Title: Mass gap for black hole formation in higher derivative and ghost-free gravity

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Abstract: The problem of the gravitational collapse of small mass in the higher derivative and ghost free theories of gravity is discussed. It will be demonstrated how higher derivative and non-local modifications of gravity equations regularizes static and dynamical solutions. Boosting a static solution of the linearized equations for the gravitational potential of a point mass we obtain a solution for the field of the ultra-relativistic source (gyraton). Using the latter we construct solutions for the collapsing spherical (thin and thick) null shell. By analysing the obtained solutions we demonstrate that for small enough value of the mass M an apparent horizon is not formed for the gravitational collapse of small mass in the higher-derivative and ghost free theories of gravity. We demonstrate that this â€œmass gapâ€• property is connected with the presence of the UV cut-off in such theories.

Mass gap for black hole formation in ghost-free gravity

Valeri P. Frolov,

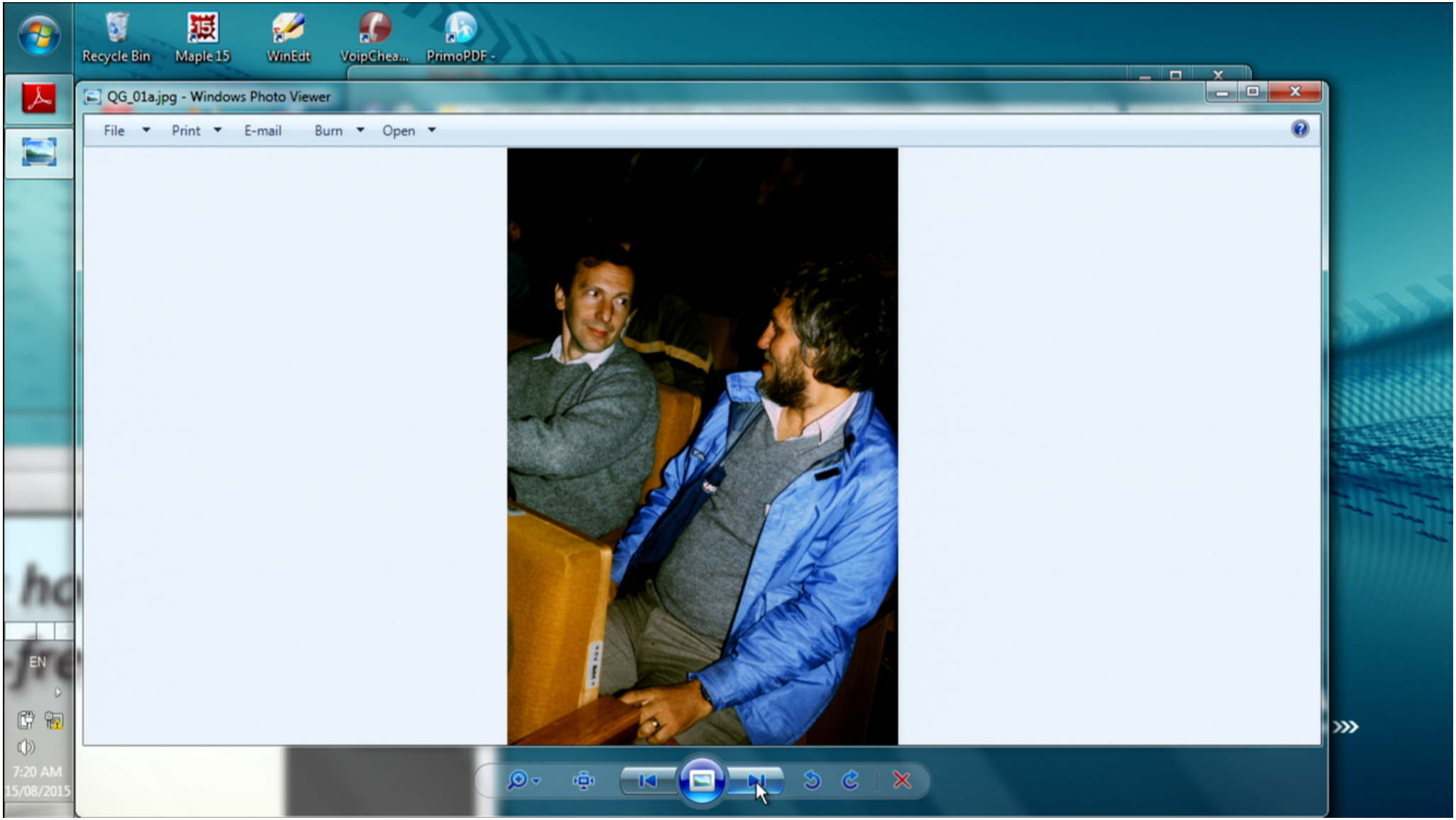
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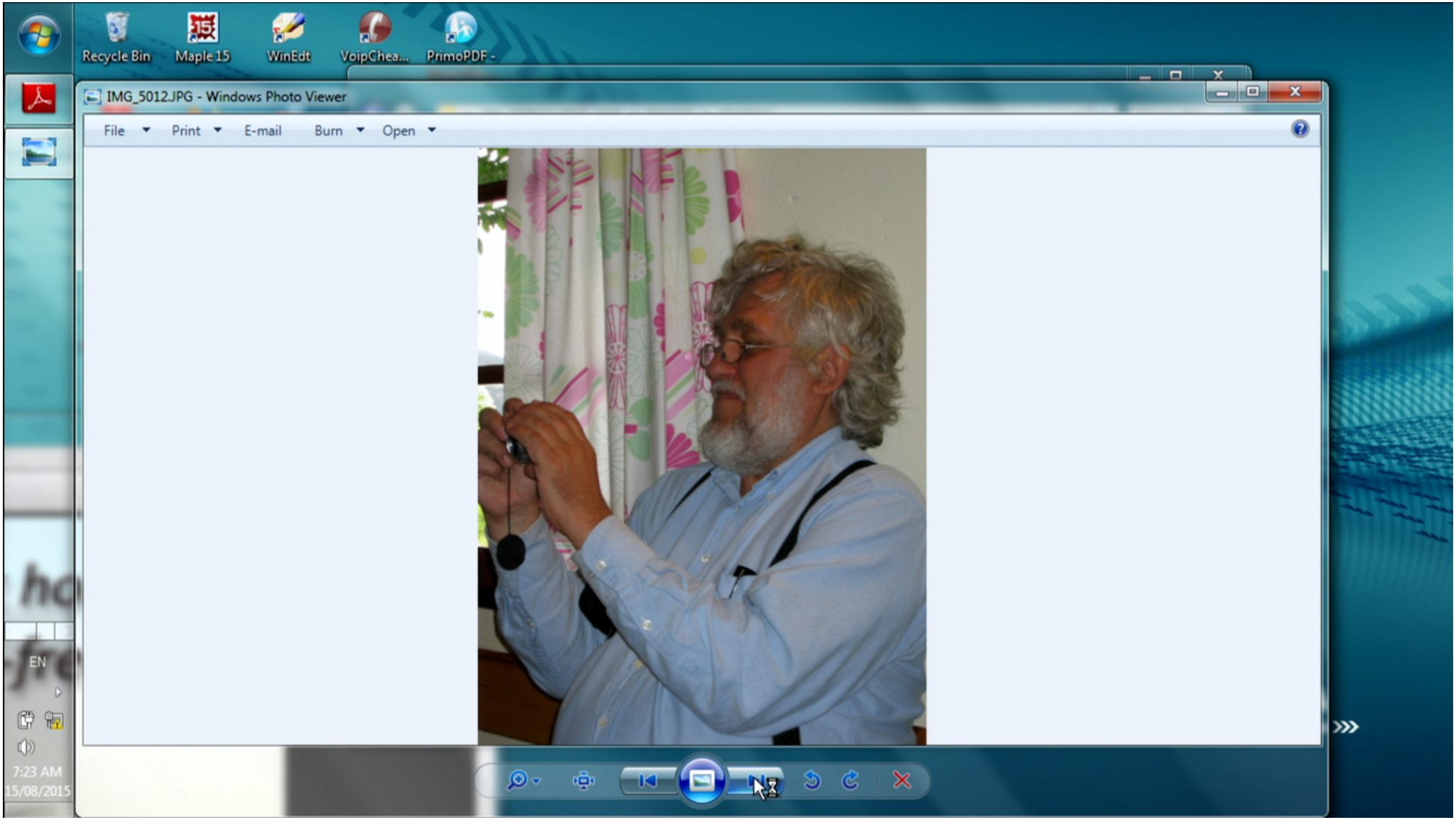
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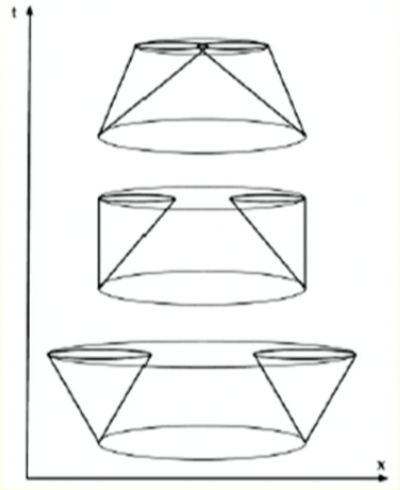
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Event horizon vs.apparent horizon

`Quasi-local definition' of BH: Apparent horizon



A compact smooth surface B is called a trapped surface if both, in- and out-going null surfaces, orthogonal to B, are non-expanding .

A trapped region is a region inside B.

A boundary of all trapped regions is called an apparent horizon.

Schwarzschild ST has a spacelike singularity.

RN and Kerr ST have a timelike singularity.

In both cases this is a curvature singularity.

Expectation 1: When curvature becomes high (e.g. reaches the Planckian value) the classical GR must be modified (quantum corrections, it is an emergent theory, etc.).

Expectation 2: Singularities of GR would be resolved.

