Title: TBA

Date: Aug 25, 2015 12:00 PM

URL: http://pirsa.org/15080045

Abstract: TBA

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

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a, $a^\dagger$'s and Anharmonic Oscillator

$(a + a^\dagger)$

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a, $a^\dagger$'s and Anharmonic Oscillator

$(a + 2\, a^\dagger)$

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a, $a^\dagger$'s and Anharmonic Oscillator

$$\left(\frac{a + 2\, a^\dagger}{\sqrt{2}}\right)$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \quad \frac{a + 2\,a^\dagger}{\sqrt{2}} \quad \frac{a + 2\,a^\dagger}{\sqrt{2}}$$

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a, $a^\dagger$'s and Anharmonic Oscillator

$$\frac{a + 2\, a^\dagger}{\sqrt{2}} \quad \frac{a + 3\, a^\dagger}{\sqrt{4}} \quad \frac{a - I\, a^\dagger}{\sqrt{2}\; I}$$

- Hidden

a, $a^\dagger$'s and Anharmonic Oscillator

In[302]:= $\frac{a + 2\, a^\dagger}{\sqrt{2}} \frac{a + 3\, a^\dagger}{\sqrt{4}} \frac{a - I\, a^\dagger}{\sqrt{2}\; I}$ // Expand

Out[302]= $-\frac{i\, a^3}{4} - \left(\frac{1}{4} + \frac{5\, i}{4}\right) a^2\, a^\dagger - \left(\frac{5}{4} + \frac{3\, i}{2}\right) a\, (a^\dagger)^2 - \frac{3\, (a^\dagger)^3}{2}$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\; I}$$

Out[303]=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a + 3\,a^\dagger\right)\right) \cdot \left(-\frac{i\left(a - i\,a^\dagger\right)}{\sqrt{2}}\right)$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[304]:=
```
a · b // FullForm
```

Out[304]//FullForm=
```
CenterDot[a, b]
```

$$\frac{a + 2\, a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\, a^\dagger}{\sqrt{4}} \cdot \frac{a - I\, a^\dagger}{\sqrt{2}\, I}$$

$$\frac{a + 2\, a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\, a^\dagger}{\sqrt{4}} \cdot \frac{a - I\, a^\dagger}{\sqrt{2}\, I}$$

Out[303]= $\frac{a + 2\, a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a + 3\, a^\dagger\right)\right) \cdot \left(-\frac{i\left(a - i\, a^\dagger\right)}{\sqrt{2}}\right)$

Out[303]= $\frac{a + 2\, a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a + 3\, a^\dagger\right)\right) \cdot \left(-\frac{i\left(a - i\, a^\dagger\right)}{\sqrt{2}}\right)$

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a, $a^\dagger$'s and Anharmonic Oscillator

In[304]:=

```
a · b // FullForm
```

Out[304]//FullForm=

```
CenterDot[a, b]
```

In[306]:=

```
? CenterDot
```

CenterDot[x, y, …] displays as $x \cdot y \cdot \ldots$. »

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\; I}$$

Out[303]=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a + 3\,a^\dagger\right)\right) \cdot \left(-\frac{i\left(a - i\,a^\dagger\right)}{\sqrt{2}}\right)$$

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Lecture

In[304]:= a · b // FullForm

Out[304]//FullForm= CenterDot[a, b]

In[306]:= ? CenterDot

CenterDot[x, y, …] displays as $x \cdot y \cdot \ldots$. >>

In[307]:= ? CircleTimes

CircleTimes[x] displays as $\otimes x$.
CircleTimes[x, y, …] displays as $x \otimes y \otimes \ldots$. >>

In[308]:= CircleTimes[a, b]

Out[308]= $a \otimes b$

$$\frac{a + 2 a^{\dagger}}{\sqrt{2}} \cdot \frac{a + 3 a^{\dagger}}{\sqrt{4}} \cdot \frac{a - I a^{\dagger}}{\sqrt{2}\, I}$$

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a, $a^\dagger$'s and Anharmonic Oscillator

In[310]:=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\;I} \text{ // FullForm}$$

Out[310]//FullForm=

```
CenterDot[Times[Power[2, Rational[-1, 2]], Plus[a, Times[2, SuperDagger[a]]]],
 Times[Rational[1, 2], Plus[a, Times[3, SuperDagger[a]]]], Times[Complex[0, -1],
  Power[2, Rational[-1, 2]], Plus[a, Times[Complex[0, -1], SuperDagger[a]]]]]
```

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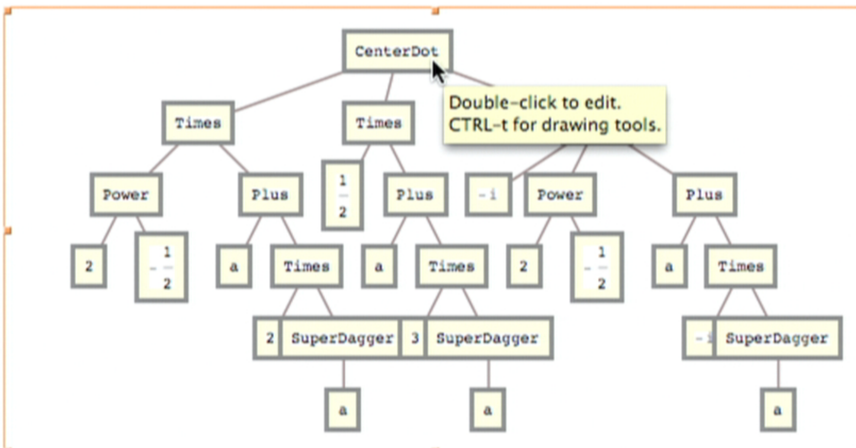
Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[311]:= $\frac{a + 2\, a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\, a^\dagger}{\sqrt{4}} \cdot \frac{a - I\, a^\dagger}{\sqrt{2}\; I}$ // TreeForm

Out[311]//TreeForm=



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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[312]:= $\frac{a+2\,a^\dagger}{\sqrt{2}}\cdot\frac{a+3\,a^\dagger}{\sqrt{4}}\cdot\frac{a-I\,a^\dagger}{\sqrt{2}\;I}$ // Expand

Out[312]= $\frac{a+2\,a^\dagger}{\sqrt{2}}\cdot\left(\frac{1}{2}\left(a+3\,a^\dagger\right)\right)\cdot-\frac{i\left(a-i\,a^\dagger\right)}{\sqrt{2}}$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\;I} \;//.\; \text{rules}$$

Out[312]=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a + 3\,a^\dagger\right)\right) \cdot -\frac{i\left(a - i\,a^\dagger\right)}{\sqrt{2}}$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[1]:=
```
rules = {
  before___ · (number_ operator_) · after___ /; NumericQ[number] → number (before · operator · after)
 }
```

Out[1]=
```
{before___ · (number_ operator_) · after___ /; NumericQ[number] → number before · operator · after}
```

$$\frac{a+2\,a^\dagger}{\sqrt{2}} \cdot \frac{a+3\,a^\dagger}{\sqrt{4}} \cdot \frac{a-I\,a^\dagger}{\sqrt{2}\;I} \;\text{//. rules}$$

Out[312]=

$$\frac{a+2\,a^\dagger}{\sqrt{2}} \cdot \left(\frac{1}{2}\left(a+3\,a^\dagger\right)\right) \cdot -\frac{i\left(a-i\,a^\dagger\right)}{\sqrt{2}}$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[4]:=
```
rules = {
   before___ · (number_ operator_) · after___ /; NumericQ[number] →
    number (before · operator · after),
   before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after
  };
```

In[6]:=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\ I} \ \texttt{//. rules // Expand}$$

Out[6]=

$$-\frac{1}{4}\, i\, a\cdot a\cdot a - \frac{1}{4}\, a\cdot a\cdot a^\dagger - \frac{3}{4}\, i\, a\cdot a^\dagger\cdot a - \frac{3}{4}\, a\cdot a^\dagger\cdot a^\dagger - \frac{1}{2}\, i\, a^\dagger\cdot a\cdot a - \frac{1}{2}\, a^\dagger\cdot a\cdot a^\dagger - \frac{3}{2}\, i\, a^\dagger\cdot a^\dagger\cdot a - \frac{3}{2}\, a^\dagger\cdot a^\dagger\cdot a^\dagger$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[9]:=

```
rules = {
   before___ · (number_ operator_) · after___ /; NumericQ[number] →
    number (before · operator · after),
   before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
   before___ · a · a† · after___ → before · a† · a · after + before · after,
   CenterDot[a_] → a
  };
```

