Title: What are the laws of quantum thermodynamics?

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Abstract: TBA

#Unruhfest

- Particles are what makes particle detectors click
- Time is what clock ticks measure
- Cooling is done by a thermal machine
- Computing is done by a Turing machine
- Work is a physical process (raising weight)
- Thermodynamics is defined by what an experimenter can do (ultimate limits)

What are the laws of Quantum Thermodynamics?

J. Oppenheim (UCL)

Masanes, J.O., quant-ph/1412.3828

Brandao, Horodeck, Ng, J.O., Wehner PNAS (2014)
quant-ph/1305.5278

Horodeck:, J.O.; Nature Communications (2013)
quant-ph/1111.3834

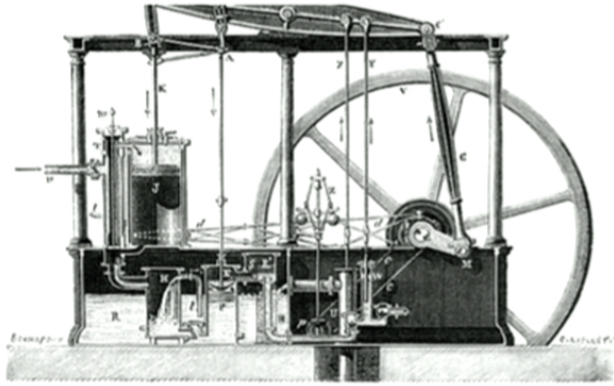
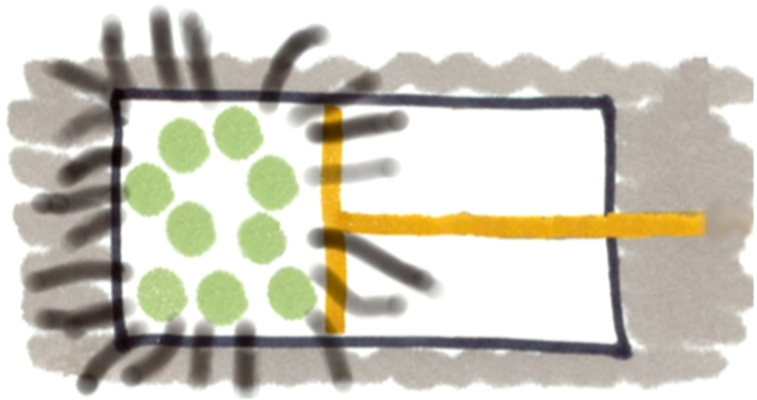


Fig. 59. — Machine à balancier de Watt.

1st wave
Carnot (1824)
Joule (1843)
Kelvin (1849)
Clausius (1854)



2nd wave
(stat mech)
Maxwell (1871)
Boltzman (1875)
Gibbs (1876)

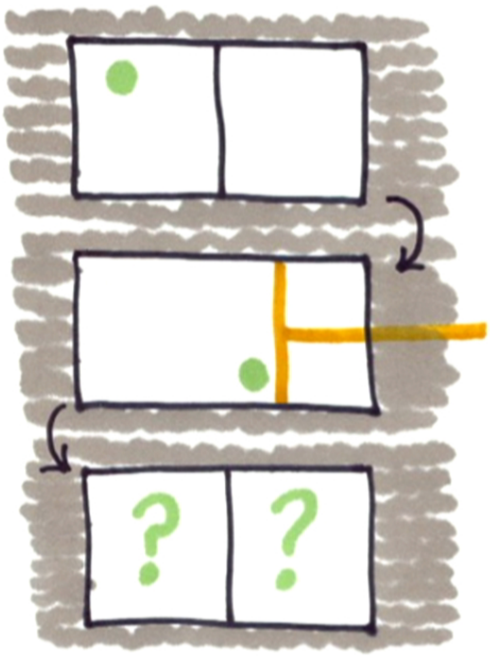
Thermodynamics as information

Maxwell

Szilard

Landauer

Bennett



Outline

- Recast laws of thermodynamics
- State transformations: 2nd laws: $F \longrightarrow F_\alpha$
- Many families of 2nd laws depending on "how cyclic" our process is
- Time for transformation: quantitative 3rd laws
- Work of transition: W: $(\rho,H) \longrightarrow (\sigma,H')$
- Class of operations: 0th, 1st law
- Tools from quantum information theory

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Quantum Thermodynamics

- **Gibbs state with full information**
  - Gemmer, Michel, Mahler (2005), Popescu, Short, Winter (2006)
- **Meaning of Negative Entropy, conditional erasure**
  - Del Rio et al. (2011), Faist et al. (2013)
- **Smallest possible fridges**
  - Linden, Popescu, Skrzypczyk (2010)
- **Deterministic tranformations**
  - HHO (2003), Dahlston et al. (2010), HO (2011), Aaberg (2011), Egloff et al. (2012)!
- **Average work extraction**
  - Brandao et. al. (2011), Skrzypczyk et. al. (2013)
- **Non-ideal heat baths, correlations, entanglement**
  - Reeb, Wolf (2013); Gallego et. al. (2013), Hovhannisyan et. al. (2013), Mueller et. al. (2014)
- **Thermalisation times**
- **Micro-engines & machines**
  - Scovil & Schultz-Dubois (1959), Howard (1997)
  - Rousselet et al. (1994), Faucheux et al. (1995), Scully (2002)
- **Pioneering works**
  - Ruch and Mead (75), Janzig et. al. (2000)

