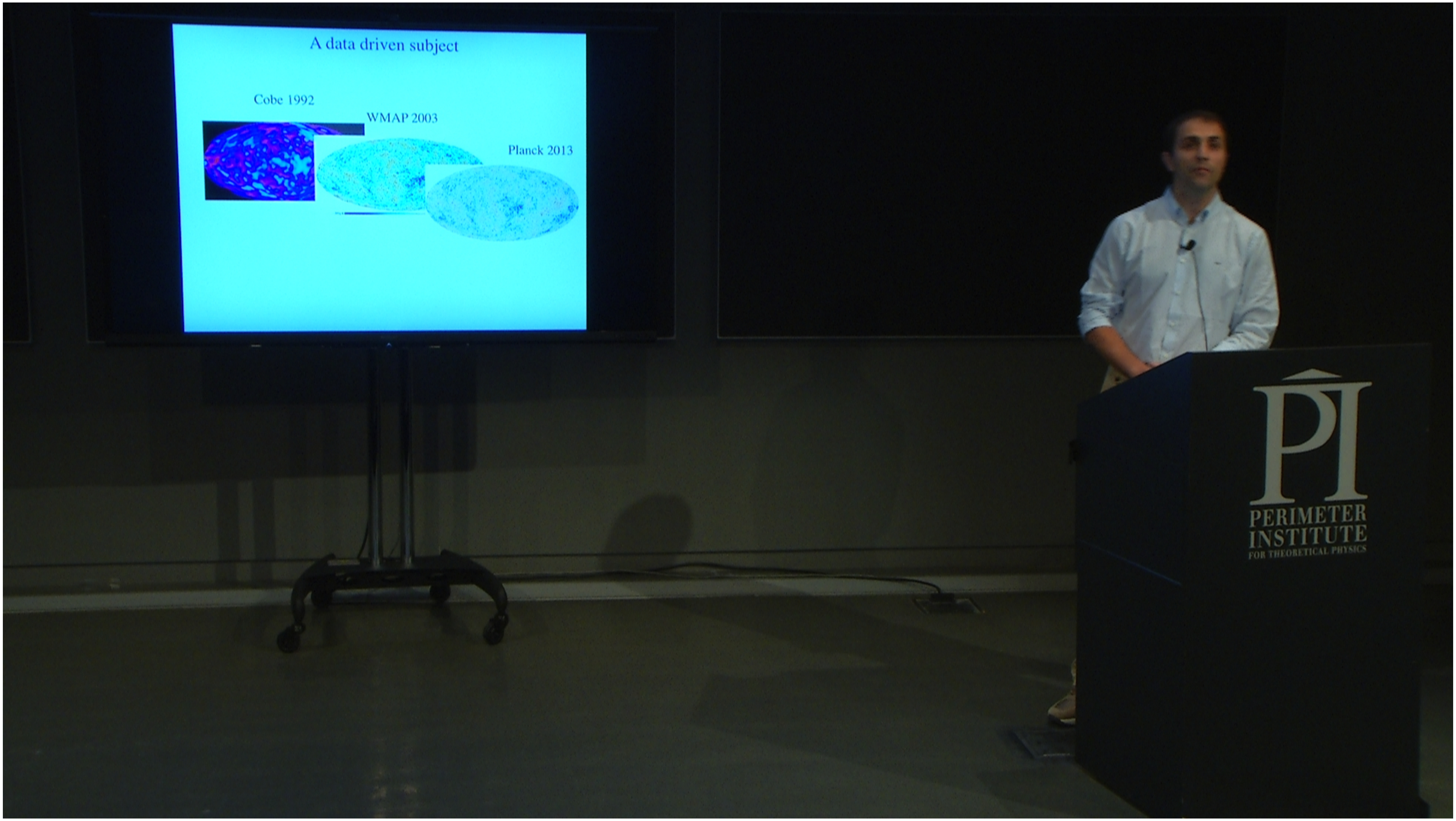
Title: The Effective Field Theory of Large Scale Structures

Date: Aug 12, 2015 02:00 PM

URL: http://pirsa.org/15080019

Abstract: After the completion of the Planck satellite, the next most important experiments in cosmology will be about mapping the Large Scale Structures of the Universe. In order to continue to make progress in our understanding of the early universe, it is essential to develop a precise understanding of this system. The Effective Filed Theory of Large Scale Structures provides a novel framework to analytically compute the clustering of the Large Scale Structures in the weakly non-linear regime in a consistent and reliable way. The theory that describes the long wavelength fluctuations is obtained after integrating out the strongly-coupled, short-distance modes, and adding suitable operators that allow us to correctly reconstruct the effect of short distance fluctuations at long distances. By using techniques that originate in the particle physics context, a few observables have been computed so far, and the results are extremely promising. I will discuss the formalism, the main results so far, and the potential implications for next generation experiments.


A data driven subject
Cobe 1992
WMAP 2003
Planck 2013
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS

What can we do with them?

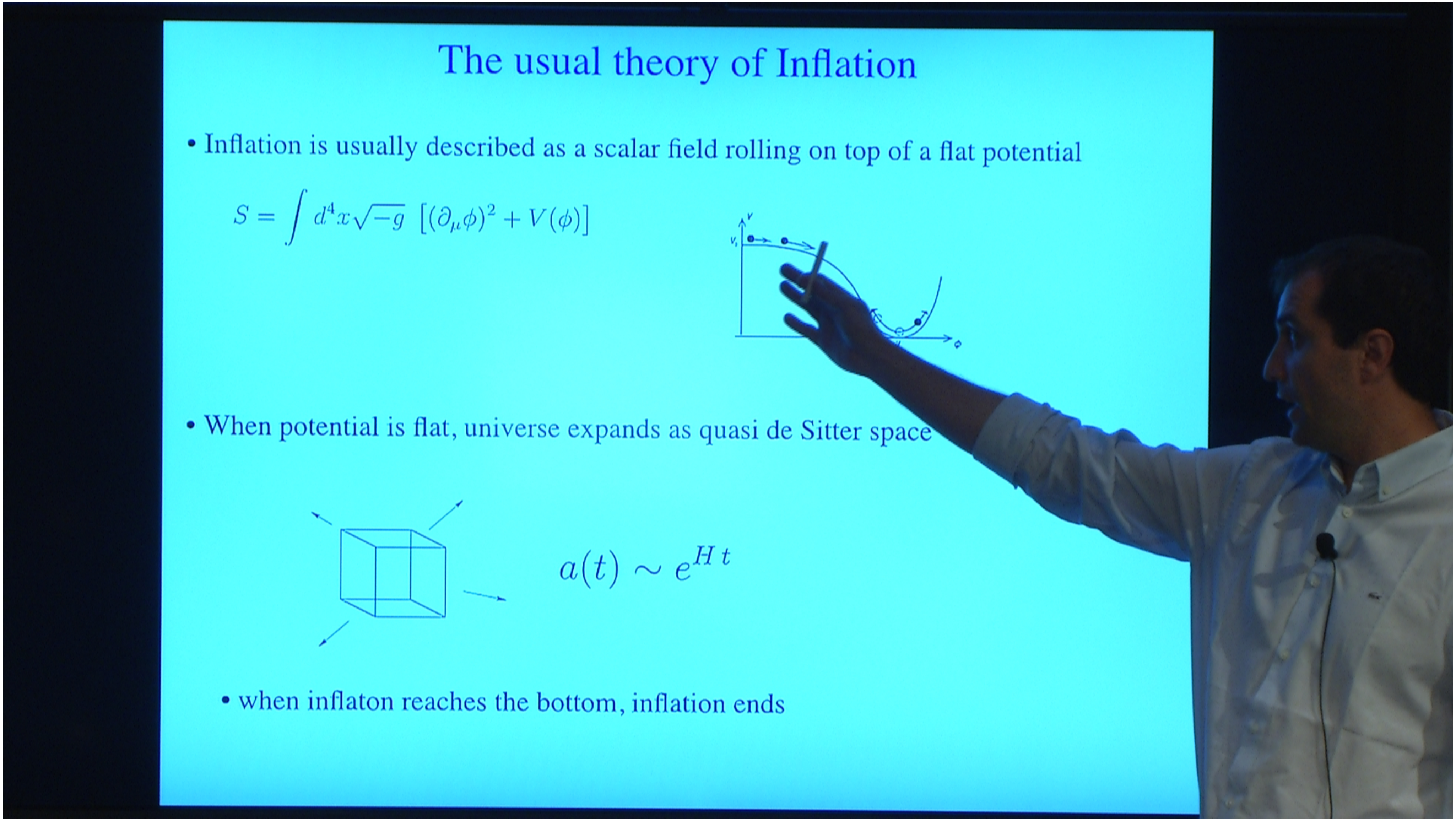
- Address Universal Question

  - How did everything begin?

  - High Energies $\Longrightarrow$ High Energy Physics

    - –we learn about High Energy
    - –we apply High Energy Physics techniques

How do we probe Inflation?


The usual theory of Inflation
• Inflation is usually described as a scalar field rolling on top of a flat potential
$S = \int d^4x \sqrt{-g}\ \left[(\partial_\mu \phi)^2 + V(\phi)\right]$
• When potential is flat, universe expands as quasi de Sitter space
$a(t) \sim e^{H\,t}$
• when inflaton reaches the bottom, inflation ends

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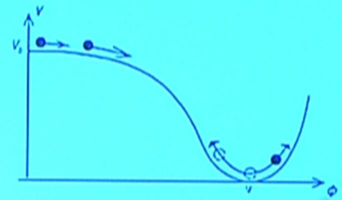

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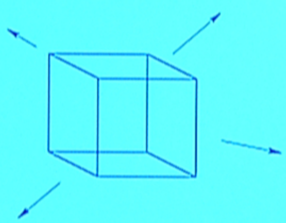

$$a(t) \sim e^{H\,t}$$

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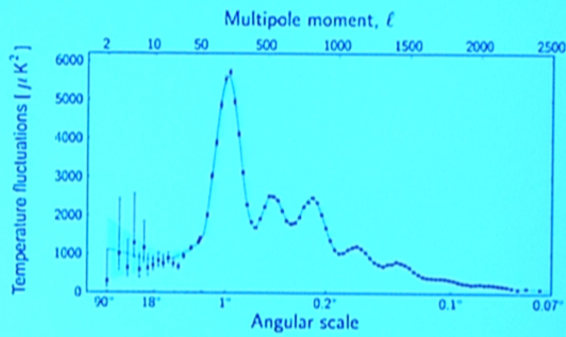
What are we seeing?

- The only observable we are testing from the background solution is

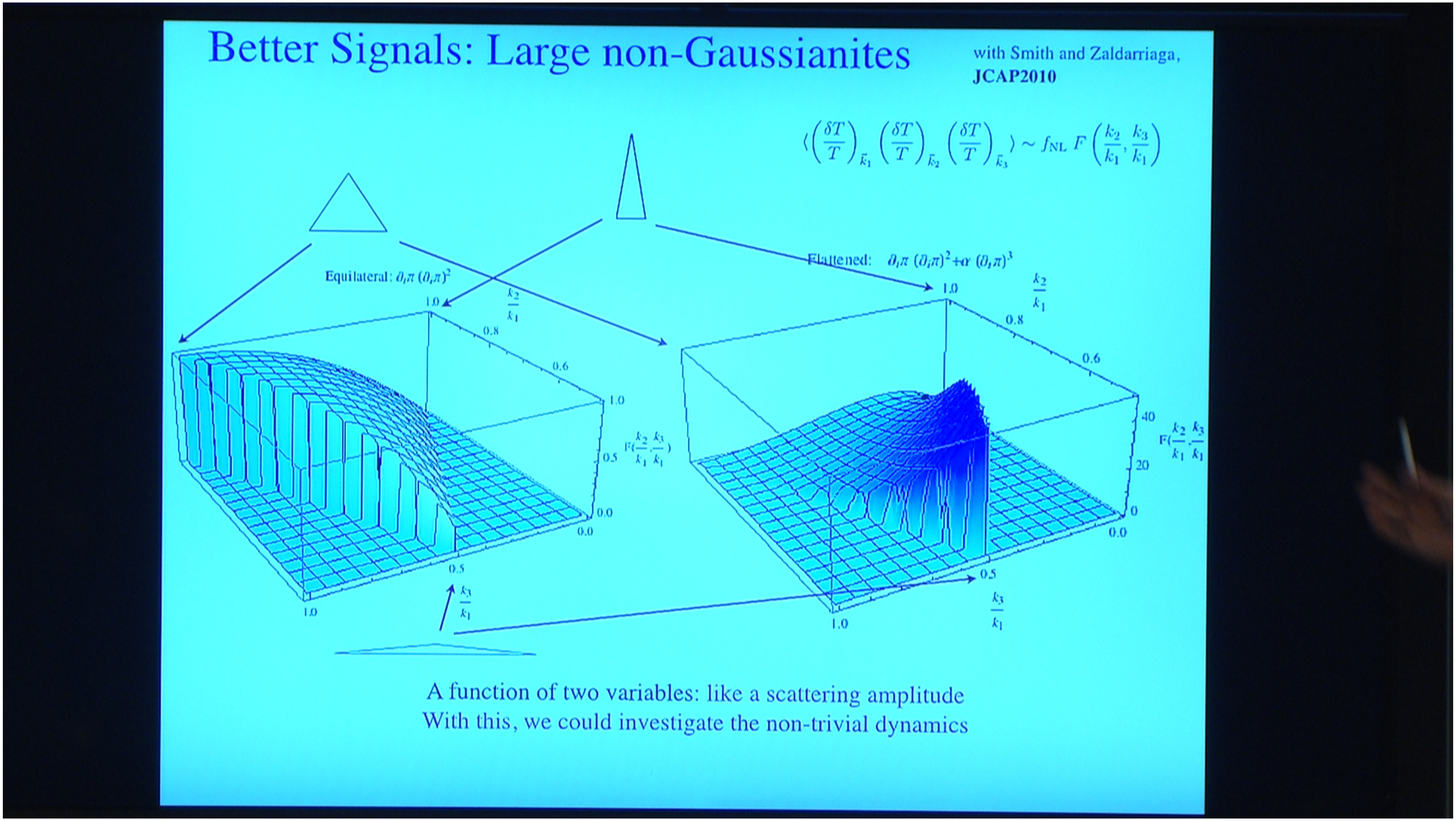
  $\Omega_K \lesssim 3 \times 10^{-3}$



- All the rest, comes from the fluctuations
- For the fluctuations
  - –they are primordial
  - –they are scale invariant
  - –they have a tilt $\quad n_s - 1 \simeq -0.04 \sim \mathcal{O}\left(\frac{1}{N_e}\right)$
  - –they are quite gaussian

$$\mathrm{NG} \sim \frac{\langle \left(\frac{\delta T}{T}\right)^3 \rangle}{\langle \left(\frac{\delta T}{T}\right)^2 \rangle^{3/2}} \lesssim 10^{-3}$$

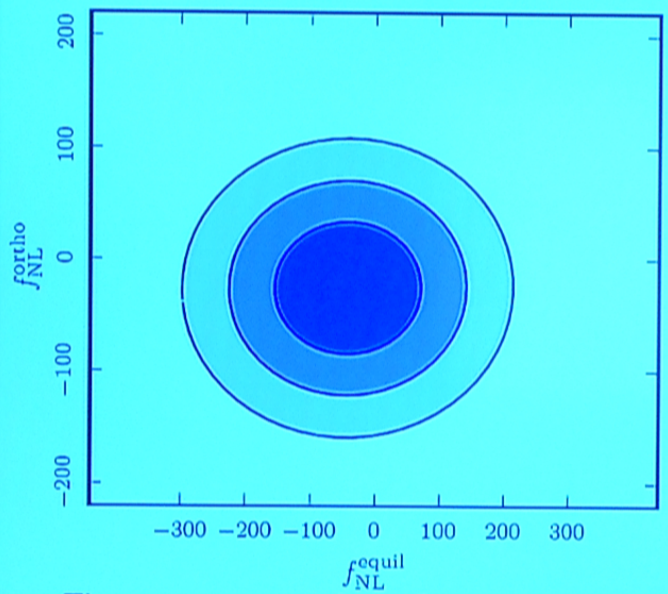
- Just 2 numbers
- Is this enough to conclude it is slow-roll Inflation? Indeed, there are other models
  - –and in general, what is the dynamics of this inflaton?