In[10]:=

$$\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\;I} \text{ //. rules // Expand}$$

Out[10]=

$$\left(-\frac{1}{2} - \frac{3\,i}{4}\right) a - \frac{1}{4}\, i\, a \cdot a \cdot a - \left(\frac{1}{4} + \frac{5\,i}{4}\right) a^\dagger \cdot a \cdot a - \left(\frac{5}{4} + \frac{3\,i}{2}\right) a^\dagger \cdot a^\dagger \cdot a - \frac{3}{2}\, a^\dagger \cdot a^\dagger \cdot a^\dagger - 2\, a^\dagger$$

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Lecture

a, $a^\dagger$'s and Anharmonic Oscillator

In[11]:=

```
rules = {
   before___ · (number_ operator_) · after___ /; NumericQ[number] →
    number (before · operator · after),
   before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
   (* before___·a·a†·after___→before·a†·a·after+before·after, *)
   CenterDot[a_] → a
  };
```

In[15]:=

$$\left(\frac{a + 2\,a^\dagger}{\sqrt{2}} \cdot \frac{a + 3\,a^\dagger}{\sqrt{4}} \cdot \frac{a - I\,a^\dagger}{\sqrt{2}\; I}\ \texttt{//. rules // Expand}\right)$$

Out[15]=

$$-\frac{1}{4}\, i\, a \cdot a \cdot a - \frac{1}{4}\, a \cdot a \cdot a^\dagger - \frac{3}{4}\, i\, a \cdot a^\dagger \cdot a -$$
$$\frac{3}{4}\, a \cdot a^\dagger \cdot a^\dagger - \frac{1}{2}\, i\, a^\dagger \cdot a \cdot a - \frac{1}{2}\, a^\dagger \cdot a \cdot a^\dagger - \frac{3}{2}\, i\, a^\dagger \cdot a^\dagger \cdot a - \frac{3}{2}\, a^\dagger \cdot a^\dagger \cdot a^\dagger$$

In[18]:=

```
SetAttributes[CenterDot, Flat]
```

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Lecture

Application: Anharmonic Oscillator by Rayleigh-Ritz

```
rules = {
   before___ · (number_ operator_) · after___ /; NumericQ[number] →
    number (before · operator · after),
   before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
   before___ · a · Ket[n_] → √n before___ · Ket[n_]
      CenterDot[a_] → a
  };
```

In[36]:=

$$\mathbb{P} = \frac{a^{\dagger} - a}{\sqrt{2}\, I};$$

$$\mathbb{X} = \frac{a^{\dagger} + a}{\sqrt{2}};$$

In[38]:=

$$\mathbb{P} \cdot \mathbb{P} \cdot |n\rangle + \mathbb{X} \cdot \mathbb{X} \cdot |n\rangle$$

Out[38]= $i\,(-a + a^{\dagger}) \quad i\,(-a + a^{\dagger}) \quad |n\rangle + a + a^{\dagger} \quad a + a^{\dagger} \quad |n\rangle$

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Lecture

Application: Anharmonic Oscillator by Rayleigh-Ritz

```
rules = {
   before___ · (number_ operator_) · after___ /; NumericQ[number] →
    number (before · operator · after),
   before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
   before___ · a · Ket[n_] → √n before___ · Ket[n - 1],
   before___ · a† · Ket[n_] → √(n + 1) before___ · Ket[n + 1],
   CenterDot[a_] → a
  };
```

In[36]:=

$$\mathbb{P} = \frac{a^\dagger - a}{\sqrt{2}\, I};$$

$$\mathbb{X} = \frac{a^\dagger + a}{\sqrt{2}};$$

In[38]:=

$$\mathbb{P} \cdot \mathbb{P} \cdot |n\rangle + \mathbb{X} \cdot \mathbb{X} \cdot |n\rangle$$

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Lecture

```
    before___ · (number_ operator_) · after___ /; NumericQ[number] →
     number (before · operator · after),
    before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
    before___ · a · Ket[n_] → √n before · Ket[n - 1],
    before___ · a† · Ket[n_] → √(n + 1) before · Ket[n + 1],
    CenterDot[a_] → a
   };
```

In[47]:=

$$\mathbb{P} = \frac{a^\dagger - a}{\sqrt{2}\, I};$$

$$\mathbb{X} = \frac{a^\dagger + a}{\sqrt{2}};$$

In[49]:= $\mathbb{P} \cdot \mathbb{P} \cdot |n\rangle + \mathbb{X} \cdot \mathbb{X} \cdot |n\rangle$ //. rules // Expand

Out[49]= $|n\rangle + 2\, n\, |n\rangle$

$|n\rangle$

Out[35]//FullForm=

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Lecture

In[53]:=
```
rules = {
    before___ · (number_ operator_) · after___ /; NumericQ[number] →
     number (before · operator · after),
    before___ · (op1_ + op2_) · after___ → before · op1 · after + before · op2 · after,
    before___ · a · Ket[n_] → √(n - 1) before · Ket[n - 1],
    before___ · a† · Ket[n_] → √(n + 0) before · Ket[n + 1],
    CenterDot[a_] → a
  };
```

In[47]:=

$$\mathbb{P} = \frac{a^\dagger - a}{\sqrt{2}\, I};$$

$$\mathbb{X} = \frac{a^\dagger + a}{\sqrt{2}};$$

In[52]:=
```
P · P · |n⟩ + X · X · |n⟩ //. rules // Expand
```

Out[52]= $|n\rangle + 2\, n\, |n\rangle$

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Lecture

```
    CenterDot[a_] → a
  };
```

In[55]:=

$$\mathbb{P} = \frac{a^\dagger - a}{\sqrt{2}\, I};$$

$$\mathbb{X} = \frac{a^\dagger + a}{\sqrt{2}};$$

In[57]:= $\mathbb{P} \cdot \mathbb{P} \cdot |n\rangle + \mathbb{X} \cdot \mathbb{X} \cdot |n\rangle$ //. rules // Expand

Out[57]= $-|n\rangle + 2\, n\, |n\rangle$

$|n\rangle$

Out[35]//FullForm=

```
Ket[n]
```

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Lecture

$$\mathbb{X} = \frac{a^\dagger + a}{\sqrt{2}};$$

In[57]:= $\mathbb{P}\cdot\mathbb{P}\cdot|n\rangle + \mathbb{X}\cdot\mathbb{X}\cdot|n\rangle$ //. rules // Expand

Out[57]= $-|n\rangle + 2\,n\,|n\rangle$

In[59]:= HonKet = $\mathbb{P}\cdot\mathbb{P}\cdot|n\rangle + \mathbb{X}\cdot\mathbb{X}\cdot\mathbb{X}\cdot\mathbb{X}\cdot|n\rangle$ //. rules

Out[59]=

$$\frac{1}{2}\left(-\sqrt{-2+n}\,\sqrt{-1+n}\,|-2+n\rangle + (-1+n)\,|n\rangle + n\,|n\rangle - \sqrt{n}\,\sqrt{1+n}\,|2+n\rangle\right) +$$
$$\frac{1}{4}\Big(\sqrt{-4+n}\,\sqrt{-3+n}\,\sqrt{-2+n}\,\sqrt{-1+n}\,|-4+n\rangle + (-3+n)\,\sqrt{-2+n}\,\sqrt{-1+n}\,|-2+n\rangle +$$
$$(-2+n)^{3/2}\sqrt{-1+n}\,|-2+n\rangle + \sqrt{-2+n}\,(-1+n)^{3/2}\,|-2+n\rangle + \sqrt{-2+n}\,\sqrt{-1+n}\,n\,|-2+n\rangle +$$
$$(-2+n)\,(-1+n)\,|n\rangle + (-1+n)^2\,|n\rangle + 2\,(-1+n)\,n\,|n\rangle + n^2\,|n\rangle + n\,(1+n)\,|n\rangle +$$
$$(-1+n)\,\sqrt{n}\,\sqrt{1+n}\,|2+n\rangle + n^{3/2}\sqrt{1+n}\,|2+n\rangle + \sqrt{n}\,(1+n)^{3/2}\,|2+n\rangle +$$
$$\sqrt{n}\,\sqrt{1+n}\,(2+n)\,|2+n\rangle + \sqrt{n}\,\sqrt{1+n}\,\sqrt{2+n}\,\sqrt{3+n}\,|4+n\rangle\Big)$$

