Title: Introduction to Majorana Fermions in Condensed Matter & Interpretation of recent Experiments in SmB6 using Majorana Fermi Sea

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Abstract: <p>A pedagogic introduction will be given to: i) Emergent Majorana fermions and Majorana zero modes in certain condensed matter models and ii) Kondo insulators. This will be followed by discussion of a remarkable Quantum Oscillation Anomaly seen in recent experiments in SmB6, a Kondo insulator, by the Cambridge group [1], as providing evidence [2] for presence of Majorana fermi sea, in an unexpected place. We show a counter intuitive result that these Majorana fermions though neutral, exhibit Landau diamagnetism.</p>

<p> </p>

<p>[1] B.S. Tan et al., Science, vol.349, pp. 287-29 (2015), arXiv:1507.01129 [1] G. Baskaran, arXiv:1507.03477</p>

Majorana Fermions in Condensed Matter and
Interpretation of a Recent Experimental Anomaly in SmB_6 using
Majorana Fermi Sea

G. Baskaran

The Institute of Mathematical Sciences
Chennai, India

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
Waterloo, Canada

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Introduction

Majarona Fermions, Majorana Zero Modes

An Electron Fermi Sea = Four (1 scalar + 3 vector) Majorana Fermi Seas

How a Scalar Majorana Fermi Sea Survives Kondo Coupling
Coleman, Miranda, Tsvelik (1993)

Landau diamagneism and dHvA Quantum Oscillations in Majorana fermi sea basis

Kondo lattice in the presence of magnetic field for Kondo lattice

Resistivity Saturation and Violation of Lifshitz Kosevich formula

Surface Majorana Band, Majorana Zero Modes

Other Kondo Insulators & Heavy Fermion system

Works on

Kitaev Spin Model, Majorana Fermi/Dirac Sea etc.

from Matscience, Chennai

R. Shankar, S. Hassan, R Ganesh, Mukul Laad, GB
Saptarshi Mandal, Naveen Surendran, G Santhosh
Abhinav Saket, Sruluckshmi, Sandeep Goyal

Exact results for spin correlation functions and quantum number fractionization
GB, S. Mandal and R Shankar, Phys. Rev. Lett. 98, 247201 (2007)

Spin-S Kitaev Model: Classical ground states, order from disorder and Exact Correlation Functions
GB, Diptiman Sen and R Shankar, Phys. Rev. B 78, 115116 (2008)

Exact quantum spin liquid with a fermi surface in a spin-1/2 model
GB, G Santhosh and R Shankar, arXiv: 0908.1614

Exactly solvable Kitaev model in three dimensions
S. Mandal and N. Surendran, Phys. Rev. B 79, 024426 (2009)

Manipulating unpaired Majorana fermions in a quantum spin chain
A. Saket, S. R. Hassan, and R. Shankar, Phys. Rev. B82, 174409 (2010)

Spin-1 Kitaev Model in One Dimension
D. Sen, R. Shankar, D. Dhar and K. Ramola, Phys. Rev. B82, 195435 (2010)

Confinement-deconfinement transition and spin correlations in a generalized Kitaev Model
S Mandal, S Bhattacharya, K Sengupta, R Shankar and GB (2011 PRB)

RVB gauge theory and the topological degeneracy in the honeycomb Kitaev model
S Mandal, R. Shankar and G. Baskaran, *J. Phys. A*, 45, 335304 (2012)

Topological aspects of an Exactly Solvable Spin Chain
A. Saket, S.R. Hassan and R. Shankar, Phys. Rev., B 87, 174414 (2013)

A stable Algebraic Spin Liquid in a Hubbard model
S.R. Hassan, P.V. Sriluckshmy, S.K. Goyal, R. Shankar and D. Senechal, Phys. Rev. Lett., 110, 037201 (2013)

Quarter-filled Kitaev-Hubbard model: A quantum Hall state in an optical lattice
S. R. Hassan, S. Goyal, R. Shankar, and D. Sénéchal, Phys. Rev. B 88, 045301 (2013)

Quantum oscillations without a Fermi surface and the anomalous de Haas-van Alphen effect

Johannes Knolle and Nigel R. Cooper

arXiv:1507.00885

Majorana Fermi Sea in Insulating SmB_6:
A proposal and a Theory of Quantum Oscillations in Kondo Insulators

G. Baskaran

arXiv:1507.03477

Liquids

a Strongly Correlated state of Matter

We are a bag of liquid mixture !

Intra cellular fluid ….. blood flow

Classical	Quantum
Liquids	Fermi liquid metals, He3, cold atoms
Liquid crystals	Bose liquid He4, cold atoms
Biological Fluids	Superconductors
	Paired Superfluids He3, Neutron Stars, cold atoms
	Luttinger Liquid
	Quantum Hall fluids
	Liquid of light
Paramagnet	Quantum Spin Liquids a big family
(a classical spin gas, liquid)	Majorana Fermi Liquids

Majorana Zero Modes

2 Channel Kondo Effect Emery and Kivelson 1992; Affleck, Ludwig, Hewson, …

Kitaev Wire (Ends of 1d p-wave superconductor)

Vortices in p-wave superconductor Volovik , Read, Green ..
Half vortices in Sr_2RuO_4 Ivanov, Budakian ..

Ends of Spin Chains on a Superconducting Surface Yazdani ..

Majorana Fermi Sea

Quantum Spin Liquid
GB, Zou, Anderson; Shastry, Tsvelik, Sen, Khaliulin, Soda, …, ; S Sachdev, R R Biswas, L Fu, C. Laumann ..

Kondo Lattice Coleman, Miranda, Tsvelik

Kitaev Model on Honeycomb Lattice Dirac Sea (Iridates) Khialiulin, Jackeli, …
(Majorana Fermi Surface) Kivelson, Yao; GB, Santhosh, Shankar; Feigelman, … YB Kim, … Trebst, ..
(3 dimension) Mandal, Sundar …

Formalism

Spin-1/2 operators interms of Majorana fermions Books: Mattis, Tsvelik

1D Spinful Fermi sea interms of Majorana fermions (Non-Abelian Bosonization) Affleck, Ludwig, Maldecena, Hewson …

2D and Higher Dimensional Fermi Sea in terms of Majorana Fermions Coleman, Miranda, Tsvelik

Hubbard model Ho, Coleman; Cenke Xu, Sachdev; Brijesh Kumar

Dirac or complex Fermion

$\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$

$\psi = \frac{1}{2}(\zeta + i\eta)$

$\{\zeta_i, \zeta_j\} = \delta_{ij}$ $\eta^2 = \zeta^2 = 1$

$|0\rangle, |1\rangle$

Majorana or Real fermions

Hilbert space dimensions

$D_\zeta = \sqrt{2}$ $D_F = D_\zeta \times D_\xi = \sqrt{2} \times \sqrt{2} = 2$

$D_\xi = \sqrt{2}$

Ising Model in a Transverse Field

&

1D Superconductor

$$H = -J \sum_i S_i^x S_{i+1}^x - h \sum S_i^z$$

2^N dim Hilbert space

$h = 0$ $\quad |A\rangle = |\uparrow \uparrow \uparrow \cdots \uparrow\rangle$, $|B\rangle = |\downarrow \downarrow \downarrow \cdots \downarrow\rangle$

$h \neq 0$ allows for Tunnelling between $|A\rangle$ and $|B\rangle$

JORDAN WIGNER TRANSFORMATION

PAULI SPIN OPERATORS $S_i^x S_i^y + S_i^y S_i^x = 0$ ANTI COMMUTE

$S_i^x S_j^y - S_j^y S_i^x = 0$ COMMUTE for $i \neq j$

We wish to construct anticommuting operators c_i's $\{c_i, c_j^+\} = \delta_{ij}$ using the set $[S_i^x, S_i^y, S_i^z]$

$$c_i = S_i^+ \prod_{l=1}^{i-1} S_l^z \quad ; \quad c_i^+ = S_i^- \prod_{l=1}^{i-1} S_l^z$$

$$c_i c_j^+ = (S_i^+ S_{i-1}^z S_{i-2}^z \cdots S_j^z \cdots S_1^z)(S_j^- S_{j-1}^z \cdots S_1^z)$$

