

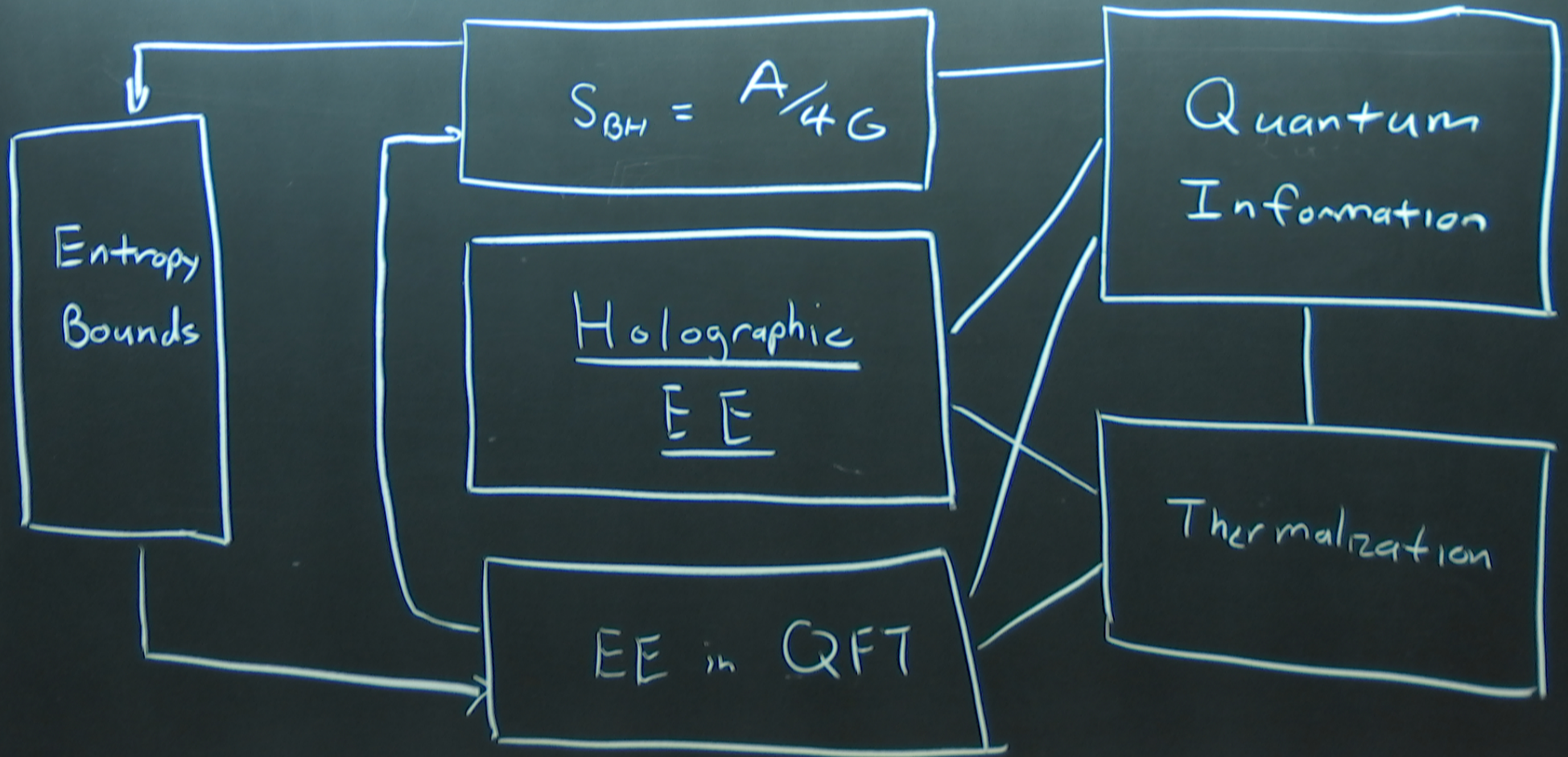
Title: Holographic entanglement entropy

Date: Aug 26, 2015 09:15 AM

URL: <http://pirsa.org/15080046>

Abstract: TBA

EE landscape (Casini):



Bekenstein + Hawking:

$$S_{BH} = \frac{k_B c^3}{h} \frac{A}{4G}$$

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$$S(\text{BH}) \sim 10^{77} k_B$$

→ originally S_{BH} appeared in black hole thermodynamics

→ statistical mechanics $S = -\text{Tr}(\rho \log \rho)$

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→ black hole microstates

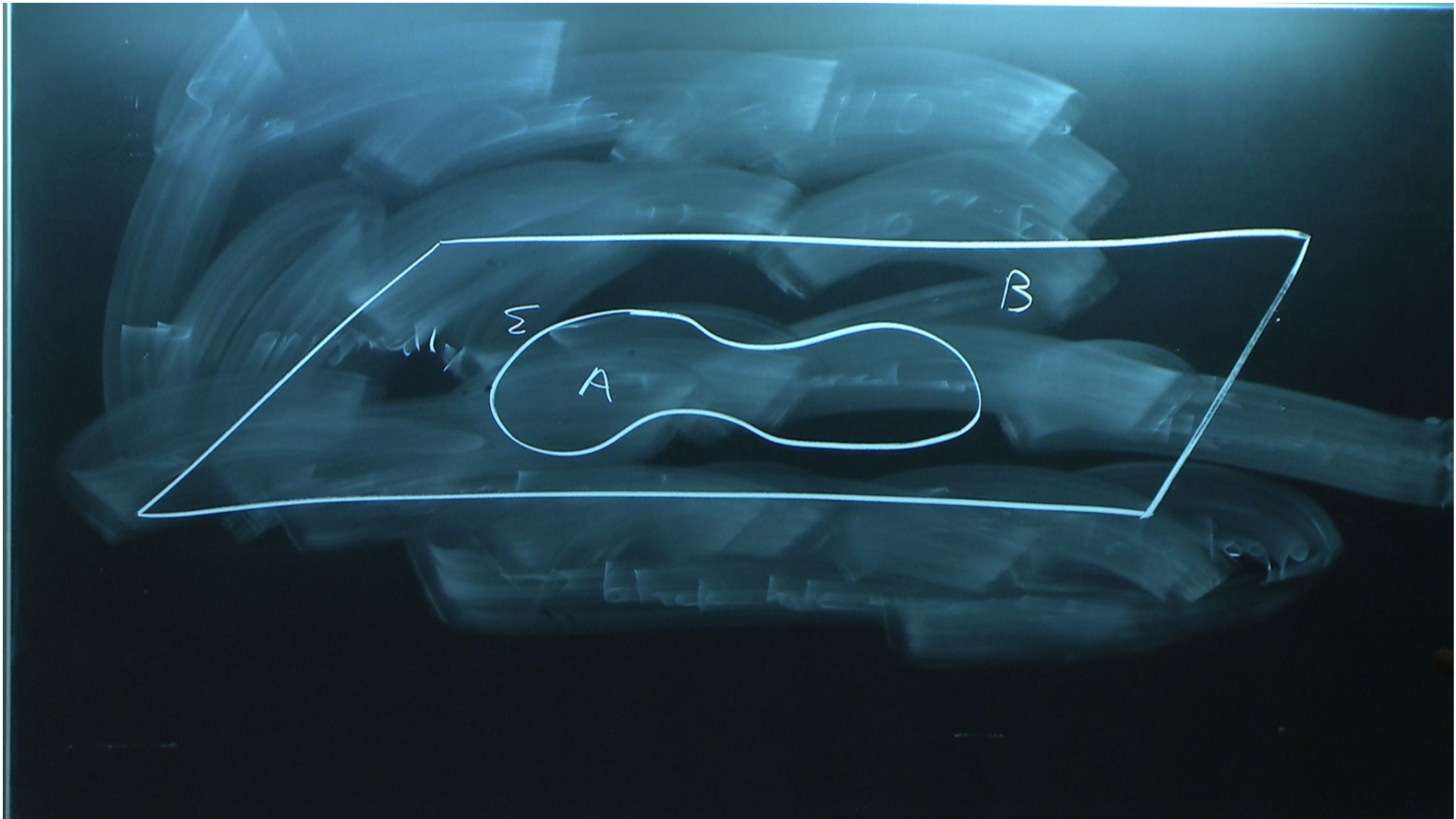
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spacetime geometry

