Title: Fine Tuning May Not Be Enough

Date: Aug 11, 2015 09:30 AM

URL: http://pirsa.org/15080020

Abstract: This talk is based on arXiv:1506.07306, with Shun-Pei Miao. We argue that the fine tuning problems of scalar-driven inflation may be worse than is commonly believed. The reason is that reheating requires the inflaton to be coupled to other matter fields whose vacuum fluctuations alter the inflaton potential. The usual response has been that even more fine-tuning of the classical potential $V(\varphi)$ can repair any damage done in this way. We point out that the effective potential in de Sitter background actually depends in a complicated way upon the dimensionless combination of φ/H. We also show that the factors of H which occur in de Sitter do not even correspond to local functionals of the metric for general geometries, nor are they Planck-suppressed.

Fine Tuning May Not Be Enough

arXiv:1506.07306 with S. P. Miao

Perimeter Institute

August 11, 2015

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$$

$$H(t) \equiv \dot{a}/a \quad \& \quad \epsilon(t) \equiv -\dot{H}/H^2$$

- Hard to avoid an early phase of accelerated expansion
  - but what caused it is a mystery
- Scalar potential models work

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - \tfrac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$$

1. $3H^2 = 8\pi G[\tfrac{1}{2}\dot{\varphi}_0^2 + V(\varphi_0)]$

2. $-2\dot{H} - 3H^2 = 8\pi G[\tfrac{1}{2}\dot{\varphi}_0^2 - V(\varphi_0)]$

- (1)+(2) ➔ $\varphi_0(t) = \varphi_i - \int_{t_i}^{t} dt' \sqrt{\frac{-\dot{H}(t')}{8\pi G}}$
- Rotate the graph ➔ $t(\varphi_0)$
- (1)-(2) ➔ $V(\varphi) = \frac{1}{8\pi G}\left[\dot{H} + 3H^2\right]_{t(\varphi)}$

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$$

$$H(t) \equiv \dot{a}/a \quad \& \quad \epsilon(t) \equiv -\dot{H}/H^2$$

- Hard to avoid an early phase of accelerated expansion
  - but what caused it is a mystery
- Scalar potential models work

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - ½\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$$

1. $3H^2 = 8\pi G[½\dot{\varphi}_0^2 + V(\varphi_0)]$
2. $-2\dot{H} - 3H^2 = 8\pi G[½\dot{\varphi}_0^2 - V(\varphi_0)]$

- (1)+(2) ➔ $\varphi_0(t) = \varphi_i - \int_{t_i}^{t} dt' \sqrt{\frac{-\dot{H}(t')}{8\pi G}}$
- Rotate the graph ➔ $t(\varphi_0)$
- (1)-(2) ➔ $V(\varphi) = \frac{1}{8\pi G}\left[\dot{H} + 3H^2\right]_{t(\varphi)}$

Fine Tuning Problems

1. Initial Conditions
   - Potential energy domination over more than a Hubble volume
2. Duration
   - $N = -8\pi G \int_{\varphi_i}^{\varphi_f} d\varphi \frac{V(\varphi)}{V'(\varphi)} \geq 50$
3. Scalar Perturbations
   - $\frac{(GV)^3}{V'^2} \sim 10^{-11}$
4. Tensor Perturbations
   - $G^2 V \leq 5 \times 10^{-12}$
5. Reheating
   - Reheating requires coupling to normal matter
6. Cosmological Constant
   - $G^2 V_{min} \approx 10^{-123}$

Reheating ➔ $\Delta V(\varphi)$

& an obstacle to more fine tuning

- Flat Space: $\Delta V(\varphi) \sim \pm\,(cc \times \varphi^2)^2 ln\left[\frac{cc^2 \times \varphi^2}{\mu^2}\right]$
  - Not Planck suppressed & typically too steep
  - Just fine tune it away . . .
- On de Sitter: $\Delta V(\varphi) \sim \pm\, H^4 f\left[\frac{cc^2 \times \varphi^2}{H^2}\right]$
  - $f(x) \rightarrow x^2 \ln(x)$ gives Coleman-Weinberg
  - But small x is relevant: $f(x) = \alpha x + \beta x^2 + O(x^3)$
  - Set $\alpha = 0$ with $\delta\xi\varphi^2 R\sqrt{-g}$ & $\beta = 0$ with $\delta\lambda\varphi^4\sqrt{-g}$
- Factors of ``H^2" are not constant & not even local

