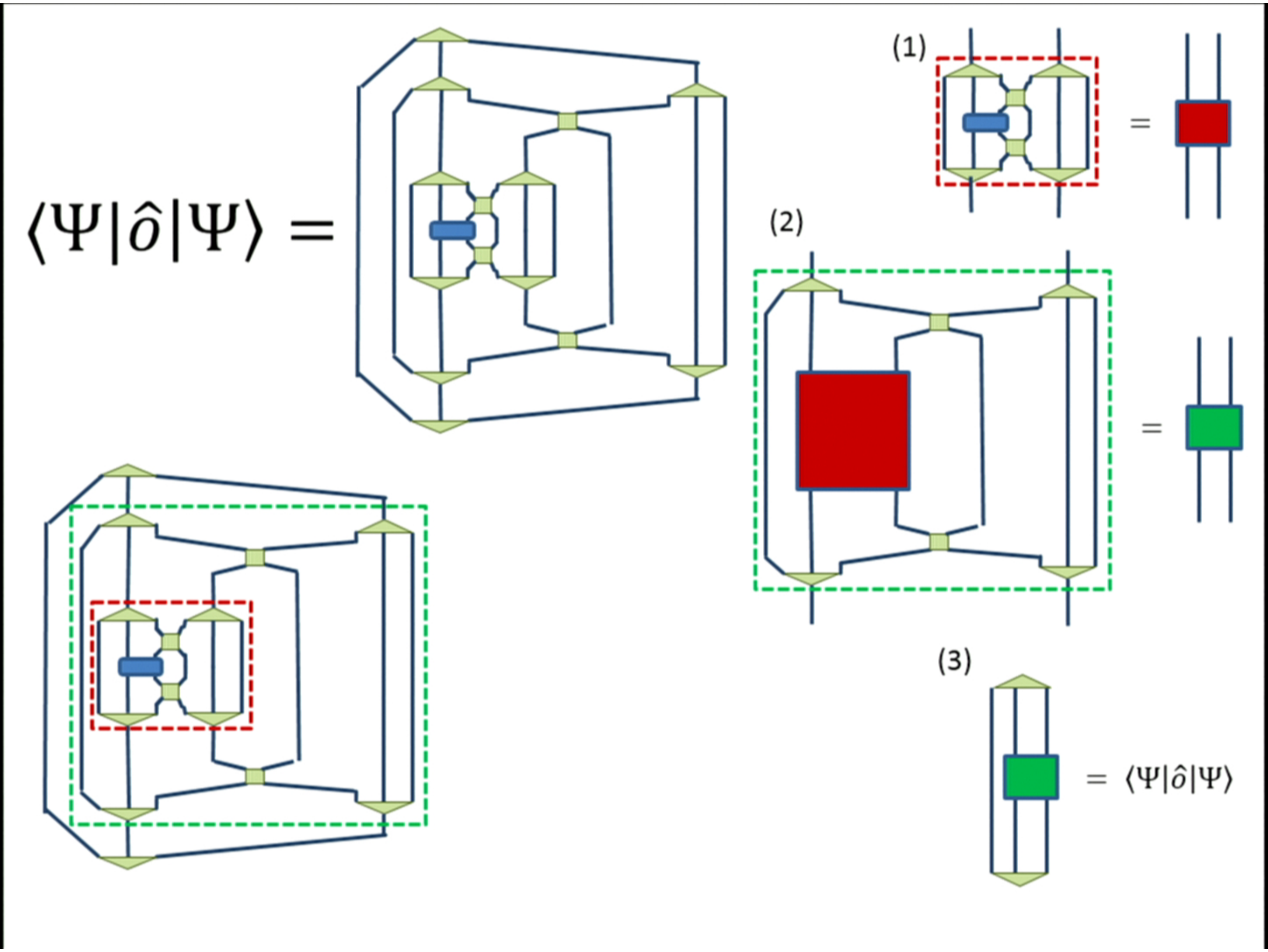
(1)



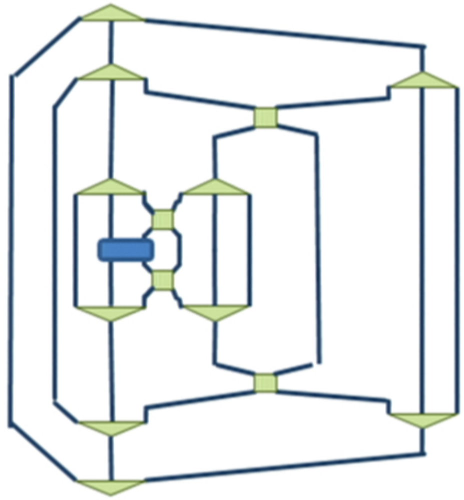
(2)

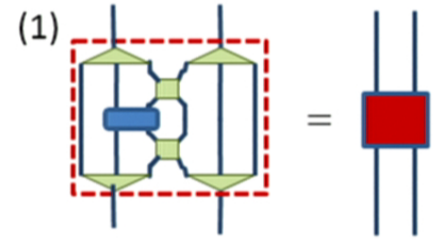


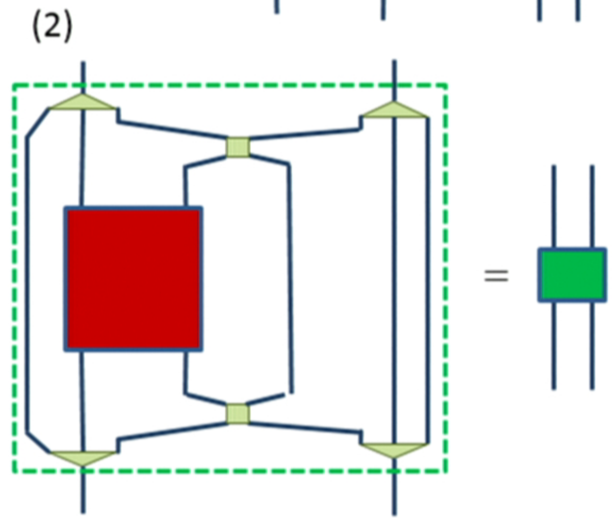


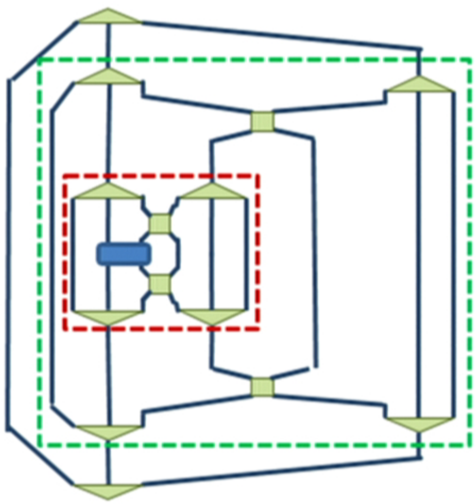

$\langle\Psi|\hat{o}|\Psi\rangle =$
(1)
=
(2)
=
(3)
$= \langle\Psi|\hat{o}|\Psi\rangle$

$$\langle\Psi|\hat{o}|\Psi\rangle =$$

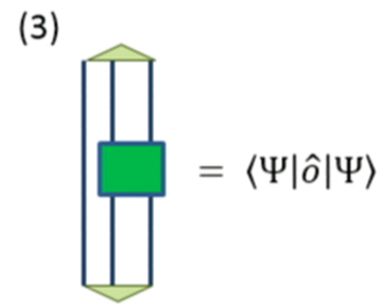








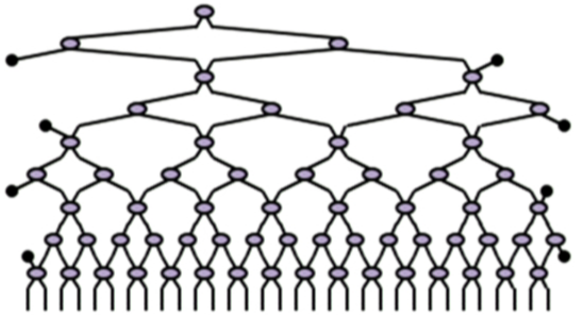
cost $O(\log(N))$



Structural properties

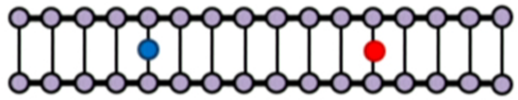
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

d^N complex numbers



- Decay of correlations
- Scaling of entanglement

$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$ MPS

$=$ 

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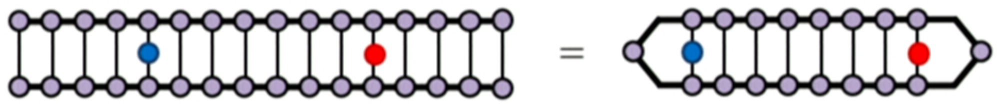
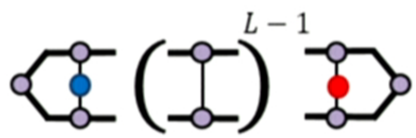
= 

