Title: Properties of a non-Abelian chiral spin-1 spin liquid

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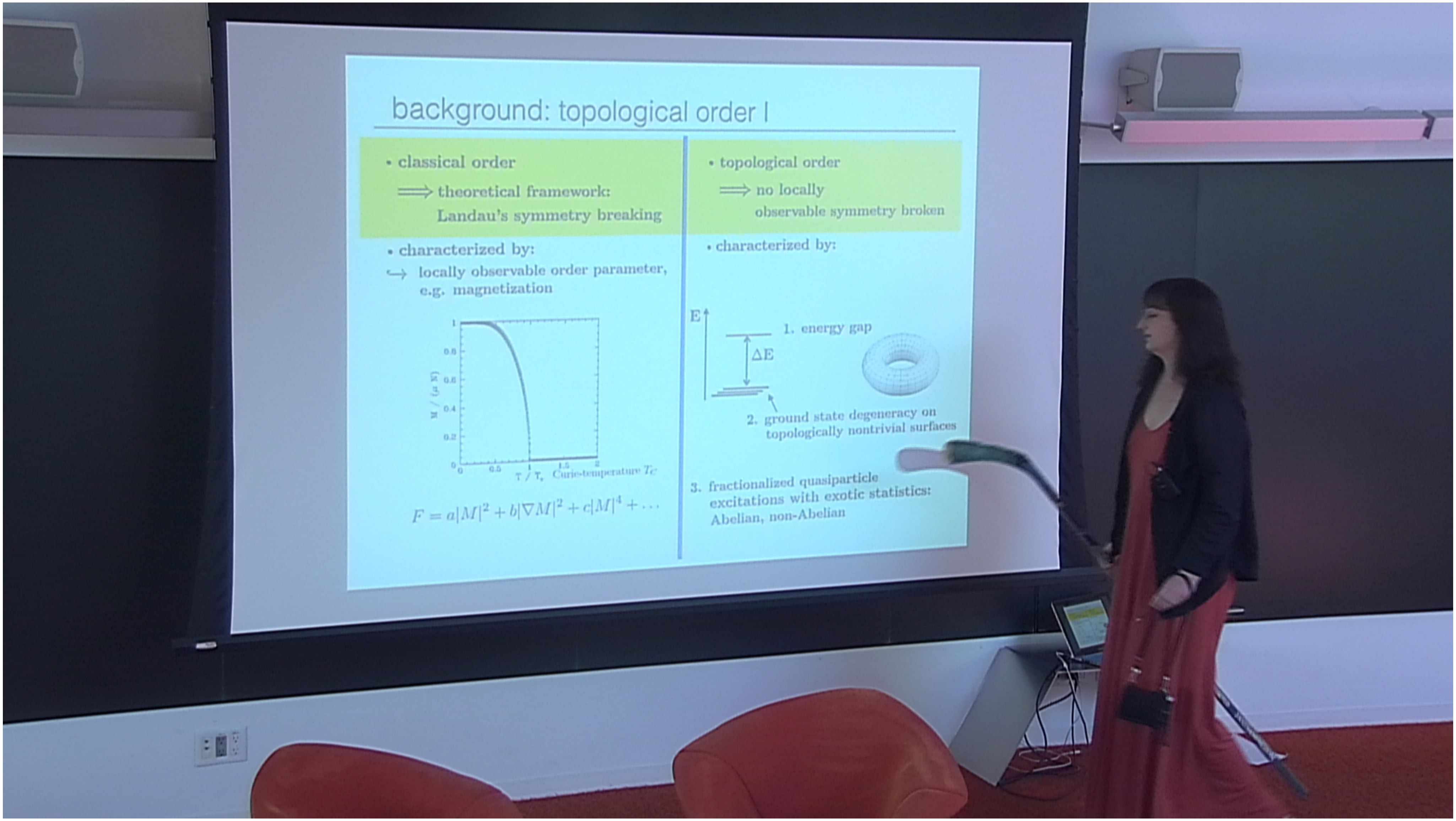
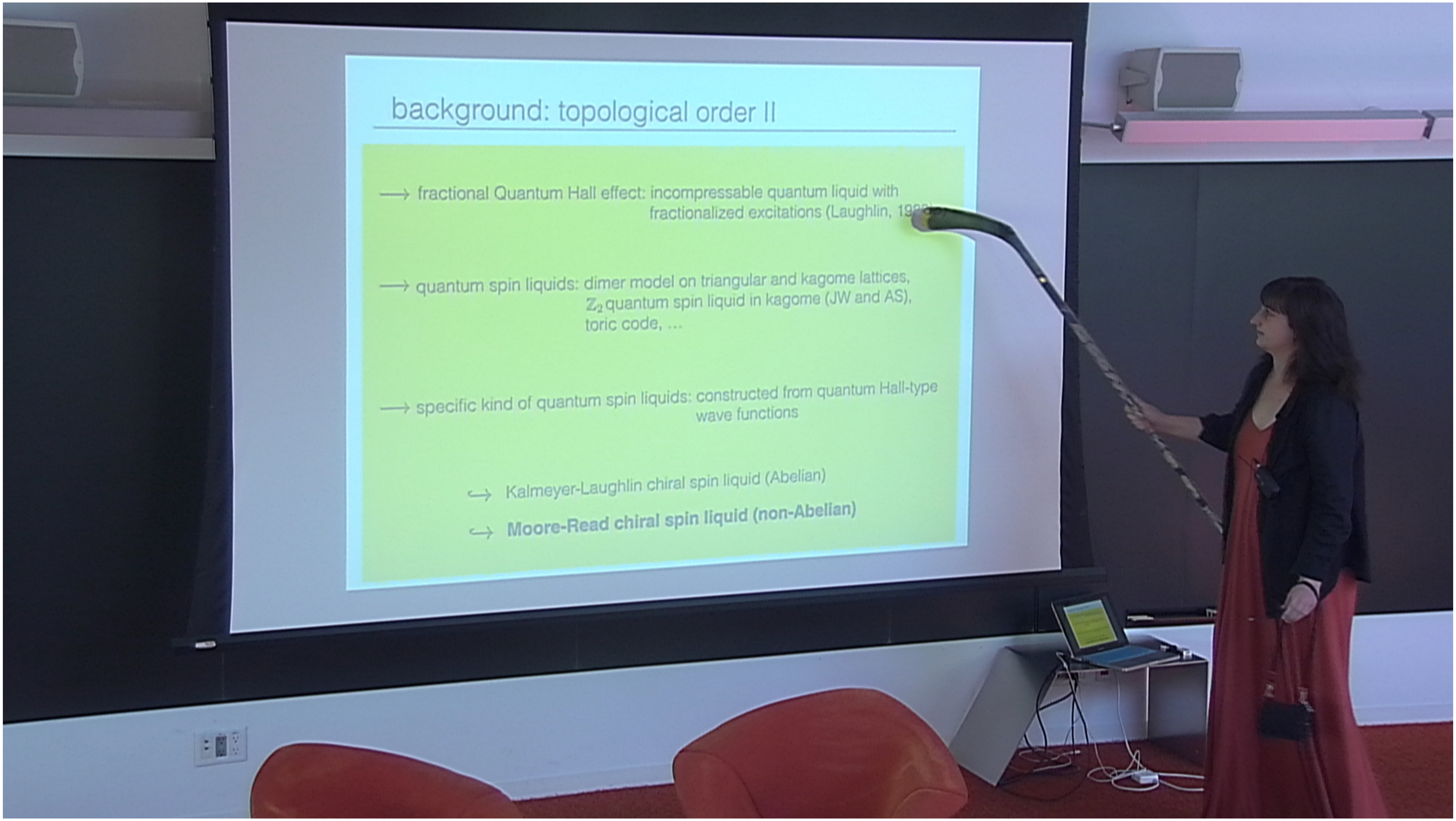
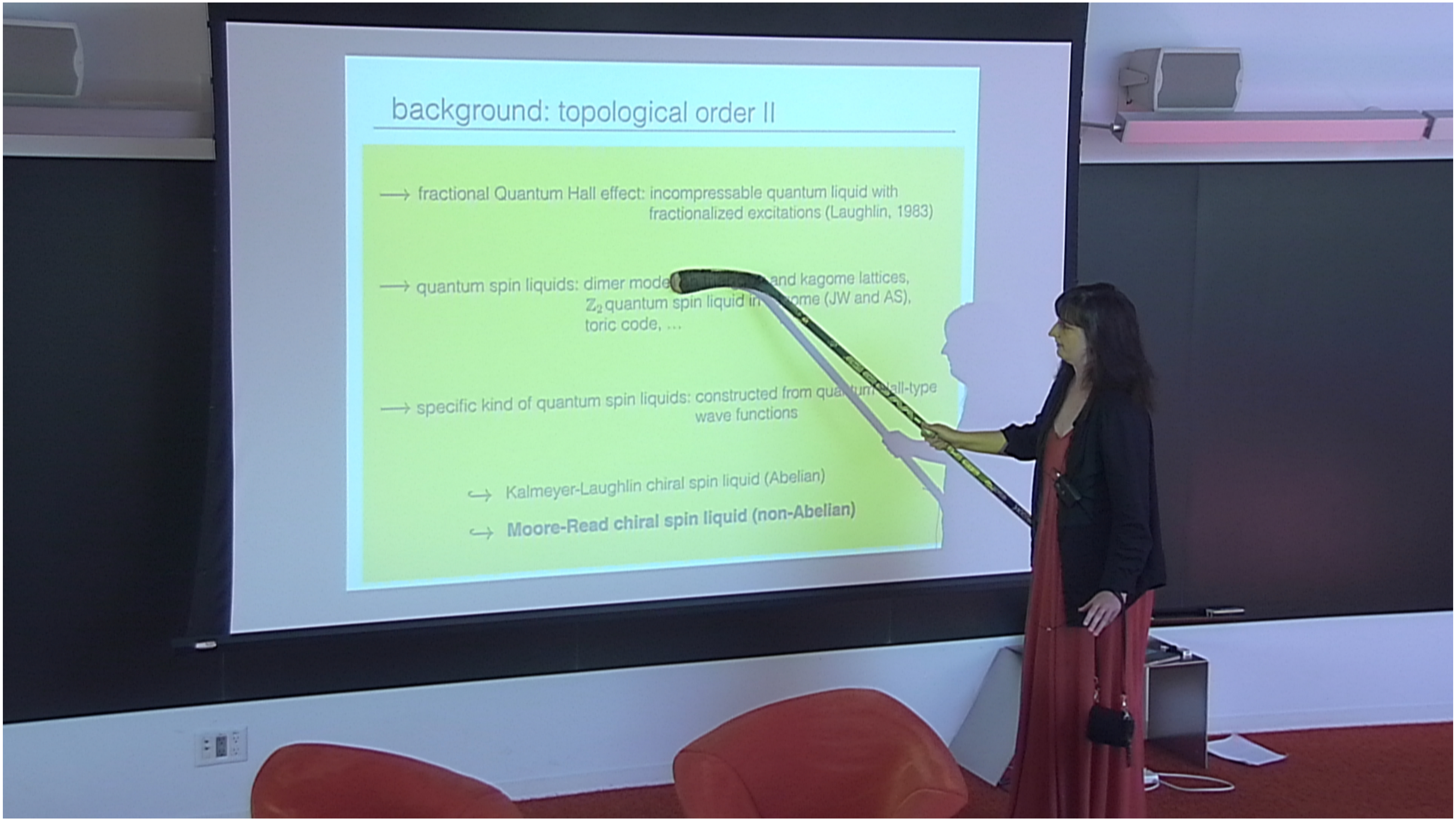
Abstract: <p>In this talk, we will analyze the properties of the bosonic $\nu = 1$ Moore-Read state when used to build a state
which is strongly believed to be a non-Abelian spin-1 chiral spin liquid state [1]. In this state the bosonic $\nu = 1$
Moore-Read Pfaffian wavefunction is interpreted as a wavefunction for a gas of bosons on a 2D square lattice with one flux quantum per plaquette. We investigate the properties of this wavefunction for the case of planar geometry and for the case of the system living on a torus. For the latter case, there are three distinct states corresponding to the three-fold degeneracy of the $\nu = 1$
bosonic Moore-Read state [2]. Our results show that correlation functions in these states become identical in the limit of large system size. Further issues investigated include the result that the three wavefunctions on the torus are linearly independent and orthogonal in the thermodynamic limit. Additionally, the Renyi entanglement entropy is also calculated for different system partitions in order to extract the topological entanglement entropy γ.