  ➔ stuck with $O(x^3)$ terms

Reheating $\rightarrow \Delta V(\varphi)$
& an obstacle to more fine tuning

- Flat Space: $\Delta V(\varphi) \sim \pm\,(cc \times \varphi^2)^2 ln\left[\frac{cc^2 \times \varphi^2}{\mu^2}\right]$
  - Not Planck suppressed & typically too steep
  - Just fine tune it away . . .
- On de Sitter: $\Delta V(\varphi) \sim \pm\, H^4 f\left[\frac{cc^2 \times \varphi^2}{H^2}\right]$
  - $f(x) \rightarrow x^2 \ln(x)$ gives Coleman-Weinberg
  - But small x is relevant: $f(x) = \alpha x + \beta x^2 + O(x^3)$
  - Set $\alpha = 0$ with $\delta\xi\varphi^2 R\sqrt{-g}$ & $\beta = 0$ with $\delta\lambda\varphi^4\sqrt{-g}$
- Factors of ``H^2" are not constant & not even local

  $\rightarrow$ stuck with $O(x^3)$ terms

Three Models

1. Another scalar ϕ

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{[1+\Delta\xi]}{12}\phi^2 R\sqrt{-g} - \tfrac{1}{4}h^2\phi^2\varphi^2\sqrt{-g}$$

2. A fermion ψ

$$\mathcal{L} = \bar{\psi}\gamma^b e_b^\mu\left(i\partial_\mu - \tfrac{1}{2}A_{\mu cd}J^{cd}\right)\psi\sqrt{-g} - f\varphi\bar{\psi}\psi\sqrt{-g}$$

3. A vector gauge boson A_μ (with complex inflaton)

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} - \left(\partial_\mu - ieA_\mu\right)\varphi(\partial_\nu + ieA_\nu)\varphi^* g^{\mu\nu}\sqrt{-g}$$

For each model:

- Give general (de Sitter) form for $\Delta V(\varphi)$
- Give large field expansion
- Give small field expansion

Limit on backgrounds is PROPAGATORS

1. Mass M scalar with conformal coupling ξ

$$\sqrt{-g}[\Box - \xi R - M^2] i\Delta[\xi, M^2](x;x') = i\delta^D(x-x')$$

2. Mass m fermion on $ds^2 = a^2[-d\eta^2 + d\vec{x}\cdot d\vec{x}]$

$$i[{}_iS_j](x;x') = \frac{1}{a^{\frac{D+1}{2}}}[i\gamma^\mu \partial_\mu + am]\frac{a^{\frac{D-1}{2}}}{\sqrt{aa'}}\mathcal{S}(x;x')$$

$$\mathcal{S} = ½(1+\gamma^0) i\Delta[\xi_c, M_+^2](x;x') + ½(1-\gamma^0) i\Delta[\xi_c, M_-^2](x;x')$$

- With $\xi_c \equiv \frac{1}{2(D-1)}$ & $M_\pm^2 \equiv m(m \mp iH)$

3. Mass M_V vector with $\xi_v \equiv \frac{(D-2)}{D(D-1)}$

$$i[{}_\mu\Delta_\rho](x;x') = [\Box_\mu^\nu - D^\nu D_\mu][\Box'^\sigma_\rho - D'^\sigma D'_\rho]\left[\frac{\partial^2 \ell^2(x;x')}{\partial x^\nu \partial x'^\sigma} S(x;x')\right]$$

$$S(x;x') = \frac{i\Delta[\xi_v, M_V^2] - i\Delta[\xi_v, 0]}{M_V^4} - \frac{1}{M_V^2}\frac{\partial i\Delta[\xi_v, N^2]}{\partial N^2}[N^2 = 0$$

$$V'_{scalar} = \delta\xi R\varphi + \tfrac{1}{6}\delta\lambda\varphi^3 + \tfrac{1}{2}h^2\varphi i\Delta\left[\tfrac{1+\Delta\xi}{12}, \tfrac{1}{2}h^2\varphi^2\right](x;x)$$

- $\nu_{\pm} \equiv \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1-8\Delta\xi}\ ,\quad z^2 \equiv \frac{h^2\varphi^2}{H^2}$
- $\psi(x) \equiv \frac{d}{dx}\, ln[\Gamma(x)]$