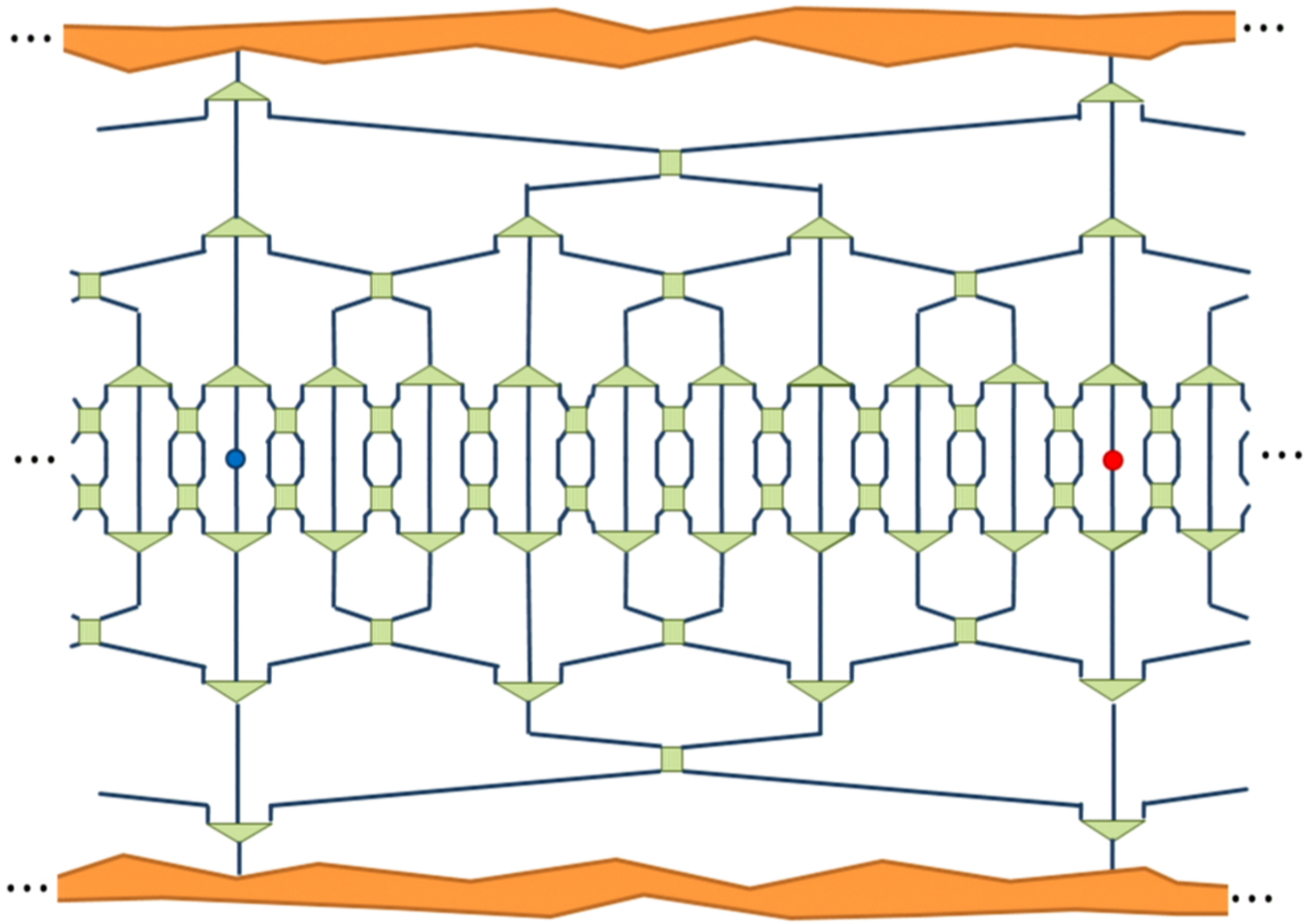
= 

$$\approx a\lambda^L = ae^{-L/\xi}$$

$$\xi \equiv -\frac{1}{\log \lambda}$$

MERA

$$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle\ =$$



MERA

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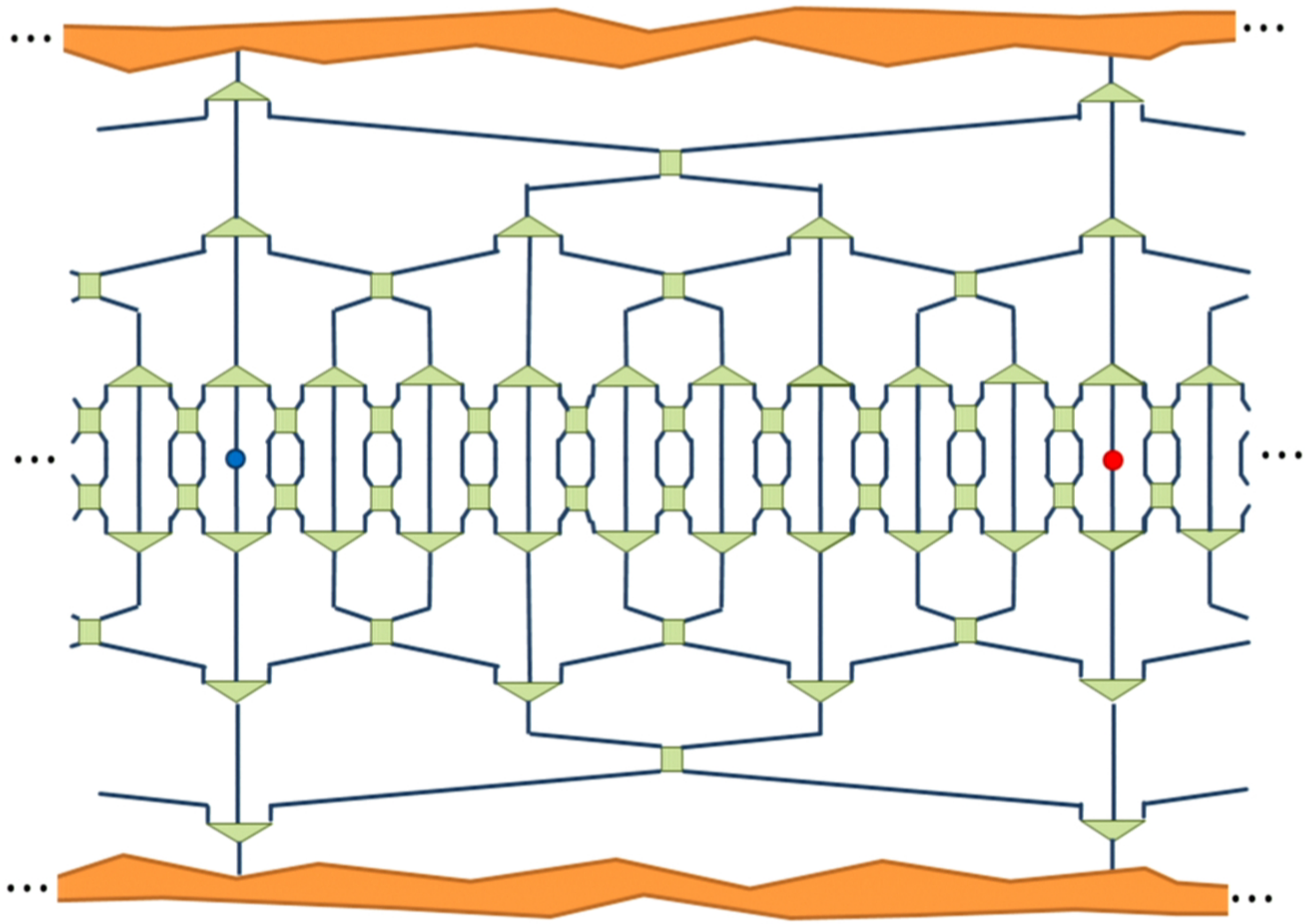
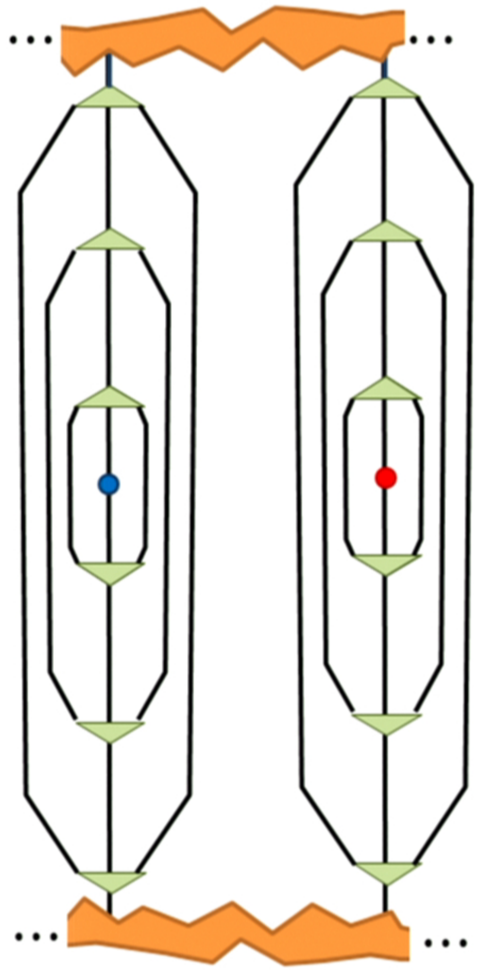