Classical thermodynamics

- It's called thermodynamics because we take the thermodynamic limit!
- System size, number of particles $\longrightarrow \infty$
- Small fluctuations
- Thermodynamics in the opposite extreme? Finite size (micro, nano) and/or quantum

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Quantum thermodynamics

- No thermodynamic limit
- Coherences in energy eigenbasis
- More precise control
- Rigorous theory
- Large fluctuations

3 laws of thermodynamics

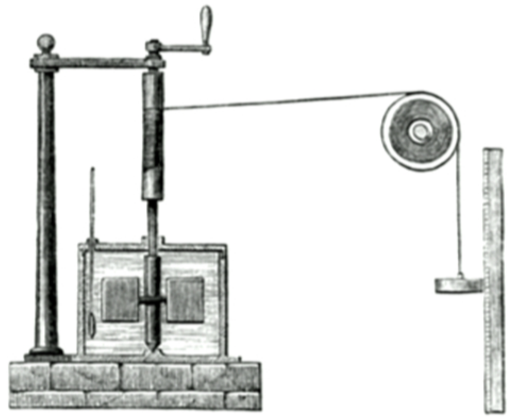
0) If R_1 is in equilibrium with R_2 and R_3 then R_2 is in equilibrium with R_3

1) $dE = dQ - dW$ (energy conservation)

2) Heat can never pass from a colder body to a warmer body without some other change occuring. – Clausius

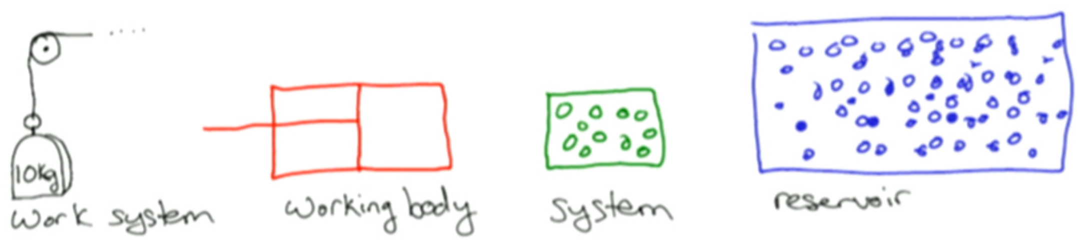
3) One can never attain T=0 in a finite number of steps

1) $dE = dQ - dW$ (energy conservation)



Joule (1843)

Not a consequence, but part of the class of operations



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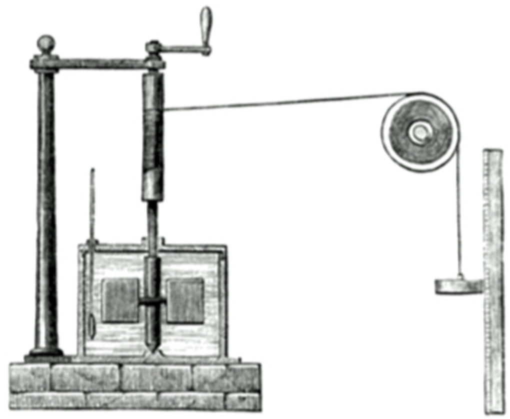

