Title: Novel Fractional Quantum Hall States in the Lowest Landau Level

Date: Jul 06, 2015 03:00 PM

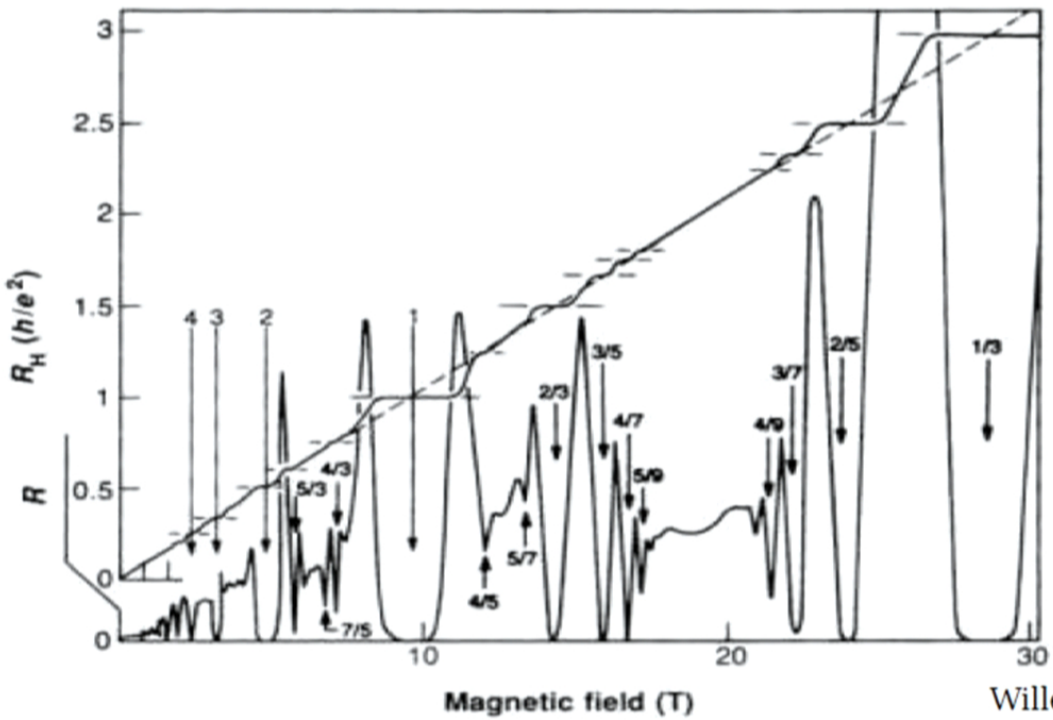
URL: http://pirsa.org/15070076

Abstract: <p>Fractional quantum Hall effect in the sequence of filling factors n/(2np +- 1) is well understood by the integer quantum Hall effect of the composite fermions at the filling factor n. A composite fermion (CF) is a bound state of an electron and 2p number of quantized vortices. However, the experimentally observed states such as 4/11, 5/13, and 3/8 which are between 1/3 and 2/5 cannot be accommodated in the conventional noninteracting theory of composite fermions. The interaction between CFs in partially filled second effective Landau levels of CFs is important for these states. However, conventional fractional quantum Hall effect of these interacting composite fermions do not reproduce these incompressible states. By diagonalizing Coulomb Hamiltonian in a restricted Hilbert space of the CFs, we find that these states are incompressible for flux-particle relationships that are different from the conventional considerations. We interpret that 4/11 and 5/13 states occur due to fractional quamtum Hall effect of the composite fermions for Haldane Pseudopotential V_3 rather than V_1. And the panti-Pfaffian pair correlation between CFs in the half-filled second effective Landau level creates incompressible 3/8 state. The unconventional topologies of these states are reflected in anomalously low magneto-roton energies for these states.</p>

Plan of Talk

- **Introduction to fractional quantum Hall effect.**
- **Observation of "second generation" FQHE in between two parent-FQHE states.**
- **Composite-fermion diagonalization: failure of conventional wisdom.**
- **New correlations that describe these novel FQHE states.**
- **Prediction of anomalously low magneto-roton energies in neutral collective modes of these FQHE states.**
- **Conclusion.**

Fractional quantum Hall effect



Willett et al, PRL,1987

$$\nu = \frac{n}{2n-1}, [2/3, 3/5, 4/7, ..., 1/2] \qquad \nu = \frac{n}{2n+1}, [1/3, 2/5, 3/7, ..., 1/2]$$

Filling factor, $\nu = \frac{N}{N_\varphi}$, **# / fraction of LL filled by the electrons.**

Hamiltonian of the FQHE problem

The problem is to find a quantum mechanical solution of interacting many electrons in presence of strong magnetic field.

$$H=\frac{1}{2m_b}\sum_j^N\left(-i\hbar\vec{\nabla}_j+\frac{e}{c}\vec{A}_j\right)^2+\frac{e^2}{\epsilon}\sum_{i<j}^N\frac{1}{r_{ij}}+g\mu_B\sum_j^N\vec{S}_j\cdot\vec{B}$$

In the limit of high magnetic field,

$$H_{eff}=\frac{e^2}{\epsilon}\sum_{i<j}^N\frac{1}{r_{ij}}$$ **(restricted in the LLL Hilbert space)**

1) Conventional perturbative method does not work here, due to the lack of any small parameter.