$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

MERA

$=$



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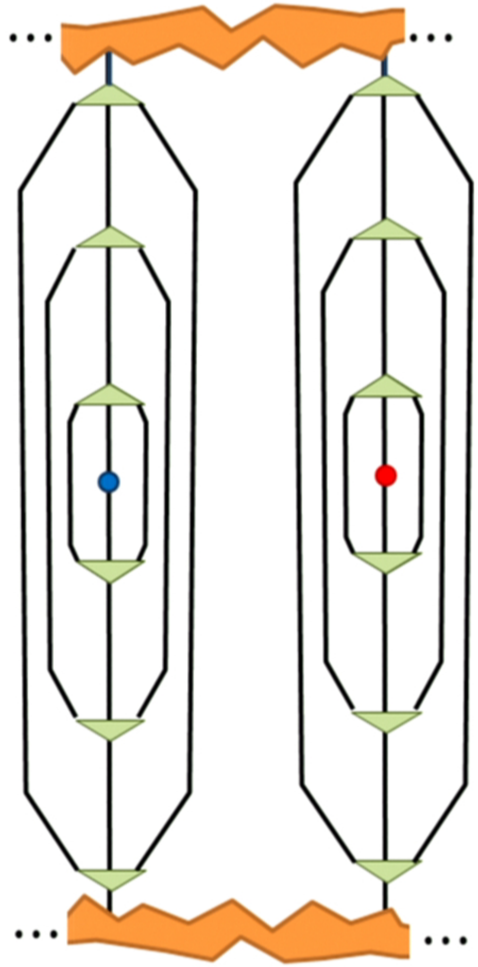
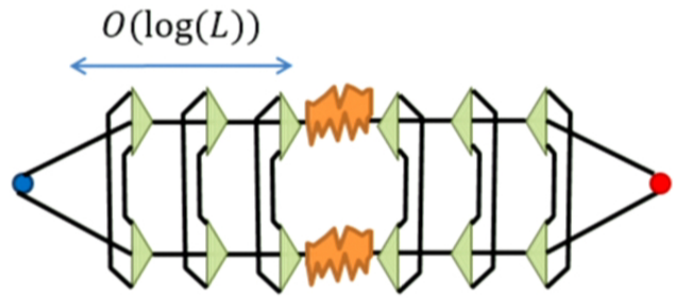
MERA

$=$



$=$



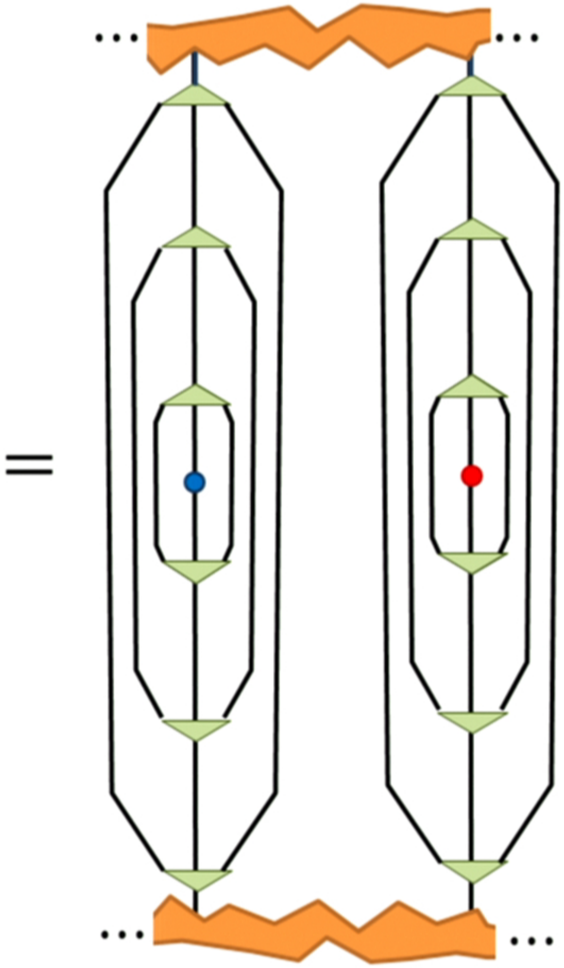
$$\approx (\lambda)^{\log_3(L)}(\lambda)^{\log_3(L)}$$

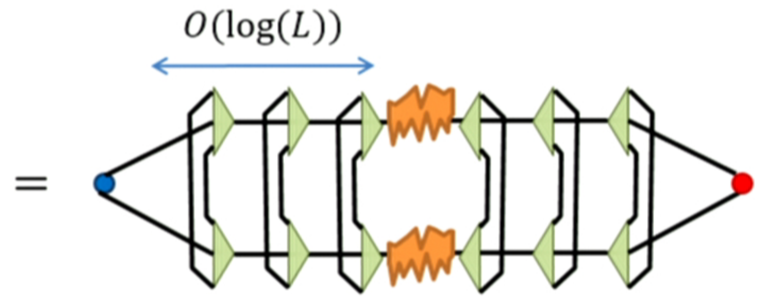
$$= \lambda^{2\log_3(L)} = L^{2\log_3(\lambda)} = L^{-p}$$

$x^{\log_3(y)} = y^{\log_3(x)}$ $\qquad$ $p \equiv -2\log_3(\lambda)$

$\langle\Psi|\hat{o}(0)\ \hat{o}(L)|\Psi\rangle$

MERA





$\approx (\lambda)^{\log_3(L)}(\lambda)^{\log_3(L)}$

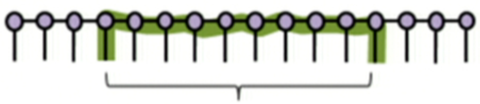
$= \lambda^{2\log_3(L)} = L^{2\log_3(\lambda)} = L^{-p}$

$x^{\log_3(y)} = y^{\log_3(x)}$ $\quad p \equiv -2\log_3(\lambda)$

$\Rightarrow$ polynomial decay of correlations

Correlations: summary and interpretation

matrix product state
(MPS)

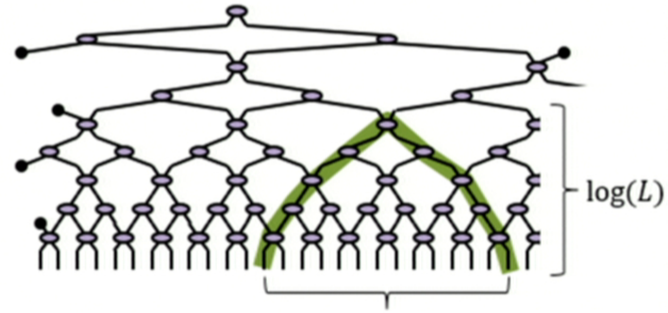


structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

exponential

multi-scale entanglement renormalization ansatz
(MERA)



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power-law

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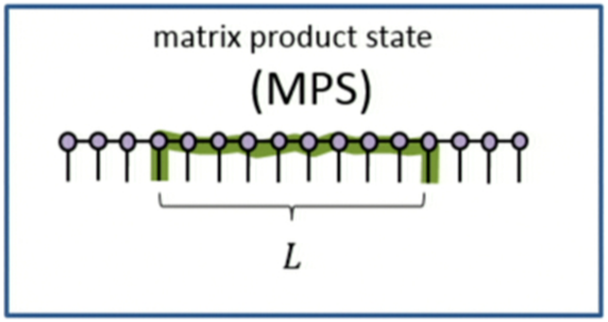

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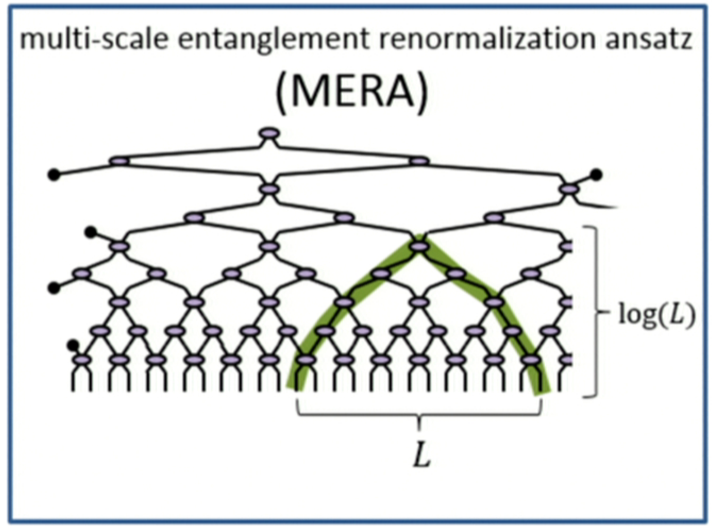

structure of geodesics:

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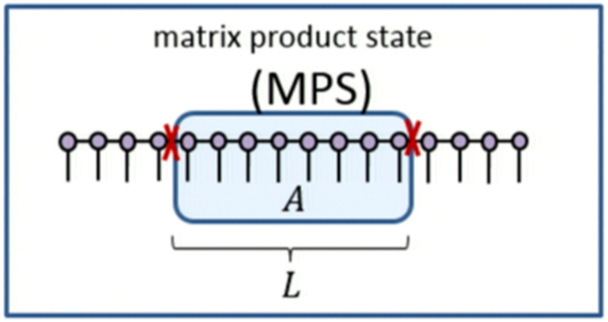
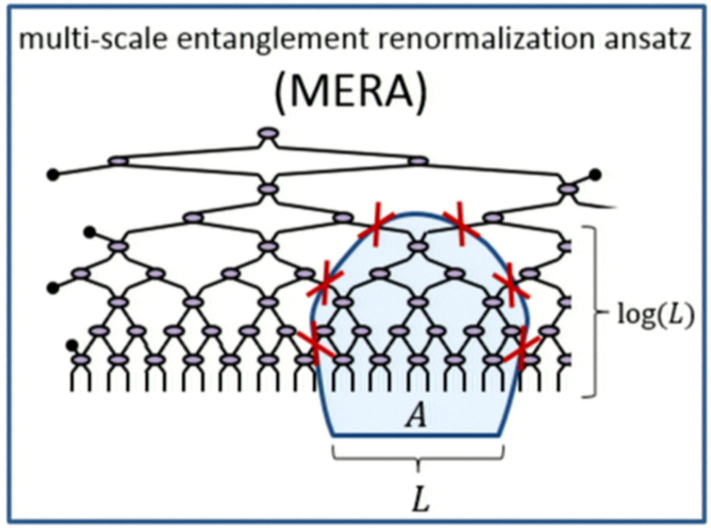
power-law

Entanglement entropy

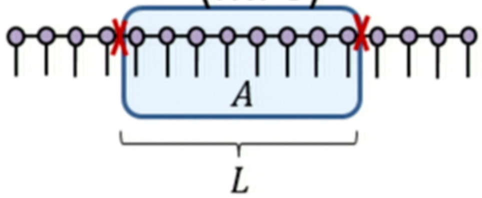




Entanglement entropy

matrix product state

(MPS)



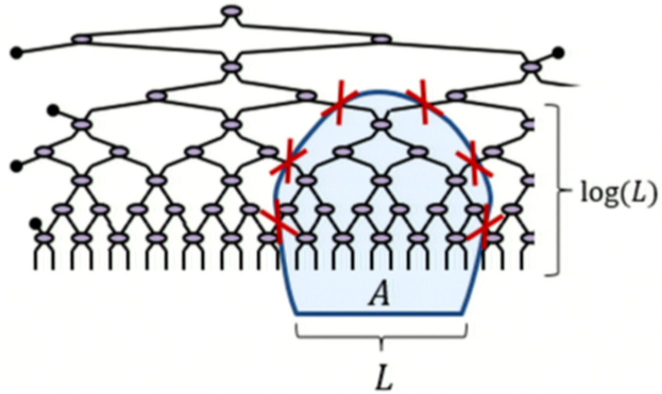
connectivity:

$S(A) \leq const$

boundary law!

multi-scale entanglement renormalization ansatz

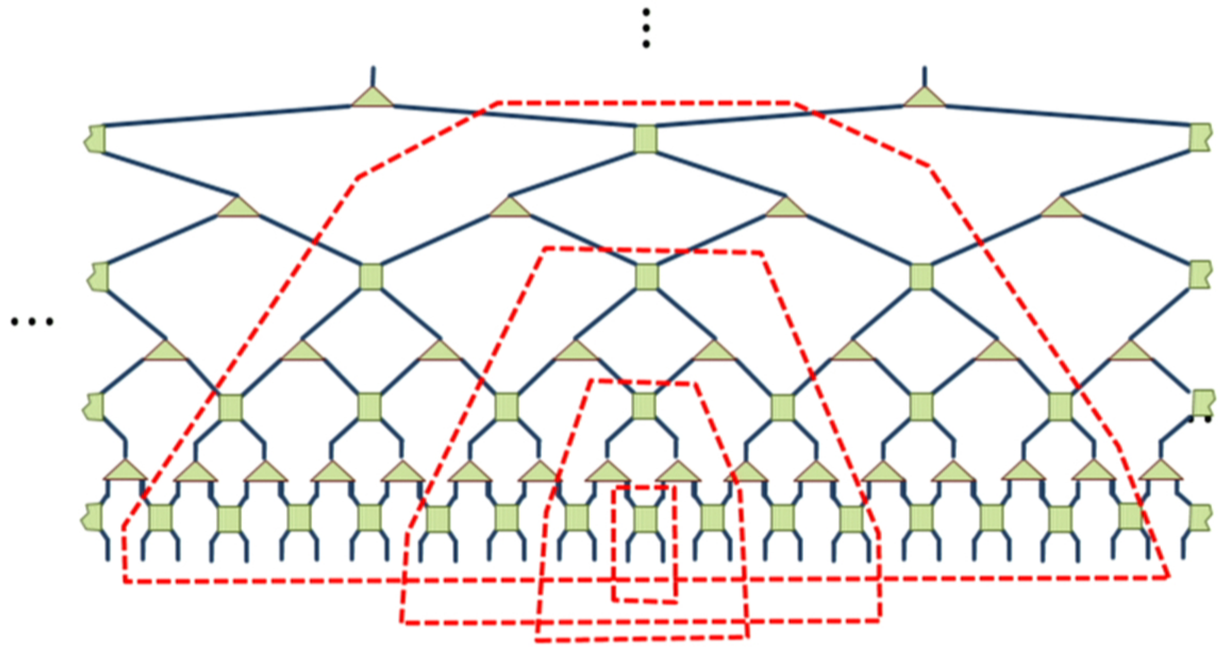
(MERA)



connectivity:

$S(A) \leq \log L$

logarithmic correction!



$n(A) \approx \log L$

$L = 2, \quad n(A) = 2$

$L = 6, \quad n(A) = 4$

$L = 14, \quad n(A) = 6$

$L = 30, \quad n(A) = 8$

Conclusions

- What is a tensor network state?

- Important aspects of a TN?

  - efficient representation
    and computation

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- Important aspects of a TN?
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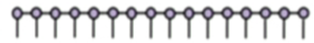
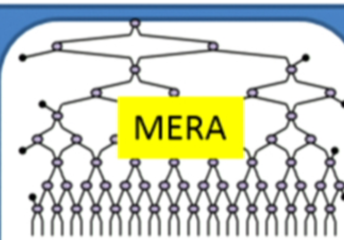


$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

$$S(A) \leq const$$

ground state
gapped Hamiltonian



$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx \frac{1}{L^p}$$

$$S(A) \approx \log L$$

ground state
gapless Hamiltonian

Conclusions

- What is a tensor network state?
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  - ➢ efficient representation and computation
  - ➢ structural properties (correlations and entanglement)

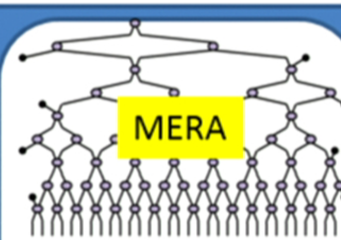
MPS

$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx e^{-L/\xi}$$

$$S(A) \leq const$$

ground state
gapped Hamiltonian

MERA



$$\langle \hat{o}(0)\hat{o}(L)\rangle \approx \frac{1}{L^p}$$

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ground state
gapless Hamiltonian

Aspects not covered

e.g. DMRG !!!!

- How to optimize variational parameters (energy minimization; imaginary time evolution)

Conclusions

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  - efficient representation and computation
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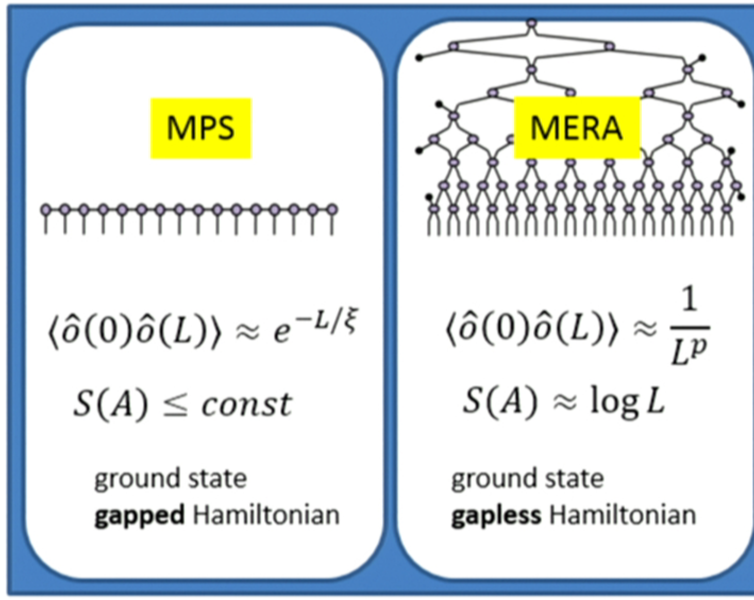

Aspects not covered

e.g. DMRG !!!!