[1] M. Greiter and R. Thomale, Phys. Rev. Lett. 102, 207203 (2009).
[2] S. B. Chung and M. Stone, J. Phys. A: Math. Theor. 40, 4923 (2007).</p>


background: topological order I
• classical order
⟹ theoretical framework:
Landau's symmetry breaking
• characterized by:
↪ locally observable order parameter,
e.g. magnetization
M / (μ N)
T / T_c   Curie-temperature T_C
$F = a|M|^2 + b|\nabla M|^2 + c|M|^4 + \dots$
• topological order
⟹ no locally
observable symmetry broken
• characterized by:
E
ΔE
1. energy gap
2. ground state degeneracy on
topologically nontrivial surfaces
3. fractionalized quasiparticle
excitations with exotic statistics:
Abelian, non-Abelian


background: topological order II
⟶ fractional Quantum Hall effect: incompressable quantum liquid with
fractionalized excitations (Laughlin,
⟶ quantum spin liquids: dimer model on triangular and kagome lattices,
$\mathbb{Z}_2$ quantum spin liquid in kagome (JW and AS),
toric code, ...
⟶ specific kind of quantum spin liquids: constructed from quantum Hall-type
wave functions
↪ Kalmeyer-Laughlin chiral spin liquid (Abelian)
↪ **Moore-Read chiral spin liquid (non-Abelian)**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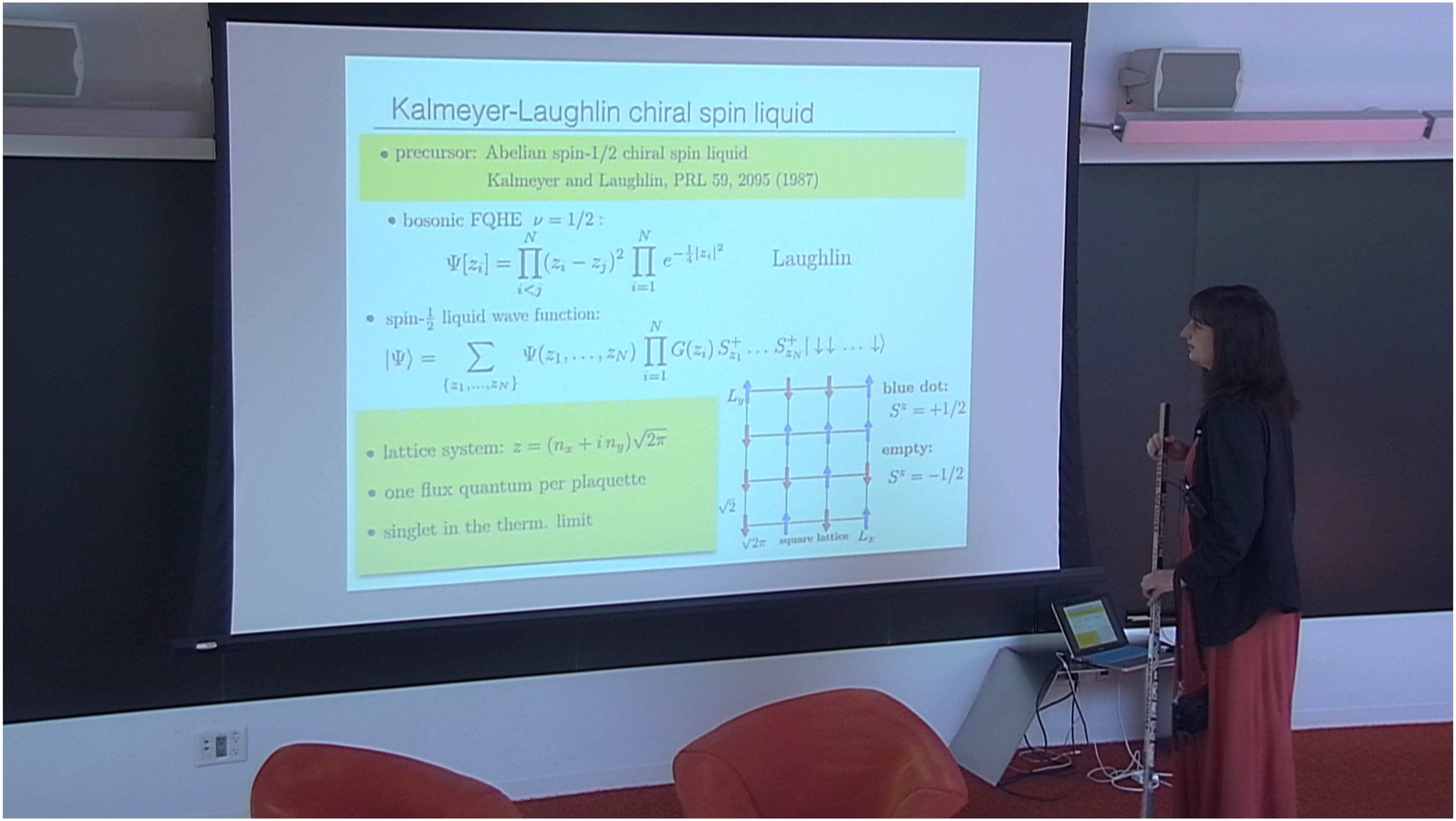
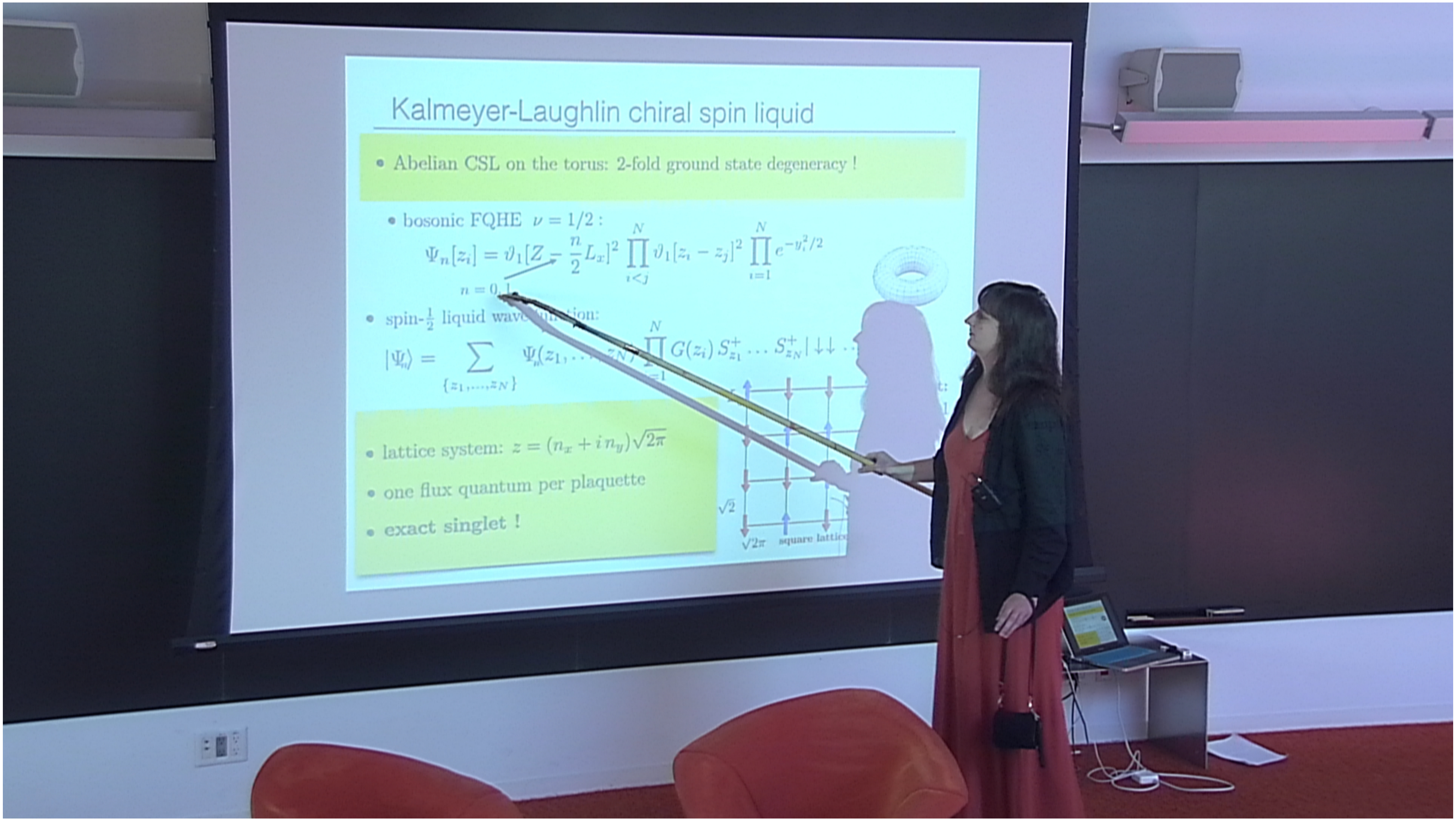
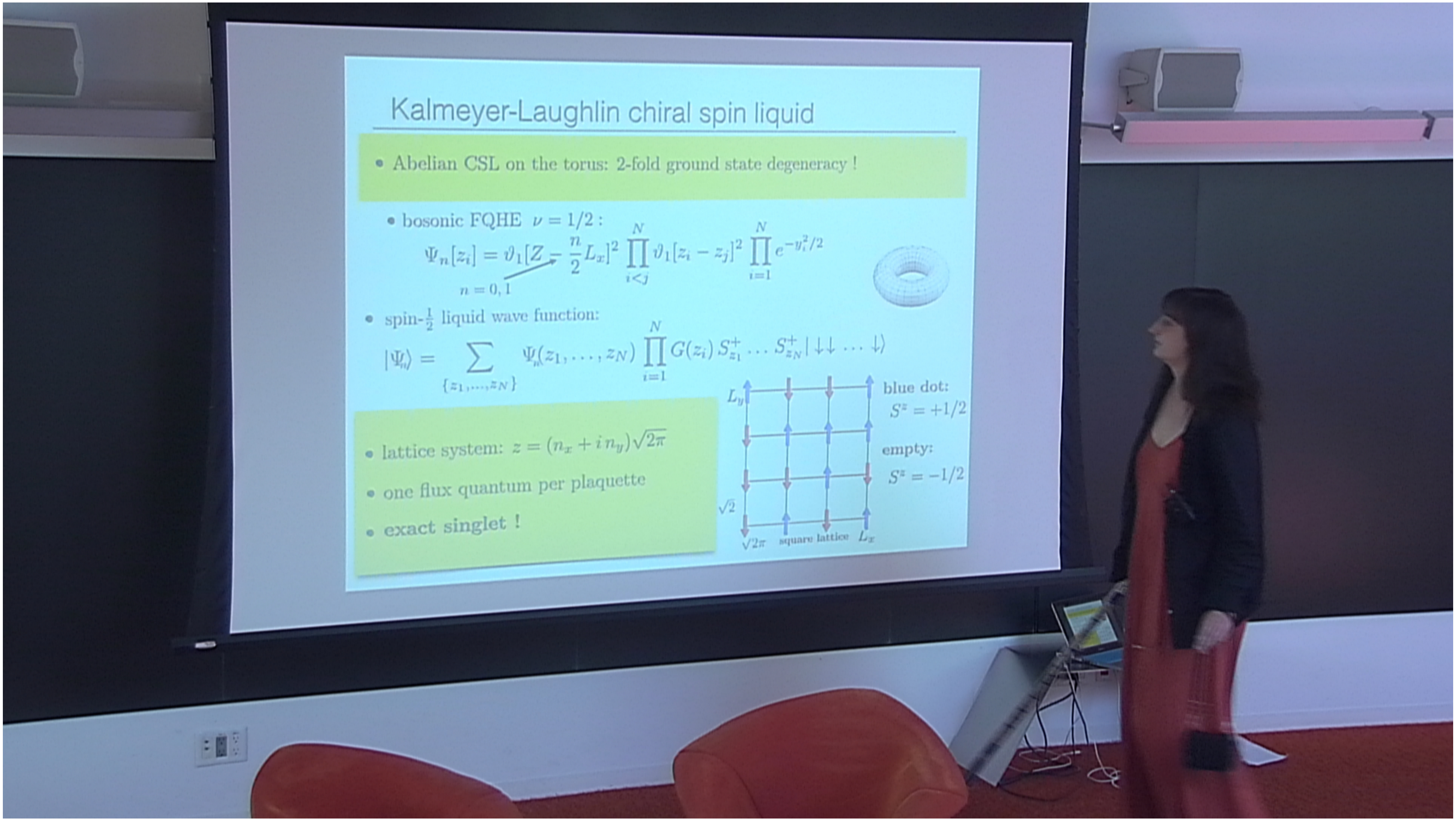