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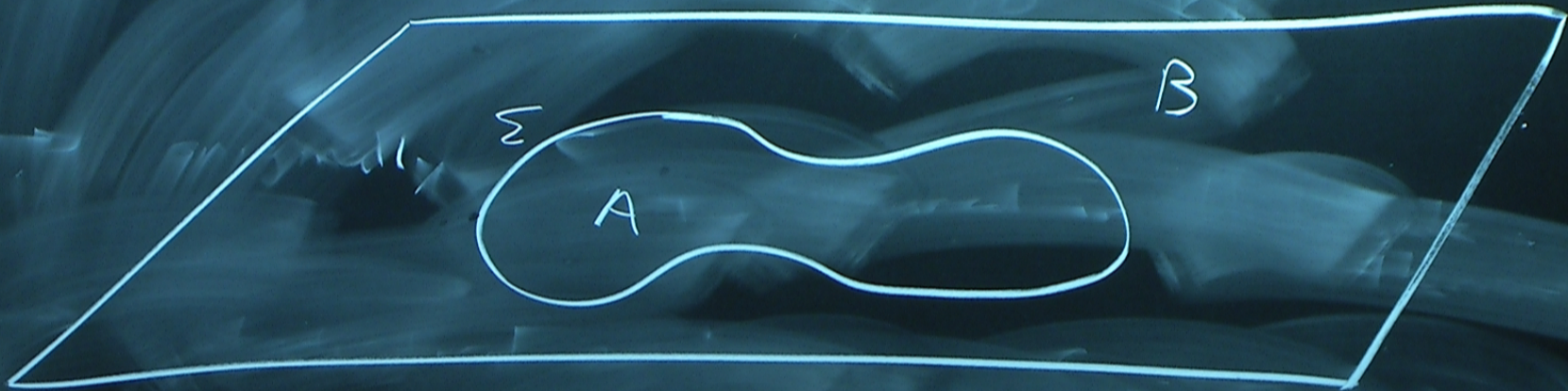
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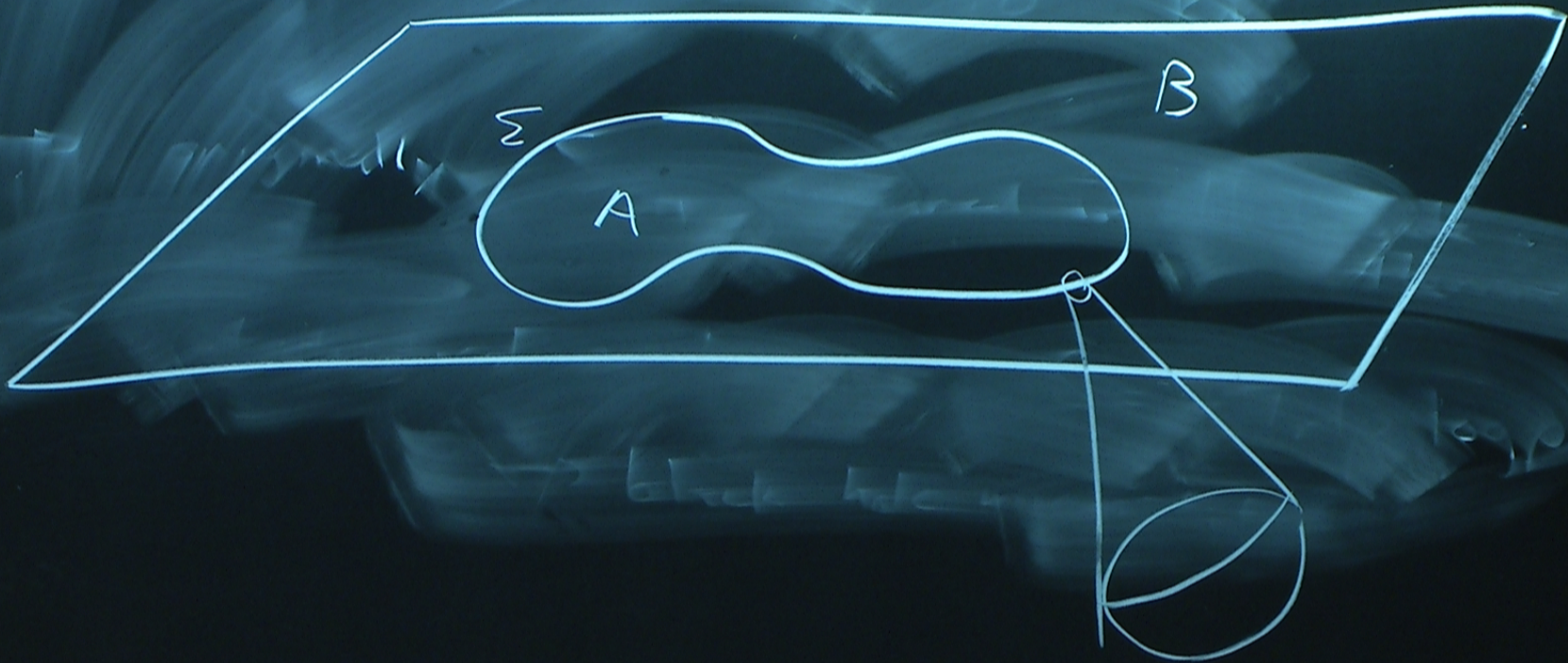
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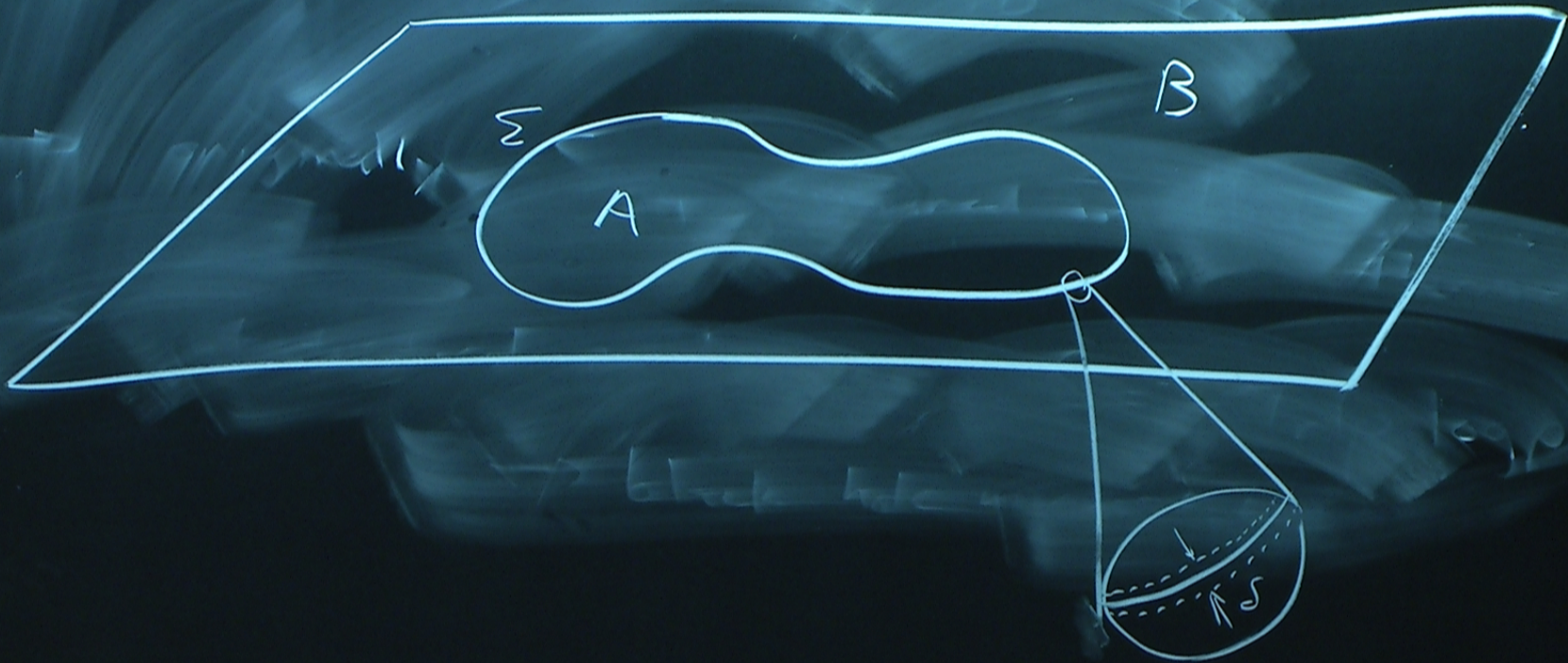


