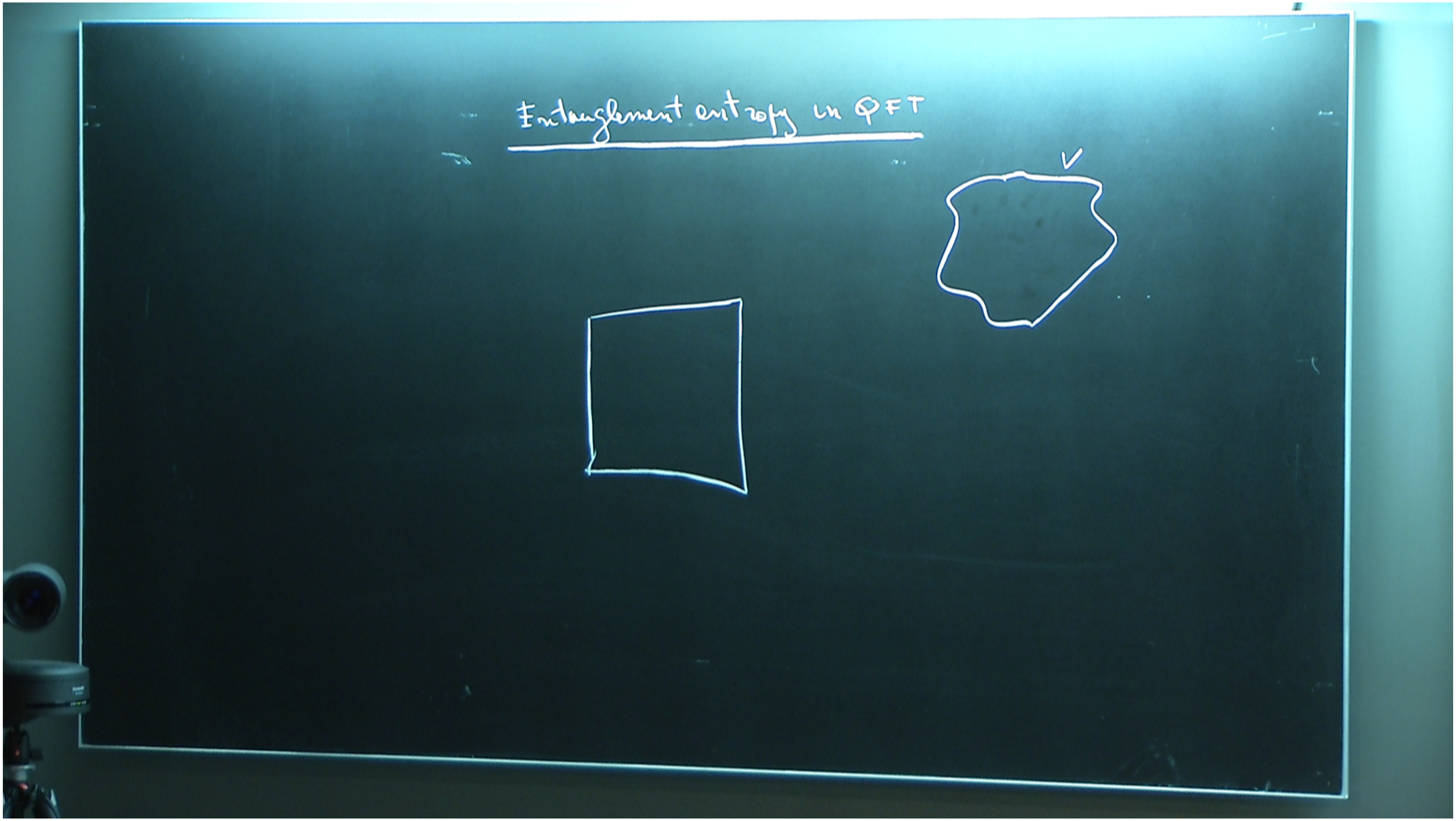
Title: TBA

Date: Aug 24, 2015 09:15 AM

URL: http://pirsa.org/15080043

Abstract: TBA


Entanglement entropy in QFT
V


Entanglement entropy in QFT
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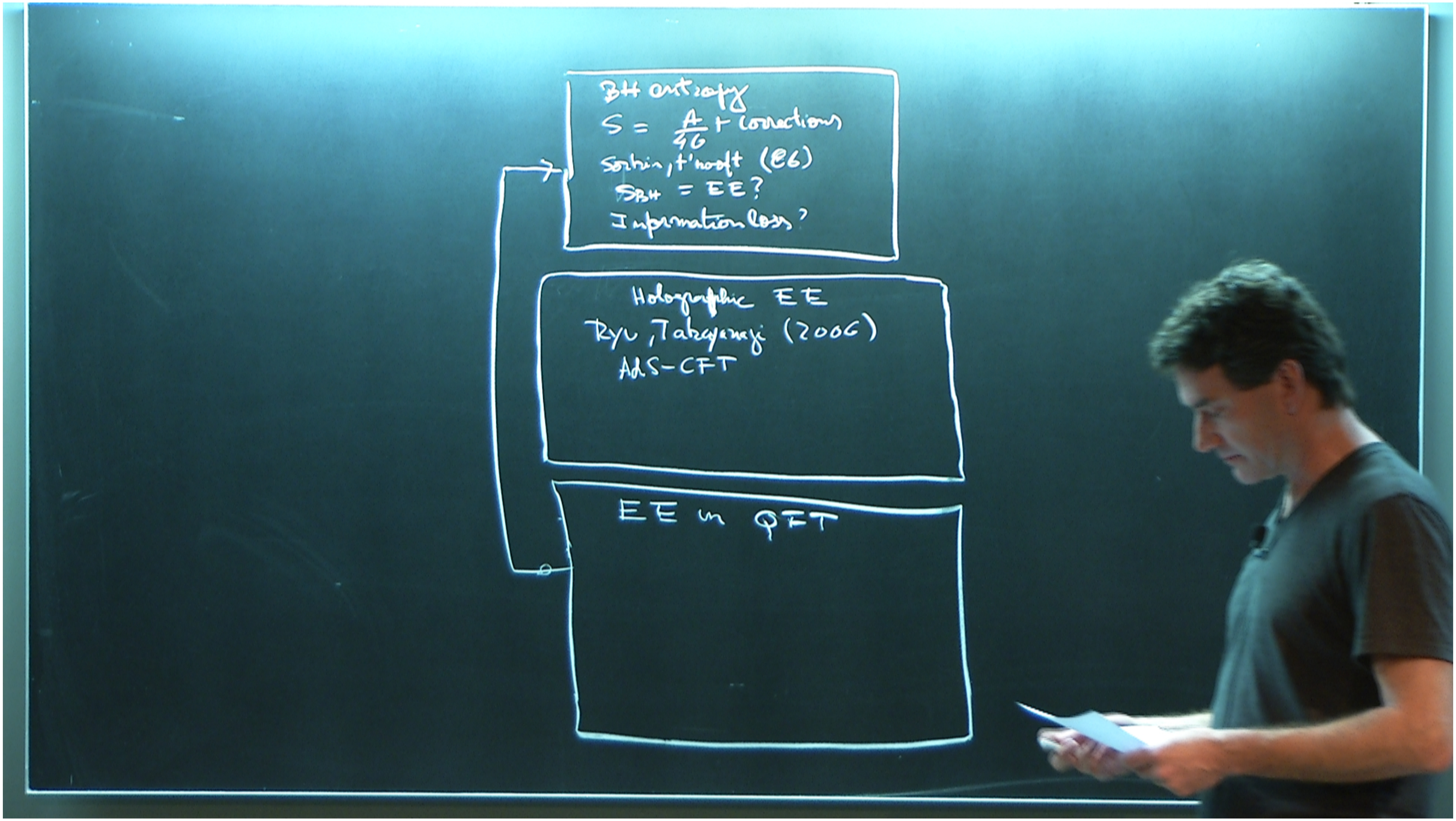
BH entropy

$$S = \frac{A}{4G} + \text{corrections}$$

Sorkin, t'Hooft (86)

$S_{BH} = EE$?

Information loss?


BH entropy
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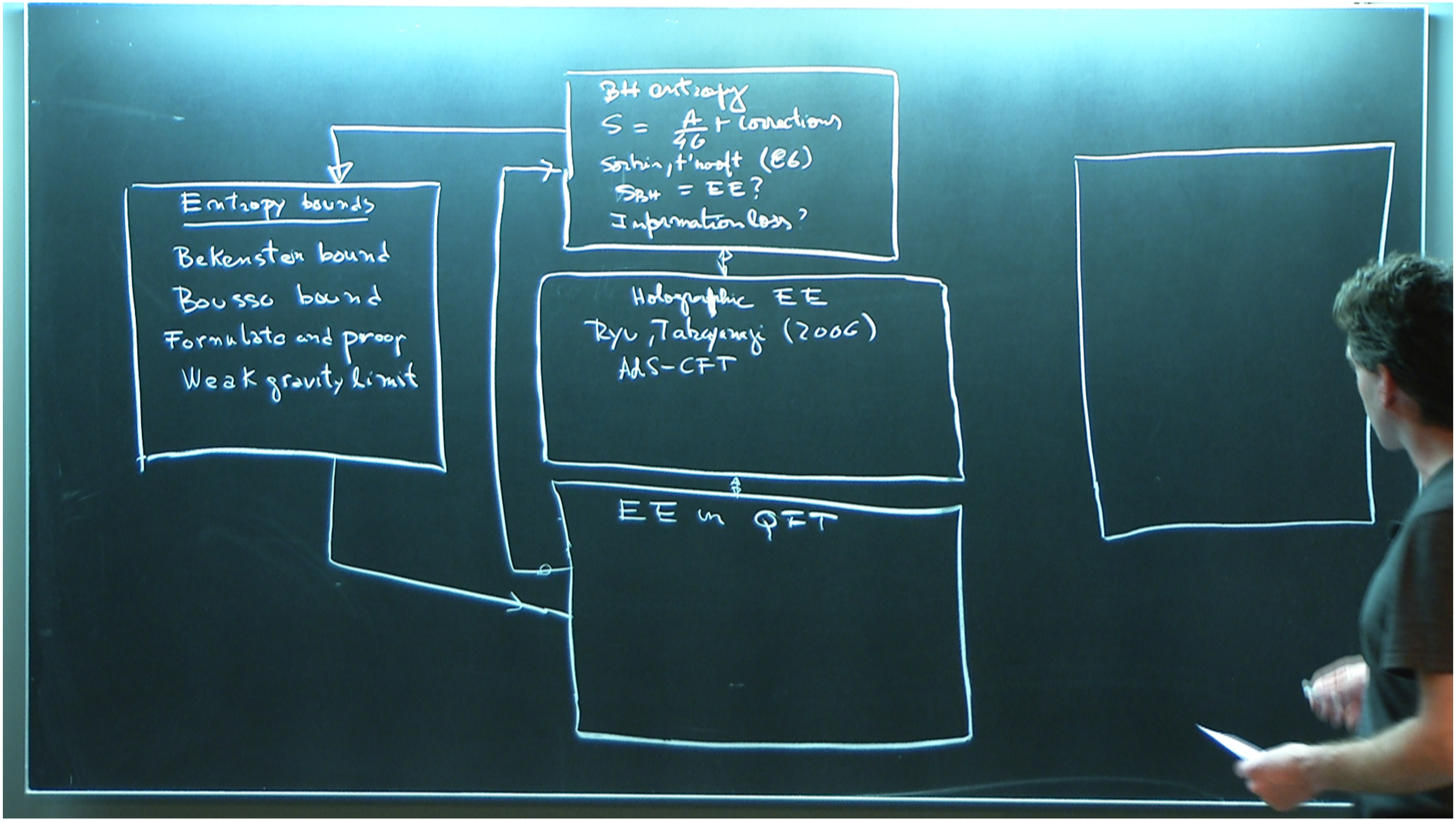
Holographic EE
Ryu, Takayanagi (2006)
AdS-CFT
EE in QFT