Regularity at r=0 and AH

$$ds^2 = -F(t,r)\,dt^2 + \frac{dr^2}{g(t,r)} + r^2 d\omega^2,$$

$$F(t,r) \sim F_0(t) + F_1(t)\,r^2,\ g(t,r) \sim g_0(t) + g_1(t)\,r^2,$$

$$\Re^2 \sim \frac{4(g_0-1)^2}{r^4} = 4\left(\frac{[(\nabla r)^2 - 1]}{r^2}\right)^2.$$

Apparent horizon: g=$(\nabla r)^2 = 0$. If an AH crosses $r = 0$, then before this the curvature singularity is developed at $r = 0$.

Schwarzschild metric: $\varphi = -GM / r.$

$$ds^2 = -F dt^2 + \frac{dr^2}{F} + r^2 d\omega^2, \; F = 1 + 2\varphi.$$

Apparent (event) horizon at $F = 0, \; r = 2GM.$

Kretschmann scalar $\mathbb{R}^2 = \dfrac{48\,(GM)^2}{r^6}.$

Linearized version

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\varphi)(dr^2 + r^2 d\omega^2).$$

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Three connected problems:

1. Regularity of potential φ at $r = 0$;

2. Finiteness of the self-energy of a point charge;

3. Existence of AH: $|\varphi| \leq CM$. For $M < C/2,\ F > 0$.

Regularization :

$$\Delta\varphi = 4\pi GM\delta(\vec{r}) \rightarrow \varphi = \frac{GM}{r},$$

$$(\Delta + \mu^2)\tilde{\varphi} = 4\pi GM\delta(\vec{r}) \rightarrow \tilde{\varphi} = \frac{GMe^{-\mu r}}{r},$$

$$\varphi_{reg}(r) = \varphi(r) - \tilde{\varphi}(r) = \frac{GM(1-e^{-\mu r})}{r},$$

$$\varphi_{reg}(0) = GM\mu \rightarrow \text{Pauli-Villars regularization}$$

$$\Delta G = -I,\ (\Delta + \mu^2)\tilde{G} = -I,$$

$$G_{reg} = G - \tilde{G} = \frac{1}{\Delta + \mu^2} - \frac{1}{\Delta} = -\frac{1}{\Delta(1 + \Delta/\mu^2)};$$

$$\Delta(1 + \Delta/\mu^2)\varphi_{reg} = 4\pi G M \delta(\vec{r}) \rightarrow$$

Higher-derivative theory.

Source-smearing vs non-locality:

$$\Delta\varphi_{reg} = 4\pi G\tilde{\rho},\ \tilde{\rho} = (1 + \Delta/\mu^2)^{-1}\rho = Me^{-\mu r}/r.$$

2. Ghost Free Theories of Gravity

Quadratic in Curvature Action

Biswas,Koivisto,Mazumdar(2013):

$$S = \int dx\sqrt{-g}\left[R/2 + R_{\bullet}\hat{O}^{\bullet\bullet}R_{\bullet} \right],$$

$\hat{O}^{\bullet\bullet}$ is an operator constructed from ∇ and g.

1. All derivatives to the right - 14 invariants
2. Symmetries and Bianchi identities - 6 invariants
3. Covariant derivatives commute in flat ST
   - 3 invariants

$$S = \frac{1}{16\pi G}\int dx\sqrt{-g}\left[R + RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}F_3(\Box)R^{\mu\nu\lambda\sigma} \right]$$

$$ {}^{*}R^{\alpha\beta\gamma\sigma} \nabla^{k} {}^{*}R_{\alpha\beta\gamma\sigma} = R^{\alpha\beta\gamma\sigma} \nabla^{k} R_{\alpha\beta\gamma\sigma} - 4R^{\alpha\beta} \nabla^{k} R_{\alpha\beta} + R \nabla^{k} R $$

$$ = O(R^3) + \text{div}, k \geq 1 $$

4.There remains 2 non-trivial form-factors

$$S \sim \int d^4x \left[\frac{1}{2} h_{\mu\nu} a(\Box) h^{\mu\nu} + h_\mu^{\ \sigma} b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} \right.$$

$$+ hc(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} hd(\Box) h$$

$$\left. + \frac{1}{2} h^{\lambda\sigma} f(\Box) \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \right].$$

$$a + b = 0, c + d = 0, b + c + f\Box = 0.$$

IR GR limit: $a(0) = c(0) = -b(0) = -d(0) = 1$

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There are no extra poles (ghosts) if a and c are entire functions $\sim \exp(P(\Box))$.

Special cases: $\mathrm{GF_n}$ $\quad a = c = \exp(q(\Box/\mu^2)^n)$

1. Ghost free gravity I ($\mathrm{GF_1}$): $a = c = \exp(-\Box/\mu^2)$;

2. Ghost free gravity II ($\mathrm{GF_2}$): $a = c = \exp(\Box^2/\mu^4)$.

All such theories have a fundamental length scale $\lambda=\mu^{-1}$, where modification of the standard GR becomes important. We assume that this parameter is different from and much larger than the other scale parameter $\tilde{\lambda}$, where effects of quantum fluctuation becomes important. Under this assumption we can still use effective metric $g_{\mu\nu}=\langle g_{\mu\nu} \rangle$.