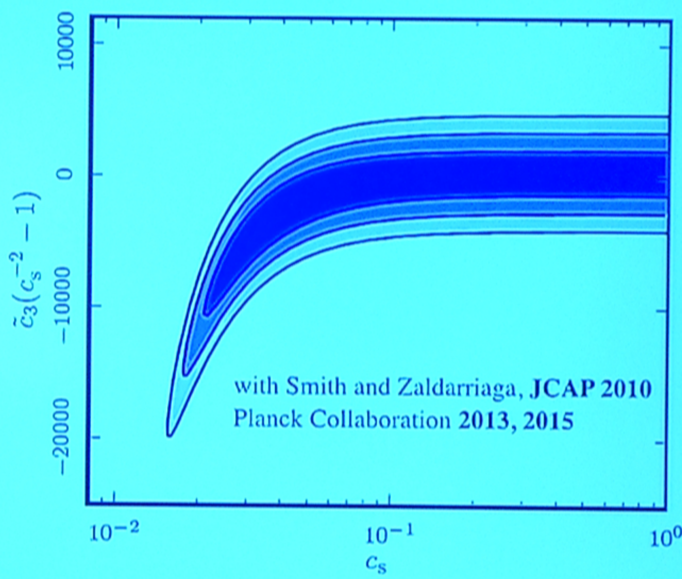

Better Signals: Large non-Gaussianites
with Smith and Zaldarriaga,
JCAP2010
$\langle \left(\frac{\delta T}{T}\right)_{\vec{k}_1} \left(\frac{\delta T}{T}\right)_{\vec{k}_2} \left(\frac{\delta T}{T}\right)_{\vec{k}_3} \rangle \sim f_{\rm NL}\, F\left(\frac{k_2}{k_1}, \frac{k_3}{k_1}\right)$
Equilateral: $\partial_t\pi\,(\partial_i\pi)^2$
Flattened: $\partial_t\pi\,(\partial_i\pi)^2 + \alpha\,(\partial_t\pi)^3$
$\frac{k_2}{k_1}$
$\frac{k_3}{k_1}$
$F(\frac{k_2}{k_1}, \frac{k_3}{k_1})$
1.0
0.8
0.6
0.5
0.0
40
20
0
A function of two variables: like a scattering amplitude
With this, we could investigate the non-trivial dynamics

Limits in terms of parameters of a Lagrangian

- $$S_\pi = \int d^4\sqrt{-g}\left[\frac{\dot{H}M_{\rm Pl}^2}{c_s^2}\left(\dot\pi^2 - c_s^2(\partial_i\pi)^2\right) + \frac{\dot{H}M_{\rm Pl}^2}{c_s^2}\left[\dot\pi(\partial_i\pi)^2 + \tilde{c}_3\,\dot\pi^3\right]\right]$$

with Cheung, Creminelli, Fitzpatrick and Kaplan, **JHEP 2008**

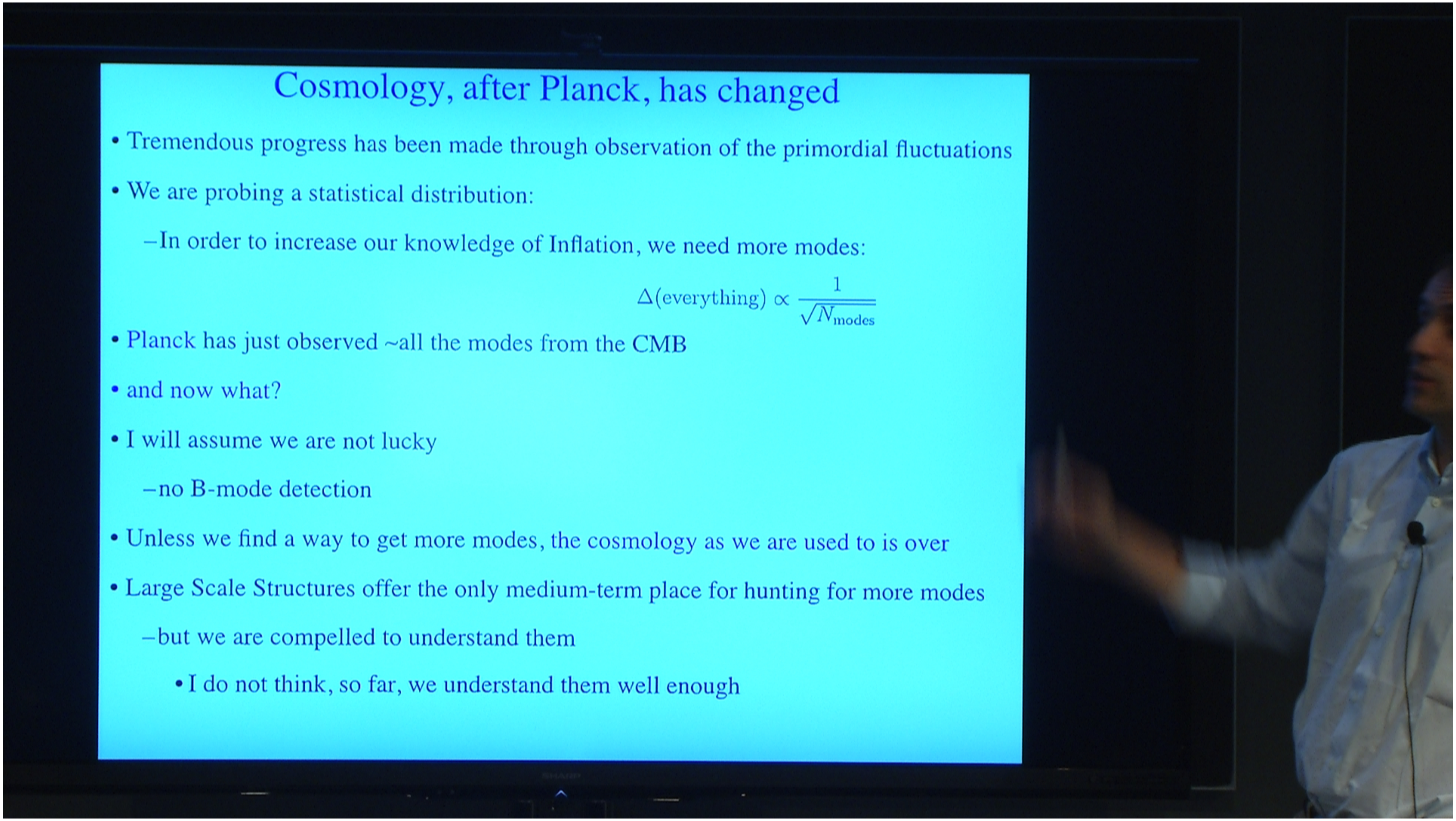




- These are contour plots of parameters of a fundamental Lagrangian
- Same as in particle accelerator Precision Electroweak Tests. see Barbieri, Giudice, Isidori, ...
- Thanks to the EFT: A qualitatively new (and superior) way to use the cosmological data

What has Planck done to theory?

- Planck improve limits wrt WMAP by a factor of ~3.
- Since $\mathrm{NG} \sim \dfrac{H^2}{\Lambda^2} \quad \Rightarrow \quad \Lambda^{\mathrm{min,Planck}} \sim \sqrt{3}\,\Lambda^{\mathrm{min,WMAP}}$
- Given the absence of known or nearby threshold, this is not much.
- Planck was great
  - but CMB did not have enough modes
- Planck was an opportunity for a detection, not much an opportunity to change the theory in absence of detection
  - We crossed the tilt-threshold (luckily WMAP had a tilt a 2.5 σ , so we got to $6\,\sigma$)
- On theory side, little changes
  - contrary for example to LHC, which was crossing thresholds
    - Any result from LHC is changing the theory


Cosmology, after Planck, has changed
• Tremendous progress has been made through observation of the primordial fluctuations
• We are probing a statistical distribution:
–In order to increase our knowledge of Inflation, we need more modes:
$\Delta(\text{everything}) \propto \frac{1}{\sqrt{N_{\text{modes}}}}$
• Planck has just observed ~all the modes from the CMB
• and now what?
• I will assume we are not lucky
–no B-mode detection
• Unless we find a way to get more modes, the cosmology as we are used to is over
• Large Scale Structures offer the only medium-term place for hunting for more modes
–but we are compelled to understand them
• I do not think, so far, we understand them well enough

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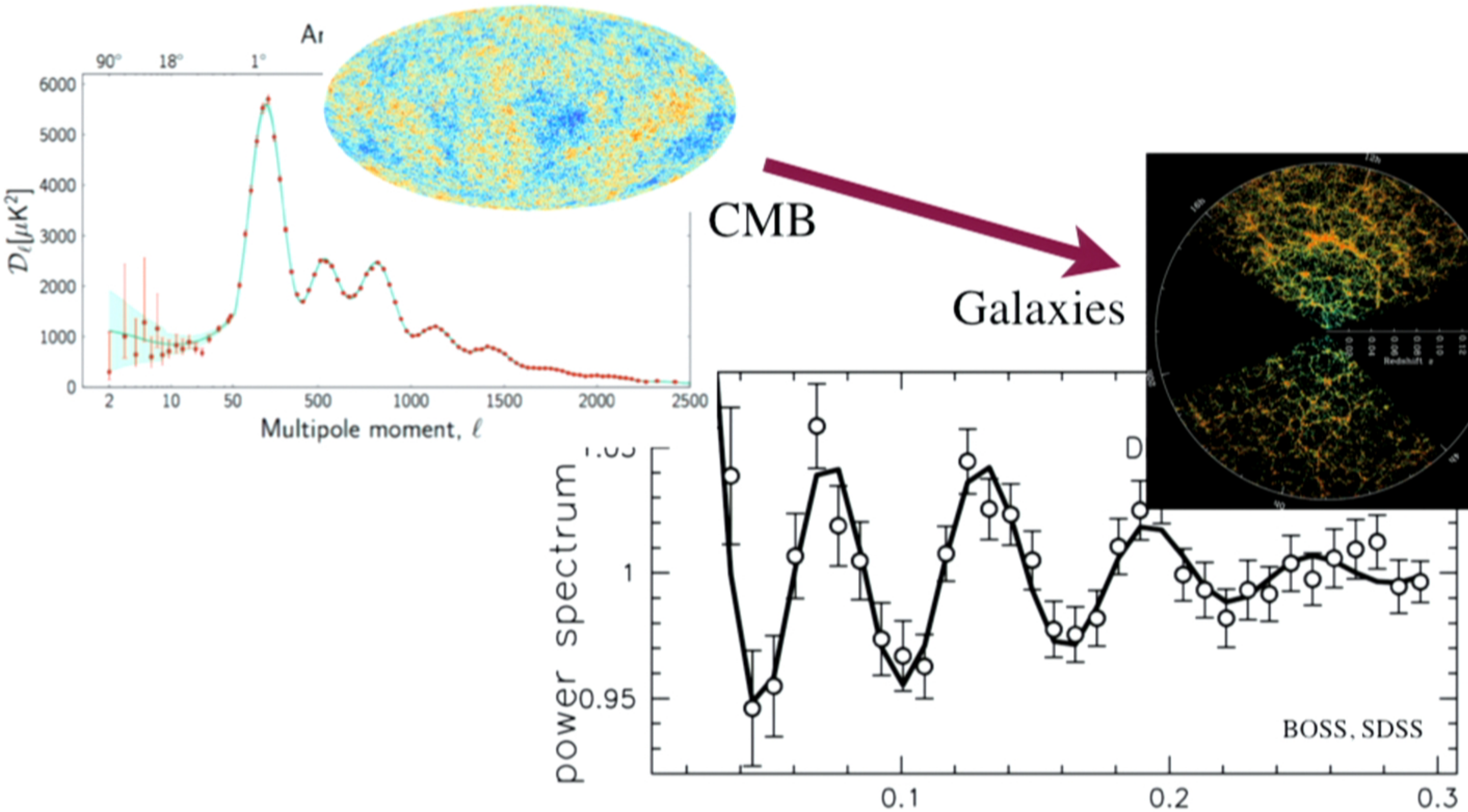
Some things already done

- Baryon Acoustic Oscillations in Galaxies distribution



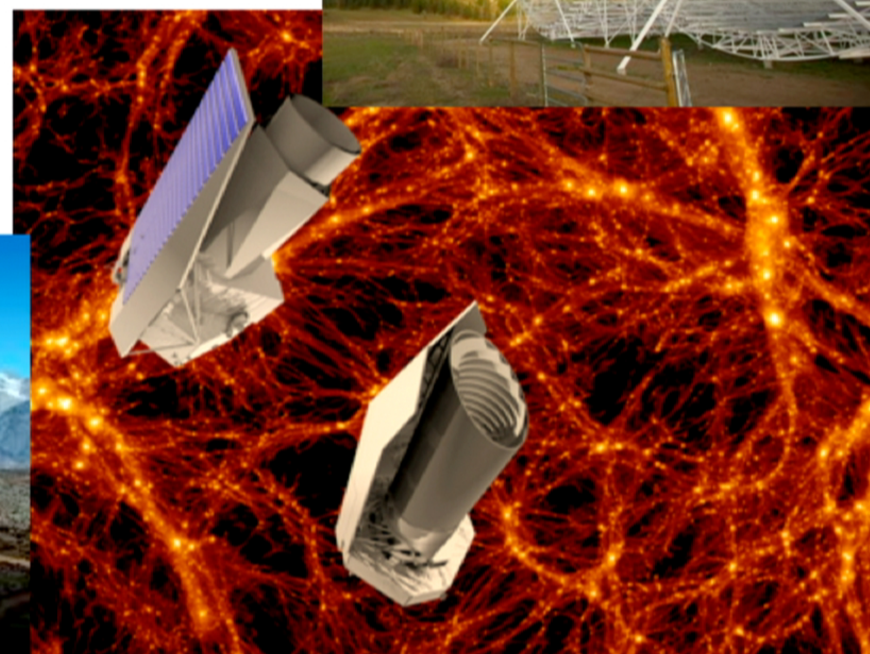
- But most new information is just about low-z universe k (h Mpc^{-1})
- Not much about early universe

What is next?

