

Title: The multi-scale entanglement renormalization ansatz

Date: Aug 26, 2015 11:00 AM

URL: <http://pirsa.org/15080047>

Abstract: TBA

## Summary of matrix product state (MPS)

$$|\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$$

## Summary of matrix product state (MPS)

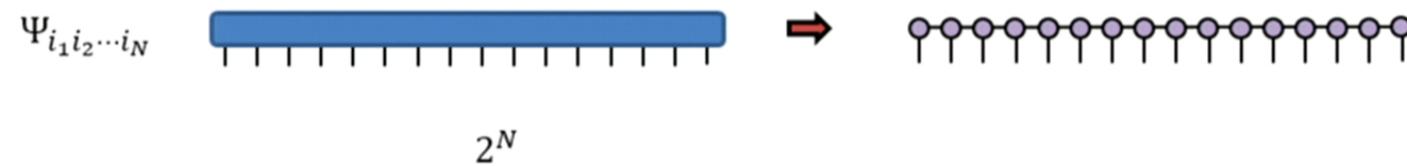
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$$\Psi_{i_1 i_2 \cdots i_N}$$


$$2^N$$

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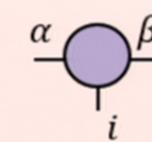


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$\Psi_{i_1 i_2 \cdots i_N}$    $\rightarrow$  

$$2^N$$



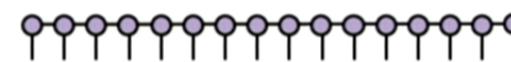
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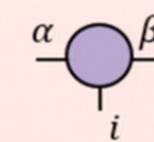


$2^N$



$O(d\chi^2)$

Efficient representation!



$|\alpha| = |\beta| = \chi$

$|i| = d$

$O(d\chi^2)$  parameters

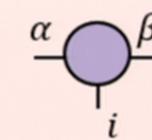
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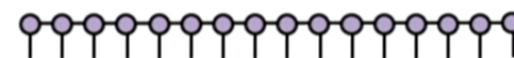
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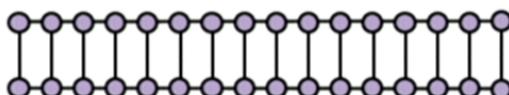
$$O(d\chi^2) \text{ parameters}$$



$$O(Nd\chi^2)$$

Efficient representation!

Efficient computation?

$$\langle \Psi | \Psi \rangle =$$


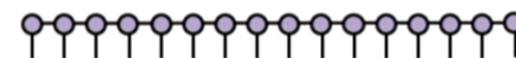
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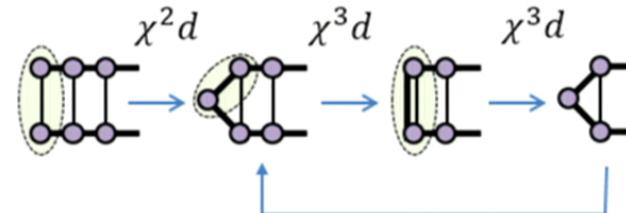


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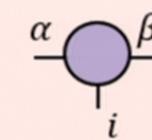
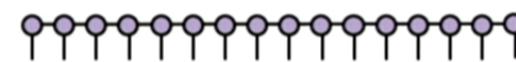
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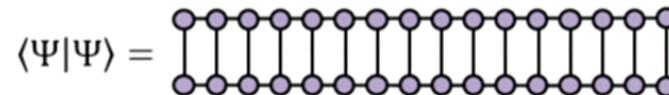
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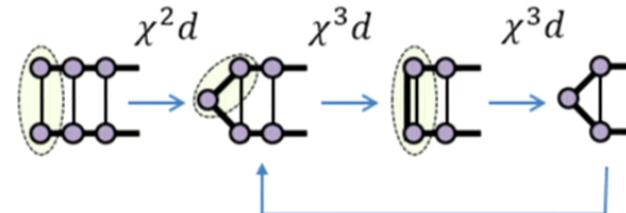
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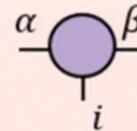


$$O(Nd\chi^3) !!!$$



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 $|\alpha| = |\beta| = \chi$   
 $|i| = d$   
 $O(d\chi^2)$  parameters

$$\Psi_{i_1 i_2 \dots i_N} \quad \xrightarrow{\hspace{1cm}} \quad \text{A chain of } d \text{ circles connected by vertical lines}$$

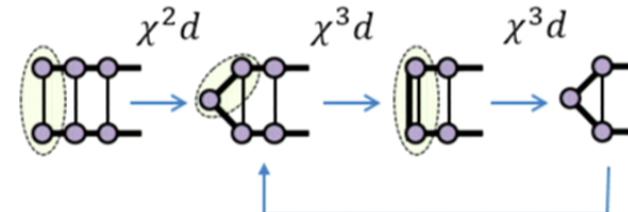
$2^N$

$O(Nd\chi^2)$

Efficient representation!

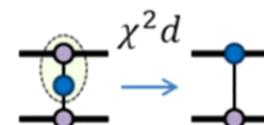
Efficient computation?

$$\langle \Psi | \Psi \rangle = \text{A double chain of } d \text{ circles connected by vertical lines}$$



$O(Nd\chi^3) !!!$

$$\langle \Psi | \hat{o} | \Psi \rangle = \text{A double chain of } d \text{ circles connected by vertical lines, with a blue dot representing an operator at one site}$$



2.B Physics? Structural properties:

- (a) entanglement entropy
- (b) correlations

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➤ entanglement entropy

$$|\Psi\rangle\langle\Psi| = \begin{array}{c} \text{---} \\ | \text{---} | \end{array}$$



$$\rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

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➤ entanglement entropy

- (a) entanglement entropy  
(b) correlations

$$|\Psi\rangle\langle\Psi| = \begin{array}{ccccccccccccc} & & & & & & & & & & & & & & \\ \text{---} & \text{---} \\ | & | & | & | & | & | & | & | & | & | & | & | & | & | & | \end{array}$$

$$\downarrow \quad \rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

$$\rho_A = \begin{array}{c} \text{---} & \text{---} \\ | & | & | & | & | & | & | & | & | & | & | & | & | & | & | \\ \text{---} & \text{---} \\ | & | & | & | & | & | & | & | & | & | & | & | & | & | & | \\ \text{---} & \text{---} \\ | & | & | & | & | & | & | & | & | & | & | & | & | & | & | \end{array}$$

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$$\rho_A = \text{tr}_B |\Psi\rangle\langle\Psi|$$

$$\begin{aligned} \rho_A &= \begin{array}{c} \text{Diagram showing a chain of circles (qubits) with two regions highlighted by dashed red boxes. The left region is enclosed in a rectangle, and the right region is also enclosed in a rectangle. Dashed red lines connect the top and bottom boundaries of these regions.} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \\ &= \begin{array}{c} \text{Diagram showing two horizontal blue bars representing subsystems A and B. A vertical line connects them. Dashed red dots are placed at the top and bottom of this vertical line. An arrow points from the text } \mu = (\alpha, \beta) \text{ to this vertical line.} \\ \mu = (\alpha, \beta) \end{array} \end{aligned}$$

2.B Physics? Structural properties:

➤ entanglement entropy

- (a) entanglement entropy  
(b) correlations

$$\begin{aligned} |\Psi\rangle\langle\Psi| &= \text{[Diagram of a 2D lattice with purple circles connected by horizontal and vertical lines]} \\ \downarrow & \\ \rho_A &= tr_B |\Psi\rangle\langle\Psi| \\ &= \text{[Diagram of a 2D lattice with a central region highlighted in red dashed lines, representing a subsystem A, with two horizontal bars below it labeled with } \mu = (\alpha, \beta) \text{]} \end{aligned}$$
$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

2.B Physics? Structural properties:

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➤ entanglement entropy

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$$\mu = (\alpha, \beta)$$

$$\rho_A = \sum_{\mu=1}^{\chi^2} |\phi_\mu\rangle\langle\tilde{\phi}_\mu|$$

$$\begin{aligned} S(\rho_A) &= S(p_1, p_2, \dots, p_{\chi^2}) \\ &\leq S\left(\frac{1}{\chi^2}, \frac{1}{\chi^2}, \dots, \frac{1}{\chi^2}\right) \\ &= \log \chi^2 = 2 \log \chi \end{aligned}$$

## 2.B Physics? Structural properties:

➤ entanglement entropy

$$|\Psi\rangle\langle\Psi| = \begin{array}{c} \text{Diagram of a 2D lattice with } N \text{ sites, each with two horizontal neighbors.} \\ \text{A red dashed line indicates a boundary or cut.} \end{array}$$

$$\rho_A = tr_B |\Psi\rangle\langle\Psi|$$

$$\rho_A = \begin{array}{c} \text{Diagram showing the reduced density matrix } \rho_A \text{ for the left half of the chain.} \\ \text{The right half is traced out.} \\ \text{A red dashed line marks the boundary.} \\ = \begin{array}{c} \text{Diagram of a 1D chain with } L \text{ sites.} \\ \text{A vertical line at position } \mu \text{ separates the chain into two parts: } (\alpha, \beta). \end{array} \end{array}$$

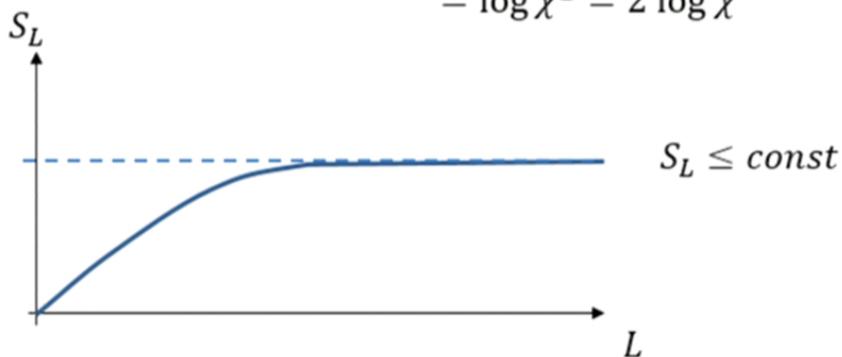
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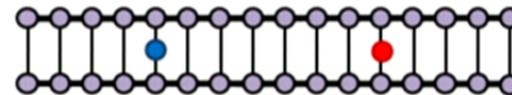
$$\leq S\left(\frac{1}{\chi^2}, \frac{1}{\chi^2}, \dots, \frac{1}{\chi^2}\right)$$

$$= \log \chi^2 = 2 \log \chi$$



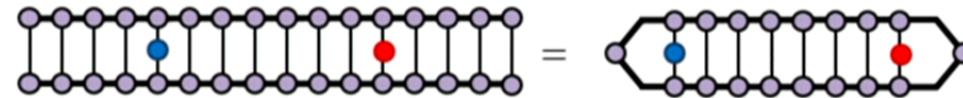
➤ correlations

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle =$$



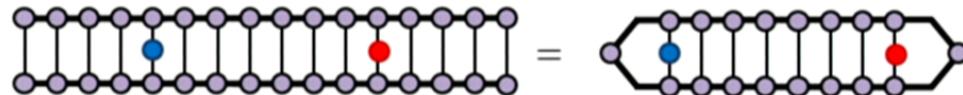
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➤ correlations

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$$= \text{Diagram showing a local interaction between two sites, with a factor } (L-1) \text{ indicated above it.}$$

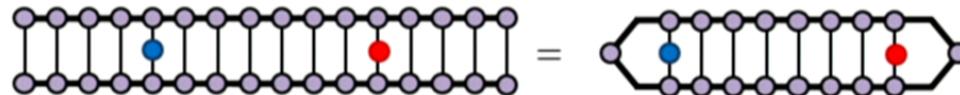
$$\approx a \lambda^L$$

$$a e^{-L/\xi}$$

$\xi \equiv -\frac{1}{\log \lambda}$

➤ correlations

$$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle =$$



$$= \left( \text{local operator} \right)^{L-1}$$

$$\approx a \lambda^L$$

$$\xi \equiv -\frac{1}{\log \lambda}$$

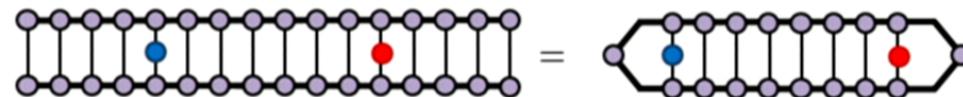
### Structural properties of MPS

correlations       $C(L) \approx e^{-L/\xi}$

entanglement       $S_L \leq 2 \log \chi$

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### Structural properties of MPS

correlations  $C(L) \approx e^{-L/\xi}$

entanglement  $S_L \leq 2 \log \chi$

match with  
ground states of 1D  
gapped Hamiltonians

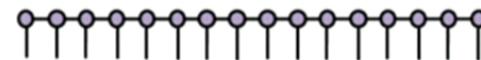
## MERA: definition

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N} \quad d^N \text{ complex numbers}$$

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Matrix product state  
(MPS)



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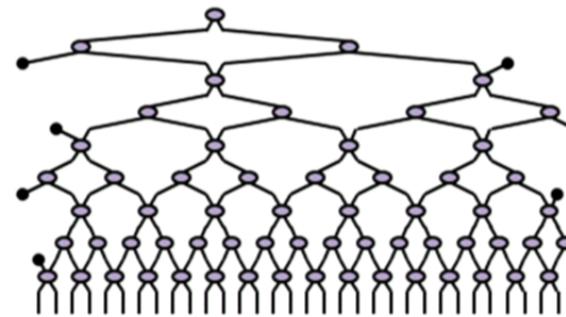
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

$d^N$  complex numbers

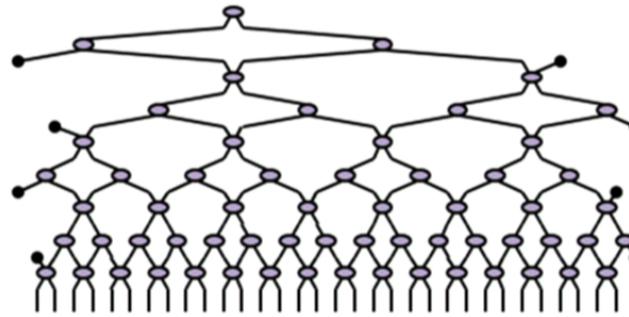
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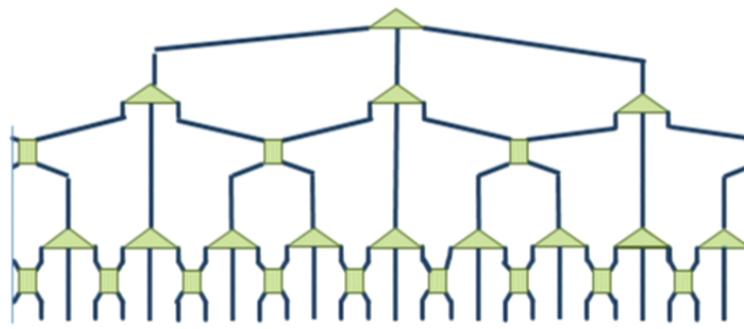
Multi-scale entanglement  
renormalization ansatz  
(MERA)



MERA



also MERA !