BH entropy
S = A/4G + corrections
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SBH = EE?
Information loss?
Entropy bounds
Bekenstein bound
Bousso bound
Formulate and proof
Weak gravity limit
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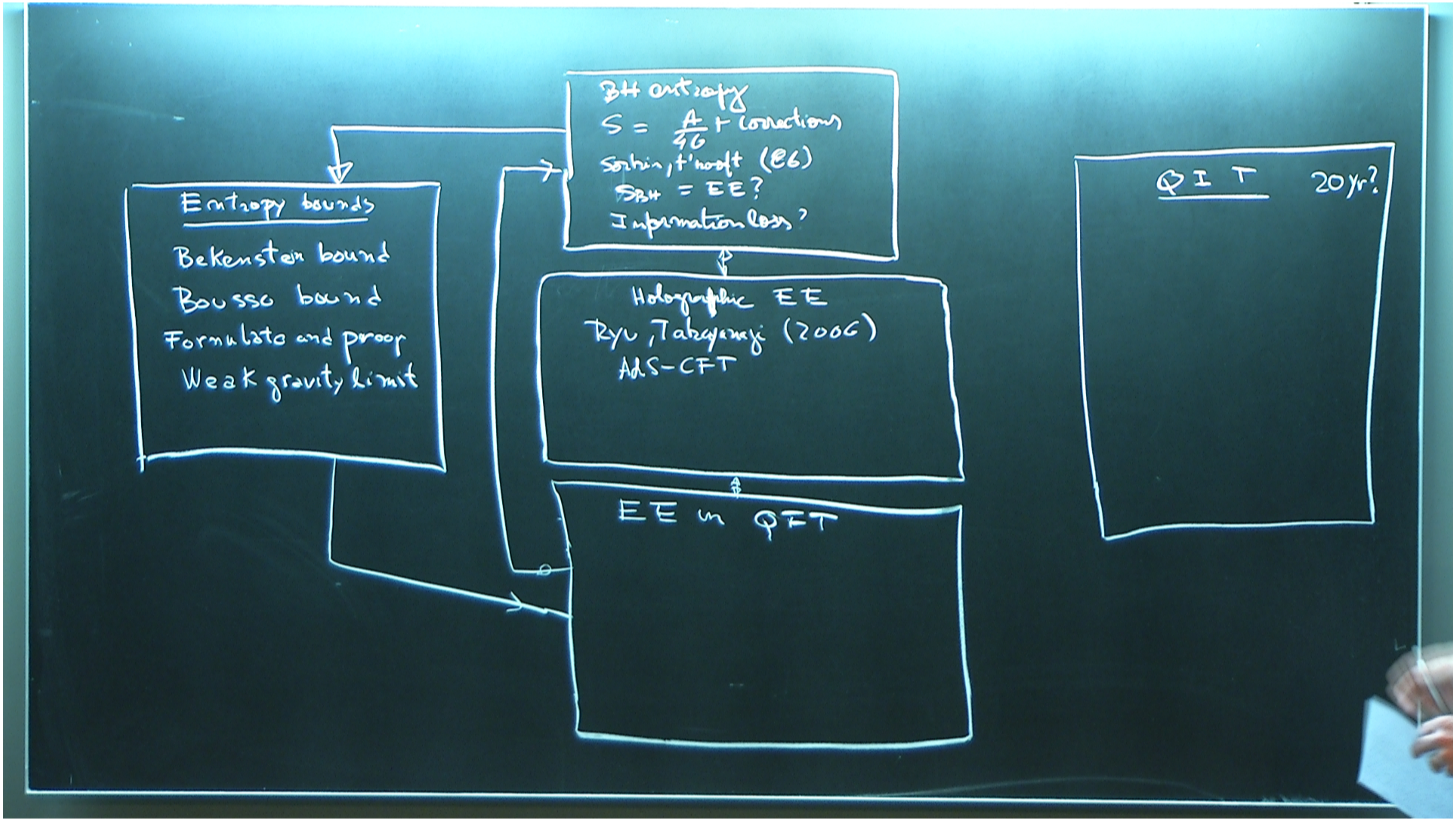

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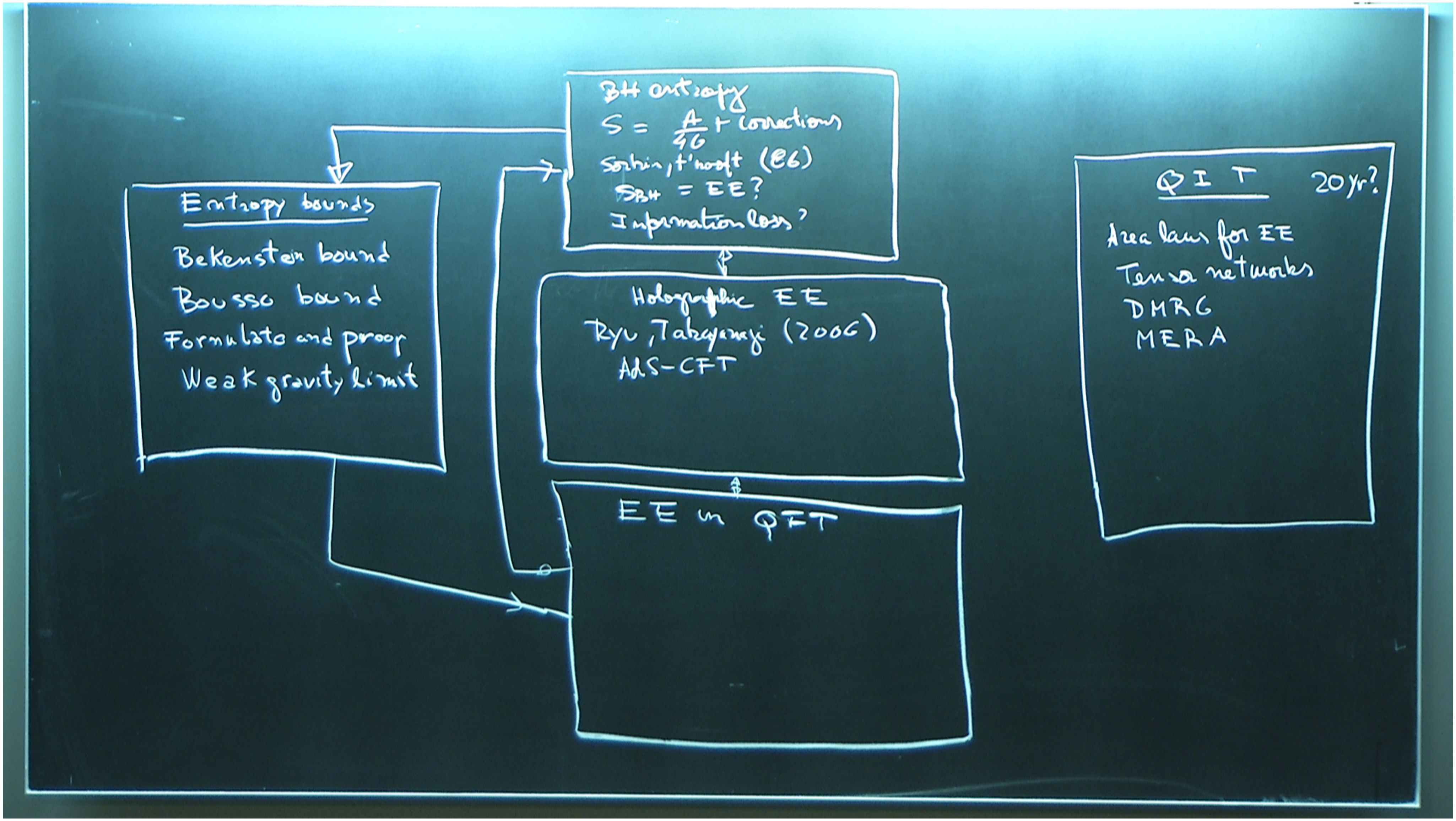
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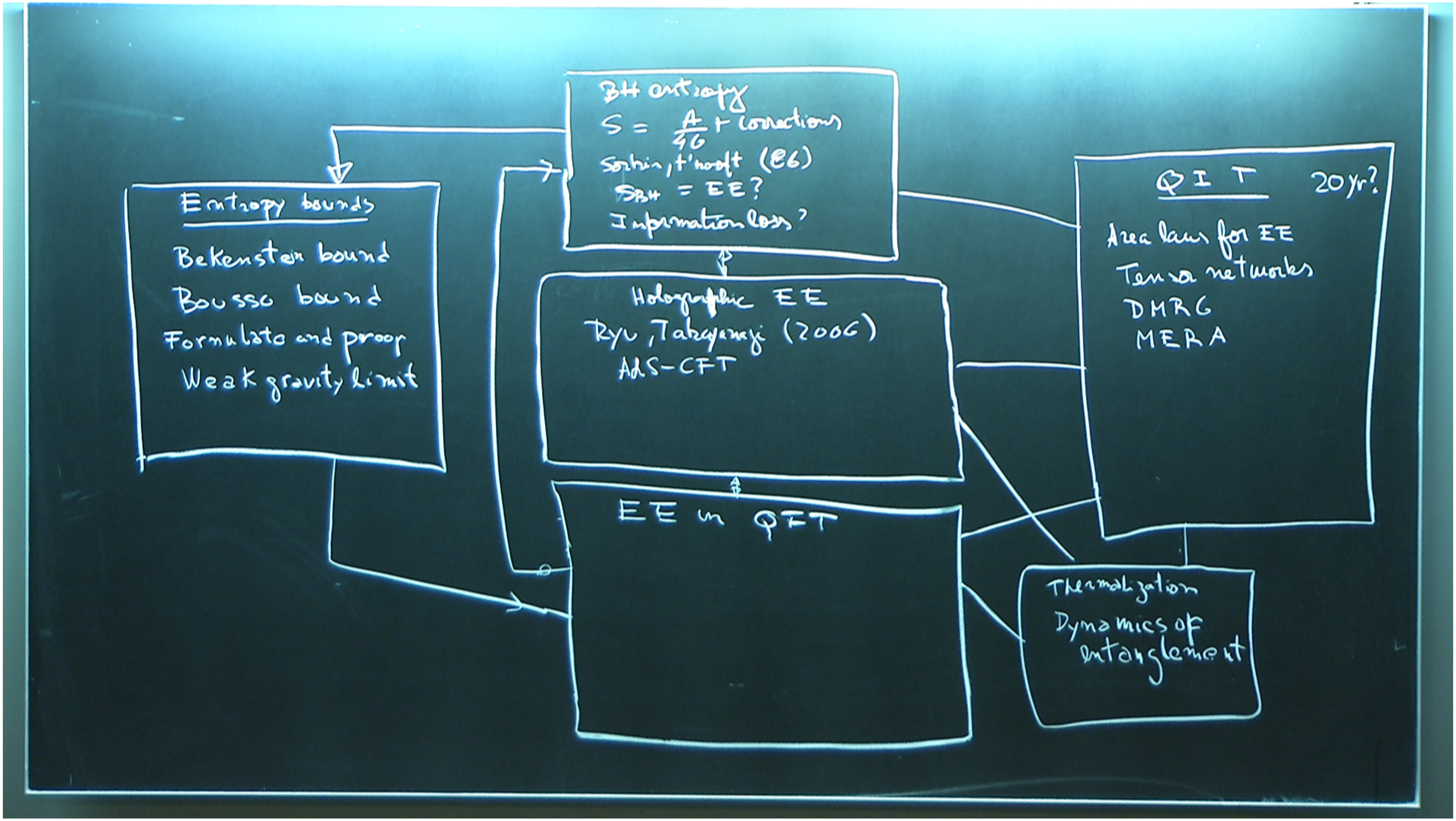
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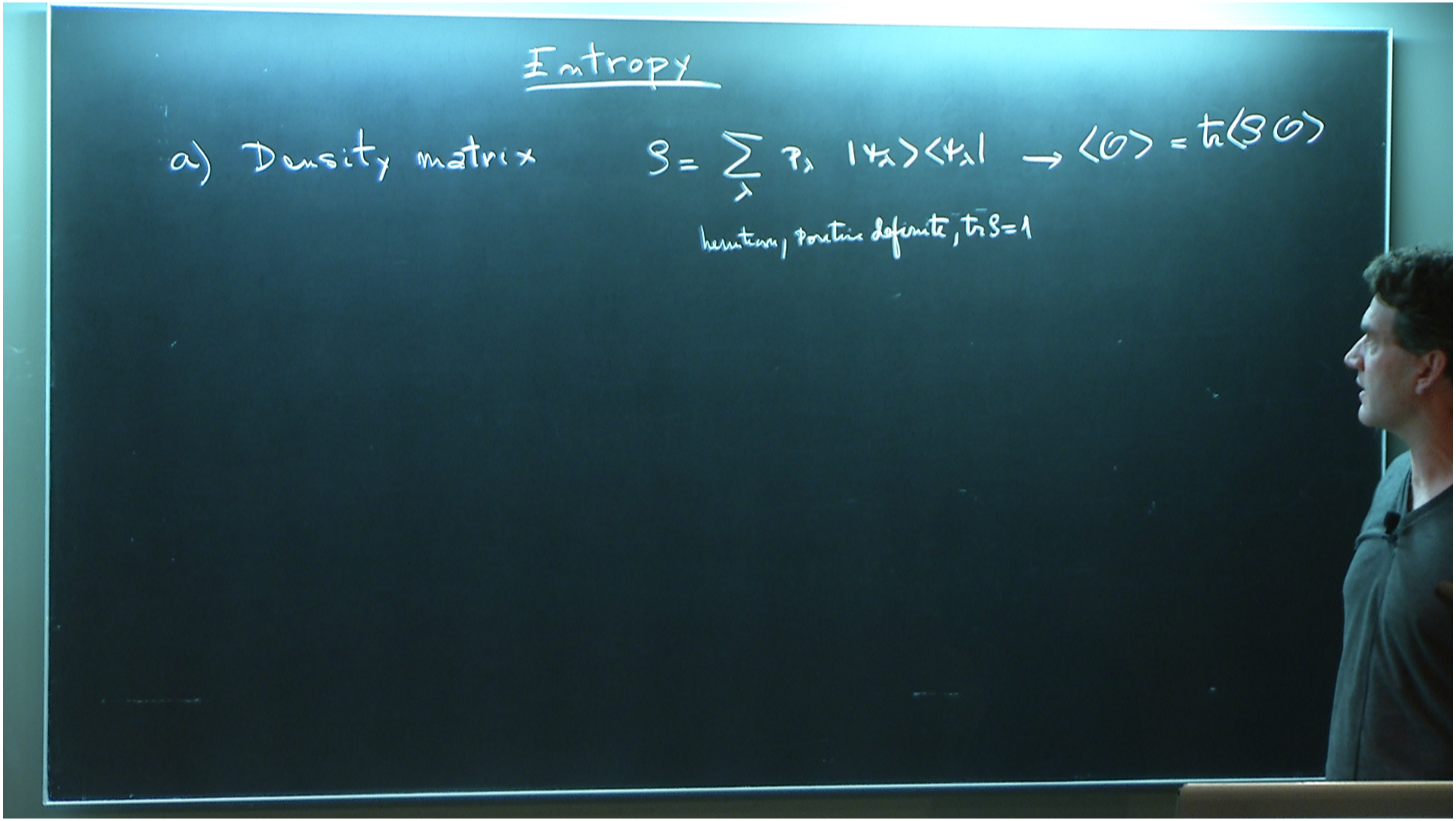
Thermalization
Dynamics of entanglement