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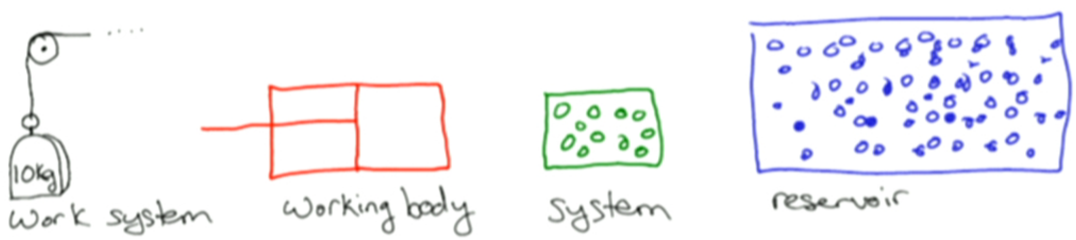
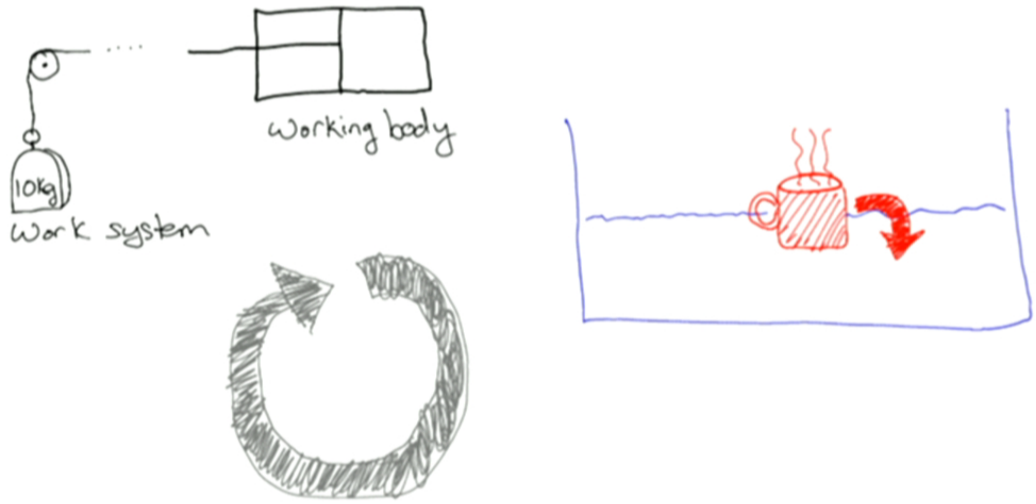


The second law

Heat can never pass from a colder body to a warmer body without some other change occurring – Clausius



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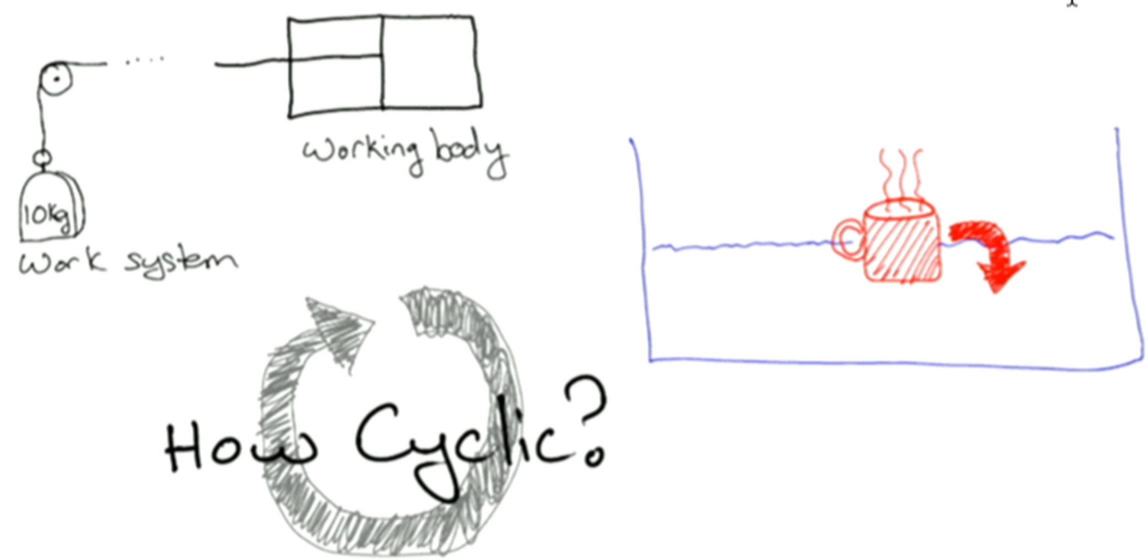
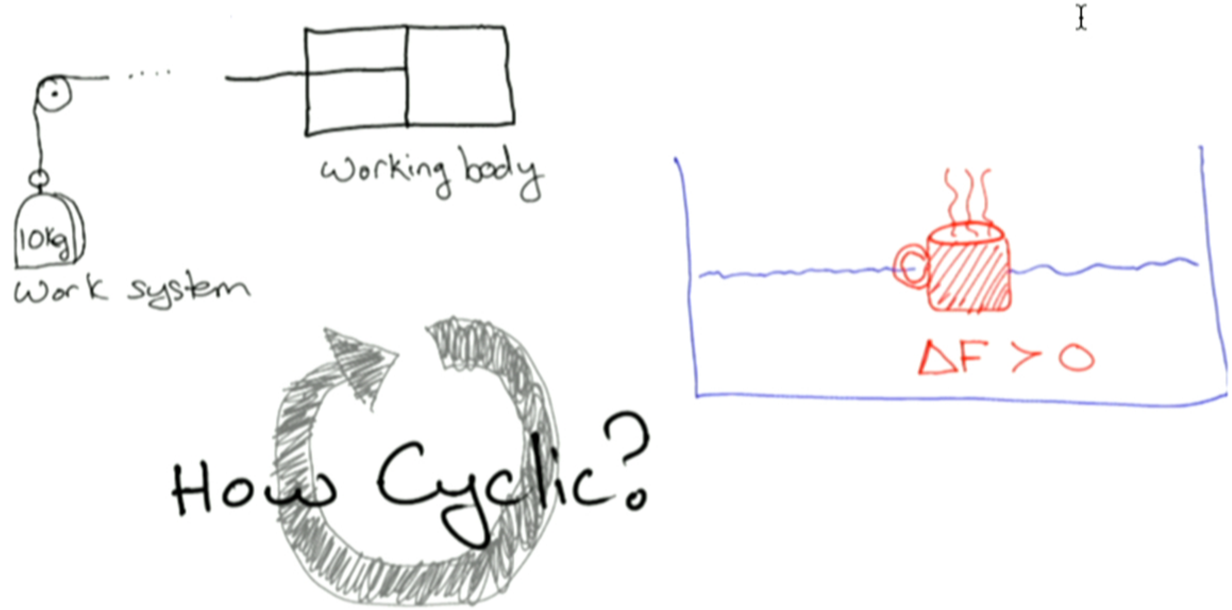
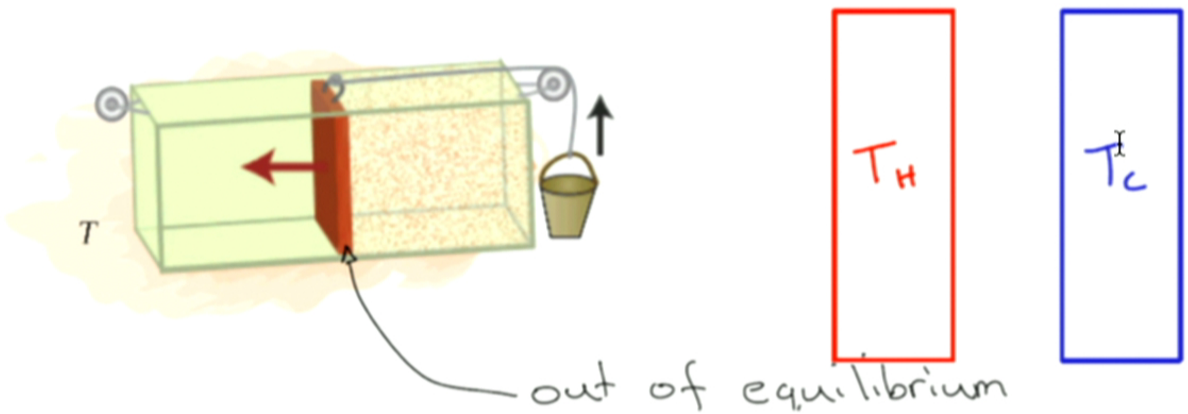