## Efficiency

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

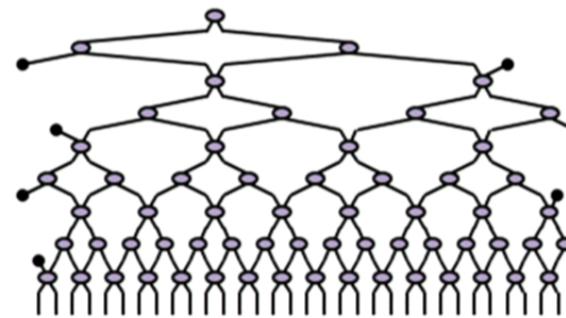
$d^N$  complex numbers

Matrix product state  
(MPS)



$N$  spins

Multi-scale entanglement  
renormalization ansatz  
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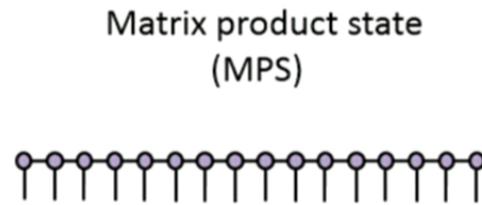


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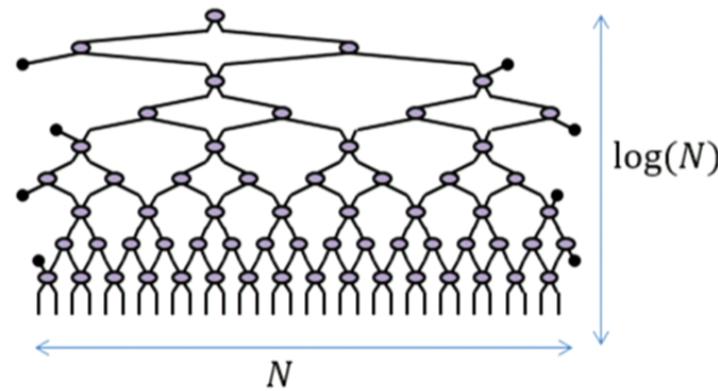
$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N} \quad d^N \text{ complex numbers}$$

$$N + \frac{N}{2} + \frac{N}{4} + \dots = N \left( 1 + \frac{1}{2} + \frac{1}{4} + \dots \right) \leq 2N$$

Multi-scale entanglement  
renormalization ansatz  
(MERA)



$N$  spins  $\Rightarrow N$  tensors  
 $\Rightarrow O(N)$  parameters

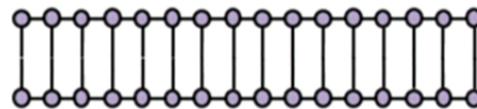


$N$  spins  $\Rightarrow N \log(N)$  tensors ?

## efficiency

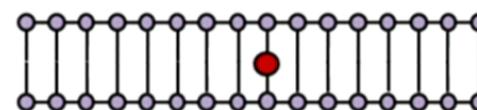
Matrix product state  
(MPS)

$$\langle \Psi | \Psi \rangle$$

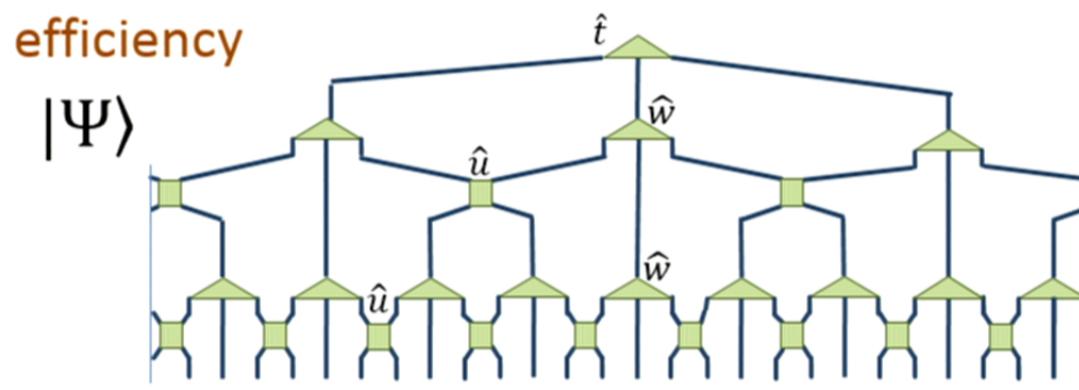


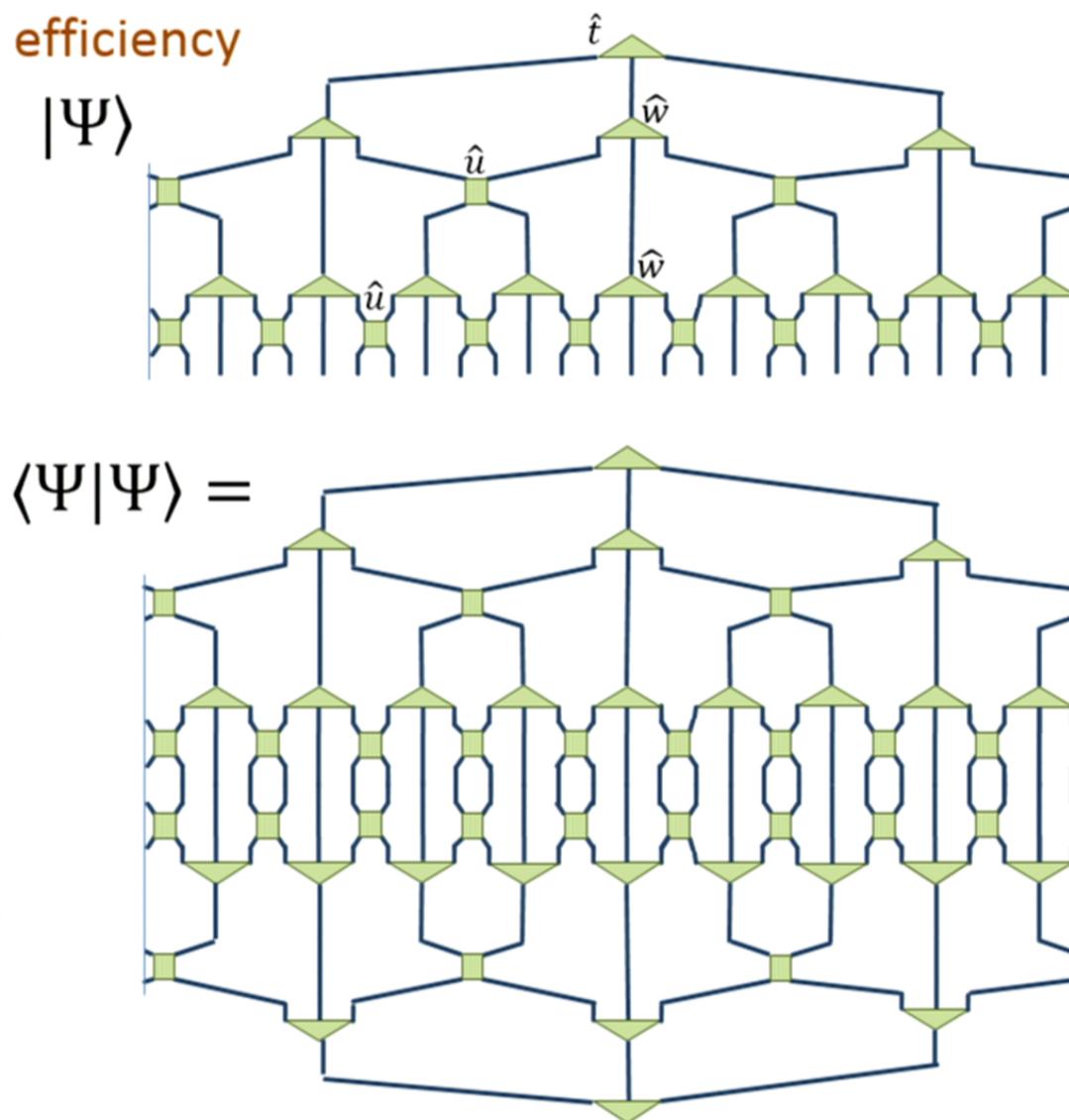
cost  $O(N)$

$$\langle \Psi | \hat{o} | \Psi \rangle$$



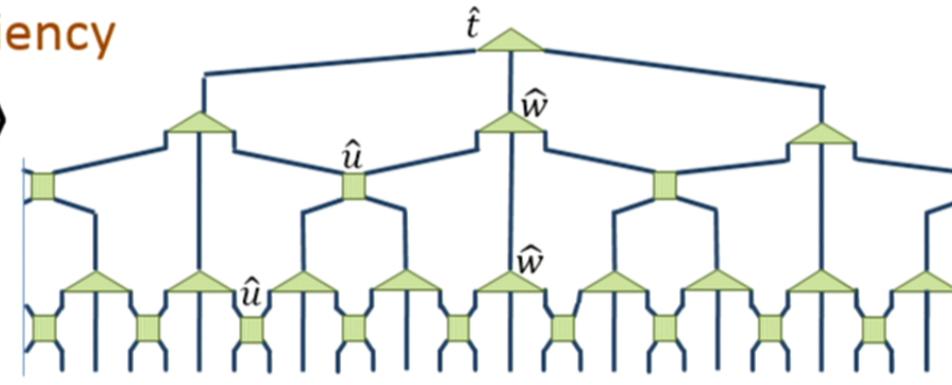
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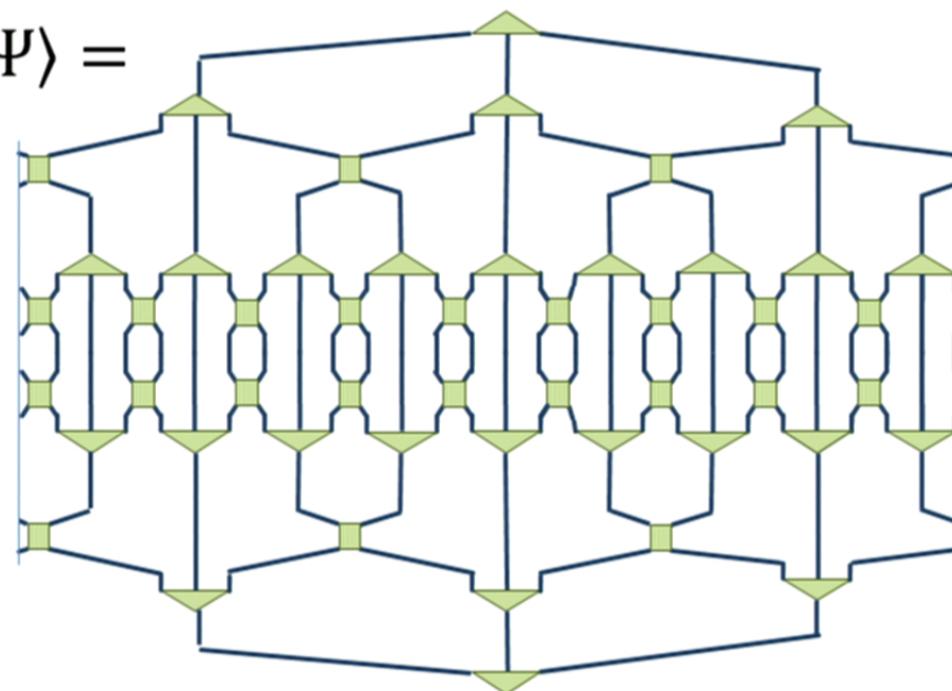


efficiency

$|\Psi\rangle$



$\langle \Psi | \Psi \rangle =$



isometric tensors!

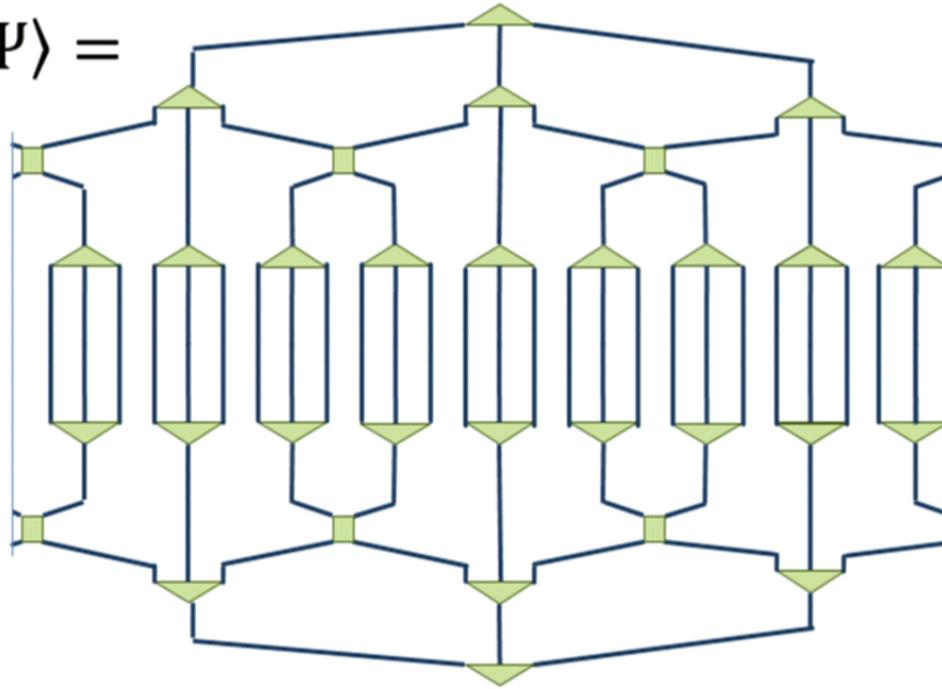
$$\begin{array}{c} \hat{t} \\ \hat{t}^\dagger \end{array} = 1$$

$$\begin{array}{c} \hat{w} \\ \hat{w}^\dagger \end{array} = \mid$$

$$\begin{array}{c} \hat{u} \\ \hat{u}^\dagger \end{array} = \mid\mid$$

efficiency

$$\langle \Psi | \Psi \rangle =$$



isometric tensors!

$$\hat{t} \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} = 1$$

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efficiency

$$\langle \Psi | \Psi \rangle = 1$$

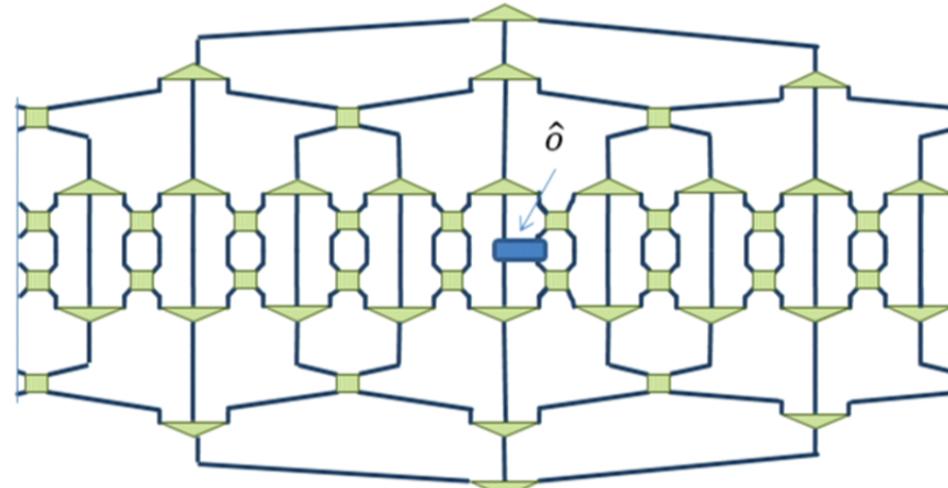
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$$\langle \Psi | \hat{o} | \Psi \rangle =$$