- How to optimize variational parameters (energy minimization; imaginary time evolution)
- Simulation of time evolution (MPS)
- continuous MPS, continuous MERA for quantum field theories

Conclusions

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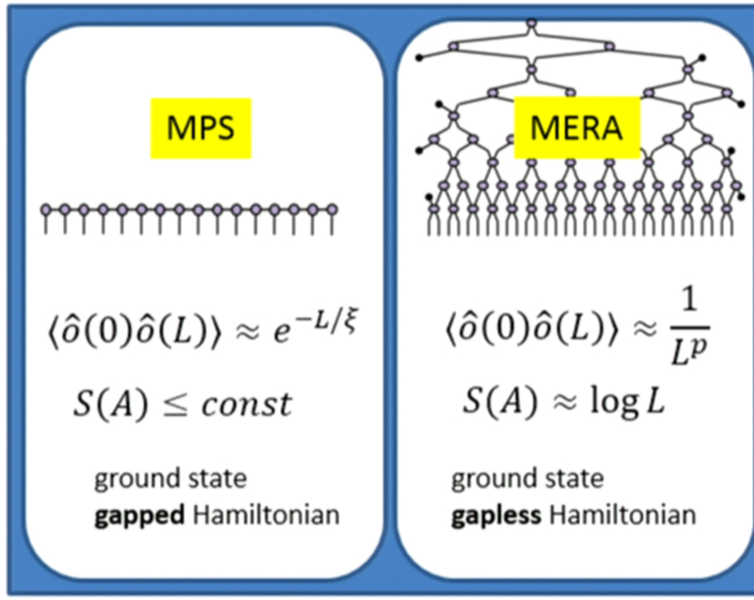
- Important aspects of a TN?
  - ➢ efficient representation and computation
  - ➢ structural properties (correlations and entanglement)

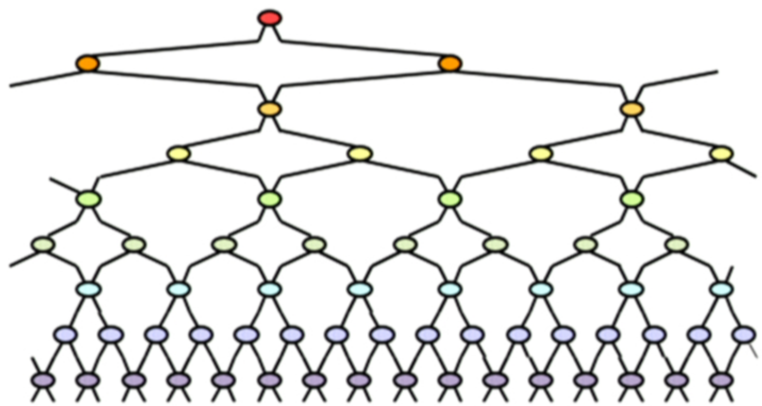


Aspects not covered

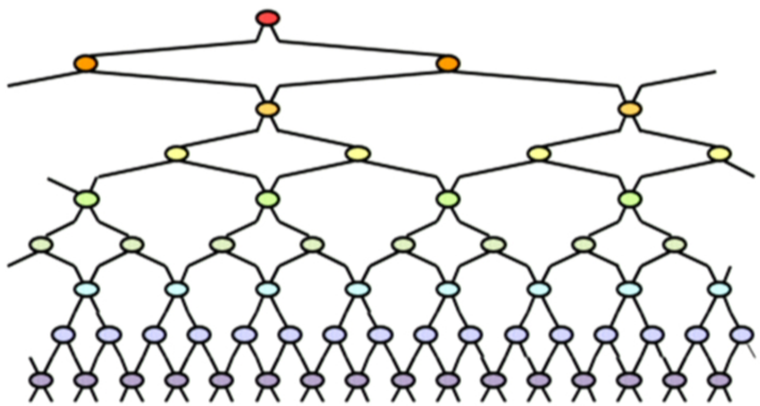
e.g. DMRG !!!!

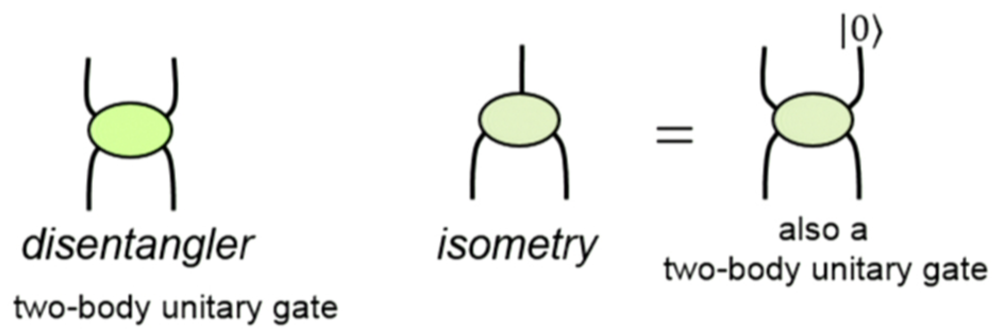
- How to optimize variational parameters (energy minimization; imaginary time evolution)
- Simulation of time evolution (MPS)
- continuous MPS, continuous MERA for quantum field theories
- D>1 spatial dimensions (PEPS, MERA, branching MERA)

MERA as a quantum circuit



MERA as a quantum circuit





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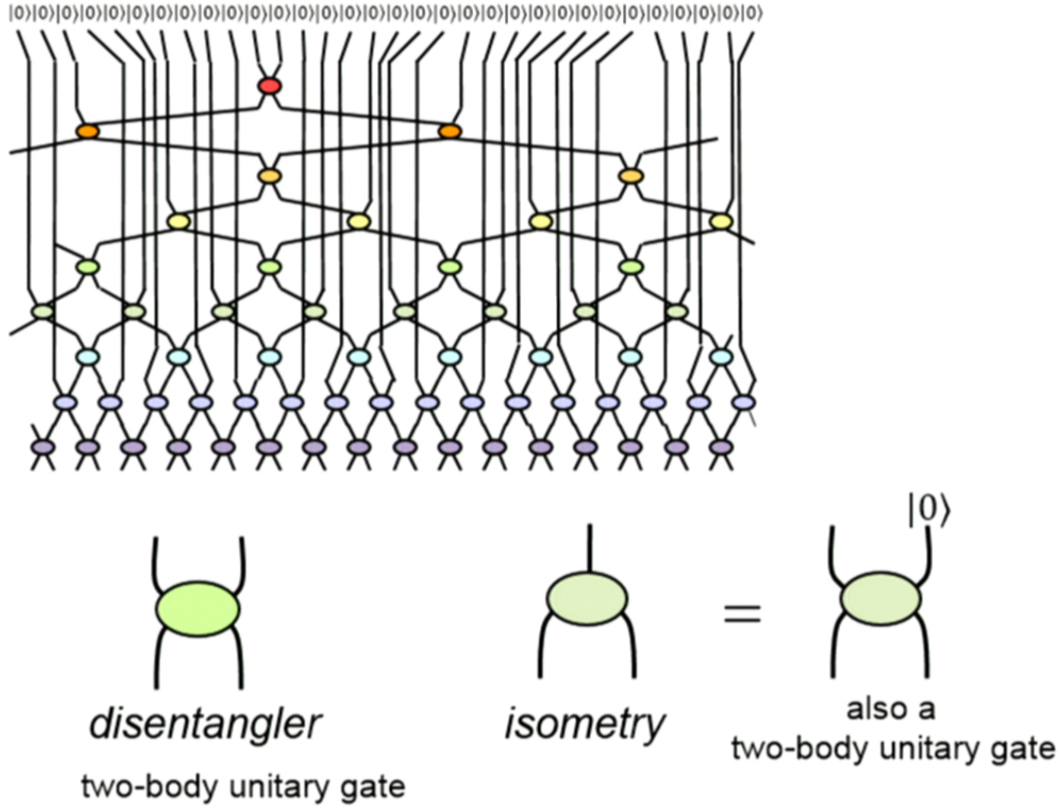

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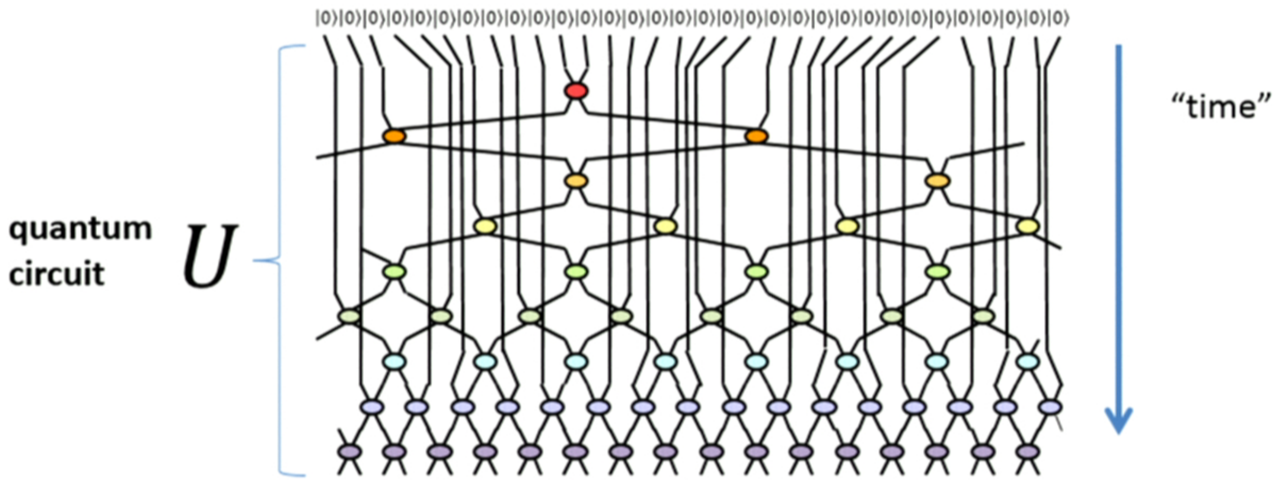

