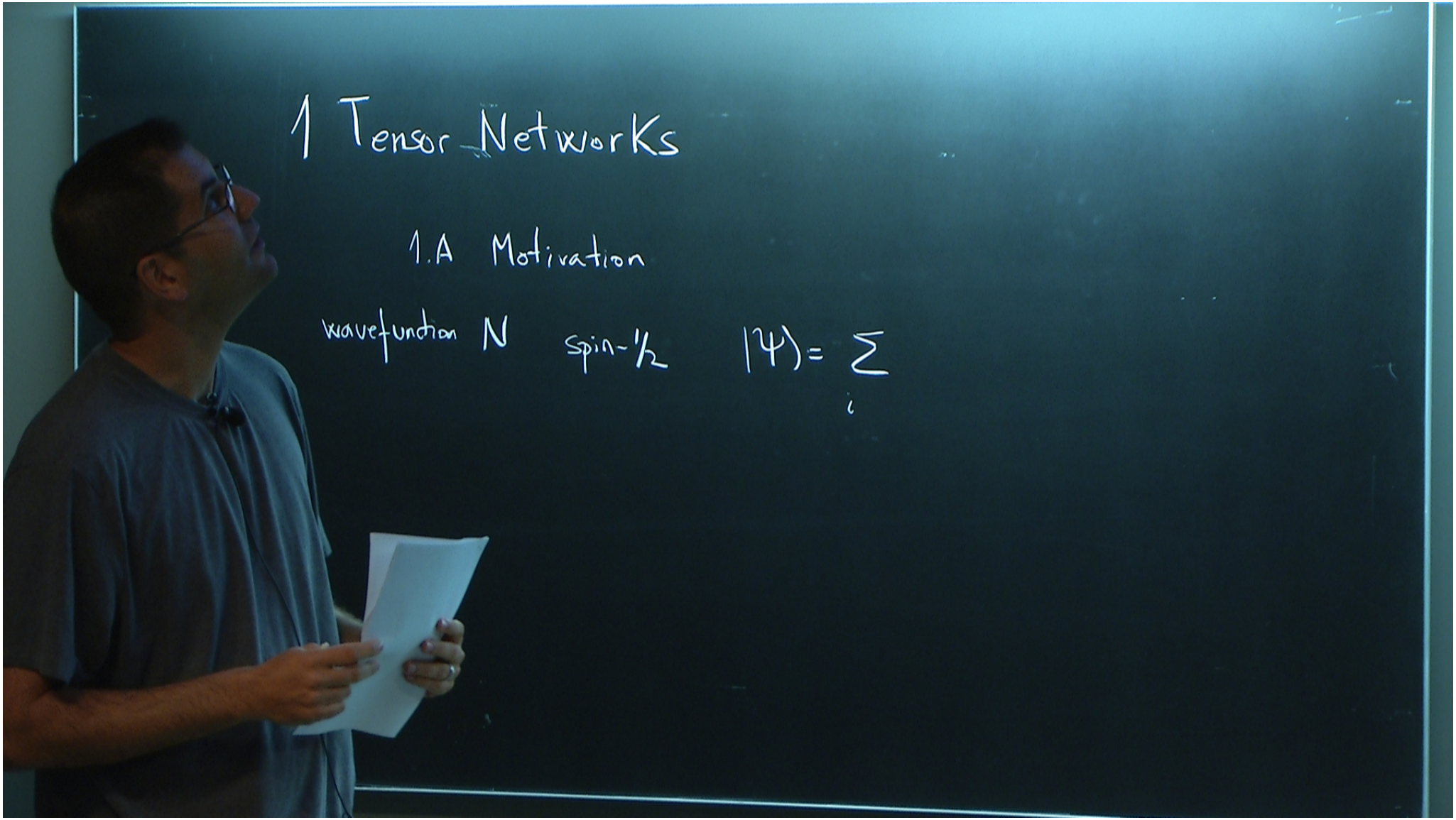
Title: Ground state entanglement and tensor networks

Date: Aug 25, 2015 09:15 AM

URL: http://pirsa.org/15080041

Abstract: TBA


1 Tensor Networks
1.A Motivation
wavefunction N spin-½ |ψ⟩ = Σ
i

1 Tensor Networks

1.A Motivation

wavefunction N spin-$\frac{1}{2}$ $|\psi\rangle = \sum_{i_1 i_2 \ldots i_N} \psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$

$i_1 i_2 \ldots i_N = \uparrow, \downarrow$

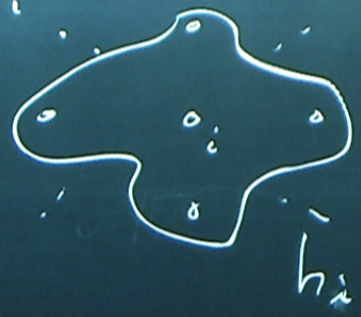
$(0, 1)$

$N = 3$

$|\psi\rangle = \psi_{000}|000\rangle + \psi_{001}|001\rangle + \psi_{010}|010\rangle + \ldots + \psi_{111}|111\rangle$

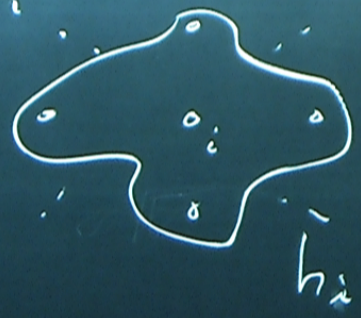
2^N complex coefficients

local Hamiltonian $H = \sum_i h_i$ $O(N)$ parameters



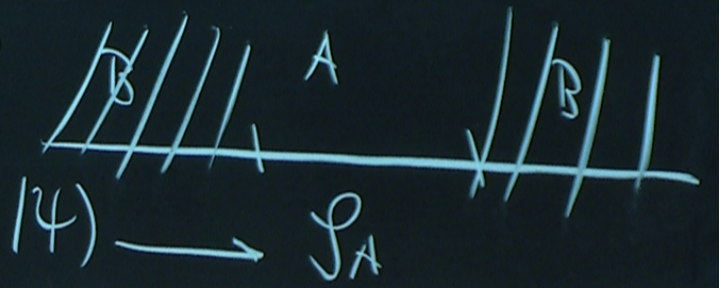
GOAL find efficient representation / computational scheme for ground states of local Hamiltonians

local Hamiltonia $H = \sum_i h_i$ $O(N)$ parameters

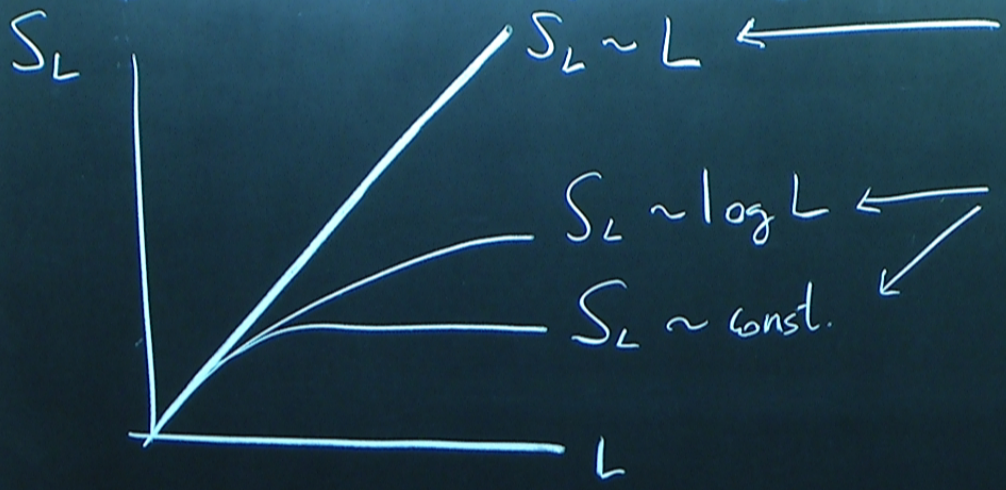


GOAL find efficient representation / computational scheme for ground states of local Hamiltonians

HINT Scaling of entanglement entropy (EE) 1D



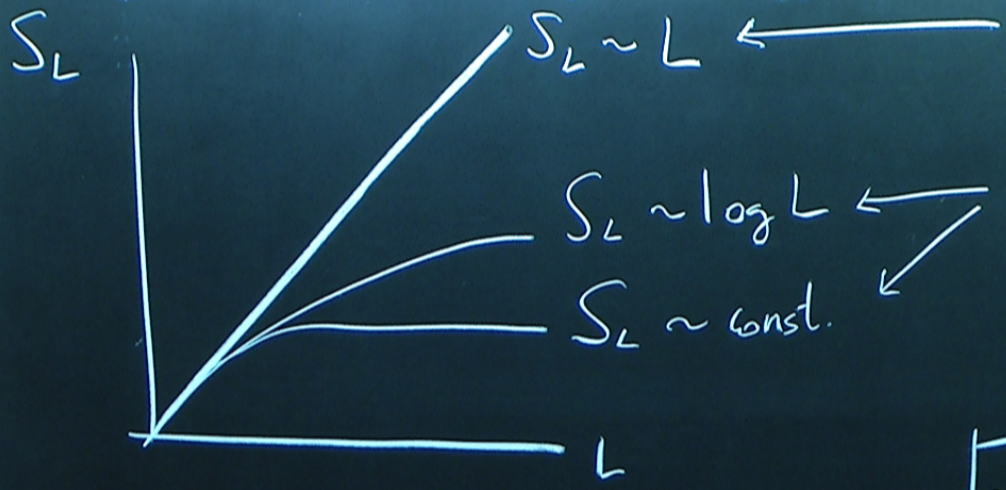
$$S_L = S(\rho_A) = -\mathrm{tr}(\rho_A \log \rho_A)$$



$S_L \sim L$ ← generic state

ground state of local H { - gapless H $\Delta E = 0$; - gapped H $\Delta E > 0$

$$S_L = S(\rho_A) = -\mathrm{tr}(\rho_A \log \rho_A)$$



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ground state of local H { - gapless H $\Delta E = 0$; - gapped H $\Delta E > 0$