Kalmeyer-Laughlin chiral spin liquid
• precursor: Abelian spin-1/2 chiral spin liquid
Kalmeyer and Laughlin, PRL 59, 2095 (1987)
• bosonic FQHE $\nu = 1/2$:
$\Psi[z_i] = \prod_{i<j}^{N} (z_i - z_j)^2 \prod_{i=1}^{N} e^{-\frac{1}{4}|z_i|^2}$
Laughlin
• spin-$\frac{1}{2}$ liquid wave function:
$|\Psi\rangle = \sum_{\{z_1,\ldots,z_N\}} \Psi(z_1,\ldots,z_N) \prod_{i=1}^{N} G(z_i) S^+_{z_1} \ldots S^+_{z_N} |\downarrow\downarrow \ldots \downarrow\rangle$
• lattice system: $z = (n_x + i\, n_y)\sqrt{2\pi}$
• one flux quantum per plaquette
• singlet in the therm. limit
blue dot:
$S^z = +1/2$
empty:
$S^z = -1/2$
L_y
$\sqrt{2}$
$\sqrt{2\pi}$
square lattice
L_x


Kalmeyer-Laughlin chiral spin liquid
• Abelian CSL on the torus: 2-fold ground state degeneracy !
• bosonic FQHE $\nu = 1/2$:
$\Psi_n[z_i] = \vartheta_1[Z - \frac{n}{2}L_x]^2 \prod_{i<j}^{N} \vartheta_1[z_i - z_j]^2 \prod_{i=1}^{N} e^{-y_i^2/2}$
$n = 0, 1$
• spin-$\frac{1}{2}$ liquid wave
• lattice system: $z = (n_x + i\,n_y)\sqrt{2\pi}$
• one flux quantum per plaquette
• exact singlet !
$\sqrt{2}$
$\sqrt{2\pi}$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• lattice system: $z = (n_x + i\, n_y)\sqrt{2\pi}$
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square lattice
L_x
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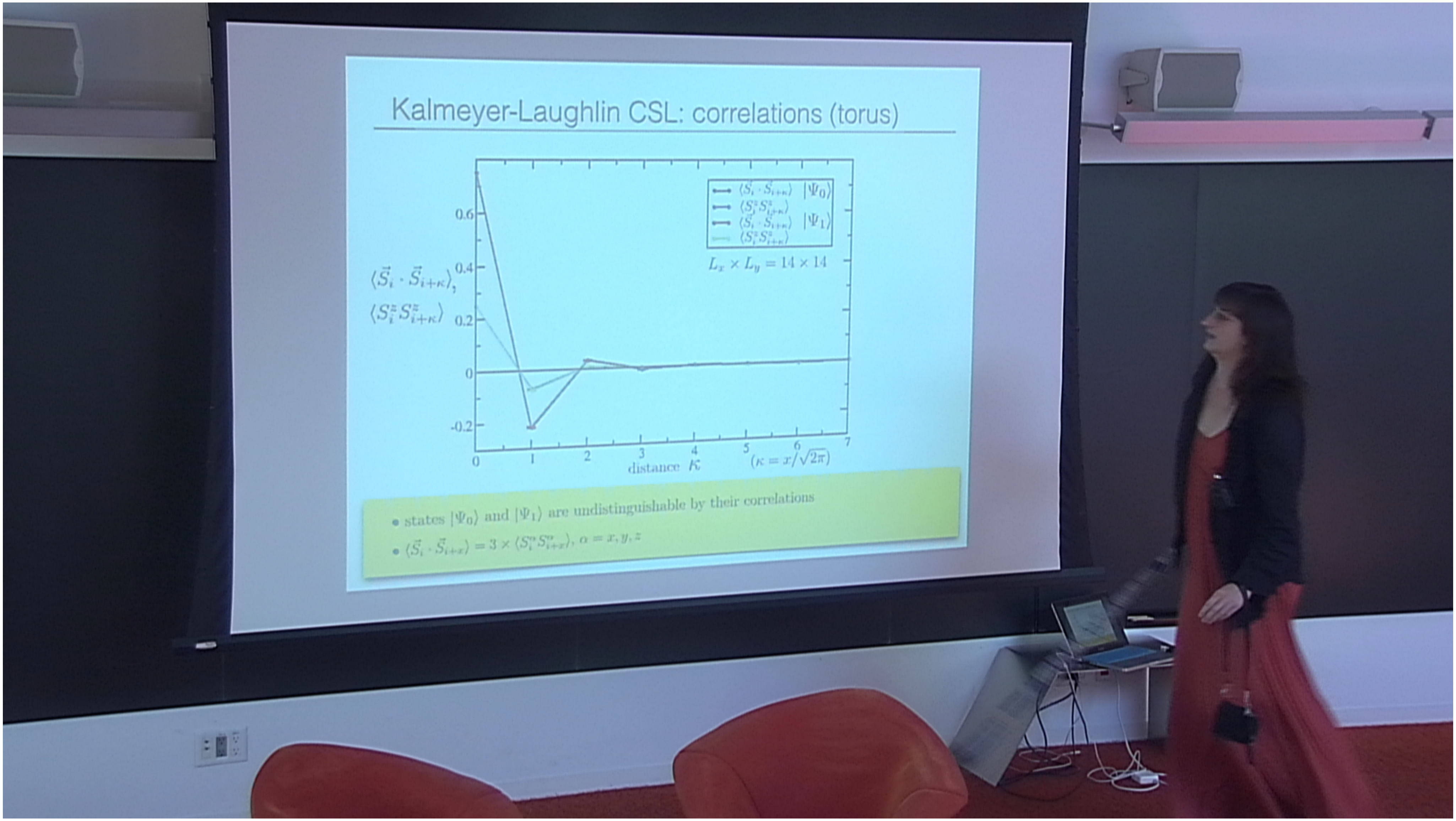
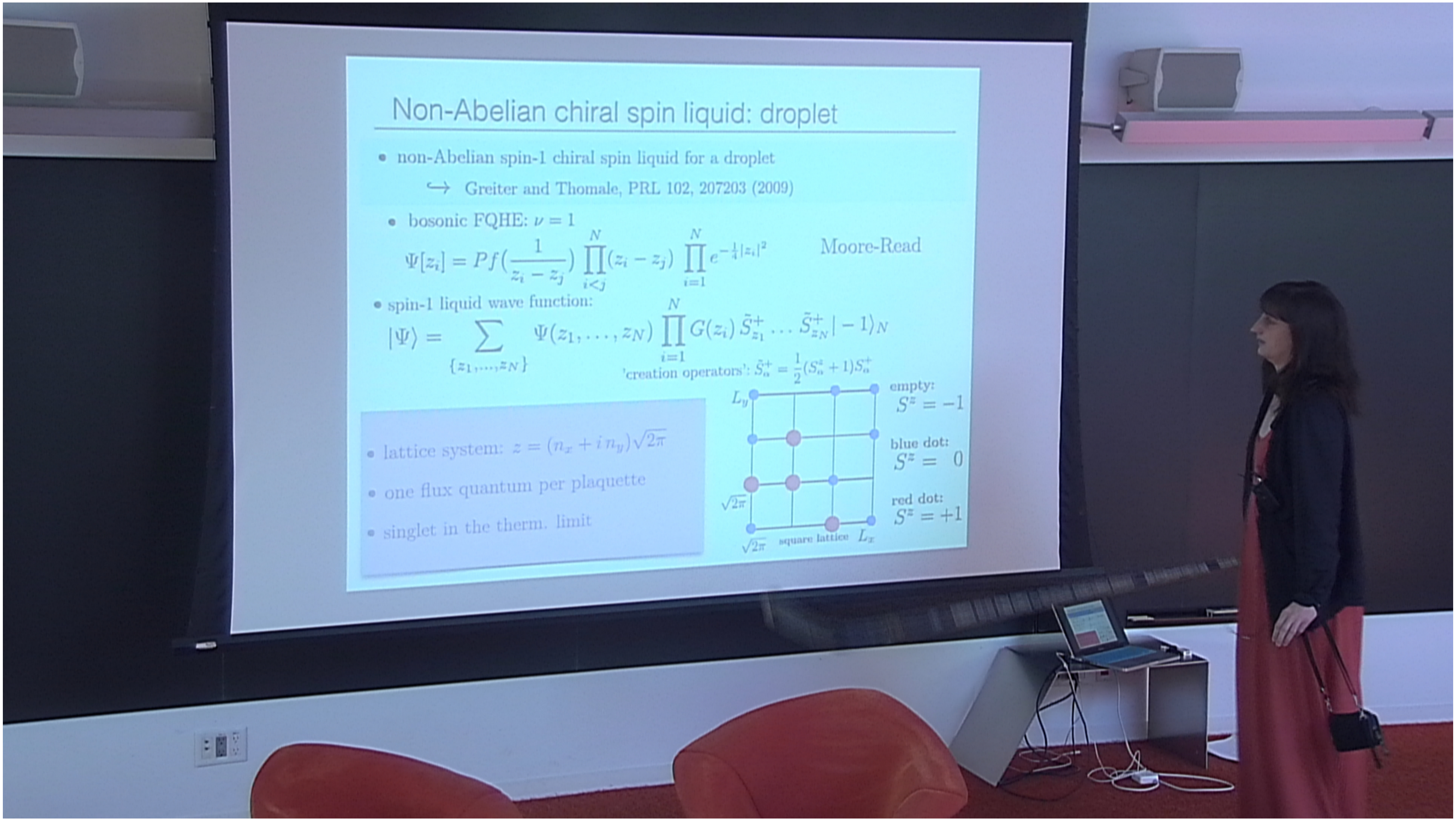
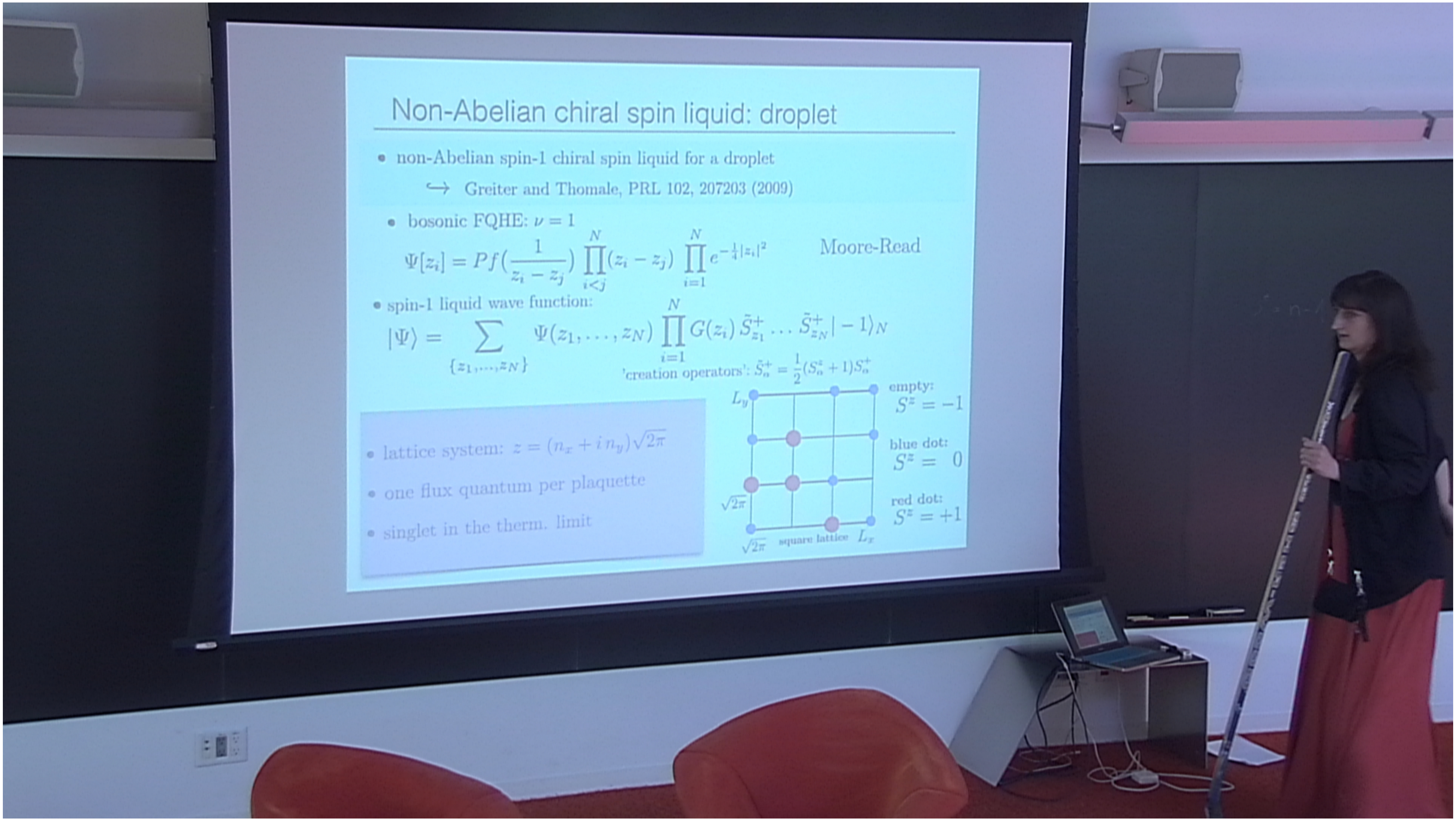
$S^z = -1/2$