- Euclid, LSST and Chime are the next big missions: this is our only next chance
  - we need to understand how many modes are available

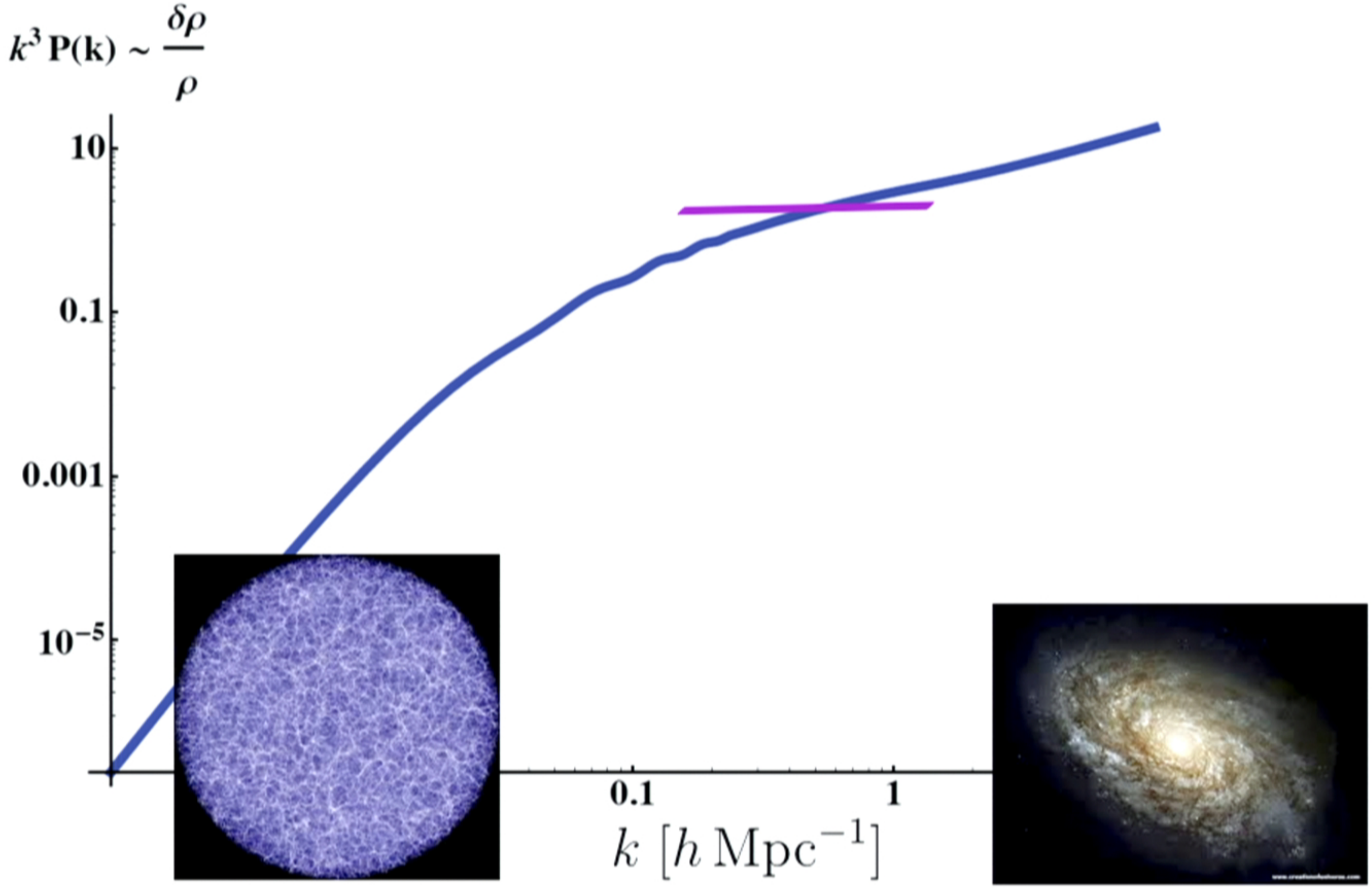
$$\text{Number of modes} \sim \left(\frac{k_{\max}}{k_{\min}}\right)^3$$

  - Need to understand short distances
  - Similar as from LEP to LHC



The EFTofLSS: A well defined perturbation theory

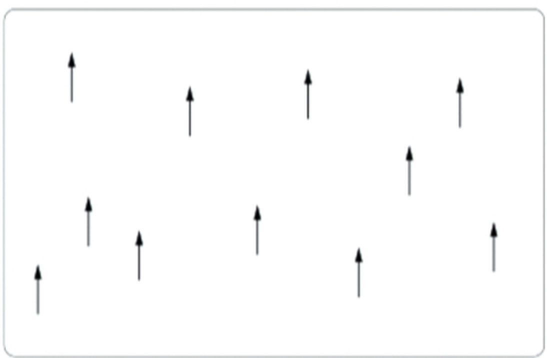
- Non-linearities at short scale



Consider a dielectric material

- Very complicated on atomic scales d_{atomic}
- On long distances $d \gg d_{\text{atomic}}$
  - we can describe atoms with their gross characteristics
    - polarizability $\vec{d}_{\text{dipole}} \sim \alpha\, \vec{E}_{\text{electric}}$: average response to electric field
  - we are led to a uniform, smooth material, with just some macroscopic properties
    - we simply solve dielectric Maxwell equations, we do not solve for each atom.
- The universe looks like a dielectric

Dielectric Fluid



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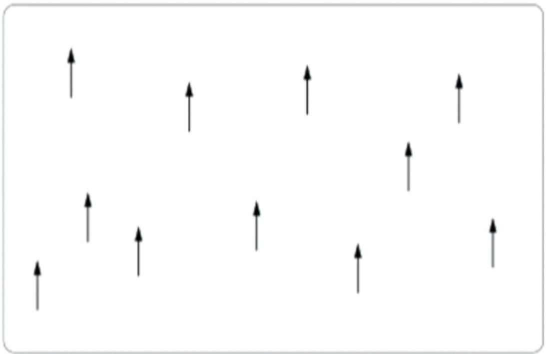
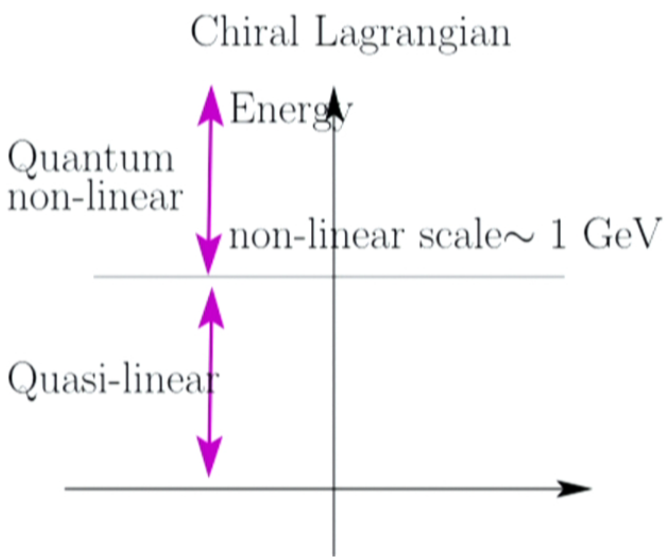


QCD Chiral Lagrangian Reminder

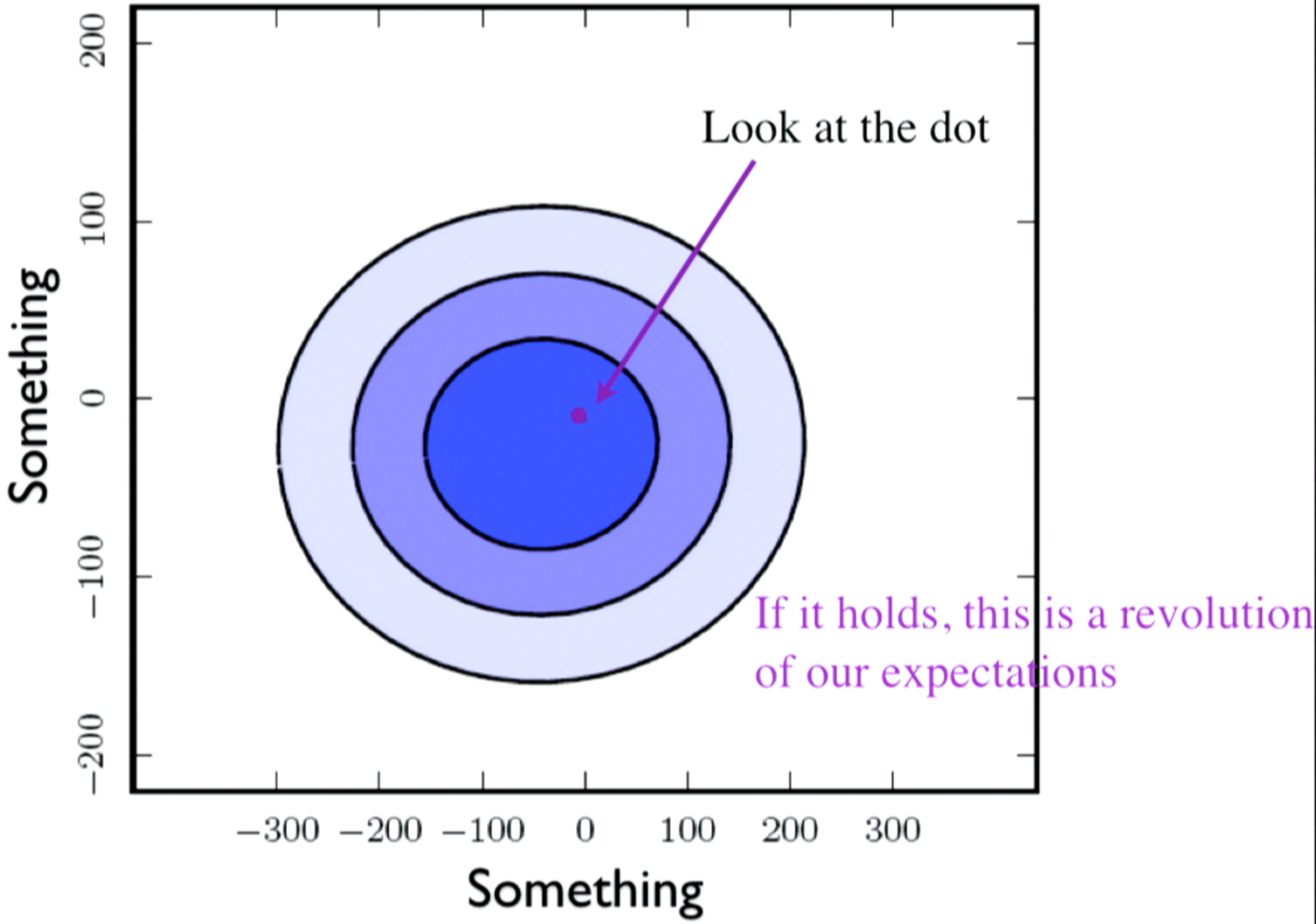
- Pions are described by

$$S = \int d^4x \left[(\partial\pi)^2 + \frac{1}{F_\pi^2}\pi^2(\partial\pi)^2 + \frac{1}{\tilde{F}_\pi^2}(\partial\pi)^4 + \dots \right]$$

- For $m_\pi \lesssim E \lesssim 4\pi F_\pi$
- Perturbative expansion in $\frac{E}{4\pi F_\pi} \ll 1$



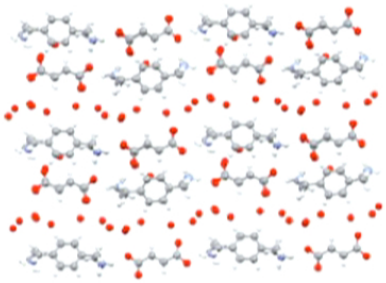
With this



Look at the dot
If it holds, this is a revolution
of our expectations
200
100
0
−100
−200
Something
−300
−200
−100
0
100
200
300
Something

The Theory of the Universe

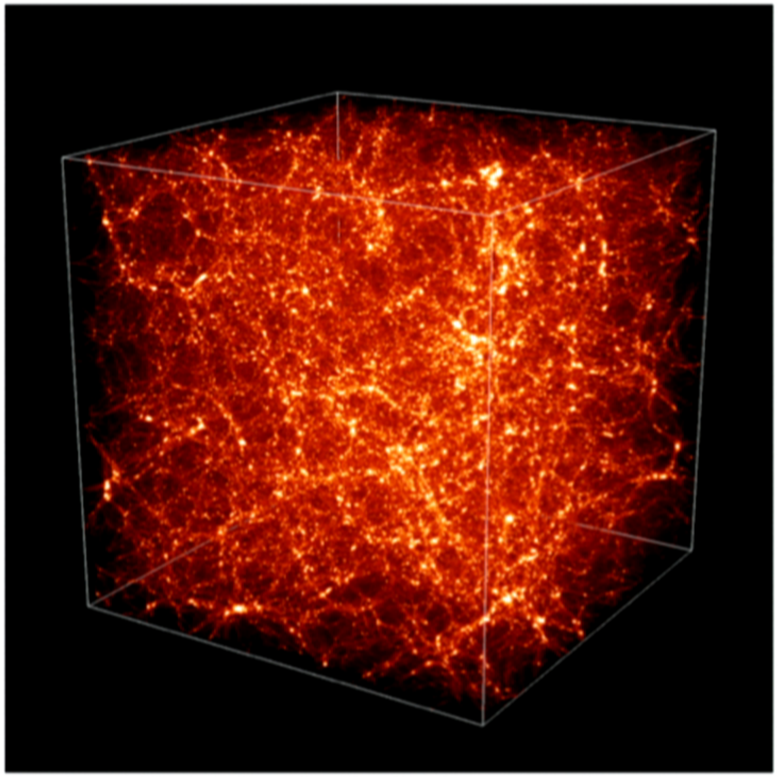
- Useful or not, this is the correct description of the long distance universe
  - as we describe water as a fluid, and not a set of molecules hitting each other



  - similarly the universe *is* the system I am going to describe

Normal Approach: numerics

- Just simulate the full universe (such as water molecules to simulate ocean waves)

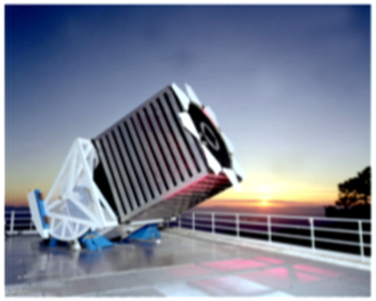


Why numerics are not enough

- they do not give the simple description of the system
- In principle, we can simulate the clustering of dark matter with N-body sims
- But
  - simulations with dark matter are very slow
    - systematic error of order 1%

A. Schneider, R. Teyssier, … V. Springel *et al*. **1503**

  - we cannot simulate baryons: we can only `model' them
  - As a proof, SDSS stops analyzing data at low k



Numerics has been great

- Do not misunderstand me:
  - numerical simulations have provided some of the most beautiful history-making discoveries:
    - dark matter is cold
      - standard model neutrinos are not the dark matter
    - structures form from small to big

- But I believe, after these giants, we live in hard times
  - and to make further progress, high precision is required
    - N-body sims do not seem, to me, the only appropriate tool.