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Lecture

In[63]:=
```
SparseArray[{{i_, i_} → 1, {2, 5} → α, {i_, j_} /; j == i + 1 → 9}, {20, 20}] // Normal //
 MatrixForm
```

Out[63]//MatrixForm=

$$\begin{pmatrix}
1&9&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&9&0&\alpha&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&9&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&9&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&9&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&9&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&9&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&9&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&9&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&9&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&9&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&9&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&9&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&9&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&9&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&9&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&9&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&9&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&9\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

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Lecture

In[79]:=
```
M = 150;
```

In[87]:=
```
H = N@SparseArray[makeMatrix, {M, M}]
```

Out[87]=
```
SparseArray[ Specified elements: 738
             Dimensions: {150, 150} ]
```

In[88]:=
```
H[[1 ;; 20, 1 ;; 20]] // Normal // MatrixForm
```

Out[88]//MatrixForm=

1.25	0.	1.41421	0.	1.22474	0.	0.	0.	0.	0.	0.	
0.	5.25	0.	4.89898	0.	2.73861	0.	0.	0.	0.	0.	
1.41421	0.	12.25	0.	10.3923	0.	4.74342	0.	0.	0.	0.	
0.	4.89898	0.	22.25	0.	17.8885	0.	7.24569	0.	0.	0.	
1.22474	0.	10.3923	0.	35.25	0.	27.3861	0.	10.247	0.	0.	
0.	2.73861	0.	17.8885	0.	51.25	0.	38.8844	0.	13.7477	0.	
0.	0.	4.74342	0.	27.3861	0.	70.25	0.	52.3832	0.	17.7482	
0.	0.	0.	7.24569	0.	38.8844	0.	92.25	0.	67.8823	0.	2
0.	0.	0.	0.	10.247	0.	52.3832	0.	117.25	0.	85.3815	
0.	0.	0.	0.	0.	13.7477	0.	67.8823	0.	145.25	0.	1
0.	0.	0.	0.	0.	0.	17.7482	0.	85.3815	0.	176.25	
0.	0.	0.	0.	0.	0.	0.	22.2486	0.	104.881	0.	2

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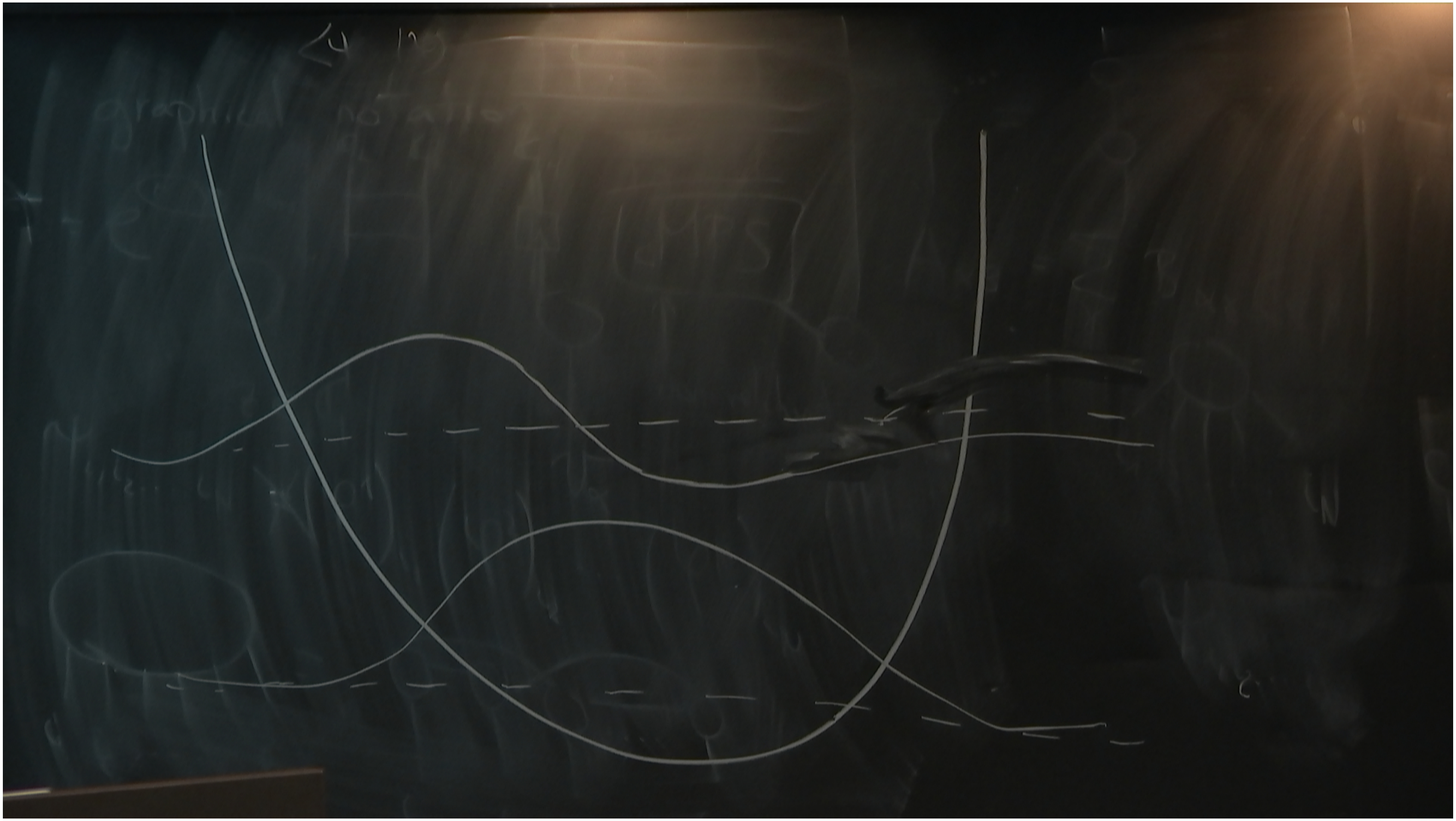

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Lecture

- **Hidden 1**
- **Hidden 2**

```
enern
```

k	E_k (TBA)	E_k (QM)
0	1.06036209048418	1.06036209048418289965[a]
1	3.79967302980139	3.79967302980[b]
2	7.45569793798672	7.45569793798673839216[a]
3	11.6447455113781 5	11.6447455114[b]
4	16.2618260188502 4	16.26182601885022593789[a]
5	21.2383729182359 5	21.2383729182[b]

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Lecture

- **Hidden 1**
- **Hidden 2**

In[93]:=

```
energies[[1 ;; 6]]
```

Out[93]=

```
{1.0603620904841828996470460167132260732483530402026,
 3.7996730298013941687830941893333275035011195886890,
 7.4556979379867383921565913605999734583352829350441,
 11.644745511378162020850373327134371754265207320332,
 16.261826018850225937894957616685662848293826549241,
 21.238372918235940024149720244018339656618211679087}
```

k	E_k (TBA)	E_k (QM)
0	1.06036209048418	1.0603620904841828965[a]
1	3.79967302980139	3.79967302980[b]
2	7.45569793798672	7.4556979379867383916[a]
3	11.644745511378[illegible]	11.64474551[illegible][b]

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Lecture

```
26.528471183682518191814006371187254277609145413768,
32.098597710968326634274194318232154263052660544519,
37.923001027033985146521551776903739669461770800935,
43.981158097289730785392927285924410377319640815217}
```

In[92]:= $\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}\ \text{HermiteH}[n-1, x];$