Entropy
a) Density matrix
$\rho = \sum_\lambda p_\lambda |\psi_\lambda\rangle\langle\psi_\lambda| \rightarrow \langle O\rangle = \mathrm{tr}(\rho O)$
hermitian, positive definite, $\mathrm{tr}\,\rho = 1$

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$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \qquad |\psi\rangle = \sum \psi_{ij} \, |i,j\rangle \qquad \sum |\psi_{ij}|^2 = 1$

$$\rho = \sum_{\substack{ij \\ kl}} \psi_{ij} \psi^*_{kl} \, |ij\rangle\langle kl| \;\to\; \rho_A = \mathrm{tr}_B\, \rho = \sum_{ik} \Big(\sum_j \psi_{ij} \psi^*_{kj} \Big) |i\rangle\langle k|$$

$$\mathrm{tr}\big(\rho \, (O_A \otimes \mathbb{1}_B)\big) = \mathrm{tr}(\rho_A O_A)$$

$\mathrm{tr}\, \rho_A^2 < 1$

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Purification

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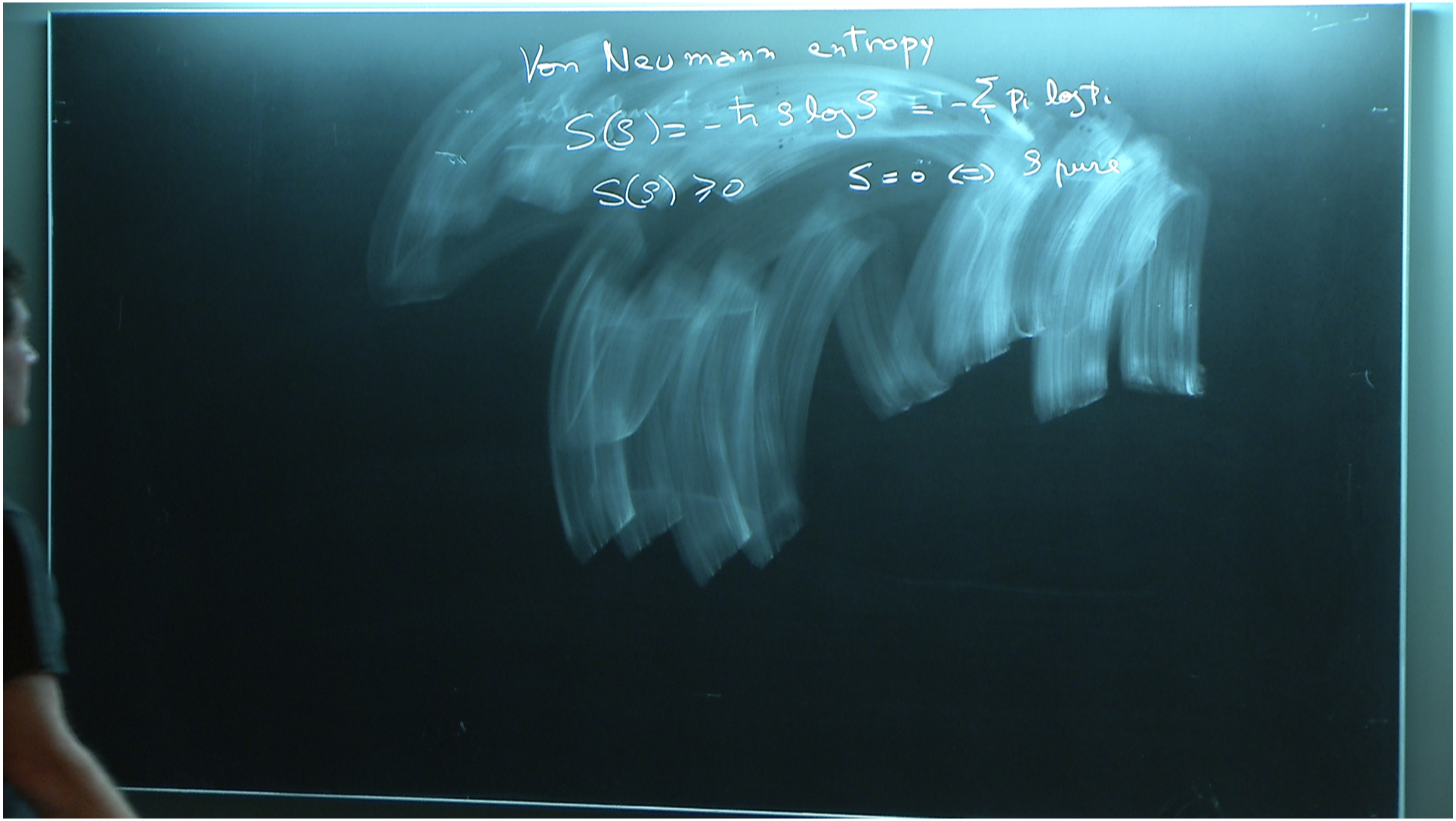
Purification


Von Neumann entropy
$S(\rho) = -\mathrm{tr}\,\rho\log\rho = -\sum_i p_i \log p_i$
$S(\rho) \geq 0$
$S = 0 \Leftrightarrow \rho$ pure

Von Neumann entropy

$$S(\rho) = -\mathrm{tr}\, \rho \log \rho = -\sum_i p_i \log p_i$$

$$S(\rho) \geq 0 \qquad S = 0 \iff \rho \text{ pure}$$

Canonical

$$p_{\varepsilon_i} = \frac{e^{-\beta \varepsilon_i}}{\sum e^{-\beta \varepsilon_i}} = \frac{e^{-\beta \varepsilon_i}}{Z}$$