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3. Weak Gravity: Gravitational Field of a Point Mass

Static solutions of linearized gravity equations in the Newtonian limit

Stress-energy tensor: $\tau_{\mu\nu} = \rho(\vec{r})\delta^0_\mu\delta^0_\nu$

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\psi+2\varphi)d\ell^2 .$$

Biswas, Gerwick, Koivisto, Mazumdar (2012);

V.F. and Zelnikov (2015):

$$a(\Delta)\Delta\psi = 8\pi G\rho,$$

$$[a(\Delta) - (D-1)c(\Delta)]\Delta\varphi +$$

$$+ (D-2)c(\Delta)\Delta\psi) = 8\pi G\rho$$

Stelle (1978); Modesto, Netto, Shapiro (2014)

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For a point mass $\rho = m\delta(\vec{r})$ the solution is spherically symmetric. We call it finite if near $r = 0$ it is of the form

$$\psi(r) \sim \psi_0 + \psi_1 r + \frac{1}{2}\psi_2 r^2 + O(r^3),$$

$$\varphi(r) \sim \varphi_0 + \varphi_1 r + \frac{1}{2}\varphi_2 r^2 + O(r^3).$$

A finite solution is not necessary regular one.

$$R^2 = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = \frac{A_2}{r^2} + \frac{A_1}{r} + O(1),$$

$$A_2 = 8(4\psi_1^2 - 5\psi_1\varphi_1 + 3\varphi_1^2),$$

$$A_1 = 16[\psi_1(5\psi_2 - 4\varphi_2) - 4\varphi_1(\psi_2 - \varphi_2)].$$

The solution is regular if $\psi_1 = \varphi_1 = 0$.

1. The gravitational collapse is regular in linearized GF theories of gravity;
2. Head-on collision of UR particles;

$\Rightarrow$ Mass gap for mini black hole production.

GF_1 theory :

$$\hat{O} = \exp(-\Delta / \mu^2)\Delta,\ f(s) = -\vartheta(s - \mu^{-2})$$

$$\exp(\Delta / \mu^2)\Delta^{-1} = \int_0^\infty ds\, f(s) e^{s\Delta},$$

Heat kernel ($d = D$ -1):

$$< x' \mid e^{s\Delta} \mid x >= K(\mid x - x' \mid; s) = \frac{e^{-|x-x'|^2/(4s)}}{(4\pi s)^{d/2}}$$

$$\psi_d(r) = 8\pi G m \int_0^\infty ds\, f(s)\, K_d(r; s)$$

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GF_2 theory :

$$\frac{1}{\Delta}\exp(-\Delta^2/\mu^4) = \frac{i}{2}\int_{-\infty}^{\infty} d\omega e^{i\omega\Delta}\,\mathrm{erf}(\mu^2\omega/2)$$

For GF_2: $K_d^{GFII}(\omega) = K_d^{GFI}(s = i\omega)$

$$\psi_d = -4\pi i G^{(d)} m \int_{-\infty}^{\infty} d\omega K_d(r \mid \omega)\,\mathrm{erf}(\mu^2\omega/2)$$

We use dimensionless quantities:

$$\tilde{G} = \mu^{d-1} G^d, \quad \tilde{m} = m/\mu, \quad \beta = \frac{1}{8}\mu^2 r^2.$$

For both theories one has

$$\psi_d^{\bullet} = -\frac{\tilde{G}\tilde{m}}{(8\pi)^{d/2-1}} S_d^{\bullet}(\beta),$$

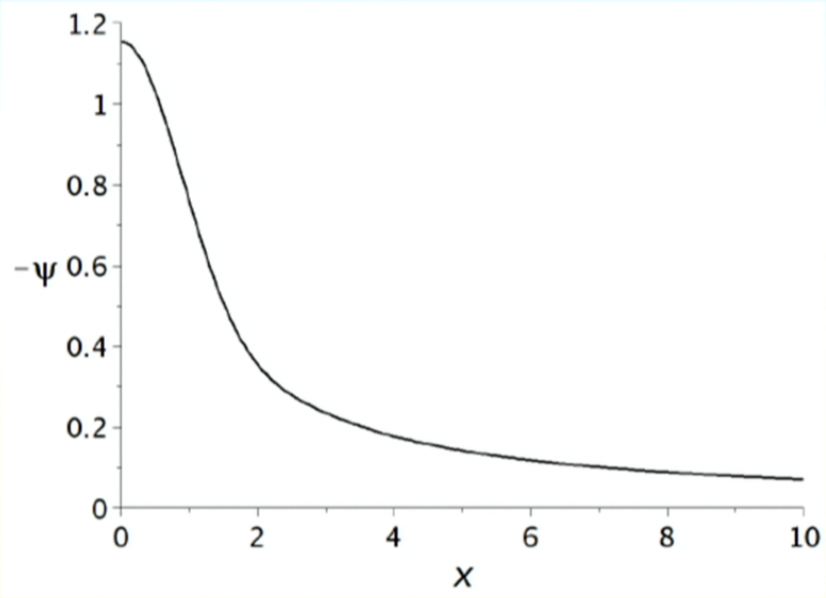
$$\partial_\beta \psi_d^{\bullet} = -8\pi \psi_{d+2}^{\bullet},$$