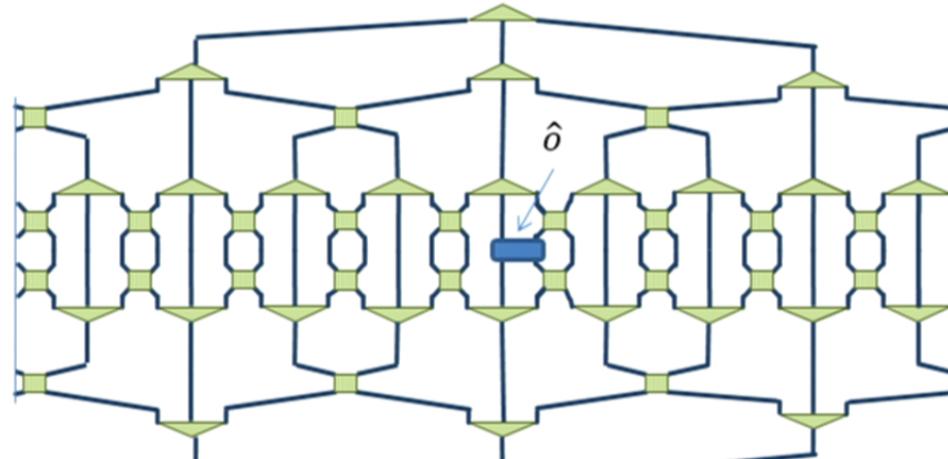
$$\langle \Psi | \hat{o} | \Psi \rangle =$$

isometric tensors!

$$\hat{t} \quad \hat{t}^\dagger \quad = 1$$

$$\hat{w} \quad \hat{w}^\dagger \quad = \quad |$$

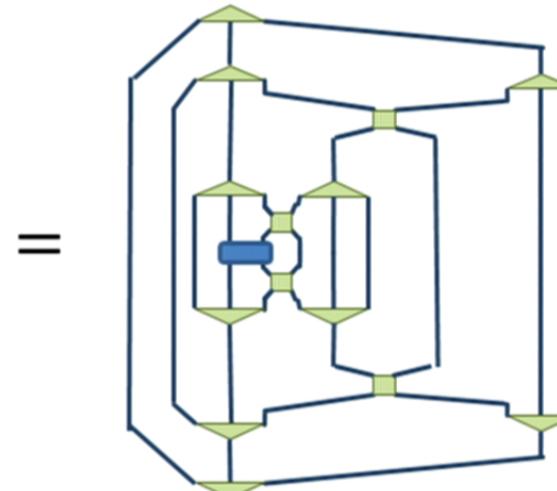
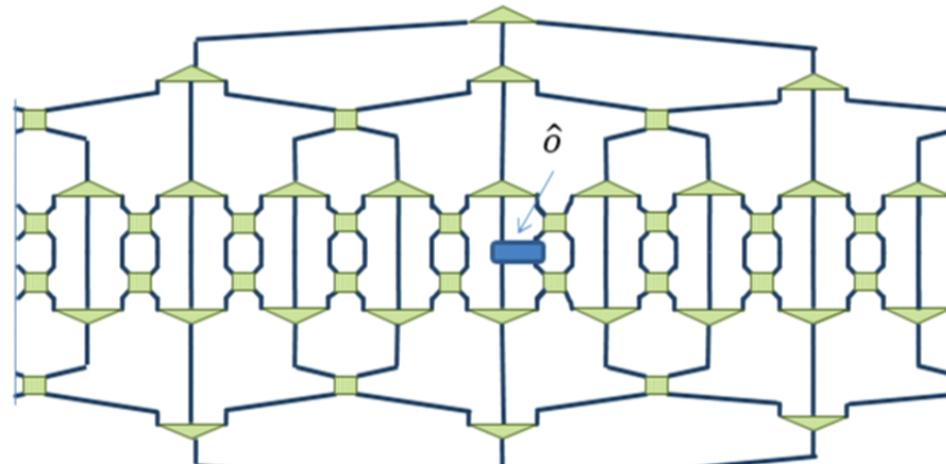
$$\hat{u} \quad \hat{u}^\dagger \quad = \quad ||$$



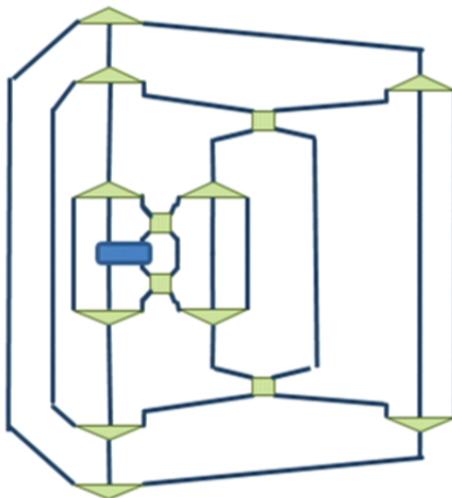
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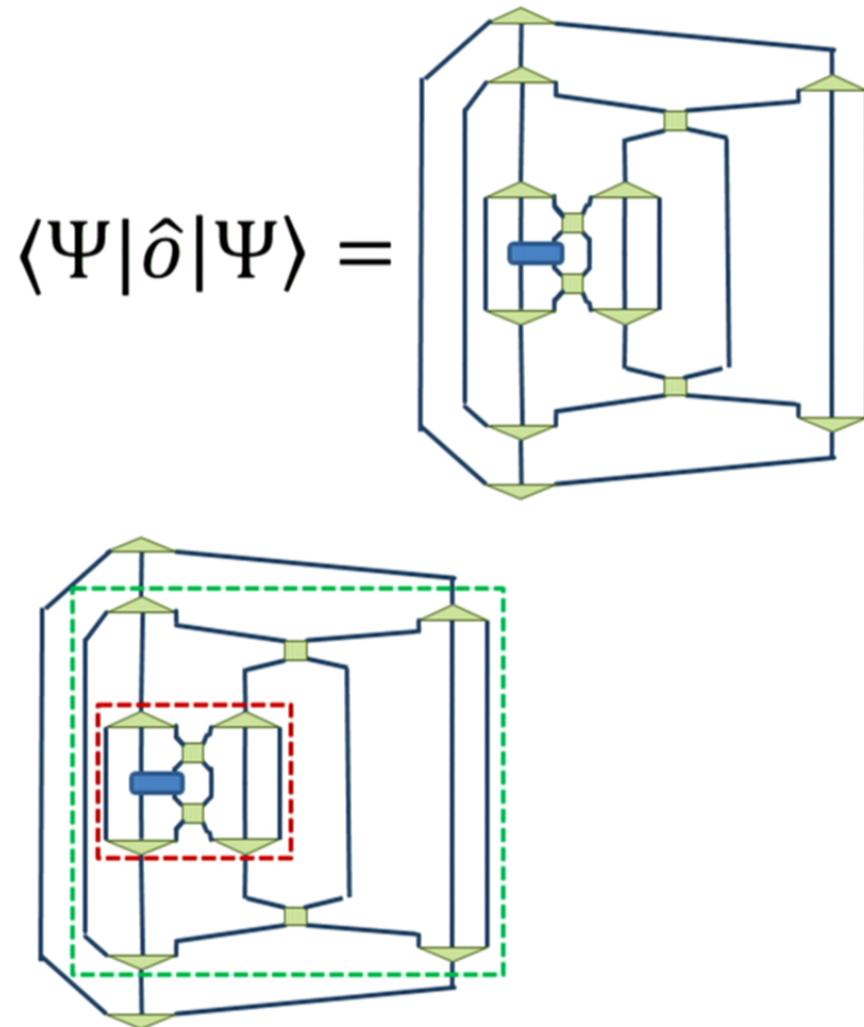
isometric tensors!

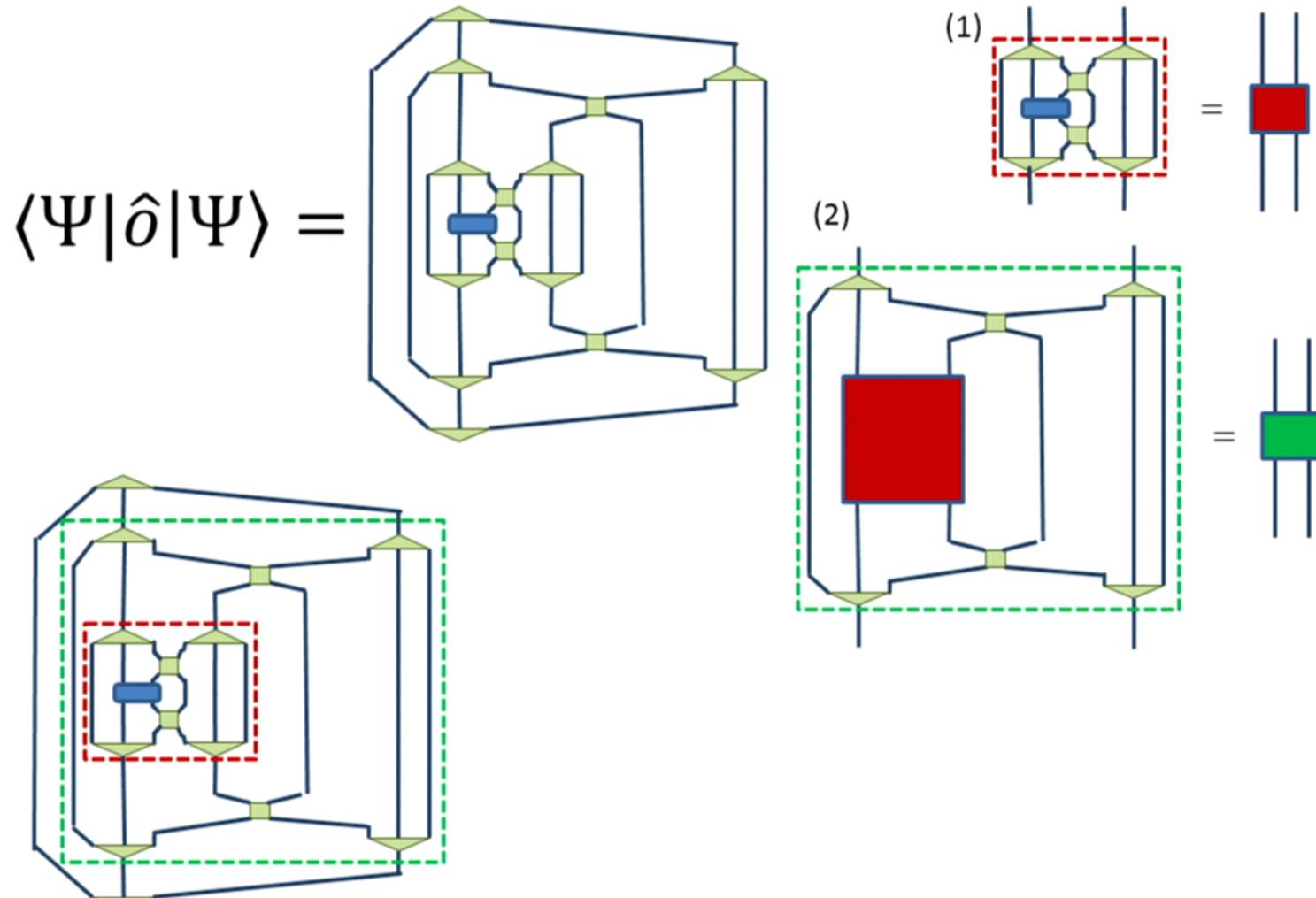
$\hat{t}$		$= 1$
$\hat{t}^\dagger$		$=$
$\hat{w}$		$=$
$\hat{w}^\dagger$		$=$
$\hat{u}$		$=$
$\hat{u}^\dagger$		$=$