Kalmeyer-Laughlin CSL: correlations (torus)
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$, $\langle S^z_i S^z_{i+\kappa} \rangle$
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$ $|\Psi_0\rangle$
$\langle S^z_i S^z_{i+\kappa} \rangle$
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$ $|\Psi_1\rangle$
$\langle S^z_i S^z_{i+\kappa} \rangle$
$L_x \times L_y = 14 \times 14$
distance κ
$(\kappa = x/\sqrt{2\pi})$
• states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are undistinguishable by their correlations
• $\langle \vec{S}_i \cdot \vec{S}_{i+x} \rangle = 3 \times \langle S^\alpha_i S^\alpha_{i+x} \rangle$, $\alpha = x, y, z$



Non-Abelian chiral spin liquid: droplet
non-Abelian spin-1 chiral spin liquid for a droplet
↪ Greiter and Thomale, PRL 102, 207203 (2009)
bosonic FQHE: $\nu = 1$
$\Psi[z_i] = Pf(\frac{1}{z_i - z_j}) \prod_{i<j}^{N} (z_i - z_j) \prod_{i=1}^{N} e^{-\frac{1}{4}|z_i|^2}$
Moore-Read
spin-1 liquid wave function:
$|\Psi\rangle = \sum_{\{z_1,\ldots,z_N\}} \Psi(z_1,\ldots,z_N) \prod_{i=1}^{N} G(z_i)\, \tilde{S}^+_{z_1} \ldots \tilde{S}^+_{z_N} |-1\rangle_N$
'creation operators': $\tilde{S}^+_\alpha = \frac{1}{2}(S^z_\alpha + 1) S^+_\alpha$
lattice system: $z = (n_x + i\, n_y)\sqrt{2\pi}$
one flux quantum per plaquette
singlet in the therm. limit
L_y
$\sqrt{2\pi}$
$\sqrt{2\pi}$
square lattice
L_x
empty:
$S^z = -1$
blue dot:
$S^z = \ 0$
red dot:
$S^z = +1$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$\Psi[z_i] = Pf\left(\frac{1}{z_i - z_j}\right) \prod_{i<j}^{N} (z_i - z_j) \prod_{i=1}^{N} e^{-\frac{1}{4}|z_i|^2}$
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• spin-1 liquid wave function:
$|\Psi\rangle = \sum_{\{z_1,\dots,z_N\}} \Psi(z_1,\dots,z_N) \prod_{i=1}^{N} G(z_i)\, \tilde{S}^+_{z_1} \dots \tilde{S}^+_{z_N} |-1\rangle_N$
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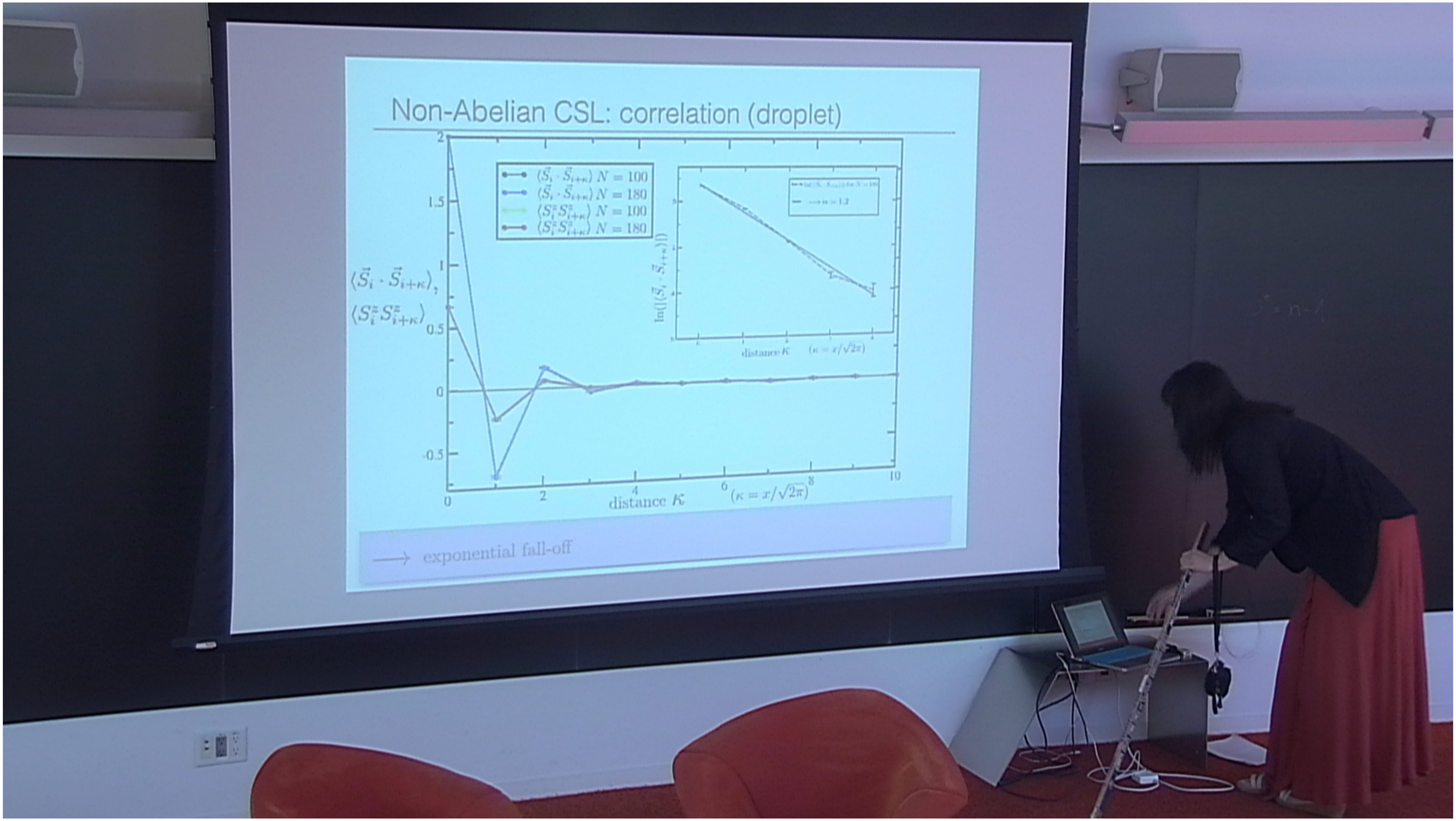
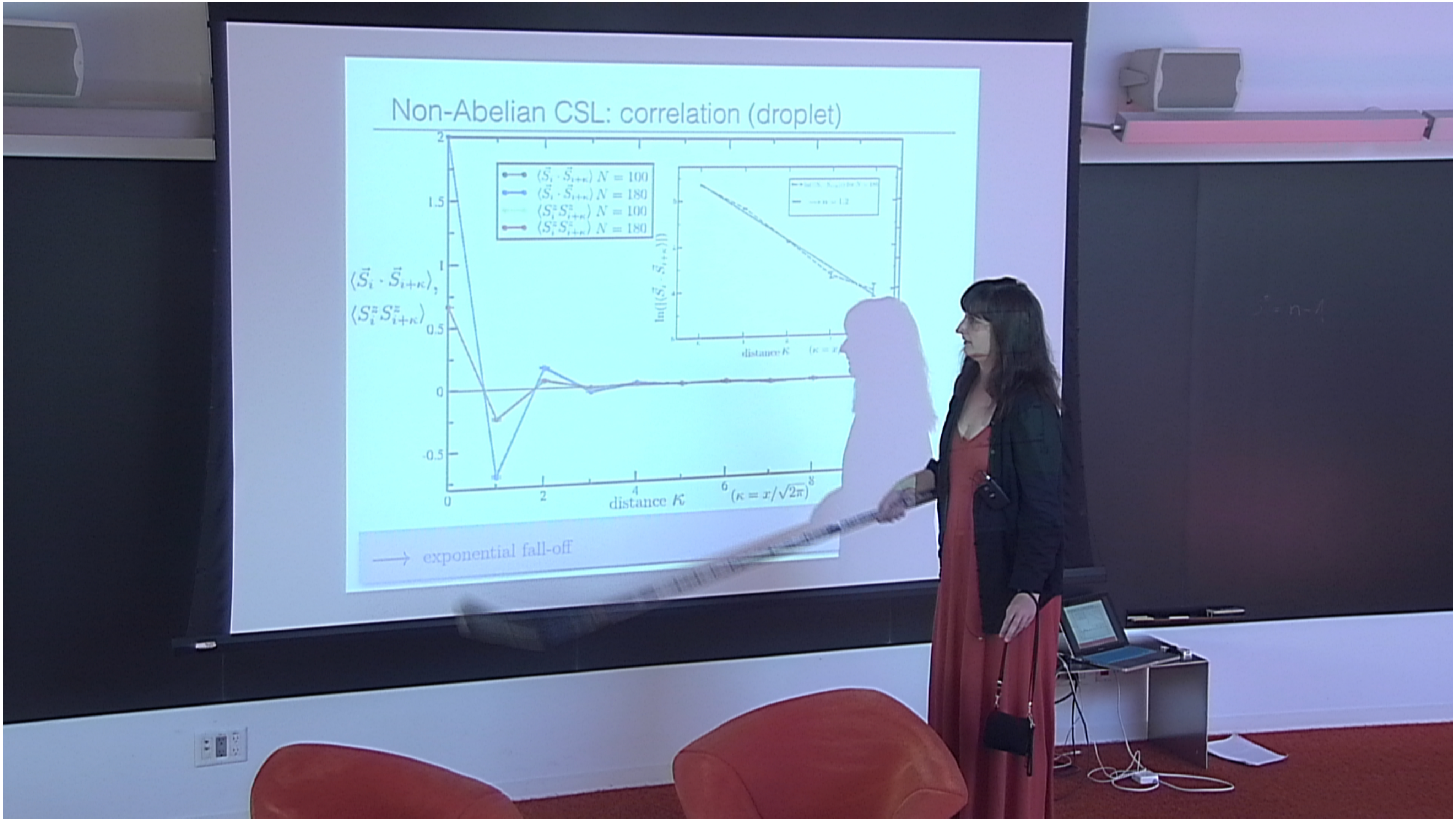