Point-like Particle versus Extended Objects

- On short distances, we have point-like particles
  - they move

$$\frac{d^2\vec{z}(\vec{q},\eta)}{d\eta^2} + \mathcal{H}\frac{d\vec{z}(\vec{q},\eta)}{d\eta} = -\vec{\partial}_x\Phi[\vec{z}(\vec{q},\eta)]$$

  - induce overdensities

$$1 + \delta(\vec{x},\eta) = \int d^3q\ \delta^{(3)}(\vec{x} - \vec{z}(\vec{q},\eta))$$

  - Source gravity

$$\partial^2\Phi(\vec{x}) = \mathcal{H}^2\delta(\vec{x})$$

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Point-like Particle versus Extended Objects

- But we cannot describe point-like particles: we need to focus on long distances.
  - We deal with Extended objects
    - they move differently:

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$$\frac{d^2\vec{z}_L(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}_L(\vec{q},\eta)}{d\eta} = -\vec{\partial}_x\left[\Phi_L[\vec{z}_L(\vec{q},\eta)]+\frac{1}{2}Q^{ij}(\vec{q},\eta)\partial_i\partial_j\Phi_L[\vec{z}_L(\vec{q},\eta)]+\cdots\right]+\vec{a}_S(\vec{q},\eta)$$

with Porto and Zaldarriaga **JCAP1405**

Point-like Particle versus Extended Objects

with Porto and Zaldarriaga **JCAP1405**

- They induce number over-densities and real-space multipole moments

$$1+\delta_{n,L}(\vec{x},\eta) \equiv \int d^3\vec{q}\; \delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta)) \;,$$

$$\mathcal{Q}^{i_1\ldots i_p}(\vec{x},\eta) \equiv \int d^3\vec{q}\; Q^{i_1\ldots i_p}(\vec{q},\eta)\delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta))$$

- they source gravity with the `overall' mass

$$\partial_x^2\Phi_L = \frac{3}{2}\mathcal{H}^2\Omega_m\left(\delta_{n,L}(\vec{x},\eta)+\frac{1}{2}\partial_i\partial_j\mathcal{Q}^{ij}(\vec{x},\eta)-\frac{1}{6}\partial_i\partial_j\partial_k\mathcal{Q}^{ijk}(\vec{x},\eta)+\cdots\right)\equiv\frac{3}{2}\mathcal{H}^2\Omega_m\delta_{m,L}(\vec{x},\eta)$$

$$\sim \text{Energy}_{\text{electrostatic}} = q\,V + \vec{d}\cdot\vec{E}+\ldots$$

- These equations can be derived from smoothing the point-particle equations
  - but actually these are the assumption-less equations

Point-like Particle versus Extended Objects

with Porto and Zaldarriaga **JCAP1405**

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$$

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The Effective ~Fluid

– Equivalently

– In history of universe Dark Matter moves about $1/k_{\rm NL} \sim 10\,{\rm Mpc}$

- – it is an effective fluid-like system with mean free path ~ $1/k_{\rm NL} \sim 10\,{\rm Mpc}$
- – it interacts with gravity so matter and momentum are conserved

• Skipping subtleties, the resulting equations are equivalent to fluid-like equations

$$\nabla^2 \Phi_l = H^2 \frac{\delta \rho_l}{\rho}$$

$$\partial_t \rho_l + H \rho_l + \partial_i \left(\rho_l v_l^i \right) = 0$$

$$\dot{v}_l^i + H v_l^i + v_l^j \partial_j v_l^i = \frac{1}{\rho} \partial_j \tau_{ij}$$

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**
with Carrasco and Hertzberg **JHEP 2012**
with Porto and Zaldarriaga **JCAP1405**

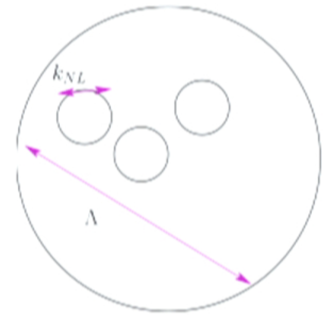
– short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} \sim \delta_{ij}\, \rho_{\rm short} \, \left(v^2_{\rm short} + \Phi_{\rm short} \right)$$

Dealing with the Effective Stress Tensor

- Take expectation value over short modes (integrate them out)

$$\langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[p_0 + c_s \, \delta\rho_l + \mathcal{O}\left(\frac{\partial}{k_{\rm NL}}, \partial_i v_l^i, \delta\rho_l^2, \dots \right) + \Delta\tau \right]$$



- We obtain equations containing only long-modes

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- How many terms to keep?
  - each term contributes as an extra factor of $\dfrac{\delta\rho_l}{\rho} \sim \dfrac{k}{k_{\rm NL}}$
    - we keep as many as required precision
    - $\Longrightarrow$ manifest expansion in $\dfrac{k}{k_{\rm NL}} \ll 1$

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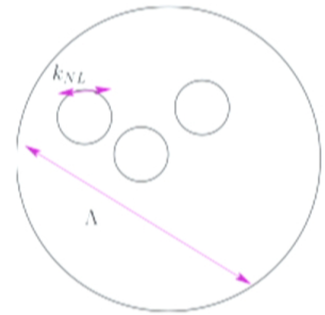

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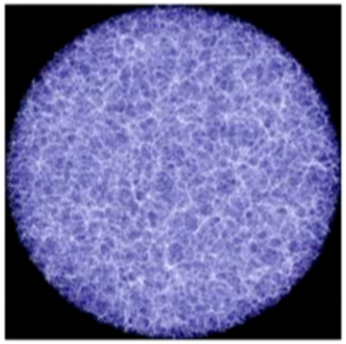
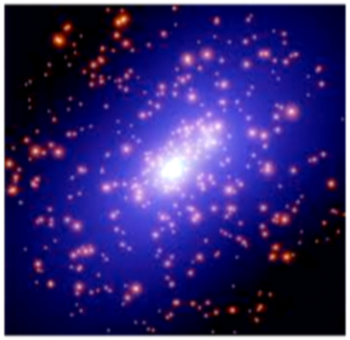
A subtlety: non-locality in Time

This EFT is non-local in time

- For local EFT, we need hierarchy of scales.
  - In space we are ok





  - In time we are not ok: all modes evolve with time-scale of order Hubble

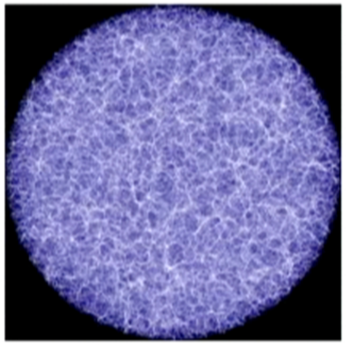
with Carrasco, Foreman and Green **1310**
Carroll, Leichenauer, Pollak **1310**
Mirbabahi, Schmidt, Zaldarriaga **1412**

- $\Longrightarrow$ The EFT is local in space, non-local in time

$$\langle \tau_{ij} \rangle_{\delta_l} \sim \int dt' \; K(t,t') \; \partial^2 \phi(x_{\rm fl}, t')$$

A non-renormalization theorem

- Can the short distance non-linearities change completely the overall expansion rate of the universe, possibly leading to acceleration without Λ ?



- In terms of the short distance perturbation, the effective stress tensor reads

$$\tau_{00} \sim (\text{mass+ kinetic energy + gravity potential energy})$$
$$\tau_{ii} \sim (\text{2 kinetic energy + gravity potential energy})$$

- when objects virialize, induced pressure vanish $\langle \rho_S \left(2v_S^2 + \Phi_S\right)\rangle_{\text{virialized}} \rightarrow 0$

  –ultraviolet modes do not contribute (like in SUSY)

- More in detail, the backreaction is dominated by modes at the virialization scale

$$\tau_{l,\,ij} \sim \partial_t^2 \left(x^2 \tau_{l,\,00}\right) \sim \frac{H^2}{{k_{\rm NL}}^2} \tau_{l,\,00} \sim 10^{-5} \tau_{l,\,00} \qquad \Rightarrow \qquad w_{\text{induced}} \sim 10^{-5}$$

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

Perturbation Theory within the EFT

- In the EFT we can solve iteratively $\delta_\ell, v_\ell, \Phi_\ell \ll 1$

$$\nabla^2 \Phi_l = H^2 \frac{\delta \rho_l}{\rho}$$
$$\partial_t \rho_l + H \rho_l + \partial_i \left(\rho_l v_l^i \right) = 0$$
$$\dot{v}_l^i + H v_l^i + v_l^j \partial_j v_l^i = \frac{1}{\rho} \partial_j \tau_{ij}$$

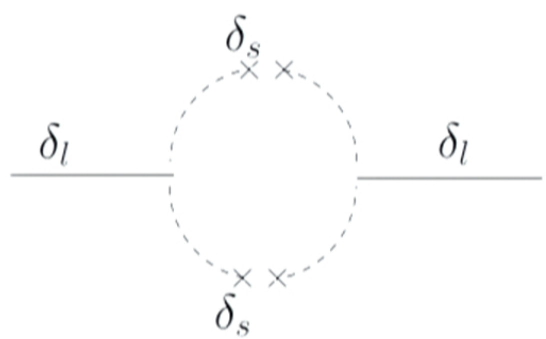
$$\langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[p_0 + c_s \, \delta \rho_l + \mathcal{O}\left(\frac{\partial}{k_{\rm NL}}, \partial_i v_l^i, \delta \rho_l^2, \dots \right) + \Delta \tau \right]$$

Perturbation Theory within the EFT

- Since equations are non-linear, we obtain convolution integrals (loops)

$$\delta^{(n)} \sim \int \text{GreenFunction} \times \text{Source}^{(n)}\left[\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(n-1)}\right]$$

$$\Rightarrow \quad \delta^{(2)}(k_l) \sim \int d^3k_s\, \delta^{(1)}(k_s)\, \delta^{(1)}(k_l - k_s)\ , \quad \Rightarrow \quad \langle \delta_l^2 \rangle \sim \int d^3k_s\, \langle \delta_s^{(1)2} \rangle^2$$



A subtlety: non-locality in Time

Consequences of non-locality in time

- The EFT is non-local in time $\Longrightarrow$ $\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} \sim \int^t dt'\ K(t,t')\ \delta\rho(\vec{x}_{\text{fl}},t') + \dots$

- Perturbative Structure has a decoupled structure

$$\delta\rho(x,t') = D(t')\delta\rho(\vec{x})^{(1)}(t) + D(t')^2\delta\rho(\vec{x})^{(2)}(t) + \dots$$

- A few coefficients for each counterterm:

$$\Rightarrow \quad \langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} \sim \int^t dt'\ K(t,t')\ \left[D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots\right]$$
$$\simeq c_1(t)\ \delta\rho(\vec{x})^{(1)}(t) + c_2(t)\ \delta\rho(\vec{x})^{(2)}(t) + \dots$$

- where $c_i(t) = \int dt'\ K(t,t')\ D(t')^i$

- Difference: Time-Local QFT: $c_1(t)\left[\delta\rho(\vec{x})^{(1)}(t) + \delta\rho(\vec{x})^{(2)}(t) + \dots\right]$
  Non-Time-Local QFT: $c_1(t)\ \delta\rho(\vec{x})^{(1)}(t) + c_2(t)\delta\rho(\vec{x})^{(2)}(t) + \dots$

  - More terms, but not a disaster

Perturbation Theory within the EFT

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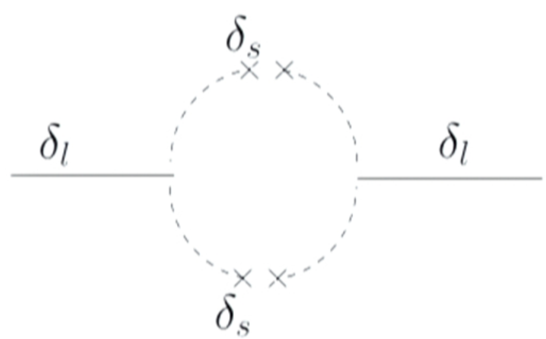

Perturbation Theory within the EFT

- Regularization and renormalization of loops (no-scale universe) $P_{11}(k) = \frac{1}{{k_{\rm NL}}^3}\left(\frac{k}{k_{\rm NL}}\right)^n$

  –evaluate with cutoff:

$$P_{1-\rm loop} = c_1^{\Lambda}\left(\frac{\Lambda}{k_{\rm NL}}\right)\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

  – divergence (we extrapolated the equations where they were not valid anymore)


Perturbation Theory within the EFT
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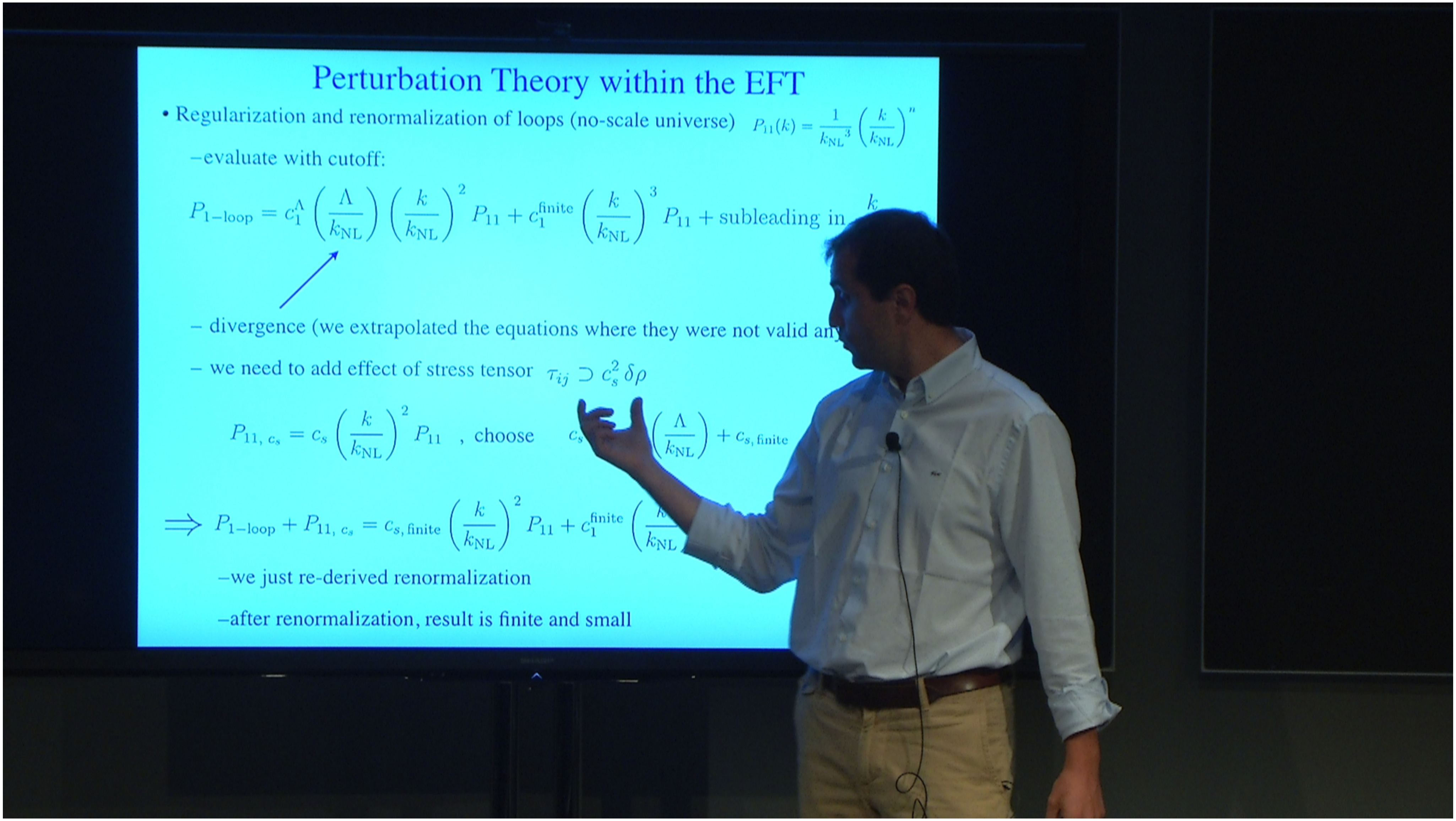
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$\Longrightarrow P_{1-\rm loop} + P_{11,\,c_s} = c_{s,\,\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite}$
–we just re-derived renormalization
–after renormalization, result is finite and small