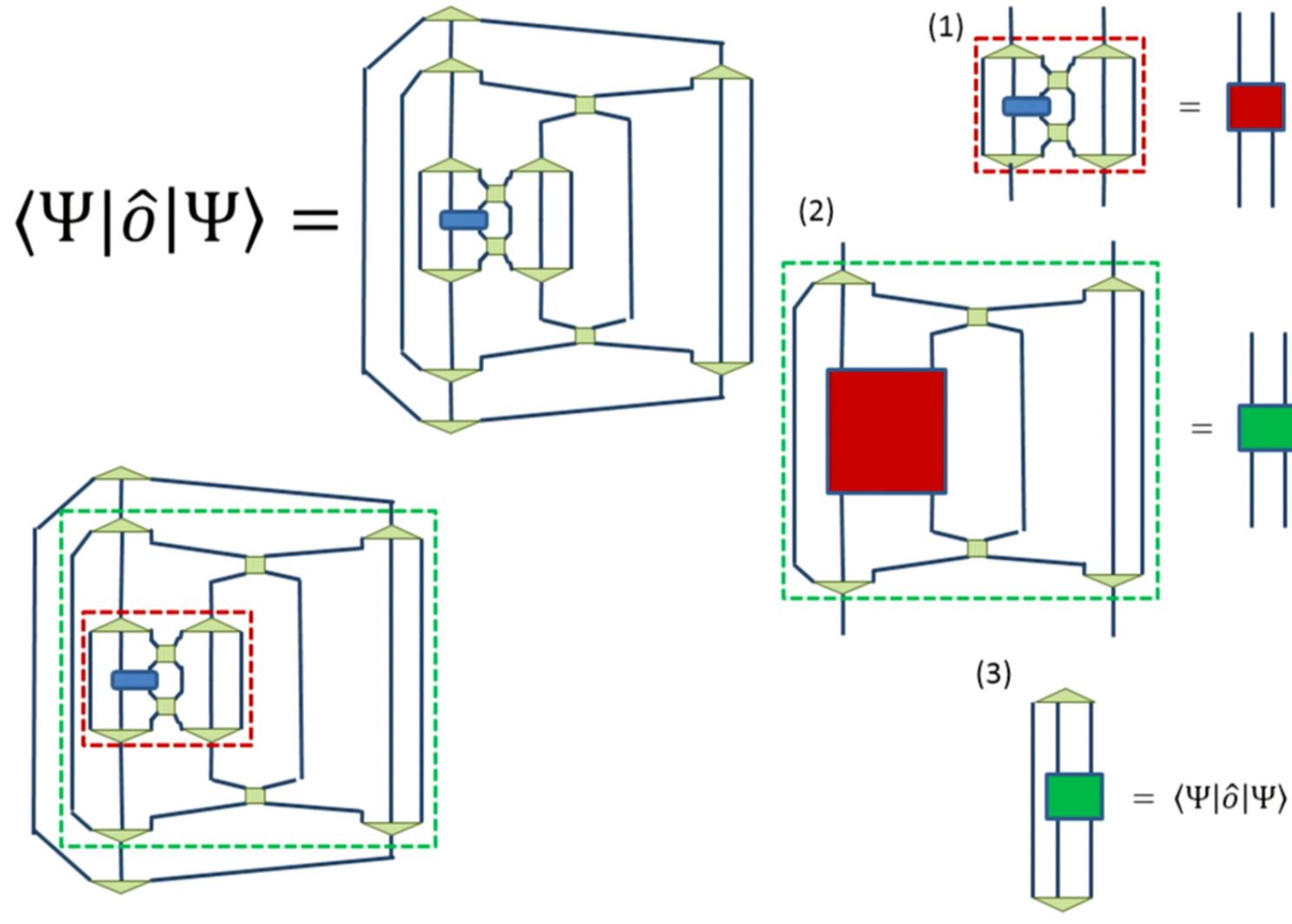


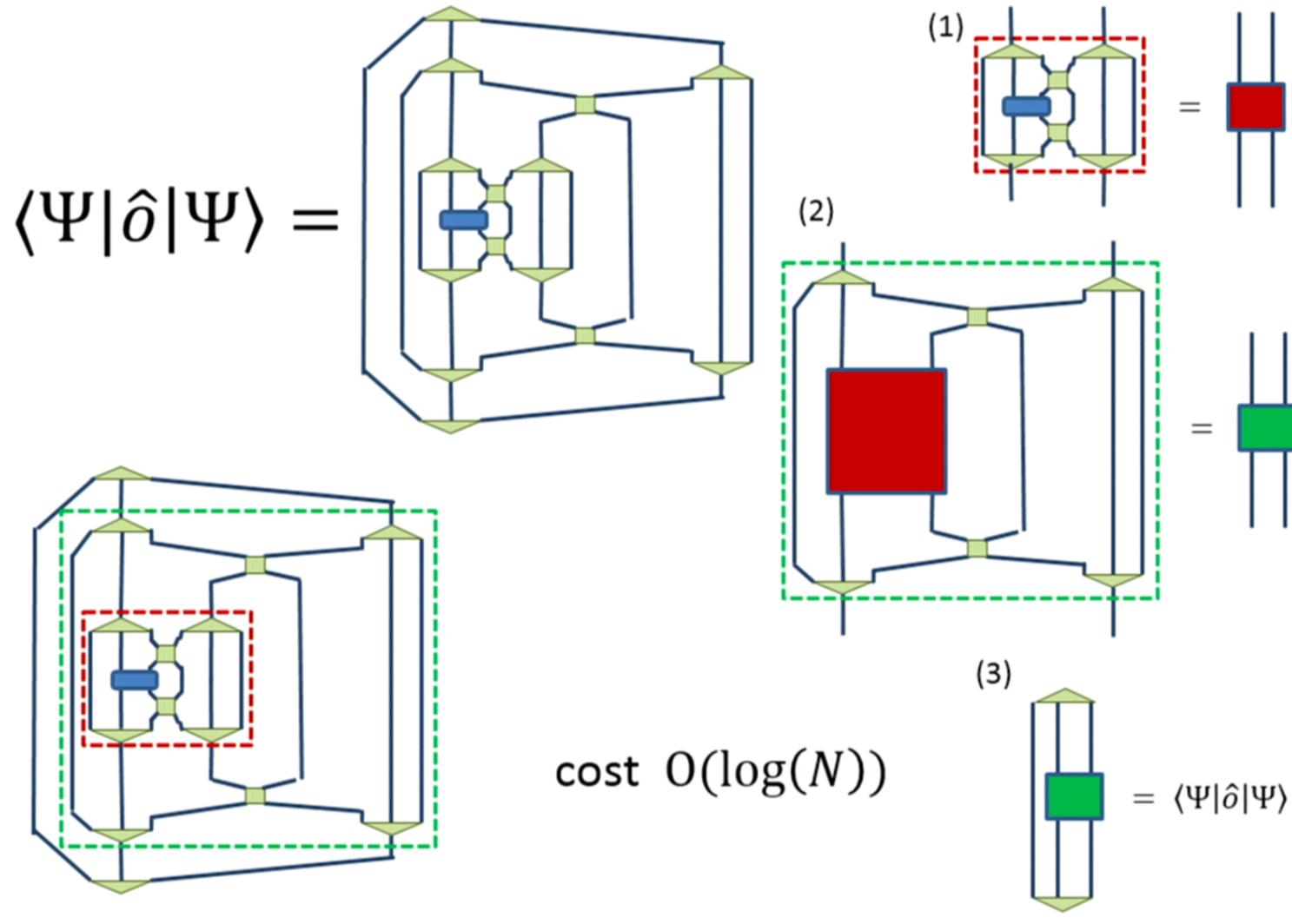
$$\langle \Psi | \hat{o} | \Psi \rangle =$$







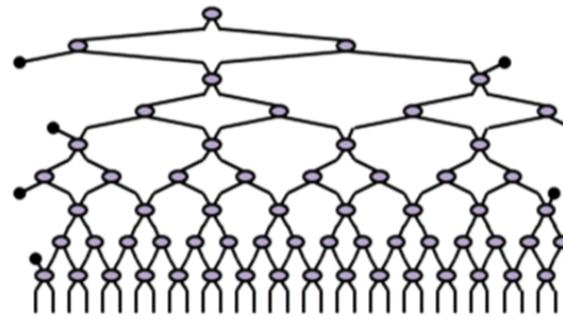




## Structural properties

$$|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$$

$d^N$  complex numbers



- Decay of correlations
- Scaling of entanglement

$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$ 

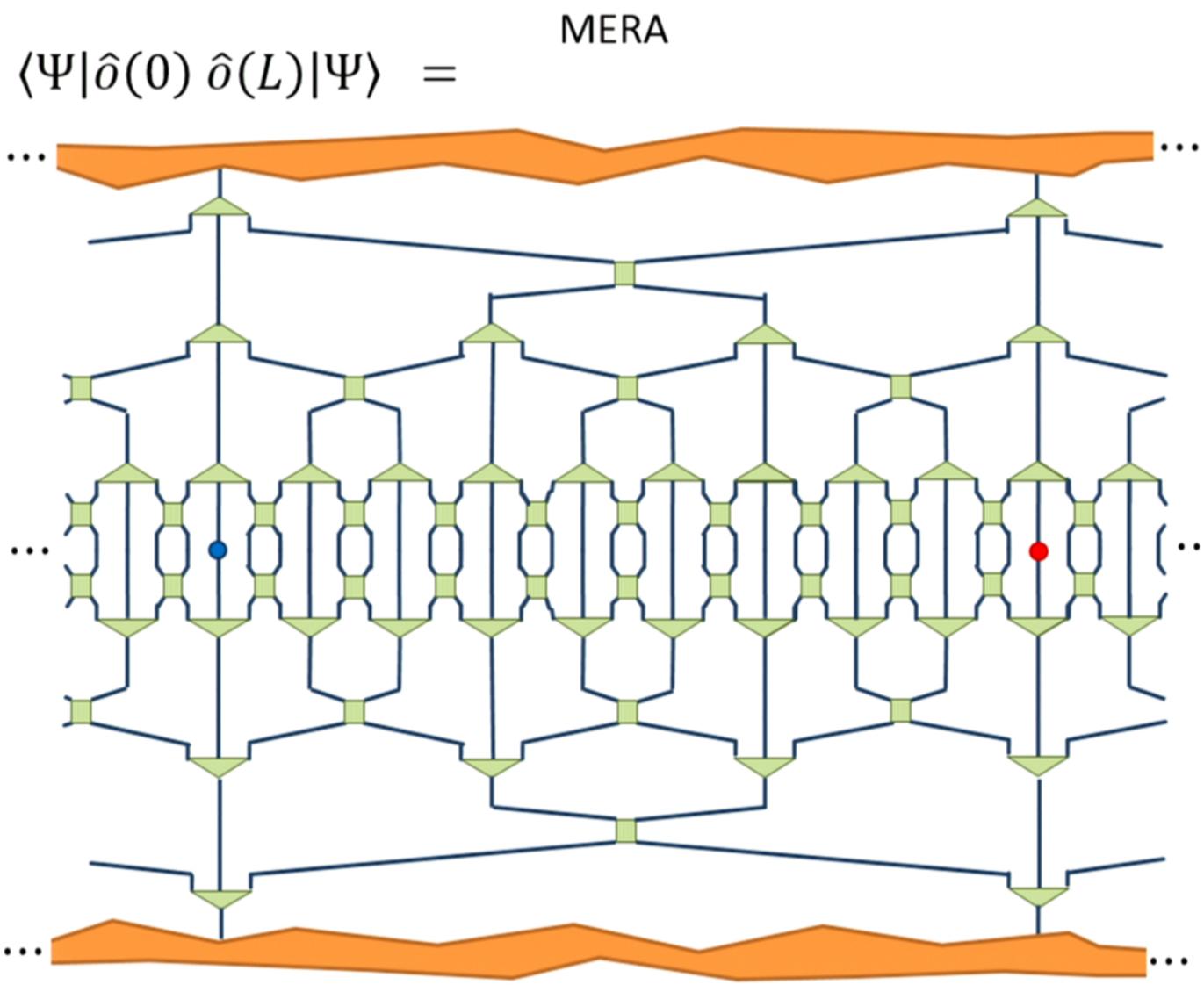
MPS

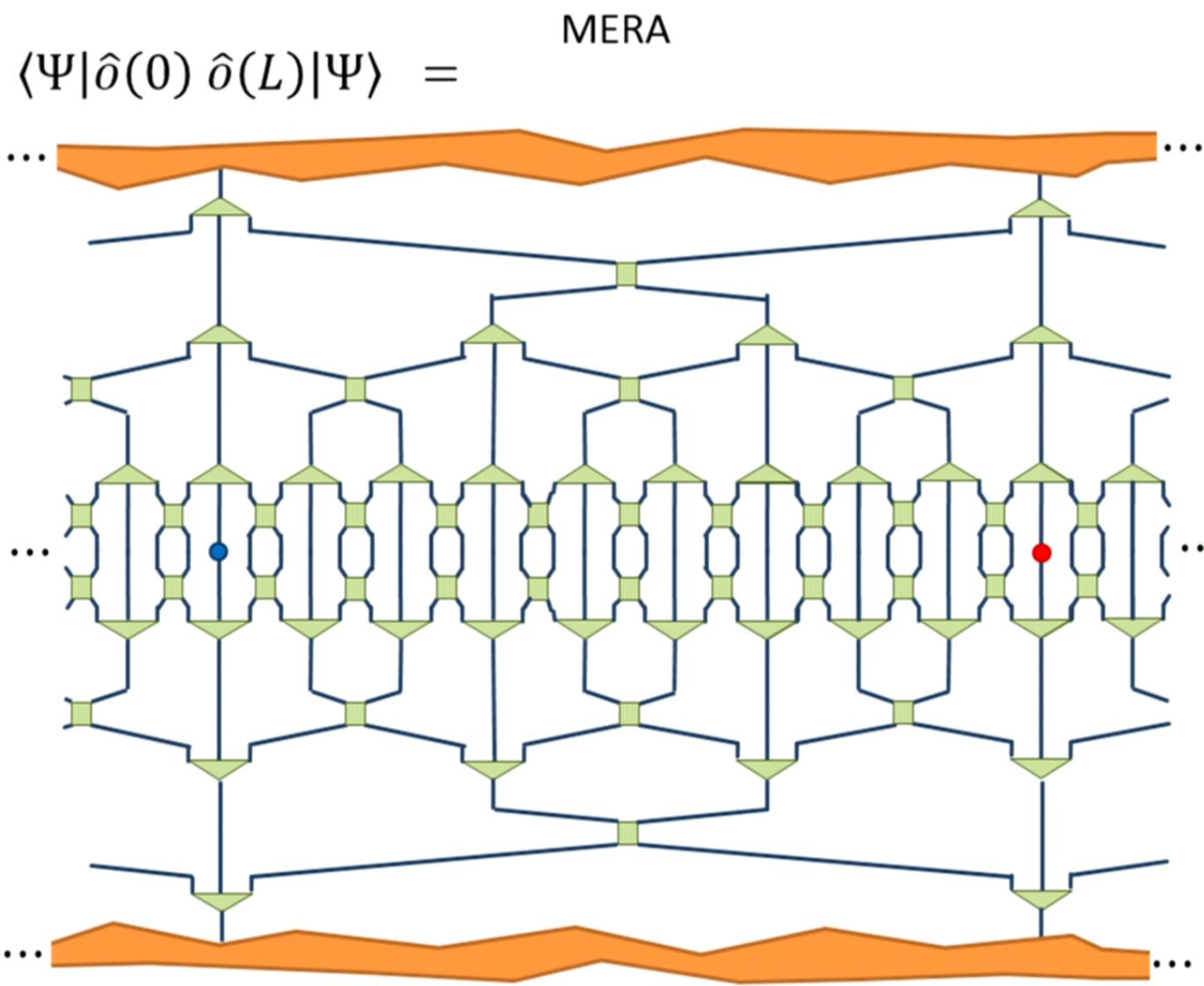
$$= \begin{array}{c} \text{Diagram of a 1D MPS with two highlighted sites: a blue circle at site 0 and a red circle at site L. The MPS consists of two horizontal rows of purple circles connected by vertical lines.} \\ \text{A diagram showing a 1D Matrix Product State (MPS) represented as a chain of circles. The chain has two highlighted sites: one blue circle at the left end (site 0) and one red circle at the right end (site L). The MPS is shown as two parallel horizontal rows of circles, where each circle is connected to its neighbors by vertical lines, forming a ladder-like structure. The blue circle is located on the top row, and the red circle is located on the bottom row. The rest of the sites in the MPS are represented by purple circles. The entire expression is preceded by an equals sign (=).} \end{array}$$