Non-Abelian CSL: correlation (droplet)
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$ $N = 100$
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$ $N = 180$
$\langle S_i^z S_{i+\kappa}^z \rangle$ $N = 100$
$\langle S_i^z S_{i+\kappa}^z \rangle$ $N = 180$
$\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle$, $\langle S_i^z S_{i+\kappa}^z \rangle$
distance κ $(\kappa = x/\sqrt{2\pi})$
$\ln(|\langle \vec{S}_i \cdot \vec{S}_{i+\kappa} \rangle|)$
→ exponential fall-off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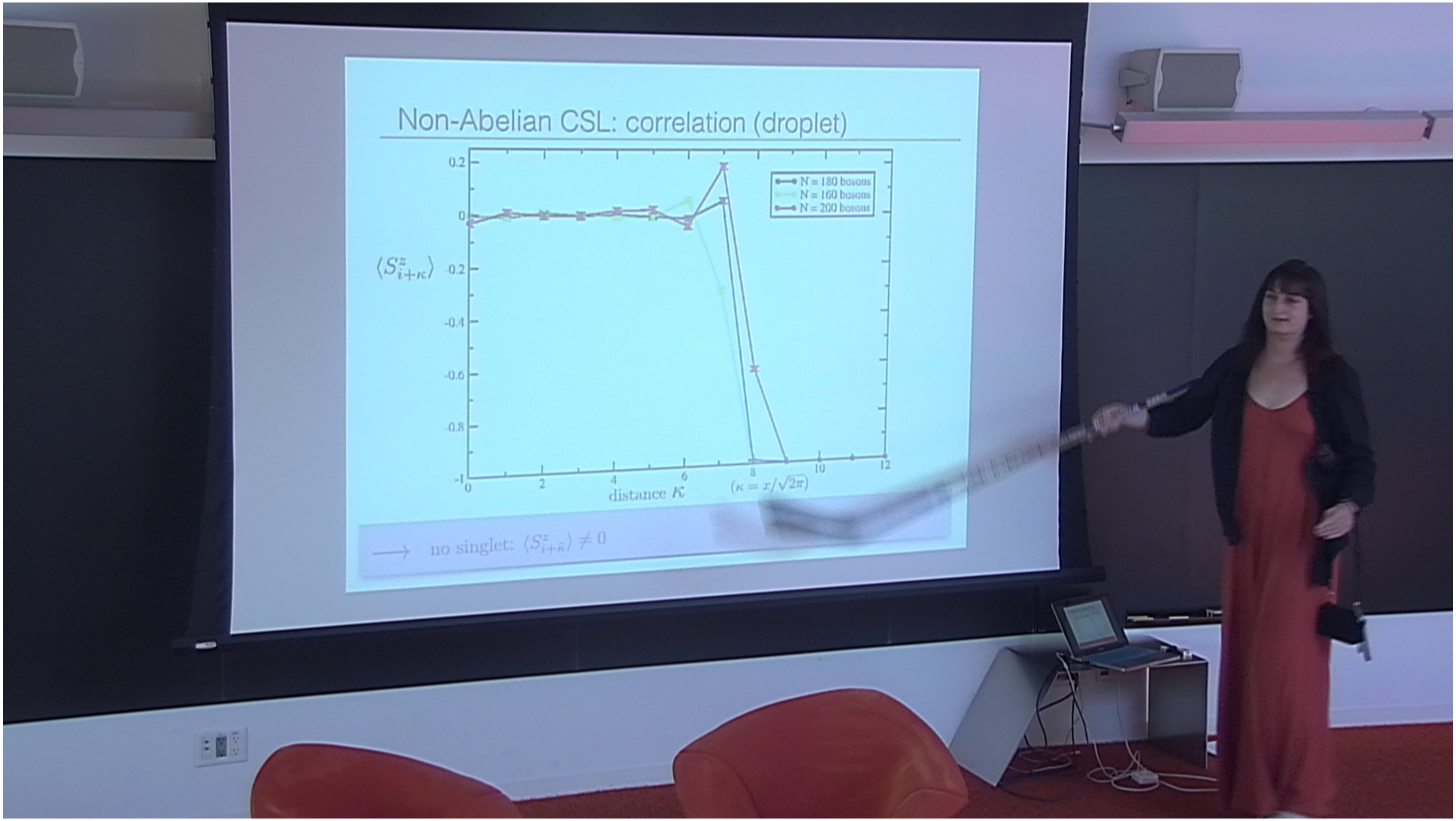
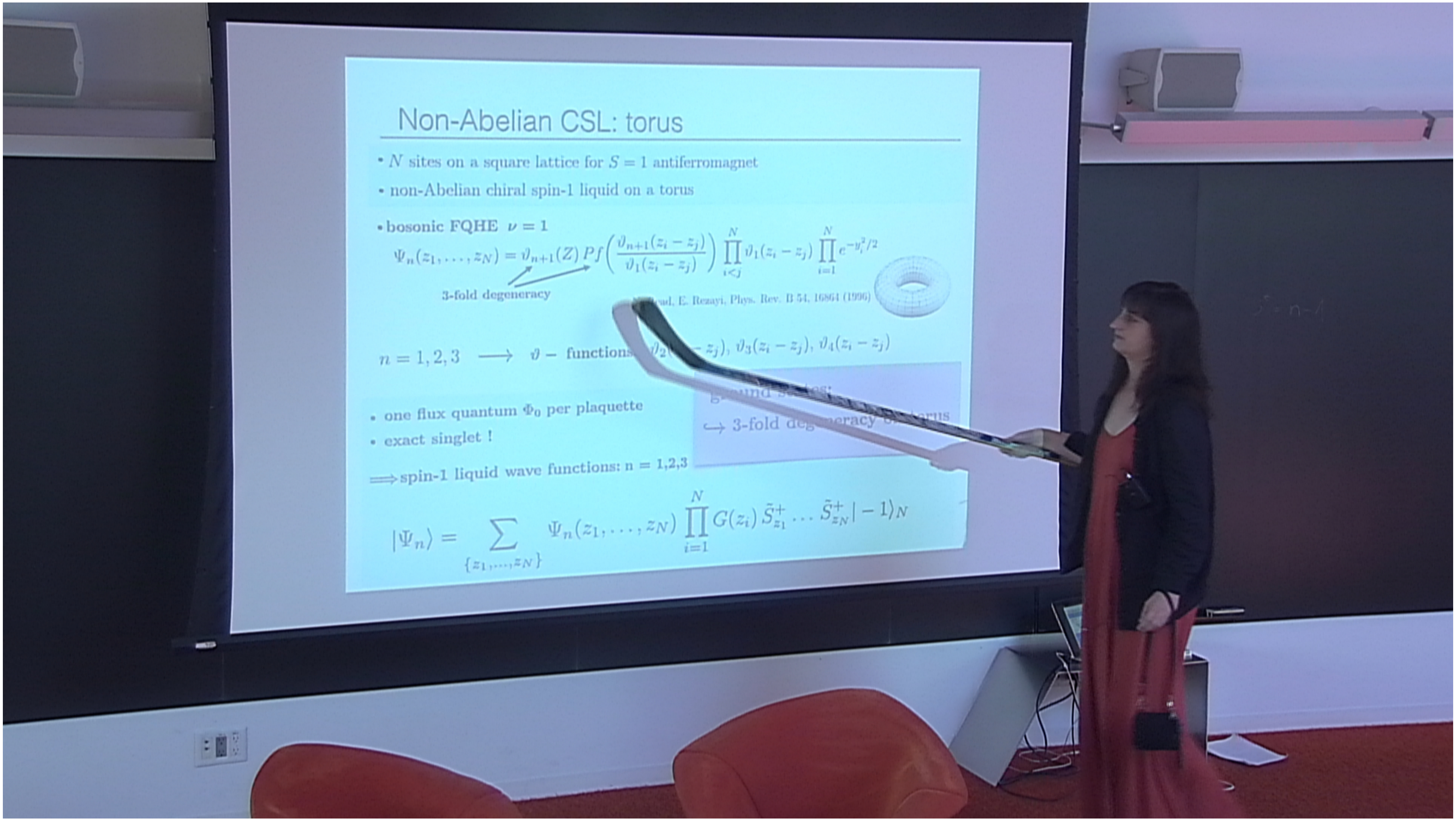
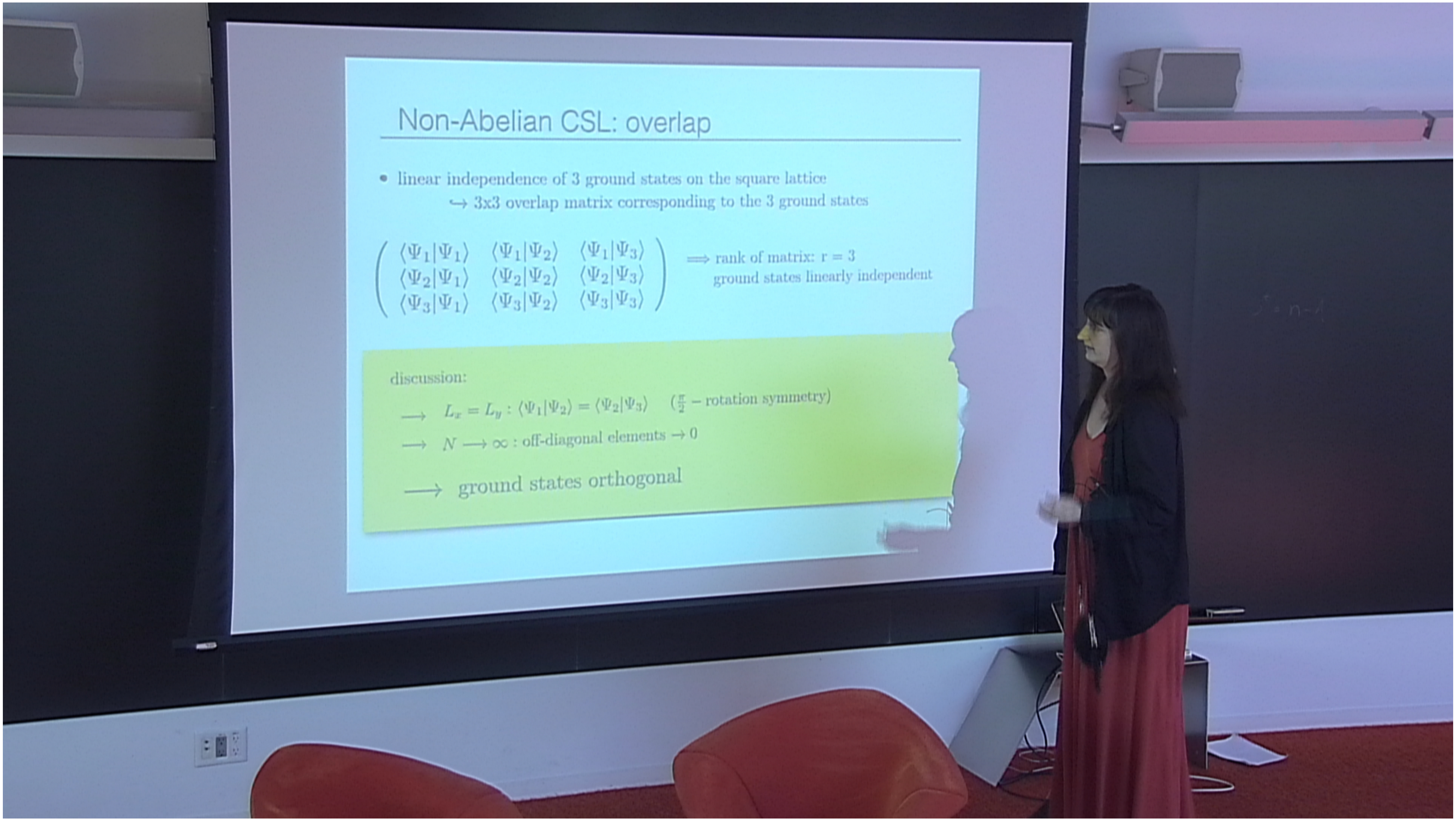
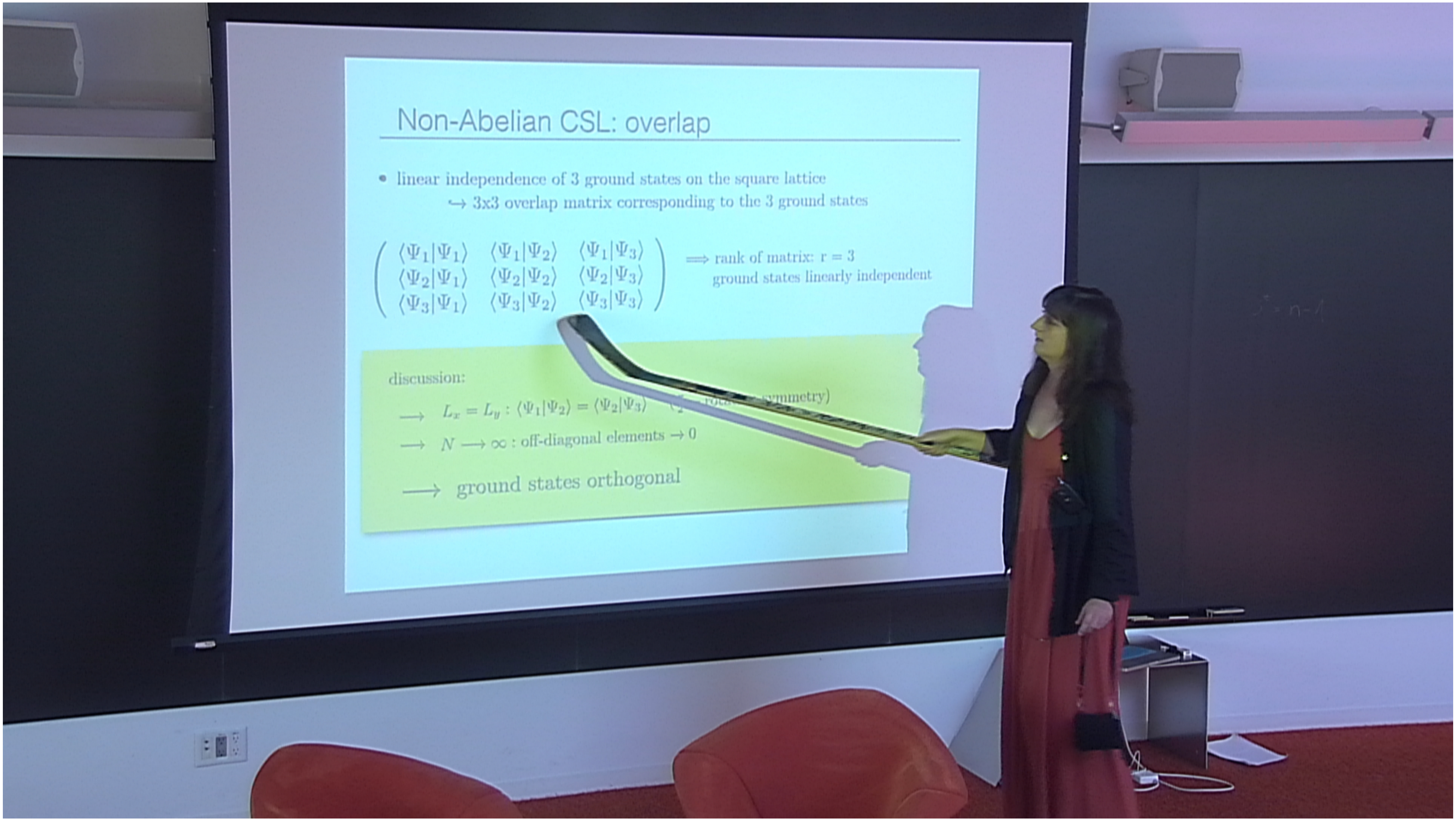