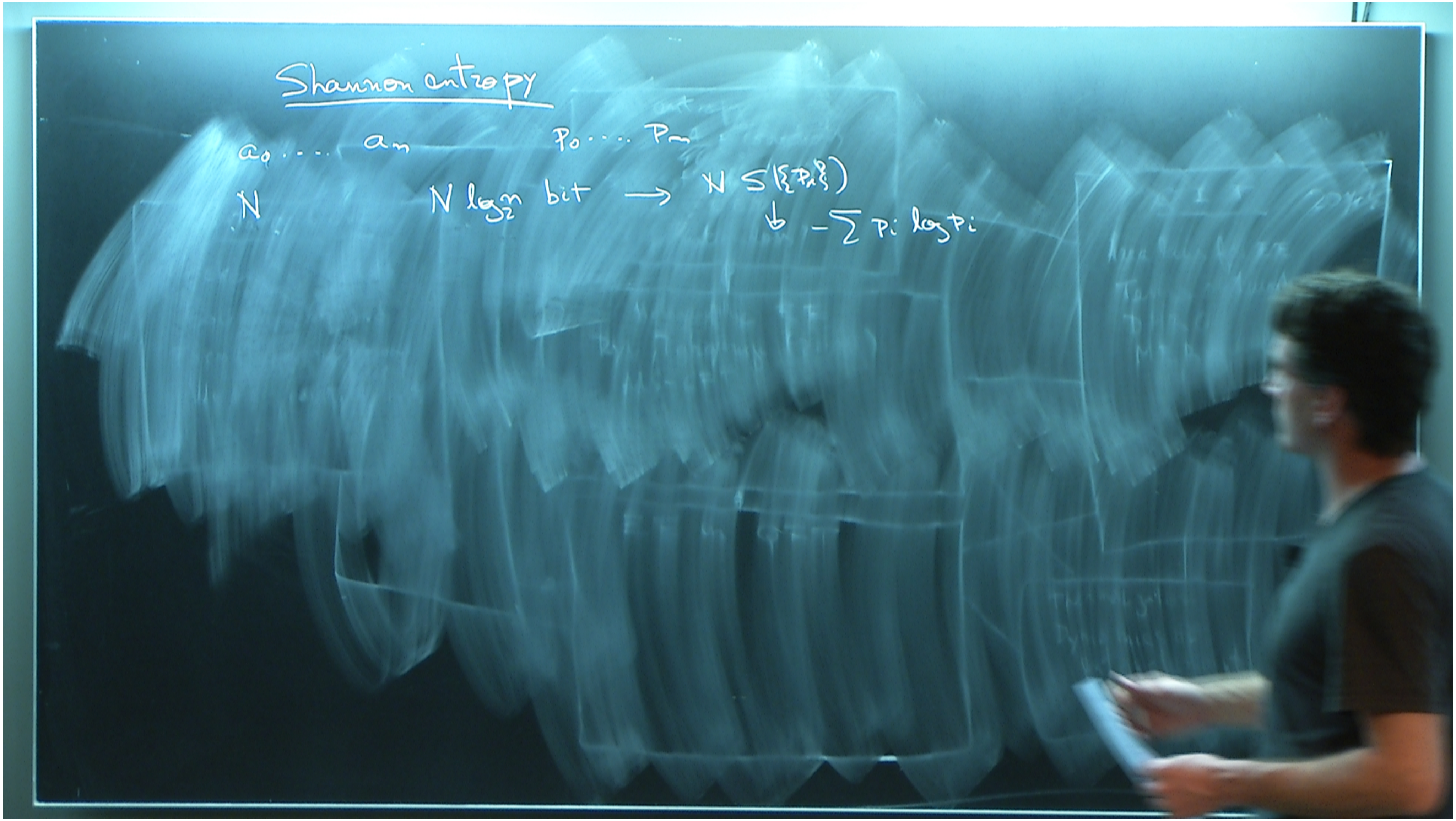
$$\rho = \frac{e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}} \to S(\rho) = \beta \langle H \rangle + \log Z$$

$$F = E - TS = -T \log Z$$

Microcanonical

$$p_i = \frac{1}{N} \qquad \rho = \frac{\mathbb{1}}{N}$$

$$S = -\mathrm{tr}\, \frac{1}{N} \log\left(\frac{\mathbb{1}}{N}\right) = \log N$$


Shannon entropy
a0 .... an
p0 .... pn
N
N log2 n bit → N S({pi})
- ∑ pi log pi

Shannon entropy

$a_0 \dots a_n$ $\quad p_0 \dots p_m$

$N \quad N \log_2 n$ bit $\longrightarrow N\, S_2(\{p_i\})$

$\downarrow\; -\sum p_i \log_2 p_i$

operational meaning

Entanglement entropy

Entanglement

Shannon entropy

$a_0 \ldots a_n \qquad p_0 \ldots p_n$

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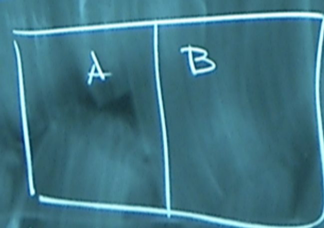

$\rho_{AB} = \sum_\lambda p_\lambda\, \rho_A^\lambda \otimes \rho_B^\lambda \rightarrow \rho_{AB}$ not entangled

Shannon entropy

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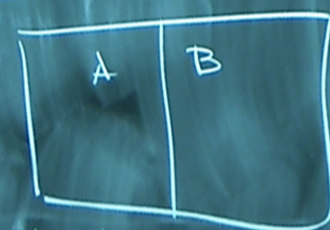
Entanglement entropy

Entanglement



$\rho_{AB} = \sum_\lambda p_\lambda\, \rho_A^\lambda \otimes \rho_B^\lambda \to \rho_{AB}$ not entangled

if ρ_{AB} is pure, not entangled $\to \rho_{AB} = \rho_A \otimes \rho_B$

Shannon entropy

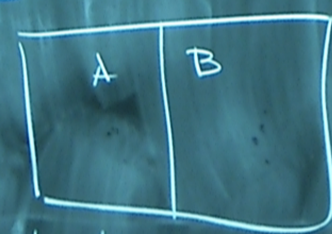
$a_0 \ldots a_n \qquad p_0 \ldots p_n$

$N \qquad N \log_2 n$ bit $\longrightarrow N\, S_2(\{p_i\})$

$\downarrow \; -\sum p_i \log_2 p_i$

operational meaning

Entanglement entropy



Entanglement

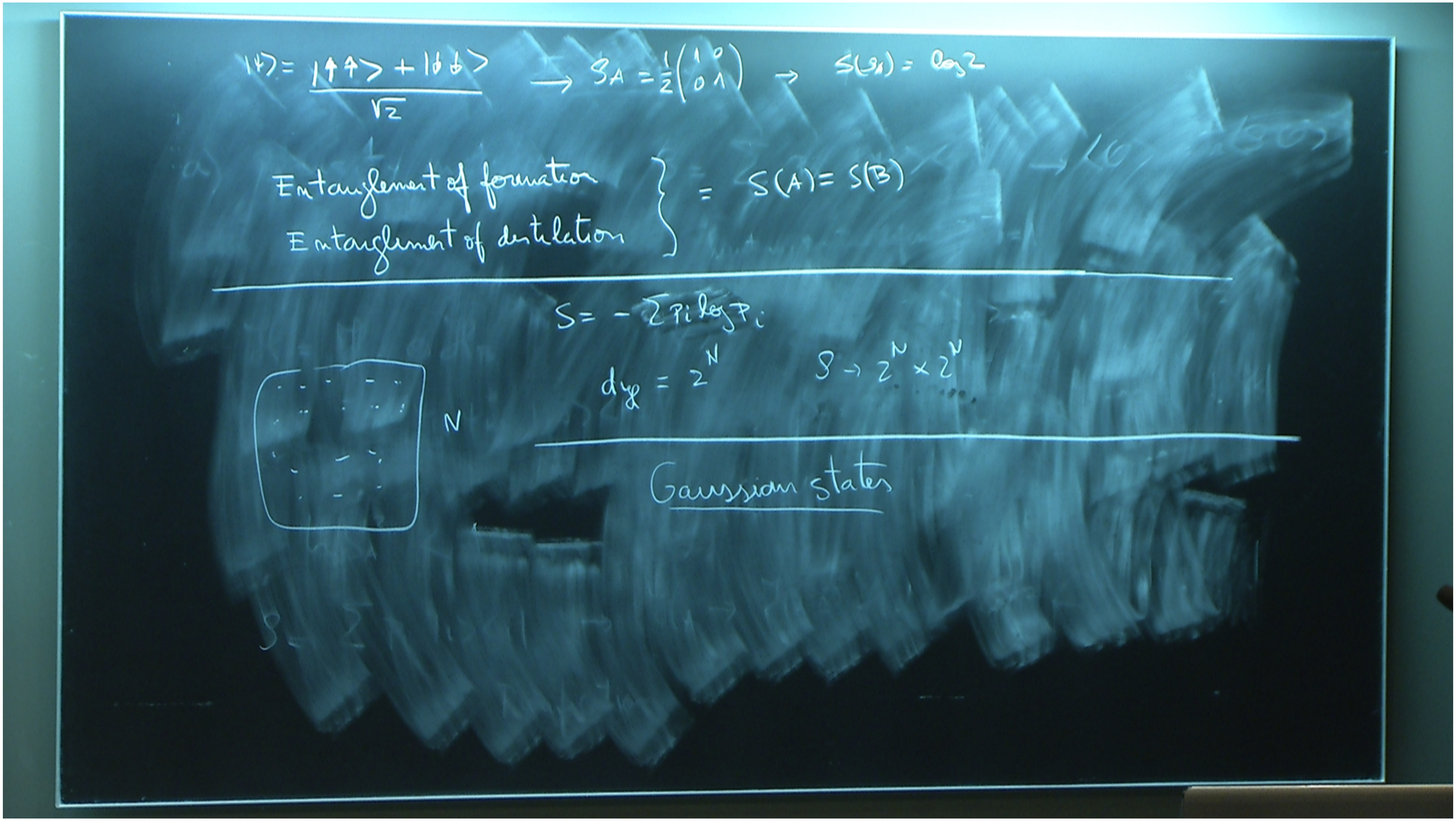
$\rho_{AB} = \sum_\lambda p_\lambda\, \rho_A^\lambda \otimes \rho_B^\lambda \to \rho_{AB}$ not entangled

if ρ_{AB} is pure, not entangled $\longrightarrow \rho_{AB} = \rho_A \otimes \rho_B \to \rho_A \to S(\rho_A) = 0$

$S(\rho_A)$ entanglement entropy $\qquad S(\rho_A) = S(\rho_B)$ (ρ_{AB} pure)

$$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \longrightarrow \rho_A = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \rightarrow S(\rho_A) = \log 2$$

$$\left.\begin{array}{l}\text{Entanglement of formation}\\ \text{Entanglement of distillation}\end{array}\right\} = S(A) = S(B)$$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Entanglement of formation
Entanglement of distillation
$= S(A) = S(B)$
$S = -\sum p_i \log p_i$
$d_{\mathcal{H}} = 2^N$
$\rho \to 2^N \times 2^N$
N
Gaussian states

$$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \longrightarrow \rho_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow S(\rho_A) = \log 2$$