$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$ 

MPS

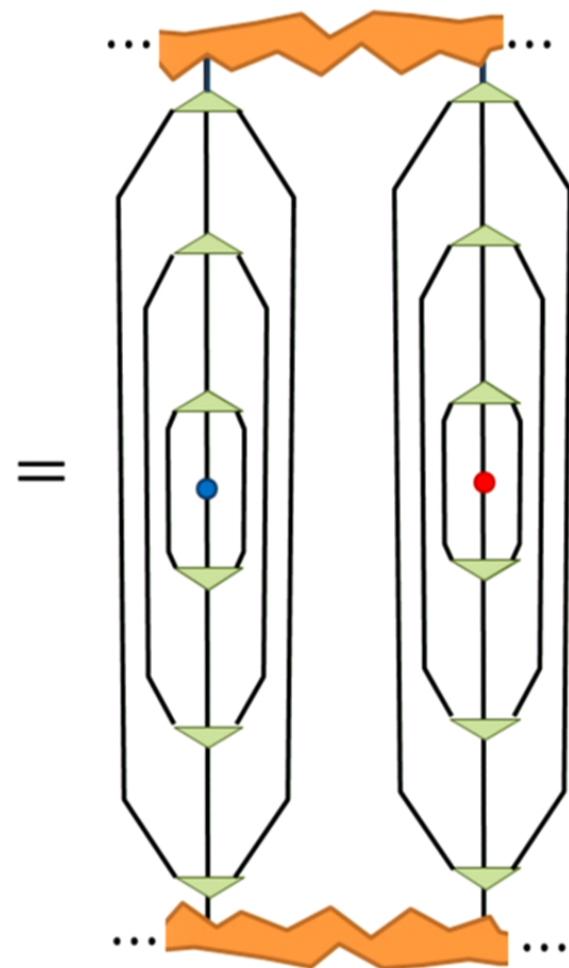
$$\begin{aligned} &= \text{Diagram of a 1D chain of sites (purple circles) with two central sites highlighted in blue and red.} \\ &= \text{Diagram of a 1D chain of sites (purple circles) with two central sites highlighted in blue and red, enclosed in a hexagonal boundary.} \\ &= \text{Diagram showing the MPS structure as a product of local operators: } \left( \text{Diagram with blue site} \right)^{L-1} \left( \text{Diagram with red site} \right) \approx a \lambda^L = a e^{-L/\xi} \end{aligned}$$
$$\xi \equiv -\frac{1}{\log \lambda}$$



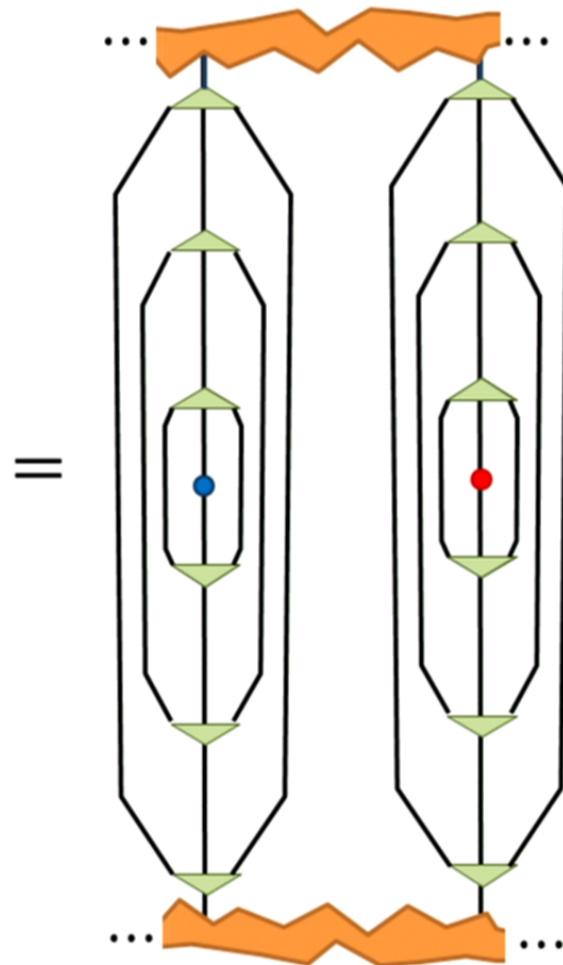


$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

MERA



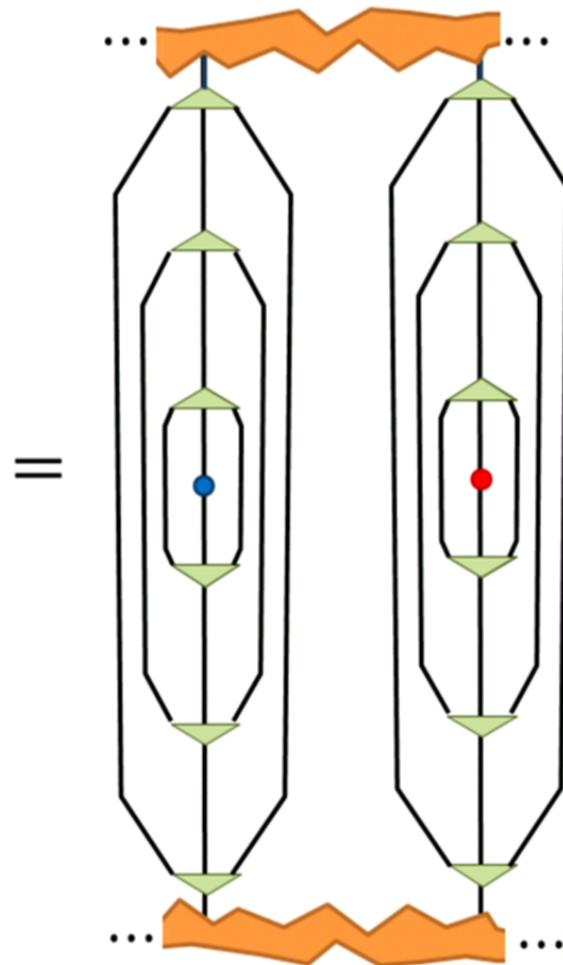
$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$



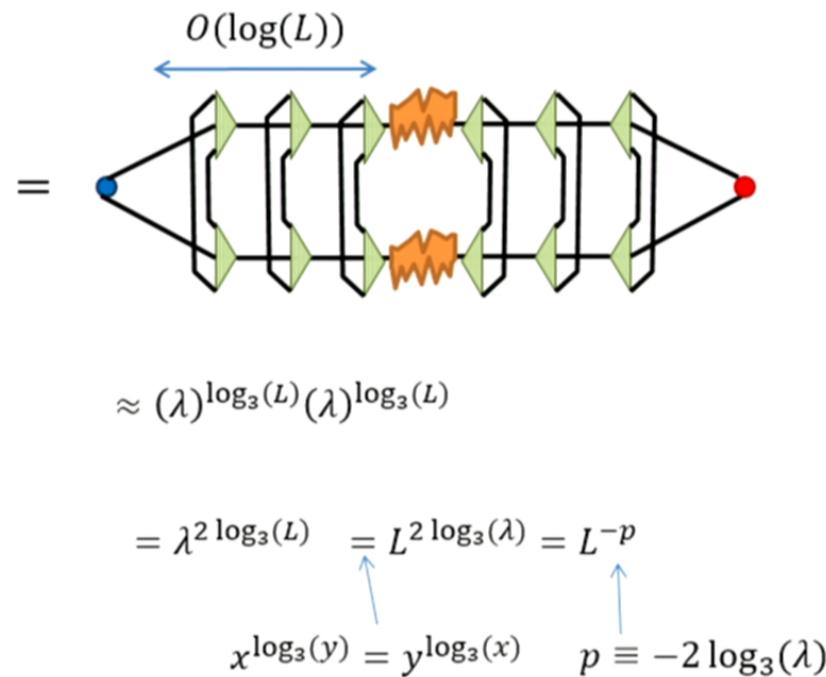
MERA

A circuit diagram representing the MERA structure. It shows a sequence of black rectangular boxes connected by horizontal lines. Between the second and third boxes, there are two orange wavy lines. Above the circuit, a double-headed arrow indicates a complexity of  $O(\log(L))$ . Below the circuit, the expression  $\approx (\lambda)^{\log_3(L)} (\lambda)^{\log_3(L)}$  is shown. Further down, the equation  $= \lambda^{2 \log_3(L)} = L^{2 \log_3(\lambda)} = L^{-p}$  is given, with arrows pointing from  $\lambda^{\log_3(y)}$  to  $L^{\log_3(x)}$  and from  $p \equiv -2 \log_3(\lambda)$  back up to the circuit.

$\langle \Psi | \hat{o}(0) \hat{o}(L) | \Psi \rangle$

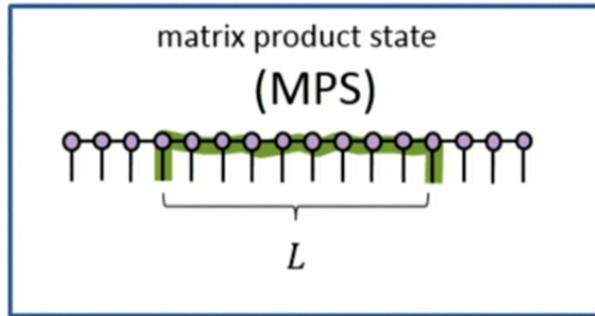


MERA



$\Rightarrow$  polynomial decay of correlations

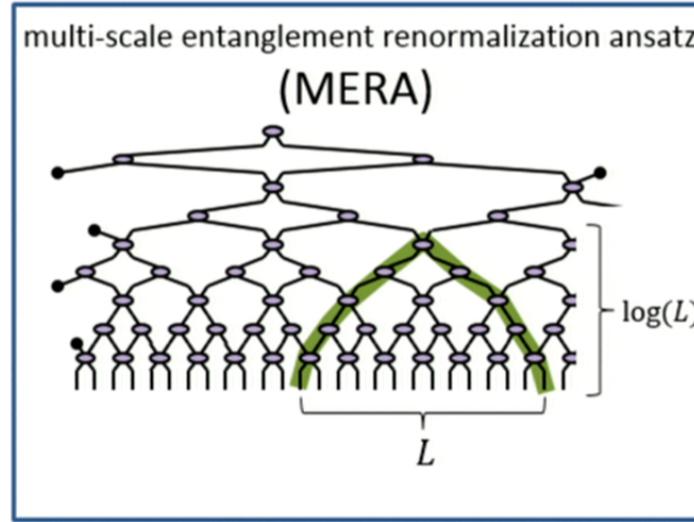
## Correlations: summary and interpretation



structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L) \rangle \approx e^{-L/\xi}$$

exponential

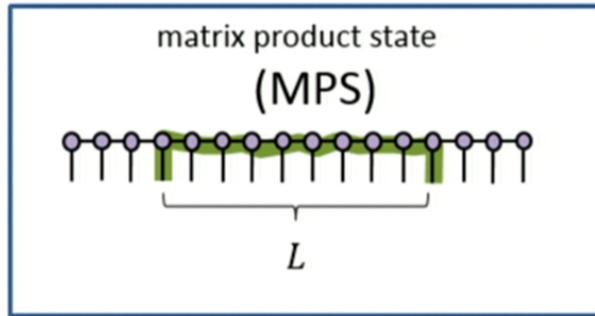


structure of geodesics:

$$\langle \hat{o}(0)\hat{o}(L) \rangle \approx L^{-p}$$

power-law

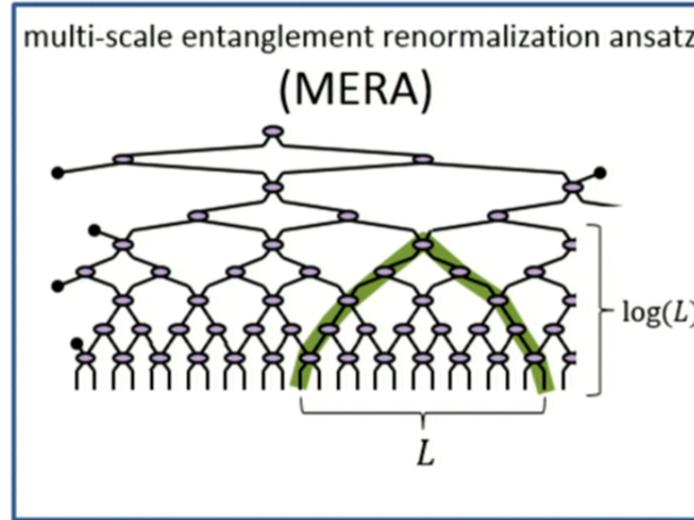
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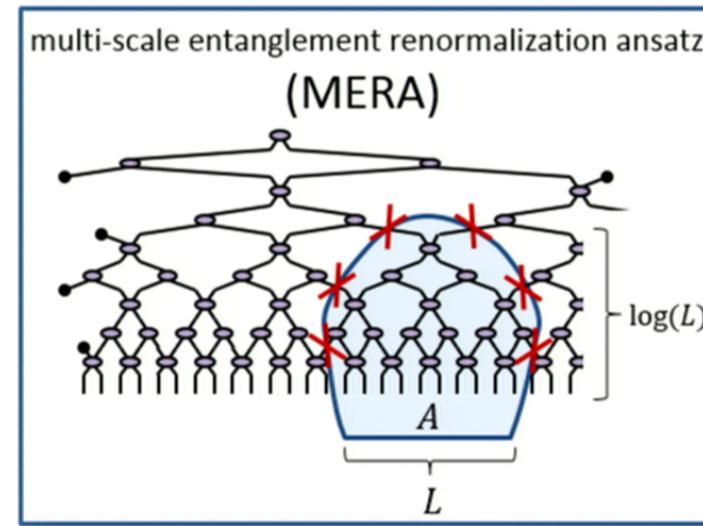
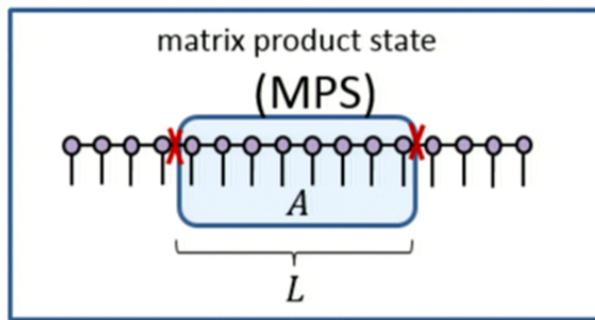


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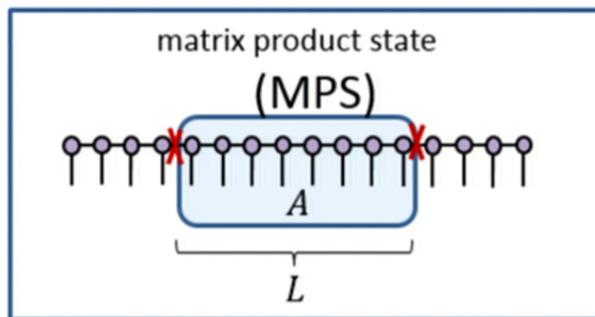
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power-law