In[94]:= `Eigensystem[H, -10]`

Out[94]=

```
{{43.981158097289730785392927285924410377319640815217,
  37.923001027033985146521551776903739669461770800935,
  32.098597710968326634274194318232154263052660544519,
  26.528471183682518191814006371187254277609145413768,
  21.238372918235940024149720244018339656618211679087,
  16.261826018850225937894957616685662848293826549241,
  11.644745511378162020850373327134371754265207320332,
  7.4556979379867383921565913605999734583352829350441,
  3.7996730298013941687830941893333275035011195886890,
  1.0603620904841828996470460167132260732483530402026},
 {{-2.2683048824898852787691668820906221382577701224487 × 10^-30,
   -0.05608934629066695983711181823173413259353147854510,
```

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Lecture

$-1.64165\times10^{-27}, -0.00022925, 1.09026\times10^{-27}, 0.00013361, -7.41083\times10^{-28}, -0.0000631605,$
$1.08795\times10^{-27}, 0.0000185837, -1.72426\times10^{-27}, 5.25342\times10^{-6}, 2.04101\times10^{-27}, -0.0000149335,$
$-1.62285\times10^{-27}, 0.0000162961, 9.83767\times10^{-28}, -0.0000136431, -4.49231\times10^{-28}, 9.71179\times10^{-6},$
$5.44264\times10^{-28}, -5.9874\times10^{-6}, -7.89839\times10^{-28}, 3.09668\times10^{-6}, 9.54826\times10^{-28}, -1.15674\times10^{-6},$
$-3.77665\times10^{-28}, 3.26734\times10^{-8}, -8.91965\times10^{-29}, 4.97187\times10^{-7}, 1.08158\times10^{-27}, -6.50062\times10^{-7},$
$-9.48163\times10^{-28}, 5.98661\times10^{-7}, 8.51941\times10^{-28}, -4.61852\times10^{-7}, 1.36819\times10^{-29}, 3.10933\times10^{-7},$
$-3.24469\times10^{-28}, -1.81692\times10^{-7}, 9.08489\times10^{-28}, 8.67284\times10^{-8}, -4.63919\times10^{-28}, -2.55295\times10^{-8},$
$5.35664\times10^{-28}, -8.4148\times10^{-9}, 7.68778\times10^{-29}, 2.31975\times10^{-8}, 4.13234\times10^{-28}, -2.61547\times10^{-8},$
$2.67224\times10^{-28}, 2.29176\times10^{-8}, 8.33148\times10^{-28}, -1.72855\times10^{-8}, 9.70269\times10^{-29}, 1.15021\times10^{-8},$
$4.5819\times10^{-28}, -6.6684\times10^{-9}, -3.25772\times10^{-29}, 3.14459\times10^{-9}, -6.45694\times10^{-28}, -8.73018\times10^{-10},$
$5.74647\times10^{-28}, -3.93741\times10^{-10}, -2.38579\times10^{-28}, 9.50725\times10^{-10}, 1.72261\times10^{-27},$
$-1.06427\times10^{-9}, 8.87124\times10^{-28}, 9.40545\times10^{-10}, 1.45484\times10^{-27}, -7.20911\times10^{-10}, 1.07499\times10^{-27},$
$4.90964\times10^{-10}, 7.17644\times10^{-28}, -2.94508\times10^{-10}, 1.21456\times10^{-27}, 1.47495\times10^{-10}, 9.44878\times10^{-28},$
$-4.95488\times10^{-11}, 1.58104\times10^{-27}, -7.65168\times10^{-12}, 1.27044\times10^{-27}, 3.49433\times10^{-11}, 1.43368\times10^{-27},$
$-4.2606\times10^{-11}, 9.74091\times10^{-28}, 3.89604\times10^{-11}, 6.8128\times10^{-28}, -3.00242\times10^{-11}, -2.04379\times10^{-28},$
$1.97512\times10^{-11}, -7.15578\times10^{-28}, -1.05226\times10^{-11}, -8.47096\times10^{-28}, 3.68235\times10^{-12}\},$

$\{0.0191936, -5.92398\times10^{-32}, 0.170524, -4.38948\times10^{-31}, 0.377817, -1.27154\times10^{-30}, 0.0894548,$
$-1.22587\times10^{-30}, -0.315758, 1.23711\times10^{-30}, 0.140454, 2.91014\times10^{-30}, 0.165093, 3.72877\times10^{-31},$
$-0.385061, -1.86245\times10^{-31}, 0.456023, 3.81818\times10^{-30}, -0.410375, 4.06901\times10^{-30}, 0.308048,$
$2.50018\times10^{-30}, -0.198172, 1.60467\times10^{-30}, 0.107995, -2.09255\times10^{-30}, -0.0461634,$
$-9.05571\times10^{-31}, 0.0102321, 8.85794\times10^{-32}, 0.00672673, -1.82214\times10^{-31}, -0.0119841,$
$2.79134\times10^{-30}, 0.0112584, -2.81093\times10^{-30}, -0.00828335, -6.85699\times10^{-31}, 0.00510182,$

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Lecture

```
26.528471183682518191814006371187254277609145413768,
32.098597710968326634274194318232154263052660544519,
37.923001027033985146521551776903739669461770800935,
43.981158097289730785392927285924410377319640815217}
```

In[92]:=

$$\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}\,\mathrm{HermiteH}[n-1, x];$$

In[101]:=

```
evs = Eigensystem[H, -10]ᵀ // Sort (* //Chop*);
```

In[103]:=

```
basis = Table[ϕ[n, x], {n, 1, M}]
```

Out[103]=

$$\left\{\frac{e^{-\frac{x^2}{2}}}{\pi^{1/4}}, \frac{\sqrt{2}\,e^{-\frac{x^2}{2}}\,x}{\pi^{1/4}}, \frac{e^{-\frac{x^2}{2}}\left(-2+4x^2\right)}{2\sqrt{2}\,\pi^{1/4}}, \frac{e^{\cdots 1 \cdots}\cdots 1\cdots}{4\sqrt{3}\,\pi^{\cdots 1 \cdots}}, \cdots 142 \cdots, \frac{\cdots 1 \cdots}{\cdots 1 \cdots}, \frac{e^{-\frac{x^2}{2}}\left(\cdots 1 \cdots + \cdots 109 \cdots\right)}{\cdots 1 \cdots}, \frac{e^{-\frac{x^2}{2}}\left(\cdots 1 \cdots\right)}{\cdots 1 \cdots}, \frac{e^{-\frac{x^2}{2}}\left(\cdots 1 \cdots\right)}{\cdots 1 \cdots}\right\}$$

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Lecture

```
26.528471183682518191814006371187254277609145413768,
32.098597710968326634274194318232154263052660544519,
37.923001027033985146521551776903739669461770800935,
43.981158097289730785392927285924410377319640815217}
```

In[92]:= $\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}$ HermiteH[n - 1, x];

```
In[101]:= evs = Eigensystem[H, -10]ᵀ // Sort (* //Chop*);

In[104]:= basis = Table[ϕ[n, x], {n, 1, M}];

In[102]:= evs[[1]]

Out[102]= {1.0603620904841828996470460167132260732483530402026,
  {0.99197594422359367501743652909153925920692468444 73,
   -3.0379468030038131840589432788728014487664947986872 × 10^-60,
   -0.12398334601434831881604766551791951359006304742450,
   -1.4492880726025860564545575415549392467972185738489 × 10^-60,
   -0.010432633937985416857063642400880663494472194214314,
   -2.3172242280861641405485099868571228052891465242270 × 10^-60,
```