The second law
(+ first law)

In any cyclic process, the free energy of a system can only decrease.



Free Energy



$F = E - TS$

$W_{\text{gain}} = F(\rho_{\text{initial}}) - F(\rho_{\text{final}})$

$\rho_{\text{initial}} \rightarrow \rho_{\text{final}}$ only if $\Delta F \geq 0$

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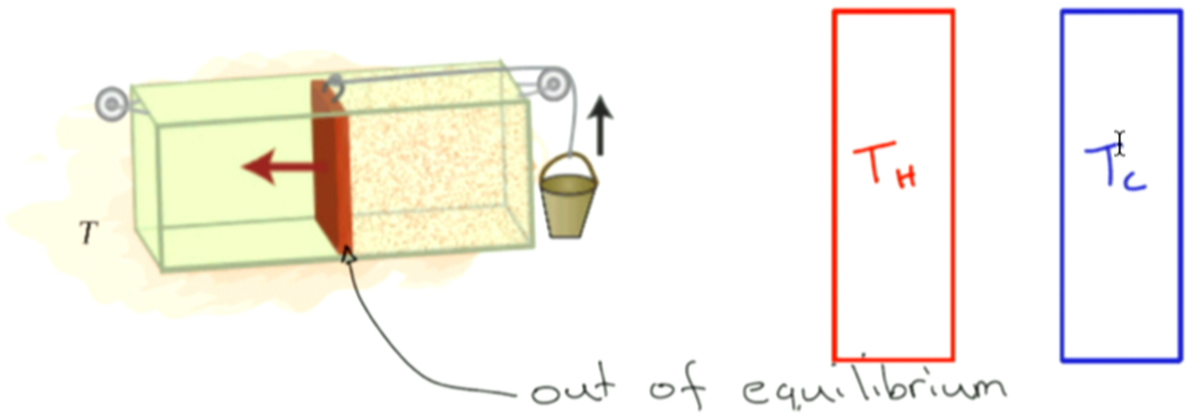

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What is Thermodynamics??