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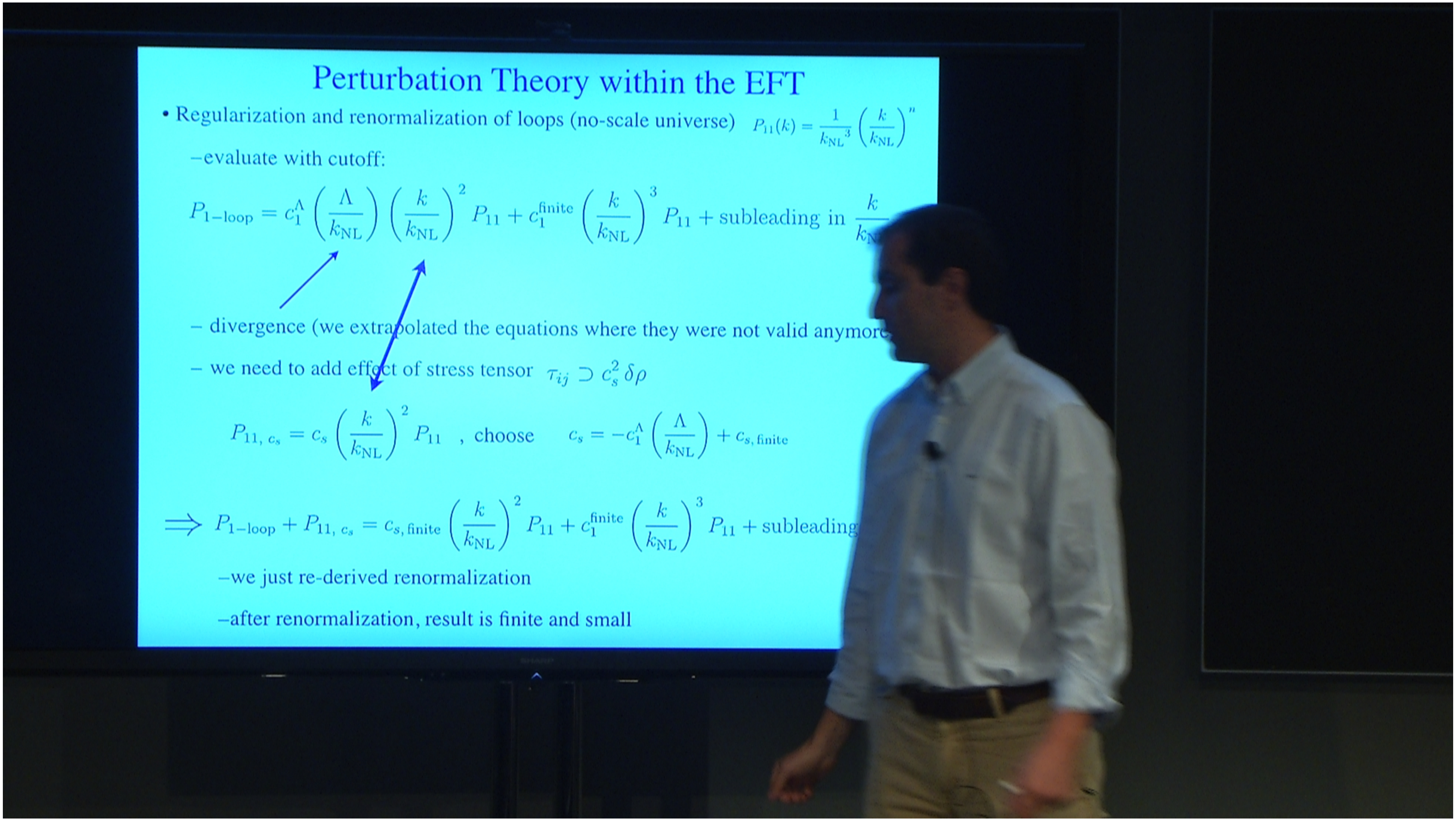
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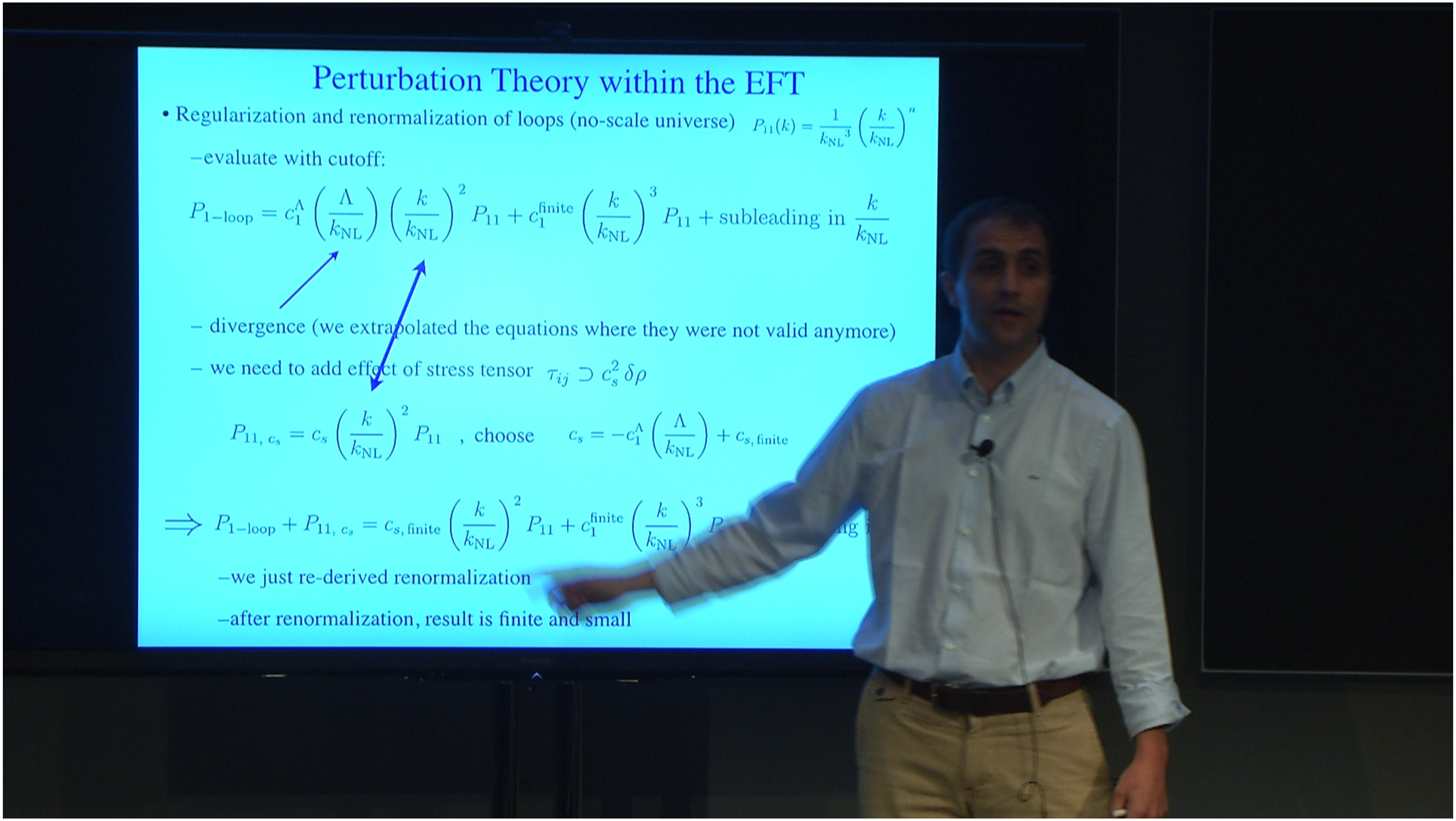
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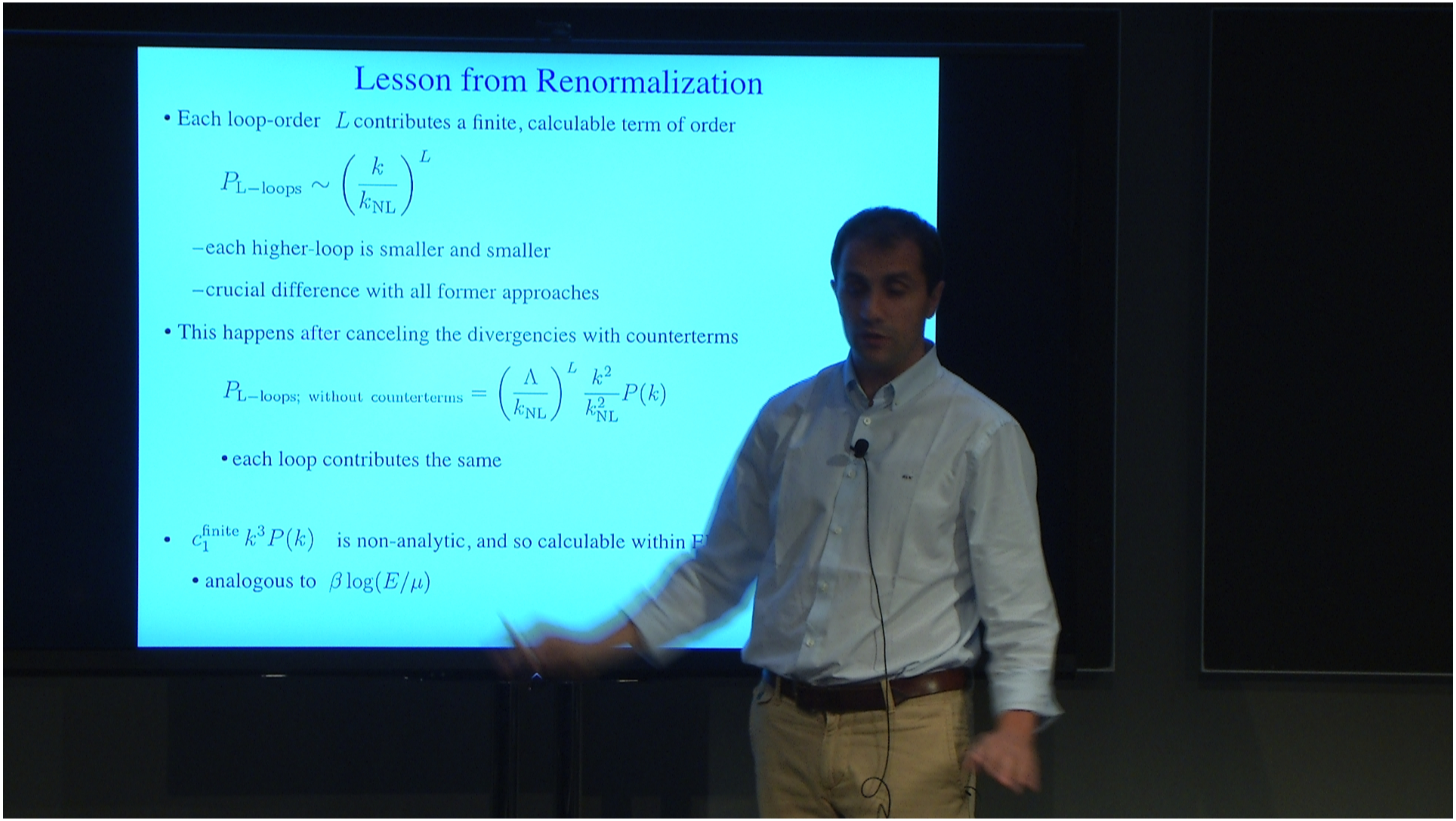
Lesson from Renormalization
• Each loop-order L contributes a finite, calculable term of order
$P_{\rm L-loops} \sim \left(\frac{k}{k_{\rm NL}}\right)^L$
–each higher-loop is smaller and smaller
–crucial difference with all former approaches
• This happens after canceling the divergencies with counterterms
$P_{\rm L-loops;\ without\ counterterms} = \left(\frac{\Lambda}{k_{\rm NL}}\right)^L \frac{k^2}{k_{\rm NL}^2} P(k)$
• each loop contributes the same
• $c_1^{\rm finite}\, k^3 P(k)$ is non-analytic, and so calculable within
• analogous to $\beta \log(E/\mu)$

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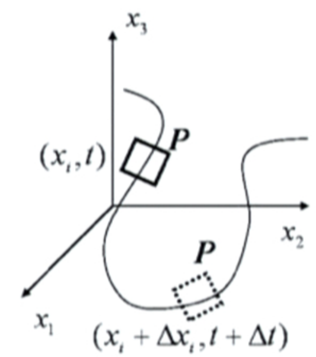
  - analogous to $\beta \log(E/\mu)$

Connecting with the Eulerian Treatment

- When we solve iteratively these equations in $\delta_\ell, v_\ell, \Phi_\ell \ll 1$,
  - this corresponds to expanding in three parameters:

$$\epsilon_{\rm tidal}(k) \sim \int^k d^3q \; P(q)$$ Effect of Long Overdensities

$$\epsilon_{\rm long\ displacement}(k) \sim k^2 \int^k d^3q \; \frac{P(q)}{q^2}$$ Effect of Long Displacements



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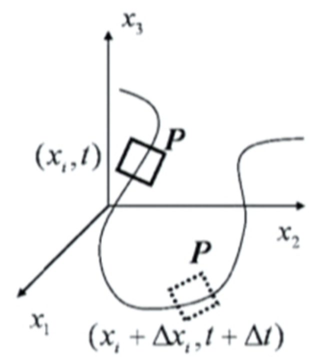

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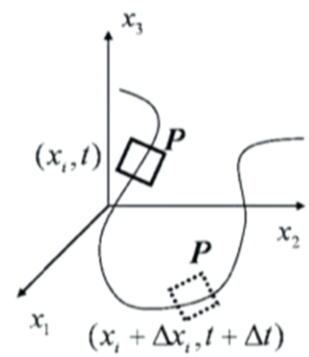
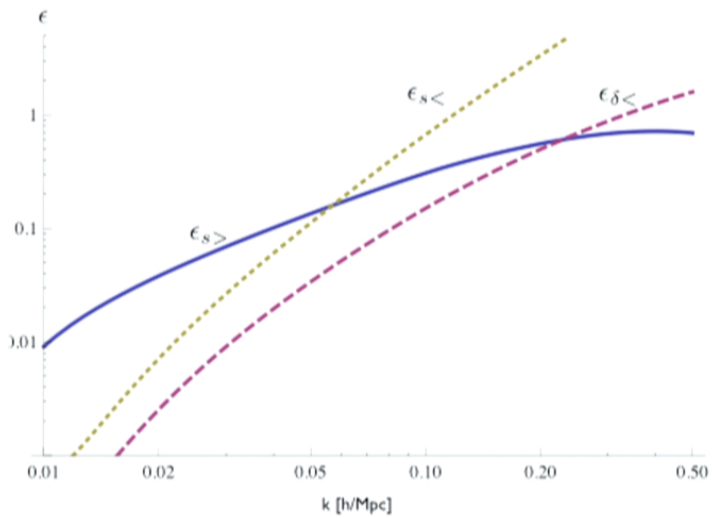


Perturbation Theory in our Universe

- In a no-scale universe $P_{11}(k) = \frac{1}{{k_{\rm NL}}^3}\left(\frac{k}{k_{\rm NL}}\right)^n$,

$$\epsilon_{\rm tidal} \sim \epsilon_{\rm long\ displacement} \sim \epsilon_{\rm short\ displacement} \sim \left(\frac{k}{k_{\rm NL}}\right)^{3+n}$$

- But our universe has features. It has more than one scale.