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{-\Big[\psi(\nu_+)+\psi(\nu_-)\Big]\Big[2\Delta\xi z^2+\frac{z^4}{4}\Big]+\Big[\psi'(\nu_+)-\psi'(\nu_-)\Big]\frac{\frac{1}{2}\Delta\xi z^4}{\sqrt{1-8\Delta\xi}} + \int_0^{z^2} dx\Big(2\Delta\xi+\frac{x}{2}\Big)\Bigg[\psi\Big(\frac{1}{2}+\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Big)+\psi\Big(\frac{1}{2}-\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Big)\Bigg]\Bigg\}. \quad (8)$$

$$V'_{scalar} = \delta\xi R\varphi + \tfrac{1}{6}\delta\lambda\varphi^3 + \tfrac{1}{2}h^2\varphi i\Delta\left[\tfrac{1+\Delta\xi}{12}, \tfrac{1}{2}h^2\varphi^2\right](x;x)$$

- $\nu_\pm \equiv \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1-8\Delta\xi}$, $z^2 \equiv \frac{h^2\varphi^2}{H^2}$
- $\psi(x) \equiv \frac{d}{dx}\, ln[\Gamma(x)]$

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Biggl\{-\Bigl[\psi(\nu_+)+\psi(\nu_-)\Bigr]\Bigl[2\Delta\xi z^2+\frac{z^4}{4}\Bigr]+\Bigl[\psi'(\nu_+)-\psi'(\nu_-)\Bigr]\frac{\frac{1}{2}\Delta\xi z^4}{\sqrt{1-8\Delta\xi}} + \int_0^{z^2} dx\Bigl(2\Delta\xi+\frac{x}{2}\Bigr)\Biggl[\psi\Bigl(\frac{1}{2}+\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Bigr)+\psi\Bigl(\frac{1}{2}-\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Bigr)\Biggr]\Biggr\}. \quad (8)$$

Large field and small field expansions

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\left\{\frac{1}{4}z^4\ln\left(\frac{1}{2}z^2+2\Delta\xi\right) - \left[\frac{1}{8}+\frac{[\psi(\nu_+)+\psi(\nu_-)]}{4}\right.\right.$$
$$\left. -\frac{\Delta\xi[\psi'(\nu_+)-\psi'(\nu_-)]}{2\sqrt{1-8\Delta\xi}}\right]z^4 + 2\Delta\xi z^2\ln\left(\frac{1}{2}z^2+2\Delta\xi\right) - \left[\frac{1}{3}+\Delta\xi\right.$$
$$\left.\left. +2\Delta\xi\left[\psi(\nu_+)+\psi(\nu_-)\right]\right]z^2 + \left[4\Delta\xi^2-\frac{2}{15}\right]\ln\left(\frac{1}{2}z^2+2\Delta\xi\right) + O(z^0)\right\}$$

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\left\{\left[\frac{(1-6\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}} + \frac{\Delta\xi[\psi''(\nu_+)+\psi''(\nu_-)]}{1-8\Delta\xi}\right]\frac{z^6}{12}\right.$$
$$+\left[\frac{3(1-4\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{2(1-8\Delta\xi)^{\frac{5}{2}}} + \frac{3(1-4\Delta\xi)[\psi''(\nu_+)+\psi''(\nu_-)]}{2(1-8\Delta\xi)^2}\right.$$
$$\left.\left. +\frac{\Delta\xi[-\psi'''(\nu_+)+\psi'''(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}}\right]\frac{z^8}{96} + O(z^{10})\right\}. \quad (1$$

$$V'_{scalar} = \delta\xi R\varphi + \tfrac{1}{6}\delta\lambda\varphi^3 + \tfrac{1}{2}h^2\varphi i\Delta\left[\tfrac{1+\Delta\xi}{12}, \tfrac{1}{2}h^2\varphi^2\right](x;x)$$

- $\nu_{\pm} \equiv \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1-8\Delta\xi}$, $z^2 \equiv \frac{h^2\varphi^2}{H^2}$
- $\psi(x) \equiv \frac{d}{dx}\, ln[\Gamma(x)]$