$i \neq j$

$$= -(S_j^- S_{j-1}^z \cdots S_1^z)(S_i^+ S_{i-1}^z S_{i-2}^z \cdots S_1^z)$$

$$= -c_j^+ c_i$$

Similarly $c_i^+ c_i + c_i c_i^+ = 1$

$\therefore \{c_i, c_j^+\} = 0$

$$\boxed{\{c_i, c_j^+\} = \delta_{ij}}$$

Inverse Transformation

$$S_i^+ = c_i \, e^{i\pi \sum_{l<i} c_l^+ c_l}$$

$$S_i^z = \frac{1}{2} - c_i^+ c_i$$

Flux tube attachment
or
String attachment

JW transformation is at the heart of Bosonization in 1 Dimension

Back to Ising Model in transverse field

$$H = -J \sum_{i=1}^{N} S_i^x S_{i+1}^x - h \sum_{i=1}^{N} S_i^z$$

$$S_i^x = \frac{1}{2}(c_i^\dagger + c_i)\, e^{i\pi \sum_{l<i} n_l} \; ; \; S_i^y = \frac{i}{2}(c_i^\dagger - c_i)\, e^{i\pi \sum_{l<i} n_l}$$

$$S_i^x S_{i+1}^x = \frac{1}{4}(c_i^\dagger + c_i)\, e^{i\pi n_i}\, (c_{i+1}^\dagger + c_{i+1})$$

$$= \frac{1}{4}(c_i^\dagger - c_i)(c_{i+1}^\dagger + c_{i+1})$$

$$H = -\frac{J}{4} \sum (c_i^\dagger - c_i)(c_{i+1}^\dagger + c_{i+1}) + \frac{h}{2} \sum c_i^\dagger c_i + \text{Const.}$$

$$H = -J \sum (c_i^\dagger c_{i+1} + h.c) - \frac{J}{4} \sum (c_i^\dagger c_{i+1}^\dagger + h.c.) + \frac{h}{2} \sum c_i^\dagger c_i$$

MAJORANA FERMIONS

$$c_i = \frac{1}{2}(a_i + i b_i) \qquad c_i^\dagger = \frac{1}{2}(a_i - i b_i)$$

$$\{c_i, c_j^\dagger\} = \delta_{ij} \qquad a_i = (c_i + c_i^\dagger)$$

$$b_i = i(c_i - c_i^\dagger)$$

$$\boxed{\begin{array}{c} a_i^2 = 1 \\ b_i^2 = 1 \end{array}} \qquad a_i^2 = (c_i + c_i^\dagger)(c_i + c_i^\dagger) = 0 + 0 + c_i^\dagger c_i + c_i c_i^\dagger = 1$$

$$\boxed{\{a_i, a_j\} = 2\delta_{ij}} \qquad \boxed{c_i^\dagger c_i = \frac{1}{2} a_i b_i}$$

Hilbert Space Dimension of a Fermi Oscillator is 2

$$|0\rangle, \; |1\rangle$$

Nominally $\quad \mathcal{H}_F = \mathcal{H}_{MF} \otimes \mathcal{H}_{MF}$

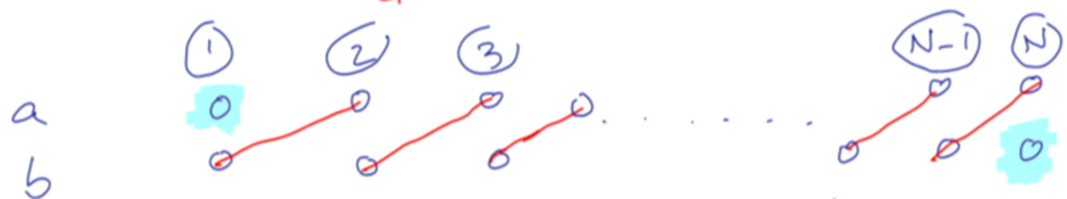
$$\mathcal{D}_F = \mathcal{D}_{MF} \times \mathcal{D}_{MF} = \sqrt{2} \times \sqrt{2}$$

$$H = -\frac{J}{4}\sum_{i=1}^{N}(c_i^{\dagger} - c_i)(c_i^{\dagger} + c_i) + \frac{h}{2}\sum c_i^{\dagger} c_i$$

$$= \frac{Ji}{4}\sum_{i=1}^{N} b_i a_{i+1} + \frac{hi}{4}\sum_{i=1}^{N} a_i b_i$$

Let us look at $h=0$ case

$$H = \frac{Ji}{4}\left(b_1 a_2 + b_2 a_3 + \cdots + b_{N-2} a_N + b_{N-1} a_N\right)$$



Now we regroup (or pair) Majorana fermions

$$d_i = \frac{1}{2}(b_i + i\, a_{i+1})$$
$$d_i^{\dagger} = \frac{1}{2}(b_i - i\, a_{i+1})$$

$$d_i^{\dagger} d_i = \frac{1}{2} b_i a_{i+1}$$

$$H = 0\, d_0^\dagger d_0 + \sum_{n=1}^{N-1} \epsilon_n d_n^\dagger d_n$$

N-1 finite energy fermi oscillators

1 ZERO ENERGY FERMI OSCICCATOR

$$H = \frac{J}{2} \sum_{i=1}^{N-1} d_i^\dagger d_i$$

One MISSING FERMI OSCILLATOR!

a_1 and b_N are MISSING

$$Z^\dagger = \frac{1}{2}(a_1 + i\, b_N)$$

$$Z = \frac{1}{2}(a_1 - i\, b_N)$$

$Z^\dagger, Z$: ZERO MODE FERMI OSCILLATOR

a_1 and b_N are Majorana Oscillators that reside at the boundary

$|\uparrow\uparrow\uparrow\uparrow \ldots \uparrow\rangle$

$|\downarrow\downarrow\downarrow \ldots \ldots \downarrow\rangle$

DUALITY →

boundary degree of freedom

For well separated edges the eigen vector of left edge mode is

$$\begin{pmatrix} 1 \\ 0 \\ \left(\frac{h}{J}\right) \\ 0 \\ \left(\frac{h}{J}\right)^2 \\ 0 \\ \left(\frac{h}{J}\right)^3 \\ 0 \\ \vdots \end{pmatrix}$$

the term $h \sum S_i^z$ causes quantum tunnelling

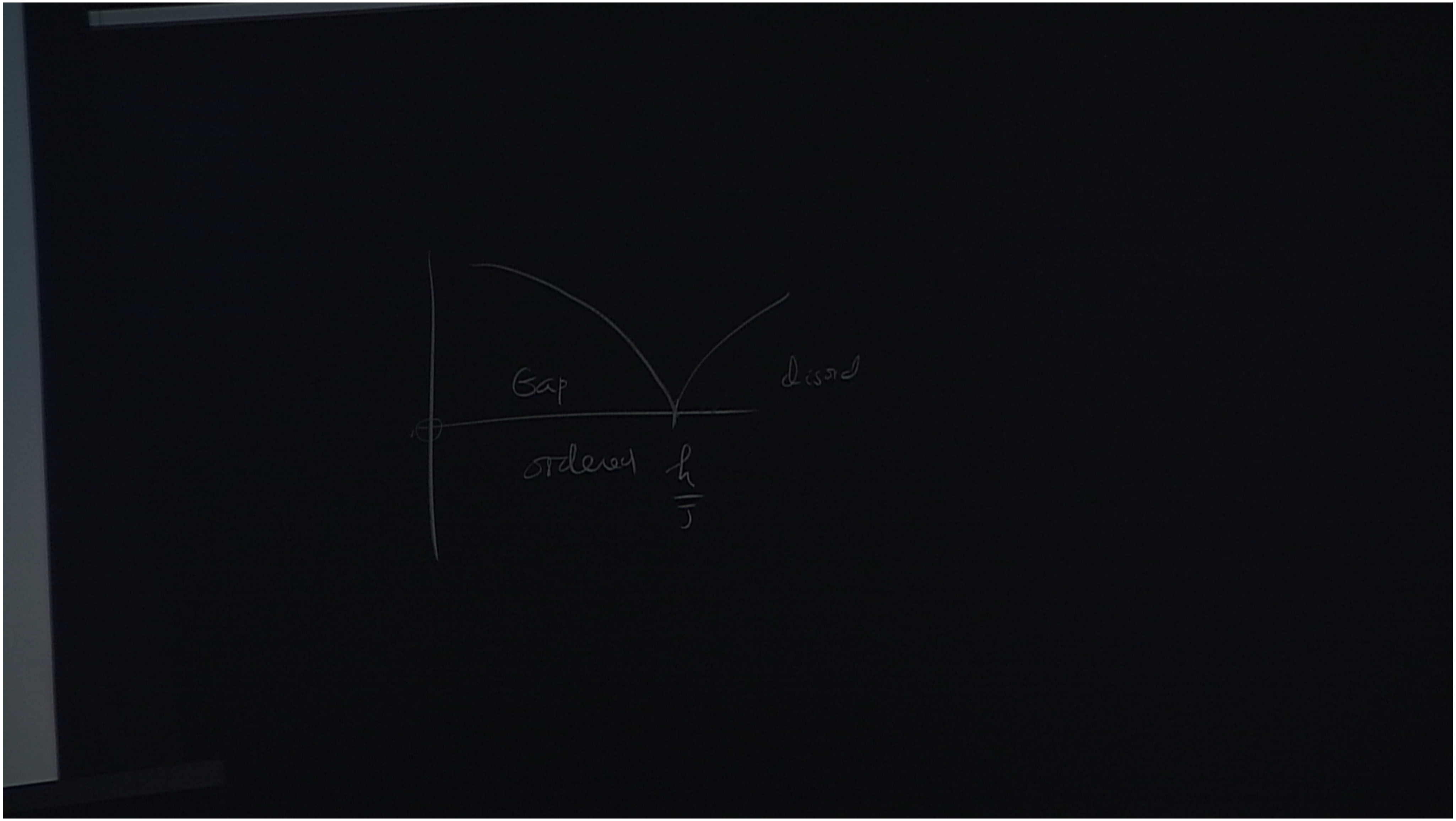
The boundary mode is slightly delocalized