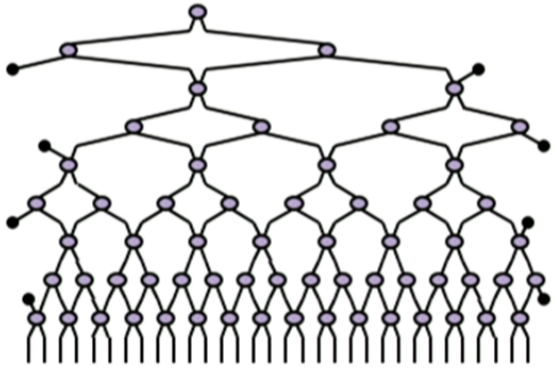
$S_L \sim \log L$ ←

$S_L \sim$ const. ←

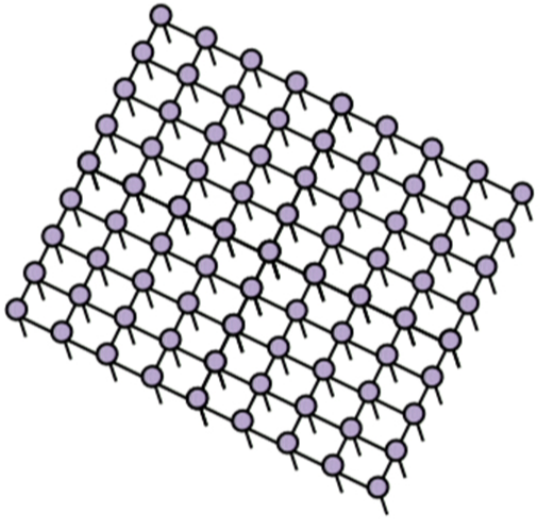
ground states are weakly entangled



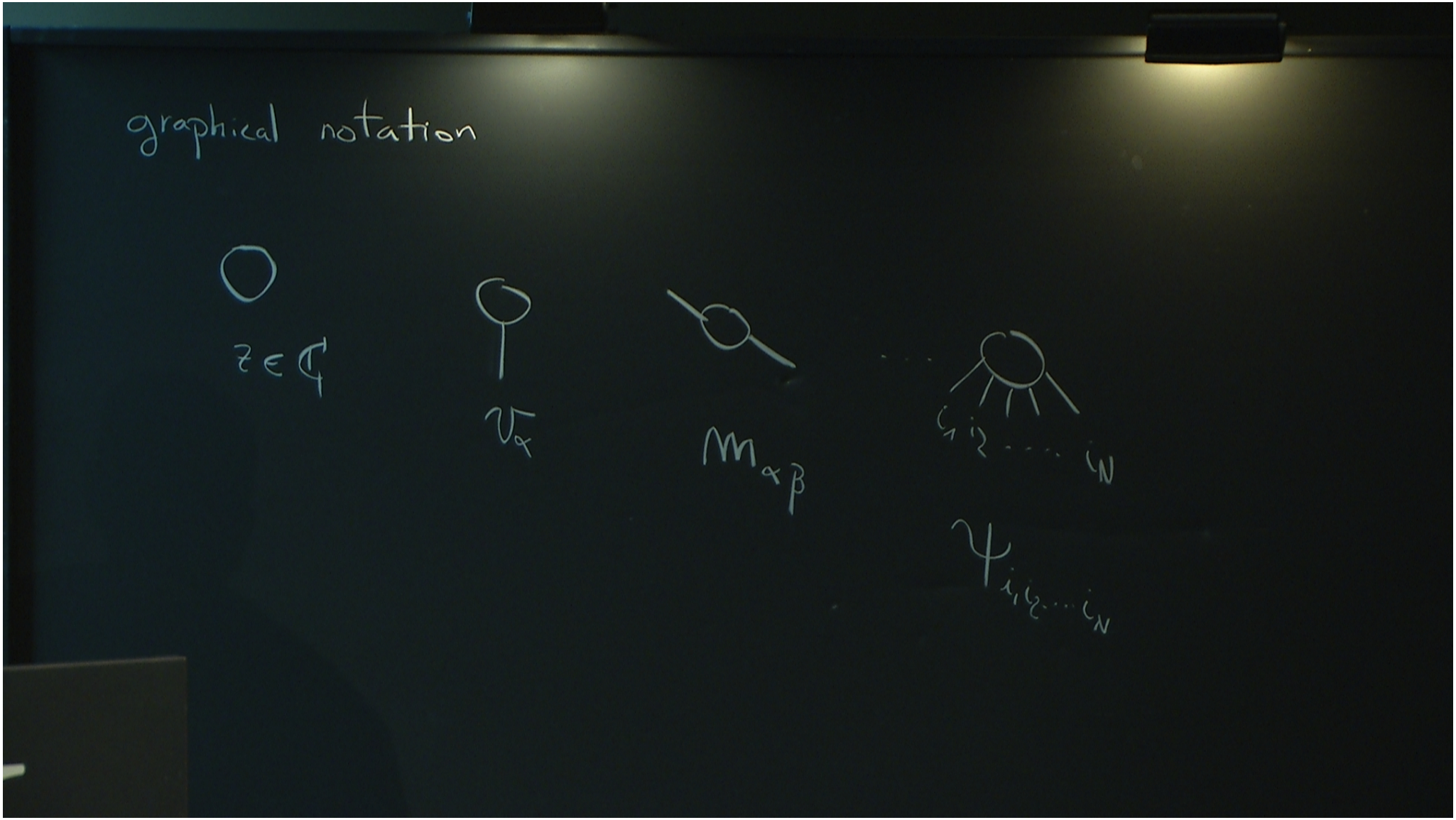
Matrix product state
MPS

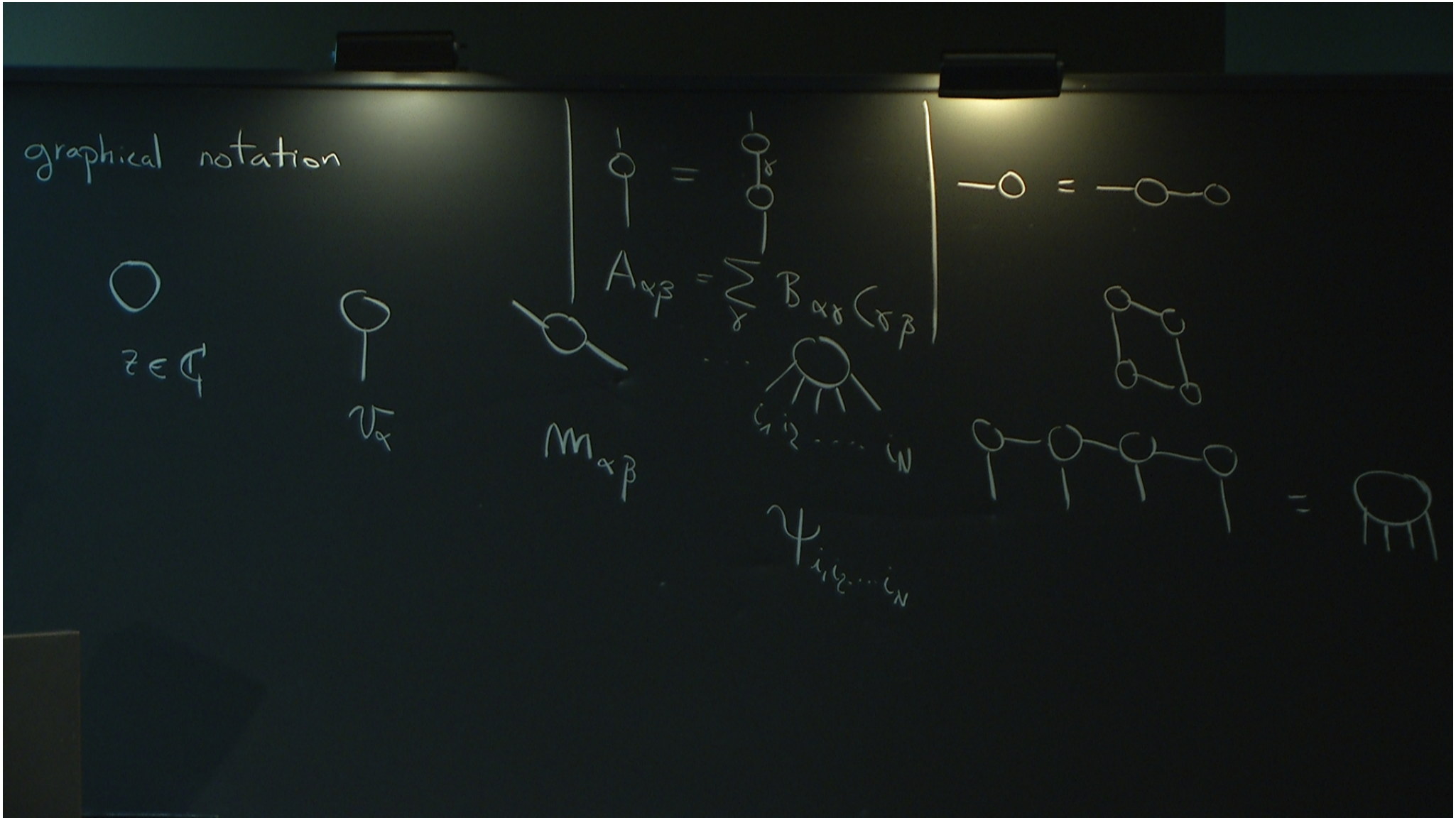


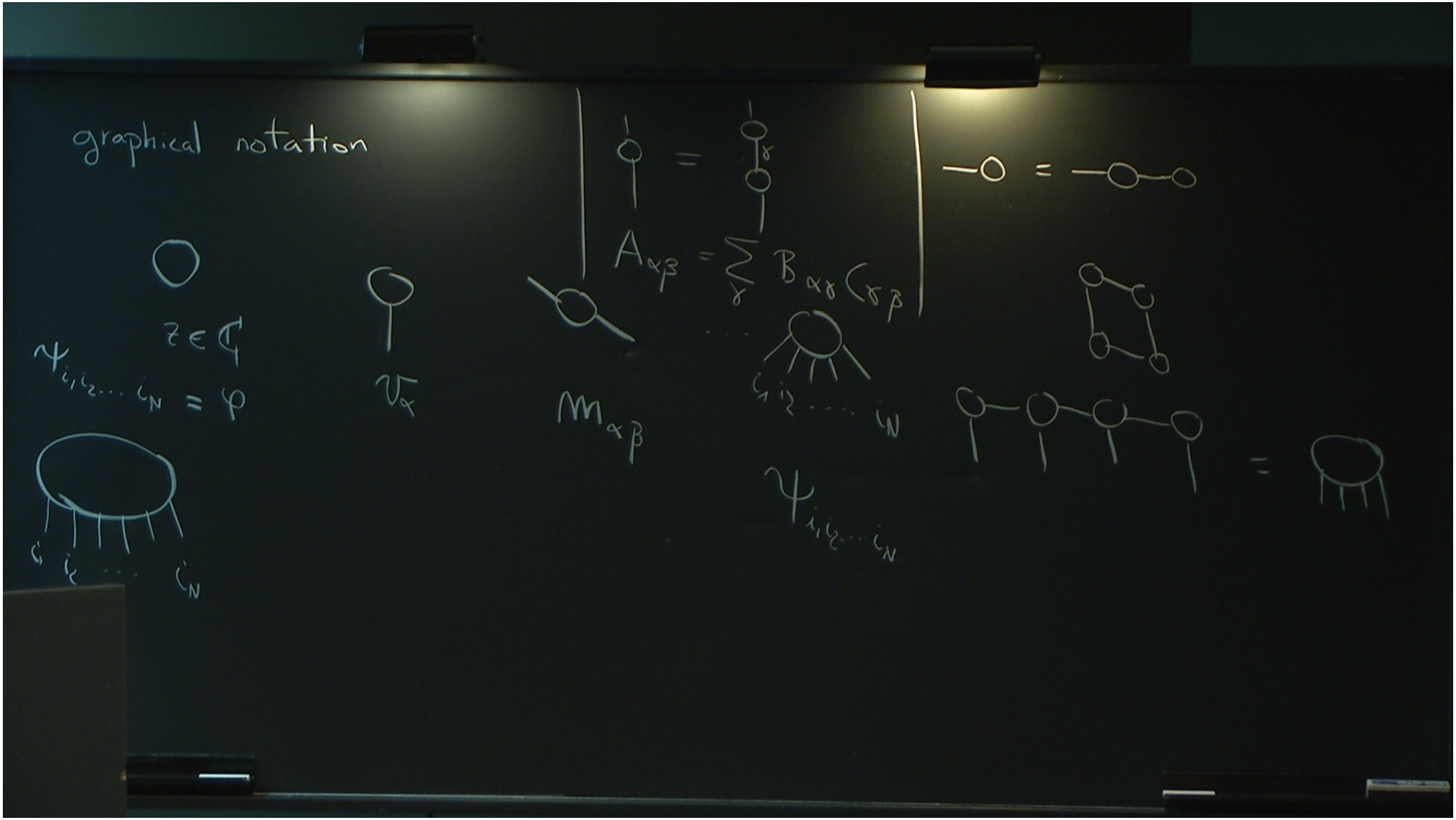
Multi-scale entanglement renormalization ansatz
MERA

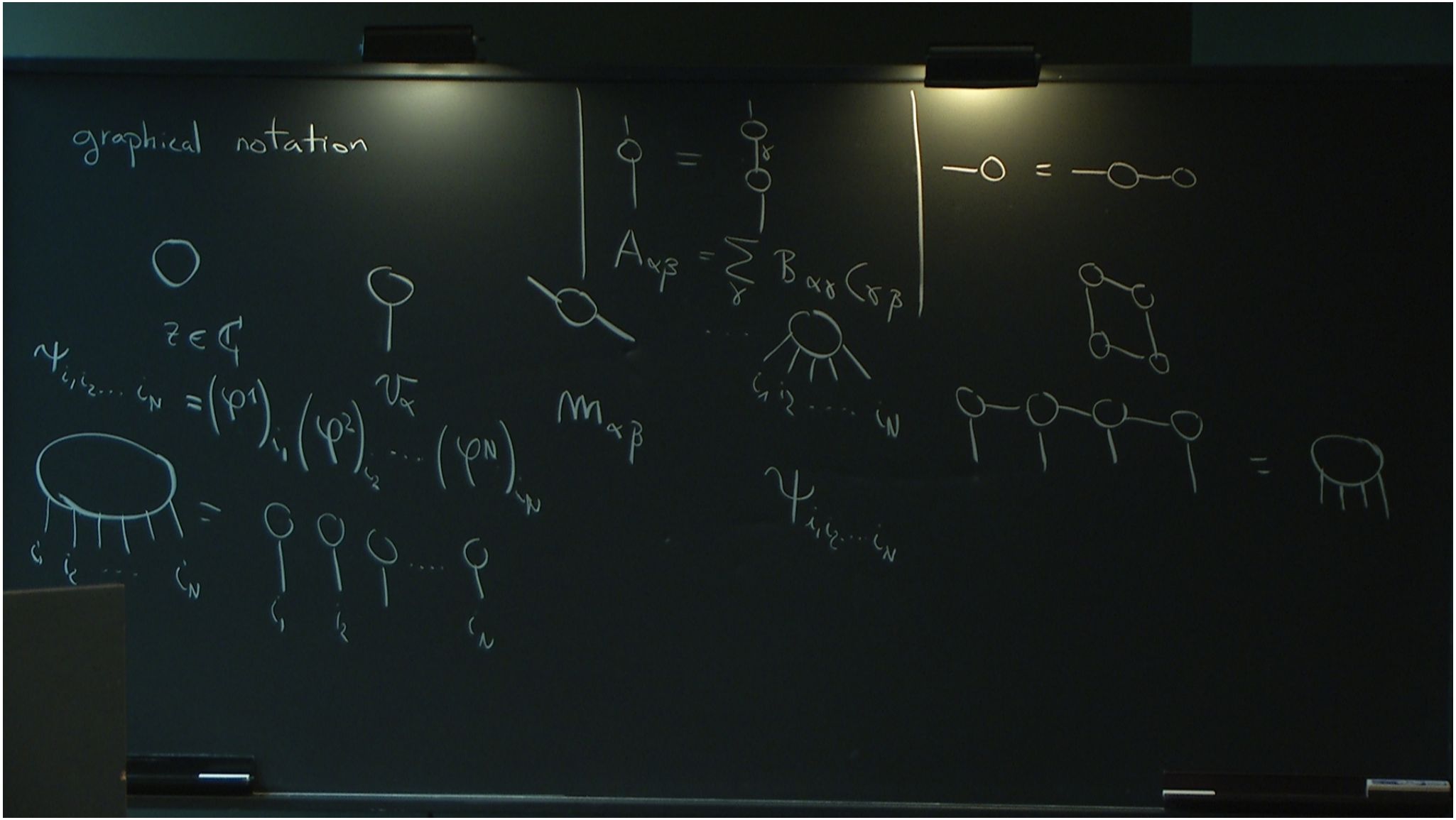


Projected entangled-pair state
PEPS


graphical notation
$z \in \mathbb{C}$
v_α
$m_{\alpha\beta}$
$i_1 i_2 \cdots i_N$
$\psi_{i_1 i_2 \cdots i_N}$


graphical notation
$z \in \mathbb{C}$
v_α
$A_{\alpha\beta} = \sum_\gamma B_{\alpha\gamma} C_{\gamma\beta}$
$M_{\alpha\beta}$
$\psi_{i_1 i_2 \cdots i_N}$


graphical notation
$z \in \mathbb{C}$
$\psi_{i_1 i_2 \ldots i_N} = \varphi$
v_α
$A_{\alpha\beta} = \sum_\gamma B_{\alpha\gamma} C_{\gamma\beta}$
$M_{\alpha\beta}$
$C_{i_1 i_2 \ldots i_N}$
$\psi_{i_1 i_2 \ldots i_N}$
$i_1 \; i_2 \; \cdots \; i_N$


graphical notation

TENSOR NETWORKS



TENSOR NETWORKS

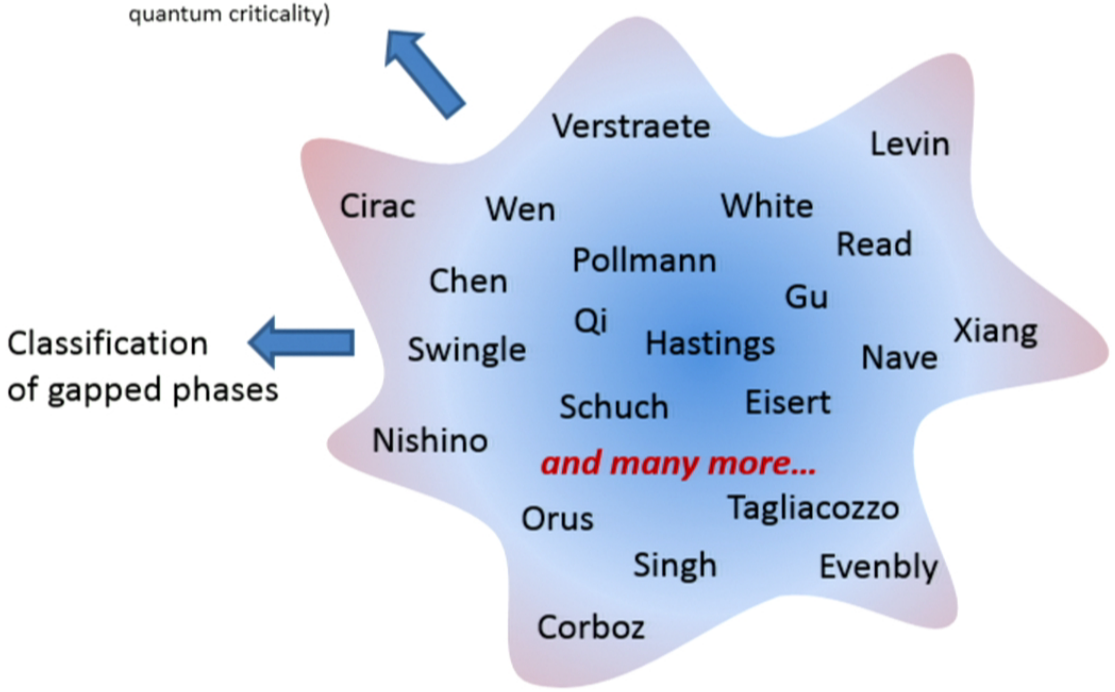
Computational condensed matter

(Frustrated magnets, interacting fermions, quantum criticality)