Catalytic Thermal Operations Λ_τ

- (ρ_s, H_s)

  ρ_s = **resource**

  H_s = Hamiltonian
- adding **free states** τ_R
- work system W
- borrowing ancillas (working body) and returning them in the "same" state σ_c
- energy conserving unitaries U
  (**1st law**) $[U, H_s + H_w + H_R + H_c] = 0$
- tracing out (trash)

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Thermal Operations

or in the micro - regime

1 law of quantum thermodynamics

Class of operations

0) The only free state τ which doesn't enable arbitrary transitions is the Gibbs state
1) Energy conserving unitaries

State Transitions

2) [cyclic]* ρ_s must get closer to τ_s in terms of free energy type distances $F_\alpha(\rho_s \| \tau_s)$ $\alpha \geq 0$

2') [single system] ρ_s must get closer to τ_s in terms thermo-majorisation

*depends on how cyclic

Zeroeth Law

After decohering in the energy eigenbasis, one can extract work from many copies of any state which is not passive

($p_i \leq p_j$ iff $E_i \geq E_j$)

[just swap levels i and j, while raising the weight, and repeat over many blocks]

Many copies of any state except the thermal state results in a state which is not passive after decohering (Pusz and Woronowicz (78), Brandao et. al. 2011)

This gives us an equivalence class, of allowed free states labelled by (τ_β , H_R)

Any other free state allows arbitrary transitions.

1 law of quantum thermodynamics

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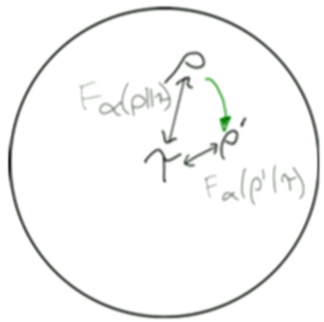
The second laws (psuedo-classical)



Thermal monotones:

$$F_\alpha(\rho||\tau) = \frac{kT}{1-\alpha}\log\, tr\,\rho^\alpha\tau^{1-\alpha} - kT\log Z \quad \alpha \geq 0$$

$$F_1(\rho||\tau) = F(\rho)$$

- Ordinary 2nd law is one of many
- In macroscopic limit, weak interactions, all $F_\alpha \simeq F$
- For ρ block diagonal, 2nd law is iff

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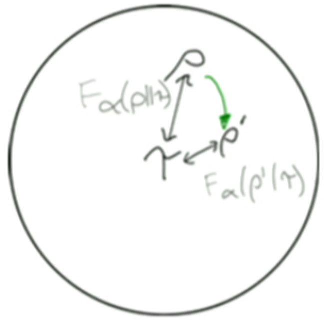

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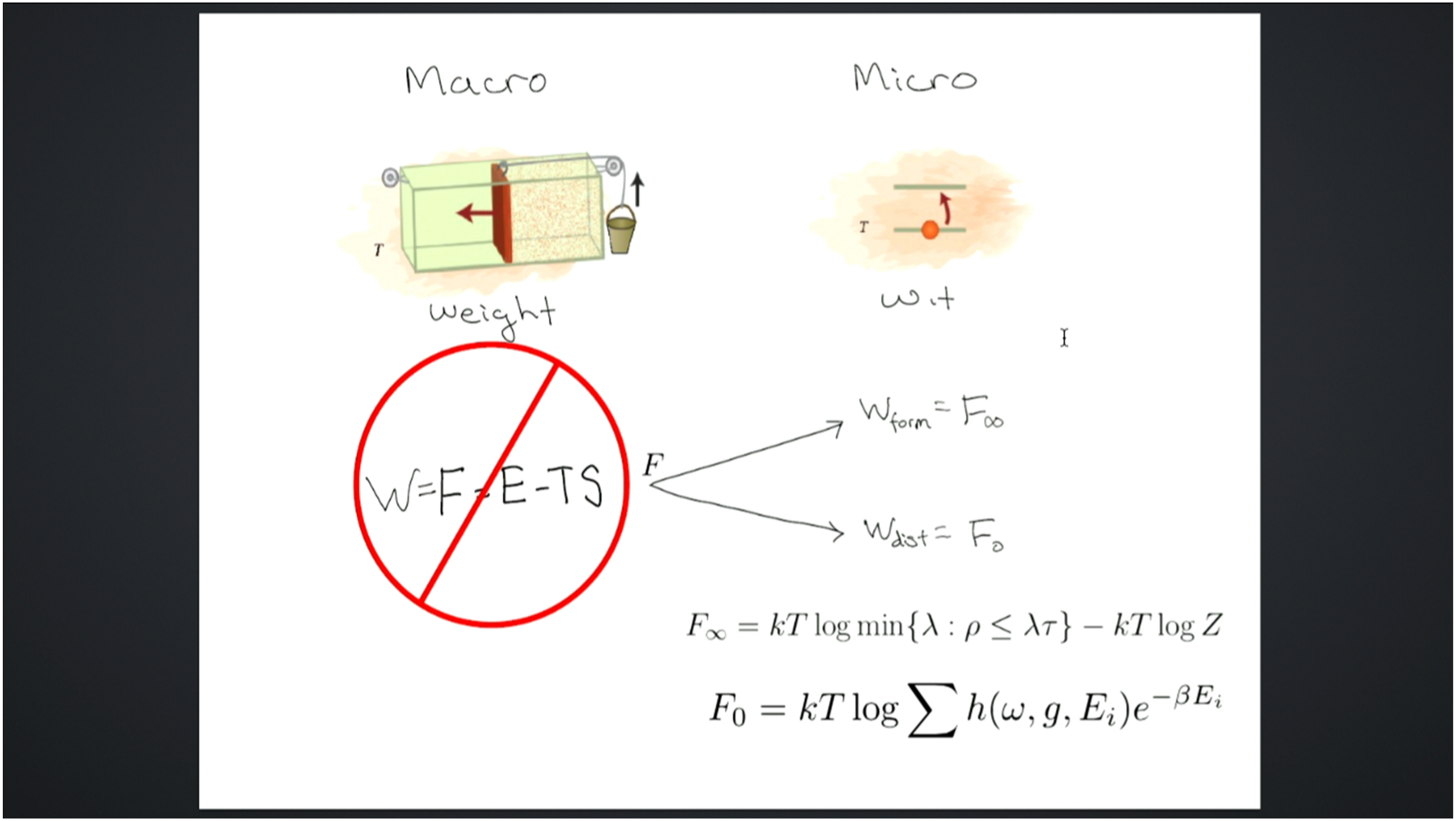