Localization length

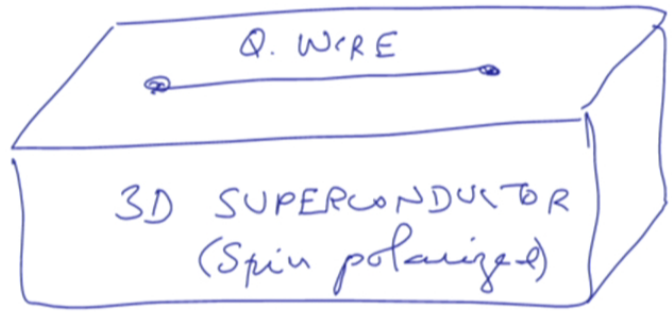
$$\xi \sim \frac{1}{\ln\left(\frac{h}{J}\right)}$$

$$\mathcal{H}_G \sim h\left(\frac{h}{J}\right)^N i\, a_1\, b_N$$

$$= e^{-N/\xi}\, Z^\dagger Z$$


Gap
disord
ordered
h/J

1-D Quantum Wire



$$\mathcal{H} = \sum_j \Big[-w\,(c_j^\dagger c_{j+1} + h.c) - \mu\,(c_j^\dagger c_j - \tfrac{1}{2}) + \Delta\, c_j c_{j+1} + \Delta^* c_{j+1}^\dagger c_j^\dagger \Big]$$

$$\Delta = |\Delta|\, e^{i\theta}$$

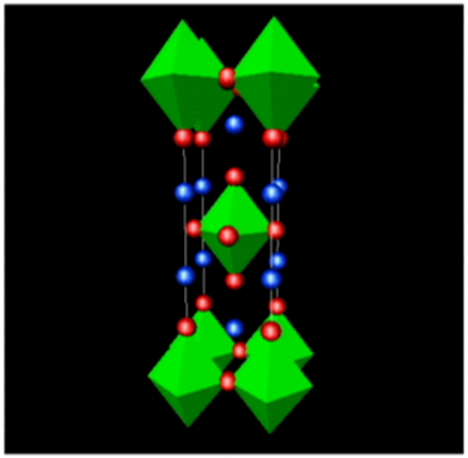
$$a_i = e^{i\frac{\theta}{2}} c_i + e^{-i\frac{\theta}{2}} c_i^\dagger$$

$$b_i = -i e^{i\frac{\theta}{2}} c_i + i\, e^{-i\frac{\theta}{2}} c_i^\dagger$$

Sr_2RuO_4 structurally similar to La_2CuO_4

Discovery of superconductivity by Maeno et al. (1994)

Prediction of p-Wave Superconductivity by
Rice and Sigrist (1995) and GB (1995,96)



Superconducting Tc ~ 1 K, very low !

Orbital part can have p_x , p_y or p_z symmetry or
linear combinations such as $p_x + i\, p_y$ or $p_x - i\, p_y$
(in 2D this will be favored, because of in plane orbital motion)

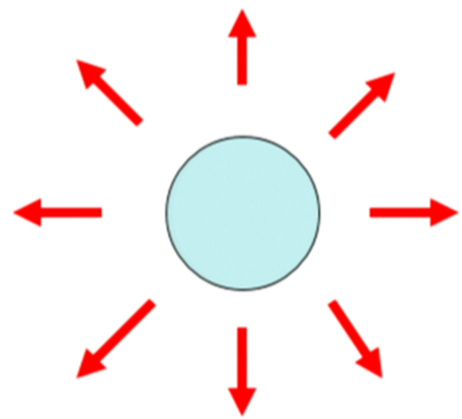
Spin part has to be symmetric under interchange. So it will be one of the three triples or linear combinations.

Cooper pair amplitude (Superconducting order parameter is not a scalar

$$\Psi = e^{i\varphi}[d_x(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + i d_y(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) + d_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)](k_x + i k_y).$$

direction $\hat{\mathbf{d}}$ of triplet pairing

Excitations of 2D supercondctors



Traditionally one views the phase as a 2d vector

Bogoliubov quasi particles and quantized vortices

In a quantized vorex carrying flux quanta $\frac{hc}{2e}$

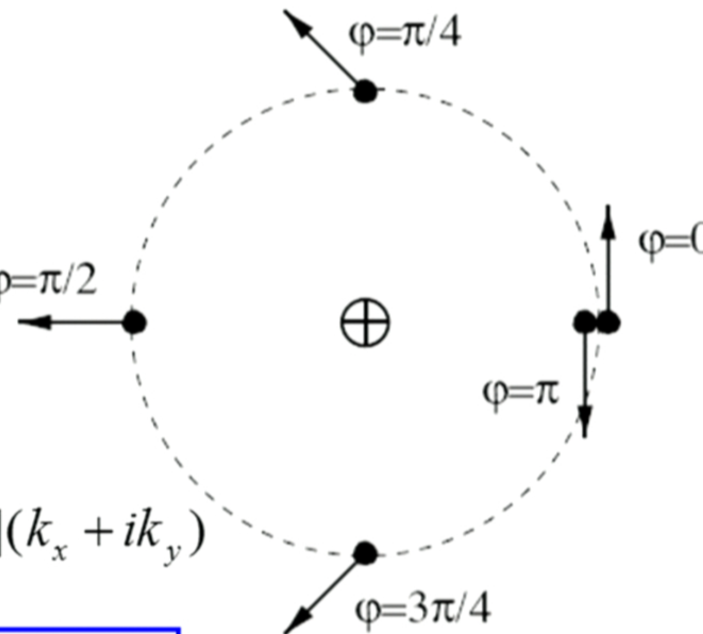
The phase of the order parameter φ **winds by** 2π
as we go around the vortex once

There is a normal core at the center of the vortex of dimension ξ **, the coherence length.**

The size of the magnetic flux is λ **the London peneteration length**

$$(\varphi, \hat{\mathbf{d}}) \mapsto (\varphi + \pi, -\hat{\mathbf{d}})$$

Half Quantum Vortex



$$\Psi(r,\theta) = \Delta(r) e^{\frac{i\theta}{2}} [e^{\frac{i\theta}{2}} \left|\uparrow\uparrow\right\rangle + e^{\frac{-i\theta}{2}} \left|\downarrow\downarrow\right\rangle](k_x + ik_y)$$

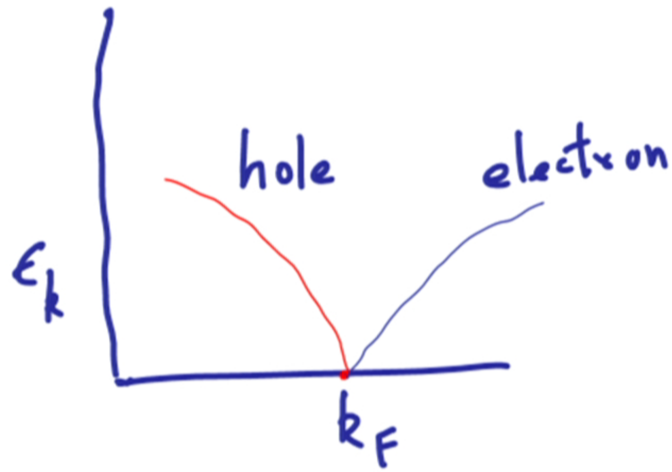
$$\Psi(r,\theta) = \Delta(r) [e^{i\theta} \left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle](k_x + ik_y)$$

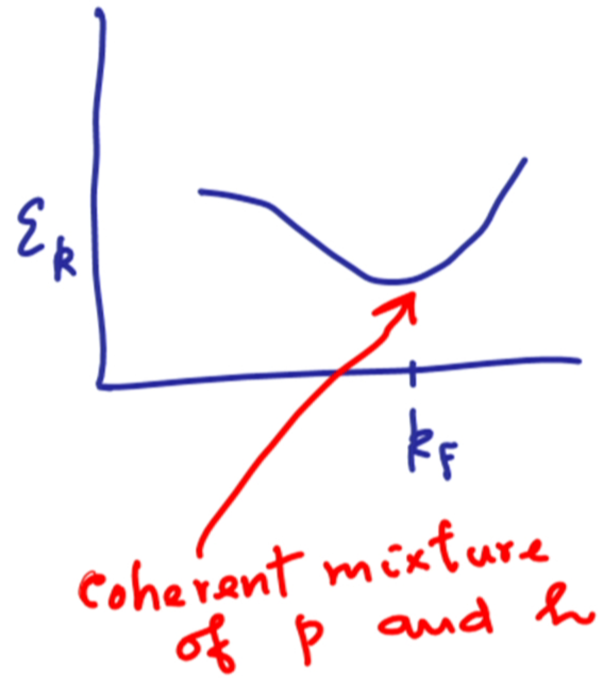
Order parameter remains single valued

Bogoliubov quasi particle is a linear combinations of an electron and a hole of opposite spin Their chrges are defined only module 2