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$$S = -\sum p_i \log p_i$$

N

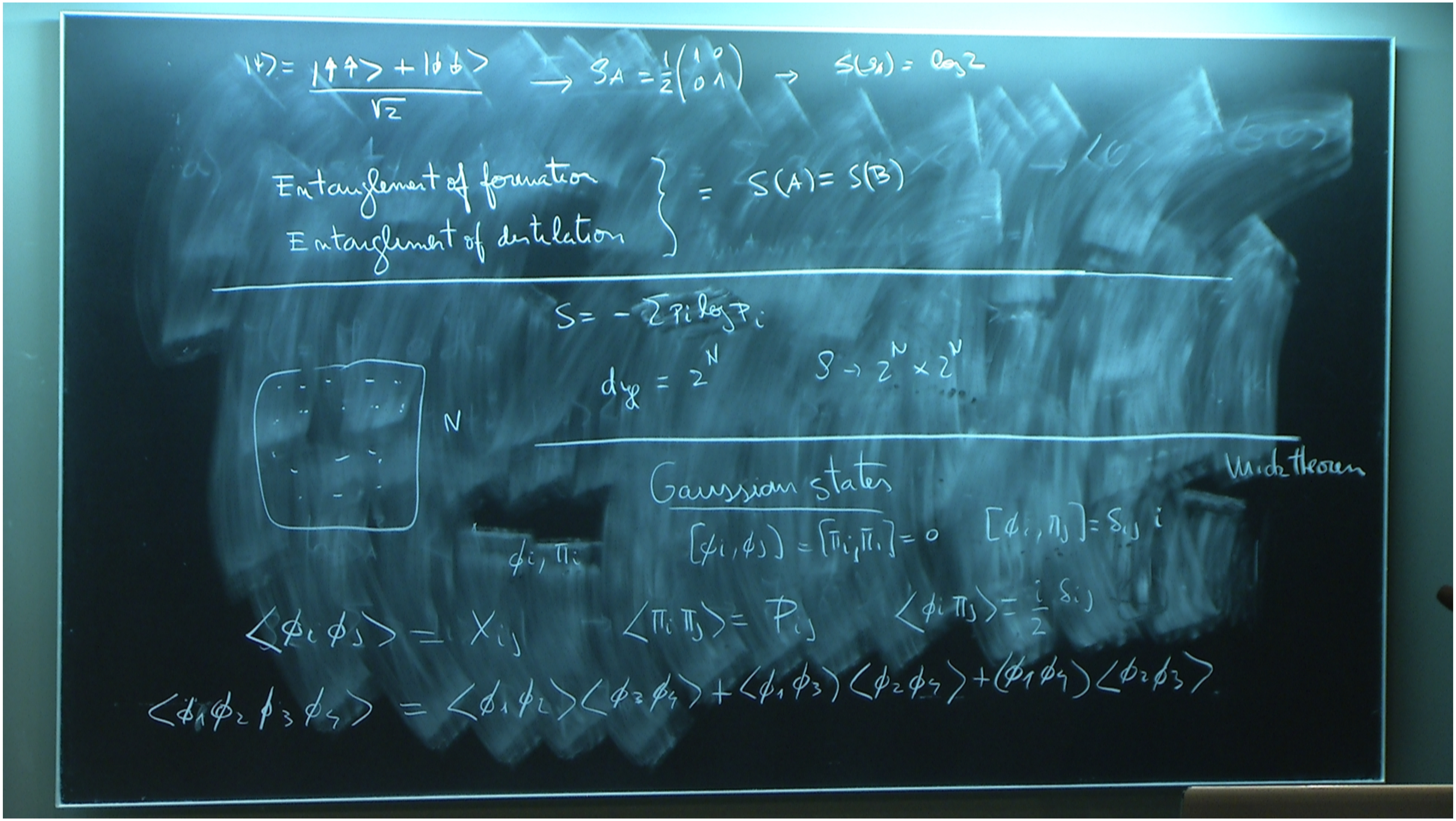
$$d_\psi = 2^N \qquad \rho \to 2^N \times 2^N$$

Gaussian states

ϕ_i, π_i

$$[\phi_i, \phi_j] = [\pi_i, \pi_j] = 0 \qquad [\phi_i, \pi_j] = \delta_{ij}\, i$$

$$\langle \phi_i \phi_j \rangle = X_{ij} \qquad \langle \pi_i \pi_j \rangle = P_{ij} \qquad \langle \phi_i \pi_j \rangle = \frac{i}{2}\delta_{ij}$$


$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \longrightarrow \rho_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow S(\rho_A) = \log 2$
Entanglement of formation
Entanglement of distilation
$= S(A) = S(B)$
$S = -\sum p_i \log p_i$
$d_\psi = 2^N$
$\rho \rightarrow 2^N \times 2^N$
N
Gaussian states
Wick theorem
ϕ_i, π_i
$[\phi_i, \phi_j] = [\pi_i, \pi_j] = 0$
$[\phi_i, \pi_j] = \delta_{ij}\, i$
$\langle \phi_i \phi_j \rangle = X_{ij}$
$\langle \pi_i \pi_j \rangle = P_{ij}$
$\langle \phi_i \pi_j \rangle = \frac{i}{2}\delta_{ij}$
$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \langle \phi_1 \phi_2 \rangle \langle \phi_3 \phi_4 \rangle + \langle \phi_1 \phi_3 \rangle \langle \phi_2 \phi_4 \rangle + \langle \phi_1 \phi_4 \rangle \langle \phi_2 \phi_3 \rangle$

$$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \longrightarrow \rho_A = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \rightarrow S(\rho_A) = \log 2$$

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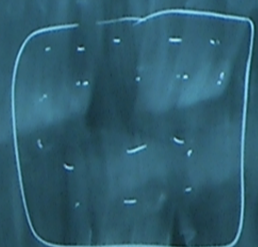
$$S = -\sum p_i \log p_i$$