Macro
Micro
T
T
weight
w.t
W=F=E-TS
F
$W_{form} = F_\infty$
$W_{dist} = F_0$
$F_\infty = kT \log \min\{\lambda : \rho \leq \lambda\tau\} - kT \log Z$
$F_0 = kT \log \sum h(\omega, g, E_i) e^{-\beta E_i}$

2^{nd} law examples

For H=0, diagonal states:

- largest p(E) can't get larger
- rank(ρ) can't get smaller
- sum of the largest k p(E)'s can't get larger (majorisation)

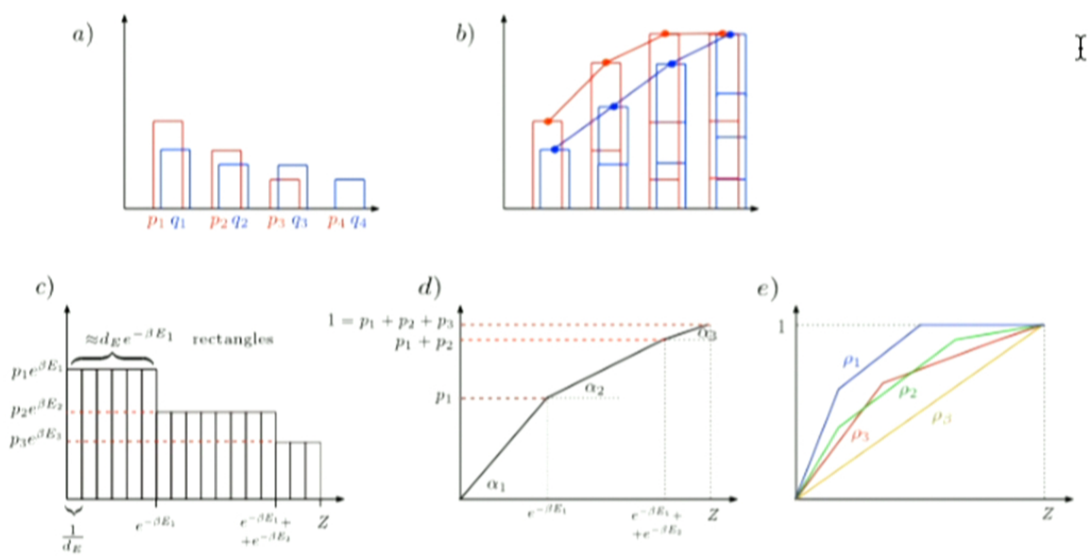
For diagonal states:

- largest p(E)$e^{\beta E}$
- $F_0 = kT \log \sum h(\omega, g, E_i) e^{-\beta E_i}$ can't get larger

Quasi-classical 2nd laws

For degenerate energy levels

$x \xrightarrow{\text{N.O.}} y$ iff $x \succ y$ $\Sigma p \geq \Sigma q$ forall k Horodecki JO (2003)

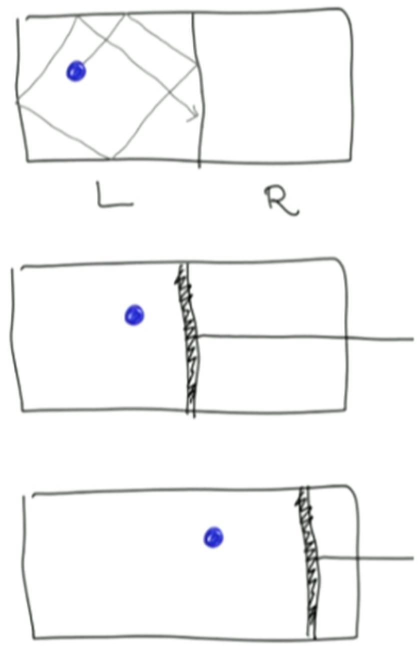


$P(E_1, g_1)e^{\beta E_1} \geq P(E_2, g_2)e^{\beta E_2} \geq \ldots \beta$ - ordered conjecture of Ruch & Mead (75)