## Entanglement entropy



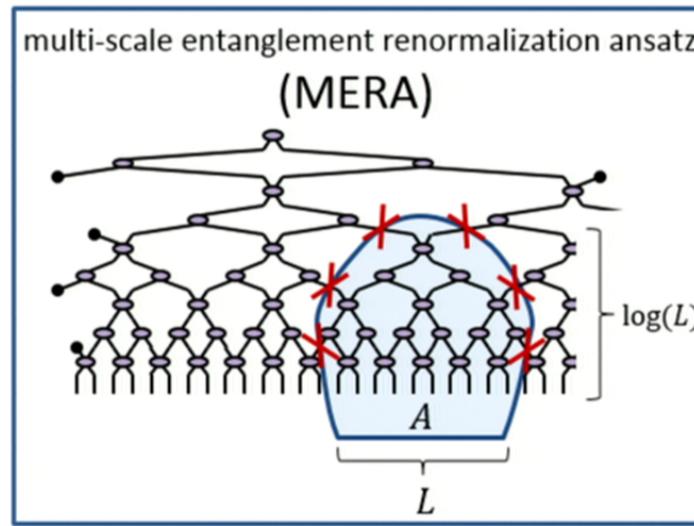
## Entanglement entropy



connectivity:

$$S(A) \leq \text{const}$$

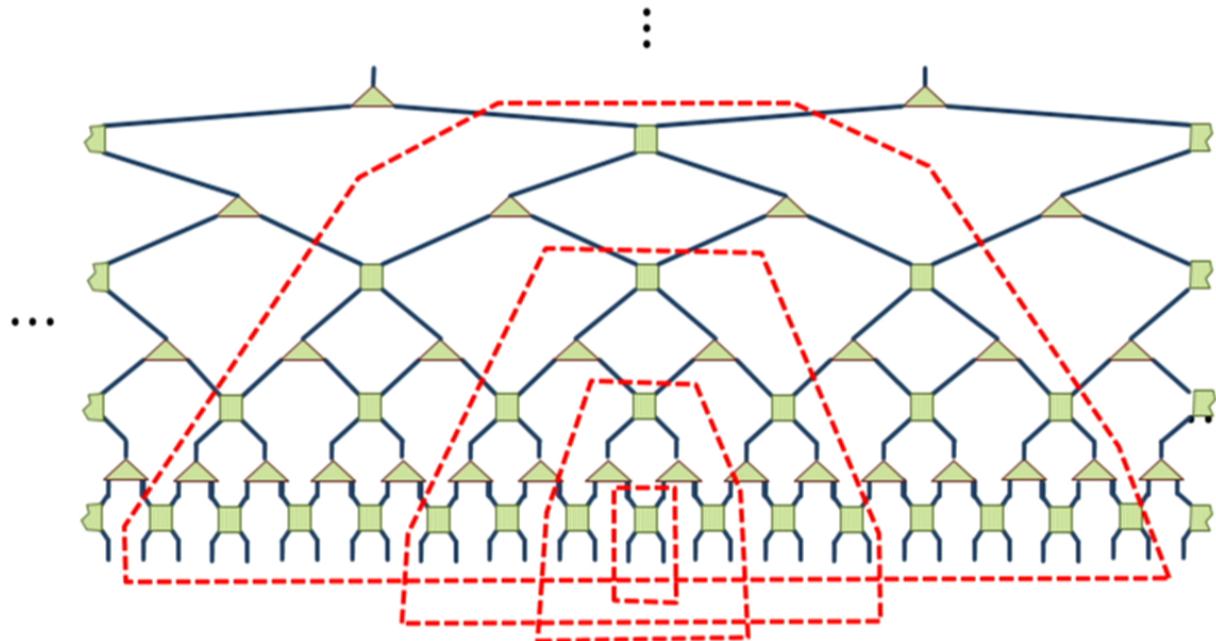
boundary law!



connectivity:

$$S(A) \leq \log L$$

logarithmic correction!



$$n(A) \approx \log L$$

$$L = 2, \quad n(A) = 2$$

$$L = 6, \quad n(A) = 4$$

$$L = 14, \quad n(A) = 6$$

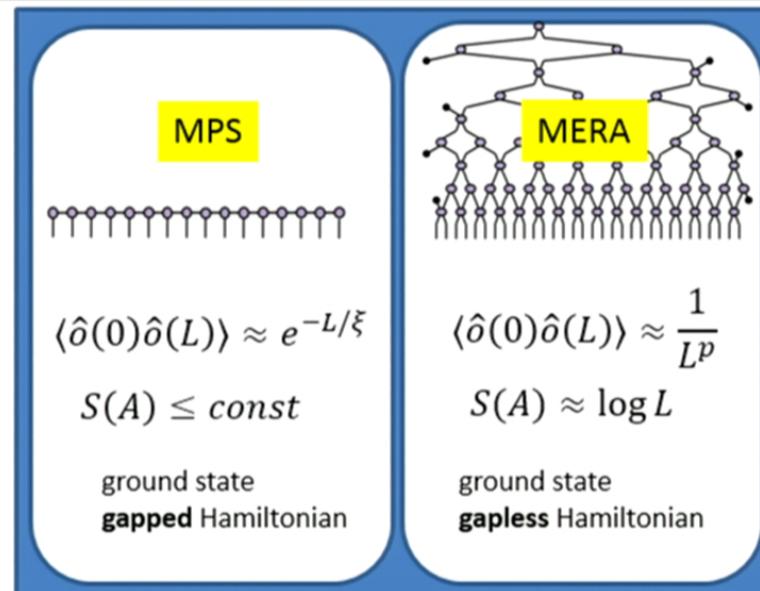
$$L = 30, \quad n(A) = 8$$

## Conclusions

- What is a tensor network state?
- Important aspects of a TN?
  - efficient representation and computation

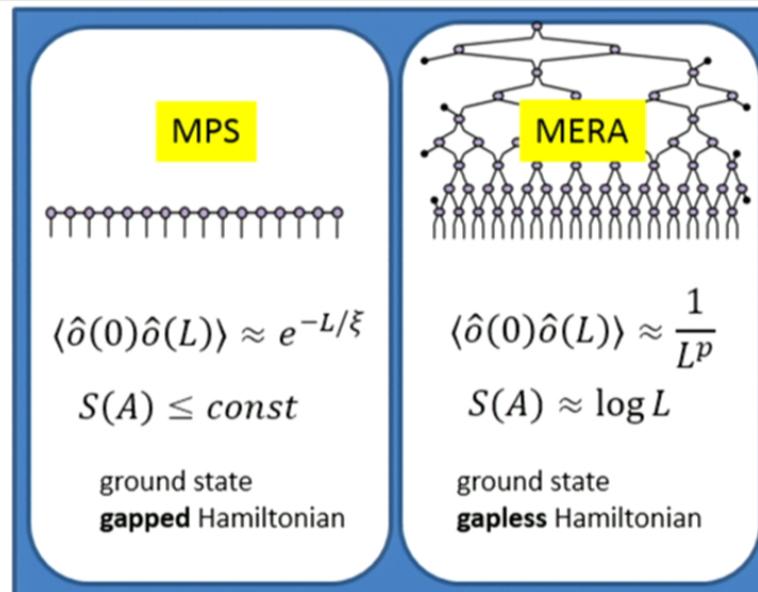
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## Aspects not covered

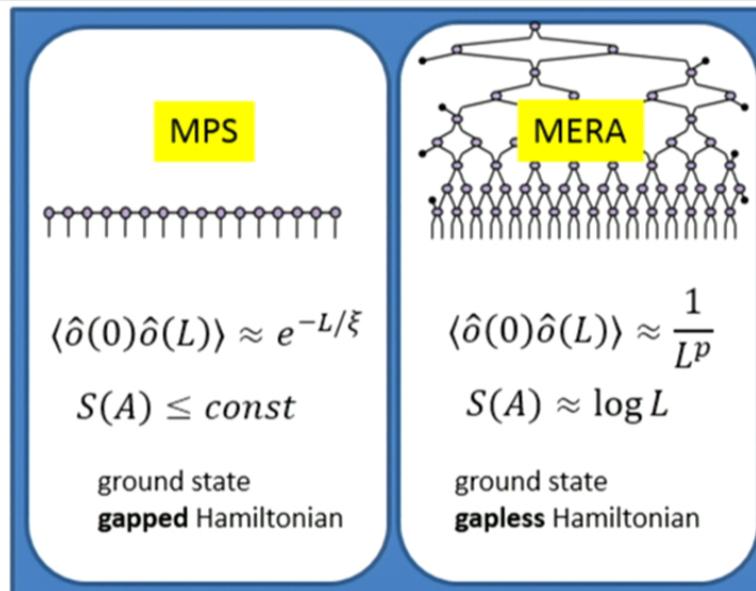
- How to optimize variational parameters (energy minimization; imaginary time evolution)

e.g. DMRG !!!!



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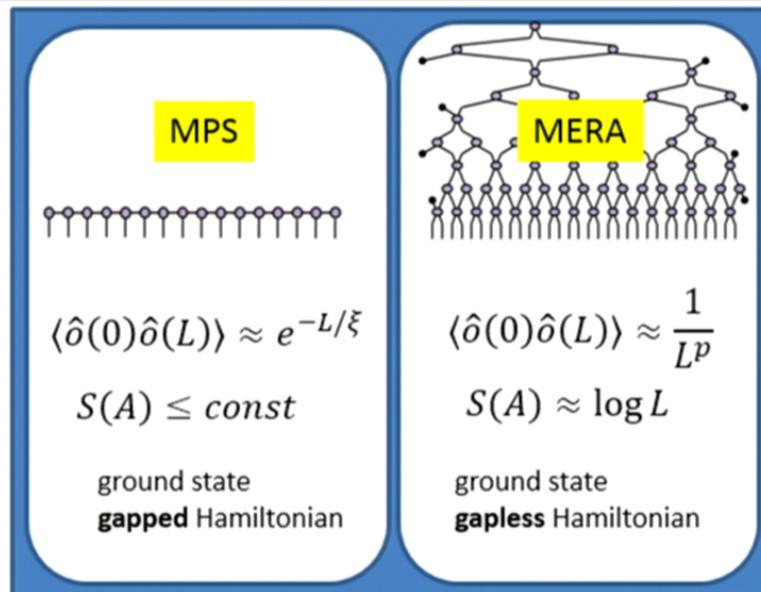
- How to optimize variational parameters (energy minimization; imaginary time evolution)
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- continuous MPS, continuous MERA for quantum field theories

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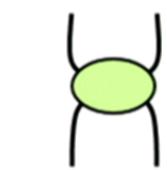
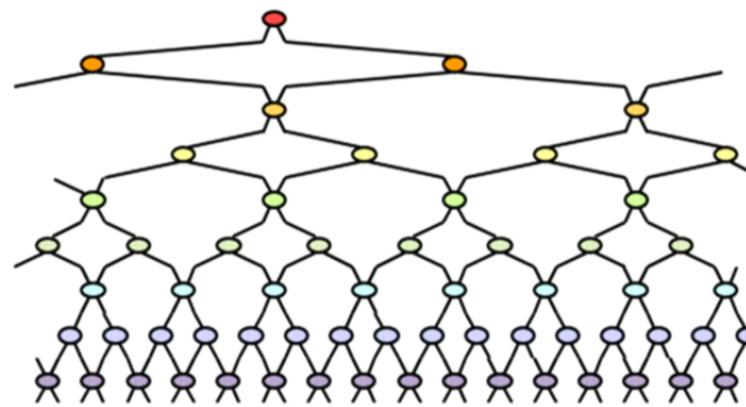
## Aspects not covered

- How to optimize variational parameters (energy minimization; imaginary time evolution)
- Simulation of time evolution (MPS)
- continuous MPS, continuous MERA for quantum field theories
- D>1 spatial dimensions (PEPS, MERA, branching MERA)

e.g. DMRG !!!!