$$\psi_d^{\bullet} \sim -\frac{2G^d m\, \Gamma(d/2-1)}{\pi^{d/2-1}} \frac{1}{r^{d-2}}, \quad \psi_d^{\bullet} \sim -\frac{2G^3 m}{r};$$

$$\psi_d^{\bullet} \sim -A_0 + A_2 r^2 + O(r^4).$$

$$\mathrm{GF}_1 : S_d = \frac{2\gamma\left(\frac{d}{2}-1, 2\beta\right)}{\beta^{\frac{d}{2}-1}}$$

GF_2: hypergeometric functions



Example of the plot of $-\psi_3^{\mathrm{GH}_2}(x)$, where $x = \sqrt{\beta}$.

4. GF Gyratons

We obtain now a solution for an ultra-relativistic particle.

In $4D$ Einstein gravity -- Aichelburg-Sexl solution.

Higher dimensional generalization for a particle with spin -- gyraton metric [V.F., Fursaev, PRD 71 104034 (2005); V.F., Israel, Zelnikov, PRD 72, 084031]

$$ds^2 = ds_0^2 + dh^2,$$

$$ds_0^2 = -dt^2 + d\ell^2, \quad d\ell^2 = dy^2 + d\zeta_\perp^2.$$

Boost transformation:

$$t = \lambda_- v + \lambda_+ u, \; y = \lambda_- v - \lambda_+ u,$$

$$\lambda_\pm = (1 \pm \beta)\gamma/2, \; \gamma = (1-\beta^2)^{-1/2}.$$

In the limit $\gamma \to \infty$ one has

$$y \sim -\gamma u,\ t \sim \gamma u,\ \ell^2 \sim \gamma^2 u^2 + \zeta_\perp^2,$$

$$ds^2 = -dudv + d\zeta_\perp^2 + dh^2\ ,$$

$$dh^2 = \Phi du^2\ , \quad \Phi = -2 \lim_{\gamma \to \infty} (\gamma^2 \psi)\ .$$

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Penrose limit: $E = \gamma m = \text{const}, \; E = \mu \tilde{E}.$

$$\lim_{\gamma \to \infty} \frac{\gamma \exp(-\gamma^2 u^2/(4s))}{\sqrt{4\pi s}} = \delta(u), \text{ (For GF}_2 \; s = i\omega).$$