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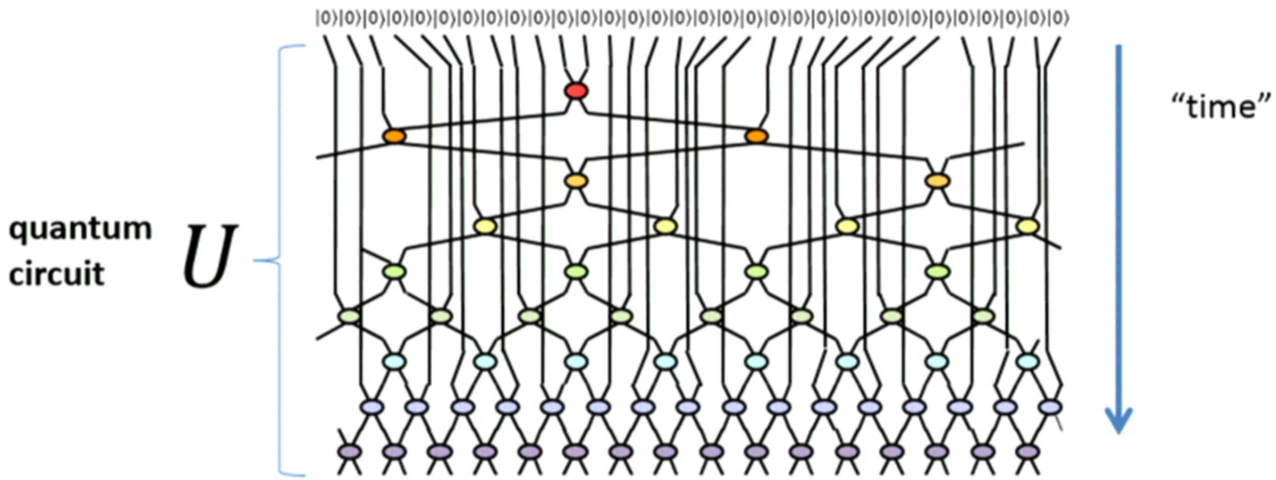
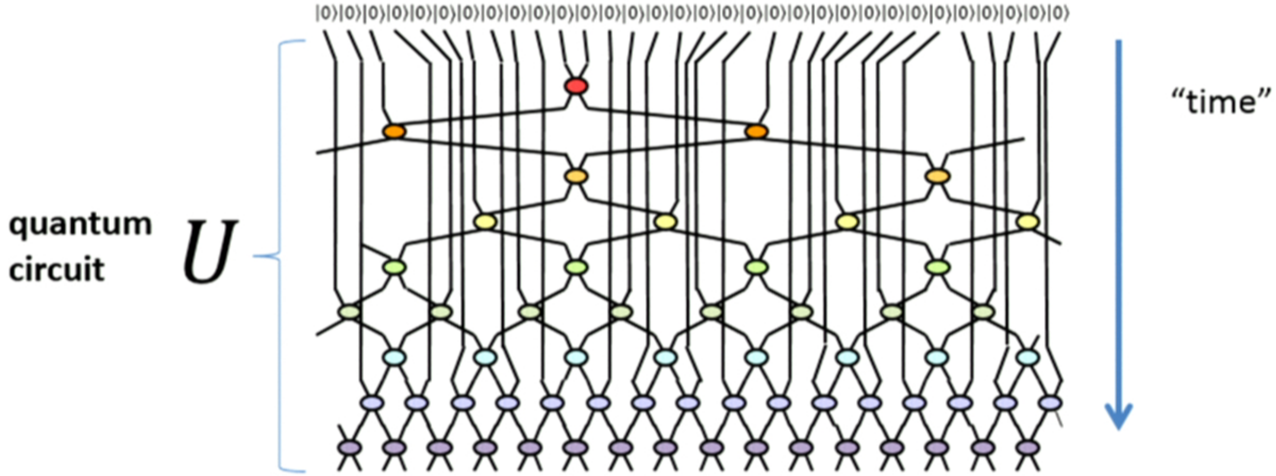


ground state ansatz $|\Psi\rangle = U\,|0\rangle^{\otimes N}$

MERA as a quantum circuit

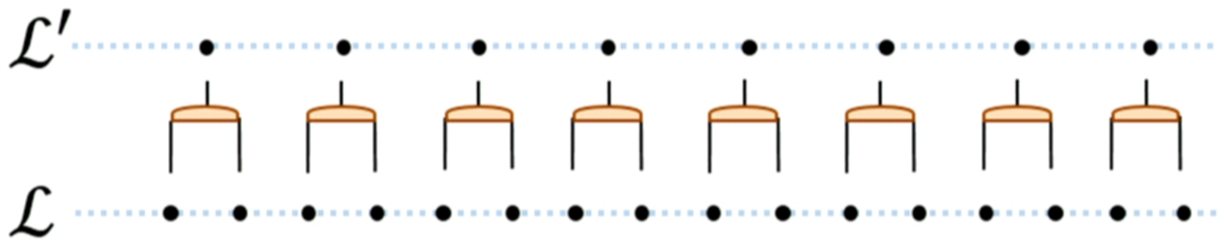


ground state ansatz $|\Psi\rangle = U\ |0\rangle^{\otimes N}$

Entanglement introduced by gates at different “times” (= length scales)

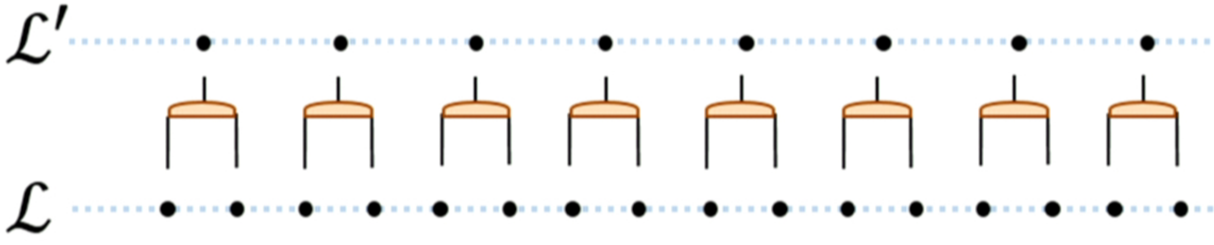
MERA as a (real space) Renormalizatin Group transformation

Kadanoff (1966) + White (1992)
blocking variational optimization

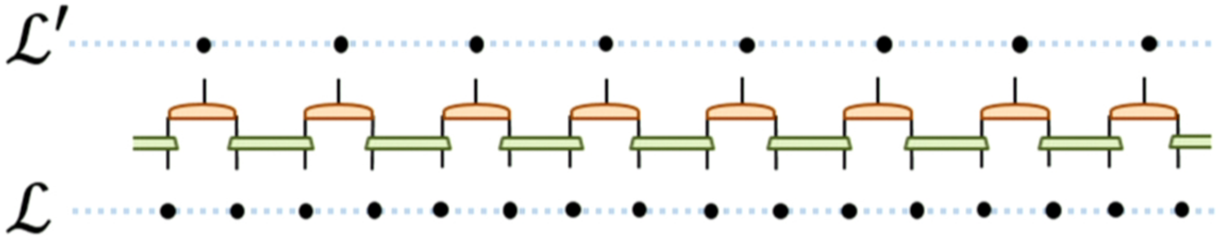


MERA as a (real space) Renormalizatin Group transformation

Kadanoff (1966) + White (1992)
blocking variational optimization



Entanglement renormalization (2005)



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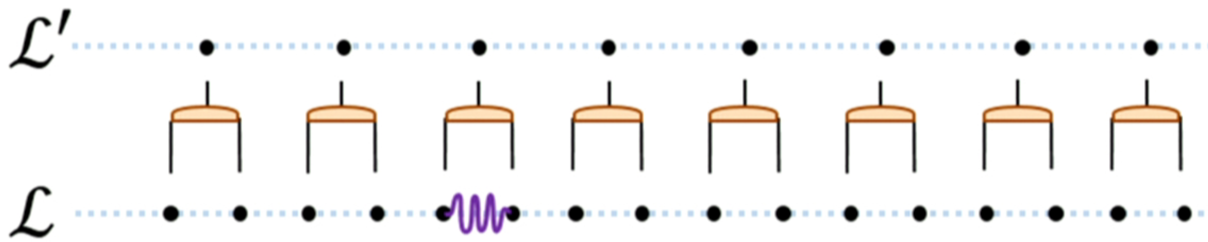
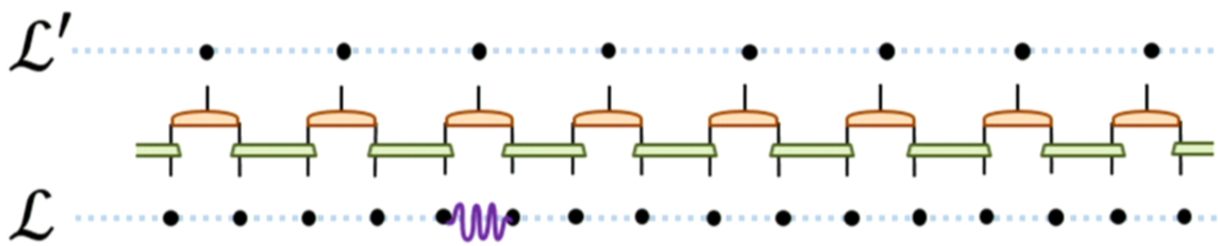