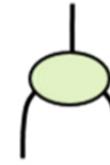


## MERA as a quantum circuit



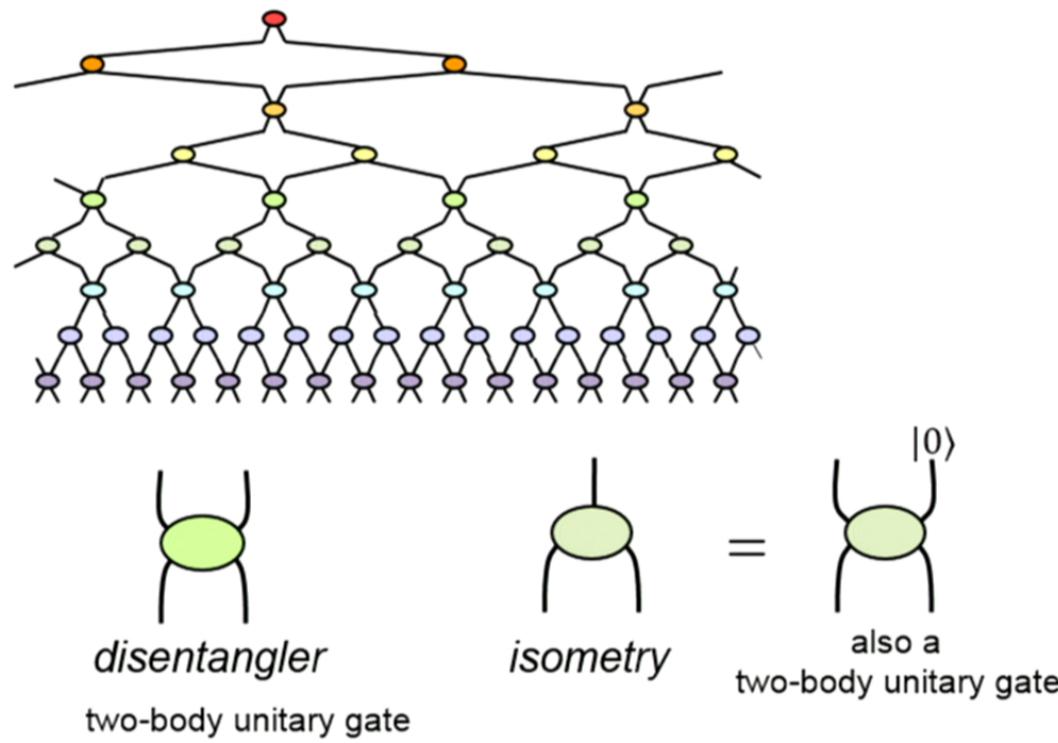
*disentangler*

two-body unitary gate

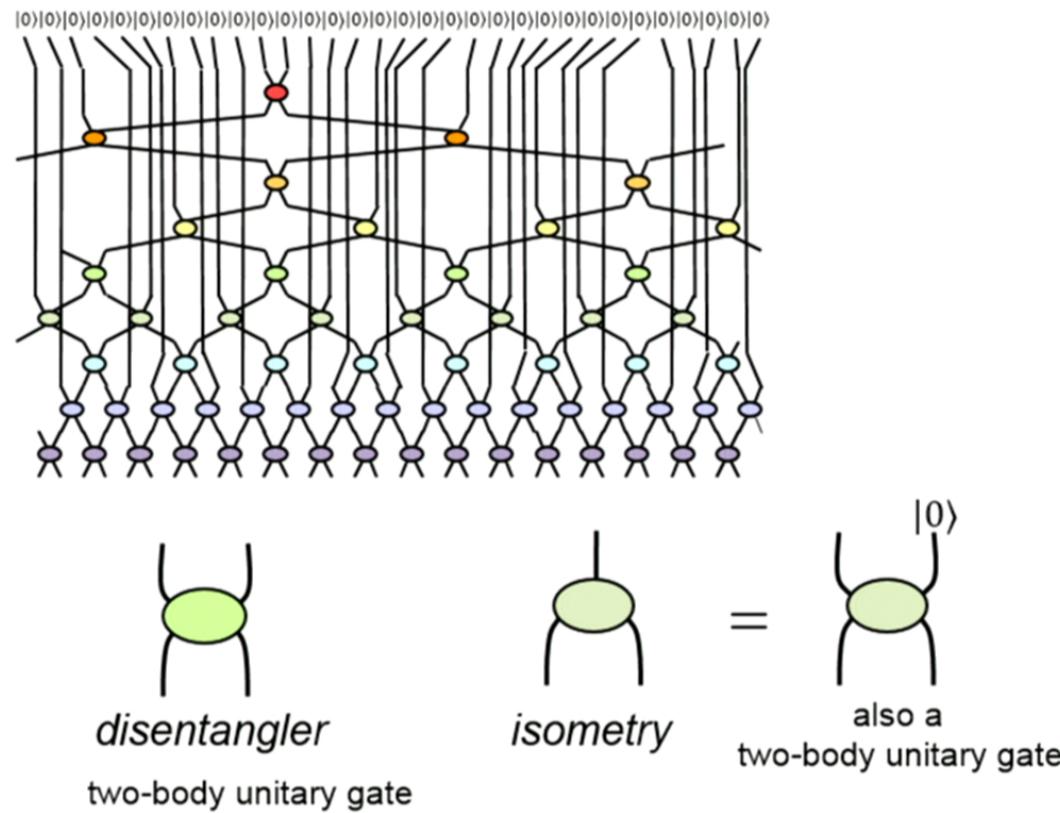


*isometry*

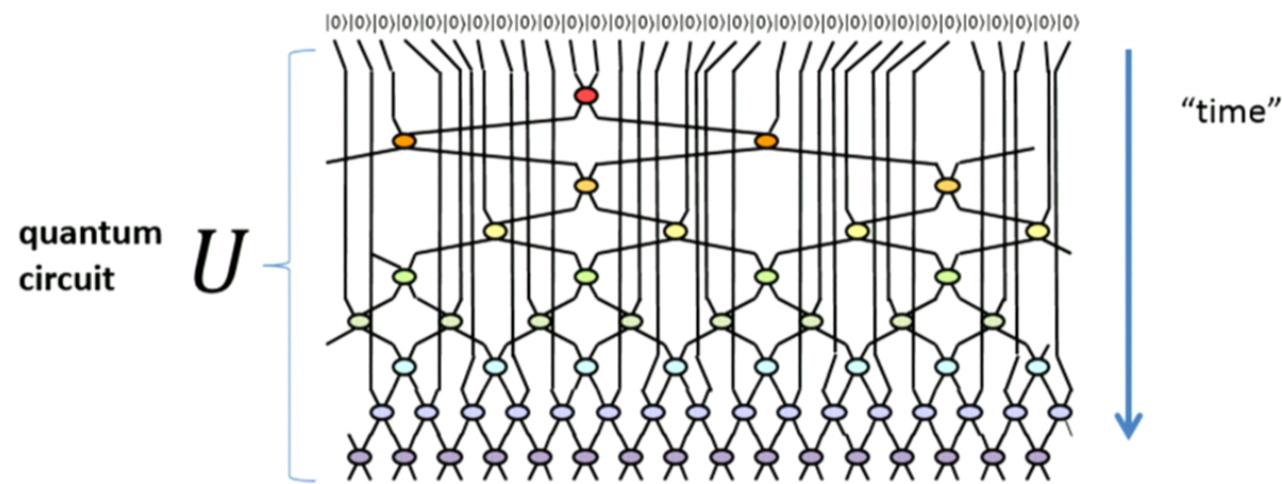
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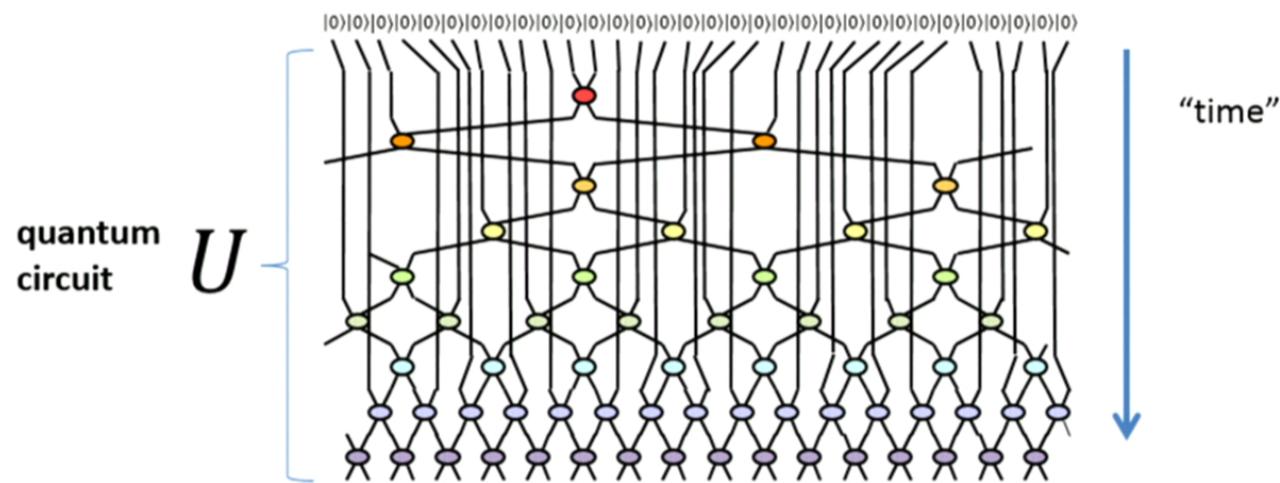
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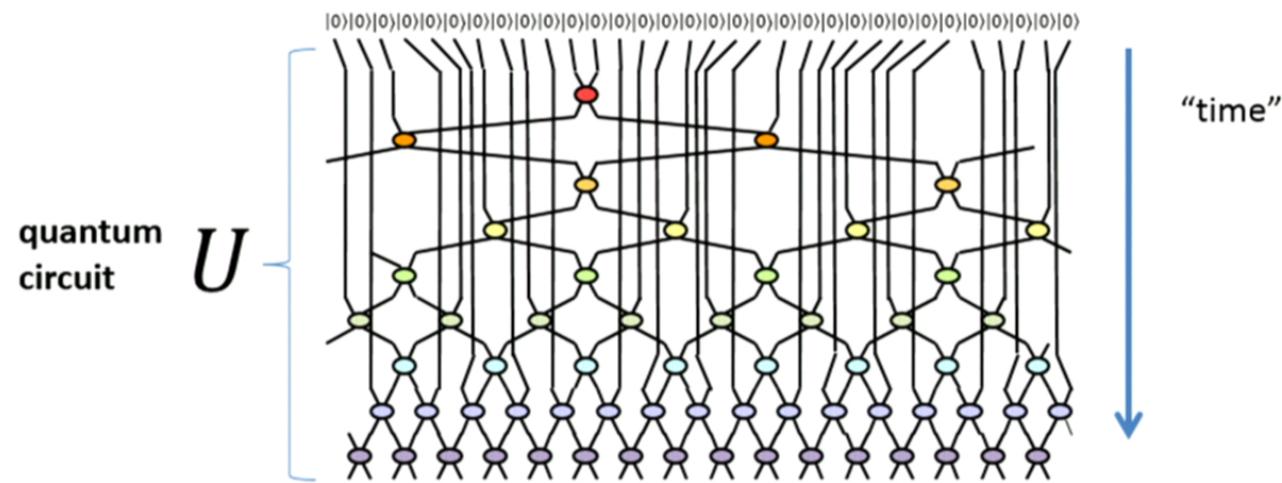


## MERA as a quantum circuit



$$\text{ground state ansatz } |\Psi\rangle = U |0\rangle^{\otimes N}$$

## MERA as a quantum circuit

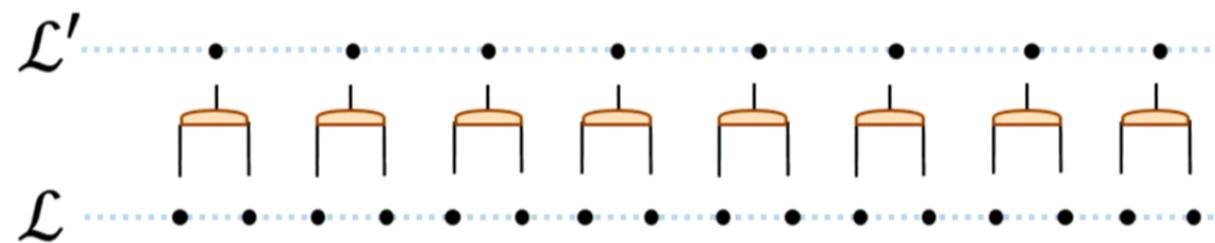


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Entanglement introduced by gates at different “times” (= length scales)

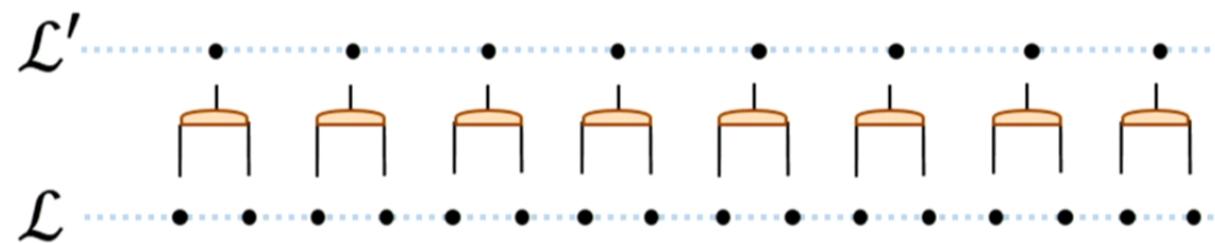
## MERA as a (real space) Renormalizatin Group transformation

Kadanoff (1966)  
blocking + White (1992)  
variational optimization

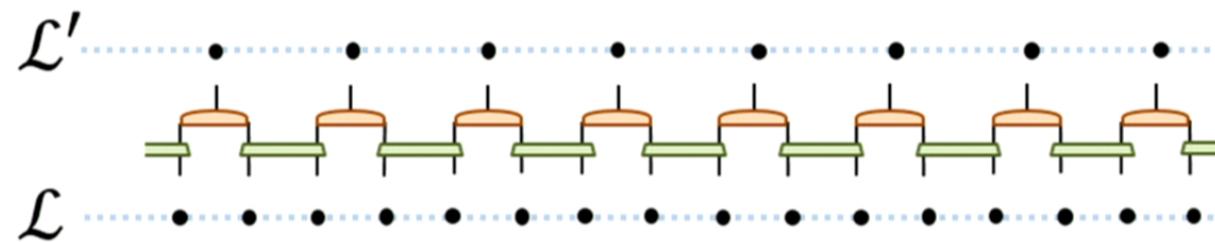


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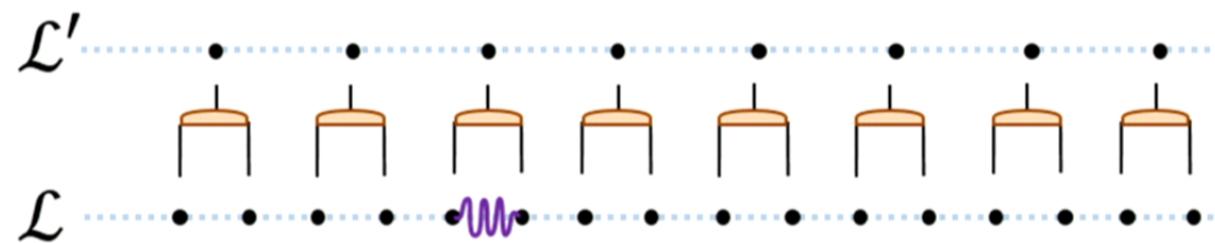


Entanglement renormalization (2005)

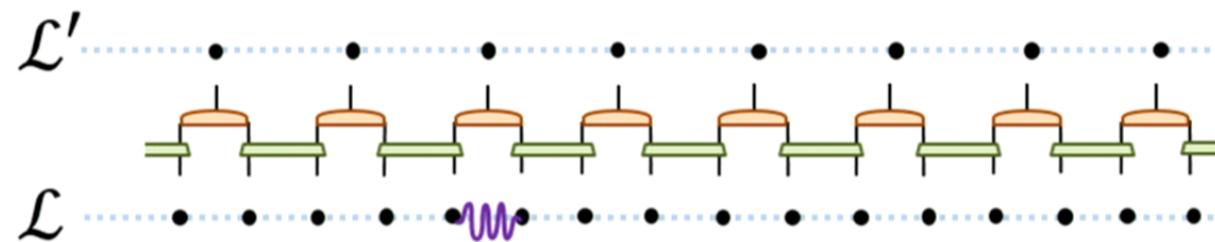


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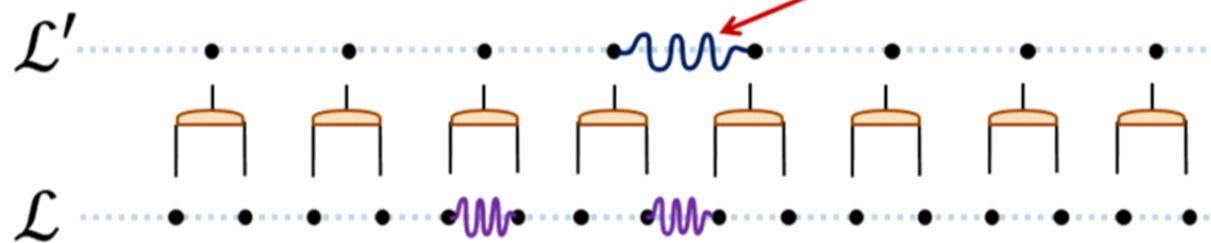


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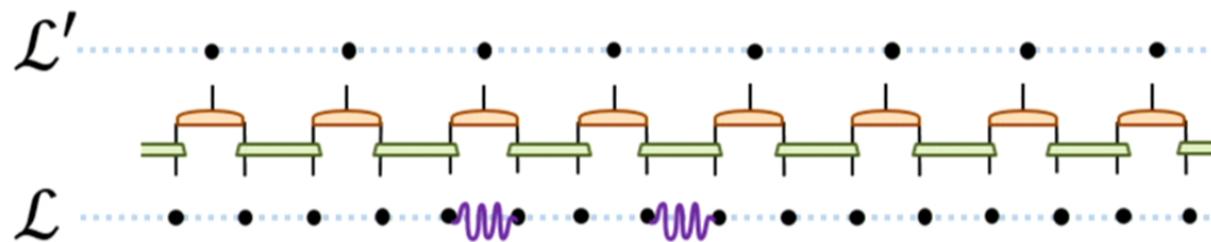
Kadanoff (1966)  
blocking

+ White (1992)  
variational optimization

failure to remove  
*some* short-range  
entanglement !