Work distillation (H=0)

$p(L)=1$

$W_{dist}=kT\ln(2)$



Work distillation

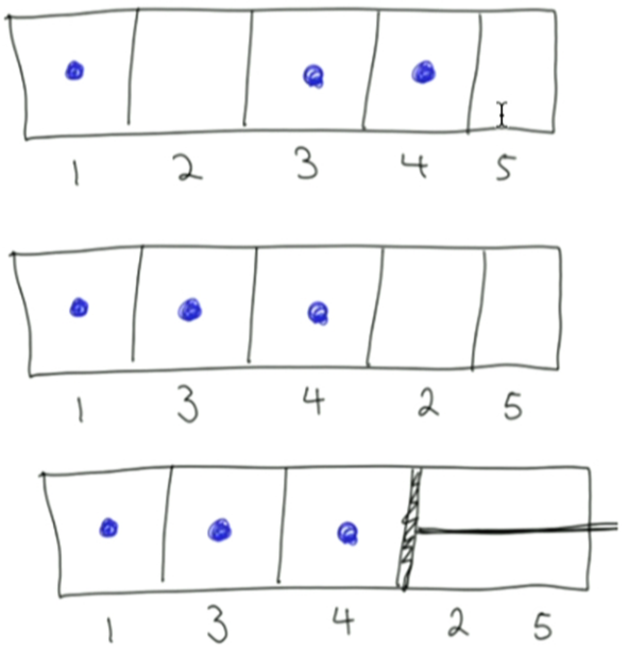
$p(1)=2/3$

$p(3)=1/6$

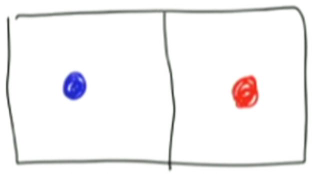
$p(4)=1/6$

$W_{dist}=kT[\ln(5)-\ln(3)]$

$=kT[\ln(d)-\ln(\text{rank})]$



Work of formation



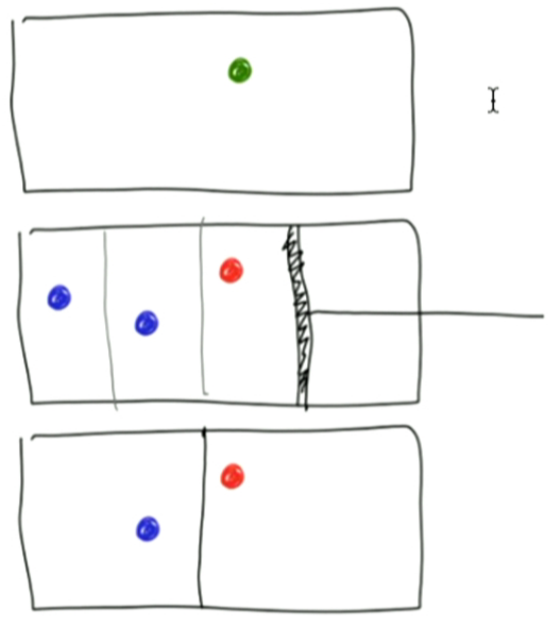
$p(L)=2/3$

$p(R)=1/3$

$W_{dist}=0$

$W_{form}=kT\ln(2/3)$

$=kT\ln(p_{largest})$



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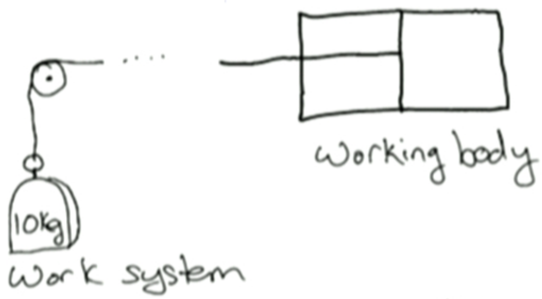

How cyclic?

$$\sigma_{in} \otimes \rho_S \to \sigma_{out} \otimes \rho'_s$$

$$|| \sigma_{in} - \sigma_{out} ||_1 \leq \epsilon$$

Embezzlement

$$\sigma_{in} = \sum_i \frac{1}{i} |i><i|$$ c.f. Van Dam, Hayden (2002)

$$||\sigma_{out} - \sigma_{in}||_1 \leq \epsilon \qquad \sigma_{in} \otimes \frac{\mathbb{I}}{2} \longrightarrow \sigma_{out} \otimes |0><0|$$