Entanglement renormalization (2005)

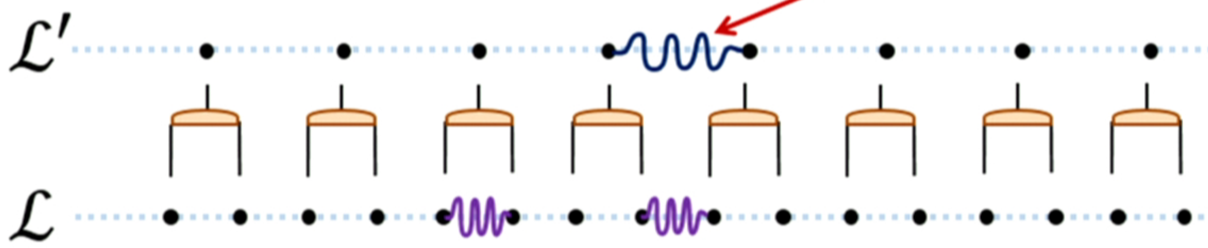


MERA as a (real space) Renormalizatin Group transformation

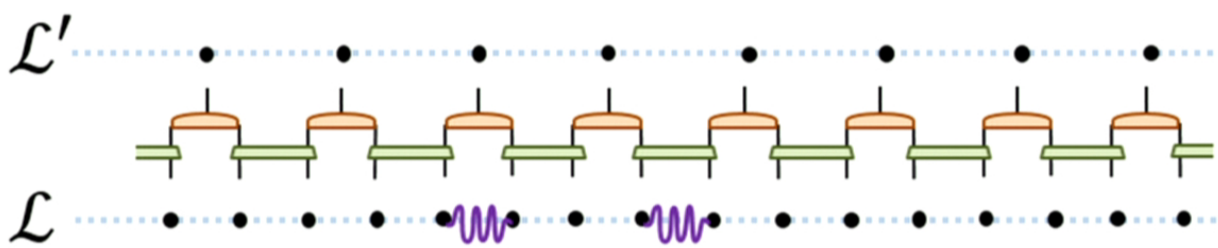
Kadanoff (1966) + White (1992)

blocking   variational optimization

failure to remove ***some*** short-range entanglement !

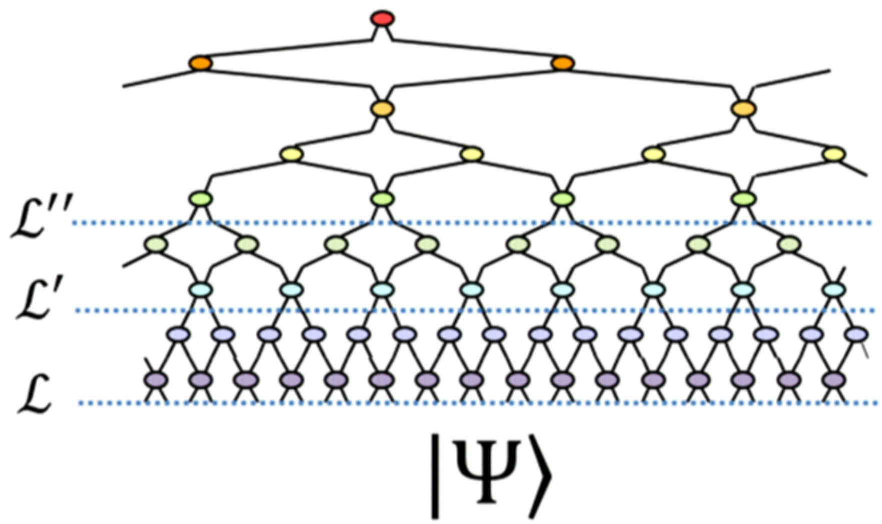


Entanglement renormalization (2005)



MERA as a (real space) Renormalizatin Group transformation

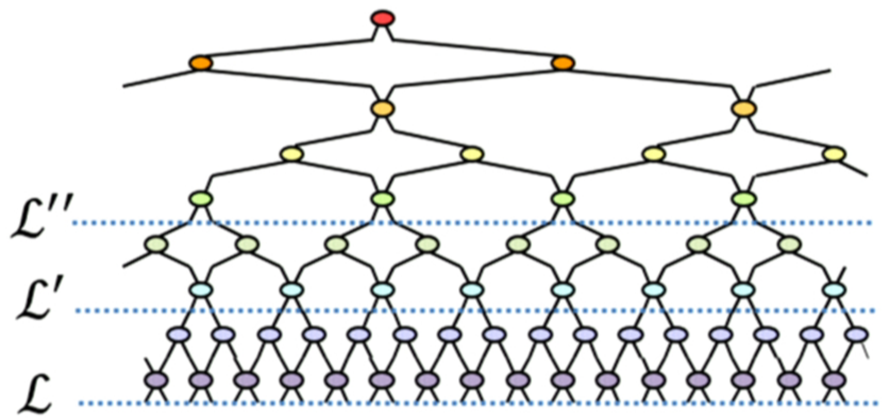
sequence of ground state wave-functions



$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

MERA as a (real space) Renormalizatin Group transformation

sequence of ground state wave-functions



$$|\Psi\rangle$$

$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

MERA defines an RG flow
in the space of wave-functions

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in the space of wave-functions

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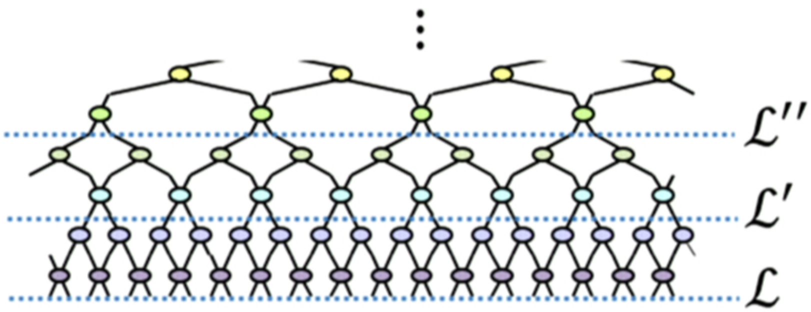

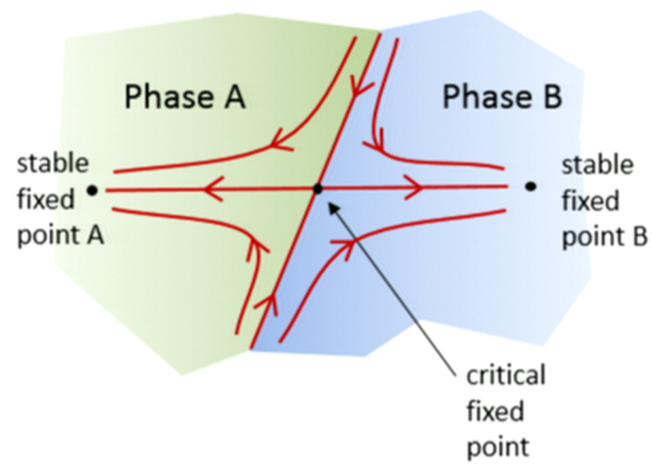

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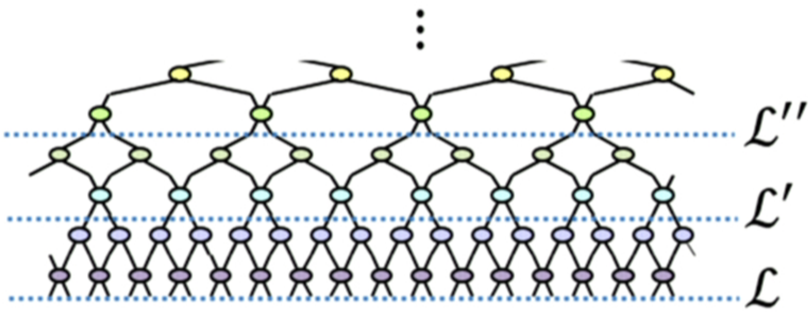

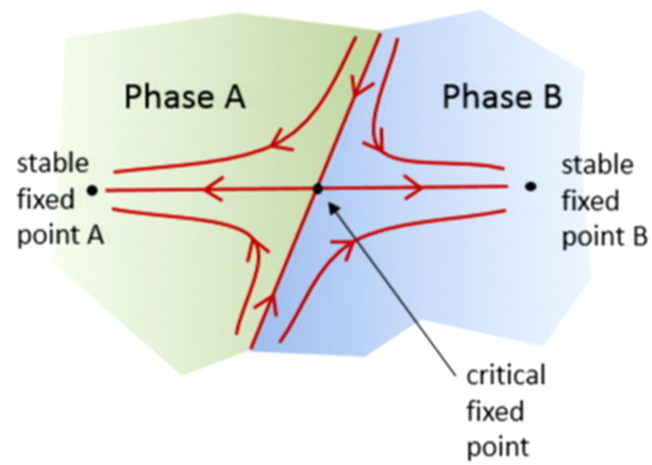

... and in the space of Hamiltonians

$H \rightarrow H' \rightarrow H'' \rightarrow \cdots$

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in the space of wave-functions