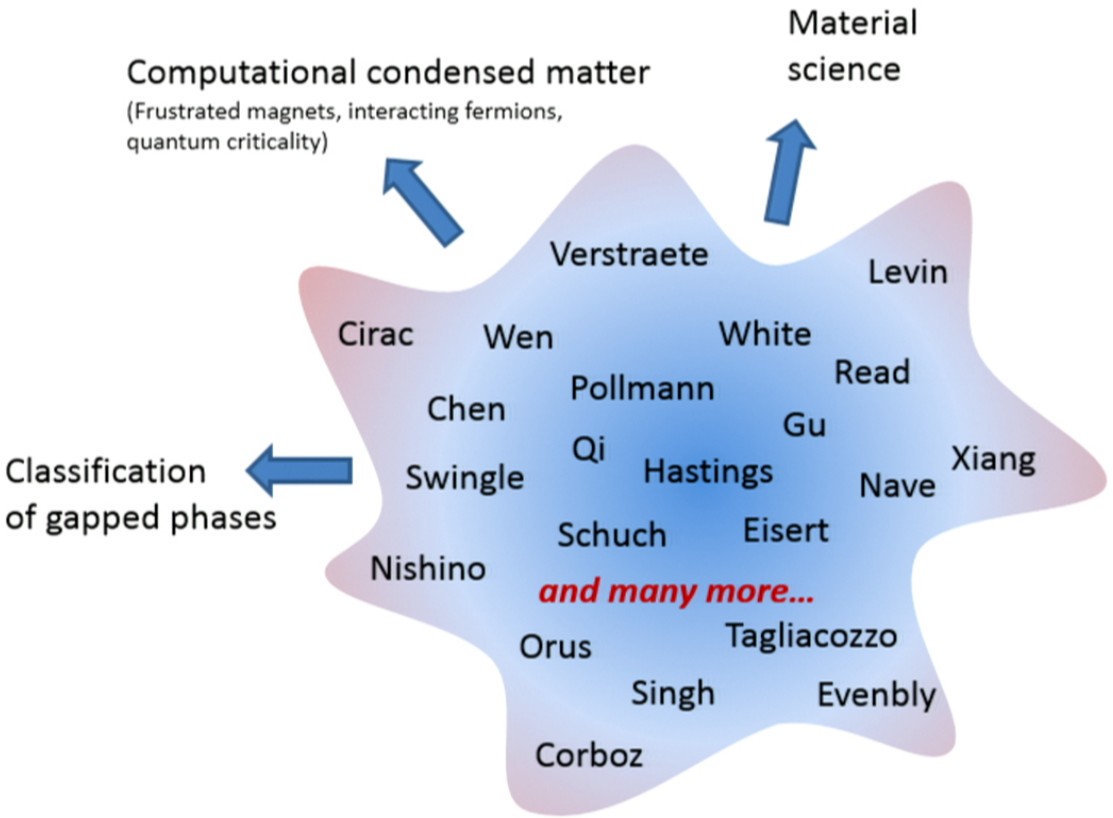
TENSOR NETWORKS

Computational condensed matter
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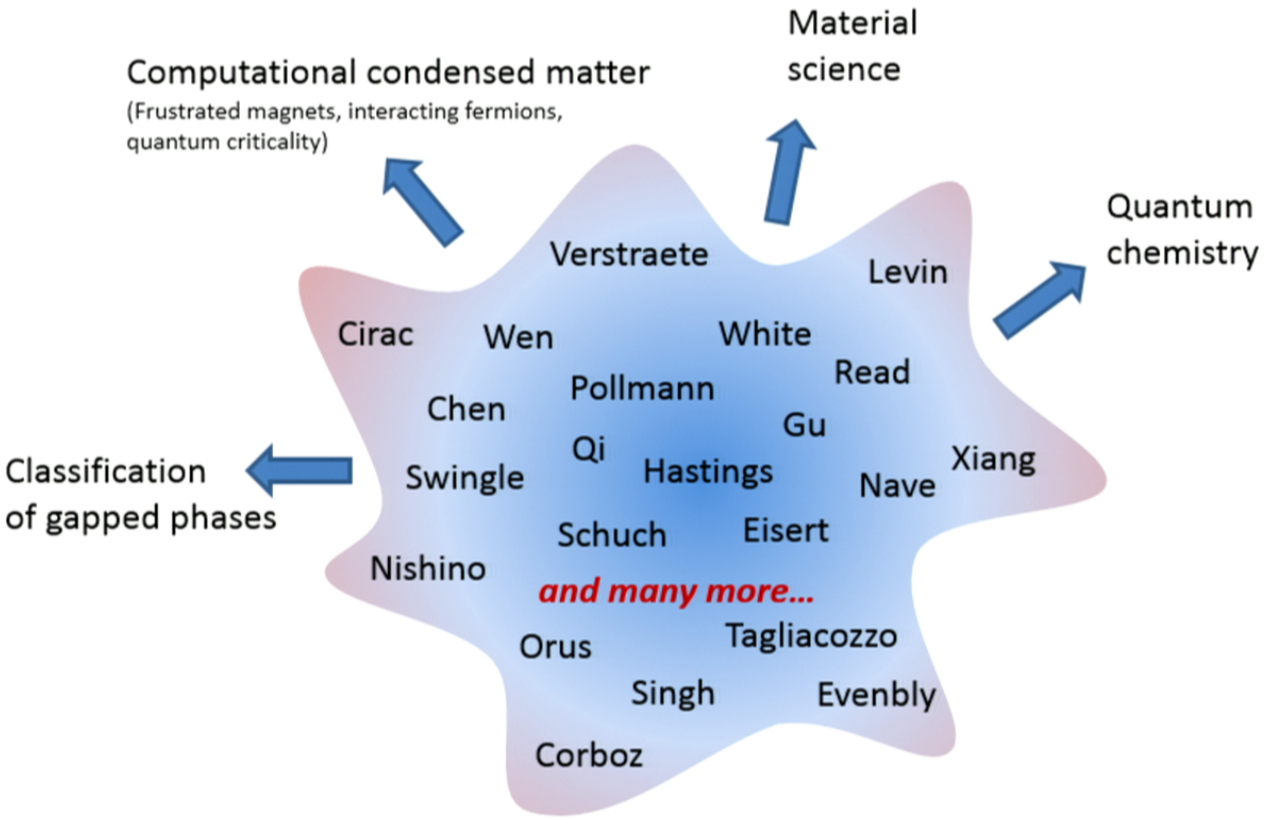