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{-\Big[\psi(\nu_+)+\psi(\nu_-)\Big]\Big[2\Delta\xi z^2+\frac{z^4}{4}\Big]+\Big[\psi'(\nu_+)-\psi'(\nu_-)\Big]\frac{\frac{1}{2}\Delta\xi z^4}{\sqrt{1-8\Delta\xi}} + \int_0^{z^2} dx\Big(2\Delta\xi+\frac{x}{2}\Big)\Big[\psi\Big(\frac{1}{2}+\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Big)+\psi\Big(\frac{1}{2}-\sqrt{\frac{1}{4}-2\Delta\xi-\frac{x}{2}}\Big)\Big]\Bigg\}. \quad (8)$$

Large field and small field expansions

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{\frac{1}{4}z^4\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) - \Bigg[\frac{1}{8}+\frac{[\psi(\nu_+)+\psi(\nu_-)]}{4}$$

$$-\frac{\Delta\xi[\psi'(\nu_+)-\psi'(\nu_-)]}{2\sqrt{1-8\Delta\xi}}\Bigg]z^4 + 2\Delta\xi z^2\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) - \Bigg[\frac{1}{3}+\Delta\xi$$

$$+2\Delta\xi\Big[\psi(\nu_+)+\psi(\nu_-)\Big]\Bigg]z^2 + \Big[4\Delta\xi^2-\frac{2}{15}\Big]\ln\Big(\frac{1}{2}z^2+2\Delta\xi\Big) + O(z^0)\Bigg\}$$

$$\Delta V_{\rm scalar} = \frac{H^4}{64\pi^2}\Bigg\{\Bigg[\frac{(1-6\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}}+\frac{\Delta\xi[\psi''(\nu_+)+\psi''(\nu_-)]}{1-8\Delta\xi}\Bigg]\frac{z^6}{12}$$

$$+\Bigg[\frac{3(1-4\Delta\xi)[-\psi'(\nu_+)+\psi'(\nu_-)]}{2(1-8\Delta\xi)^{\frac{5}{2}}}+\frac{3(1-4\Delta\xi)[\psi''(\nu_+)+\psi''(\nu_-)]}{2(1-8\Delta\xi)^2}$$

$$+\frac{\Delta\xi[-\psi'''(\nu_+)+\psi'''(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}}\Bigg]\frac{z^8}{96}+O(z^{10})\Bigg\}. \quad (1$$

$$\Delta V'_{fermion} = \delta\xi R\varphi + {}^{1}\!/\!_{6}\,\delta\lambda\varphi^3 - f[{}_iS_i](x;x)$$

- $z^2 \equiv \frac{f^2\varphi^2}{H^2}$

$$\Delta V_{fermion} = -\frac{H^4}{8\pi^2}\Big\{2\gamma z^2 - [\zeta(3)-\gamma]z^4 + 2\int_0^z dx(x+x^3)[\psi(1+ix)+\psi(1-ix)]\Big\}$$

$$\Delta V_{fermion} = -\frac{H^4}{8\pi^2}\Big\{\tfrac{1}{2}z^4\ln(z^2+1) - [\zeta(3)+\tfrac{1}{4}-\gamma]z^4 + z^2\ln(z^2+1) - \big[\tfrac{4}{3}-2\gamma\big]z^2 + \frac{11}{60}\ln(z^2+1) + O(z^0)\Big\}$$

$$\Delta V_{fermion} = -\frac{H^4}{8\pi^2}\Big\{\tfrac{2}{3}[\zeta(3)-\zeta(5)]z^6 - \tfrac{1}{2}[\zeta(5)-\zeta(7)]z^8 + O(z^{10})\Big\}$$