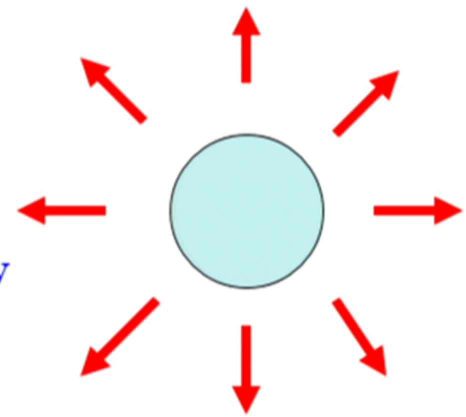
$$\alpha_{k\sigma} = u_k c_{k\sigma} + \sigma v_k c^{+}{}_{-k\bar{\sigma}}$$

$$\alpha^{+}{}_{k\sigma} = u_k c^{+}{}_{k\sigma} + \sigma v_k c_{-k\bar{\sigma}}$$

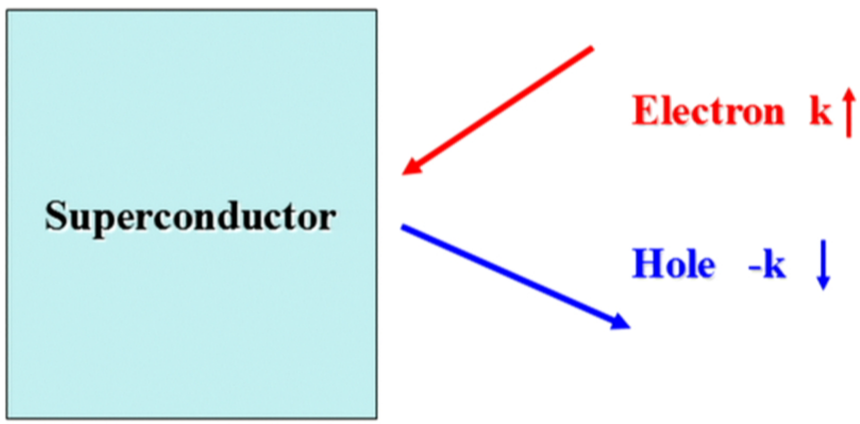


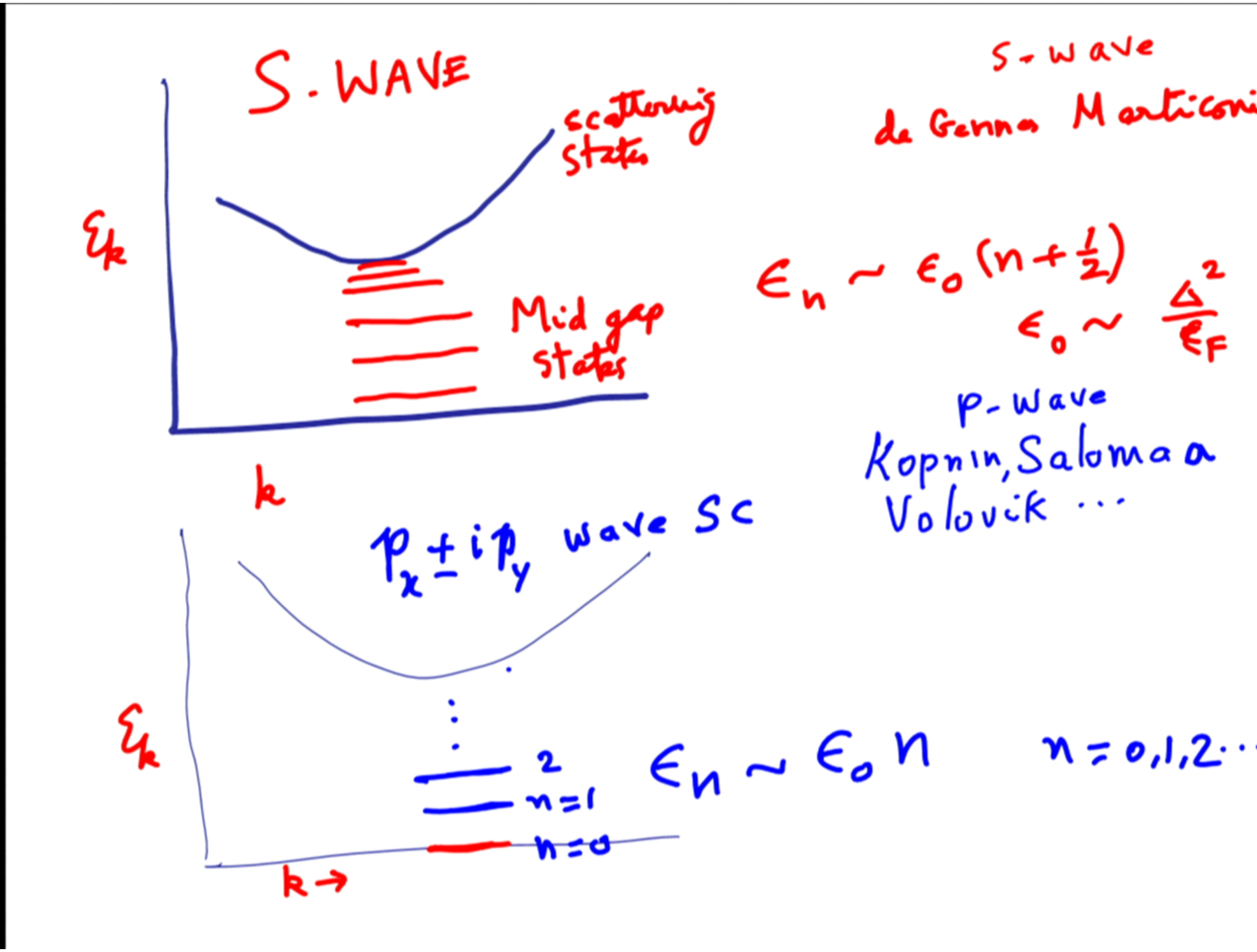


Normal state quasi particles at the vortex core get Andreev reflected at the boundary of the core and establish bound quasi particle states in the gap of the quasi particle spectrum



Because the boundary has a non trivial topology for the phase of the order parameter the bound qp-states could have non-trivial topological and robust character





S. WAVE
scattering states
$\mathcal{E}_k$
Mid gap states
k
s-wave
de Gennes Marticoni
$\epsilon_n \sim \epsilon_0 (n + \frac{1}{2})$
$\epsilon_0 \sim \frac{\Delta^2}{\epsilon_F}$
p-wave
Kopnin, Salomaa
Volovik ...
$p_x \pm i p_y$ wave SC
$\mathcal{E}_k$
2
n=1
n=0
$\epsilon_n \sim \epsilon_0 n$
$n = 0,1,2 \cdots$
k →

Nature of localized quasi particle states in Half vortices **D. A. Ivanov, PRL 2001**

$$\gamma^{\dagger}(E) = \gamma(-E)$$

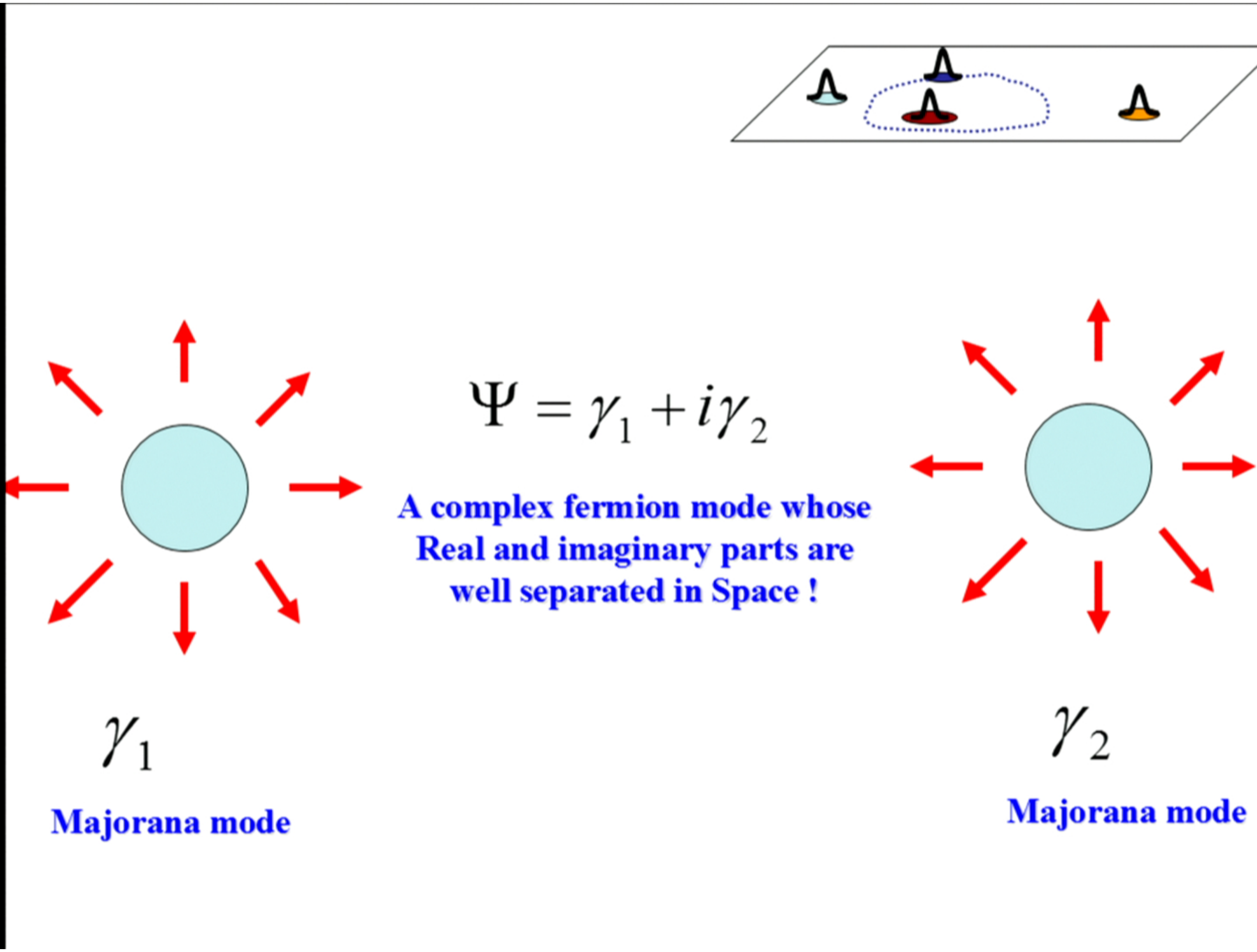
Number of degree of freedom (number of fermi oscillators) is half at a single vortex