$$K_d(r) \Rightarrow K_{d-1}(\rho), \quad \rho^2 = \zeta_\perp^2.$$

$$\Phi_d = F_d(\rho)\, \delta(u),$$

$$F_d(\rho) = -2\mu^{-1} \psi^{\bullet}_{d-1}(\rho)\Big|_{\tilde{m}=\tilde{E}}$$

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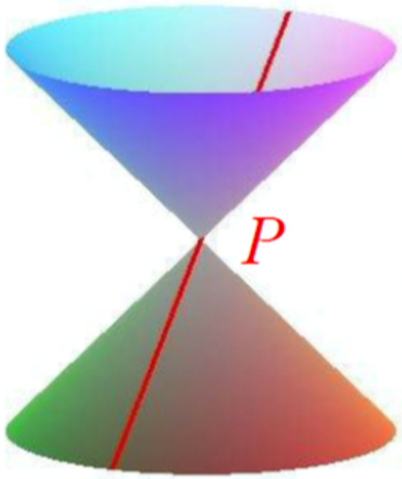
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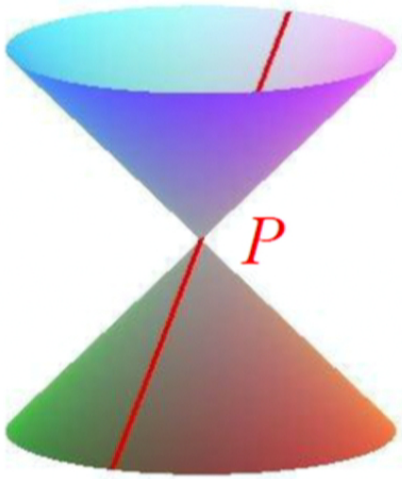
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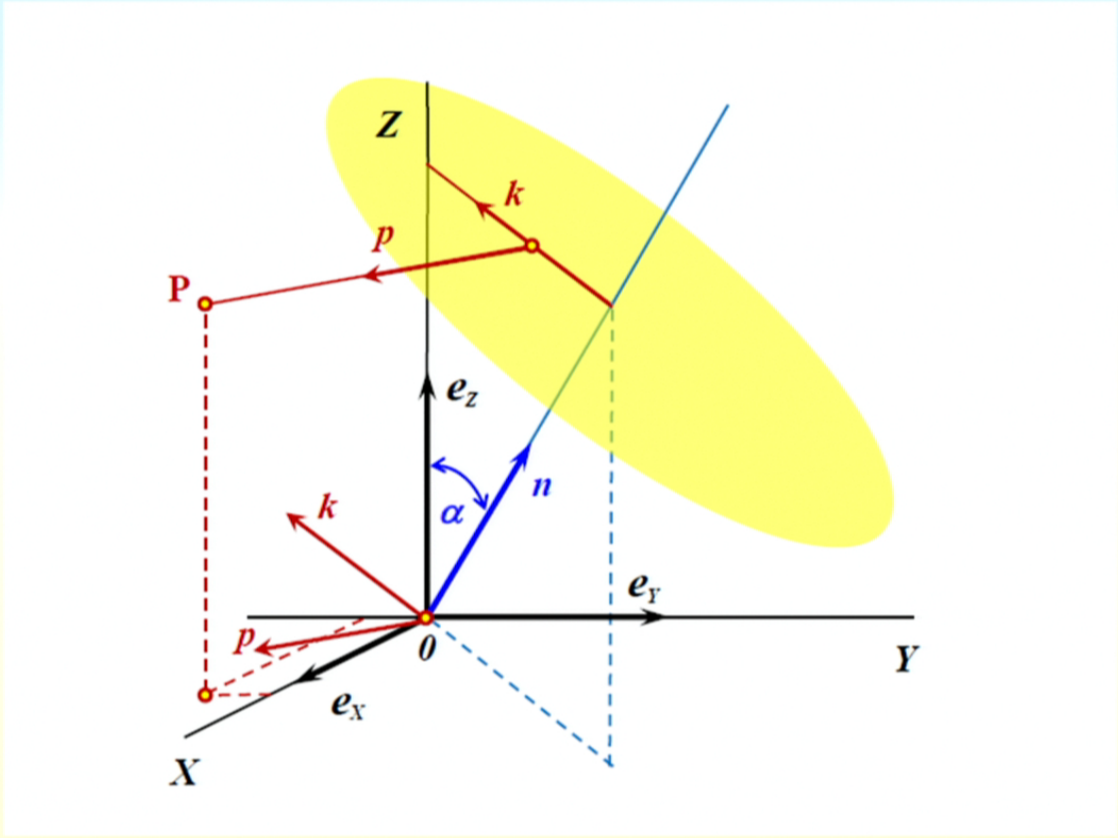
5.Null Shell Collapse in GH Gravity

Metric for Thin Null Shell



Metric for Thin Null Shell





Gyraton frame vs Minkowski frame

The result of averaging:

$$< dh^2 >= \frac{\dot{\psi}_2}{\mu r}\left[\left(dt - \frac{t}{r}dr\right)^2 + \frac{r^2 - t^2}{2}d\omega^2\right].$$

$F = 0$ inside the null cones.

For $F = \text{const}$ the metric $ds^2 = ds_0^2 + < dh^2 >$

is flat and $< dh^2 >$ can be "gauged away" .

For GR $ds^2 = ds_0^2 + \frac{2GM}{r}(dt^2 + dr^2)$

Apparent Horizon

$$(\nabla\rho)^2 \equiv g = 1 - \frac{GM}{\mu r}\beta S_4(\beta)$$

$$t = \pm\sqrt{1-\alpha^2}\,r, \quad \beta^2 = \tfrac{1}{8}\mu^2(r^2 - t^2) = \tfrac{1}{8}\mu^2\alpha^2 r^2,$$

$$g = 1 - \frac{GM\alpha}{\sqrt{8}}P_3(\beta), \quad P_3(\beta) = \sqrt{\beta}S_4^{\bullet}(\beta).$$

Critical value GM_0: when g first reaches 0.