Three families of 2nd laws

1) All transformations are possible

$||\sigma_{\text{in}} - \sigma_{\text{out}}||_1 \leq \epsilon$

2) Ordinary 2nd law: $F_1(\rho\,||\,\tau)$ goes down

$||\sigma_{\text{in}} - \sigma_{\text{out}}||_1 \leq \epsilon/\log d_{\text{catalyst}}$

3) $F_\alpha(\rho\,||\,\tau)$ must go down $\alpha \geq 0$

Small work distance

$$D_{work}\left(\sigma^{\text{in}} > \sigma^{\text{out}}\right) := \text{kT} \inf_{\alpha>0} \left[F_\alpha(\sigma_{\text{in}}||\tau) - F_\alpha(\sigma_{\text{out}}||\tau)\right]$$

2^{nd} laws : recent & future directions

- **Laws for coherences**
  - Brandao et. al. (2003)
  - Ćwikliński, Studzinski, Horodecki, JO (1405.5029) qubit etc.
  - Lostaglio et. al.(x2)
  - Thermal Machines with coherences (Korzekwa, Lostaglio, JO, Jennings)
- **Probabilistic transformations**
  - Perry, Alhambra, JO
- **Understanding catalysis**
  - Ng et. al., Lostaglio, Mueller, Pastena

Quantitative third law
(Masanes, JO; quant-ph/1412.3828)

Heat Theorem (Planck 1911): *when the temperature of a pure substance approaches absolute zero, its entropy approaches zero*

Unattainability Principle (Nernst 1912): *any thermodynamical process cannot attain absolute zero in a finite number of steps or within a finite time*

$$T' \geq \frac{\alpha T}{t^{2d+1}}$$

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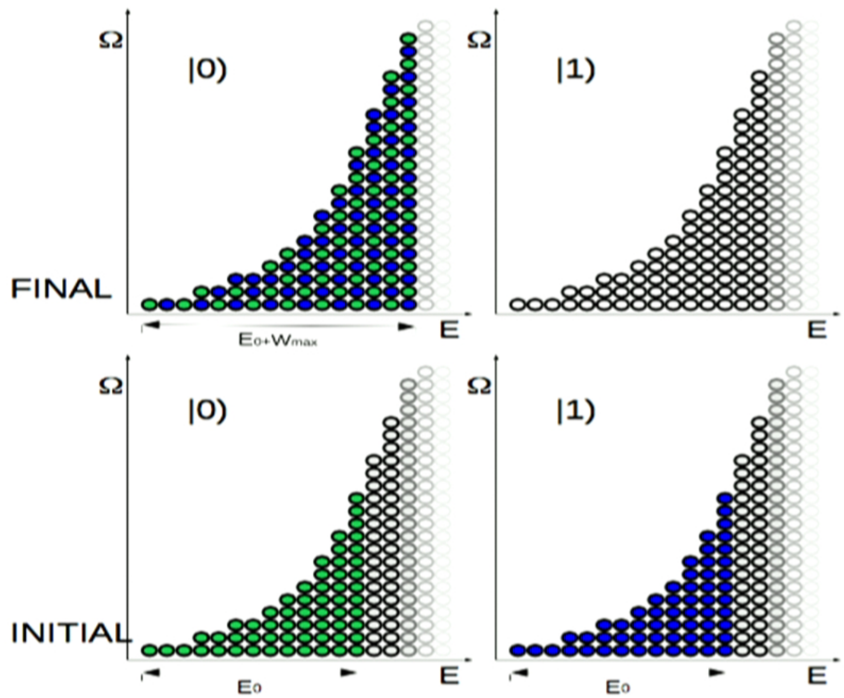
Thermal Machines

- Like Turing Machines
- In a finite time, they interact with a finite volume and inject a finite amount of work
- $t \geq \frac{1}{v} V^{1/d}$ $\qquad$ $t \geq \frac{1}{u} w_{max}$
- Bath of volume V has sub-exponential density of states Ω(E)

Erasure of a bit



Conclusions

Laws of thermodynamics 1st law

– Thermal Operations (U_E, τ, tr)

0^{th} : τ must be thermal for non-trivial theory

2^{nd} * : $F_\alpha(\rho \,||\, \tau)$ must go down $0 \leq \alpha$

* : embezzlement

Quasi-classical states: 2^{nd} laws are also sufficient

Coherences: poorly understood

Probabilistic transformations

Many free energies → irreversibility

Quantitative third law

Probabilistic transformations
(Alhambra, Perry, JO)

$\rho \xrightarrow{W} \sigma$

$\rho \longrightarrow \rho' = p\sigma + (1-p)X$

$$p = \min_{l \in \{1,2\ldots n\}} \frac{V_l(\rho)}{V_l(\sigma)} \qquad V_l(\rho) = \sum_{i=1}^{l} \eta_i$$

$$2^{W_{\rho \Rightarrow \sigma}} \leq p \leq 2^{-W_{\sigma \Rightarrow \rho}}$$