The zero-energy level becomes a self-conjugate **Majorana Fermion**

$$\gamma^{\dagger}(E=0) = \gamma(E=0)$$

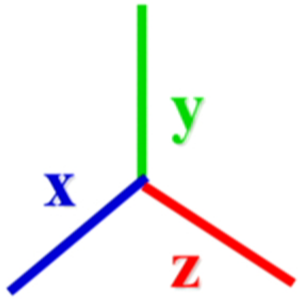
Contrast it with midgap states in domain walls in polyacetylene

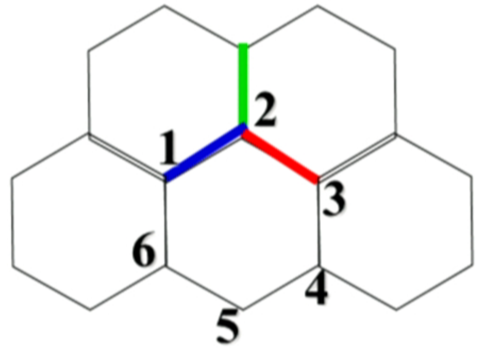
The Majorana Fermion zero mode is stable against local perturbation such external scalar, electromagnetic vector potentials, spin orbit coupling, local variation of the order parameter etc.


$\Psi = \gamma_1 + i\gamma_2$
A complex fermion mode whose
Real and imaginary parts are
well separated in Space !
γ_1
Majorana mode
γ_2
Majorana mode

Kitaev Model on a Honeycomb lattice

Kitaev 2001, 2003





Frustration in Spin Space

$$H = -J_x \sum_{\langle ij \rangle_x} \boxed{\sigma_i^x \sigma_j^x} - J_y \sum_{\langle ij \rangle_y} \boxed{\sigma_i^y \sigma_j^y} - J_z \sum_{\langle ij \rangle_z} \boxed{\sigma_i^z \sigma_j^z}$$

Flux Operator

$$\widehat{B}_P \equiv \sigma_1^y \sigma_2^z \sigma_3^x \sigma_4^y \sigma_5^z \sigma_6^x$$

2N sites

$$[\widehat{B}_P, H] = 0 \qquad \widehat{B}_P{}^2 = 1$$

Eigen value $B_P =$ ± 1

$$[\widehat{B}_P, \widehat{B}_{P'}] = 0$$

for every plaquette P

2^N possible configurations of B_P

2^N states

$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$

. . . Hilbert Space . . .

← **2^N different B_P Sectors** →

Representing a Spin-1/2 Operator by Majorana Fermions

$\{\psi, \psi^{+}\} = 1$

Introduce 4 Majorana fermions

$\psi^{+} = \xi + i\zeta$

$\{\xi, \zeta\} = 0$

$2 = \sqrt{2} \times \sqrt{2}$

$$c^{\alpha}, \ \alpha = 0, x, y, z$$

$\xi^{2} = \zeta^{2} = 1$

$$\{c^{\alpha}, c^{\beta}\} = 2\delta_{\alpha\beta}$$

Dimension of Physical Hilbert Space 2

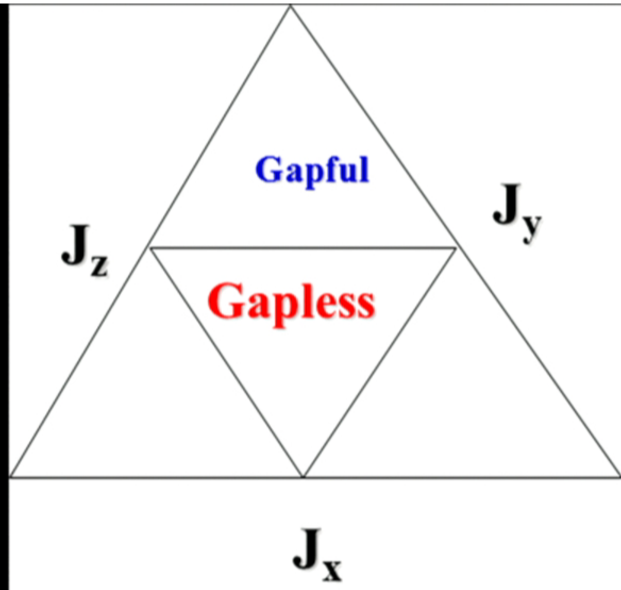
Dimension of Enlarged Hilbert Space 4

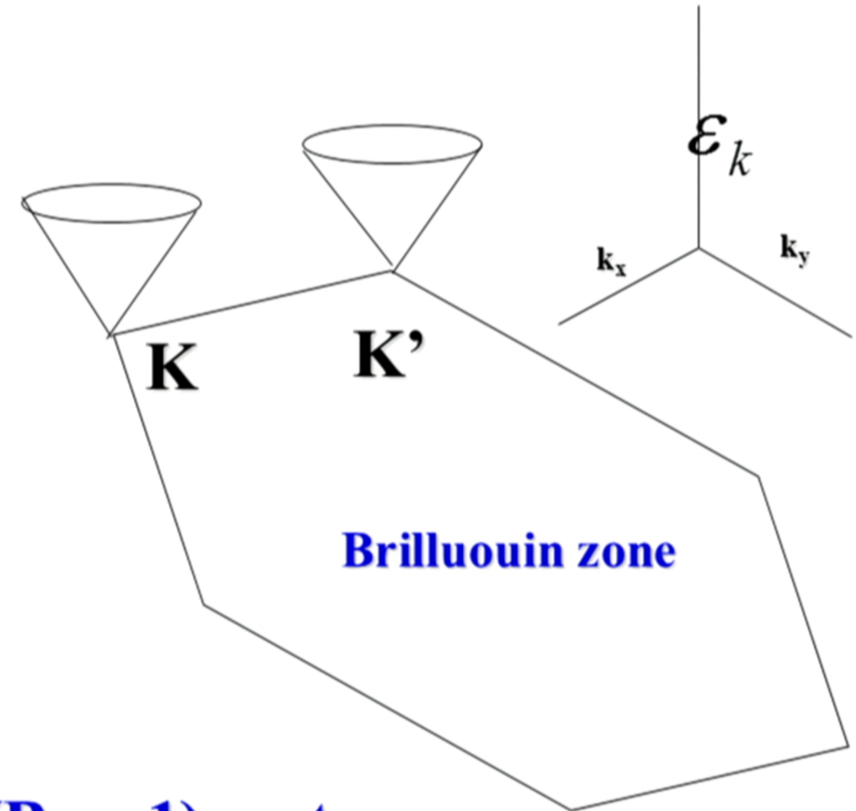
$$D_i|\Psi\rangle_{\text{phys}} = |\Psi\rangle_{\text{phys}}$$
$$D_i \equiv c_i \ c_i^x c_i^y c_i^z$$