For GF_2 it happens when $\alpha = 1$ and

$GM = GM_0 = 1.386.$

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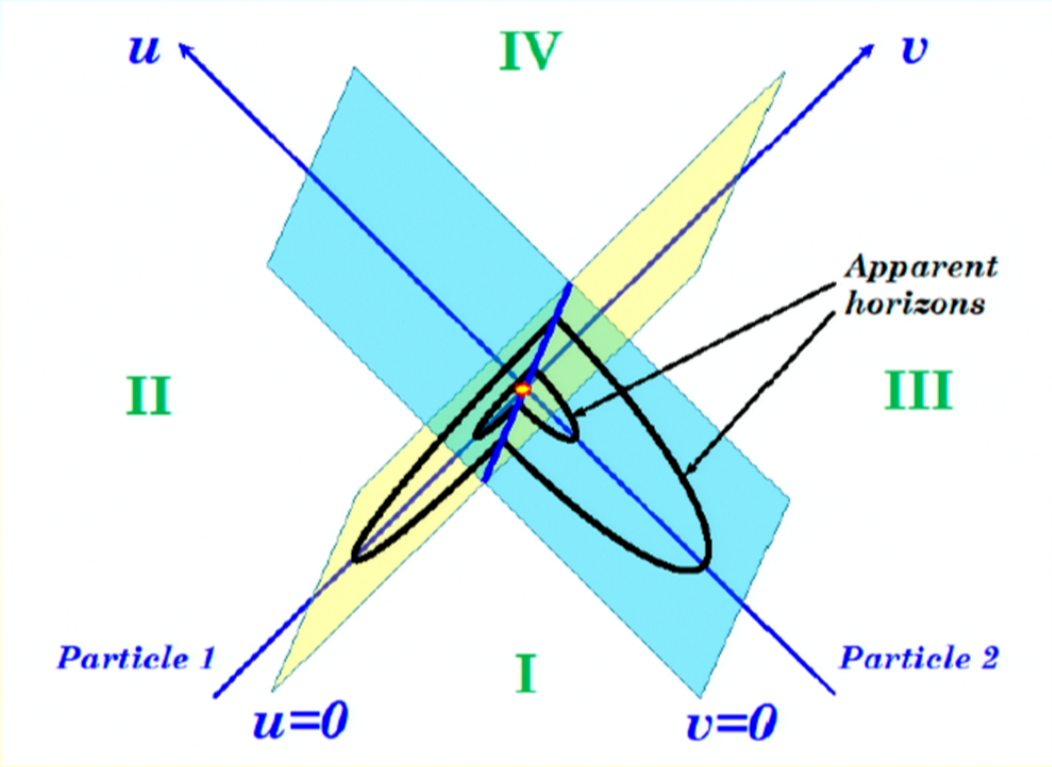
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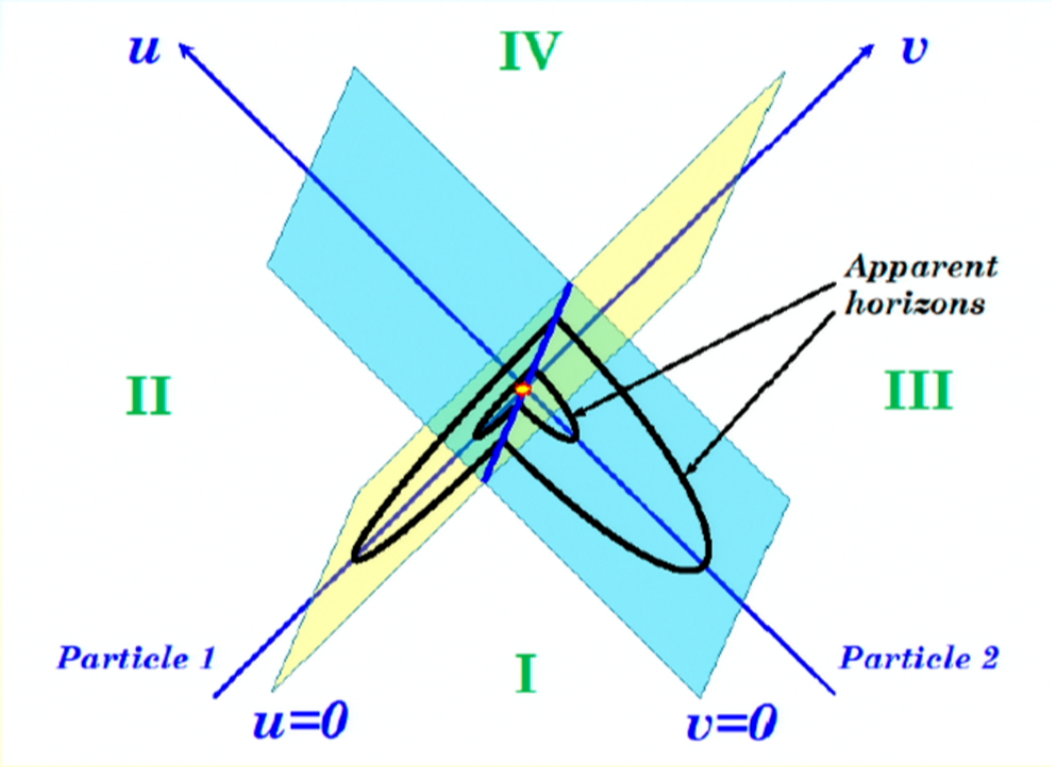
$GM = GM_0 = 1.386.$

Denote $\mu^{-1} = \lambda$. Then we have

$R \sim \frac{GM}{T\lambda^2}$. For $T \geq \lambda \Rightarrow R \leq GM\mu\frac{1}{\lambda^2}$.