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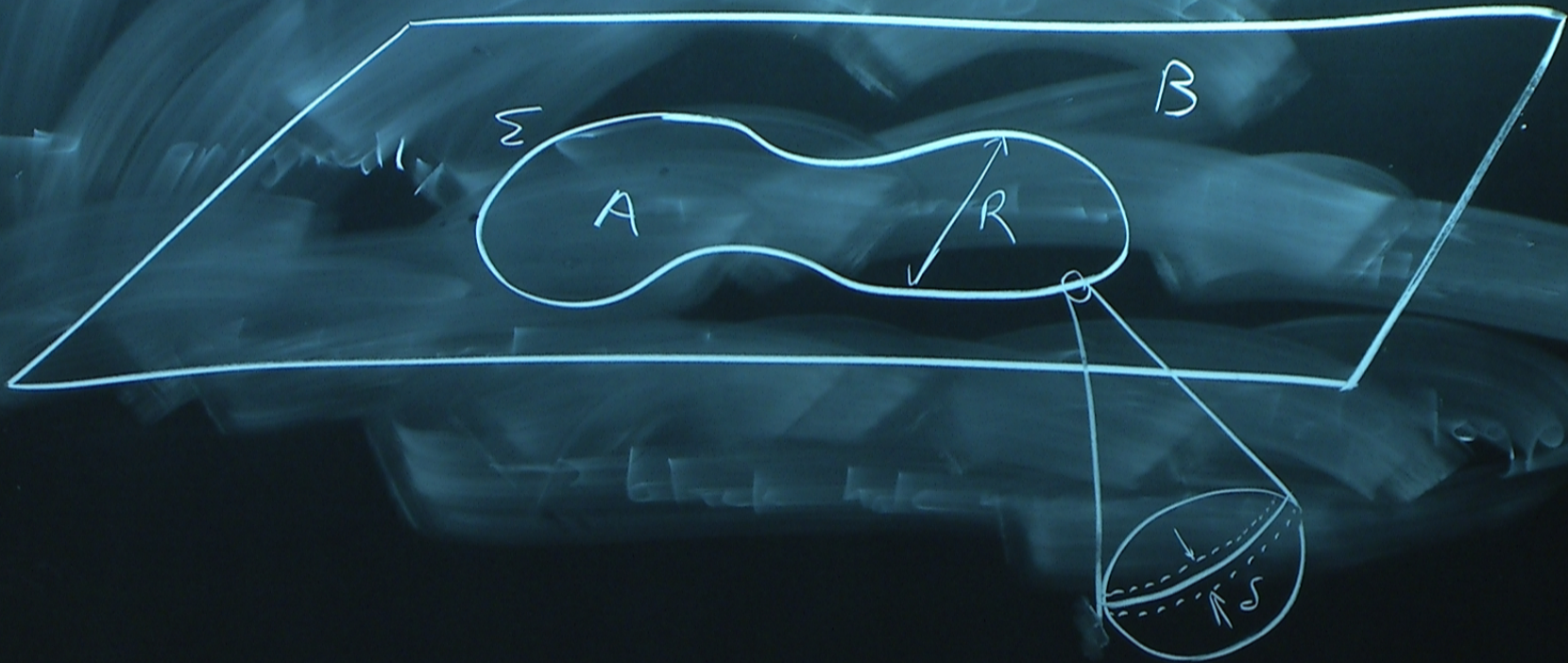


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result is ∞ ! dominated by short λ 's
regulate $\delta =$ short-distance cut-off

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$$S_{EE} = c_0 \left(\frac{R}{\delta} \right)^{d-2} + c_2 \left(\frac{R}{\delta} \right)^{d-2} + \dots$$

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$$\begin{aligned} S_{EE} &= c_0 \left(\frac{R}{\delta} \right)^{d-2} + c_2 \left(\frac{R}{\delta} \right)^{d-2} + \dots \\ &= c_0 \frac{A_{d-2}}{\delta^{d-2}} \end{aligned}$$

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universal

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$EE \propto \text{Area} \rightarrow$ unusual geometric dependence, matches S_{BH}

- Sorkin '84 $\rightarrow$ suggested connecting $S_{BH} + S_{EE}$
 $\rightarrow$ entangling surface = horizon

$\rightarrow S_{BH} = S_{EE} \leftarrow$ QG must regulate
with $\delta \sim l_{pl}$

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- active topic in 90's $\rightarrow$ Issues! with $S \sim \log$