N

$$d_\psi = 2^N \qquad \rho \to 2^N \times 2^N$$

Gaussian states

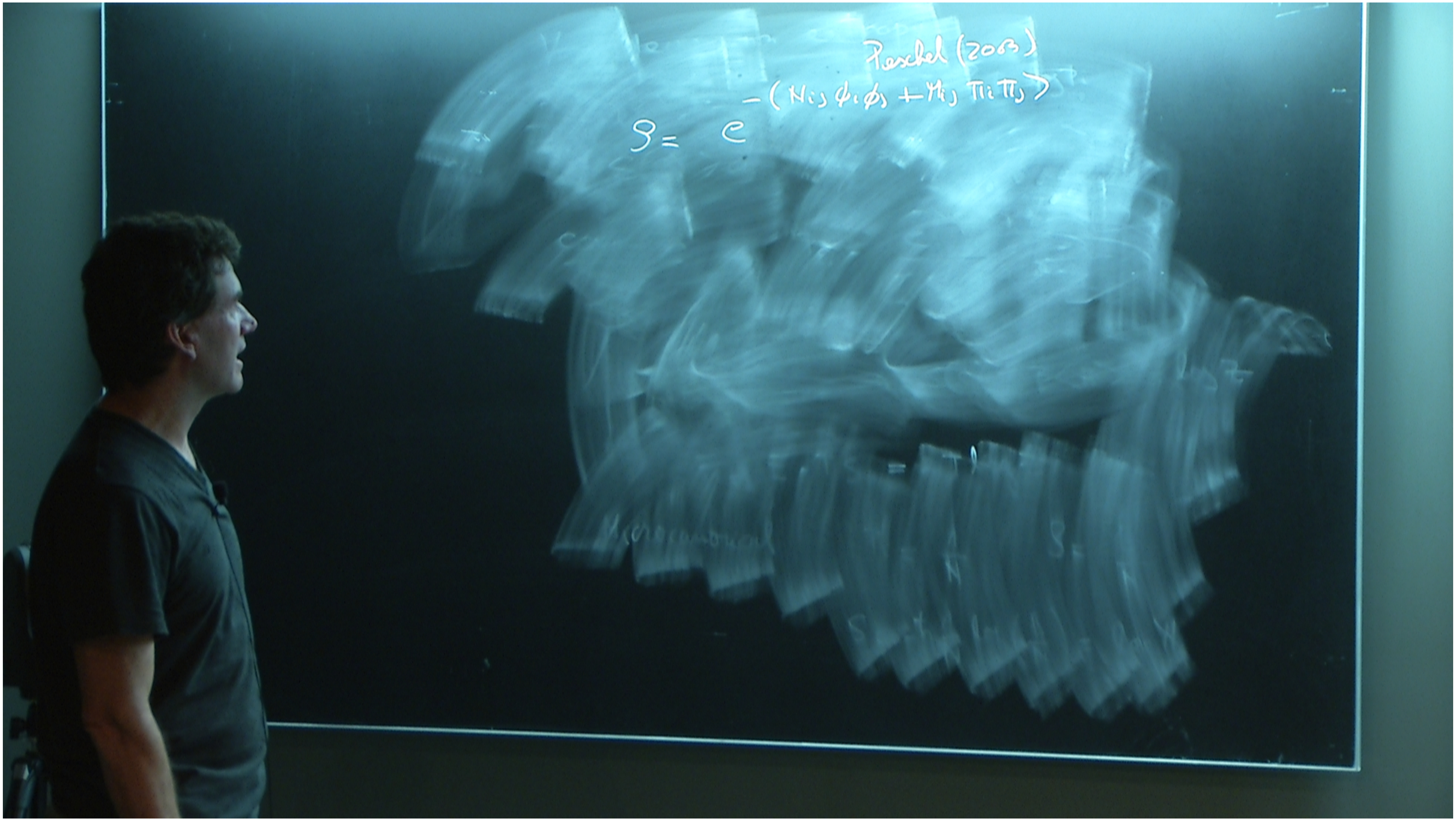
Wick theorem

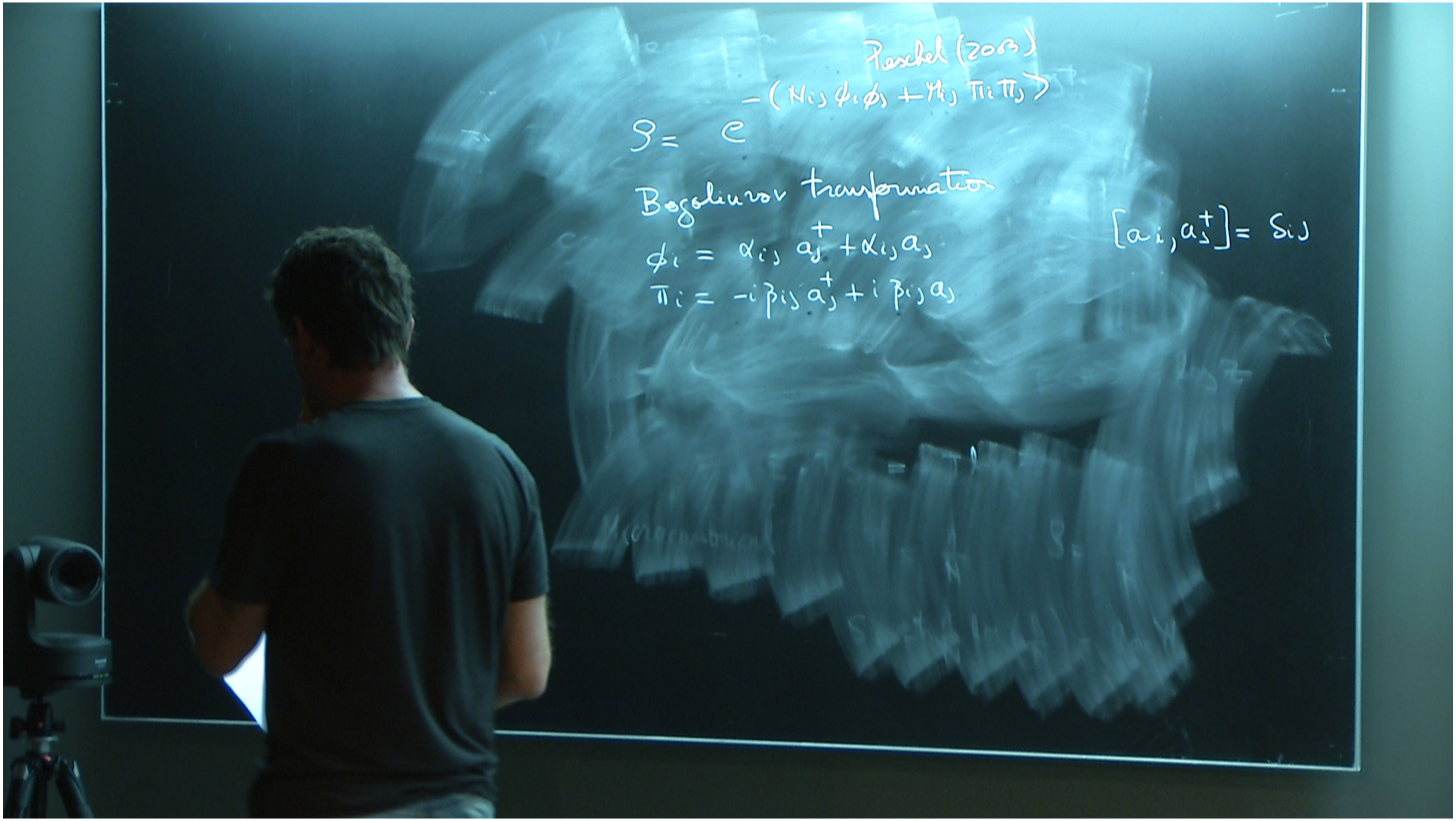
ϕ_i, π_i

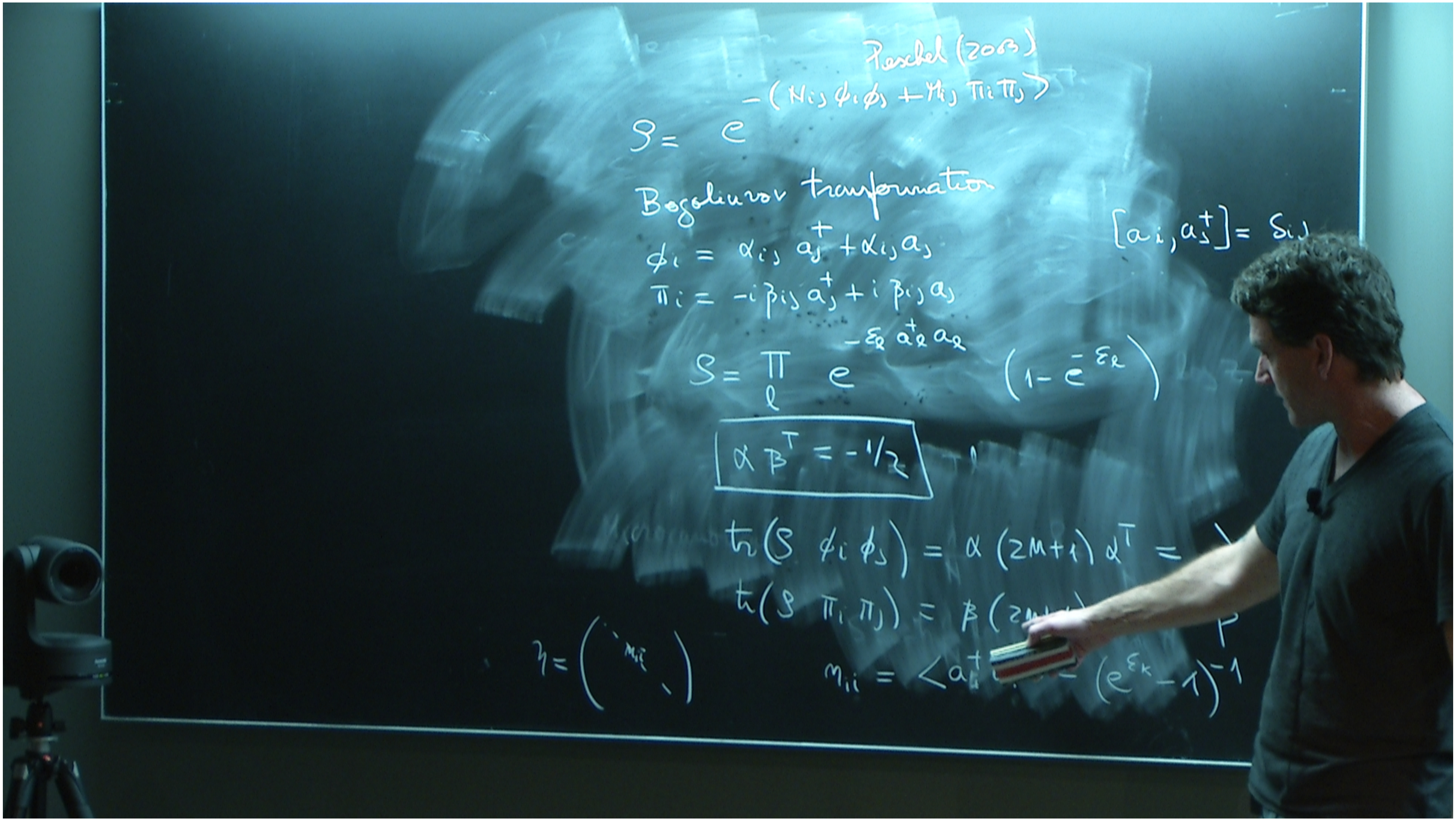
$$[\phi_i, \phi_j] = [\pi_i, \pi_j] = 0 \qquad [\phi_i, \pi_j] = \delta_{ij}\, i$$

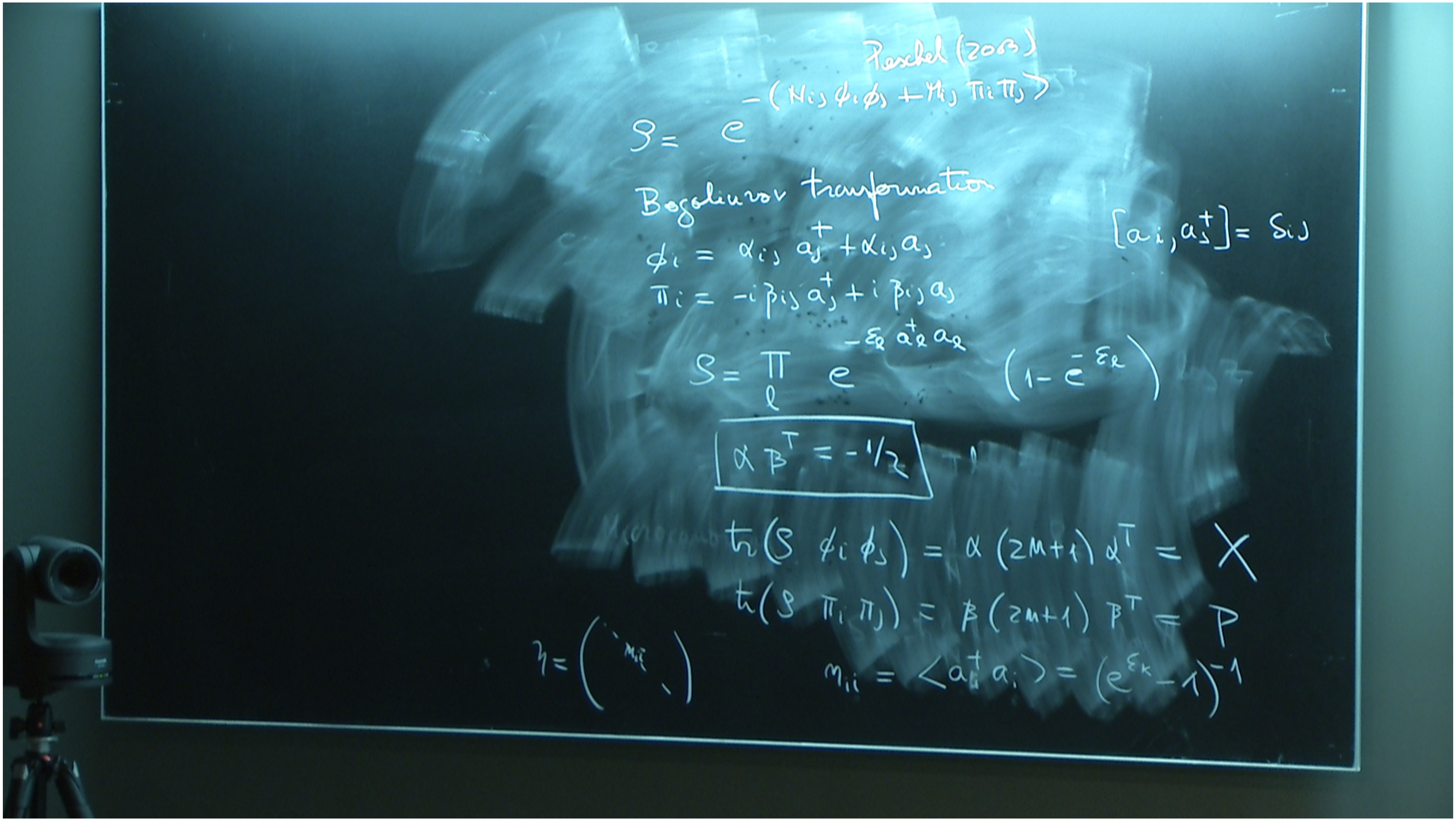
$$\langle \phi_i \phi_j \rangle = X_{ij} \qquad \langle \pi_i \pi_j \rangle = P_{ij} \qquad \langle \phi_i \pi_j \rangle = \frac{i}{2}\delta_{ij}$$

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \langle \phi_1 \phi_2 \rangle \langle \phi_3 \phi_4 \rangle + \langle \phi_1 \phi_3 \rangle \langle \phi_2 \phi_4 \rangle + \langle \phi_1 \phi_4 \rangle \langle \phi_2 \phi_3 \rangle$$