If $GM\mu \ll 1 \Rightarrow R \ll R_{crit} = \lambda^{-2}$. Then one can neglect all higher in curvature corrections (weak field regime!)


u
IV
v
Apparent
horizons
II
III
Particle 1
I
Particle 2
u=0
v=0


u
IV
v
Apparent
horizons
II
III
Particle 1
I
Particle 2
u=0
v=0

Eardley and Giddings (2002); Yoshino and Nambu (2003); Yoshino and Rychkov (2005)

$$\Phi_d = F_d(\rho)\,\delta(u);$$

$$v = -\Psi(\rho) \text{ at } u = 0 \quad \text{and } u = -\Psi(\rho) \text{ at } v = 0$$

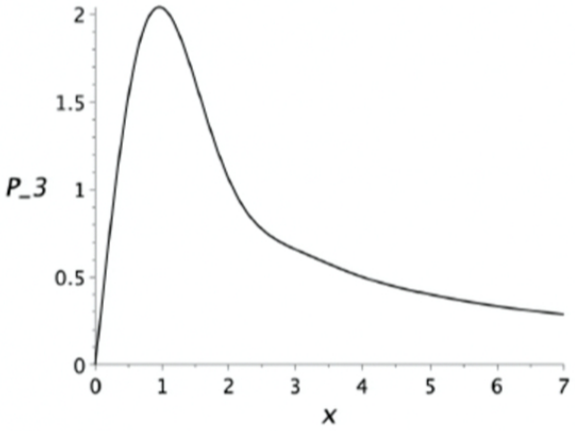
$$\nabla^2(\Psi - F_d) = 0, \quad \text{inside } C$$

$$(\nabla\Psi)^2 = 4, \quad \text{on } C.$$

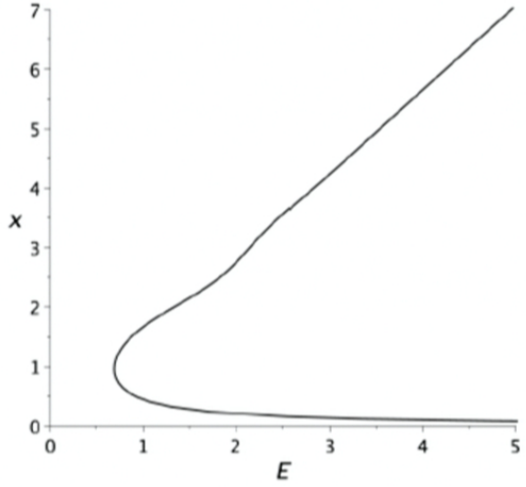
$$\Psi = F_d(\rho) - F_d(\rho_C), \quad (\nabla F_d)^2_C = 4.$$

$$2\pi\mu\rho\,|\,\psi^{\bullet}_{d+1}\,| = 1,$$

$$P^{\bullet}_d = \sqrt{\beta}\,S^{\bullet}_{d+1} = \frac{\sqrt{2}(8\pi)^{(d-3)/2}}{\tilde{G}\tilde{E}} \qquad (*)$$

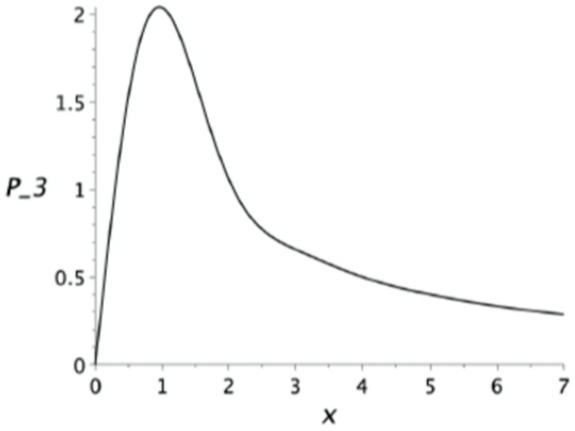


$P_3(x) = \sqrt{\beta} S_4(\beta)$ as a function of $x = \sqrt{\beta}$ for GF_2 theory in d=3 ST (left plot). $P_3(x)$ has maximum 1.451 at x=0.917.

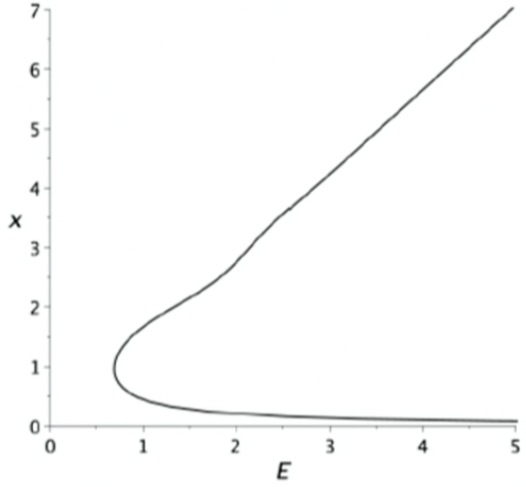


Solution of Eqn.(*) gives radii of inner and outer apparent horizons for $\tilde{G}\tilde{E} \geq 0.693$. At $\tilde{G}\tilde{E} = 0.693$ they meet one the other. For smaller energy there is no apparent horizon.

In other words, for $\tilde{G}\tilde{E} < 0.693$ a 'black hole' is not formed.



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7. Summary and Discussions

1. We considered linearized equations of GF theories of gravity.
2. Gravitational potential of a point source is regular for GF theories.
3. Taking Penrose limit we found gravitational field for these theories.
4. We obtained a solution for a spherical collapse of null shell.
5. We obtained conditions for an apparent horizon formation for the head-on collision of ultra-relativistic particles.