$$\boxed{\sigma_i^a = i c_i c_i^a,} \qquad a = x, y, z$$

$$[\sigma_i^a, \sigma_j^b] = i\epsilon_{abc}\sigma^c\delta_{ij}$$

Only particle like fermionic excitations are present



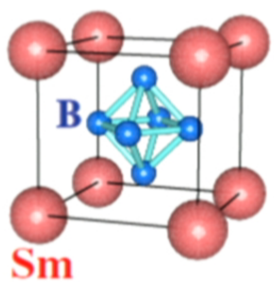


For zero flux ($B_p = 1$) sector

Kondo Effect, Kondo Insulators, Valence Fluctuations, Heavy Fermions, …

Anderson Lattice Model, Kondo Lattice Model, multi band Hubbard Model …

Spin-Orbit Coupling
Kramers Doublets

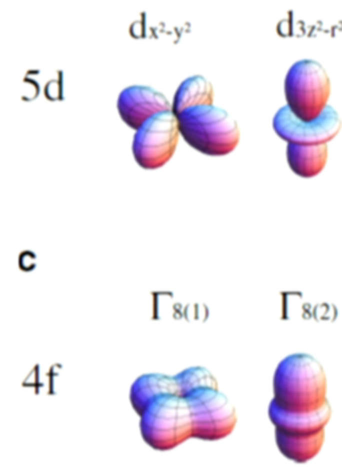


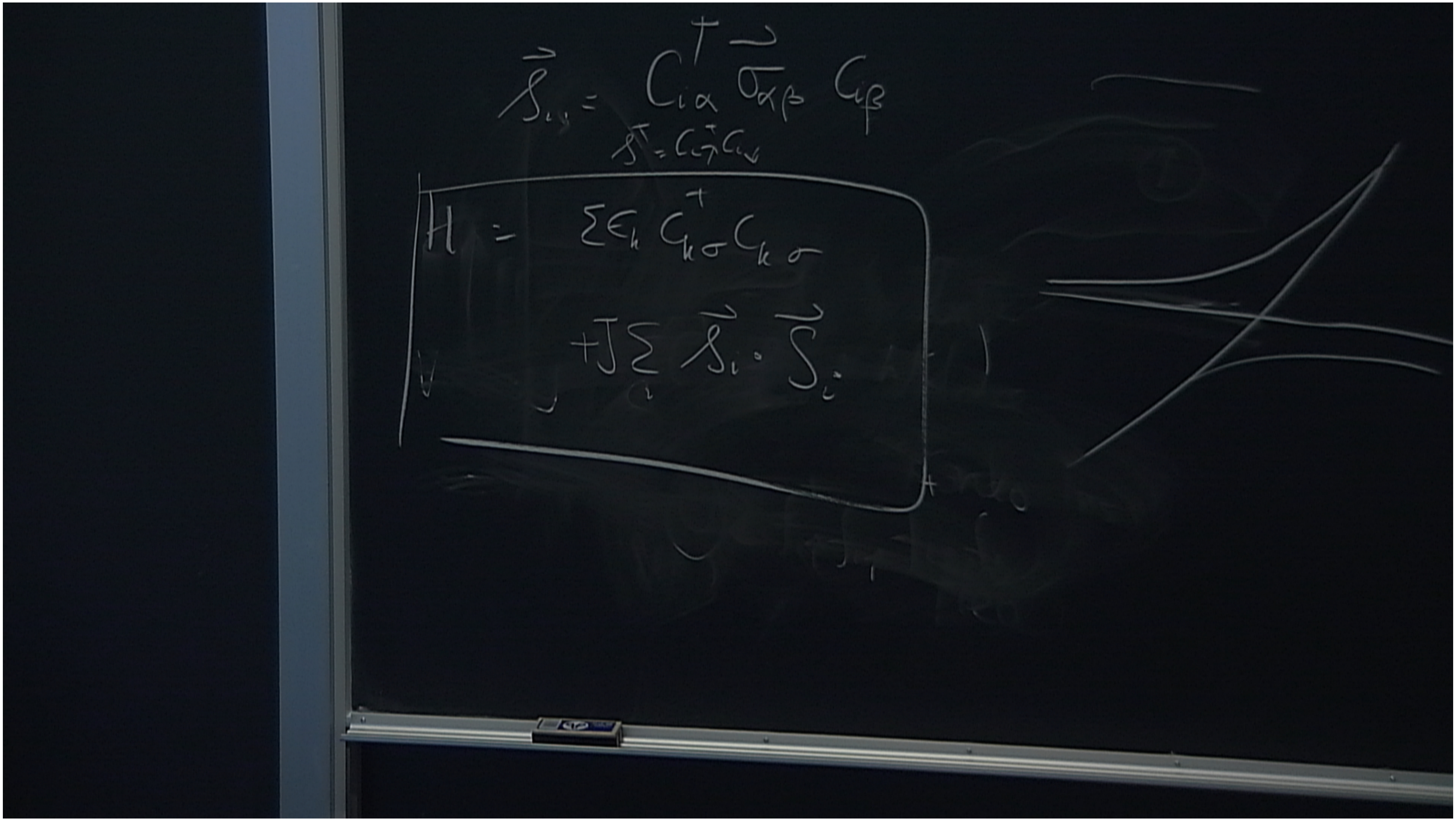
SmB_6

$(B_6)^{2-}$

Sm^{2+}

$4f^6\ 5d^0$ and $4f^5\ 5d^1$

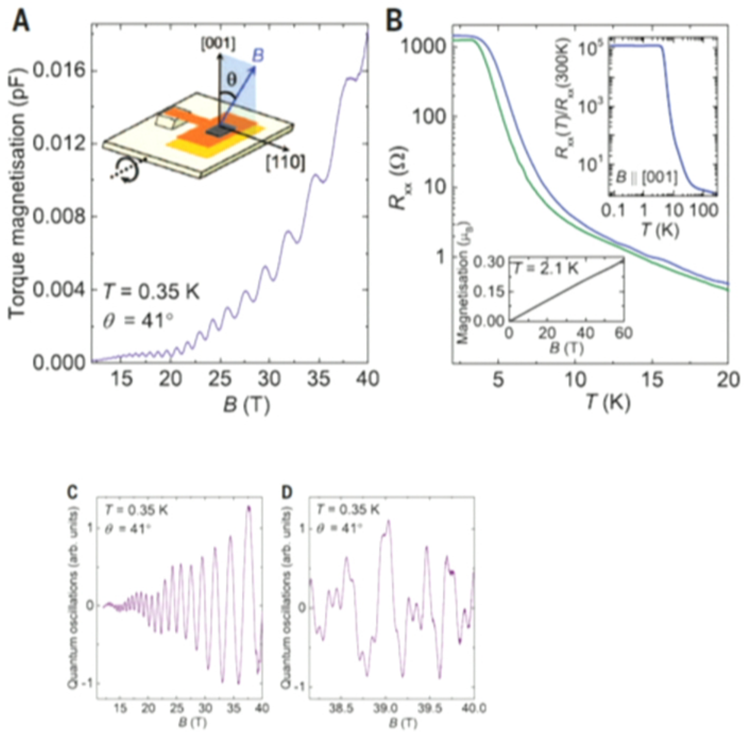


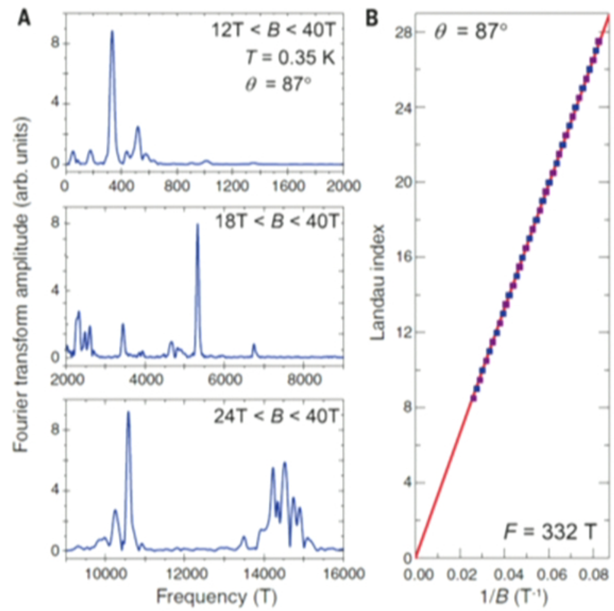

$\vec{S}_{i} = c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$
$H = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J \sum_i \vec{S}_i \cdot \vec{S}_i$

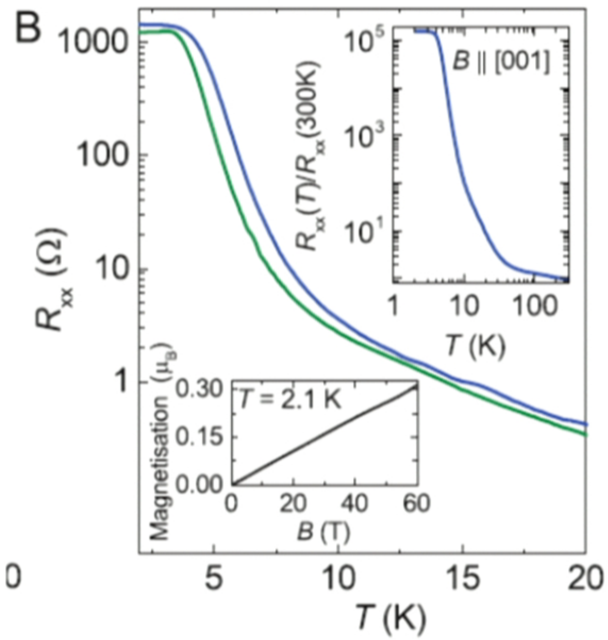
Unconventional Fermi surface in an insulating state