Classification of gapped phases

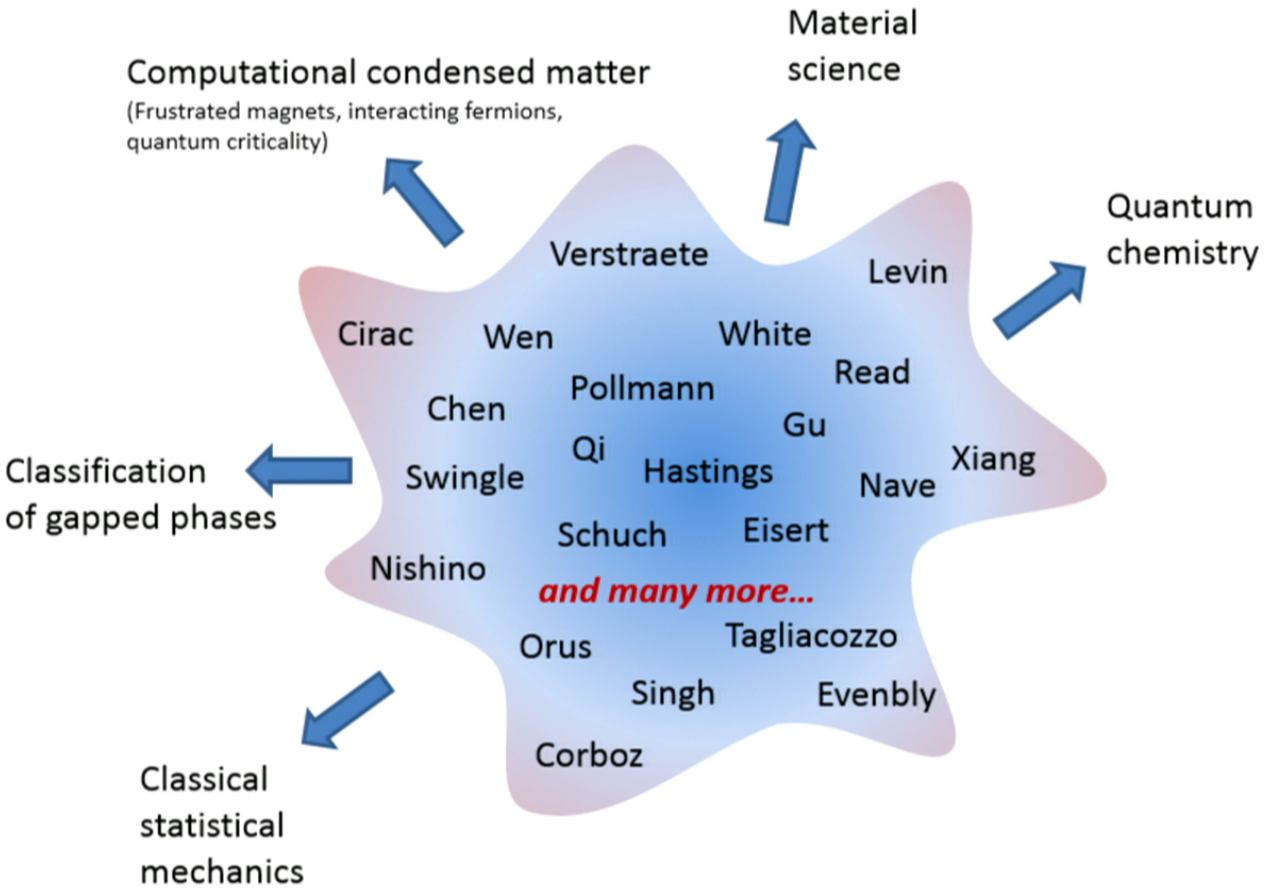
TENSOR NETWORKS



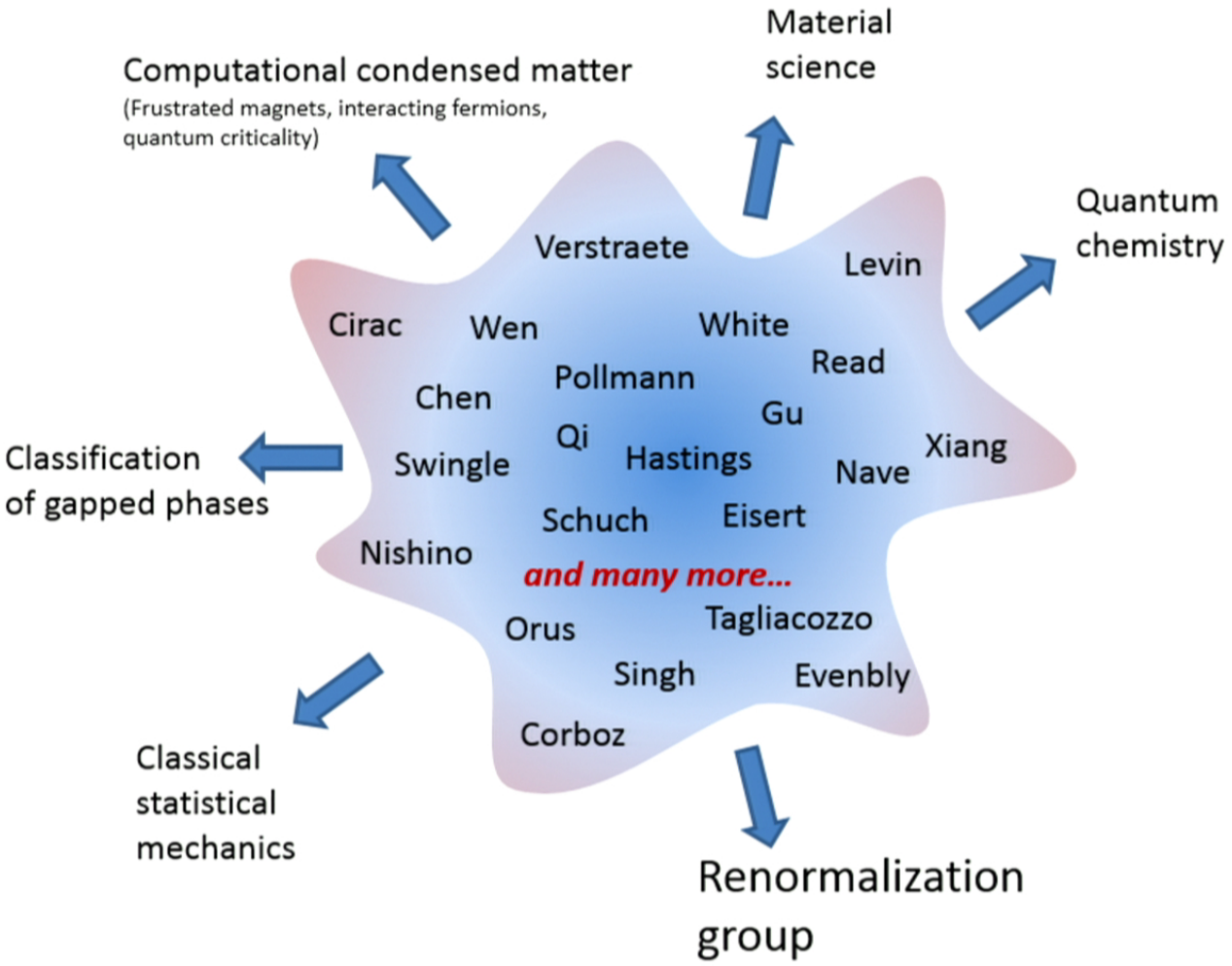
TENSOR NETWORKS



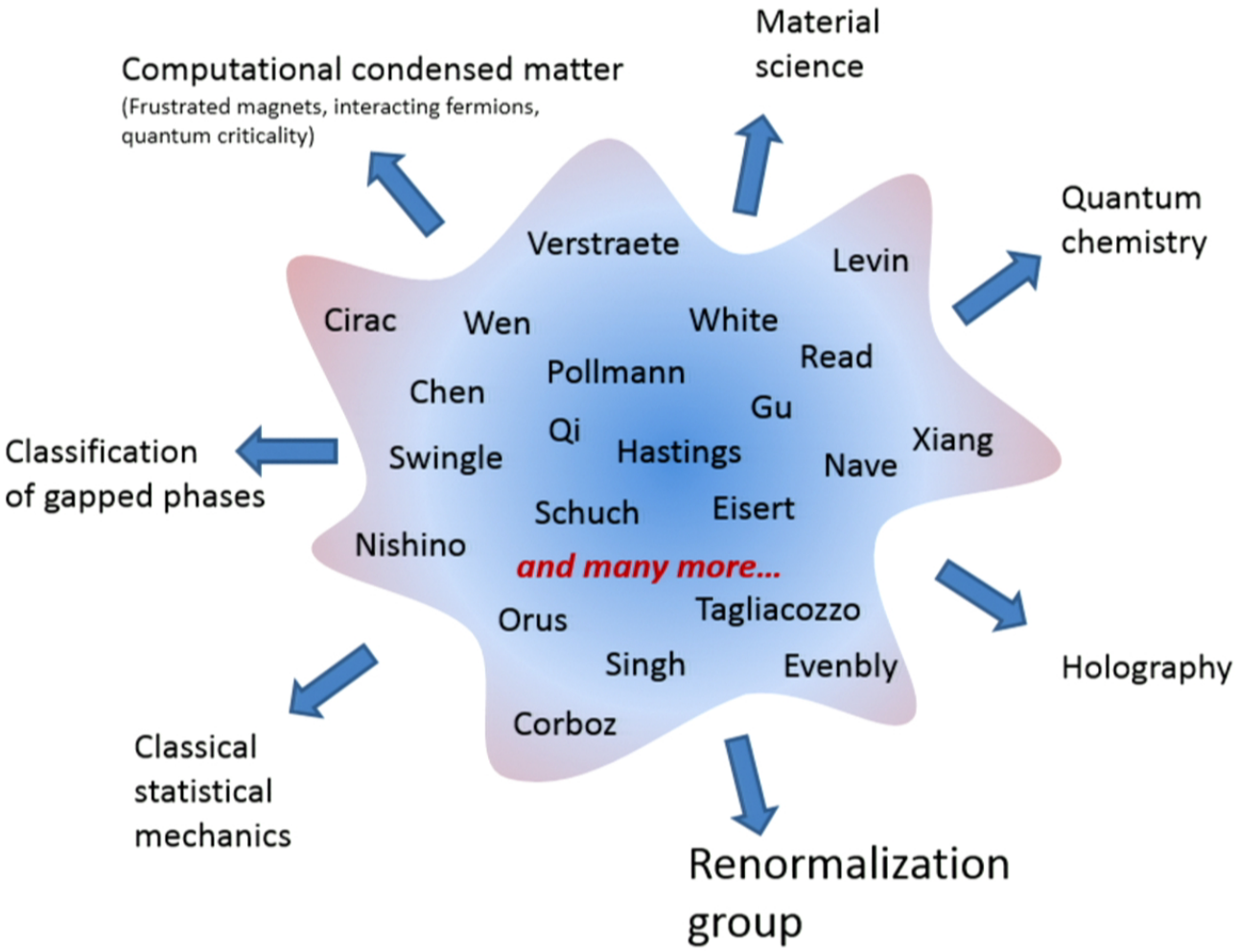
TENSOR NETWORKS

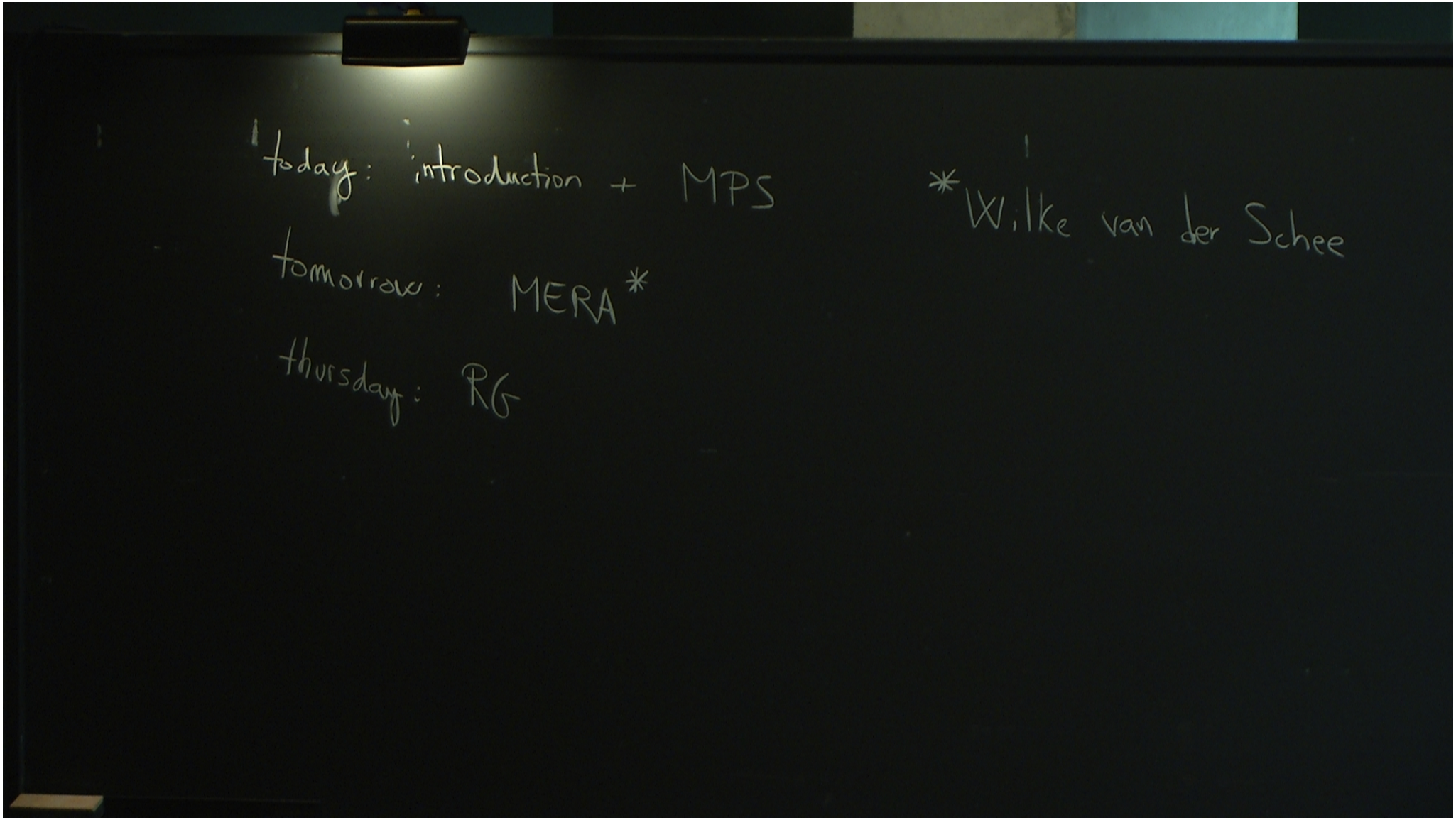


TENSOR NETWORKS

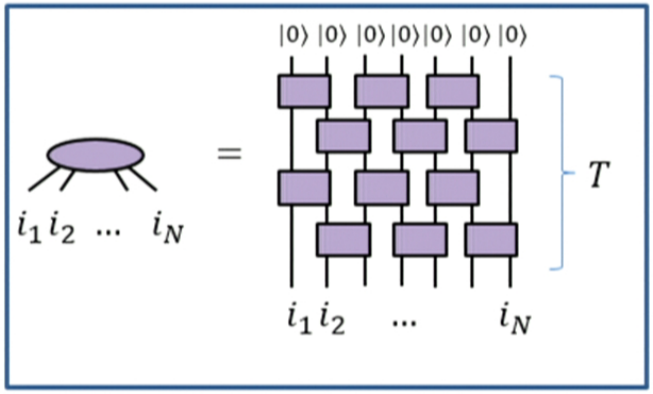


TENSOR NETWORKS

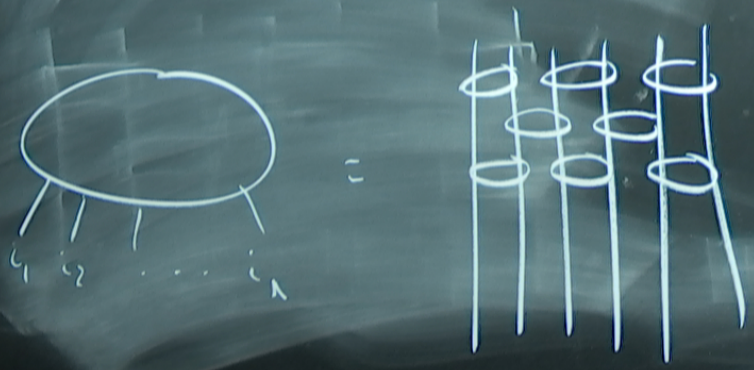



today: introduction + MPS
*Wilke van der Schee
tomorrow: MERA*
thursday: RG

Example of tensor network: Quantum Circuit



1.B Simple (=useless) Tensor Network

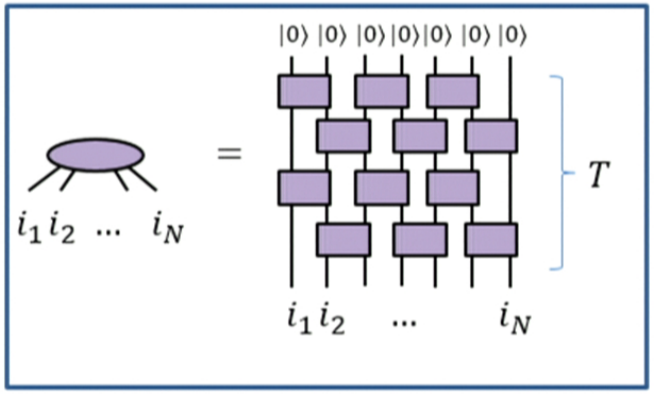


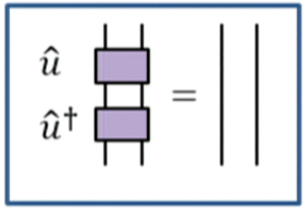
Questions

1) Computational
   - efficient representation
   - efficient computation

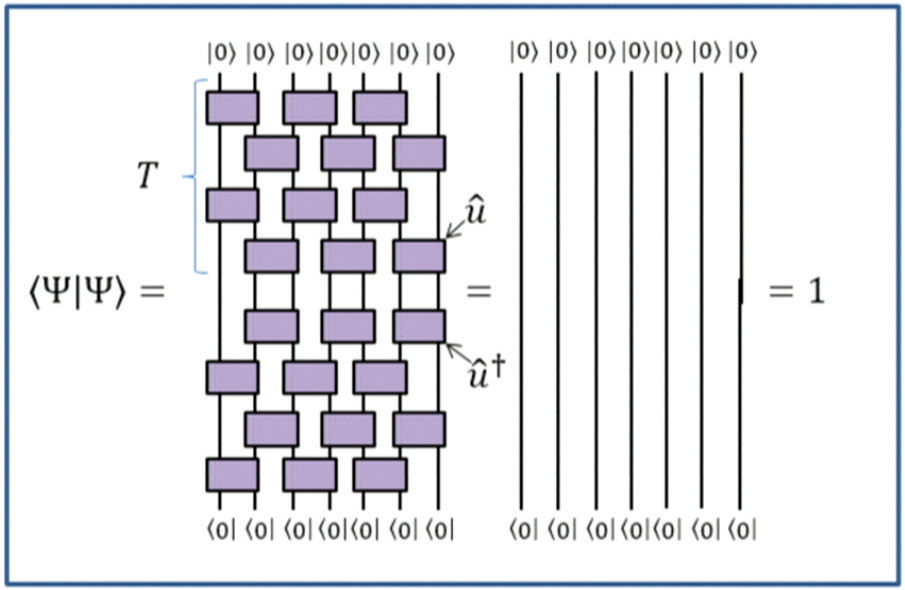
2) Physics

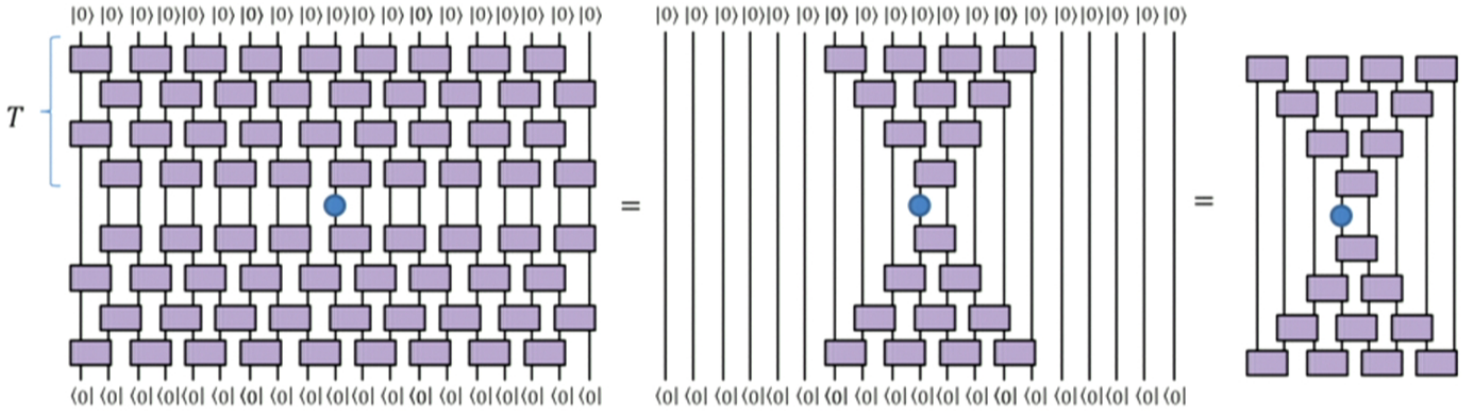
Example of tensor network: Quantum Circuit