Non-Abelian CSL: correlation (droplet)
N = 180 bosons
N = 160 bosons
N = 200 bosons
$\langle S^z_{i+\kappa} \rangle$
distance κ
$(\kappa = x/\sqrt{2\pi})$
$\longrightarrow$ no singlet: $\langle S^z_{i+\kappa} \rangle \neq 0$


Non-Abelian CSL: torus
• N sites on a square lattice for S = 1 antiferromagnet
• non-Abelian chiral spin-1 liquid on a torus
• bosonic FQHE ν = 1
3-fold degeneracy
n = 1, 2, 3 ⟶ ϑ − functions
• one flux quantum Φ0 per plaquette
• exact singlet !
⟹ spin-1 liquid wave functions: n = 1,2,3


Non-Abelian CSL: overlap
• linear independence of 3 ground states on the square lattice
↪ 3x3 overlap matrix corresponding to the 3 ground states
$\begin{pmatrix} \langle\Psi_1|\Psi_1\rangle & \langle\Psi_1|\Psi_2\rangle & \langle\Psi_1|\Psi_3\rangle \\ \langle\Psi_2|\Psi_1\rangle & \langle\Psi_2|\Psi_2\rangle & \langle\Psi_2|\Psi_3\rangle \\ \langle\Psi_3|\Psi_1\rangle & \langle\Psi_3|\Psi_2\rangle & \langle\Psi_3|\Psi_3\rangle \end{pmatrix}$
⟹ rank of matrix: r = 3
ground states linearly independent
discussion:
⟶ $L_x = L_y : \langle\Psi_1|\Psi_2\rangle = \langle\Psi_2|\Psi_3\rangle$ ($\frac{\pi}{2}$ – rotation symmetry)
⟶ $N \longrightarrow \infty$: off-diagonal elements → 0
⟶ ground states orthogonal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symmetry)
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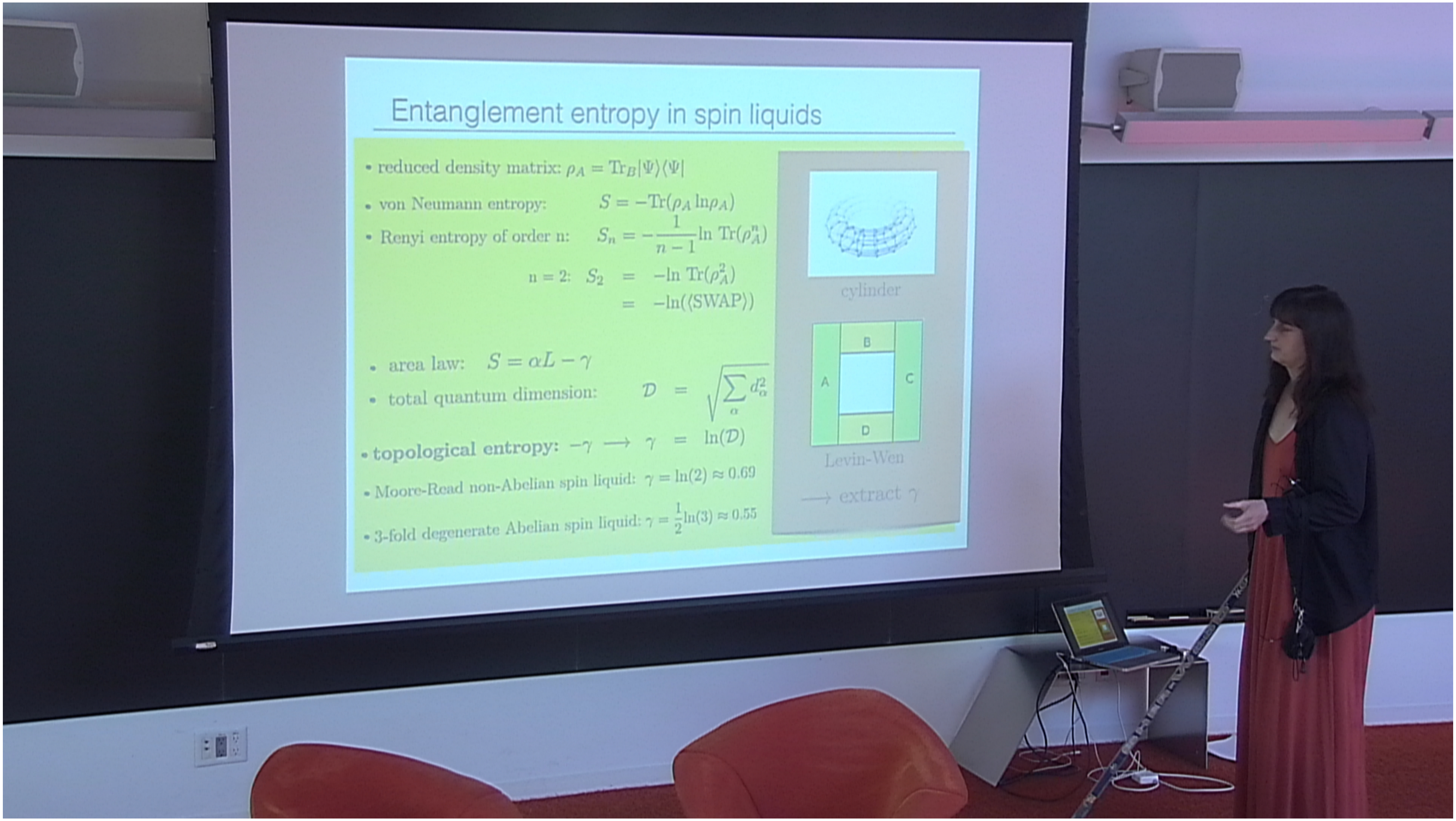
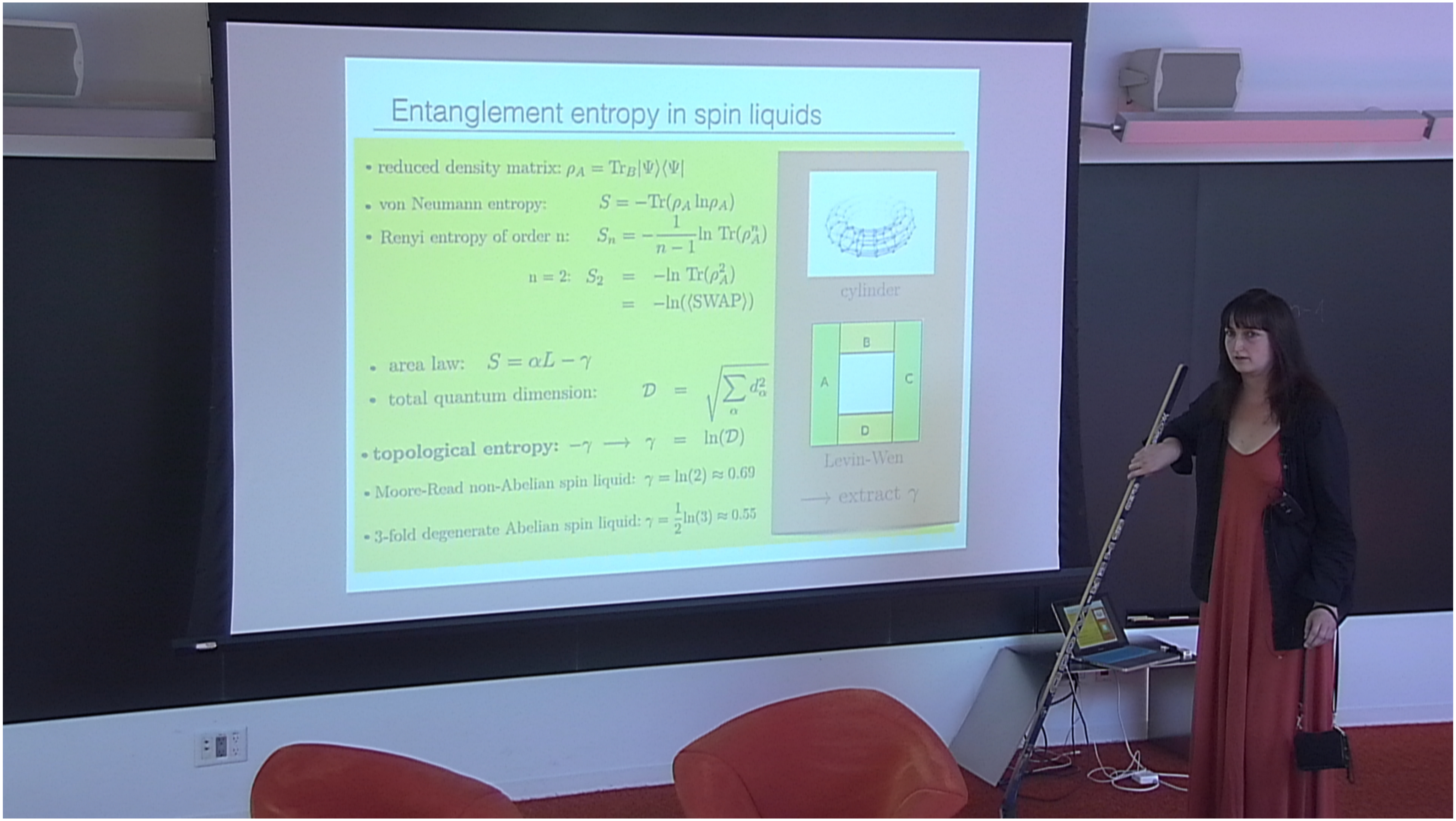
$\longrightarrow$ ground states orthogonal