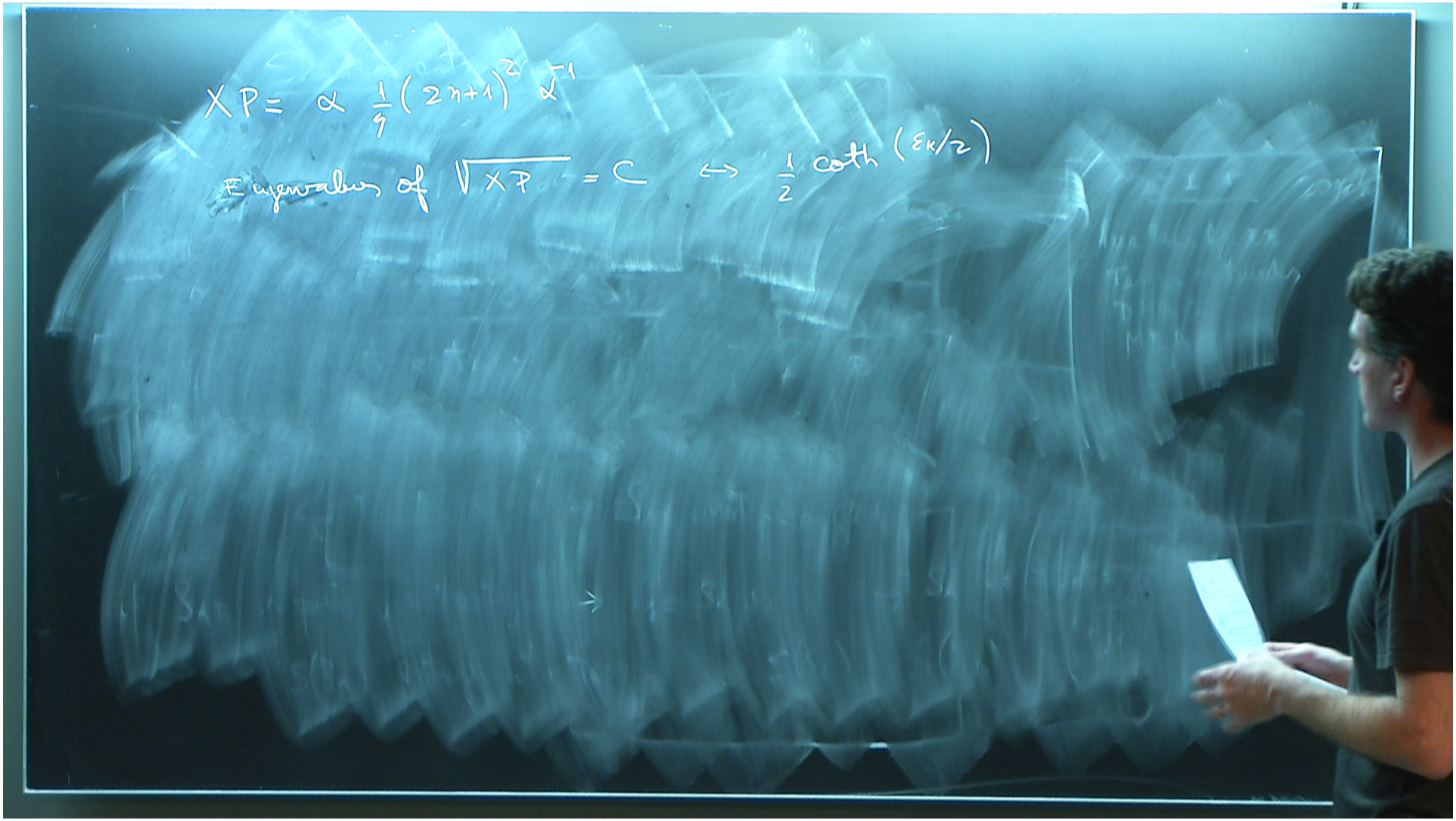

Peschel (2003)
$\rho = e^{-(M_{ij}\phi_i\phi_j + N_{ij}\pi_i\pi_j)}$


Peschel (2003)
$\rho = e^{-(N_{ij}\phi_i\phi_j + M_{ij}\pi_i\pi_j)}$
Bogoliubov transformation
$\phi_i = \alpha_{ij} a_j^{+} + \alpha_{ij} a_j$
$\pi_i = -i\beta_{ij} a_j^{+} + i\beta_{ij} a_j$
$[a_i, a_j^{+}] = \delta_{ij}$


Peschel (2003)
ρ = e^{-(N_ij φ_i φ_j + M_ij π_i π_j)}
Bogoliubov transformation
[a_i, a_j^+] = δ_ij
φ_i = α_ij a_j^+ + α_ij a_j
π_i = -i β_ij a_j^+ + i β_ij a_j
ρ = Π_l e^{-ε_l a_l^+ a_l} (1 - e^{-ε_l})
α β^T = -1/2
tr(ρ φ_i φ_j) = α (2M+1) α^T =
tr(ρ π_i π_j) = β (2M
n_ii = <a_i^+
(e^{ε_k} - 1)^{-1}



Peschel (2003)
$\rho = e^{-(N_{ij}\phi_i\phi_j + M_{ij}\pi_i\pi_j)}$
Bogoliubov transformation
$[a_i, a_j^+] = \delta_{ij}$
$\phi_i = \alpha_{ij} a_j^+ + \alpha_{ij} a_j$
$\pi_i = -i\beta_{ij} a_j^+ + i\beta_{ij} a_j$
$\rho = \prod_l e^{-\varepsilon_l a_l^+ a_l} (1-e^{-\varepsilon_l})$
$\boxed{\alpha\beta^T = -1/2}$
$tr(\rho\,\phi_i\phi_j) = \alpha(2M+1)\alpha^T = X$
$tr(\rho\,\pi_i\pi_j) = \beta(2M+1)\beta^T = P$
$\eta = \begin{pmatrix} \ddots & & \\ & n_{ii} & \\ & & \ddots \end{pmatrix}$
$n_{ii} = \langle a_i^+ a_i \rangle = (e^{\varepsilon_k} - 1)^{-1}$


$XP = \alpha \frac{1}{4}(2n+1)^2 \alpha^{-1}$
Eigenvalues of $\sqrt{XP} = C \leftrightarrow \frac{1}{2}\coth(\varepsilon_K/2)$

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$$S = \sum_{\ell} \left(-\log\left(1 - e^{-\varepsilon_\ell}\right) + \frac{\varepsilon_\ell \, e^{-\varepsilon_\ell}}{1 - e^{-\varepsilon_\ell}} \right) = \mathrm{tr}\left((C + 1/2)\log(C+1/2) - (C - 1/2)\log(C - 1/2) \right)$$

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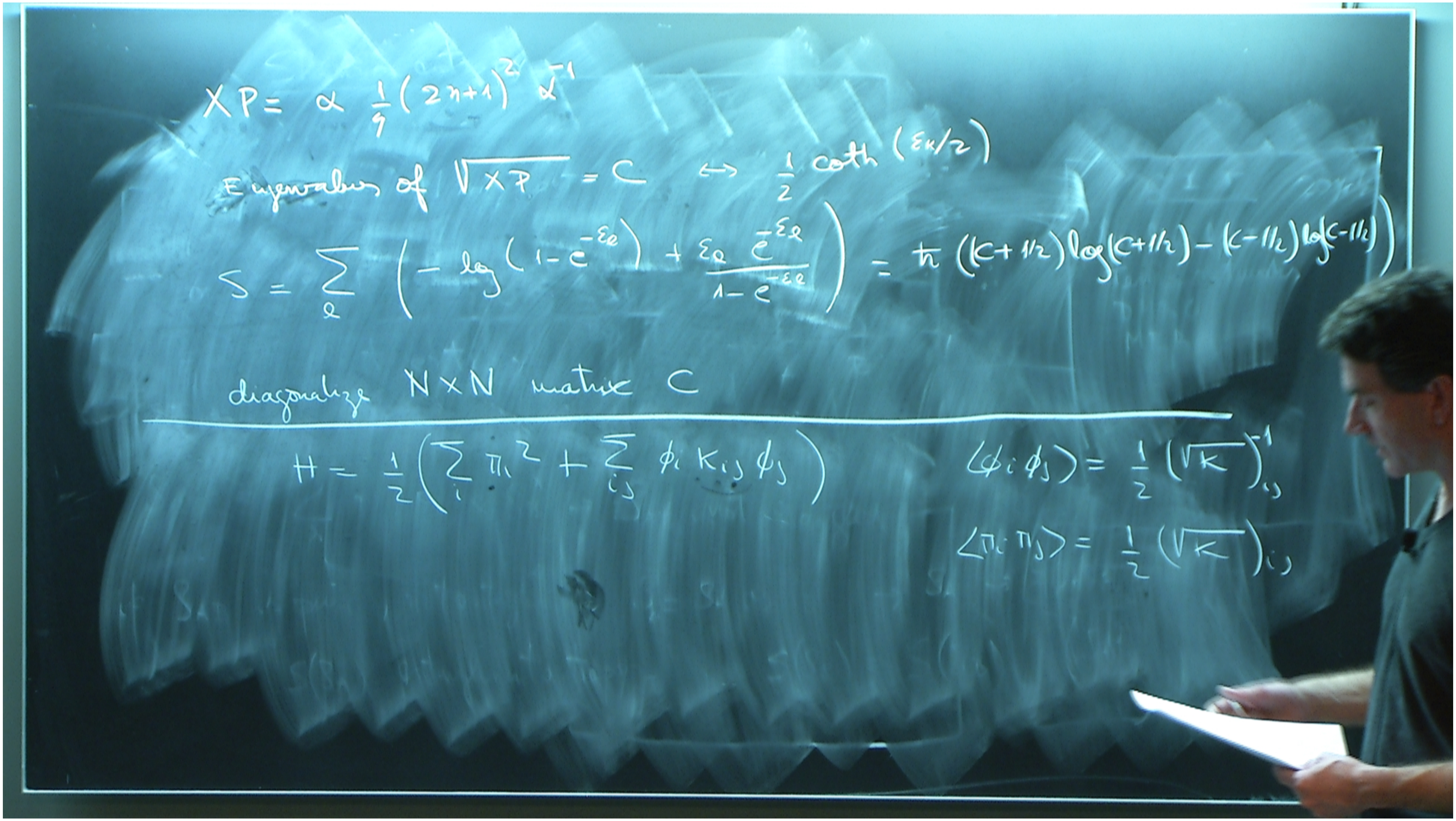
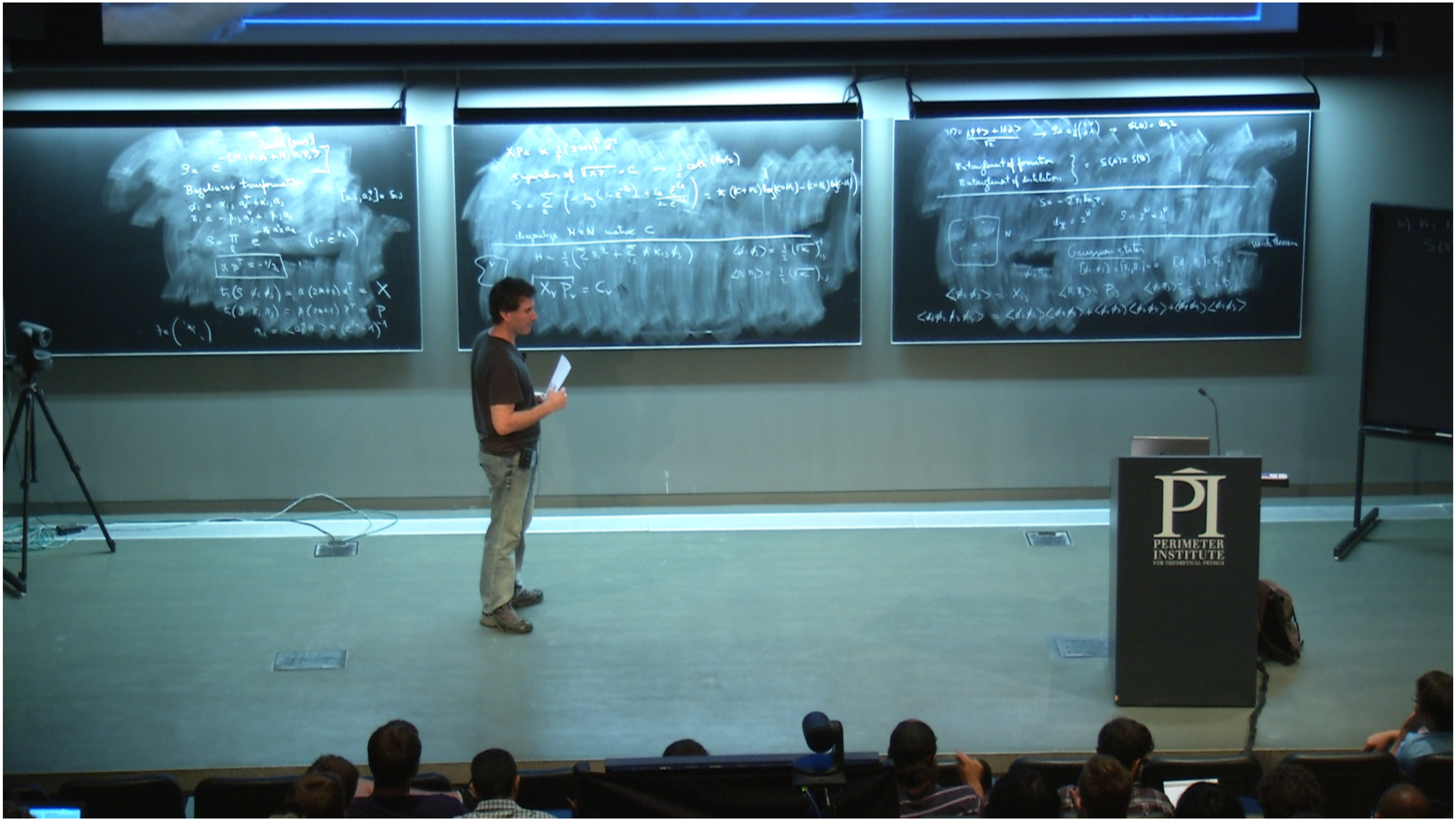
diagonalize $N \times N$ matrix C