Susskind + Uglum

- graviton propagator also suffers power law divergence

- choose fixed regulator

$$\delta(1/G) \sim \frac{G}{S^{d-2}}$$

$$S_{EE} = \frac{G}{S^{d-2}} \frac{A}{4}$$

- $(S_{EE})_{\text{QFT}}$ is a contribution to S_{BH}

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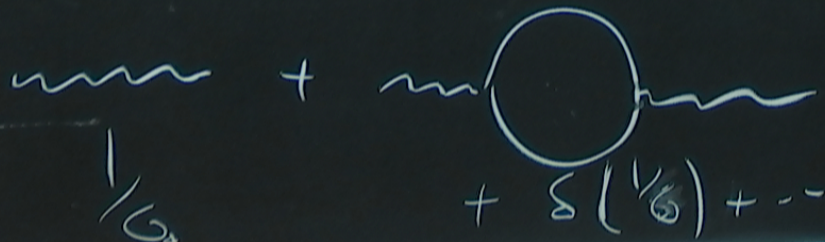
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$$\text{wavy line} + \text{wavy line} \bigcirc \text{wavy line} + \delta(1/G) + \dots$$

$1/G$

$$S_0 + S_{EE} = \delta(1/G) \frac{A_2}{4} + \frac{A_2}{4G_0}$$

contribution

AdS/CFT correspondence

- dictionary between 2 languages

anti de Sitter space

- quantum gravity
- negative Λ
- $(d+1)$ spacetime dims



conformal field theory

- quantum field theory
- no intrinsic scales
- d spacetime dim's

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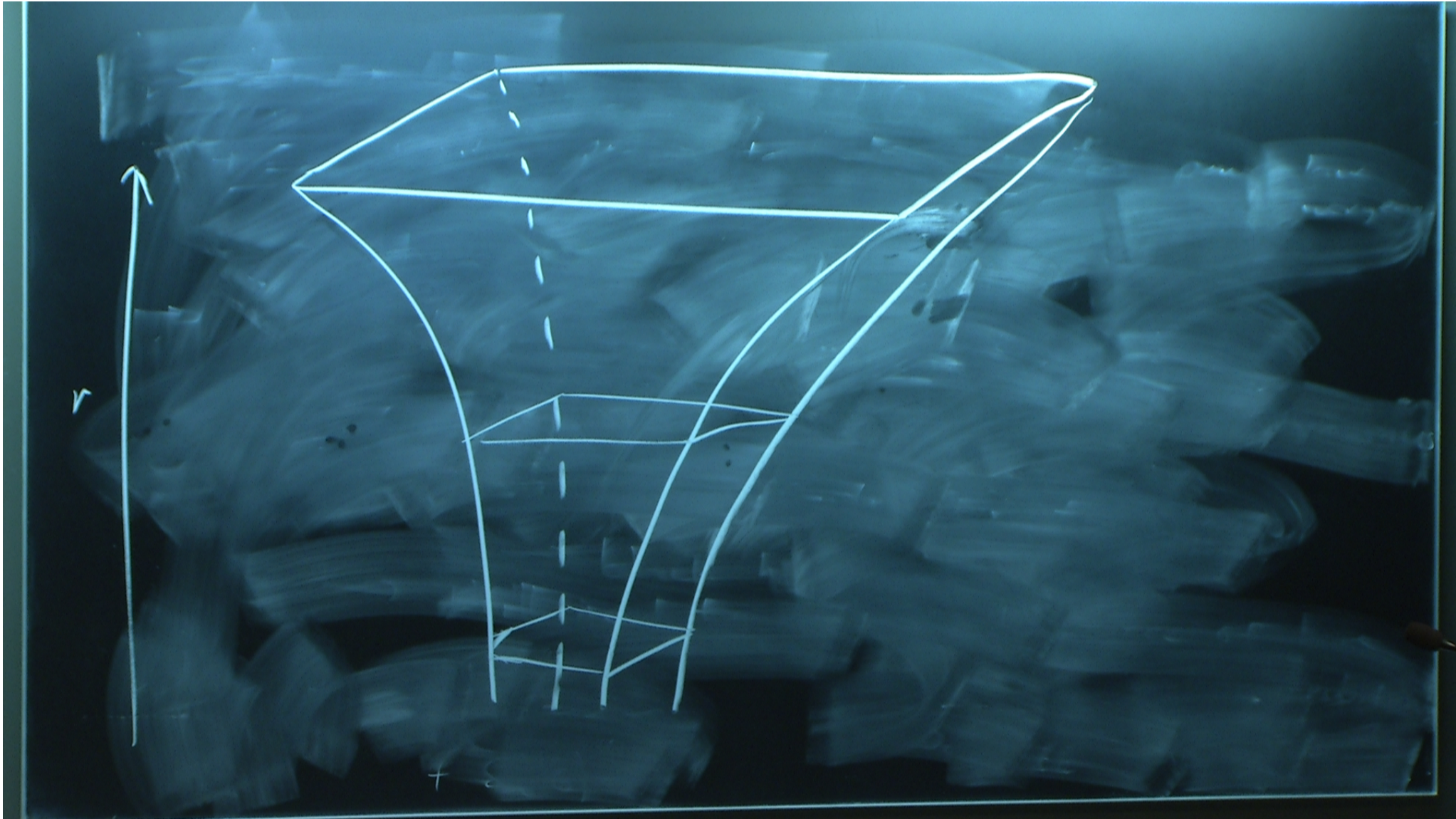
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strong coupling

AdS space

- maximally symmetric geometry with - curvature

$$ds^2 = \frac{r^2}{L^2} (-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} dr^2$$