Entanglement entropy in spin liquids
• reduced density matrix: $\rho_A = \mathrm{Tr}_B|\Psi\rangle\langle\Psi|$
• von Neumann entropy: $S = -\mathrm{Tr}(\rho_A \ln \rho_A)$
• Renyi entropy of order n: $S_n = -\frac{1}{n-1}\ln \mathrm{Tr}(\rho_A^n)$
n = 2: $S_2 = -\ln \mathrm{Tr}(\rho_A^2) = -\ln(\langle \mathrm{SWAP}\rangle)$
• area law: $S = \alpha L - \gamma$
• total quantum dimension: $\mathcal{D} = \sqrt{\sum_\alpha d_\alpha^2}$
• topological entropy: $-\gamma \longrightarrow \gamma = \ln(\mathcal{D})$
• Moore-Read non-Abelian spin liquid: $\gamma = \ln(2) \approx 0.69$
• 3-fold degenerate Abelian spin liquid: $\gamma = \frac{1}{2}\ln(3) \approx 0.55$
cylinder
B
A
C
D
Levin-Wen
$\longrightarrow$ extract γ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$= -\ln(\langle\mathrm{SWAP}\rangle)$
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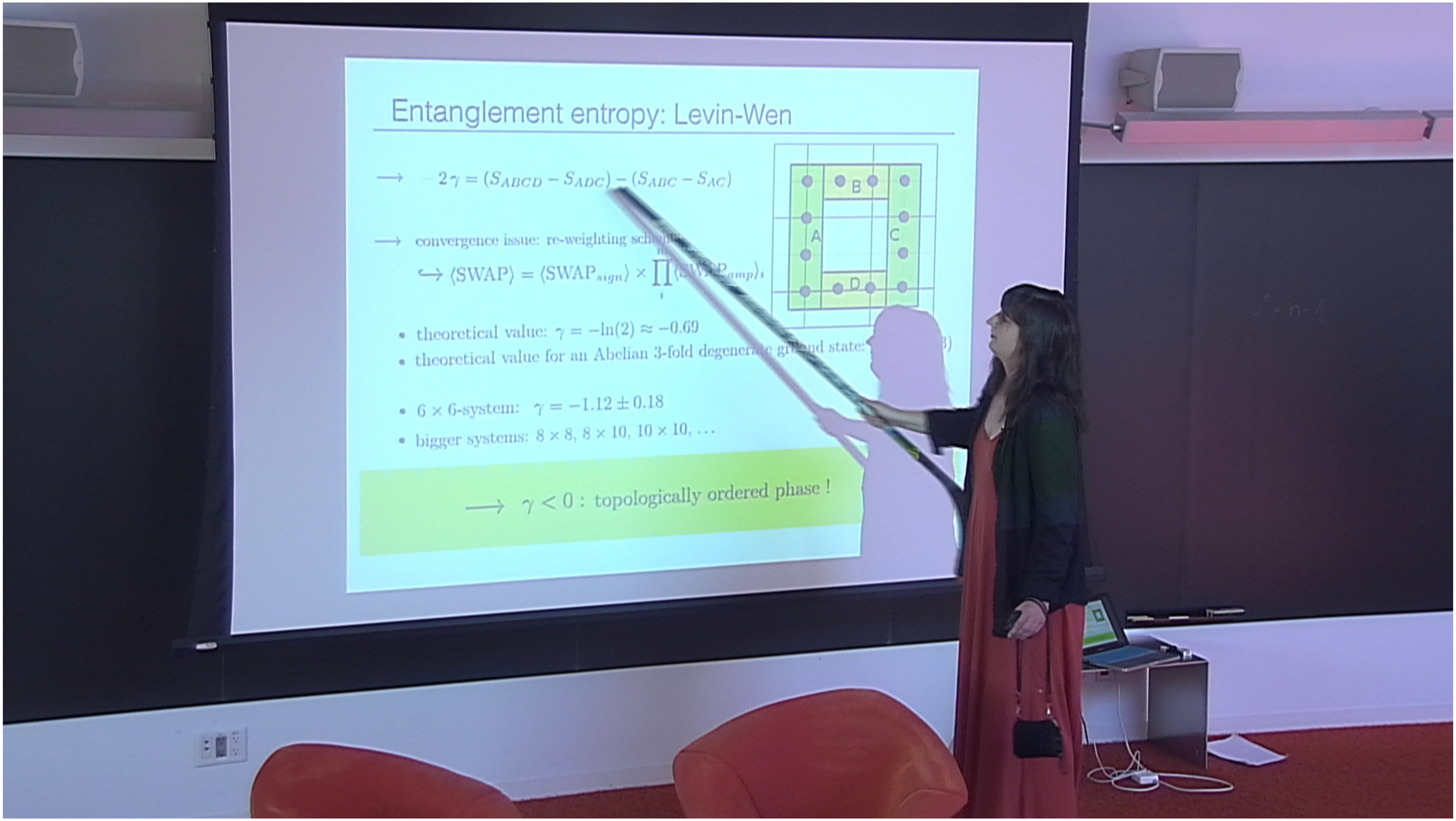
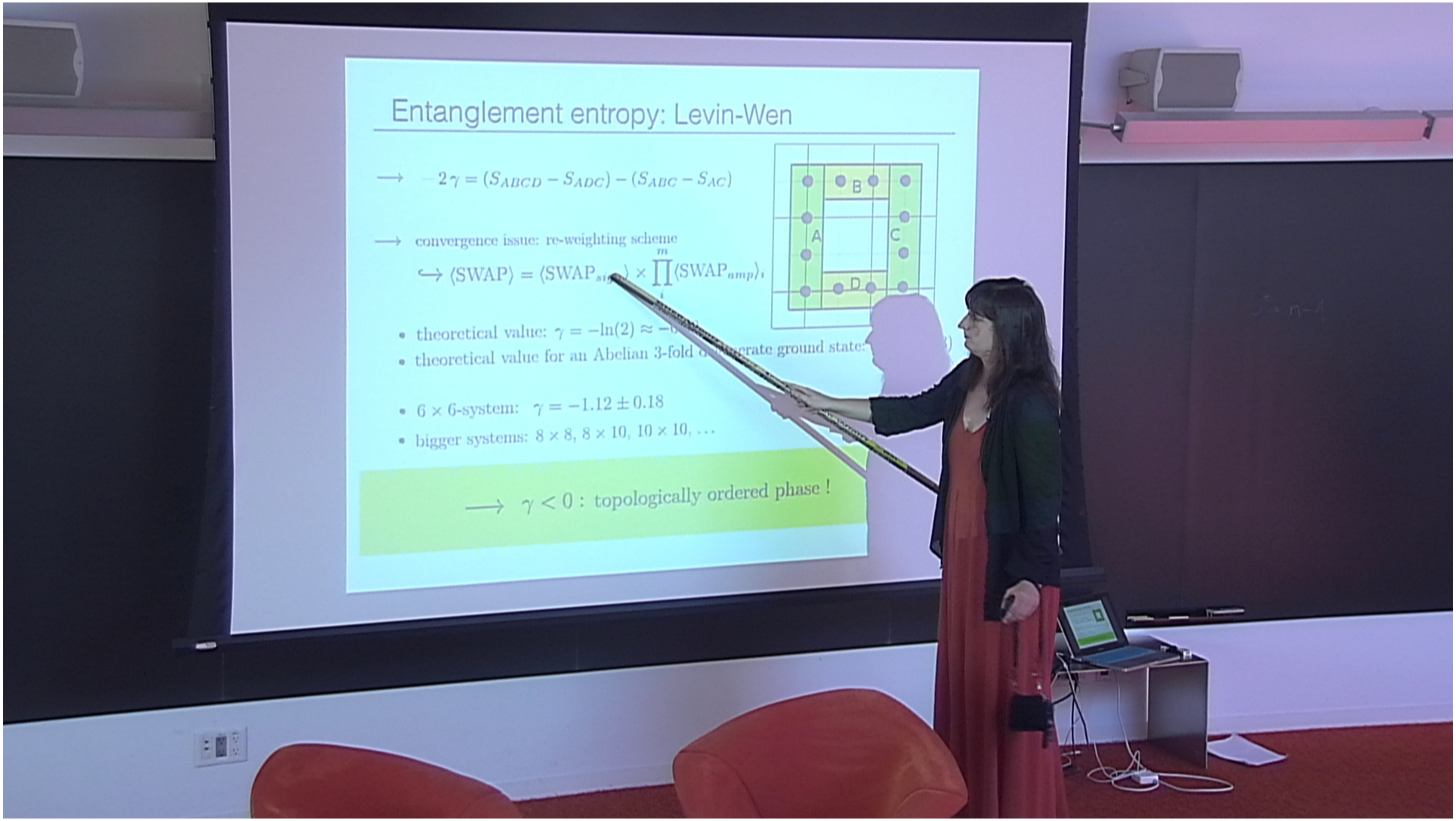
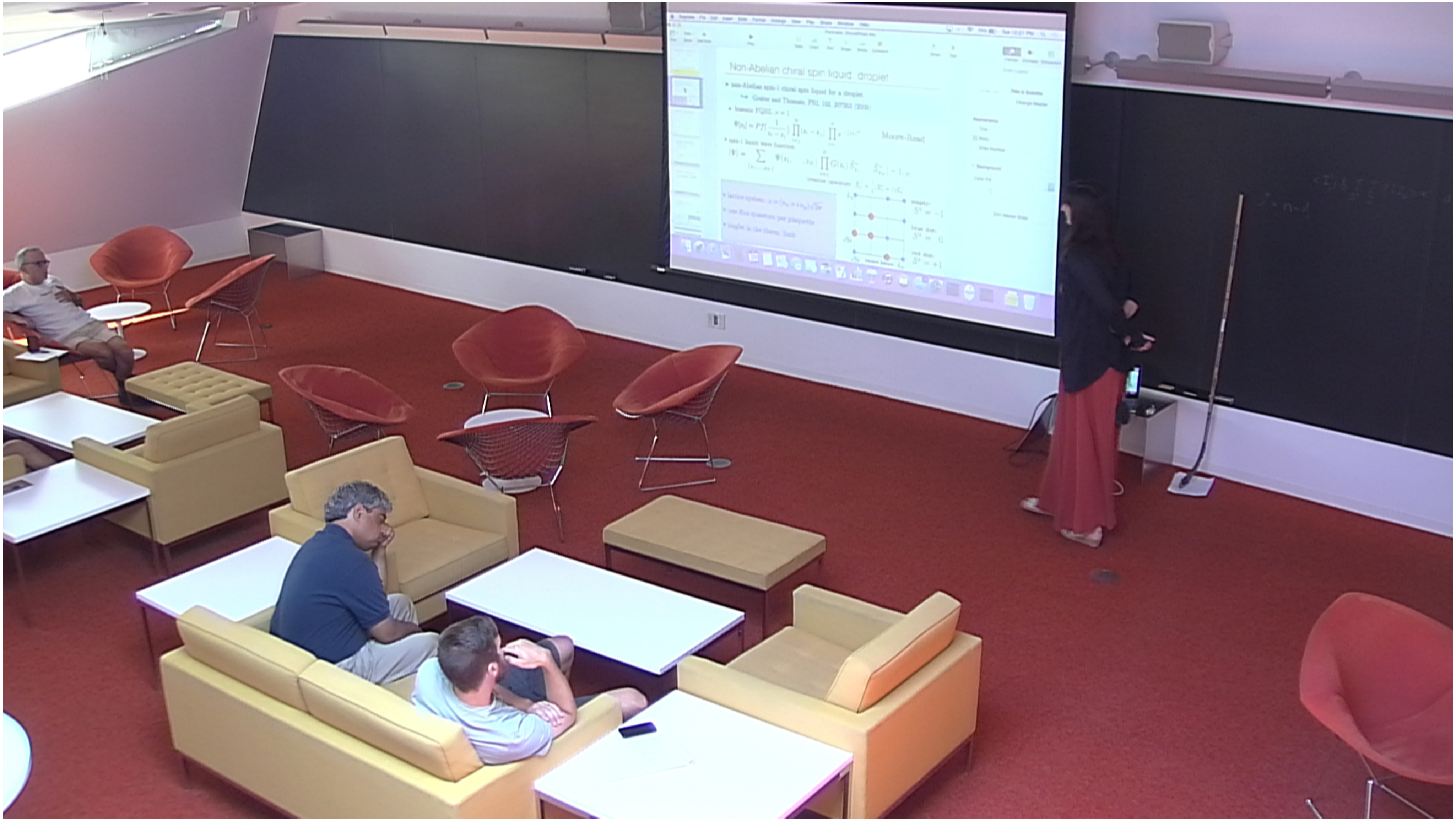
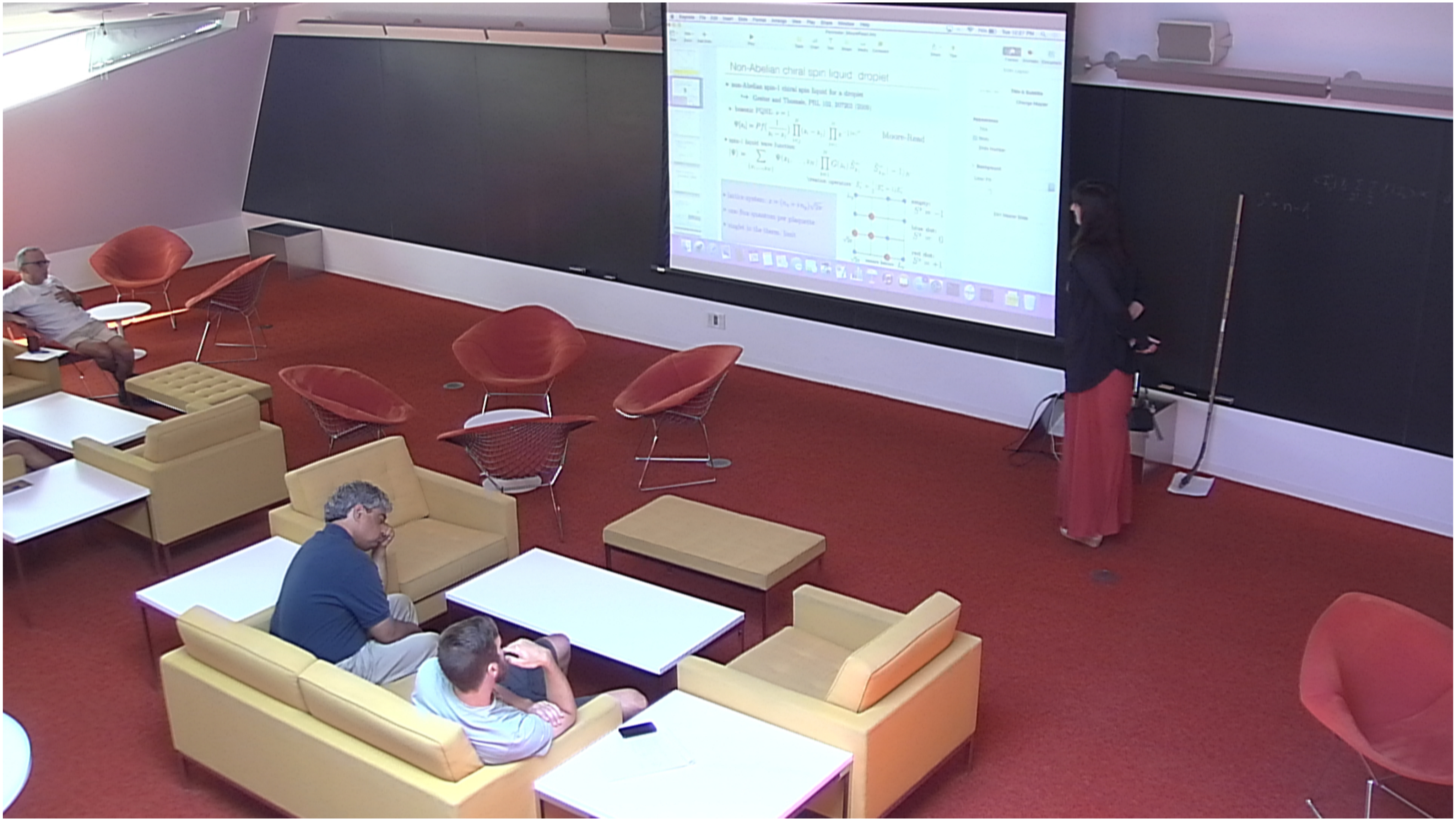
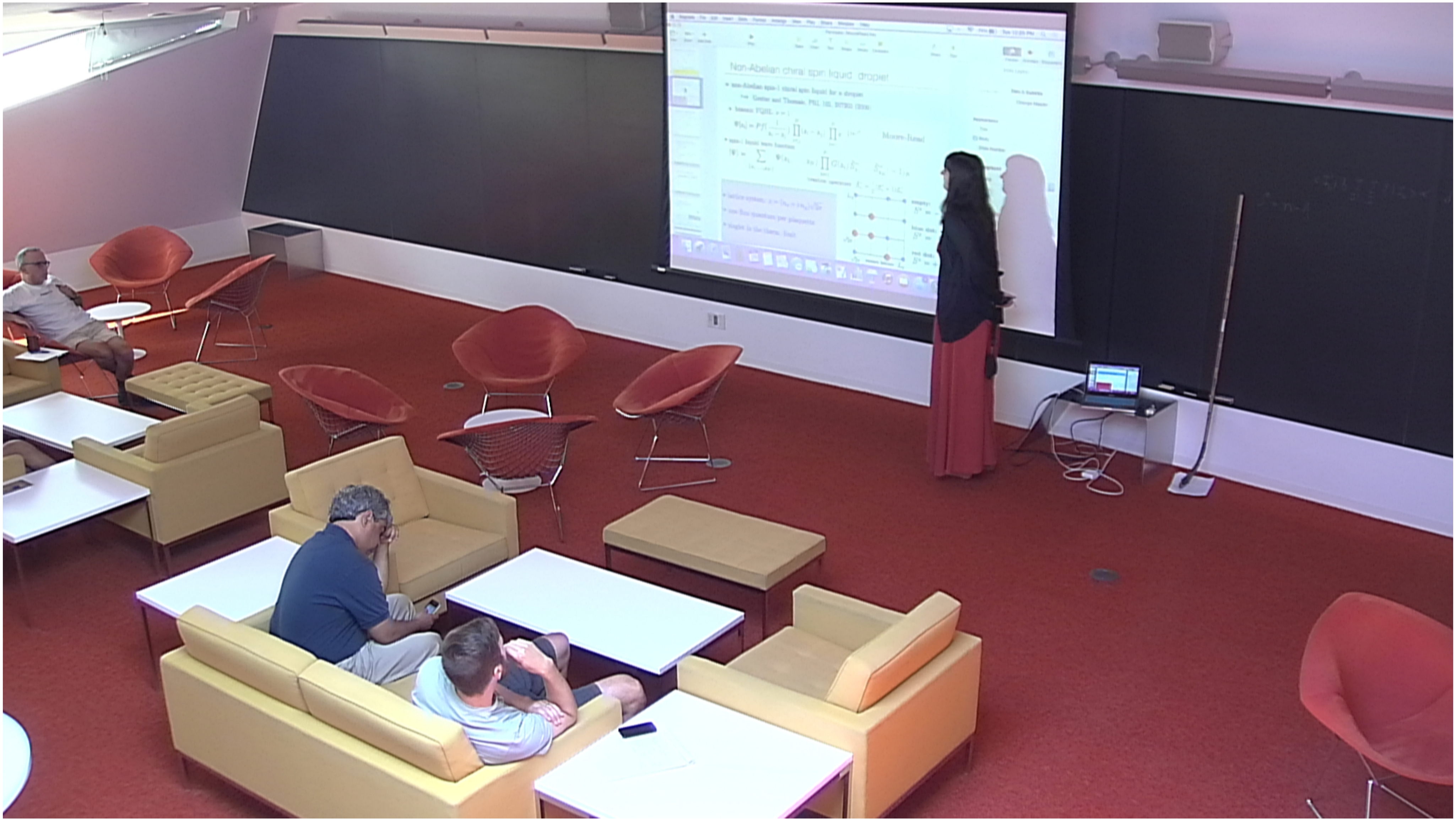