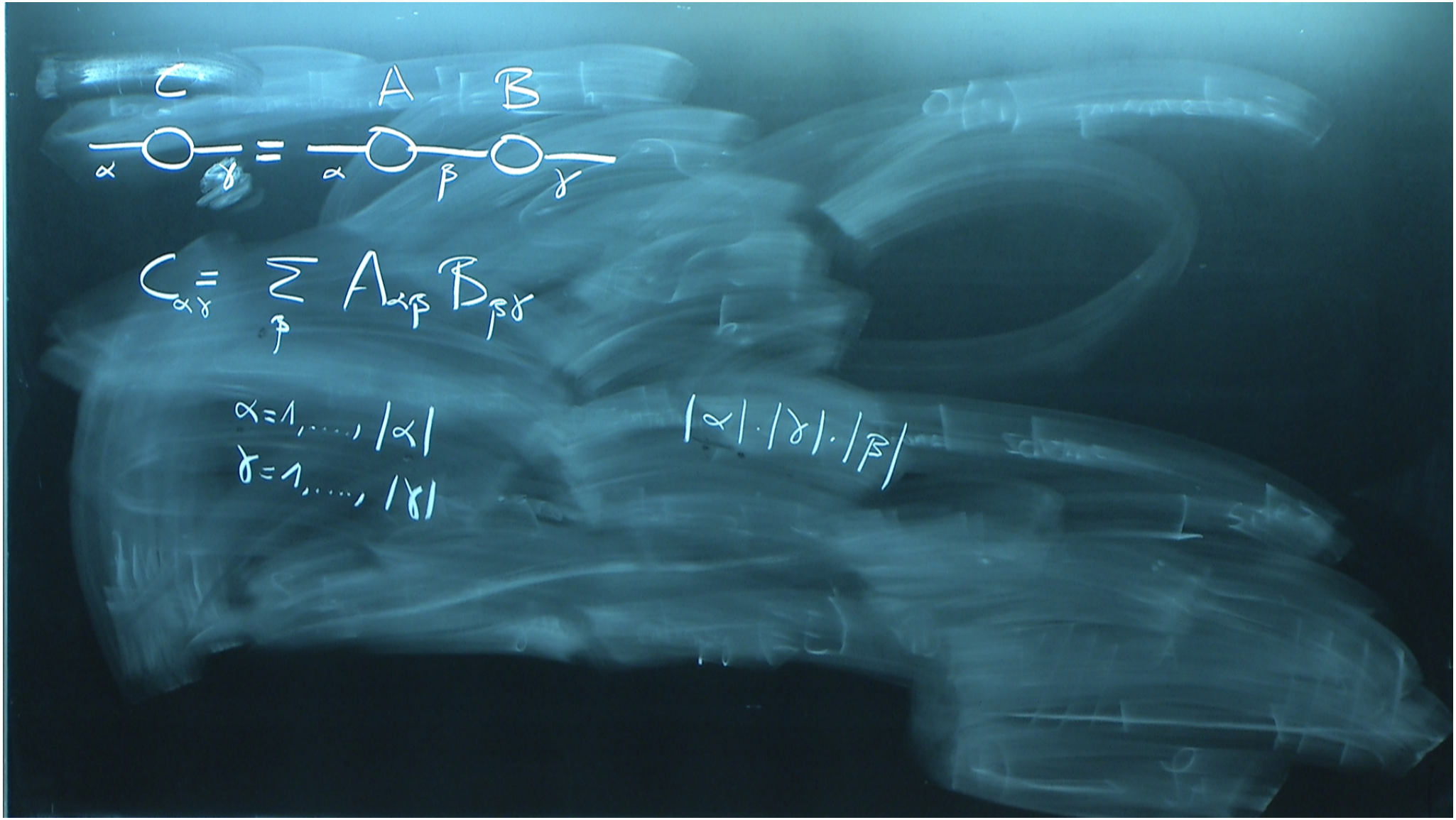


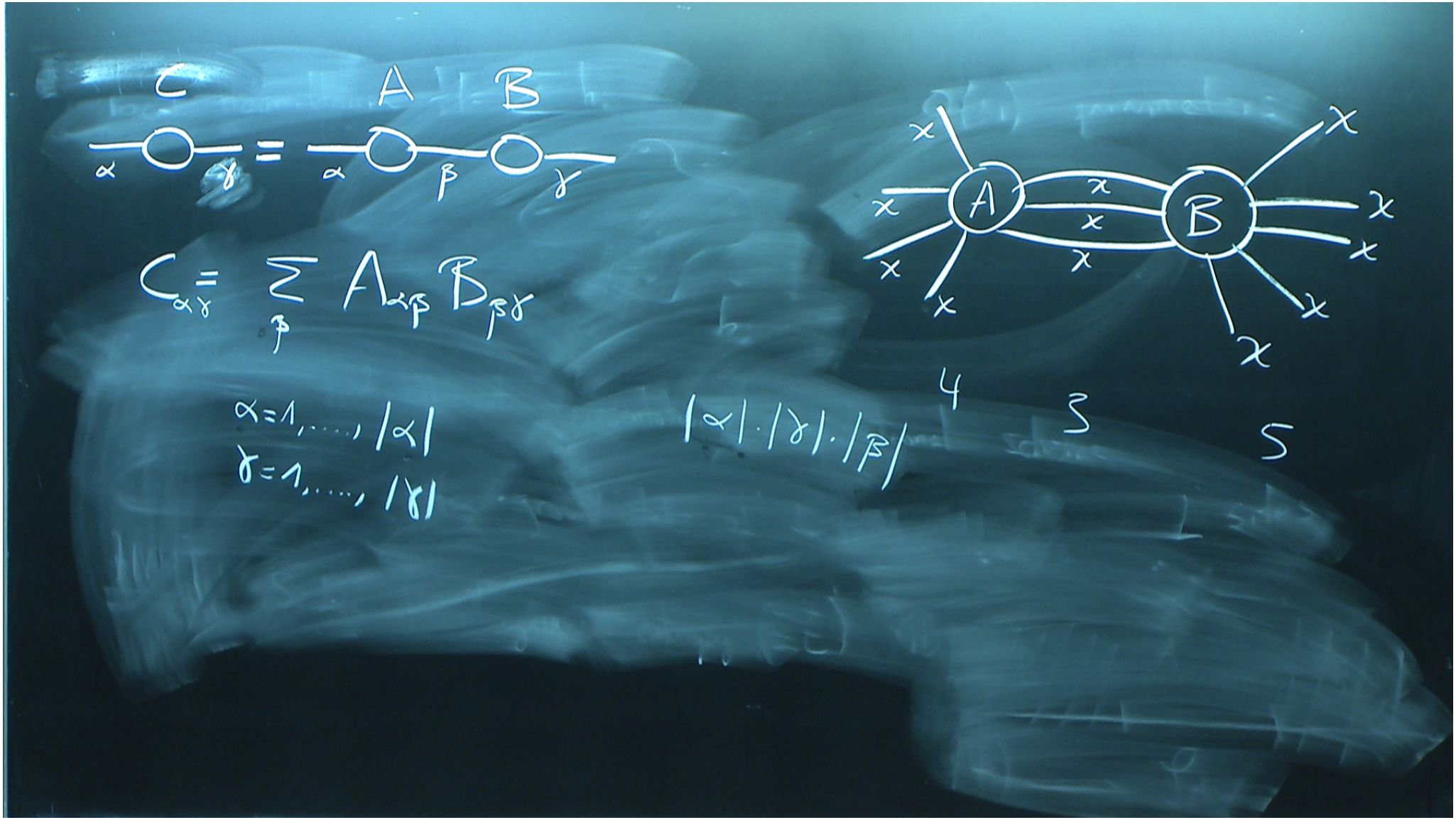


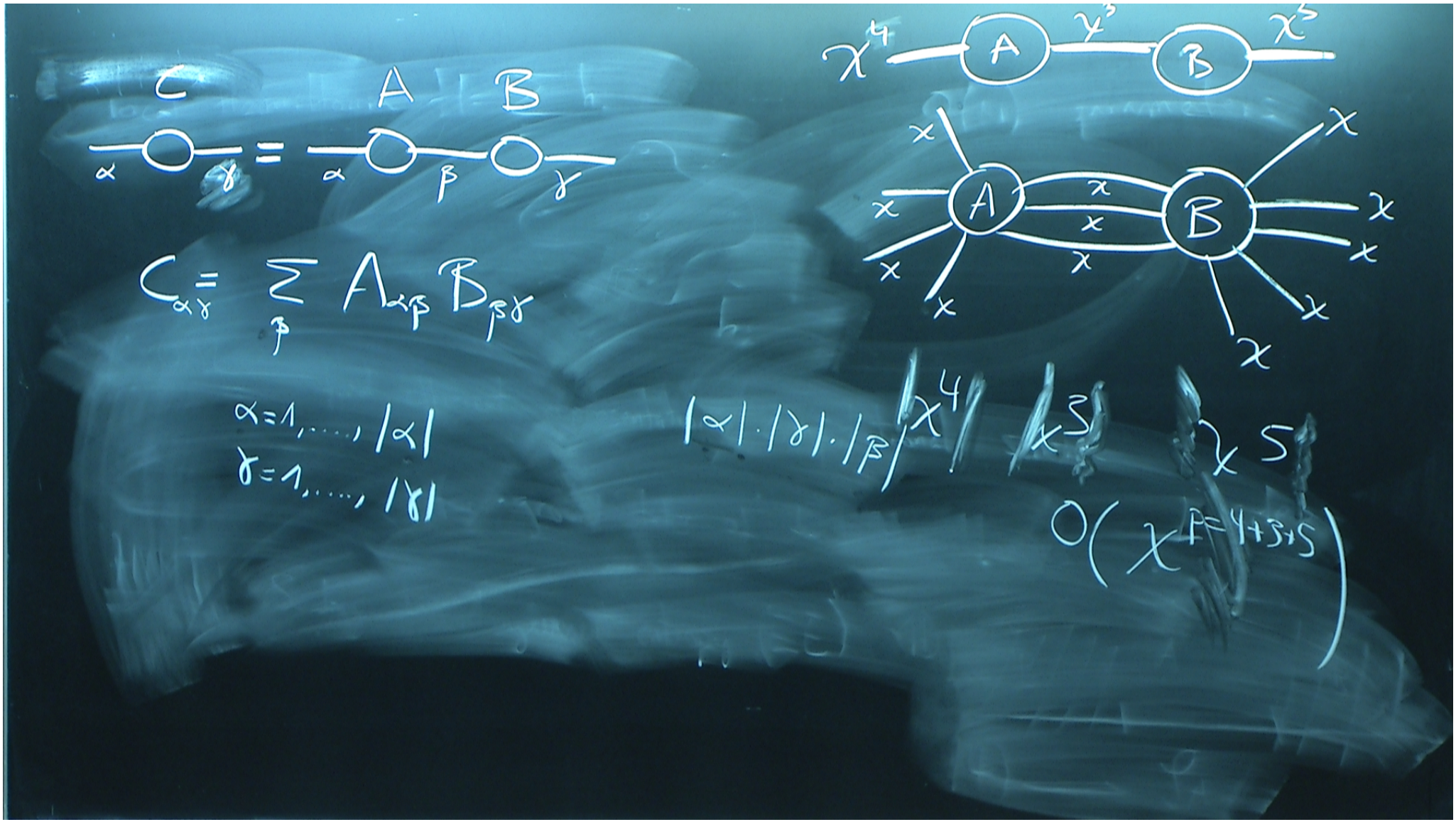
Norm

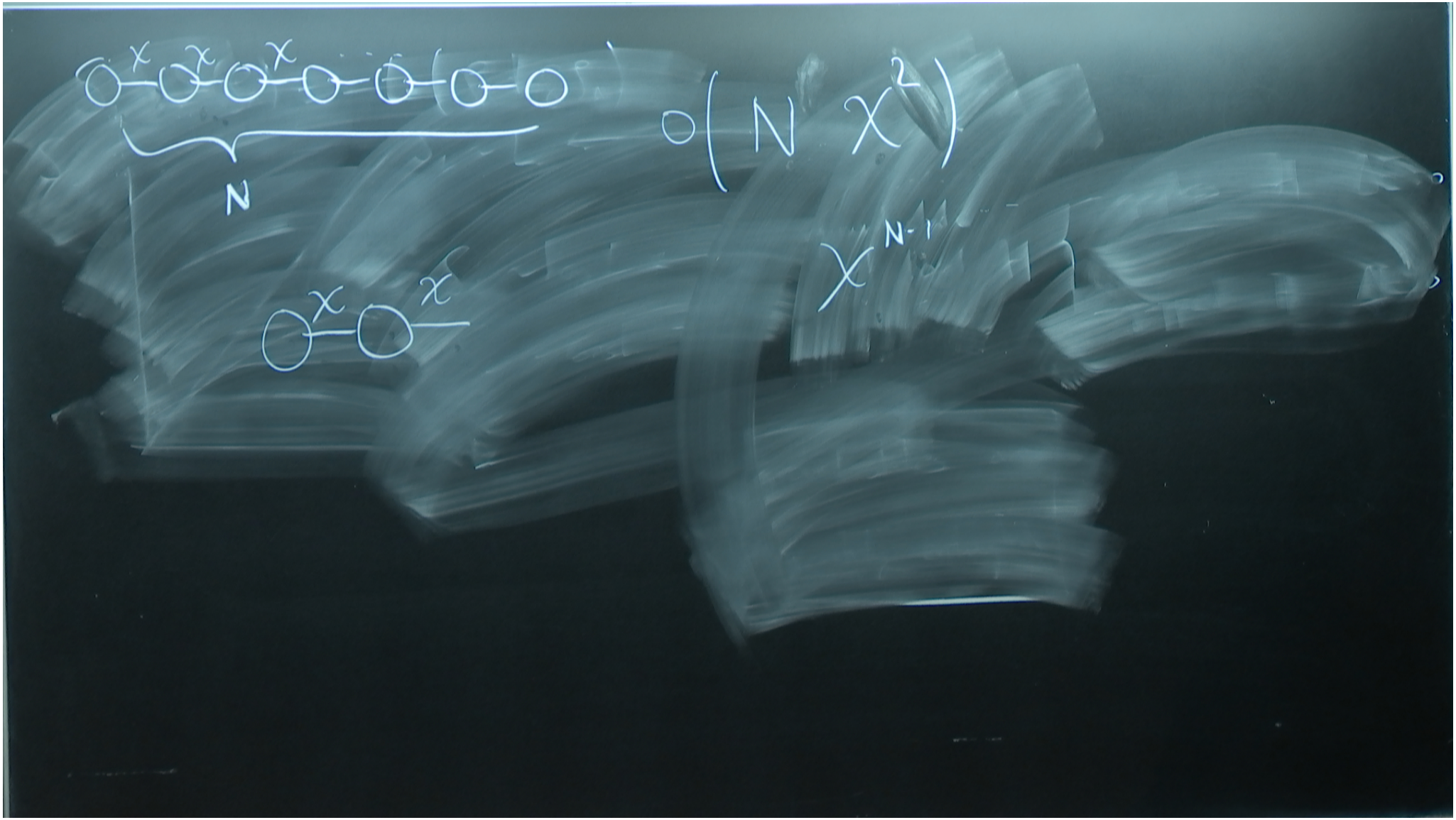


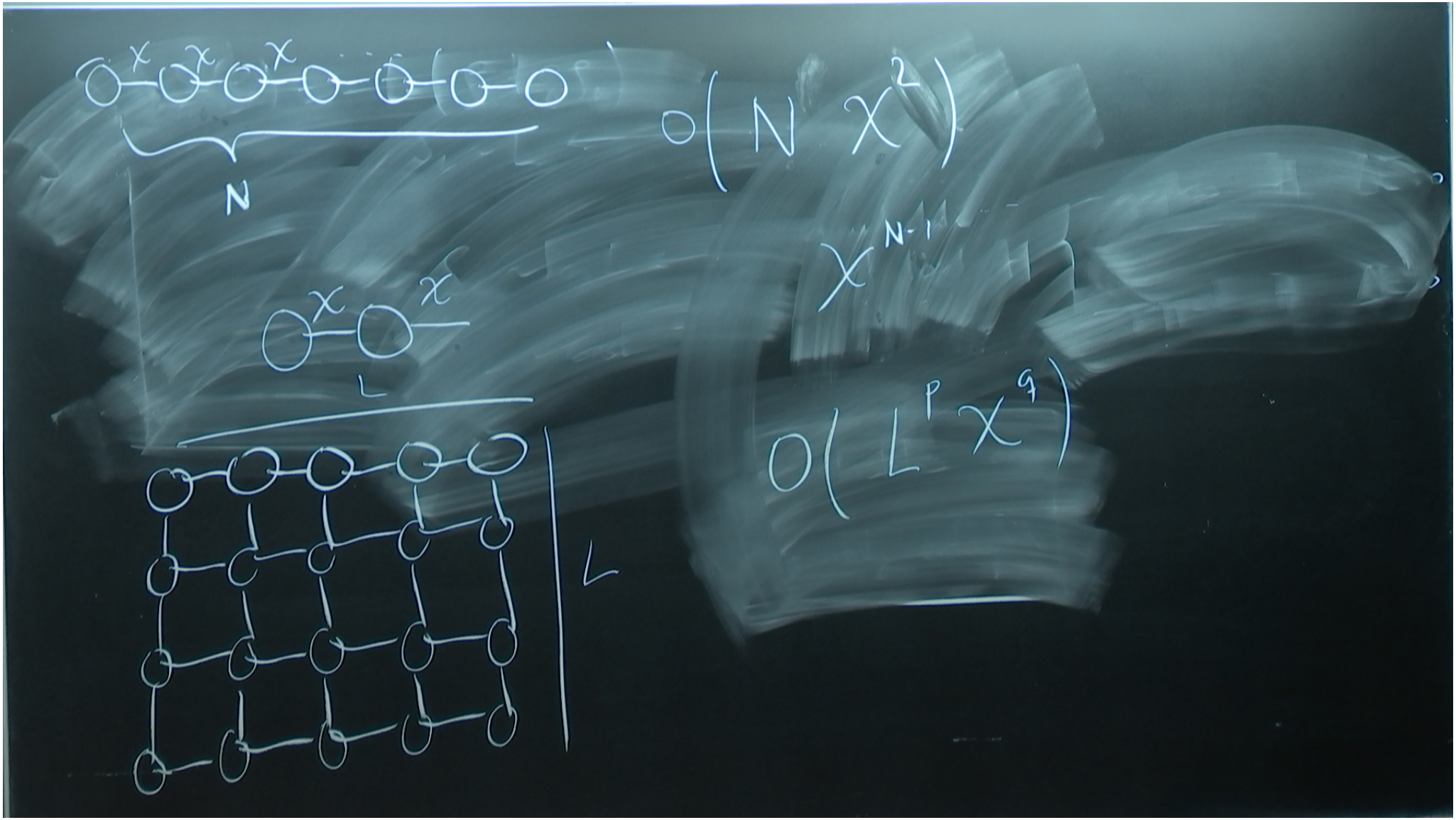

T
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⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0|
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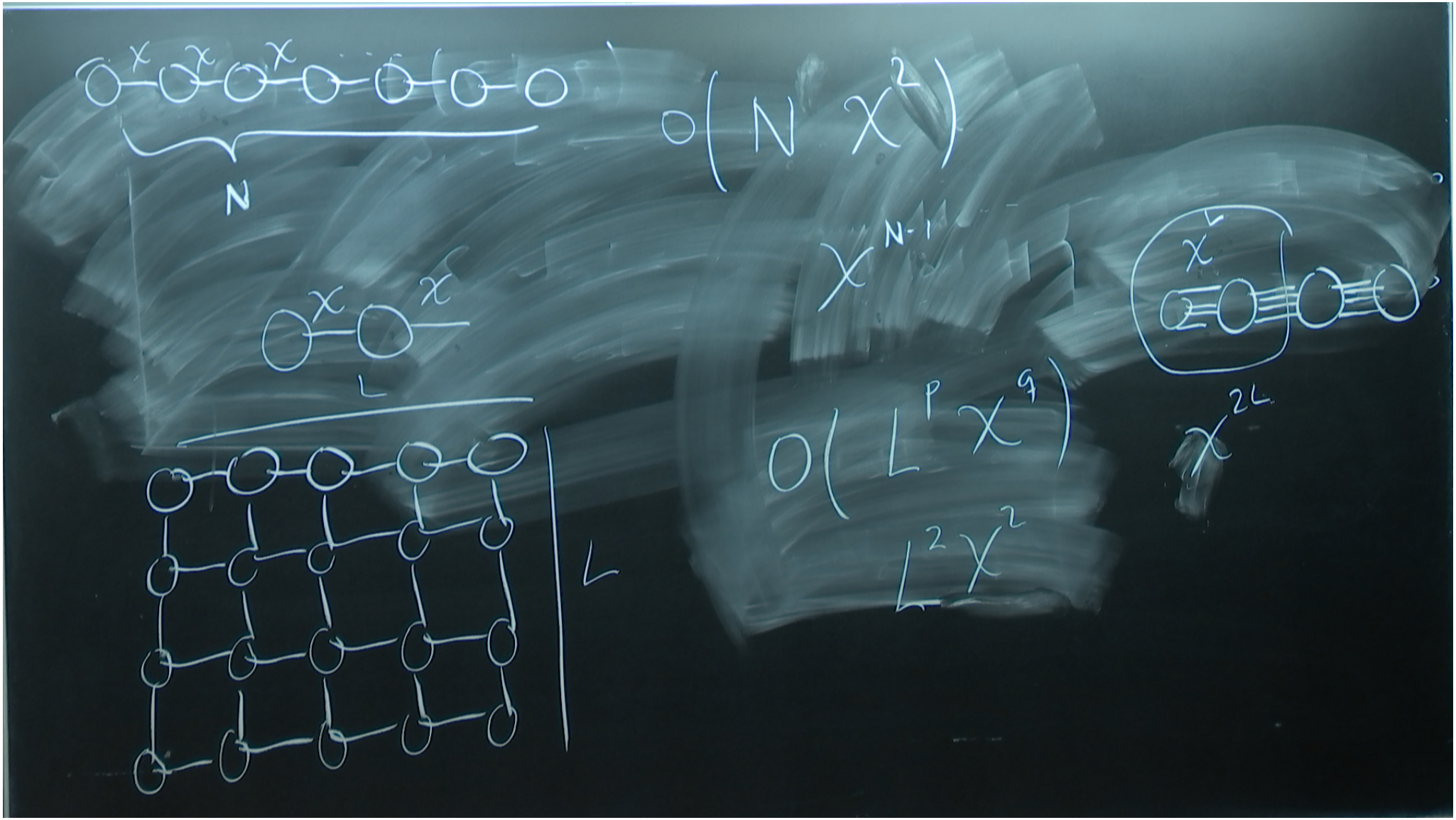