$\epsilon_{\rm long\ displacement}$ is of order one for low k 's, but being IR dominated, its contribution can be treated non-perturbatively

Since displacements displace (they do not deform) effect is kinematical and not dynamical (so conceivable to resum)

with Zaldarriaga **JCAP1502**

- After IR-resummation, and after renormalization, each loop goes as power of $\left(\epsilon_{\rm tidal}\right)^L$

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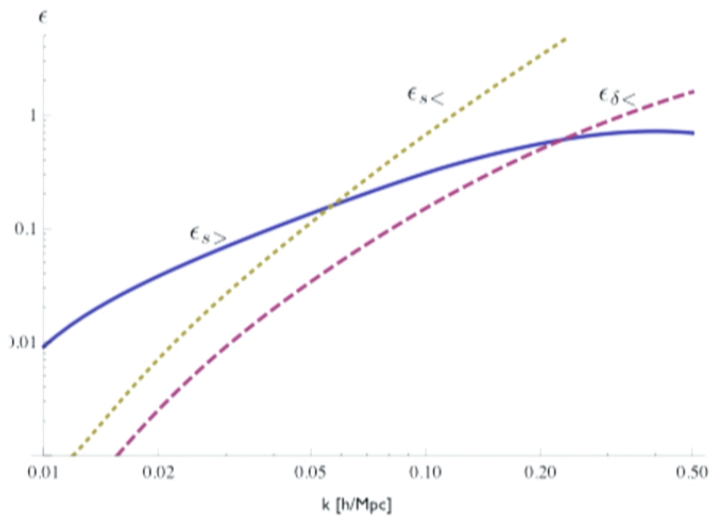

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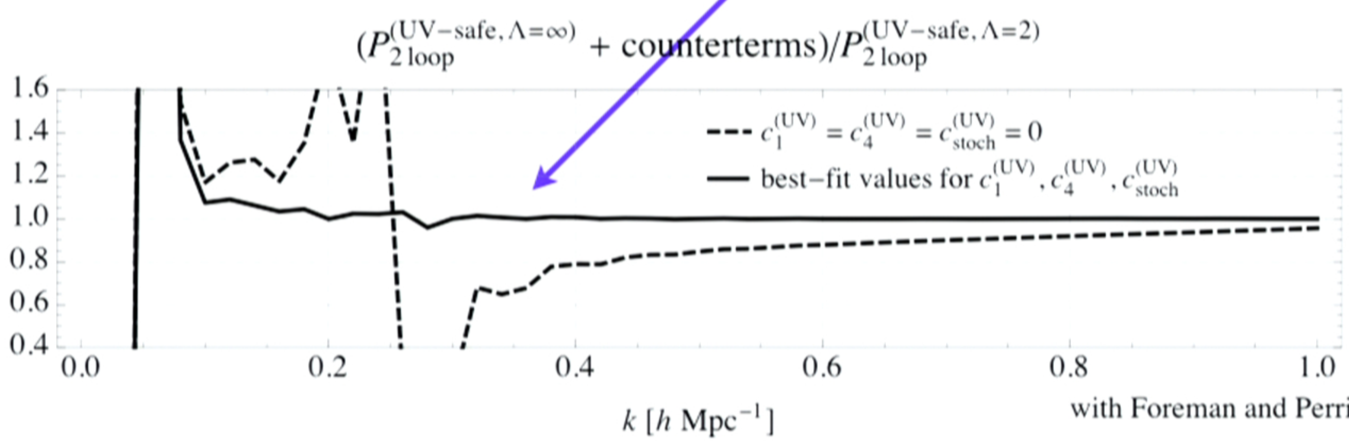
Results for Dark Matter

EFT of Large Scale Structures

- Loop contributions from non-linear modes give non-sense results: we need to correct for them: renormalization (make the calculation UV-insensitive)

- At 1-loop one counterterm is enough $\partial^2\tau_{ij} \sim c_s\, k^2\delta(k)$
- At 2-loops, consider $\partial^2\tau_{ij} \sim c_1\, k^2[\delta^2](k) + c_4\, k^4\delta(k)$

Estimate size of counterterms
by requiring cutoff independent result



with Foreman and Perrier **1507**

- $\Longrightarrow$ At two-loops, with precise data, 3 counterterms are needed, and we estimate size
- The fact that this works is another proof that the EFTofLSS is correct

EFT of Large Scale Structures

with Foreman and Perrier **1507**

- At 2-loops, we need speed of sound & quadratic & higher-derivative counterterm:

$$\partial^2 \tau_{ij} \sim c_s \, k^2 \delta(k) + c_1 \, k^2 [\delta^2](k) + c_4 \, k^4 \delta(k)$$

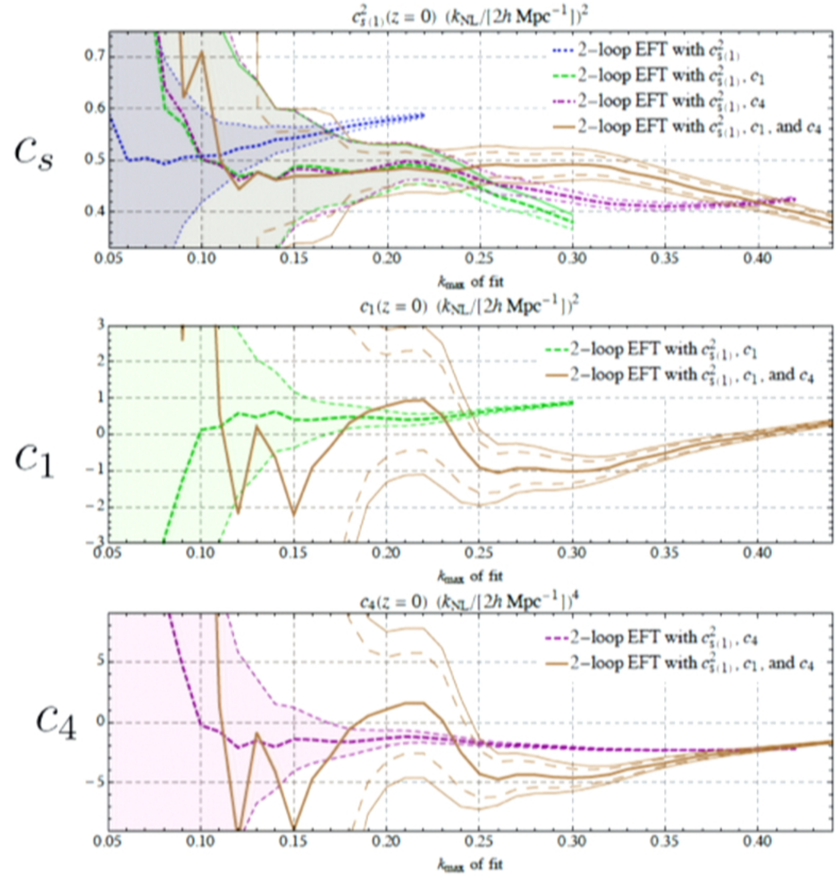
- How to choose for them?
  - Fit them to data

EFT of Large Scale Structures at Two Loops

$$\partial^2 \tau_{ij} \sim c_s\, k^2 \delta(k) + c_1\, k^2 [\delta^2](k) + c_4\, k^4 \delta(k)$$

- As data increase, the improved measurement of parameters should be compatible with measurement with less data

- If we fit until $k \sim 0.32\, h\, \mathrm{Mpc}^{-1}$ we are not overfitting



with Foreman and Perrier **1507**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c_1
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with Foreman and Perrier 1507

EFT of Large Scale Structures at Two Loops

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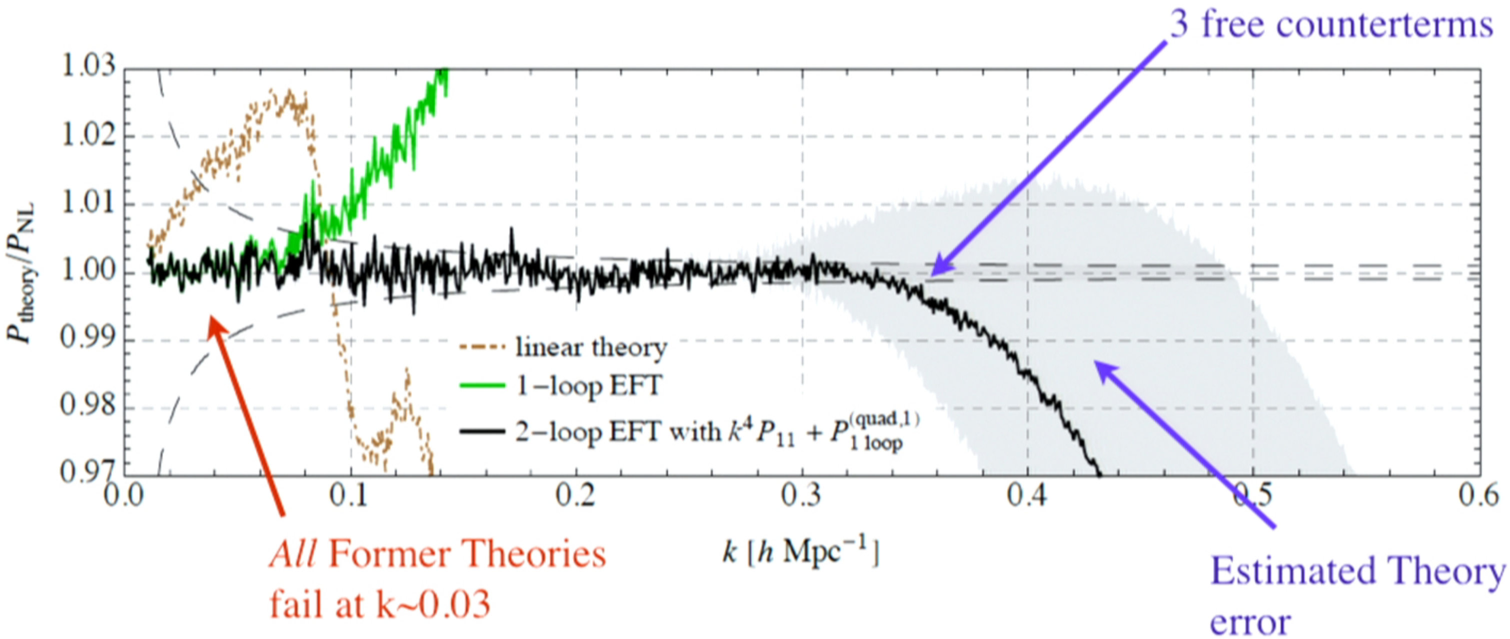
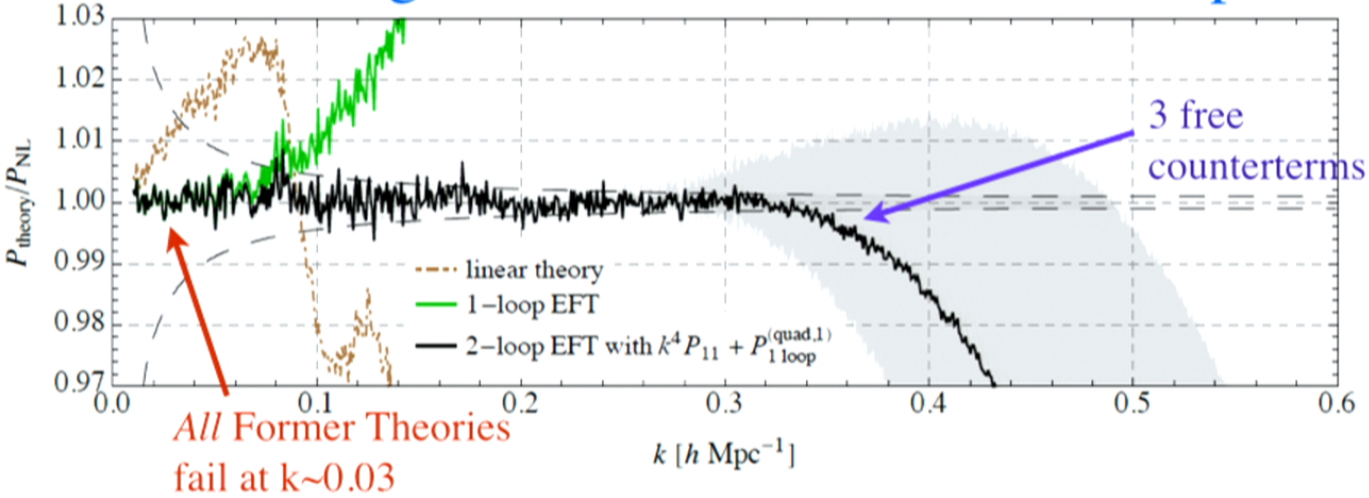


- k-reach pushed to $k \sim 0.34\, h\, \mathrm{Mpc}^{-1}$, cosmic variance $\sim 10^{-3}$
- Order by order improvement $\left(\frac{k}{k_{\rm NL}}\right)^L$
- Huge gain wrt former theories
- Theory error estimated

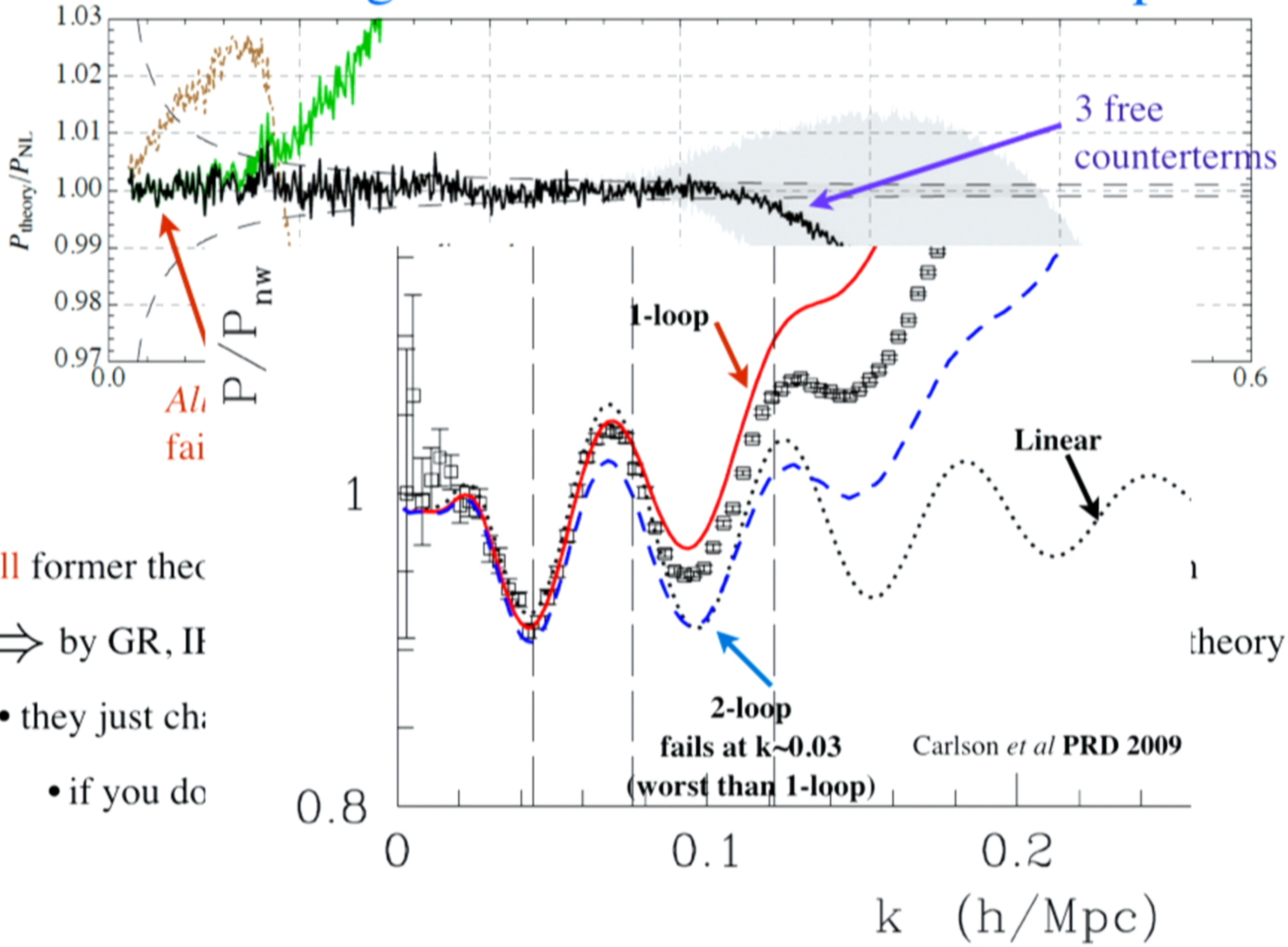
with Carrasco, Foreman and Green **JCAP1407**
with Zaldarriaga **JCAP1502**
with Foreman and Perrier **1507**
see also Baldauf, Shaan, Mercolli and Zaldarriaga **1507 , 1507**

EFT of Large Scale Structures at Two Loops



- All former theories, RPT, LPT,.... differ from SPT just by the IR-resummation
- $\Longrightarrow$ by GR, IR-modes cancel in $P(k)$, so cannot change broad k-reach of the theory
  - they just change the BAO, which are 2% oscillations in k-space
    - if you doubt this, please ask me questions