Entanglement entropy: Levin-Wen
⟶ $-2\gamma = (S_{ABCD} - S_{ADC}) - (S_{ABC} - S_{AC})$
⟶ convergence issue: re-weighting sch
↪ ⟨SWAP⟩ = ⟨SWAP$_{sign}$⟩ ×
• theoretical value: $\gamma = -\ln(2) \approx -0.69$
• theoretical value for an Abelian 3-fold degener
• 6 × 6-system: $\gamma = -1.12 \pm 0.18$
• bigger systems: 8 × 8, 8 × 10, 10 × 10, ...
⟶ $\gamma < 0$: topologically ordered phase !
B
A
C
D


Entanglement entropy: Levin-Wen
$\longrightarrow \; -2\gamma = (S_{ABCD} - S_{ADC}) - (S_{ABC} - S_{AC})$
$\longrightarrow$ convergence issue: re-weighting scheme
$\hookrightarrow \langle \text{SWAP} \rangle = \langle \text{SWAP}_{sig} \rangle \times \prod_i^m \langle \text{SWAP}_{amp} \rangle_i$
• theoretical value: $\gamma = -\ln(2) \approx$
• theoretical value for an Abelian 3-fold
ground state:
• 6×6-system: $\gamma = -1.12 \pm 0.18$
• bigger systems: 8×8, 8×10, 10×10, …
$\longrightarrow \; \gamma < 0$: topologically ordered phase !
A
B
C
D


Non-Abelian chiral spin liquid: droplet
Moore-Read


Non-Abelian chiral spin liquid droplet


Non-Abelian chiral spin liquid droplet
Moore-Read