C
A
B
α
γ
=
α
β
γ
$C_{\alpha\gamma} = \sum_{\beta} A_{\alpha\beta} B_{\beta\gamma}$
$\alpha = 1, \ldots, |\alpha|$
$\gamma = 1, \ldots, |\gamma|$
$|\alpha| \cdot |\gamma| \cdot |\beta|$

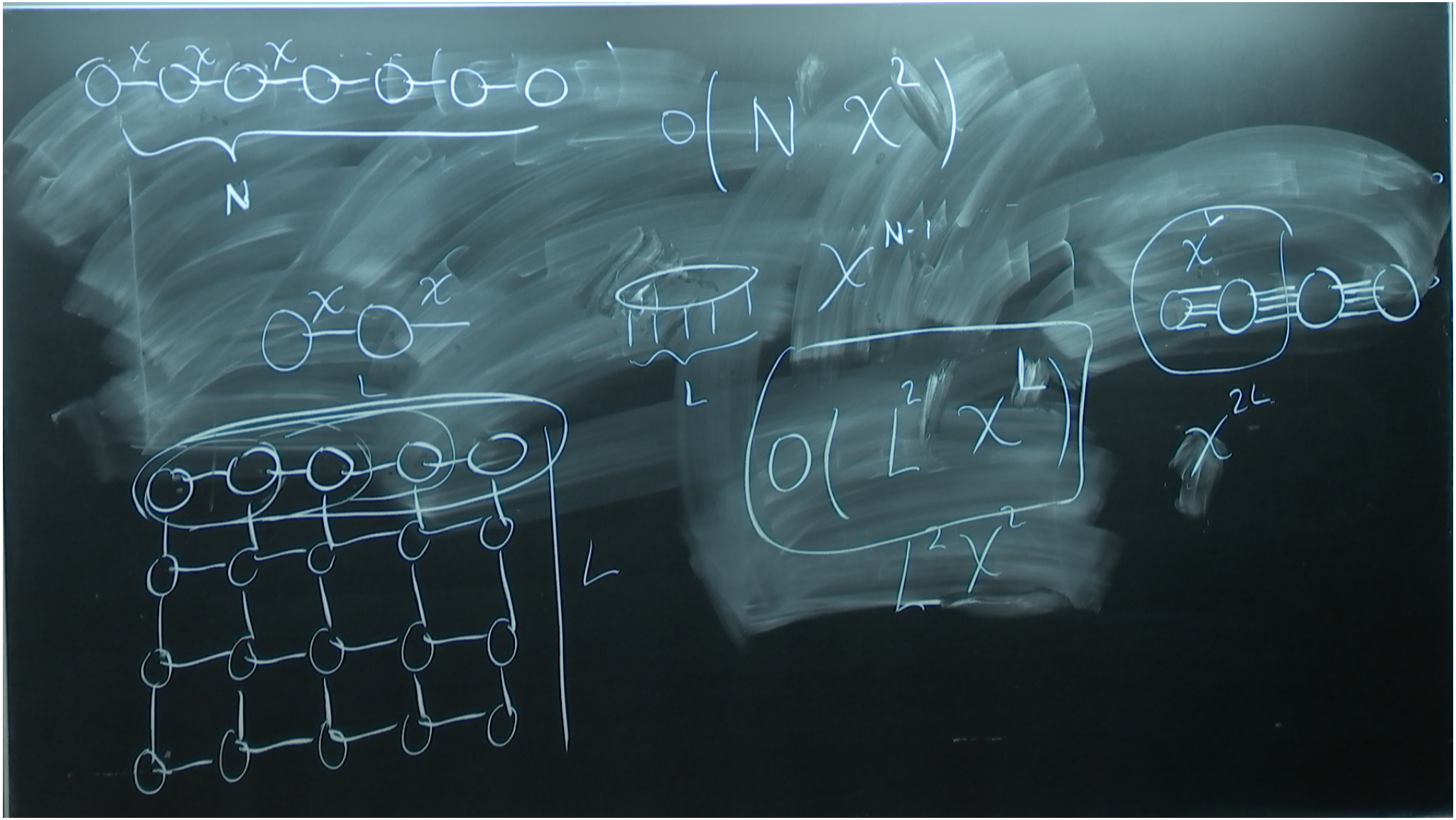

C
A
B
α
γ
=
α
β
γ
$C_{\alpha\gamma} = \sum_{\beta} A_{\alpha\beta} B_{\beta\gamma}$
$\alpha = 1, \ldots, |\alpha|$
$\gamma = 1, \ldots, |\gamma|$
$|\alpha| \cdot |\gamma| \cdot |\beta|$
A
B
4
3
5

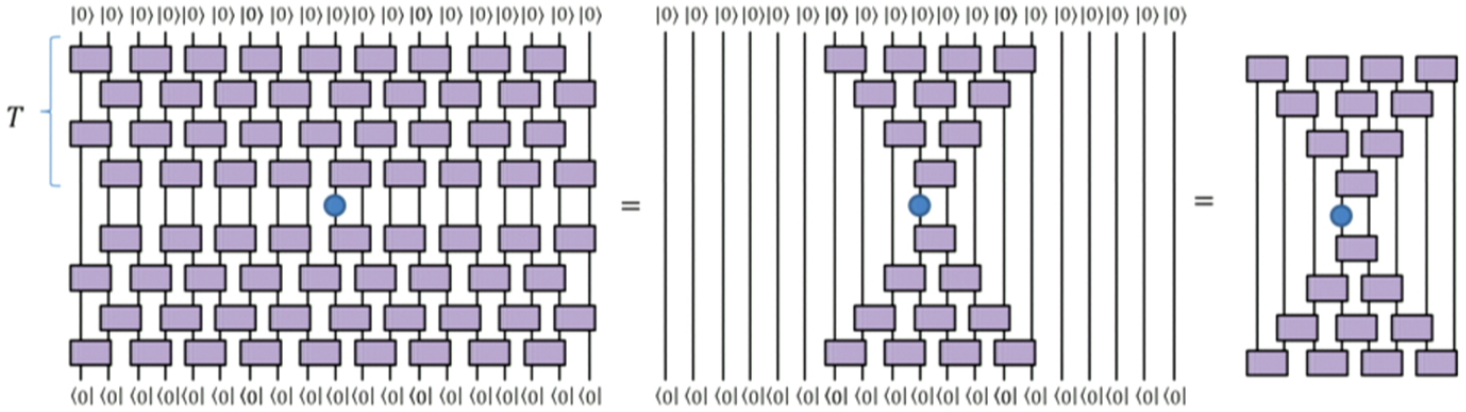

C
A
B
α
γ
=
α
β
γ
$C_{\alpha\gamma} = \sum_{\beta} A_{\alpha\beta} B_{\beta\gamma}$
$\alpha = 1, \ldots, |\alpha|$
$\gamma = 1, \ldots, |\gamma|$
x^4
A
x^3
B
x^5
x
$|\alpha| \cdot |\beta| \cdot |\gamma|$
x^4 x^3 x^5
$O(x^{P=4+3+5})$

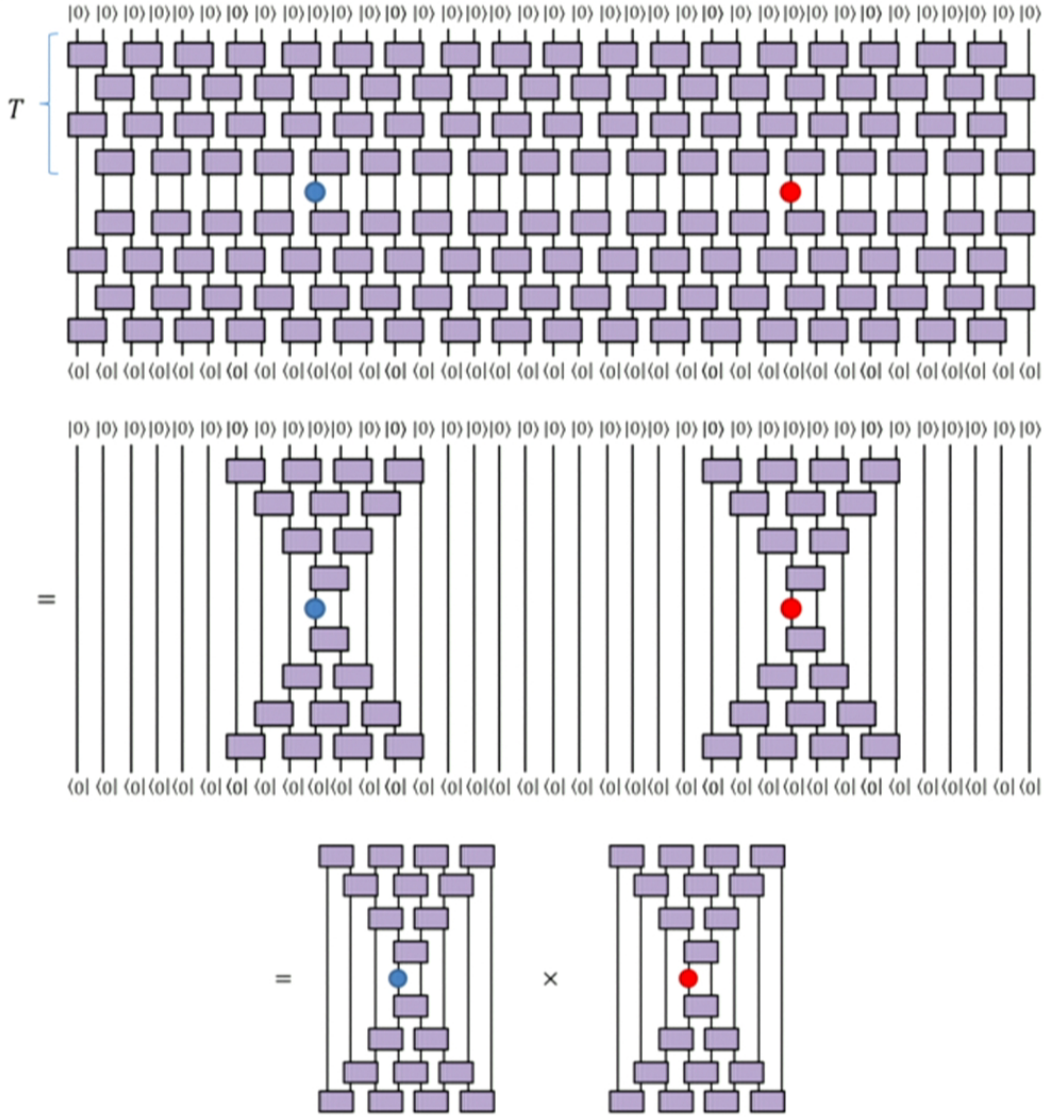

x x x
N
O(N X^2)
x x
X^{N-1}

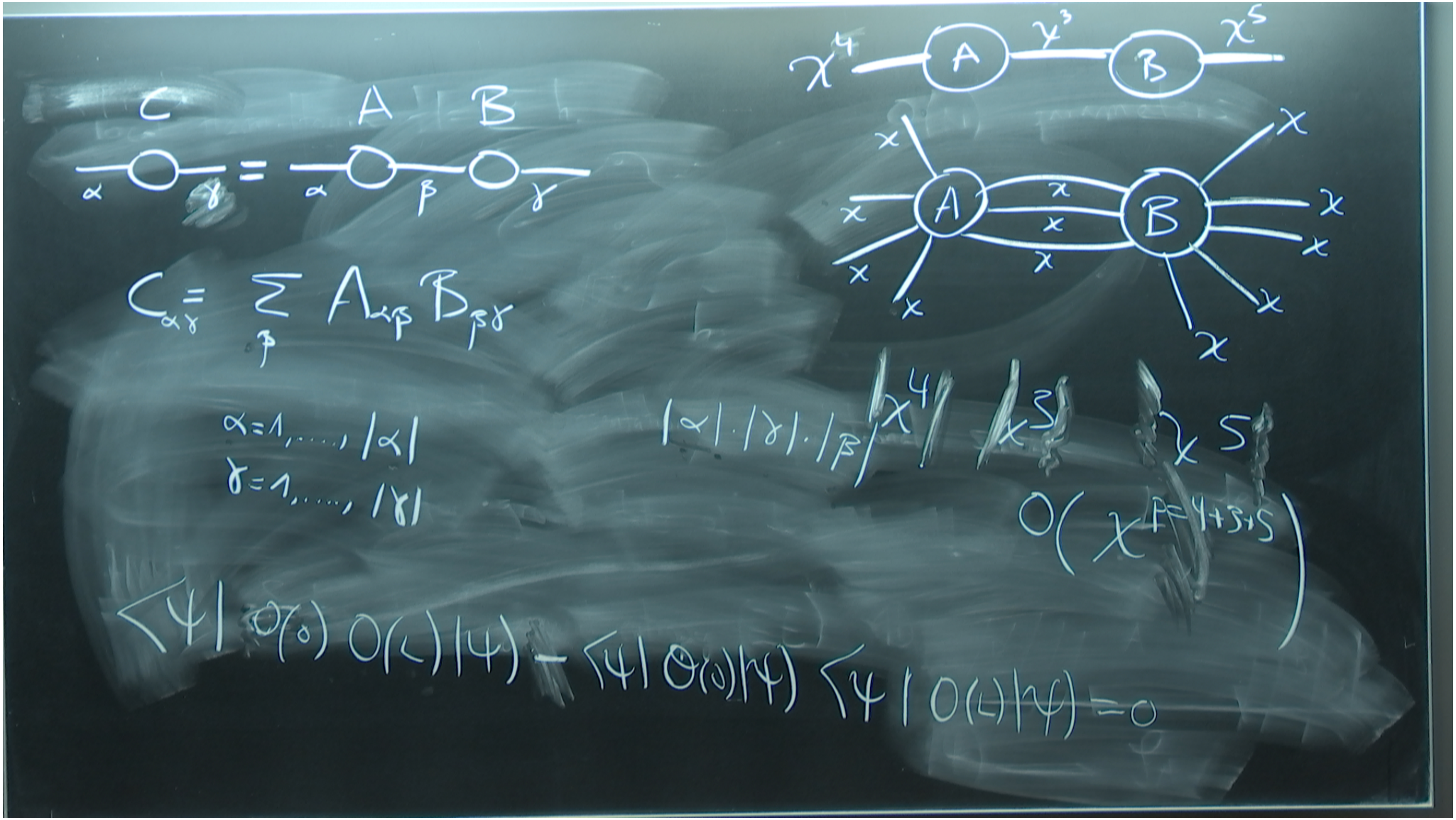

x x x
N
O(N χ²)
χ^{N-1}
x x
L
L
O(L^p χ^q)