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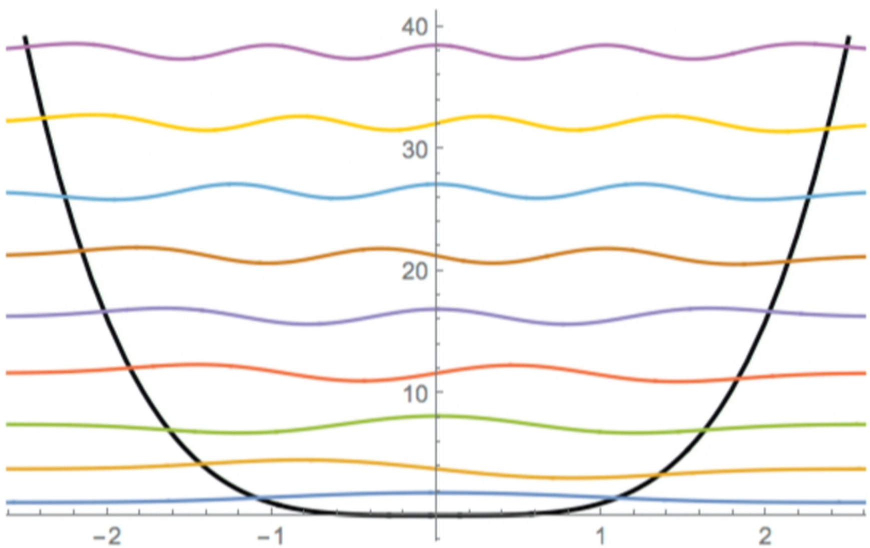
Lecture

In[107]:=
```
newPlot = Plot[toPlot, {x, -4, 4}];
```

In[108]:=
```
Show[Plot[x^4, {x, -5 / 2, 5 / 2}, PlotStyle → {{Black, Thick}}], newPlot]
```

Out[108]=



In[102]:=
```
evs[[1]]
```

Out[102]=
```
{1.0603620904841828996470460167132260732483530402026,
 {0.99197594422359367501743652909153925920692468444473,
```

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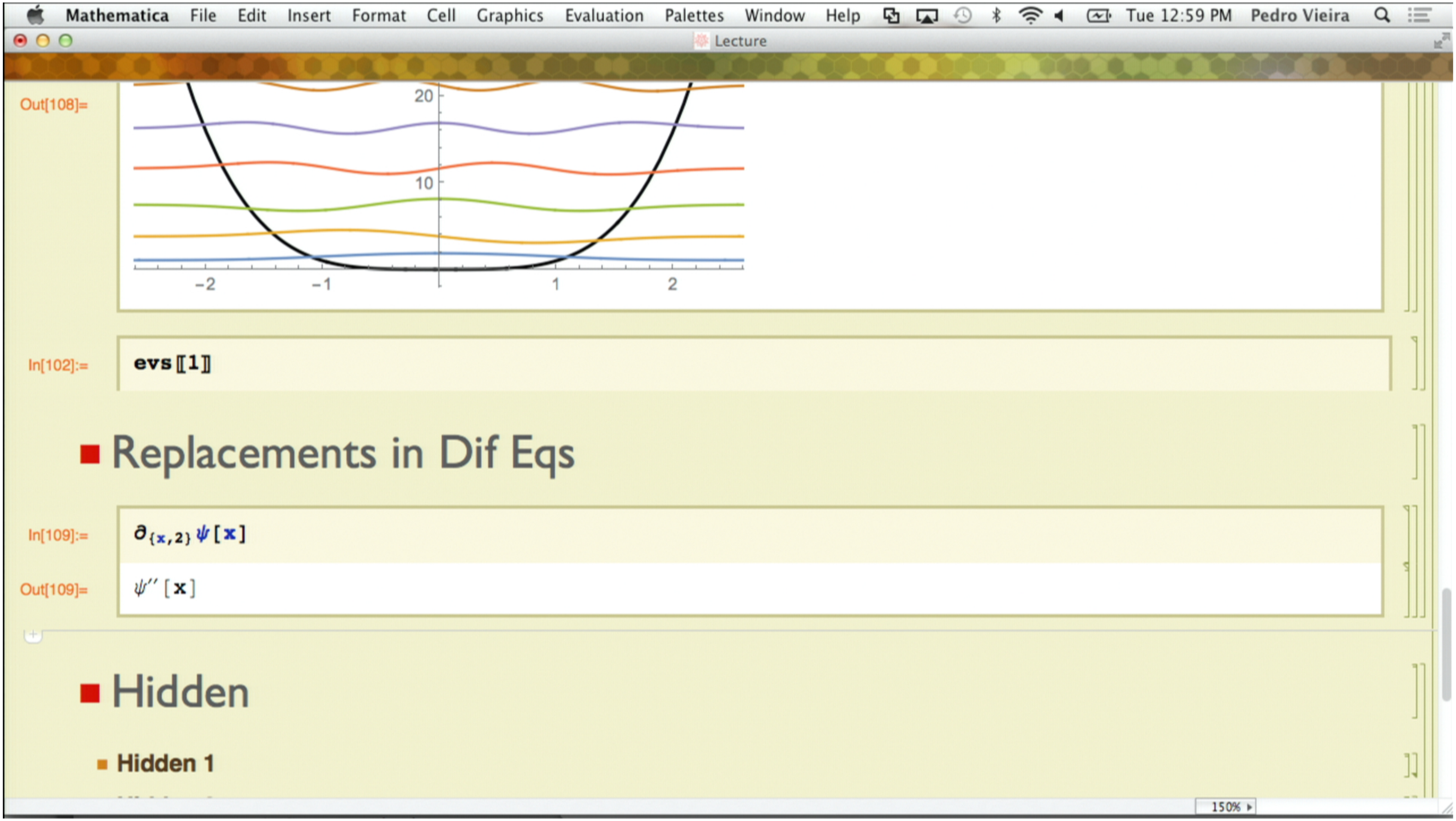

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Tue 12:59 PM Pedro Vieira
Lecture
Out[108]=
20
10
−2
−1
1
2
In[102]:=
evs[[1]]
Replacements in Dif Eqs
In[109]:=
∂{x,2} ψ[x]
Out[109]=
ψ′′[x]
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Hidden 1
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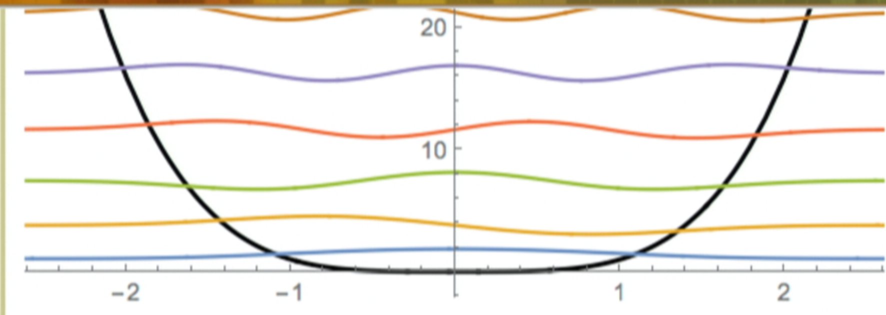
Lecture

Out[108]=



In[102]:= **evs[[1]]**

Replacements in Dif Eqs

In[111]:= $-\partial_{\{x,2\}}\psi[x] + x^2\,\psi[x] - (2\,n - 1)\,\psi[x]$

Out[111]= $-(-1 + 2\,n)\,\psi[x] + x^2\,\psi[x] - \psi''[x]$

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Lecture

In[102]:= evs[[1]]

Replacements in Dif Eqs

In[112]:= ?ϕ

Global`ϕ

$$\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}\,\text{HermiteH}[n-1,x]}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}$$

In[113]:= $\text{Schrodinger} = -\partial_{\{x,2\}}\psi[x] + x^2\,\psi[x] - (2\,n-1)\,\psi[x]$

Out[113]= $-(-1+2\,n)\,\psi[x] + x^2\,\psi[x] - \psi''[x]$

In[114]:= Schrodinger /. ψ[x_] :→ ϕ[n, x]

Out[114]= $$-\frac{e^{-\frac{x^2}{2}}\,(-1+2\,n)\,\text{HermiteH}[-1+n,x]}{\pi^{1/4}\sqrt{2^{-1+n}\,(-1+n)!}} + \frac{e^{-\frac{x^2}{2}}\,x^2\,\text{HermiteH}[-1+n,x]}{\pi^{1/4}\sqrt{2^{-1+n}\,(-1+n)!}} - \psi''[x]$$