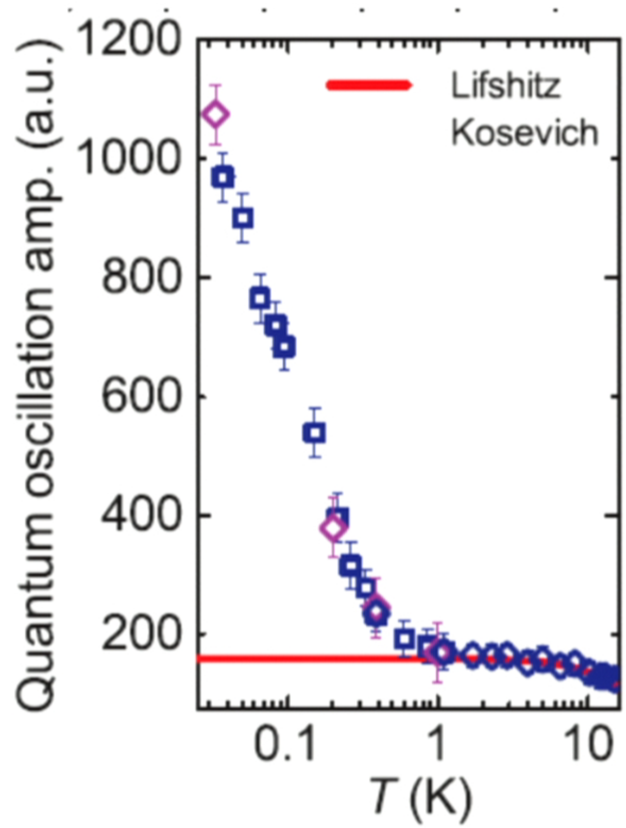
Science **349**, 287 (2015)

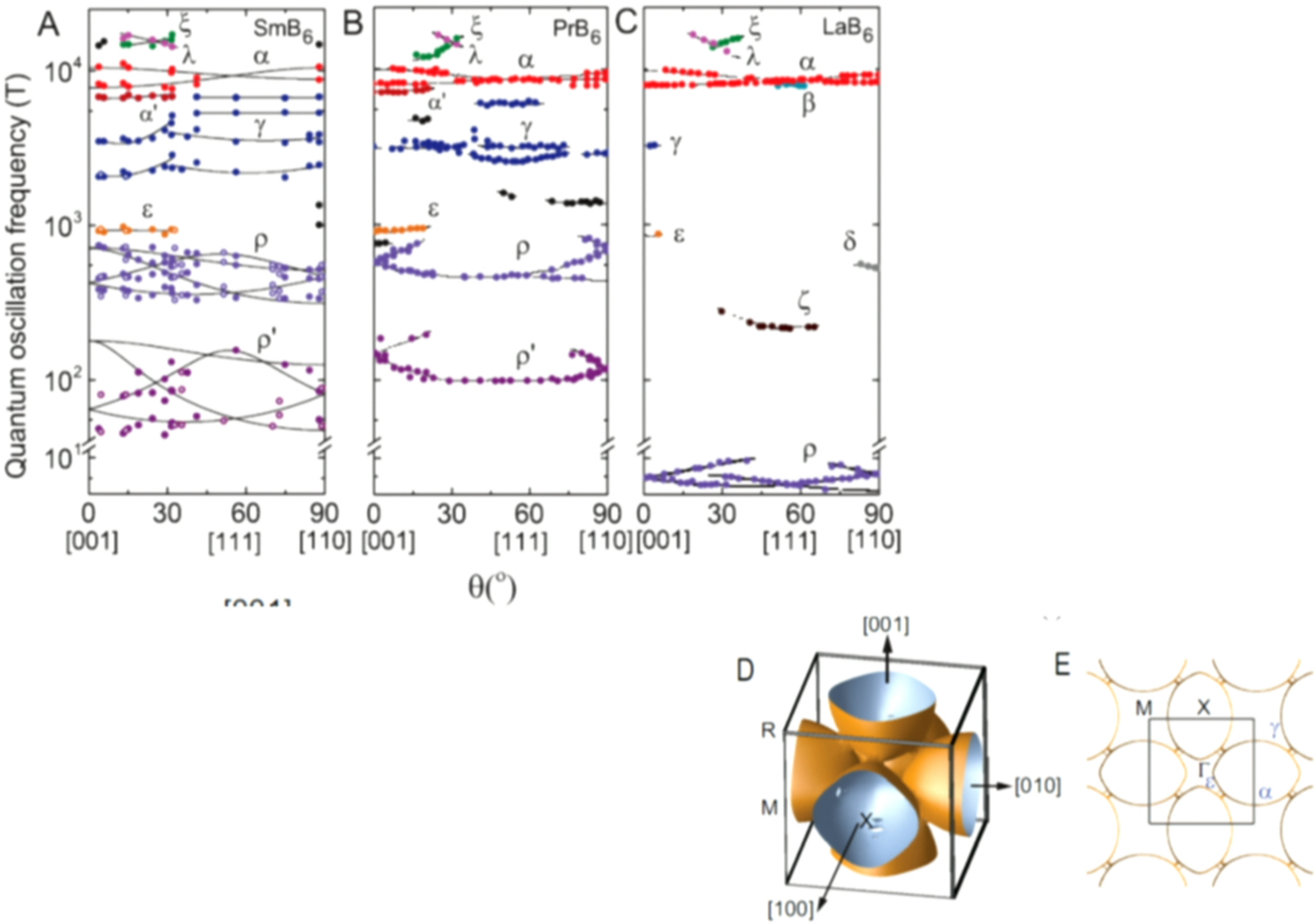
B. S. Tan,[1] Y.-T. Hsu,[1] B. Zeng,[2] M. Ciomaga Hatnean,[3] N. Harrison,[4] Z. Zhu,[4] M. Hartstein,[1] M. Kiourlappou,[1] A. Srivastava,[1] M. D. Johannes,[5] T. P. Murphy,[2] J.-H. Park,[2] L. Balicas,[2] G. G. Lonzarich,[1] G. Balakrishnan,[3] Suchitra E. Sebastian[1*]






B
1000
100
10
1
Rxx (Ω)
10^5
10^3
10^1
1
10
100
T (K)
B || [001]
Rxx(T)/Rxx(300K)
0.30
0.15
0.00
T = 2.1 K
0
20
40
60
B (T)
Magnetisation (μB)
0
5
10
15
20
T (K)


1200
1000
800
600
400
200
Quantum oscillation amp. (a.u.)
Lifshitz
Kosevich
0.1
1
10
T (K)


A
SmB6
B
PrB6
C
LaB6
Quantum oscillation frequency (T)
θ(°)
[001]
[111]
[110]
D
E
[001]
[010]
[100]
R
M
X
Γ

A spinful free Fermi sea at Half filling with particle-hole symmetry

can be rewritten as

Four Majorana Fermi seas, one scalar and three vector

Majorana fermions are their own antiparticles
So positive energy fermions exist only on one side of the Fermi surface !

in a standard Fermi sea, positive energy excitations
particles live outside the FS
holes live inside the FS

The currently popular (as qubits for topological quantum computation)
Majorana Zero Modes are Different

However, Majorana fermions offer an easy opportunity to create Majorana Zero Modes

$$H_{KE} = -t\sum_{\langle ij\rangle}(c^\dagger_{i\sigma}c_{j\sigma}+c^\dagger_{j\sigma}c_{i\sigma}) \to -it\sum_{\langle ij\rangle}(c^\dagger_{i\sigma}c_{j\sigma}-c^\dagger_{j\sigma}c_{i\sigma})$$

Transform operators in one of the two sublattices, $c^\dagger_{i\sigma} \to ic^\dagger_{i\sigma}$

$c^\dagger_{i\uparrow} = \frac{1}{2}(c_{ix} + ic_{iy})$ and $c^\dagger_{i\downarrow} = \frac{1}{2}(c_{iz} - ic_{i0})$

$\{c_{i\alpha}, c_{j\beta}\} = 0$ for $i \neq j$ and $\alpha \neq \beta$ and $c^2_{i\alpha} = 1$

$$H_{KE} = -it\sum_{\langle ij\rangle}(c_{i0}c_{j0} + \mathbf{c}_i \cdot \mathbf{c}_j)$$

$$H_{KE} = -it\sum_{\langle ij\rangle}(c_{i0}c_{j0} + \mathbf{c}_i \cdot \mathbf{c}_j)$$