Factors of ``H^2'' are not constant

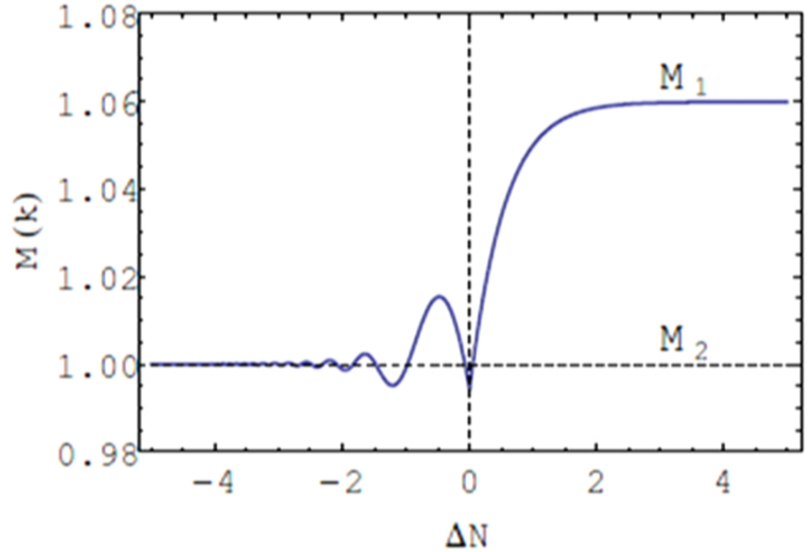
- Evaluate $\langle \Delta T_{\mu\nu} \rangle = -\frac{2}{\sqrt{-g}} \frac{\delta \Delta \Gamma[g]}{\delta g^{\mu\nu}}$ on de Sitter
- $H^2 = \frac{\Lambda}{3}$ ➔ $\langle \Delta T_{\mu\nu} \rangle_{dS} = -g_{\mu\nu} H^4 \times ½ f(z^2)$
  - Recall $\Delta V = H^4 f(z^2)$ & $z^2 = \frac{cc^2 |\varphi|^2}{H^2}$
- Actually $\langle \Delta T_{\mu\nu} \rangle_{dS} = -g_{\mu\nu} H^4 \times ½ z^2 f'(z^2)$
  - Except for $\mu^{D-4} \to H^{D-4}$ in $\delta\xi$ and $\delta\lambda$
- Consistent with ``H^2'' $= \frac{R}{12}$
  - Because $F(R)$ models are ok this could be removed

Factors of ``H^2'' are not even local!

- E.g., ``ringing'' in $\Delta^2_{\mathcal{R}}(k)$
  - $\epsilon_1 = \frac{1}{200} \rightarrow \epsilon_2 = \frac{1}{10}$
  - Starobinsky 1992
- Good analytic form in arXiv:1507.07452

$$h(t,k) = \int_0^N dn \, \sin\Big[\int_n^N dn' \, \omega(n', N_k)\Big] \frac{\mathcal{S}(n, N_k)}{\omega(n, N_k)}$$

- This stuff also can't be part of a classical action



How bad is it in practice?

- Could tune $V(\varphi)$ to cancel ΔV at one instant
  - But $H(t)$ typically changes a lot
- For $V(\varphi) = A\varphi^{\alpha}$ we have $\frac{H(t)}{H_i} = \left[\frac{\epsilon(t)}{\epsilon_i}\right]^{\frac{\alpha}{4}}$
- Could cancel with ``H^2'' $\rightarrow$ $\frac{R}{12}$
  - But then a modified gravity theory
  - E.g., Friedman equation changes
- Need a careful numerical study

Conclusions

- $V(\varphi)$ models need heavy fine tuning
- Reheating requires couplings to light matter
- These induce corrections ΔV to $V(\varphi)$
  - Not Planck-suppressed
  - Can't compute for general $\epsilon(t) = -\frac{\dot{H}}{H^2}$
  - $\epsilon = 0$ ➔ $\Delta V = H^4 f\left(\frac{cc^2|\varphi|^2}{H^2}\right)$
- Factors of ``H^2'' are not constant, or $\frac{R}{12}$, or even local
  ➔ cannot remove ΔV by more fine tuning of $V(\varphi)$
- This looks bad
  - At the least, a new constraint on model-building

Three Models

1. Another scalar ϕ

$$\mathcal{L} = -½\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{[1+\Delta\xi]}{12}\phi^2 R\sqrt{-g} - ¼h^2\phi^2\varphi^2\sqrt{-g}$$

2. A fermion ψ

$$\mathcal{L} = \bar{\psi}\gamma^b e_b^\mu(i\partial_\mu - ½A_{\mu cd}J^{cd})\psi\sqrt{-g} - f\varphi\bar{\psi}\psi\sqrt{-g}$$

3. A vector gauge boson A_μ (with complex inflaton)

$$\mathcal{L} = -¼F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g}$$
$$- (\partial_\mu - ieA_\mu)\varphi(\partial_\nu + ieA_\nu)\varphi^* g^{\mu\nu}\sqrt{-g}$$

For each model:

- Give general (de Sitter) form for $\Delta V(\varphi)$
- Give large field expansion
- Give small field expansion