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Lecture

Global`ϕ

$$\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}} \text{HermiteH}[n-1, x]}{\sqrt{2^{n-1} (n-1)! \sqrt{\pi}}}$$

In[113]:= Schrodinger = $-\partial_{\{x,2\}} \psi[x] + x^2 \psi[x] - (2 n - 1) \psi[x]$

Out[113]= $-(-1 + 2 n) \psi[x] + x^2 \psi[x] - \psi''[x]$

Schrodinger /. ψ :→ ϕ[n,]

Out[114]= $-\frac{e^{-\frac{x^2}{2}} (-1 + 2 n) \text{HermiteH}[-1 + n, x]}{\pi^{1/4} \sqrt{2^{-1+n} (-1 + n)!}} + \frac{e^{-\frac{x^2}{2}} x^2 \text{HermiteH}[-1 + n, x]}{\pi^{1/4} \sqrt{2^{-1+n} (-1 + n)!}} - \psi''[x]$

In[115]:= ψ''[x] // FullForm

Out[115]//FullForm=

```
Derivative[2][\[Psi]][x]
```

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Lecture

```
Global`ϕ
```

$$\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}\,\text{HermiteH}[n-1,x]}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}$$

In[113]:= Schrodinger = $-\partial_{\{x,2\}}\psi[x] + x^2\,\psi[x] - (2\,n-1)\,\psi[x]$

Out[113]= $-(-1+2\,n)\,\psi[x] + x^2\,\psi[x] - \psi''[x]$

Schrodinger /. ψ :→ (ϕ[n, #] &)

Out[114]= $-\frac{e^{-\frac{x^2}{2}}\,(-1+2\,n)\,\text{HermiteH}[-1+n,x]}{\pi^{1/4}\sqrt{2^{-1+n}\,(-1+n)!}} + \frac{e^{-\frac{x^2}{2}}\,x^2\,\text{HermiteH}[-1+n,x]}{\pi^{1/4}\sqrt{2^{-1+n}\,(-1+n)!}} - \psi''[x]$

In[115]:= ψ''[x] // FullForm

Out[115]//FullForm= Derivative[2][\[Psi]][x]

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Lecture

```
Global`ϕ
```

$$\phi[n\_, x\_] := \frac{e^{-\frac{x^2}{2}}\,\text{HermiteH}[n-1,x]}{\sqrt{2^{n-1}\,(n-1)!\,\sqrt{\pi}}}$$

In[113]:= Schrodinger = $-\partial_{\{x,2\}}\psi[x] + x^2\,\psi[x] - (2\,n - 1)\,\psi[x]$

Out[113]= $-(-1 + 2\,n)\,\psi[x] + x^2\,\psi[x] - \psi''[x]$

In[118]:= Schrodinger /. ψ :→ (ϕ[n, #] &) // Simplify //FullSimplify

Out[118]= 0

In[115]:= ψ''[x] // FullForm

Out[115]//FullForm=

```
Derivative[2][\[Psi]][x]
```

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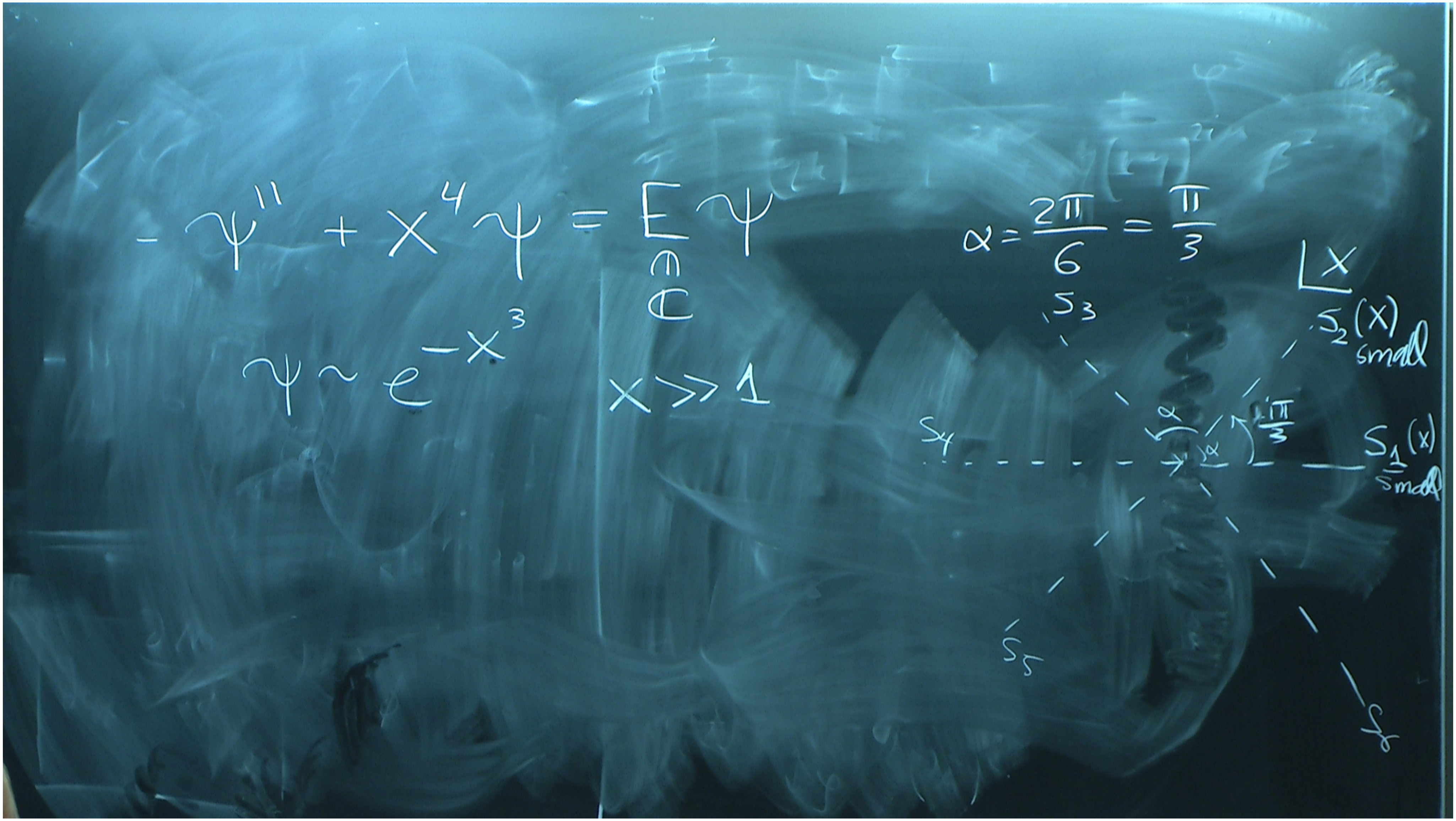
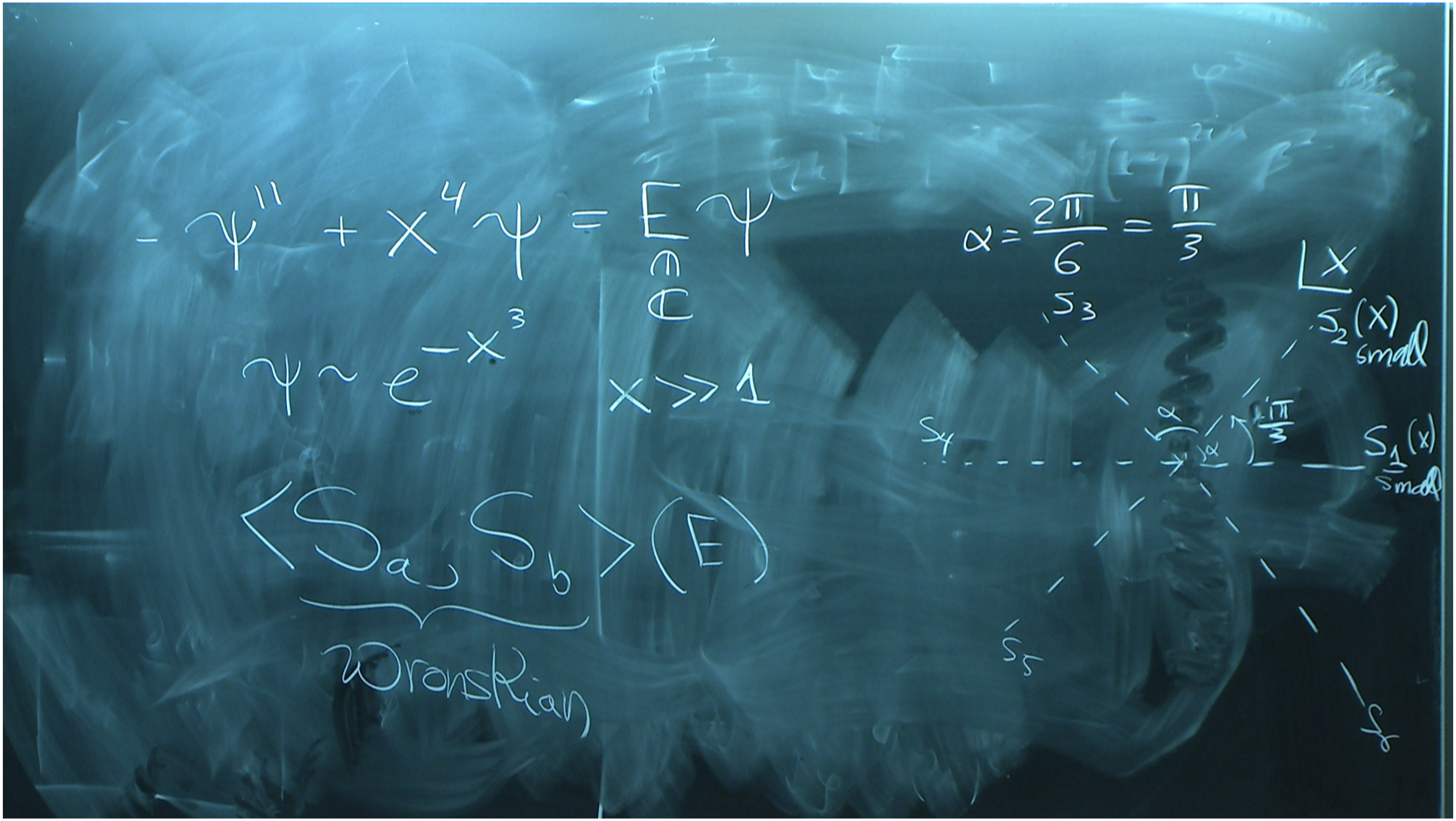
$$-\psi'' + x^4\psi = E\psi$$