$(a^\dagger_{\mathbf{k}\alpha}, a_{\mathbf{k}\alpha}),\ \alpha, = 0, x, y, z$

$$c_{i\alpha} = \frac{1}{\sqrt{N}}\sum_{\text{half BZ}}(a_{\mathbf{k}\alpha}e^{i\mathbf{k}\cdot\mathbf{R}_i} + a^\dagger_{\mathbf{k}\alpha}e^{-i\mathbf{k}\cdot\mathbf{R}_i})$$

$$H_{KE} = \sum_{k\in\text{half BZ}} \epsilon_k(a^\dagger_{k0}a_{k0} + \mathbf{a}^\dagger_k \cdot \mathbf{a}_k) + E_0$$

$$\mathrm{E}_0 = 2\sum_{\epsilon_k\leq 0}\epsilon_k = 2N\int_{-W}^{0}\epsilon\rho(\epsilon)d\epsilon$$

$$H = H_{\mathrm{KE}} + J\sum_i \mathbf{s}_i \cdot \mathbf{S}_i$$

Coleman, Miranda, Tsvelik 1993

$$\boldsymbol{S}_i \equiv -\tfrac{i}{2}\boldsymbol{\eta}_i \times \boldsymbol{\eta}_i$$

$$\mathbf{s}_i \equiv c^\dagger_{i\alpha}\boldsymbol{\sigma}_{\alpha\beta}c_{i\beta}$$

$$\mathbf{s}_i = ic_{i0}\mathbf{c}_i + \tfrac{i}{2}(\mathbf{c}_i \times \mathbf{c}_i)$$

$$(\mathbf{c}_i \times \mathbf{c}_i)\cdot(\boldsymbol{\eta}_i \times \boldsymbol{\eta}_i) = -\tfrac{1}{2}(\mathbf{c}_i \cdot \boldsymbol{\eta}_i)^2 - \tfrac{3}{2}$$

$$H = H_{\mathrm{KE}} + \tfrac{J}{2}\sum_i [c_{i0}\mathbf{c}_i \cdot (\boldsymbol{\eta}_i \times \boldsymbol{\eta}_i) - \frac{1}{2}(\mathbf{c}_i \cdot \boldsymbol{\eta}_i)^2] + \mathrm{const}$$

The above representation of Kondo lattice Hamiltonian is exact, except for the gauge redundancy. That is, many body spectrum of the original Kondo Hamiltonian is preserved. However there are $2^{\frac{N}{2}}$ extra gauge copies of the Hilbert space.

$$H = H_{\rm KE} + \frac{J}{2}\sum_i [c_{i0}\mathbf{c}_i \cdot (\boldsymbol{\eta}_i \times \boldsymbol{\eta}_i) - \frac{1}{2}(\mathbf{c}_i \cdot \boldsymbol{\eta}_i)^2] + \text{const}$$

$$\langle c_{i0}\mathbf{c}_i\rangle = 0$$

$$\langle \mathbf{S}_i\rangle = \langle \boldsymbol{\eta}_i \times \boldsymbol{\eta}_i\rangle = 0$$

$(\mathbf{c}_i \cdot \boldsymbol{\eta}_i)^2 \to 2\chi_0(\mathbf{c}_i \cdot \boldsymbol{\eta}_i) - \chi_0^2$. Here $\chi_0 \equiv \langle \mathbf{c}_i \cdot \boldsymbol{\eta}_i\rangle$

$$\begin{aligned} H_{\rm mf} &= H_{\rm KE} + -J\chi_0 \sum_i \mathbf{c}_i \cdot \eta_i + \text{const} \\ &= \sum_{\text{Half BZ}} \epsilon_k a^\dagger_{k0} a_{k0} + \sum_{BZ} \varepsilon_k \tilde{\mathbf{a}}^\dagger_k . \tilde{\mathbf{a}}_k + \text{const} \end{aligned}$$

$$\varepsilon_k = \frac{\epsilon_k}{2} \pm \sqrt{(\frac{\epsilon_k}{2})^2 + (J\chi_0)^2}$$

$$H_{\rm KE} = -t\sum_{\langle ij\rangle}(e^{i\frac{e}{\hbar c}\int_i^j \mathbf{A}\cdot d\mathbf{l}}c_{i\sigma}^\dagger c_{i\sigma} + H.c.) \qquad \mathbf{H} = \boldsymbol{\nabla}\times\mathbf{A}$$

$$= \sum_{\text{all } \alpha} \epsilon_\alpha c_{\alpha\sigma}^\dagger c_{\alpha\sigma}$$

Energy of the half filled band of (up and down spin) fermi sea gets modified from $2\sum_{\epsilon_k\leq 0}\epsilon_k$ to $2\sum_{\epsilon_\alpha<0}\epsilon_\alpha$, in the presence of a finite uniform magnetic field. Magnetic field preserves the particle hole symmetry.

$$H_{\rm KE} = \sum_{\epsilon_\alpha\geq 0}\epsilon_\alpha a_{\alpha 0}^\dagger a_{\alpha 0} + \sum_{\epsilon_\alpha\geq 0}\epsilon_\alpha \mathbf{a}_\alpha^\dagger\cdot\mathbf{a}_\alpha + E_0(\mathbf{H})$$

Vacuum energy of individual Majorana fermi seas are all identical. Total vacuum energy is, $\mathrm{E}_0(\mathbf{H}) = 4\times\frac{1}{2}\sum_{\epsilon_\alpha\leq 0}\epsilon_\alpha \equiv 2N\int_{-W}^0\rho(\epsilon,\mathbf{H})\epsilon d\epsilon$. Here $\rho(\epsilon,\mathbf{H})$ is the modified one electron density of states in the presence of an uniform magnetic field $\mathbf{H}$ and 2W is the bandwidth of one electron states.

$$H_{\rm mf}(\mathbf{H}) = \sum_{\epsilon_\alpha \geq 0} \epsilon_\alpha a^\dagger_{\alpha 0} a_{\alpha 0} + \sum_{\epsilon_\alpha \geq 0} \epsilon_\alpha \mathbf{a}^\dagger_\alpha \cdot \mathbf{a}_\alpha - J\chi_0 \sum_i \mathbf{c}_i \cdot \boldsymbol{\eta}_i + \frac{1}{4}E_0(\mathbf{H}) + \frac{3}{4}E_v(\mathbf{H}, \chi_0) \quad (10)$$

$$H_{\rm mf}(\mathbf{H}) = \sum_{\epsilon_\alpha > 0} \epsilon_\alpha a^\dagger_{\alpha 0} a_{\alpha 0} + \sum_{\text{all } \varepsilon_\alpha} \varepsilon_\alpha \tilde{\mathbf{a}}^\dagger_\alpha \cdot \tilde{\mathbf{a}}_\alpha + \frac{1}{4}E_0(\mathbf{H}) + \frac{3}{4}E_v(\mathbf{H}, \chi_0)$$

$$\varepsilon_\alpha = \frac{\epsilon_\alpha}{2} \pm \sqrt{(\frac{\epsilon_\alpha}{2})^2 + (J\chi_0)^2}$$

Majorana Fermions in a Superconductor

C. Chamon, R. Jackiw, Y. Nishida, S.Y. Pi, L. Santos

Remaining Question : Who will discover Majorana fermions first, condensed matter physicists or particle physicists ?

$$
\begin{aligned}
H_{\rm mf}(\mathbf{H}) &= \sum_{\epsilon_\alpha \geq 0} \epsilon_\alpha a^\dagger_{\alpha 0} a_{\alpha 0} + \sum_{\epsilon_\alpha \geq 0} \epsilon_\alpha \mathbf{a}^\dagger_\alpha \cdot \mathbf{a}_\alpha - J\chi_0 \sum_i \mathbf{c}_i \cdot \boldsymbol{\eta}_i \\
&+ \tfrac{1}{4} E_0(\mathbf{H}) + \tfrac{3}{4} E_v(\mathbf{H}, \chi_0) \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
H_{\rm mf}(\mathbf{H}) &= \sum_{\epsilon_\alpha > 0} \epsilon_\alpha a^\dagger_{\alpha 0} a_{\alpha 0} + \sum_{\text{all } \varepsilon_\alpha} \varepsilon_\alpha \tilde{\mathbf{a}}^\dagger_\alpha \cdot \tilde{\mathbf{a}}_\alpha \\
&+ \tfrac{1}{4} E_0(\mathbf{H}) + \tfrac{3}{4} E_v(\mathbf{H}, \chi_0)
\end{aligned}
$$

$$
\varepsilon_\alpha = \frac{\epsilon_\alpha}{2} \pm \sqrt{(\frac{\epsilon_\alpha}{2})^2 + (J\chi_0)^2}
$$

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Conclusion

SmB_6 is a Topological Insulator

Ours is a minimal model - a Spherical Cow Approximation Need for more realistic model

Nature of Surface States (ARPES, Quantum Oscillations, missing oscillation in magnetoresistance)

Charge neutral Scalar, Vector Majorana surface states and charged electron fermi surfaces (GB)

Majorana Zero Modes – Inexpensive Topological Qubits ?

Topological crystal defects, condensate defects, ….

Resistivity Saturation at Low Temperatures vs Violation of Lifshitz-Kosevich Formula

Nature of Symmetry Breaking odd frequency pairing …

Doped Kondo Insulators

Do Majorana Fermi Seas survive ?

Are Heavy Fermions Majorana Fermions ?

Doniach Phase Diagram ?