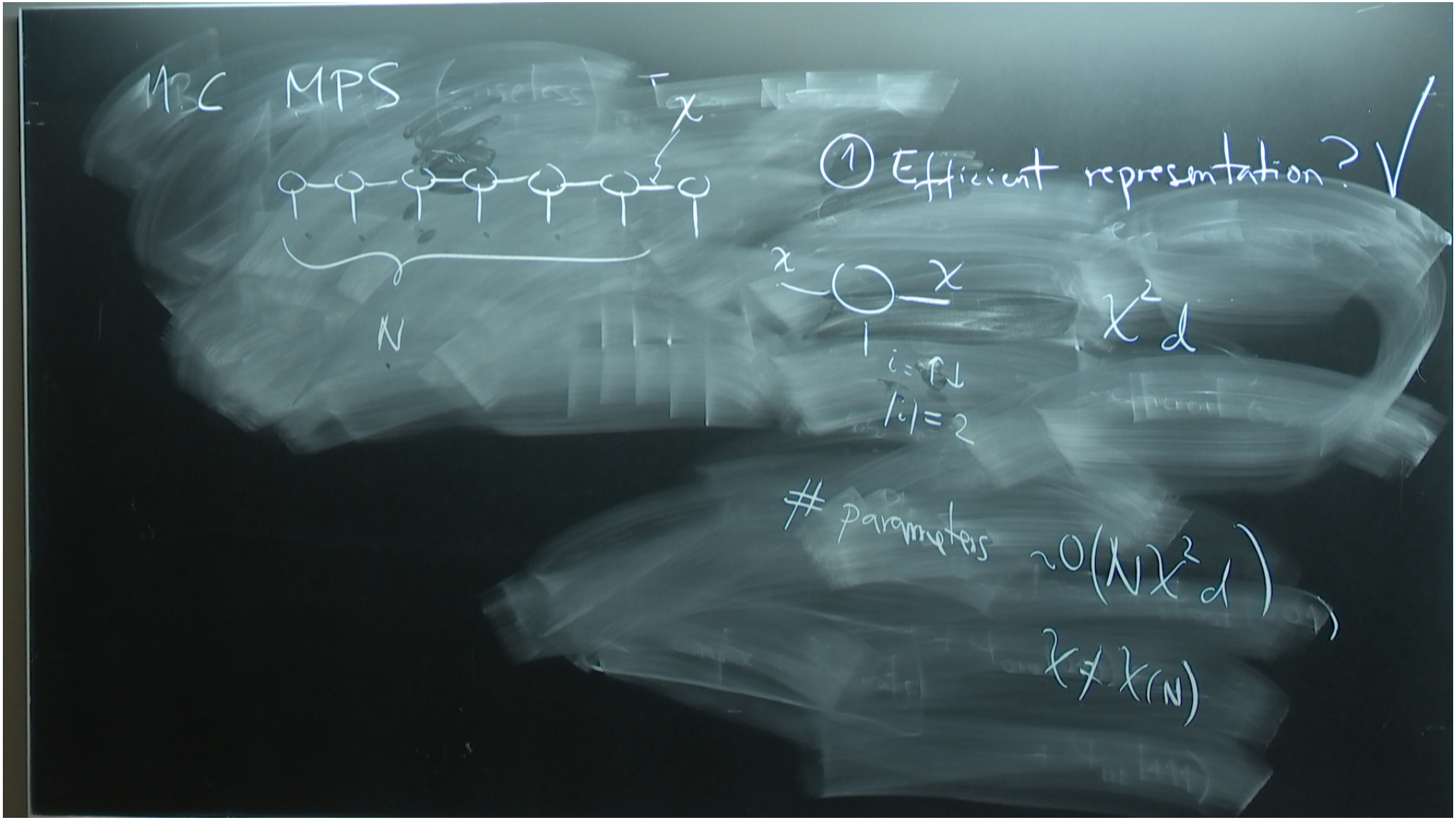

x x x
N
O(N X^2)
x x
L
X^{N-1}
x
O(L^p X^q)
L^2 X^2
X^{2L}
L


x x x
N
O(N X^2)
x x
L
L
X^{N-1}
O(L^2 X^L)
L X^L
x
X^{2L}
L

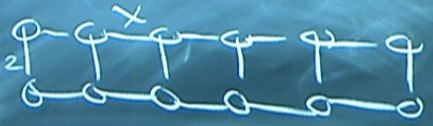

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⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0| ⟨0|
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C
A
B
α
γ
=
α
β
γ
$C_{\alpha\gamma} = \sum_{\beta} A_{\alpha\beta} B_{\beta\gamma}$
$\alpha = 1, \ldots, |\alpha|$
$\gamma = 1, \ldots, |\gamma|$
x^4
A
x^3
B
x^5
$|\alpha| \cdot |\beta| \cdot |\gamma|$
x^4 x^3 x^5
$O(x^{P=4+3+5})$
$\langle\psi| O(0) O(1) |\psi\rangle - \langle\psi| O(0) |\psi\rangle \langle\psi| O(1) |\psi\rangle = 0$

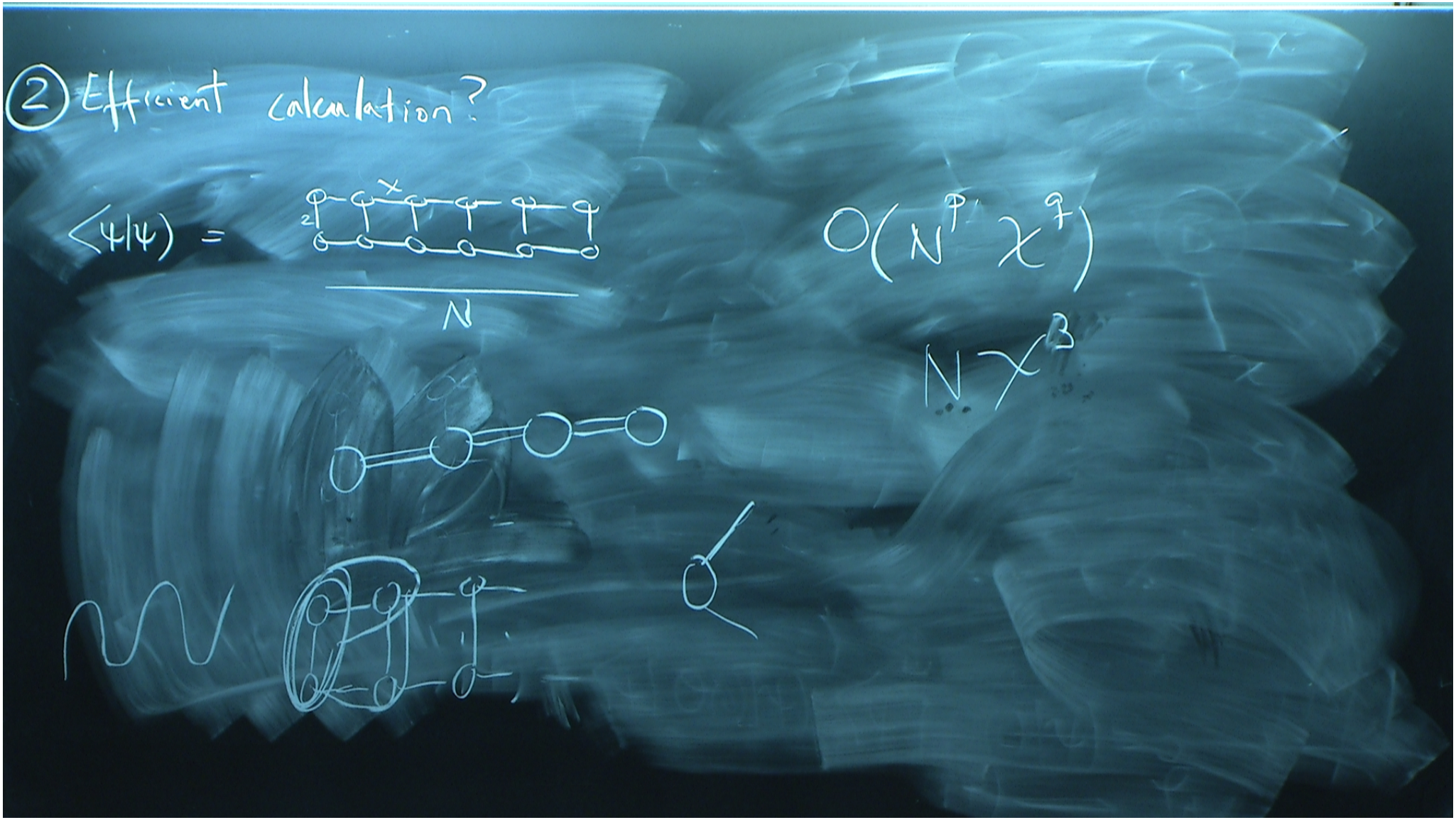

MPS
χ
N
① Efficient representation? √
χ
χ
1
i = ↑↓
|i| = 2
χ² d
parameters
~O(Nχ² d)
χ ≠ χ(N)

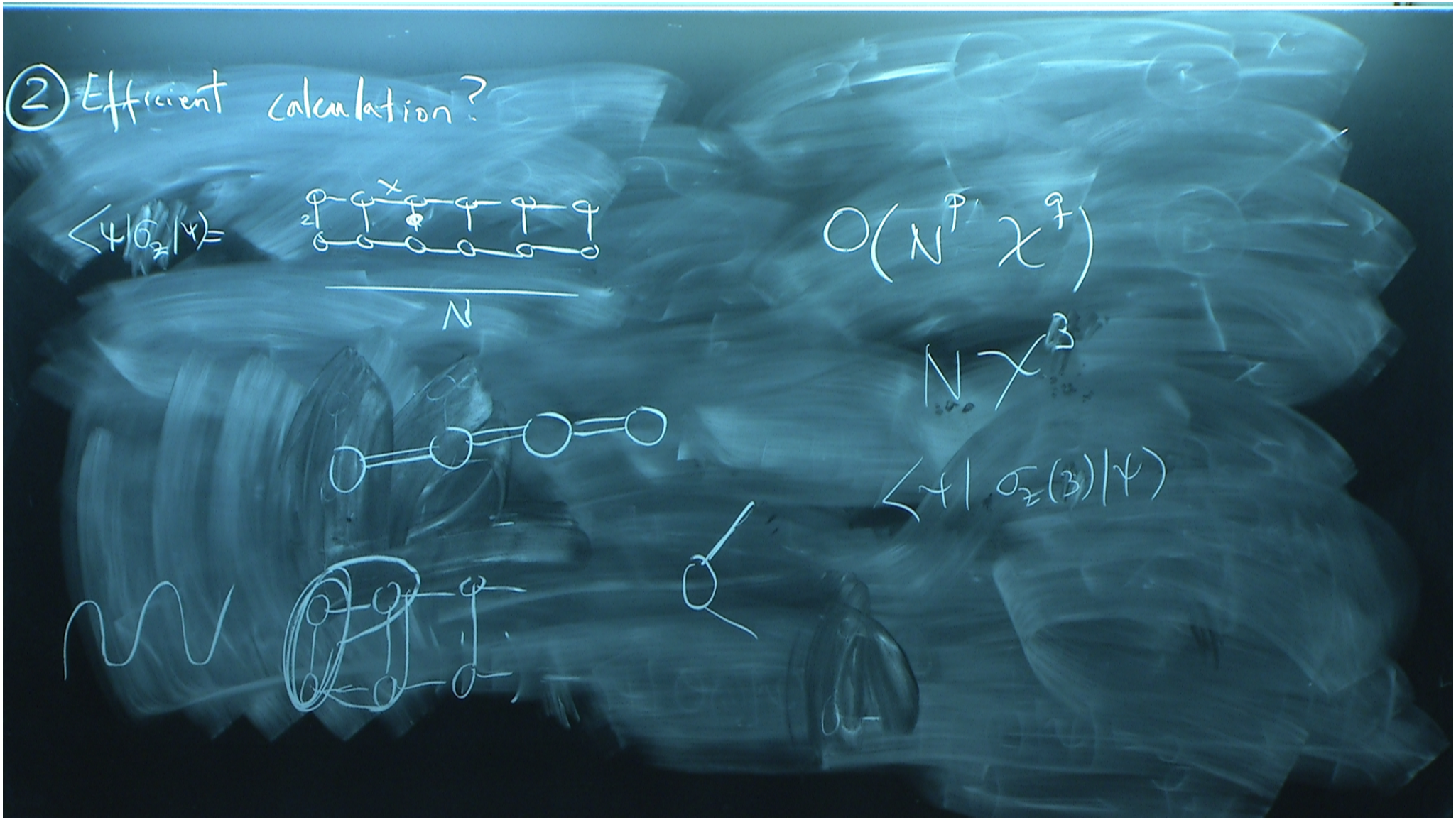
(2) Efficient calculation?