AdS space

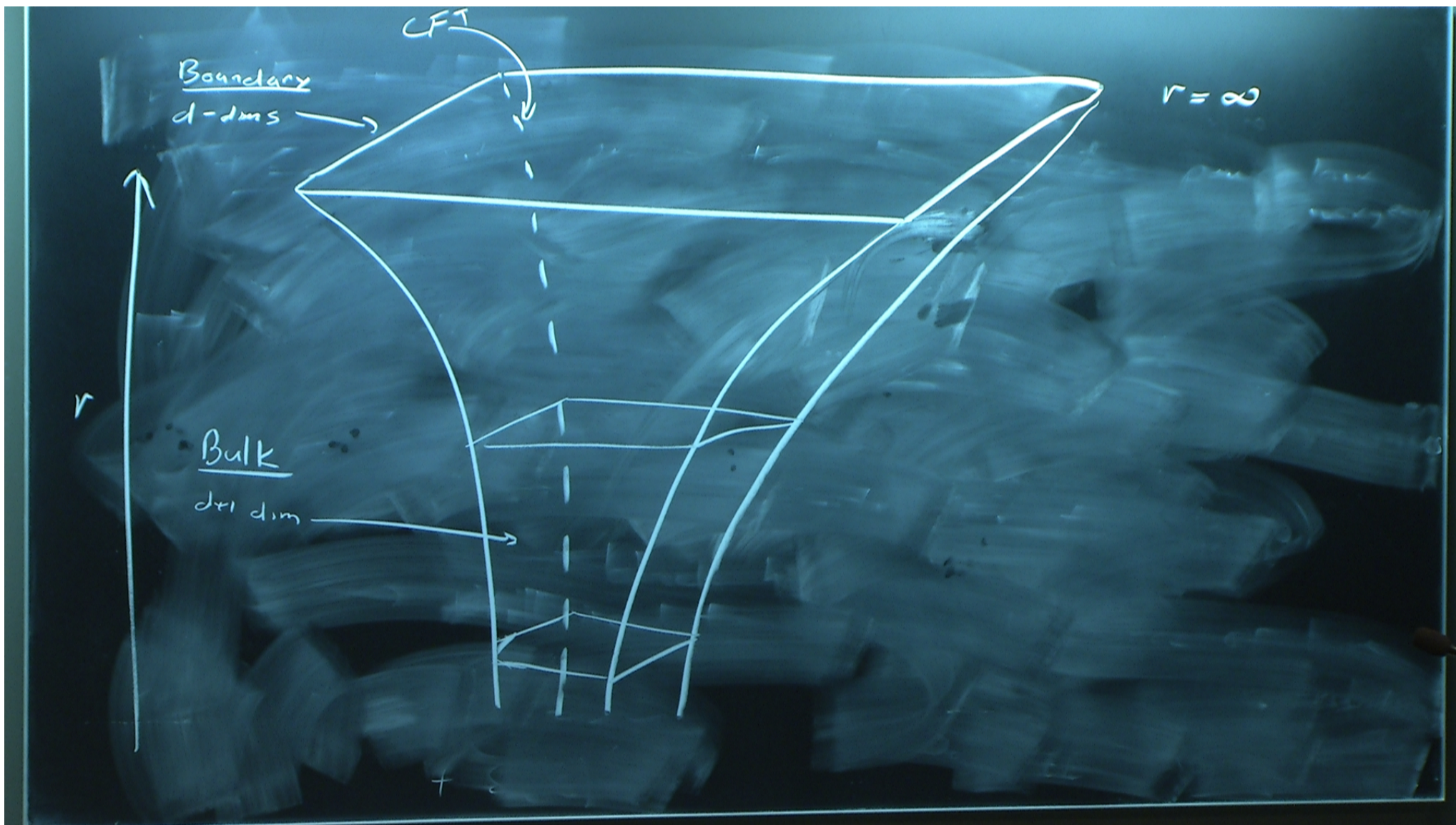
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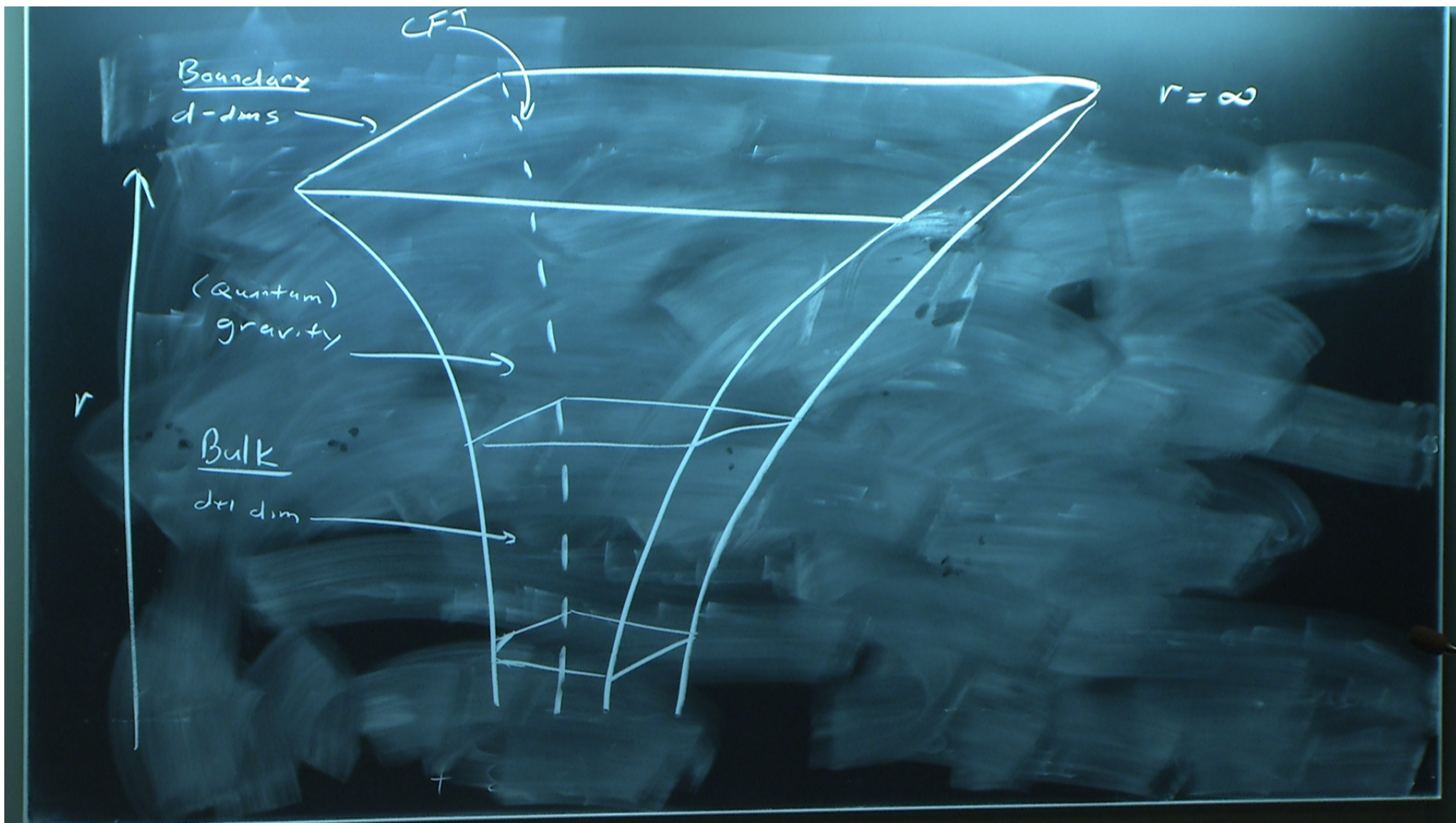
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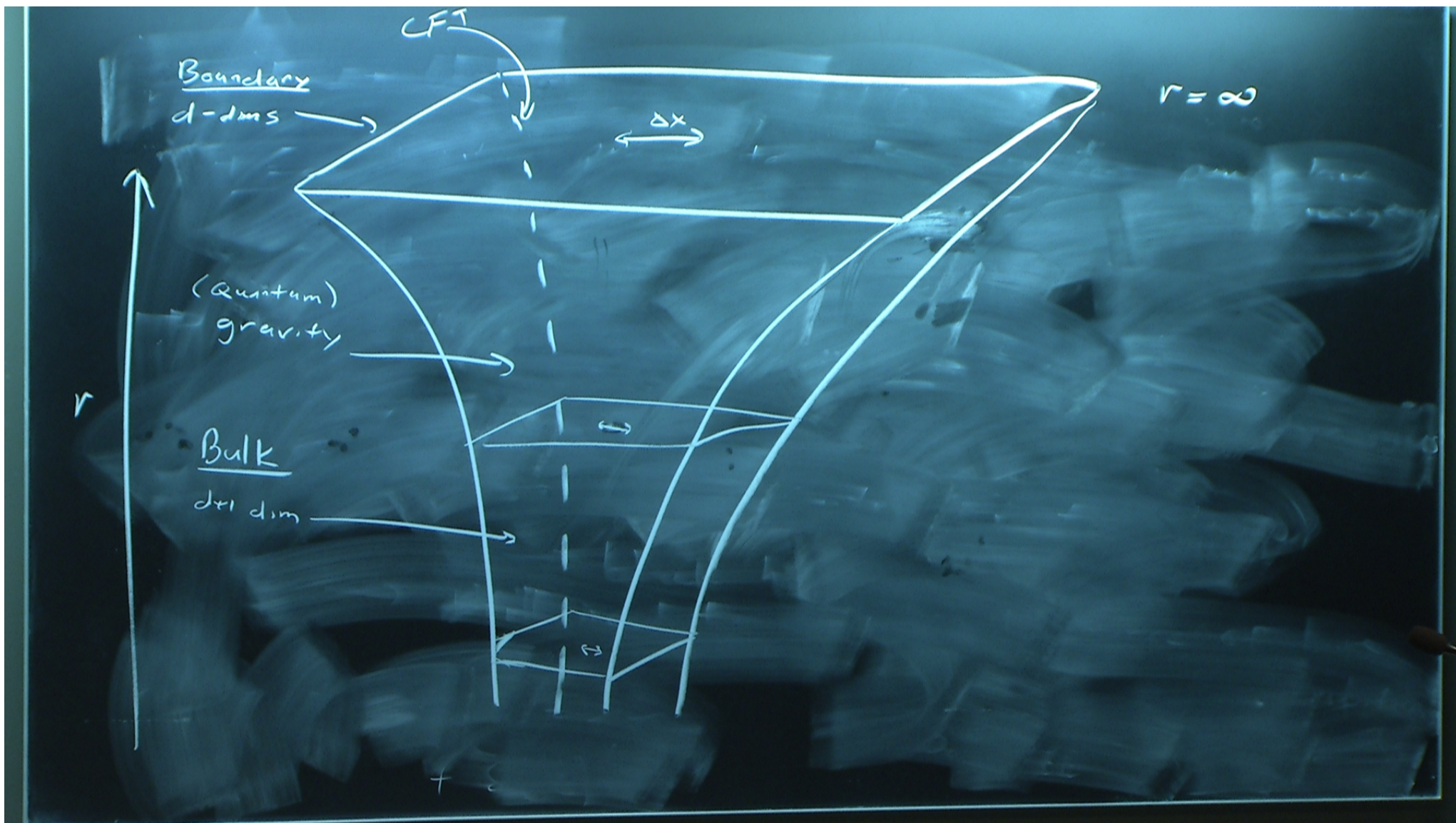
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$$R_{ab} = - \frac{d}{L^2} g_{ab}$$