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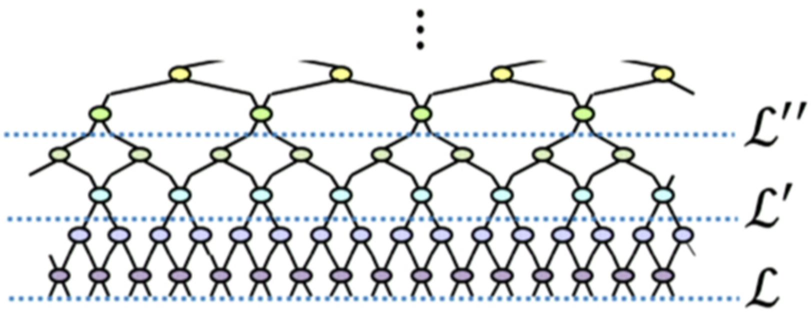

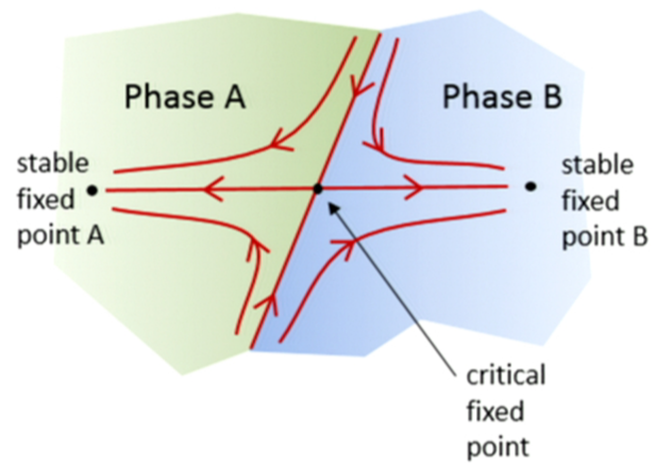

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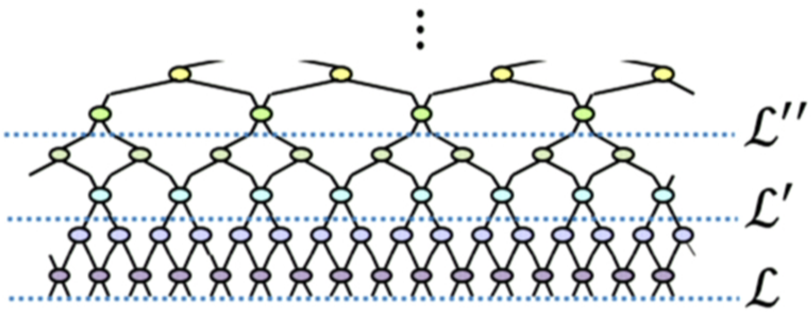

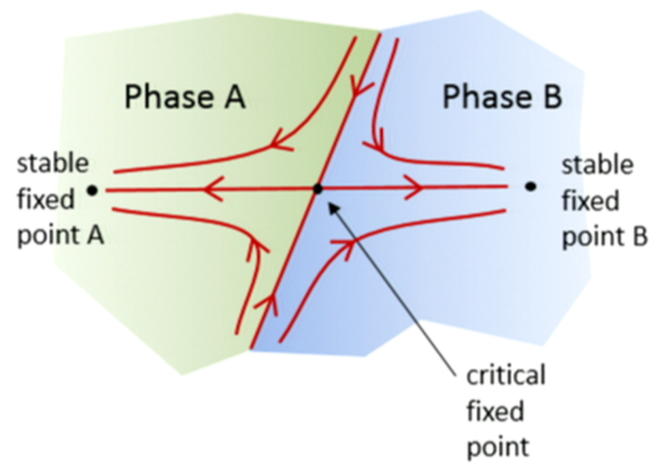

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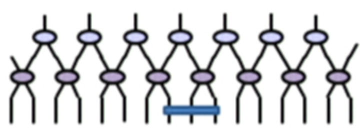

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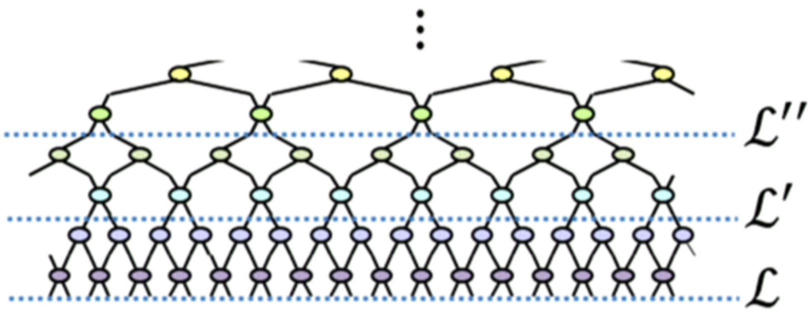

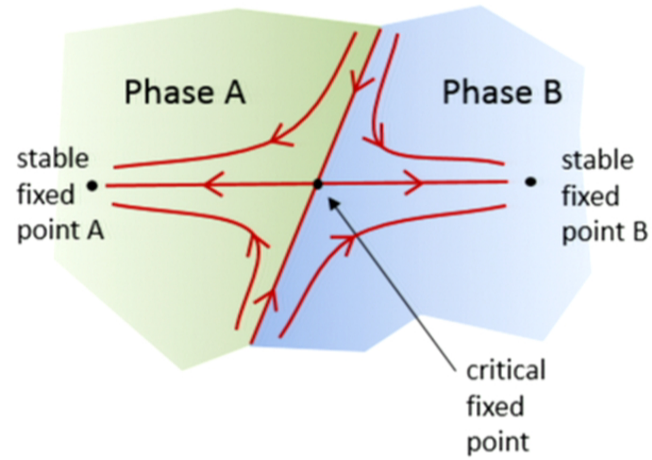

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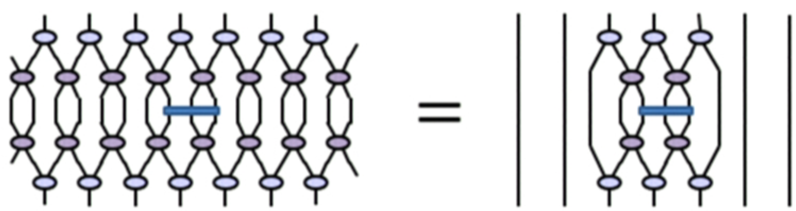

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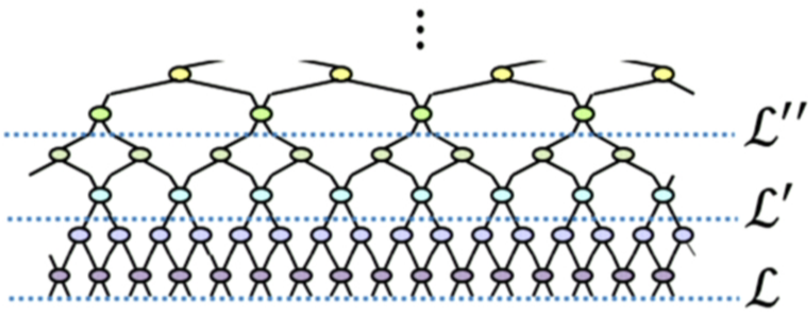

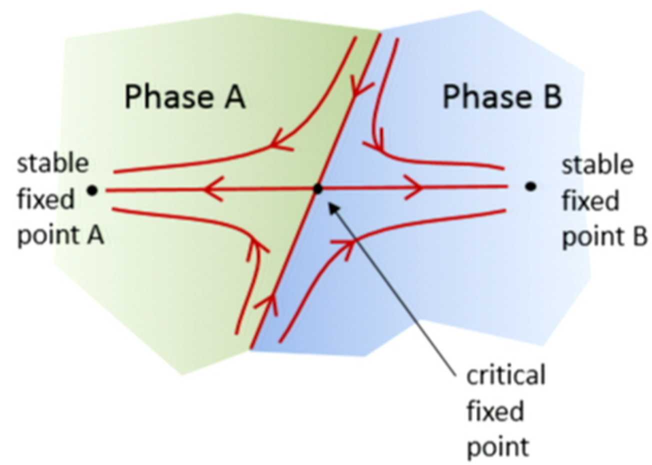

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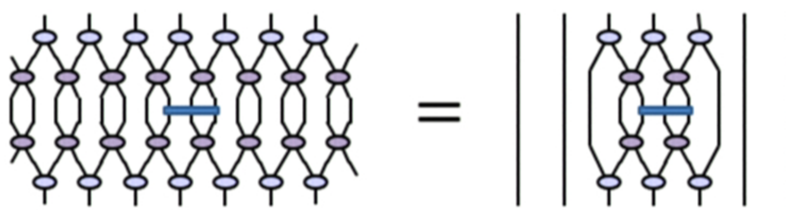

local operators are mapped into local operators !