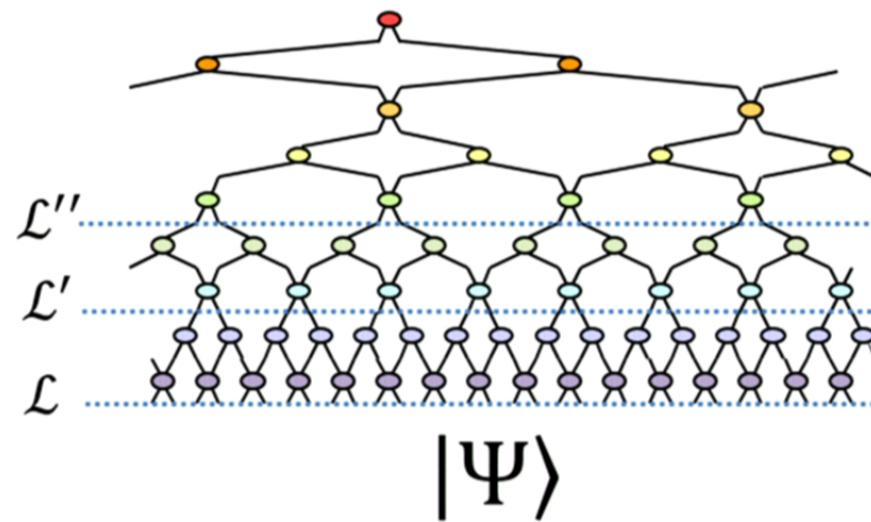


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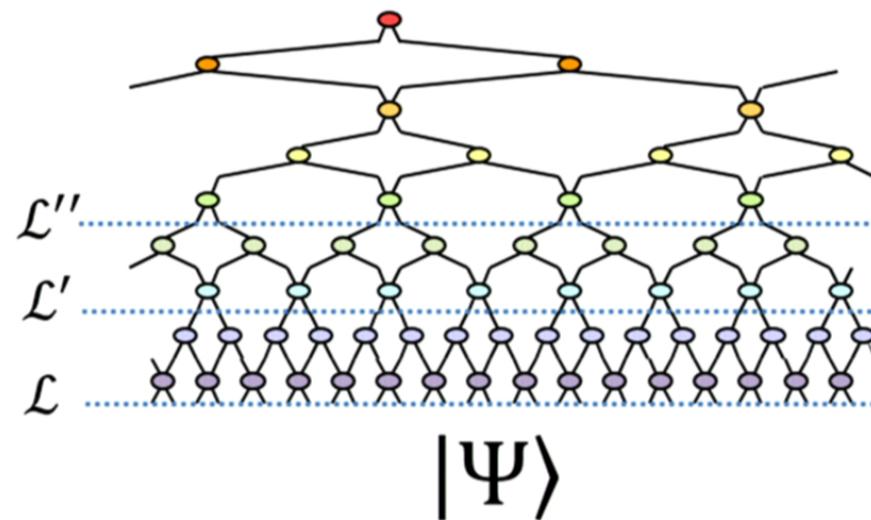
sequence of ground state wave-functions



$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \dots$$

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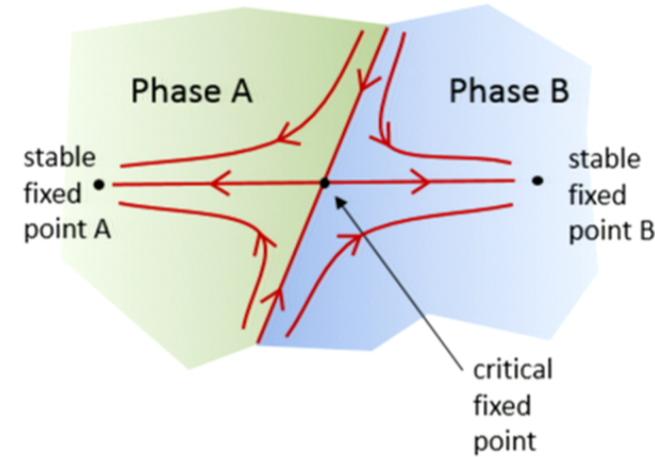
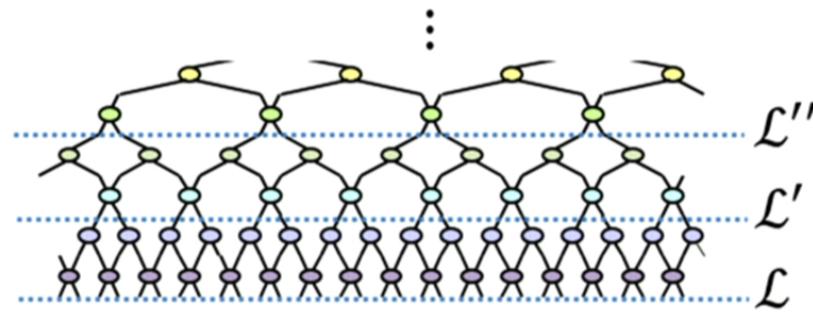


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MERA defines an RG flow  
in the space of wave-functions

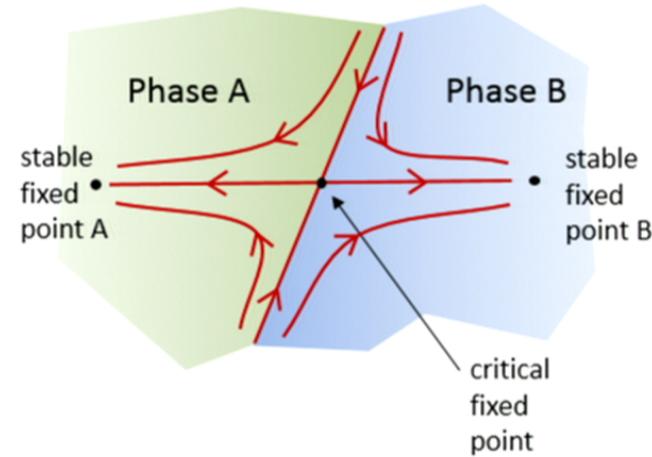
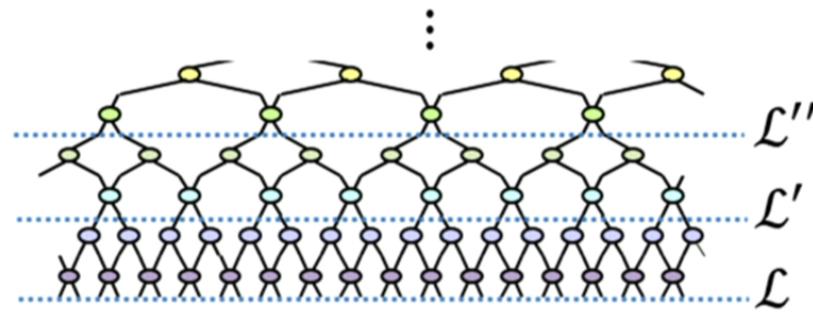
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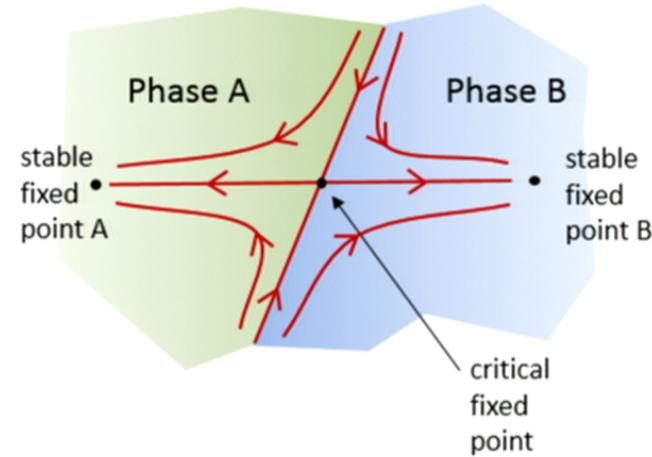
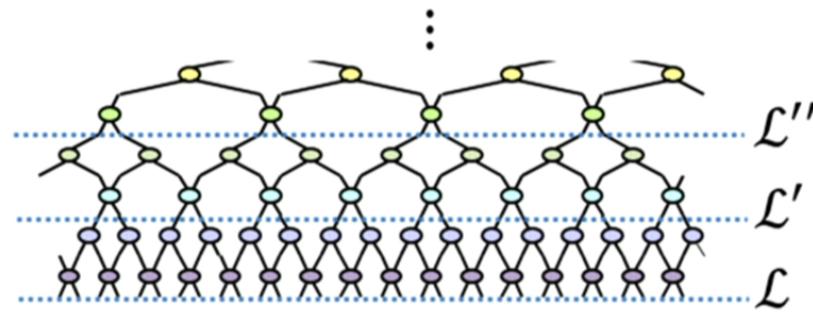


... and in the space of Hamiltonians

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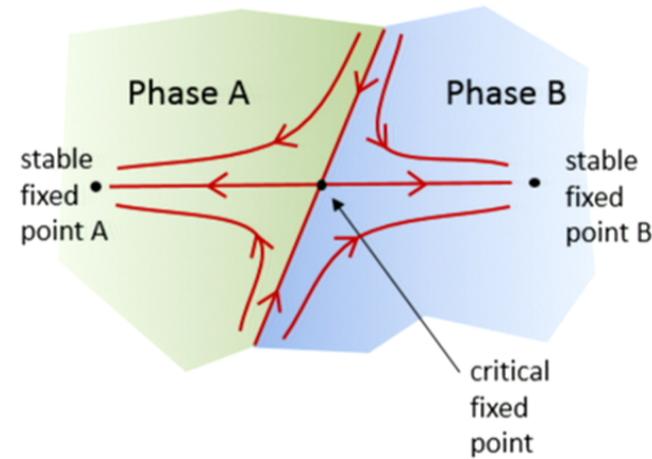
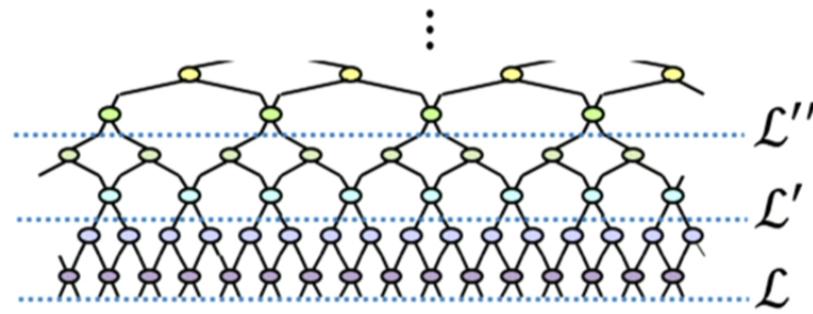
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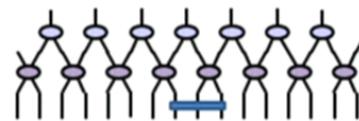
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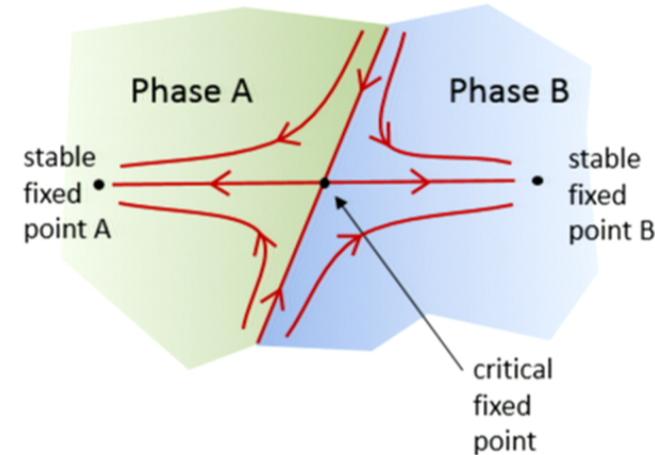
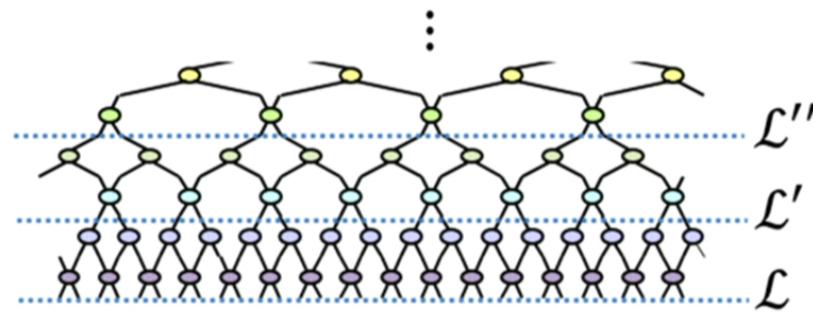
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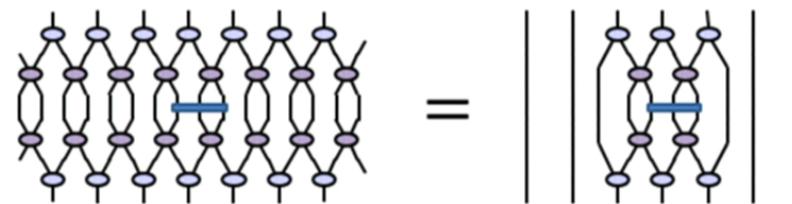
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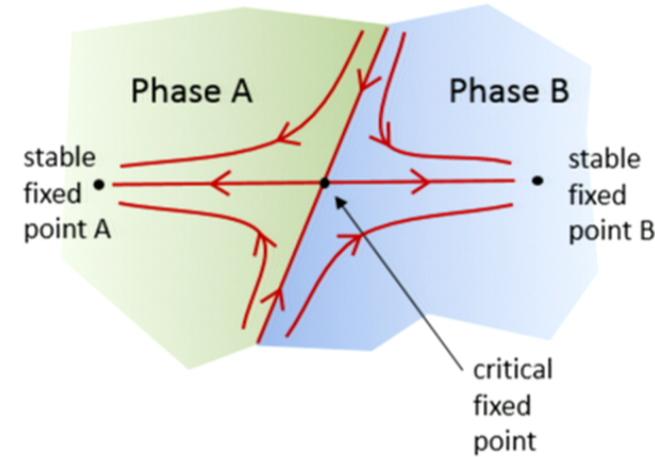
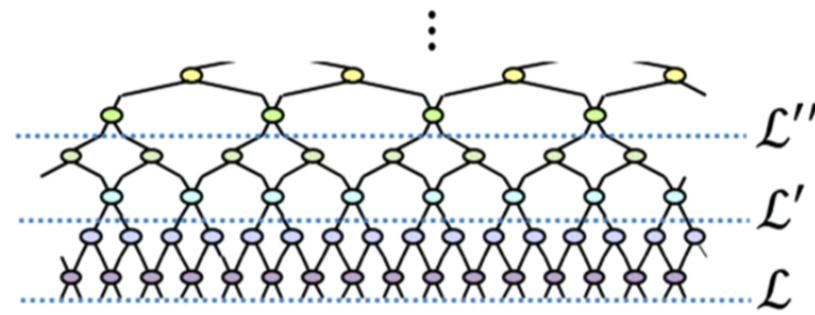
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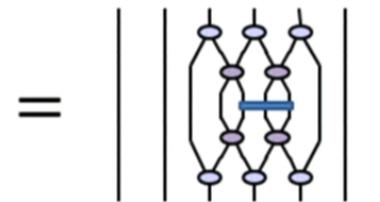
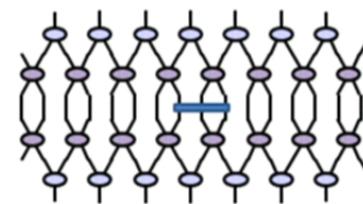
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local operators  
are mapped into  
local operators !