$XP = \alpha \frac{1}{4}(2n+1)^2 \alpha^{-1}$
Eigenvalues of $\sqrt{XP} = C \leftrightarrow \frac{1}{2}\coth(\varepsilon_k/2)$
$S = \sum_l \left(-\log(1-e^{-\varepsilon_l}) + \frac{\varepsilon_l e^{-\varepsilon_l}}{1-e^{-\varepsilon_l}} \right) = \mathrm{tr}\left((C+1/2)\log(C+1/2) - (C-1/2)\log(C-1/2)\right)$
diagonalize $N \times N$ matrix C
$H = \frac{1}{2}\left(\sum_i \pi_i^2 + \sum_{ij} \phi_i K_{ij} \phi_j\right)$
$\langle \phi_i \phi_j \rangle = \frac{1}{2}(\sqrt{K})^{-1}_{ij}$
$\langle \pi_i \pi_j \rangle = \frac{1}{2}(\sqrt{K})_{ij}$


PERIMETER
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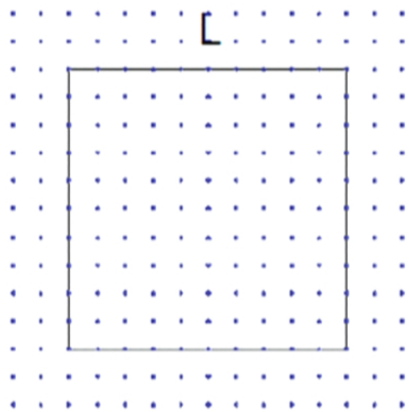
Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
Similar to phonons in a solid

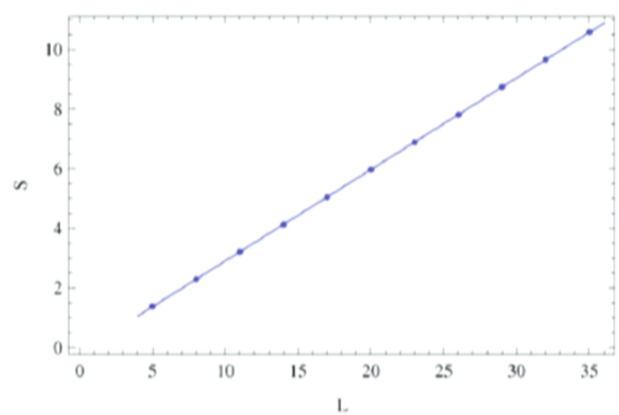


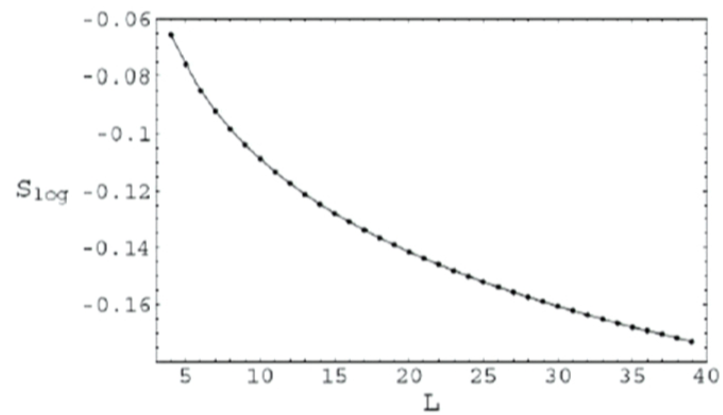
$$H = \frac{1}{2}\int d^2x \left(\dot{\phi}(x)^2 + (\nabla\phi(x))^2\right)$$

$$\to H = \frac{1}{2}\sum_i \epsilon^2 \left(\dot{\phi}_i^{\,2} + \sum_{j\sim i}\frac{(\phi_i - \phi_j)^2}{\epsilon^2}\right)$$

For interacting spin systems the Hilbert space dimension grows as 2^N

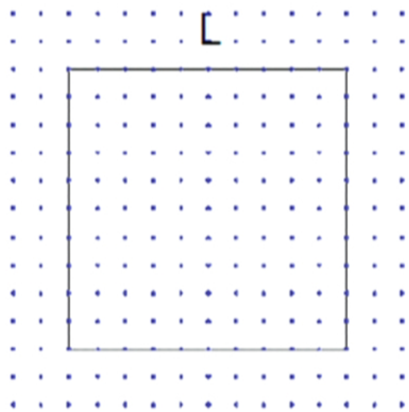
For coupled Harmonic oscillators we have only to diagonalize matrices of $N \times N$





S=.075 (4 L/ε)-0.047 Log[L/ε] +const=.075 (perimeter/ε)-0.047 Log[L/ ε]+const

Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
Similar to phonons in a solid



$$H = \frac{1}{2}\int d^2x \left(\dot{\phi}(x)^2 + (\nabla\phi(x))^2\right)$$

$$\rightarrow H = \frac{1}{2}\sum_i \epsilon^2 \left(\dot{\phi_i}^2 + \sum_{j\sim i}\frac{(\phi_i - \phi_j)^2}{\epsilon^2}\right)$$

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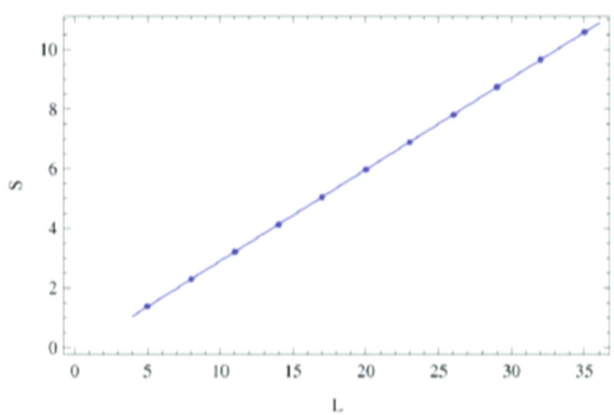

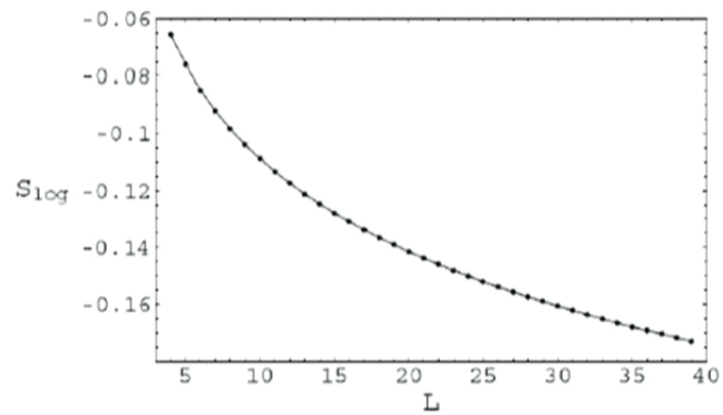

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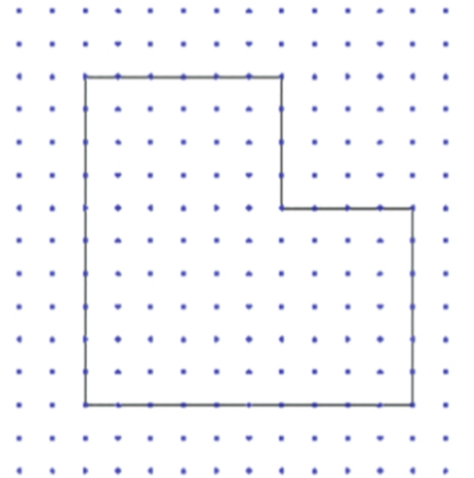
S=.075 (perimeter/ε)-(6/4) 0.047 Log[L/ε]+const



The same «area» term. A logarithmic coefficient growing with the number of vertices.
(All vertices have the same angle S(A)=S(-A) for a global pure state)

In general:

$$S(A) = c_1\,(\mathrm{perimeter}/\epsilon) - \sum_{\mathrm{vertices}} c_{\log}(\theta)\log(R/\epsilon) + \mathrm{const}$$

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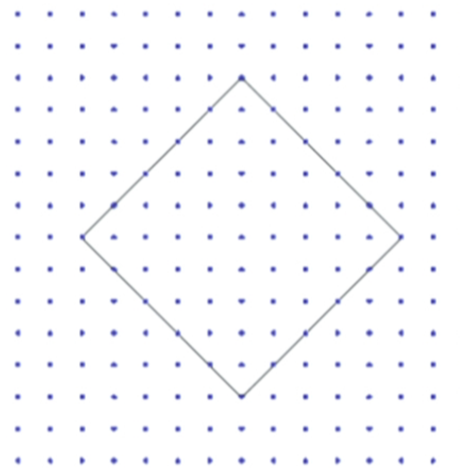

Bad: area term does not have the rotational symmetry of the theory in the continuum limit

Good: the logarithmic term does not notice the lattice

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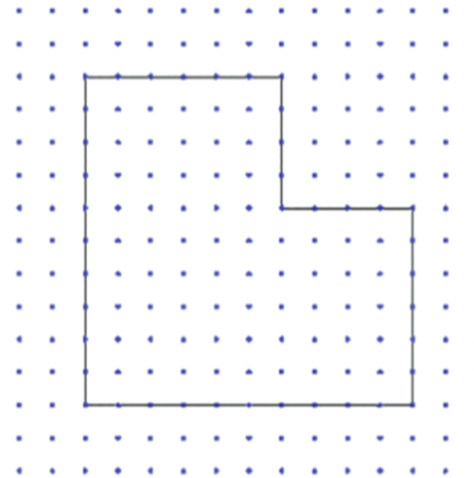

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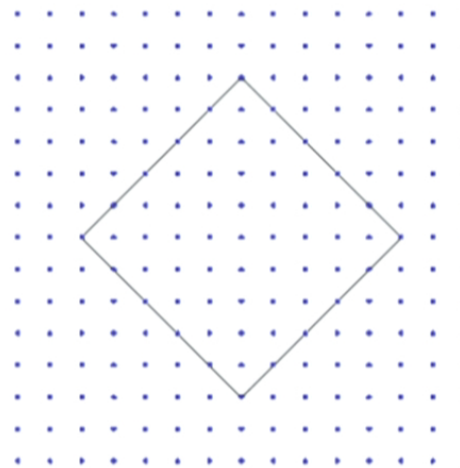

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