curvature scale







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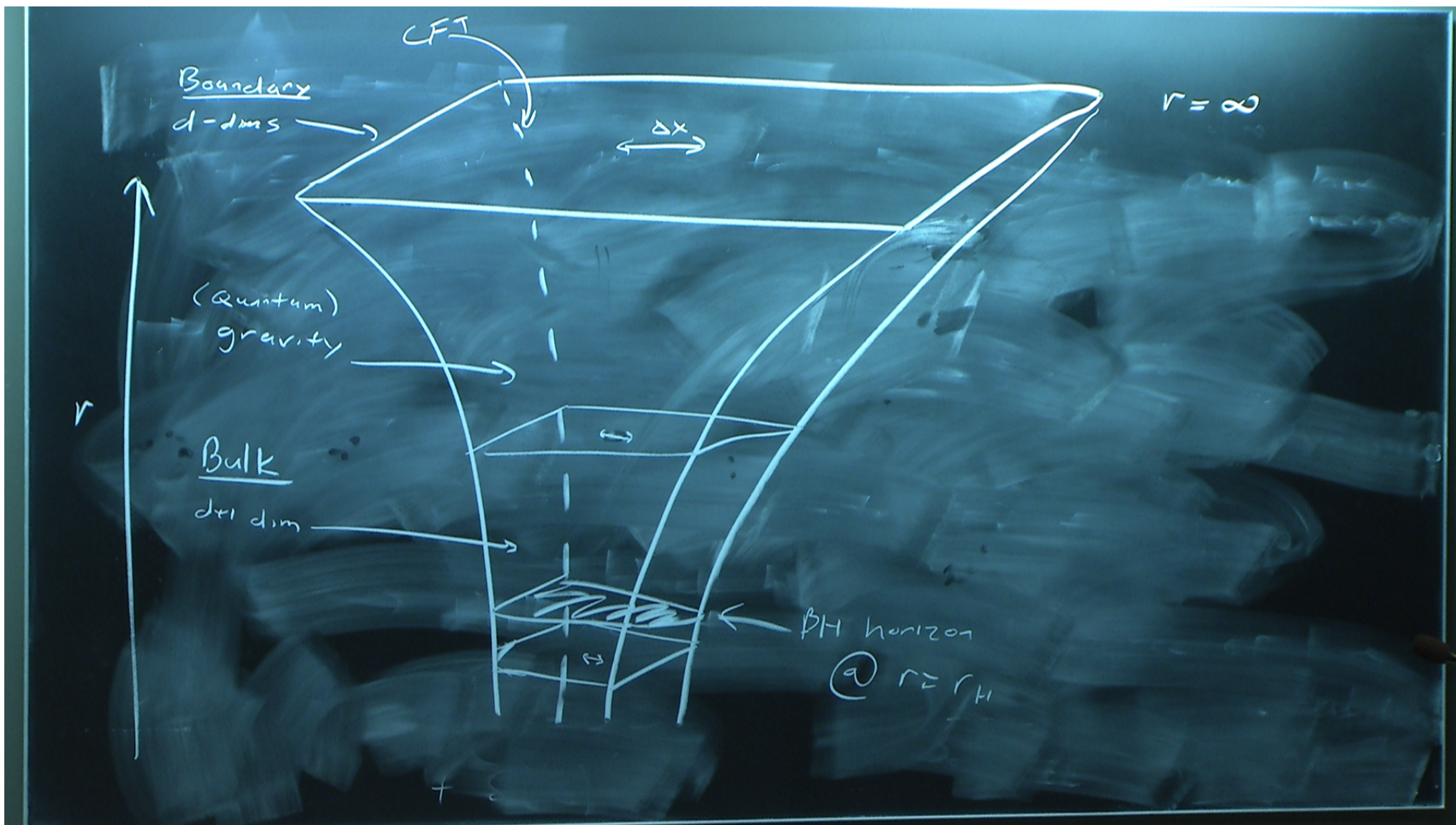
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interesting relation:

AdS Black hole $\equiv$ CFT thermal state

T_{Hawking}



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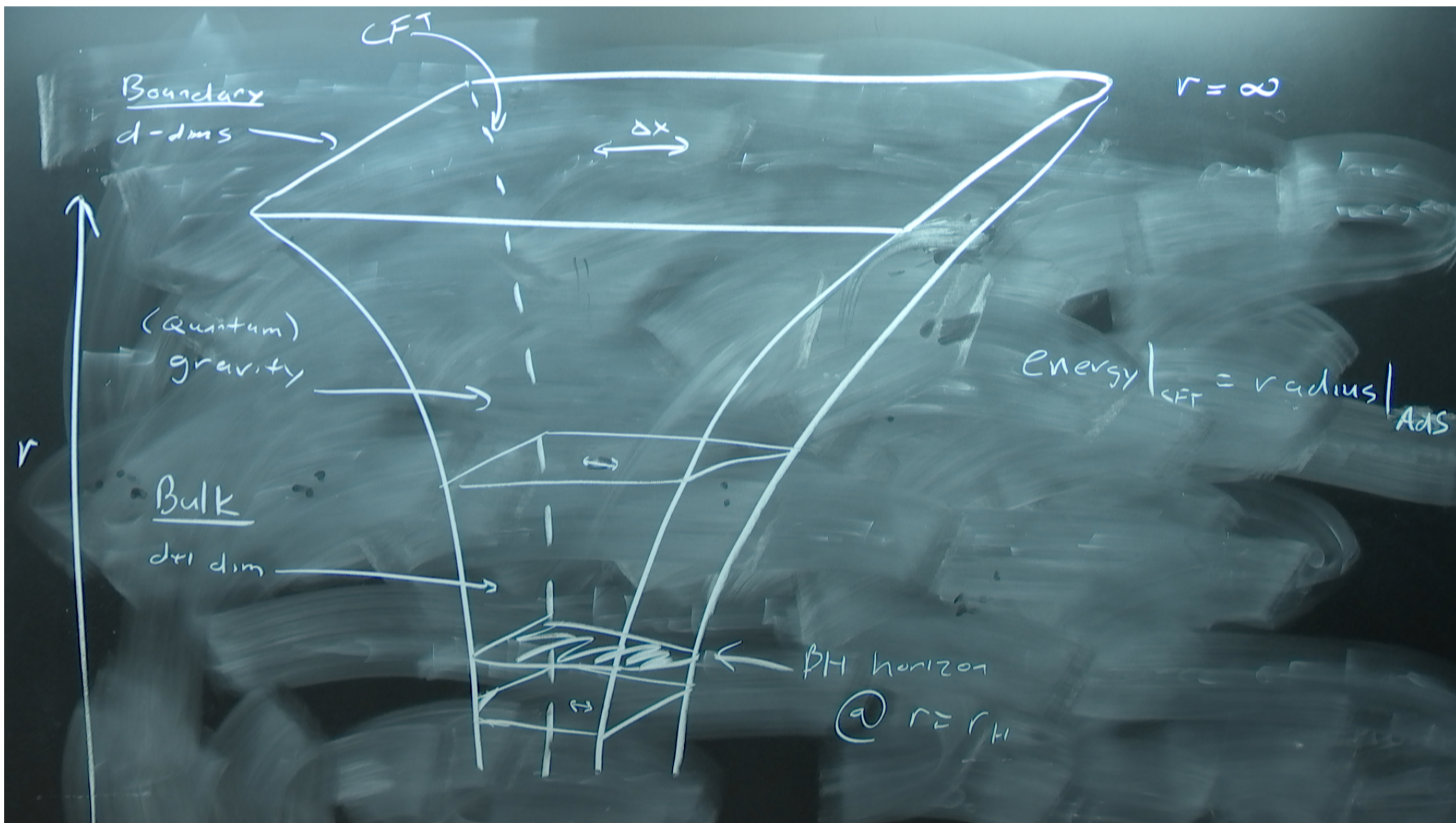
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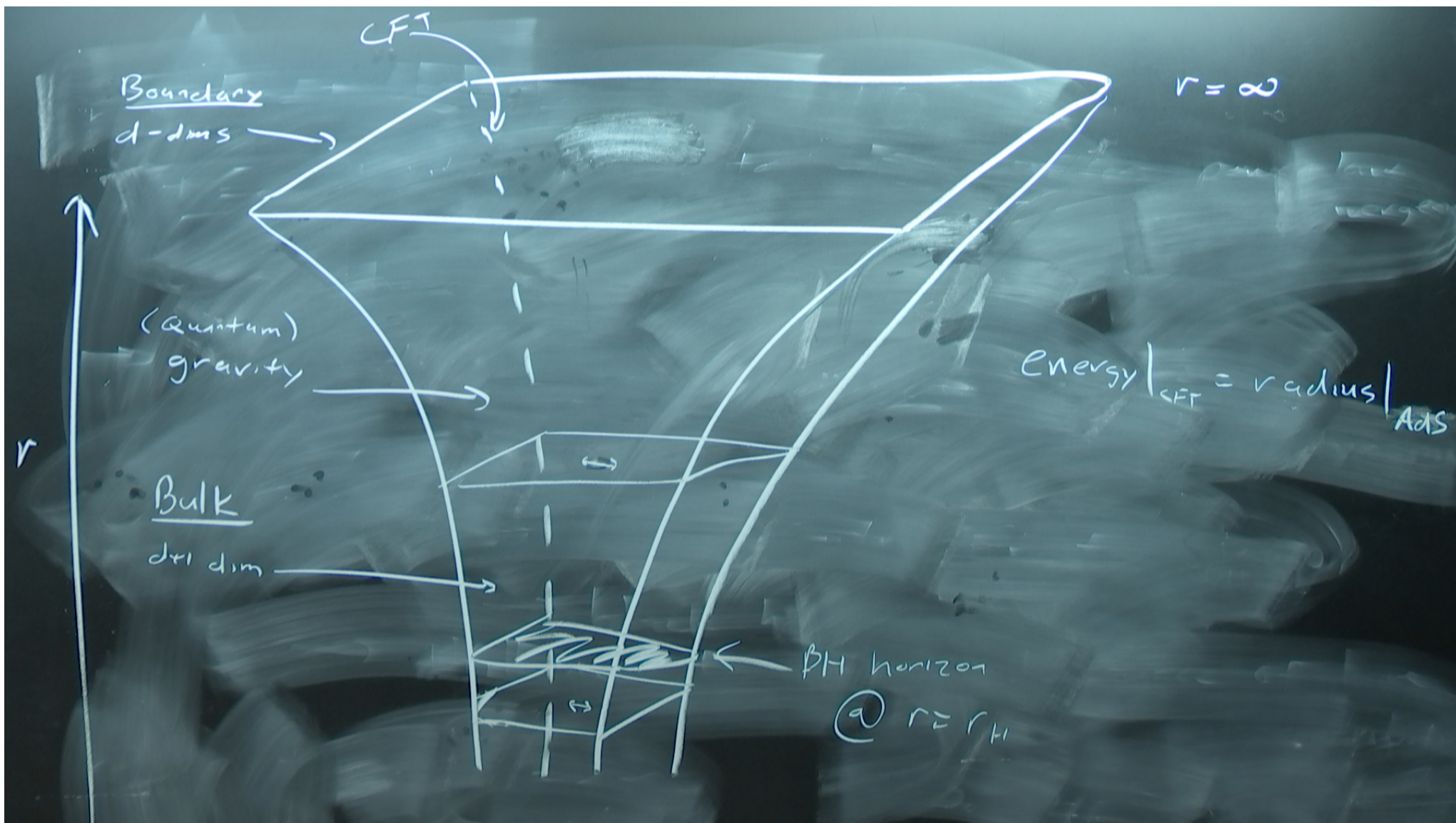
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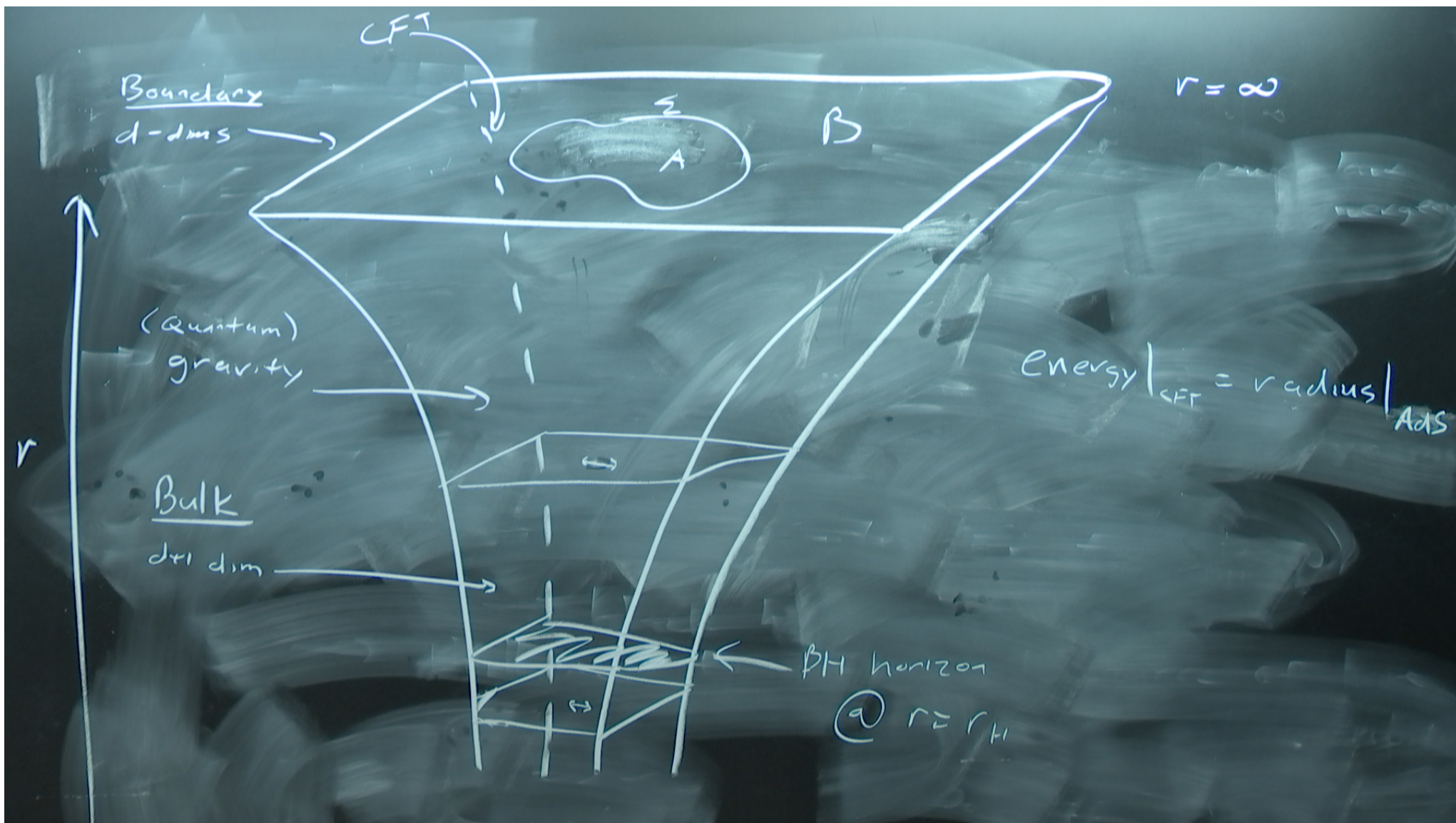