EFT of Large Scale Structures at Two Loops



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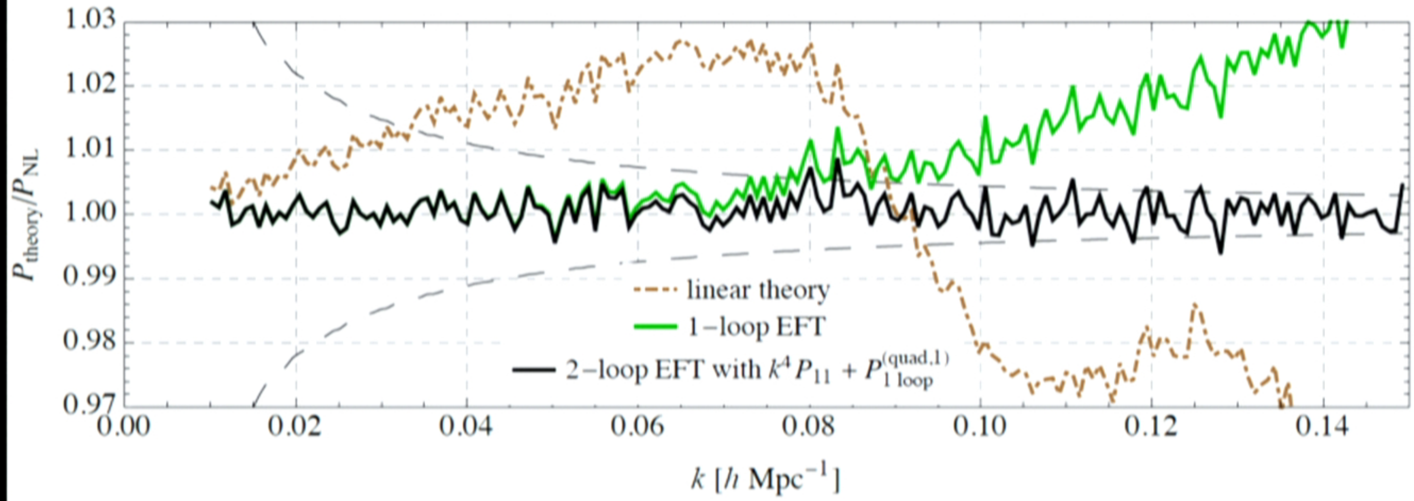
    - if you do

theory

Precision at low k's



- Precision at low k's is also important and great
- Look where linear theory fails by 1% at $k \sim 0.03\, h\, \mathrm{Mpc}^{-1}$!
- we can see that order by order, at low k's, the EFT converges!
  - former techniques and N-body sims *do not* converge to this accuracy

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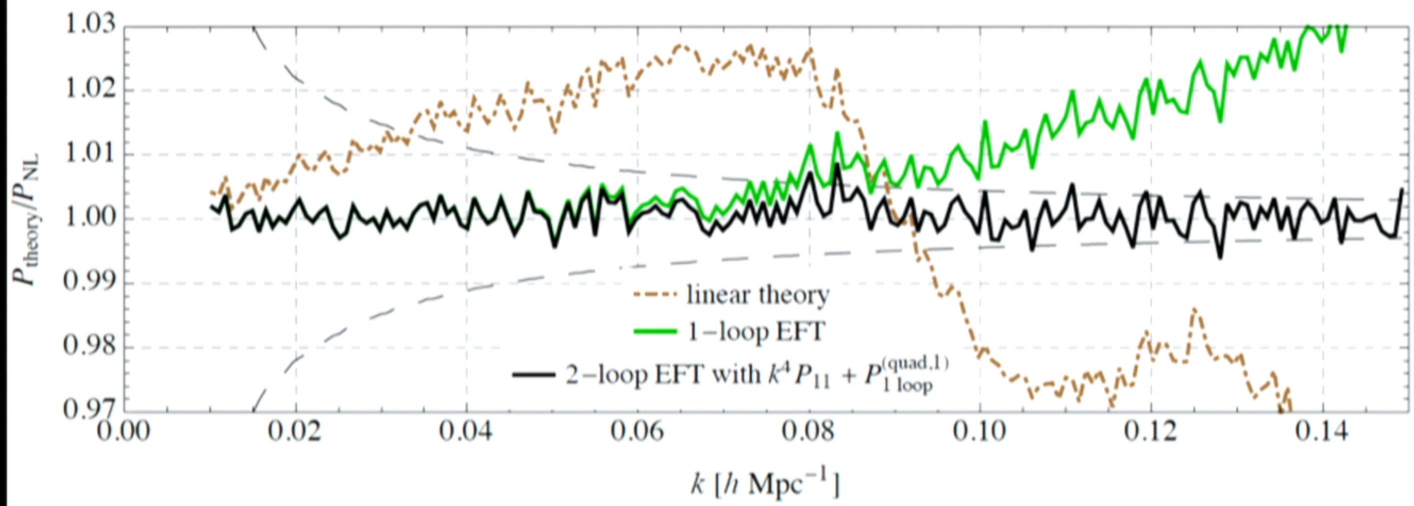

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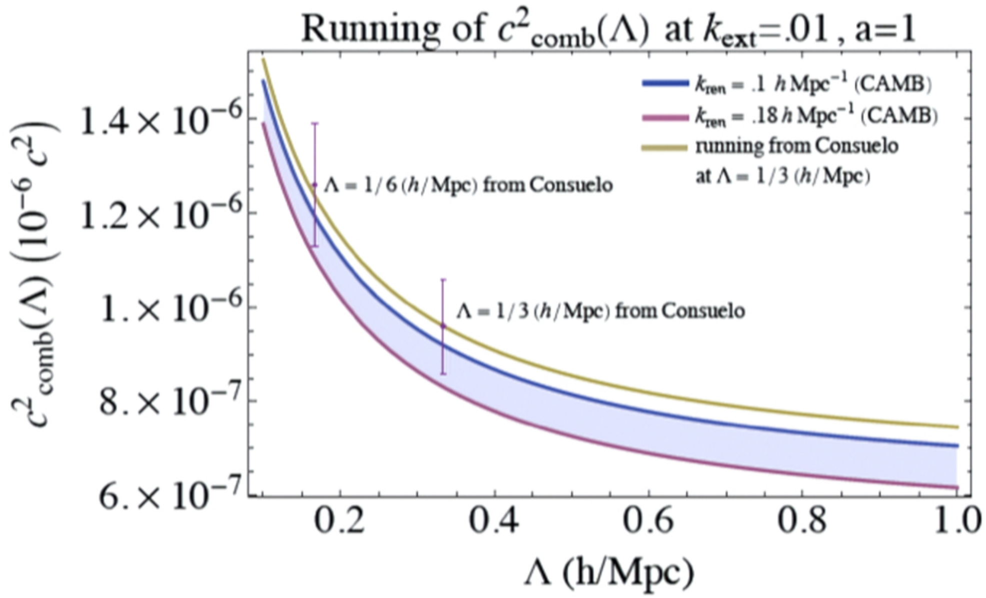
In the EFTofLSS we need parameters.
Let us measure them from
small N-body Simulations!

with Carrasco and Hertzberg **JHEP 2012**

Measuring parameters from N-body sims.

- The EFT parameters can be measured from small N-body simulations, using UV theory
  - –similar to what happens in QCD: lattice sims
- We measure c_s using the dark matter particles:

$$\tau_{ij} \sim \sum_i m_i \left(v_i^2 + \phi_i\right)$$



- Lattice running
- Agreement with fitting from Power Spectrum directly

$$\frac{d\,c_s}{d\Lambda} = \frac{d}{d\,\Lambda}\int^{\Lambda} d^3k\;P_{13}(k)$$

with Carrasco and Hertzberg **JHEP 2012**
see also McQuinn and White **1502**

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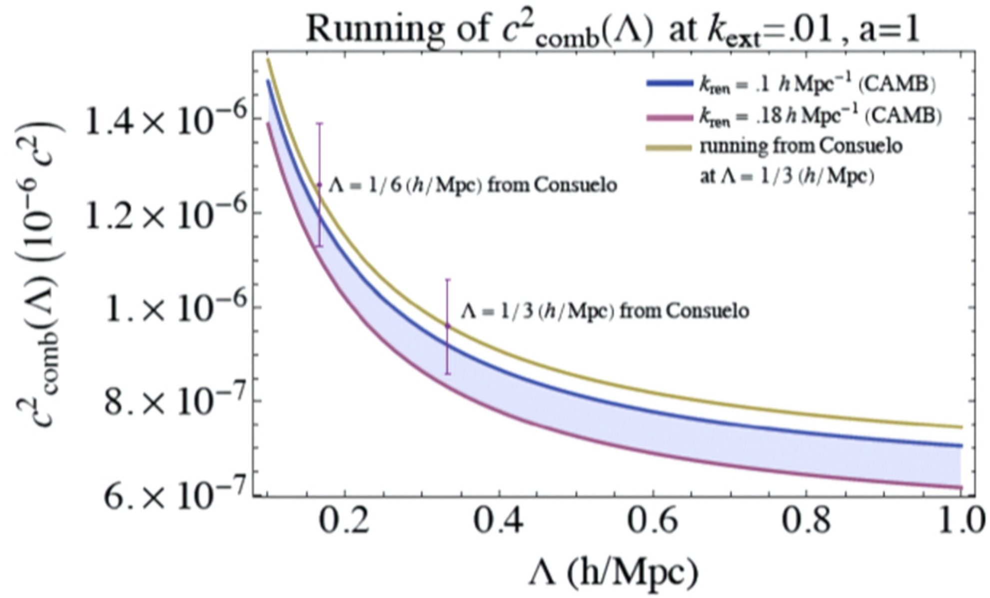

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with Carrasco and Hertzberg **JHEP 2012**
see also McQuinn and White **1502**

Other Observables

Other Observables

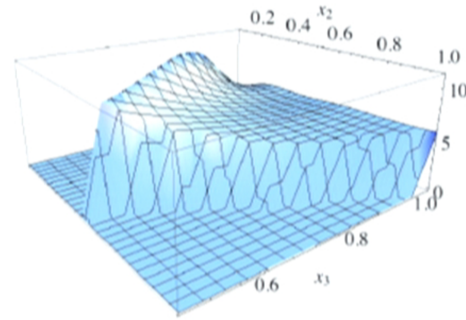
– Since this is a theory and not a model

  – prediction for other observables from same parameters

– 3point function

  – very non-trivial function of three variables!

with Angulo, Foreman and Schmittful **1406**
see also Baldauf et al. **1406**



– Momentum

  – They all work as they should

with Carrasco, Foreman and Green **JCAP 1407**
Baldauf, Mercolli and Zaldarriaga **1507**

– Vorticity Spectrum with Carrasco, Foreman and Green **JCAP1407**

  – agrees with most accurate measurements in simulations

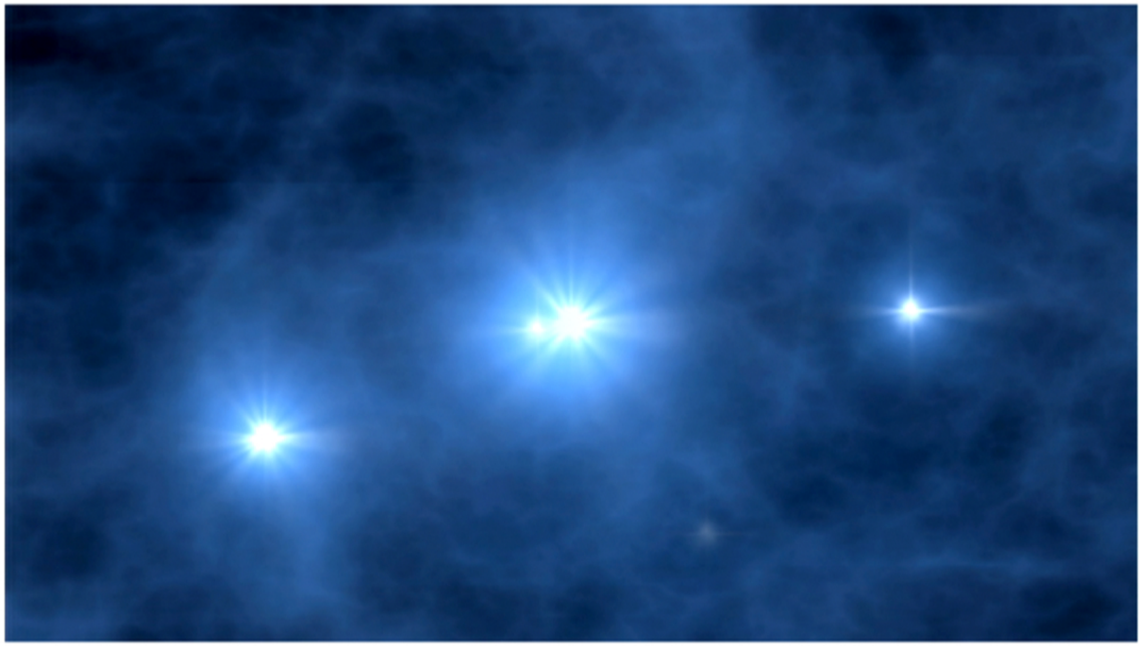
Pueblas and Scoccimarro **0809**
Hahn, Angulo, Abel **1404**

Analytic Prediction of Baryon Effects

with Lewandowski and Perko **JCAP1502**

Baryonic effects

- When stars explode, baryons behave differently than dark matter



- They cannot be reliably simulated due to large range of scales

Baryons

- Main idea for EFT for dark matter:
  - since in history of universe Dark Matter moves about $1/k_{\rm NL} \sim 10\,{\rm Mpc}$
    - $\Longrightarrow$ it is an effective fluid-like system with mean free path $\sim 1/k_{\rm NL}$
- Baryons heat due to star formation, but they do not move much:
  - indeed, from observations in clusters, we know that they move

$$1/k_{{\rm NL}(B)} \sim 1/k_{\rm NL} \sim 10\,{\rm Mpc}$$

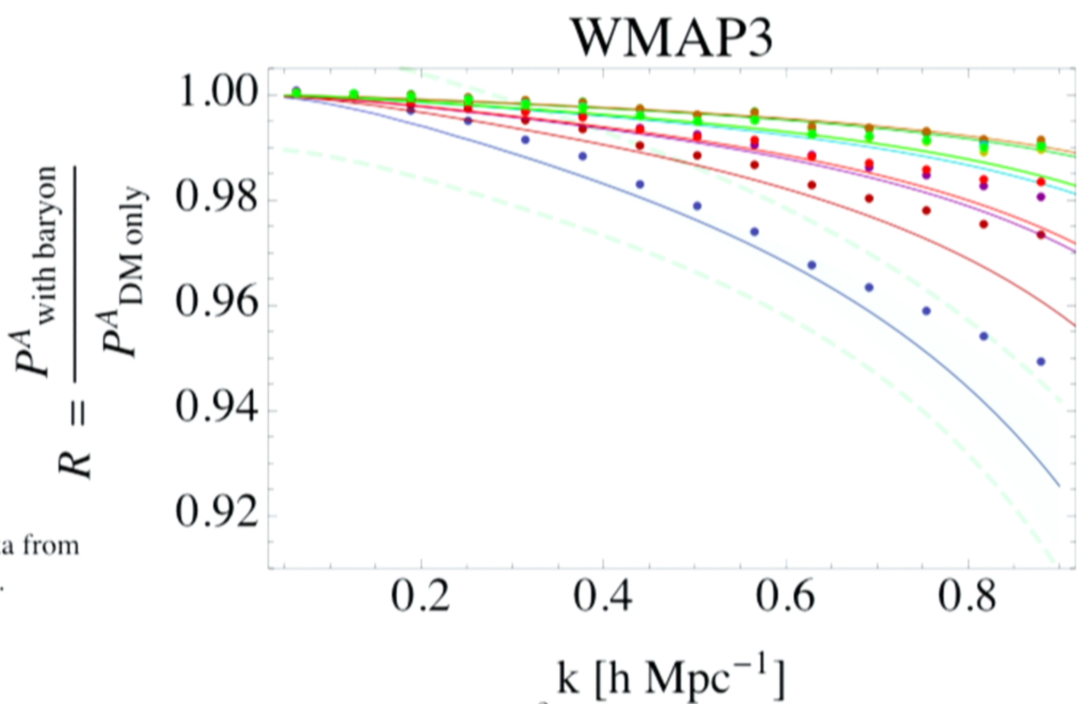
  - $\Longrightarrow$ it is an effective fluid with similar mean free path
  - Universe with CDM+Baryons $\Longrightarrow$ EFTofLSS with 2 species
- The effective force on baryons: expand force in long-wavelength fields:

$$\partial^2 \tau_b + \partial \gamma_b \sim c_s^2\, \partial^2 \delta_l + c_\star\, \partial^2 \delta_l + \dots$$