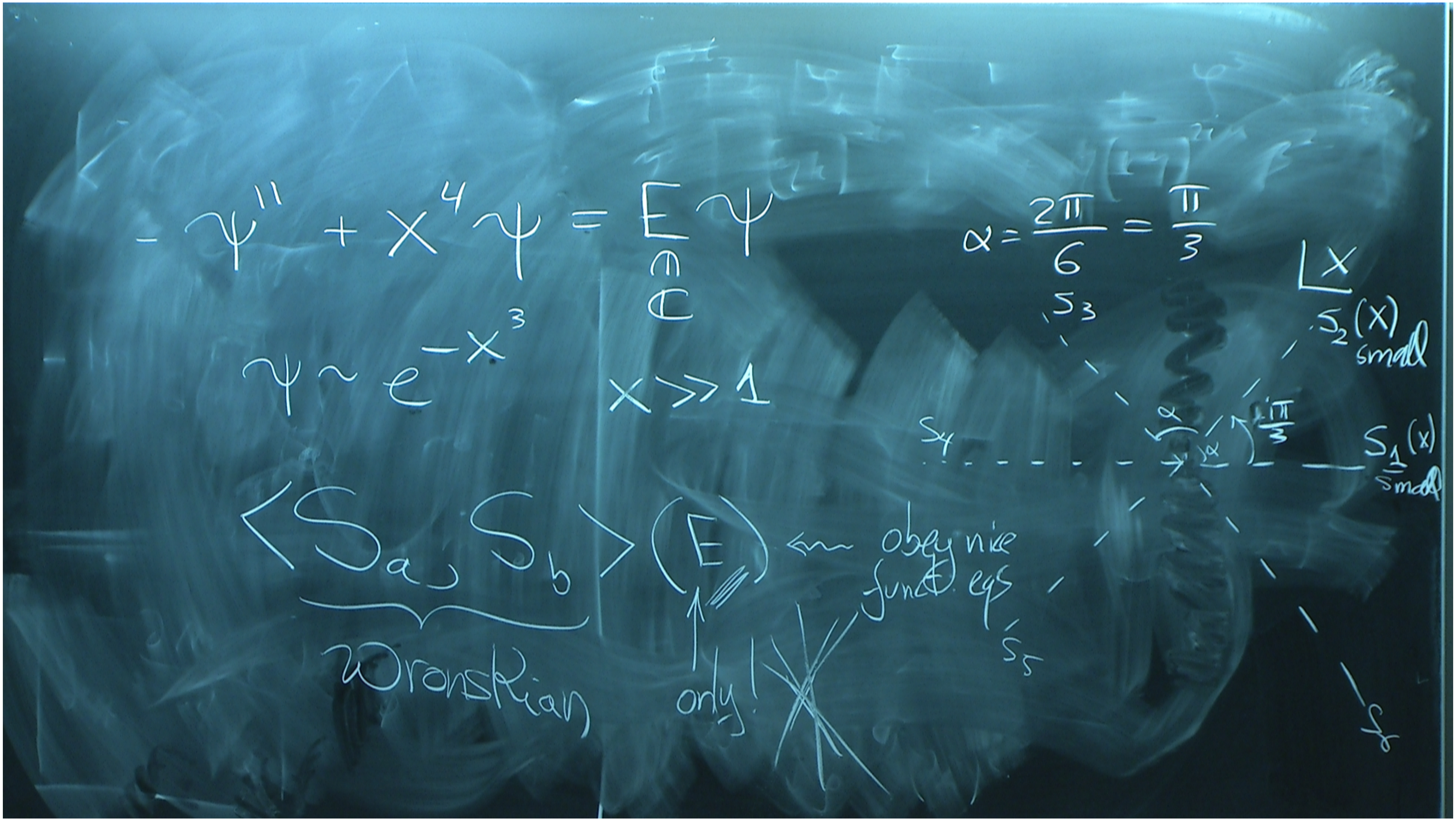
$$E \in \mathbb{C}$$

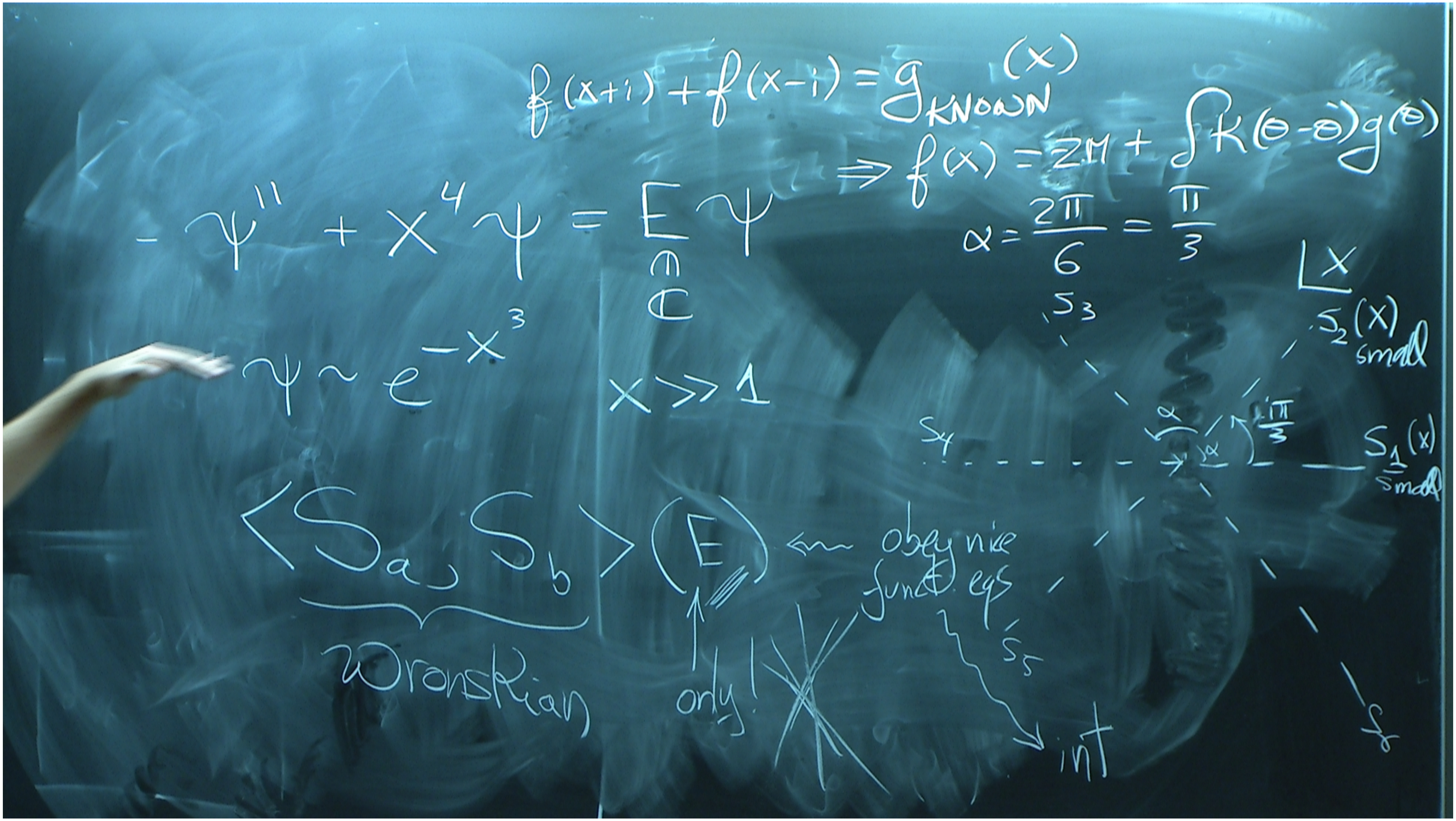
$$\psi \sim e^{-x^3} \qquad x \gg 1$$

$$\alpha = \frac{2\pi}{6} = \frac{\pi}{3}$$

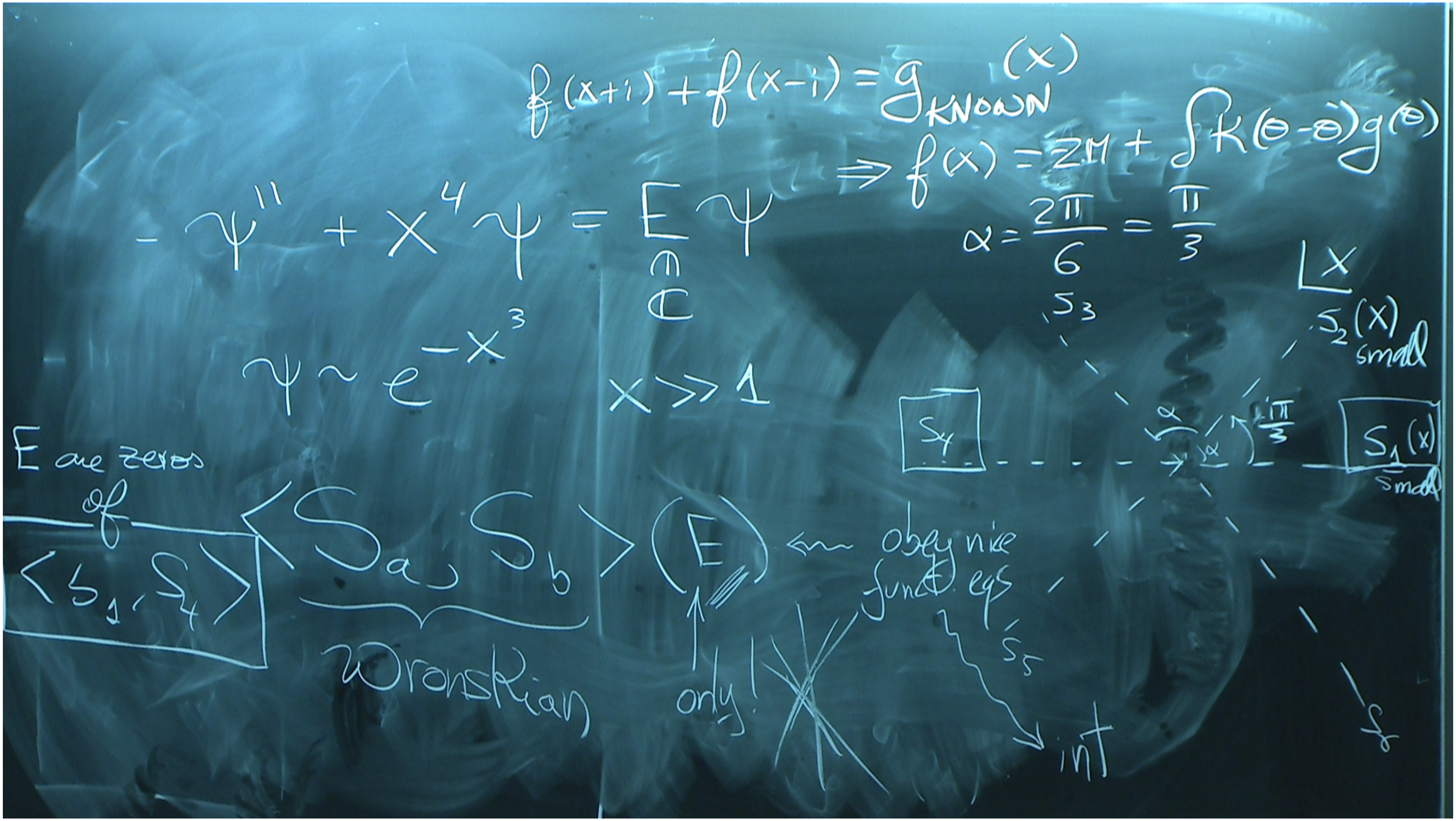

$-\psi'' + x^4\psi = E\psi$
$E \in \mathbb{C}$
$\psi \sim e^{-x^3}$
$x \gg 1$
$\alpha = \frac{2\pi}{6} = \frac{\pi}{3}$
S_3
$S_2(x)$ small
S_4
$S_1(x)$ small
S_5
S_6


-ψ'' + x⁴ψ = Eψ
E ∈ ℂ
ψ ~ e^{-x³}
x >> 1
⟨S_a, S_b⟩(E)
Wronskian
α = 2π/6 = π/3
S₃
S₂(x)
small
S₄
S₁(x)
small
S₅
α
2π/3


-ψ'' + x^4 ψ = E ψ
ψ ~ e^{-x^3}
x >> 1
⟨S_a, S_b⟩(E)
Wronskian
only!
obey nice funct. eqs
α = 2π/6 = π/3
S_3
S_4
S_5
S_2(x) small
S_1(x) small



f(x+i) + f(x−i) = g_KNOWN(x)
⇒ f(x) = 2M + ∫K(θ−θ')g(θ')
−ψ'' + x⁴ψ = Eψ
ψ ~ e^{−x³}
x >> 1
α = 2π/6 = π/3
S₃
S₂(x) small
S₄
S₁(x) small
⟨S_a, S_b⟩(E) ← obey nice funct. eqs
Wronskian
only!
S₅
int



f(x+i) + f(x−i) = g_KNOWN(x)
⇒ f(x) = 2M + ∫K(θ−θ')g(θ')
−ψ'' + x^4 ψ = E ψ
α = 2π/6 = π/3
ψ ~ e^{−x^3}
x >> 1
S_3
S_2(x) small
S_4
S_1(x) small
E are zeros of ⟨S_1, S_4⟩
⟨S_a, S_b⟩(E)
obey nice funct. eqs
Wronskian
only!
S_5
int