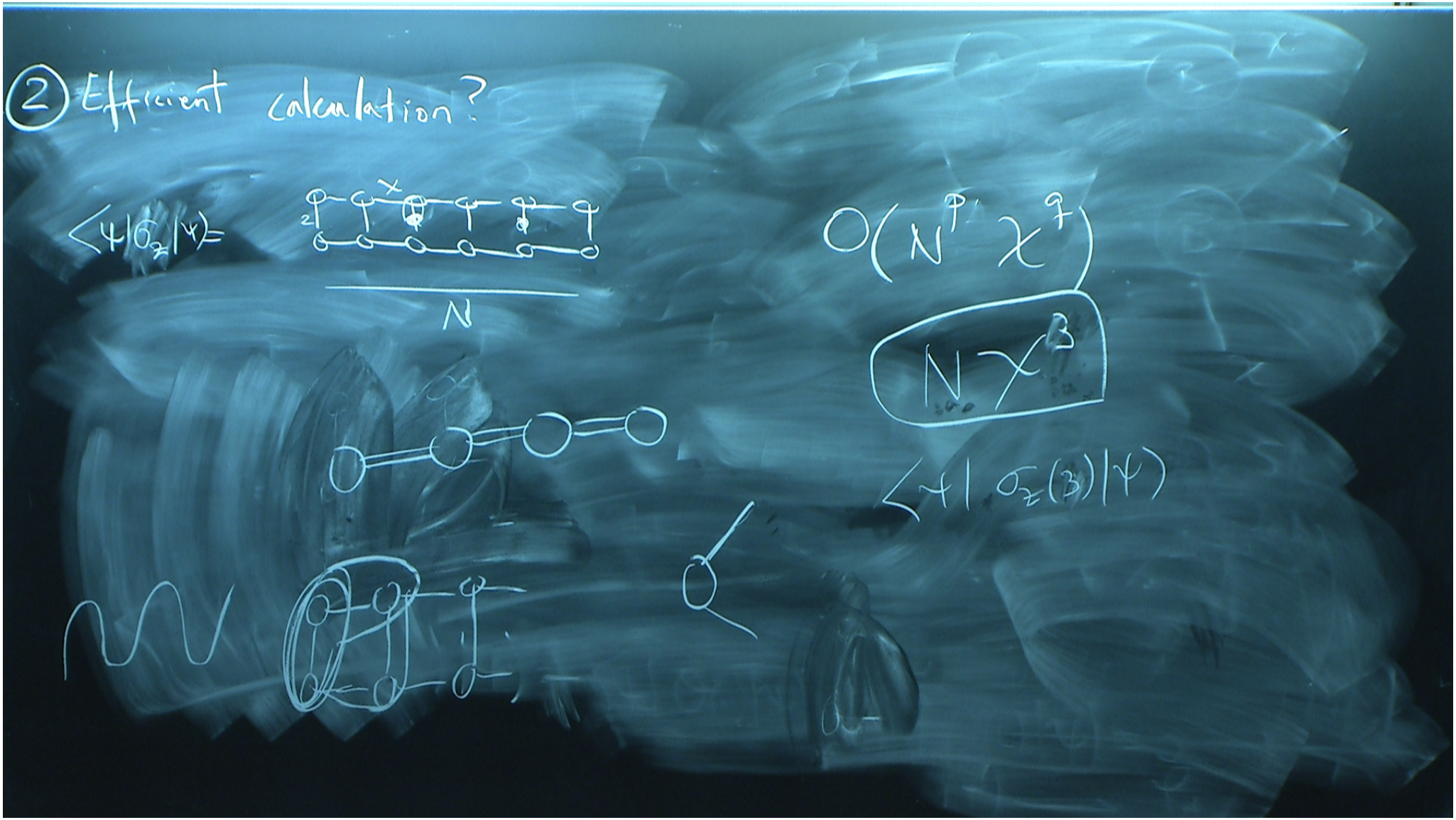
$\langle \psi | \psi \rangle =$ 

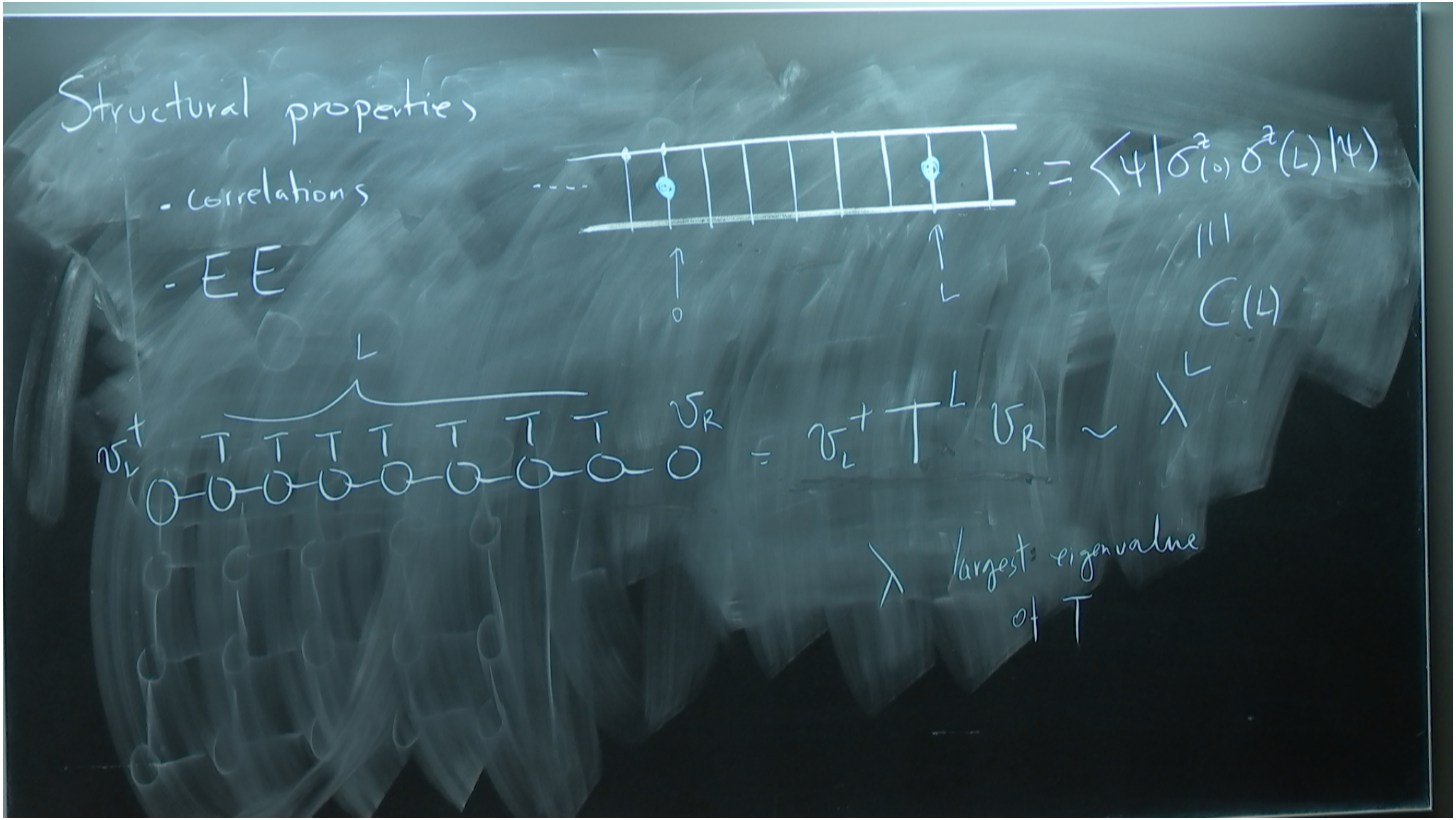
N

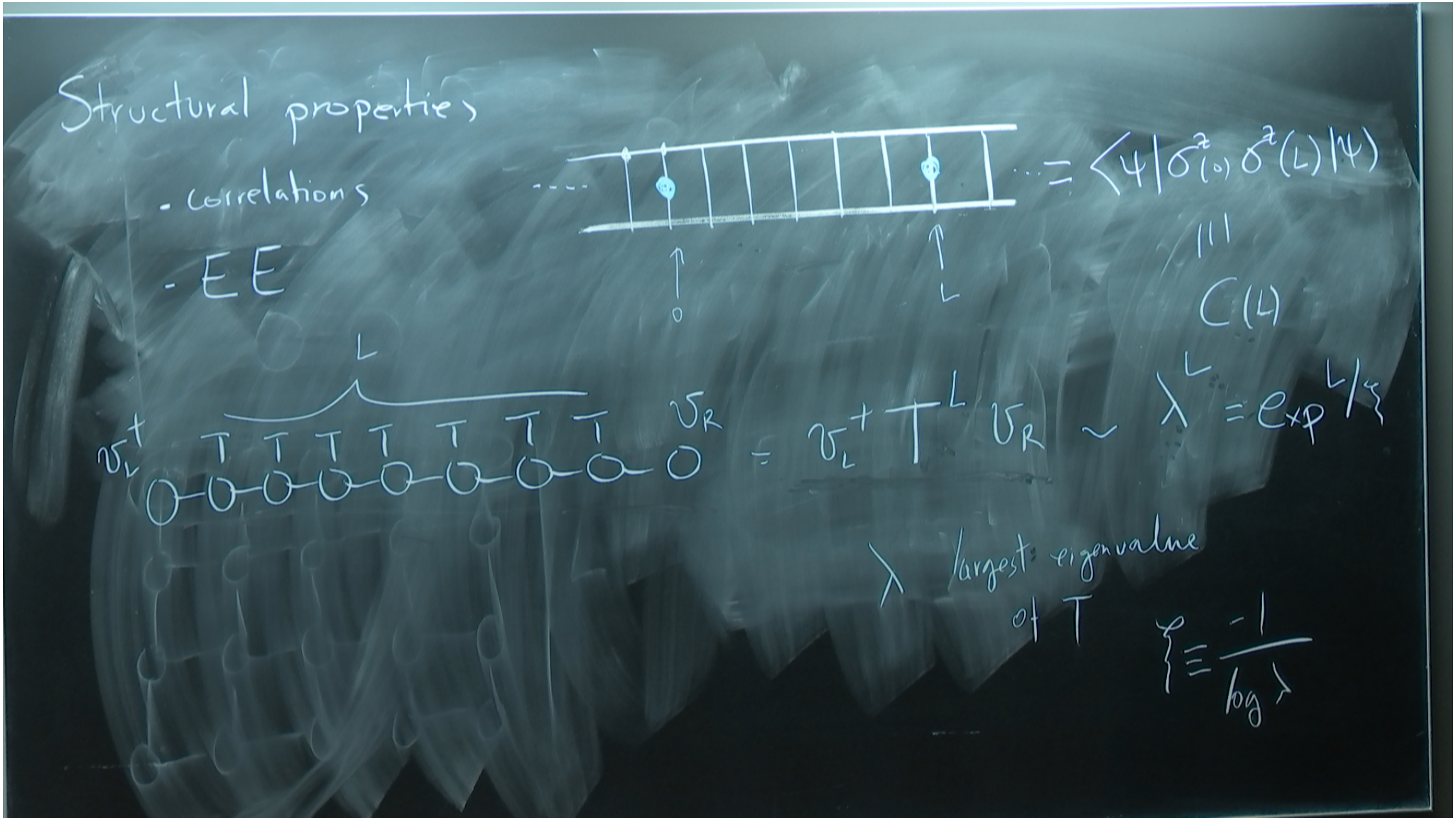
$O(N^p \chi^q)$

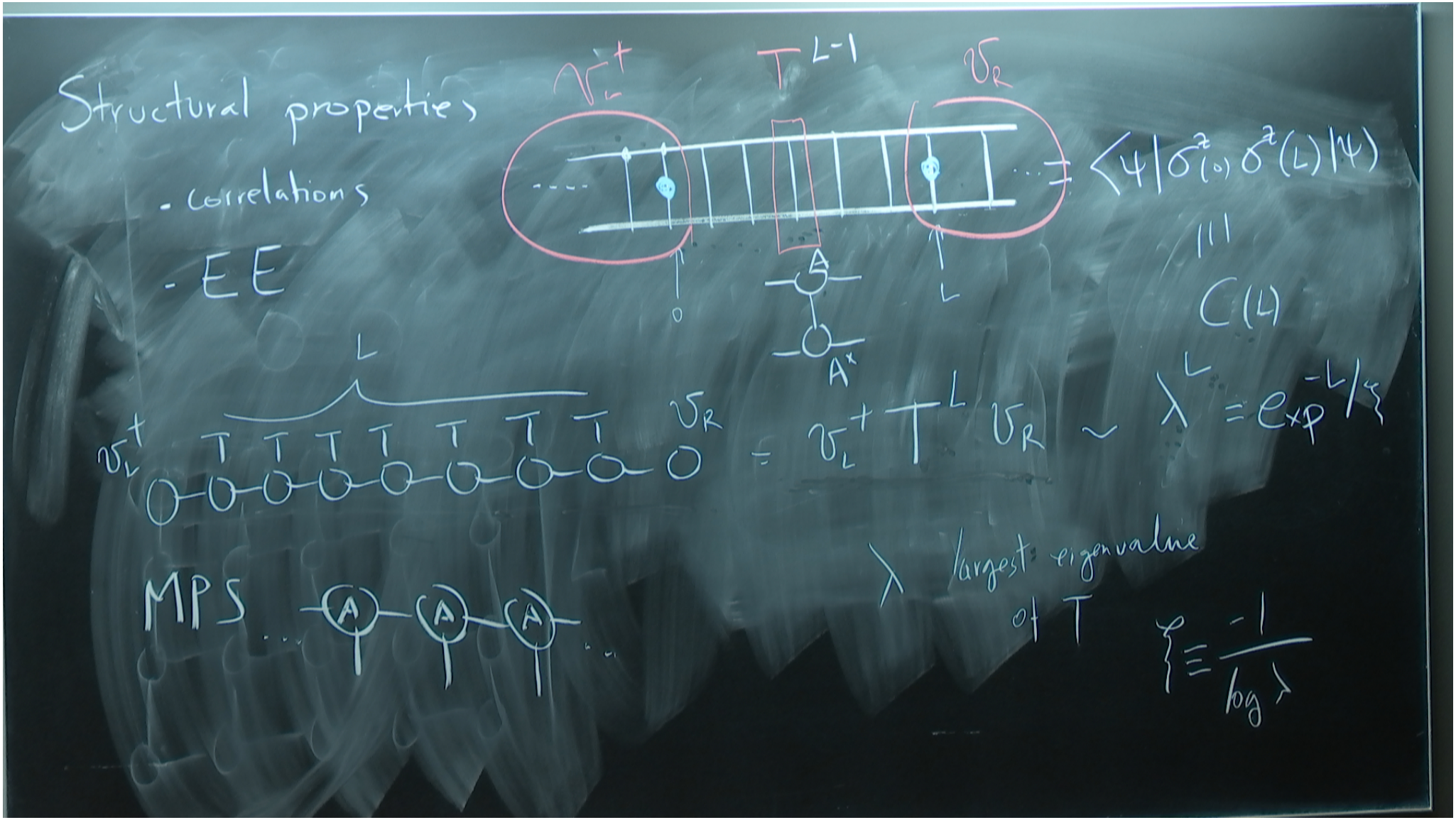

② Efficient calculation?
⟨ψ|ψ⟩ =
N
O(N^p χ^q)
N χ^β

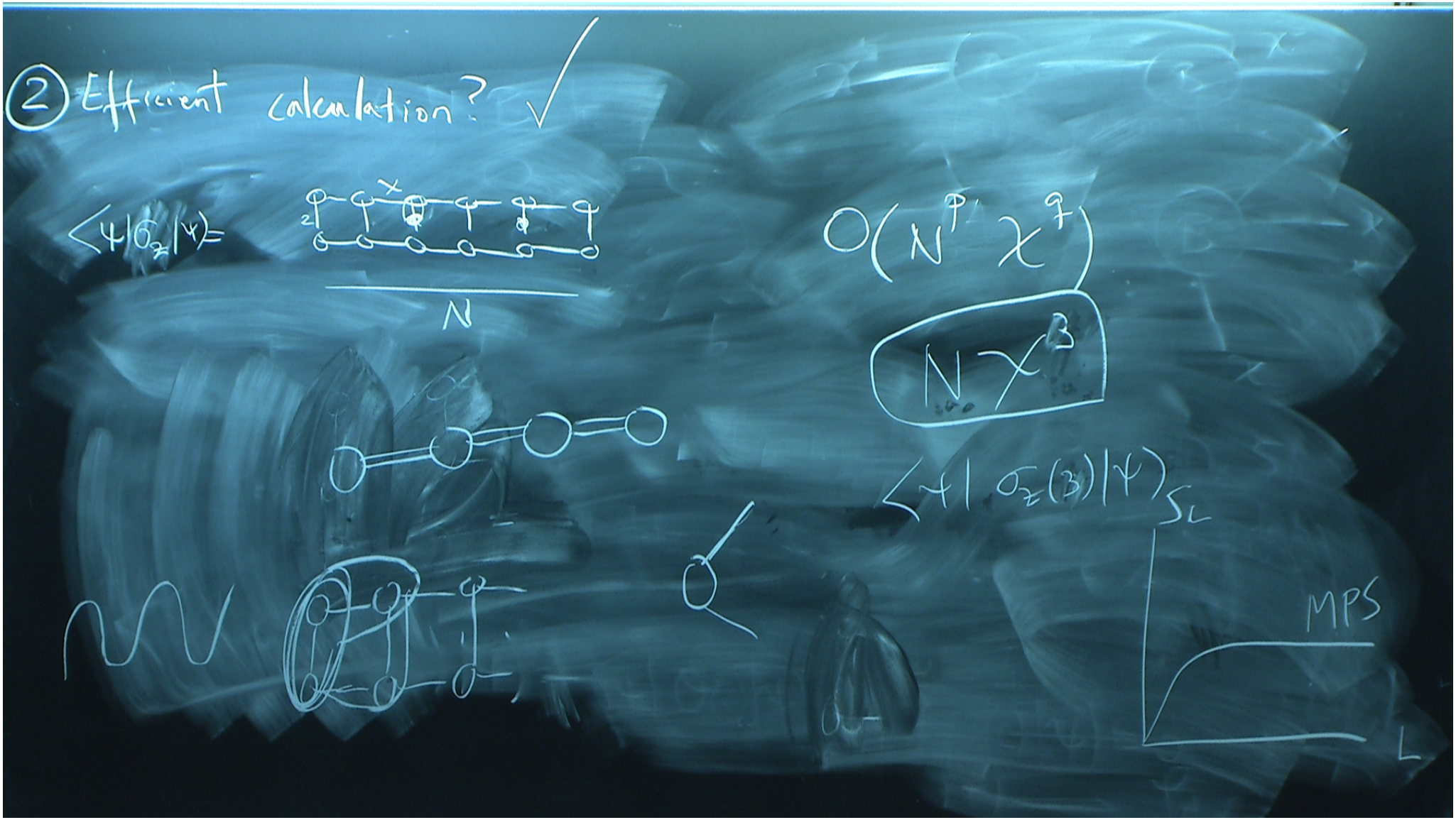

② Efficient calculation?
$\langle\psi|O_z|\psi\rangle =$
$O(N^p \chi^q)$
$N\chi^\beta$
$\langle\psi|\sigma_z(\beta)|\psi\rangle$


(2) Efficient calculation?
$\langle\psi|O_z|\psi\rangle =$
N
$O(N^p \chi^q)$
$N\chi^\beta$
$\langle\psi|\sigma_z(\beta)|\psi\rangle$


Structural properties
- correlations
- EE
$\langle\psi|\sigma^z_{(0)}\sigma^z_{(L)}|\psi\rangle$
0
L
C(L)
L
$v_L^\dagger$
T
v_R
$= v_L^\dagger T^L v_R \sim \lambda^L$
λ largest eigenvalue of T


Structural properties
- correlations
- EE
$\langle\psi|\sigma^z_{(0)}\sigma^z_{(L)}|\psi\rangle$
$C(L)$
L
$v_L^\dagger$
v_R
$= v_L^\dagger T^L v_R \sim \lambda^L = \exp L/\xi$
λ largest eigenvalue of T
$\xi \equiv \frac{-1}{\log \lambda}$


Structural properties
- correlations
- EE
$v_L^\dagger$
T^{L-1}
v_R
$\langle\psi|\sigma_{(0)}^z \sigma^z(L)|\psi\rangle$
0
L
A
A^*
C(L)
L
$v_L^\dagger$
T T T T T T T
v_R
$= v_L^\dagger T^L v_R \sim \lambda^L = e^{-L/\xi}$
MPS
A A A
λ largest eigenvalue of T
$\xi \equiv \frac{-1}{\log \lambda}$


② Efficient calculation? ✓
$\langle\psi|O_z|\psi\rangle$ =
N
$O(N\chi^3)$
$N\chi^3$
$\langle\psi|\sigma_z(3)|\psi\rangle$
S_L
MPS
L