gravity-induced pressure

star formation-induced pressure

Baryons

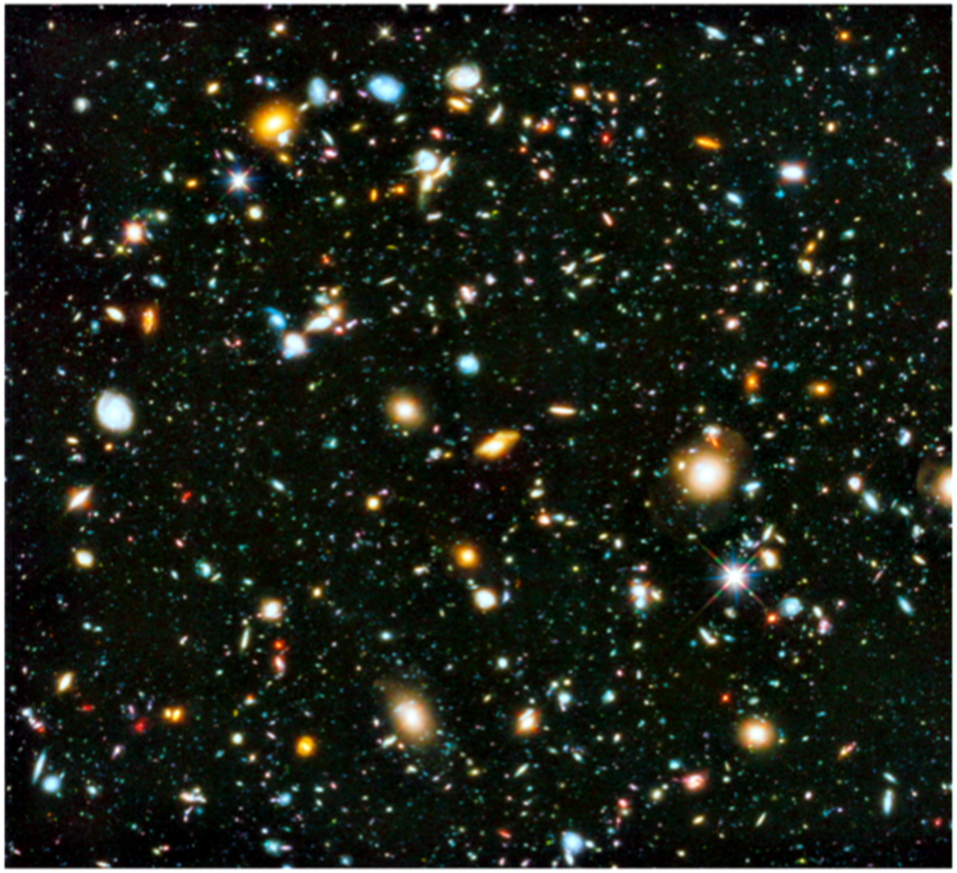


Simulation data from
J. Schaye et al.

– Analytic form of effect known: $\Delta P_b(k) \simeq c_\star^2 \left(\frac{k}{k_{\mathrm{NL}}}\right)^2 P^A_{11}(k)$

– and it seems to work as expected

Galaxies Power and Bispectrum



Galaxies in the EFTofLSS

- Similar considerations apply to biased tracers:
  - Galaxy density depends on all long fields evaluated on past history on past path

Senatore **1406**
Mirbabahi, Schmidt, Zaldarriaga **1412**

$$\delta_M(\vec{x},t) \simeq \int^t dt'\; H(t') \left[\bar{c}_{\partial^2\phi}(t,t')\, \frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\partial_i v^i}(t,t')\, \frac{\partial_i v^i(\vec{x}_{\rm fl},t')}{H(t')} + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\, \frac{\partial_i\partial_j\phi(\vec{x}_{\rm fl},t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \dots \right].$$

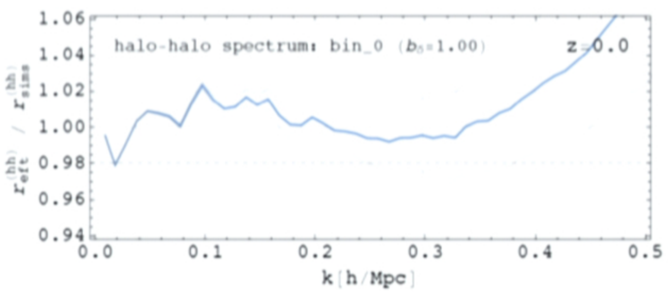
  - all terms allowed by symmetries
  - this generalizes and completes McDonald and Roy **0902**
  - this correctly parametrizes assembly bias
- Obtain only 7 parameters for
  - at 1-loop power spectrum
  - tree level bispectrum
  - tree level trispectrum

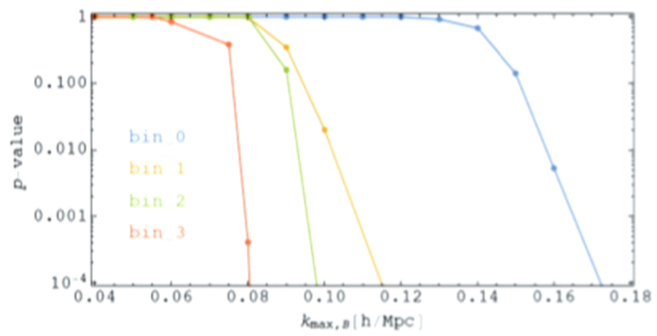
Halos in the EFTofLSS

with Angulo, Fasiello, Vlah **1503**

- We compare $P_{hh}^{1-\mathrm{loop}}$, $P_{hm}^{1-\mathrm{loop}}$, B_{hhh}^{tree}, B_{hhm}^{tree}, B_{hmm}^{tree} using 7 bias parameters

  - Fit works up to $k \simeq 0.3\, h\mathrm{Mpc}^{-1}$ for 1-loop and $k \simeq 0.15\, h\mathrm{Mpc}^{-1}$ at tree-level (for low bins, with large theory uncertainties): as it should





  - the 3pt function measures very well the bias coefficients (there is a lot of data)

  - Similar formulas just worked out for redshift space distortions

with Zaldarriaga **1409**

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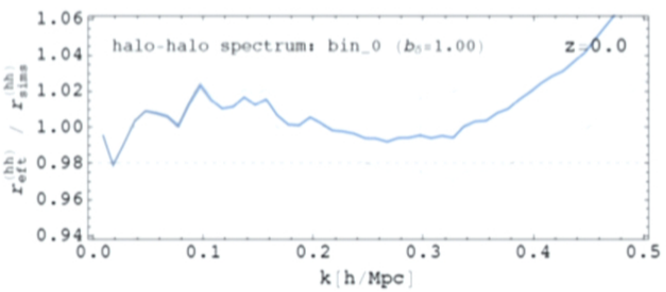

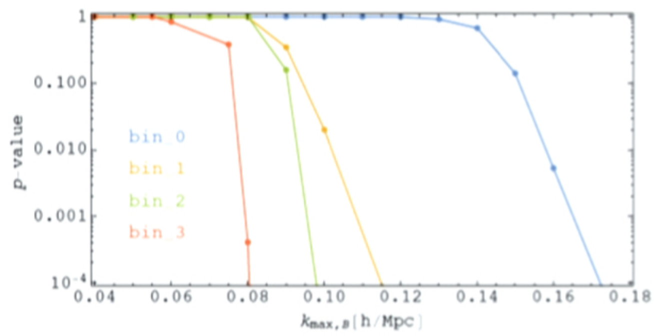

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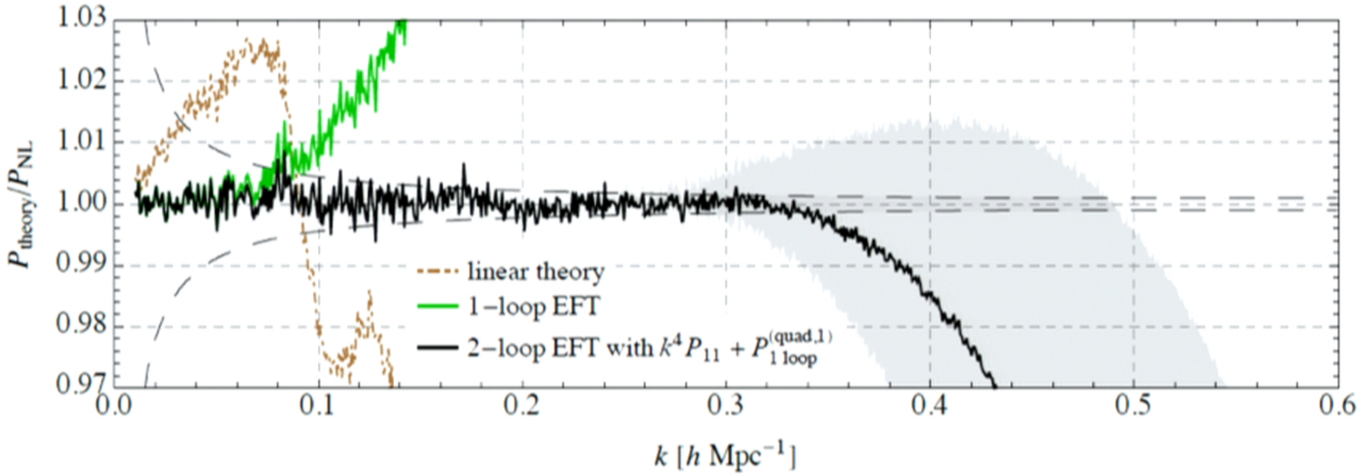
The EFT of Large Scale Structures



- A manifestly well-defined perturbation theory $\left(\frac{k}{k_{\rm NL}}\right)^L$
- we match until $k \sim 0.34\, h\, {\rm Mpc}^{-1}$, as where we should stop fitting
  - –there are ~10^2 more quasi linear modes than previously believed!
  - –huge impact on possibilities, for ex: $f_{\rm NL}^{\rm equil.,\, orthog.} \lesssim 1$, neutrinos, dark energy.
- This is an huge opportunity and a challenge for us.

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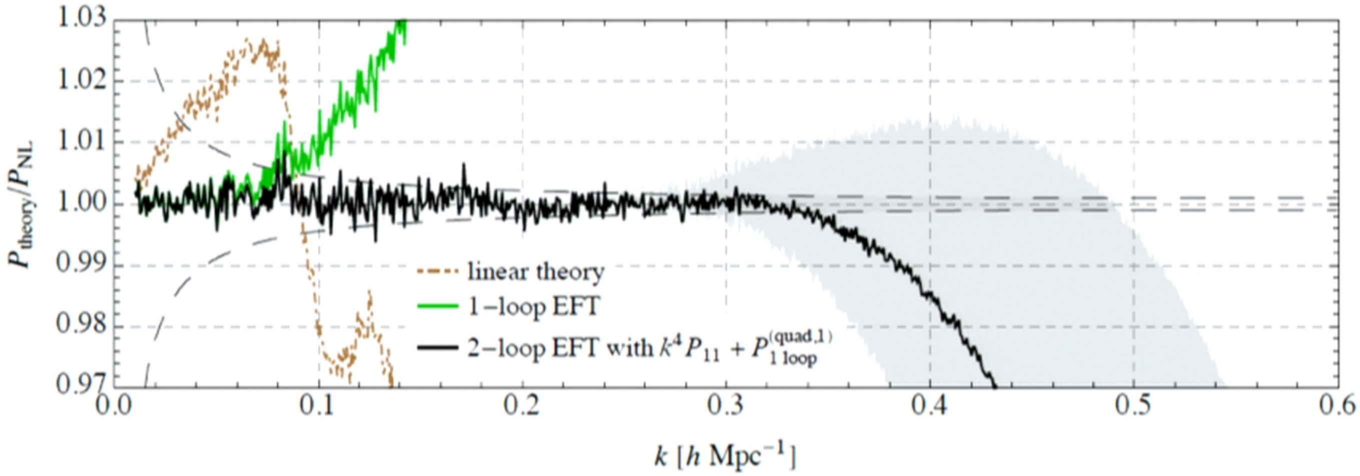

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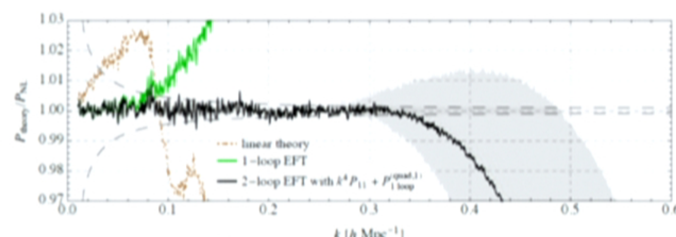
Conclusions

- The EFTofLSS: a novel and powerful way to analytically describe Large Scale Structures
  - It describes something true, the real universe: many application for astrophysics
  - It uses novel techniques that come from particle physics
    - Loops, divergencies, counterterms, renormalization & non-ren., IR divergencies
    - Measurements in Simulations (lattice) and lattice-running
- Many calculations and verifications to do
- Huge opportunity for complementarity with simulations
  - Maybe do simulations focused to convey the EFT parameters?!
- If success continues, revolution in our expectations for next generation experiments
  - on primordial cosmology

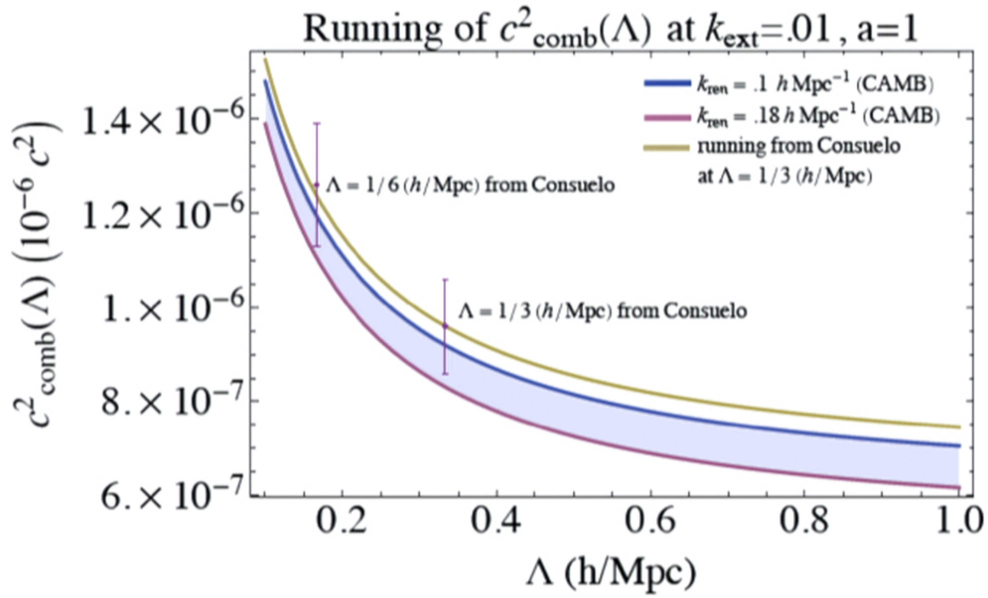


$$S_\pi = \int d^4x\,\sqrt{-g}\left[M_{\rm Pl}^2 \dot H(\dot\pi^2 - (\partial_i\pi)^2) + M_2^4\left(\dot\pi^2 + \dot\pi^3 - \dot\pi(\partial_i\pi)^2\right) - M_3^4\dot\pi^3 + \dots\right]$$

Measuring parameters from N-body sims.

- The EFT parameters can be measured from small N-body simulations, using UV theory
  - –similar to what happens in QCD: lattice sims
- We measure c_s using the dark matter particles:

$$\tau_{ij} \sim \sum_i m_i \left(v_i^2 + \phi_i\right)$$



- Lattice running
- Agreement with fitting from Power Spectrum directly

$$\frac{d\,c_s}{d\Lambda} = \frac{d}{d\,\Lambda}\int^{\Lambda} d^3k\ P_{13}(k)$$

with Carrasco and Hertzberg **JHEP 2012**
see also McQuinn and White **1502**