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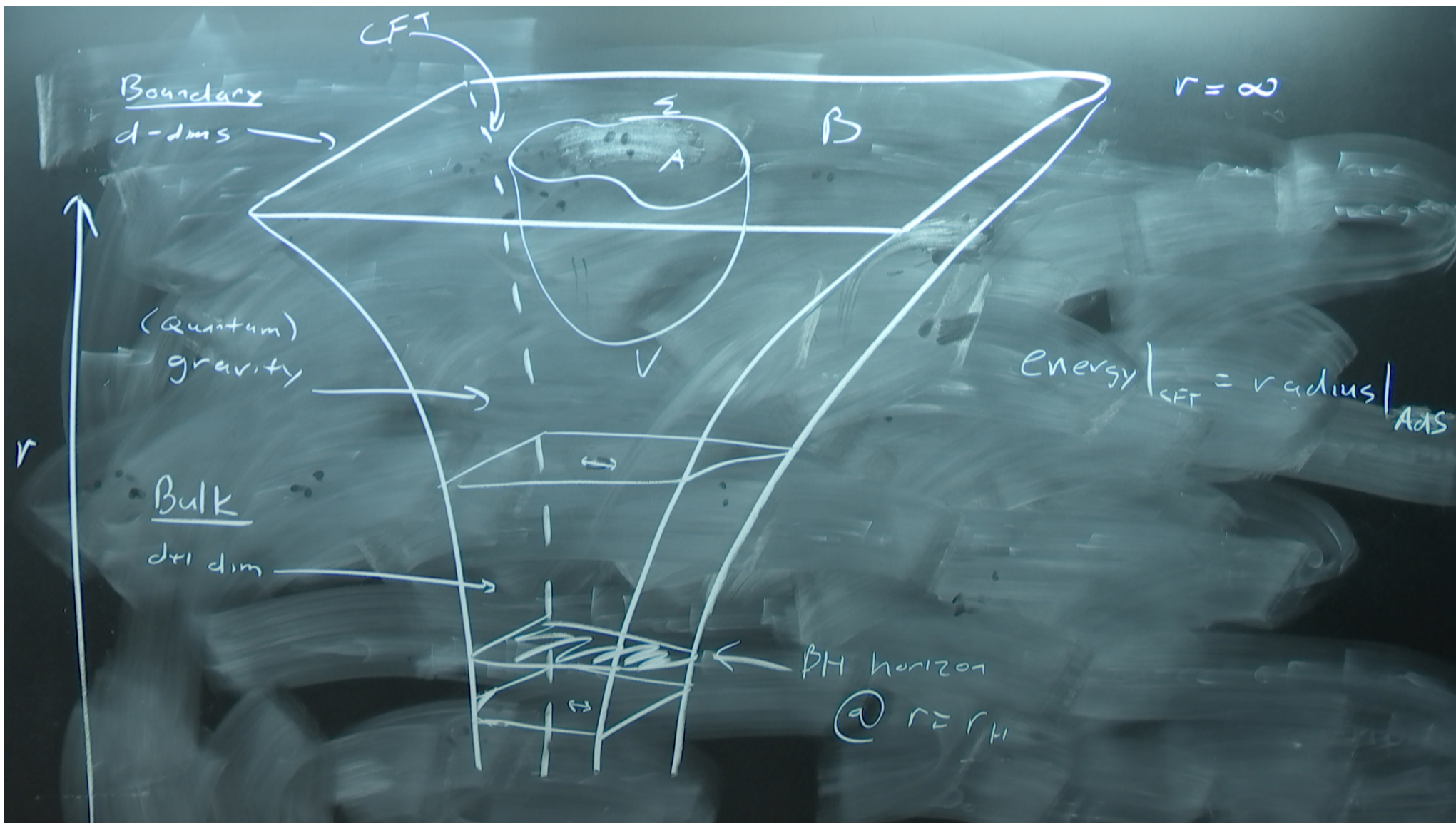


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- use AdS/CFT to translate calculation of S_{EE} in CFT to geometric calculation in AdS space

→ Ryu + Takayanagi

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