2)FQHE cannot be understood as an instability of any known state.

Laughlin Wave Function

$$\Psi(z_1, z_2, \ldots\ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)^{2n+1} \exp[-\sum_k |z_k|^2/(4l^2)]$$

A particle will feel (2n+1)(N-1) vortices for a N particle system. In the thermodynamic limit, this wave function thus corresponds to filling factor:

$$\nu = \lim_{N\to\infty} \frac{N}{(2n+1)(N-1)} = \frac{1}{(2n+1)}$$

Laughlin wave function for filling factor 1/3 has very good overlap with the exact wave function that one obtains with Coulomb interaction V(r). It can describe essential qualitative features that one obtains at filling factor 1/3.

The wave function $\Psi_{1/3}$ **is the *exact* ground state of the Haldane pseudo potential** V_1

$$\hat{V} = \sum_m V_m |m>.<m|$$

$\Psi_{1/5}$ **is *exact* for the potential** $V_1 + V_3$

Simple generalization to Laughlin wave function does not work for other observed states in the sequence, e.g., n/(2n+1).

Laughlin Wave Function

$$\Psi(z_1, z_2, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)^{2n+1} \exp[-\sum_k |z_k|^2/(4l^2)]$$

A particle will feel (2n+1)(N-1) vortices for a N particle system. In the thermodynamic limit, this wave function thus corresponds to filling factor:

$$\nu = \lim_{N\to\infty} \frac{N}{(2n+1)(N-1)} = \frac{1}{(2n+1)}$$

Laughlin wave function for filling factor 1/3 has very good overlap with the exact wave function that one obtains with Coulomb interaction V(r). It can describe essential qualitative features that one obtains at filling factor 1/3.

The wave function $\Psi_{1/3}$ is the *exact* ground state of the Haldane pseudo potential V_1

$$\hat{V} = \sum_m V_m |m>.<m|$$

$\Psi_{1/5}$ is *exact* for the potential $V_1 + V_3$

Simple generalization to Laughlin wave function does not work for other observed states in the sequence, e.g., n/(2n+1).

Composite Fermion Theory

J. K. Jain, PRL, 1989

Postulate: Electrons minimize the repulsive interaction by capturing *even number of quantized vortices* to form a bound state known as "composite fermions".

$$\nu = \frac{\nu_{CF}}{2\nu_{CF}+1} \qquad (B_{eff} = B - 2p n_e \varphi_0)$$

The FQHE of electrons is the manifestation of the IQHE of CFs.

Many body ground state wave function for the filling factor $\nu = \frac{n}{2n+1}$

$$\Psi_{CF}(z_1, z_2, \dots, z_N) = P_{LLL}\,\Phi_n(z_1, z_2, \dots, z_N) \prod_{i<j}^{N} (z_i - z_j)^2$$

$$\Phi_1(z_1, z_2, \dots, z_N) = \exp\left[-\sum_k \frac{|z_k|^2}{4l^2}\right] \begin{vmatrix} 1 & 1 & \cdots \\ z_1 & z_2 & \cdots \\ z_1^2 & z_2^2 & \cdots \\ \vdots & \vdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \ddots \end{vmatrix} \equiv \prod_{i<J}^{N} (z_i - z_j) \exp\left[-\sum_k \frac{|z_k|^2}{4l^2}\right]$$

This assumes non-interacting CFs.

Spherical Geometry

Haldane, PRL, 1983

N interacting electrons move on the surface of a sphere with 2Q flux passing through it, due to a magnetic "monopole" Q kept at the centre of the sphere. The radius of the sphere is $R=\sqrt{Q}\,l$

$$l=\left(\frac{\hbar c}{eB}\right)^{1/2}$$

$$(N,2Q)\rightarrow(N,2q);\,2q=2Q-2(N-1)$$ [Effective Flux]

ν	2Q	2q
1/3	$3N-3$	$N-1$
2/5	$\frac{5}{2}N-4$	$\frac{1}{2}N-2$

$$\nu=\lim_{N\rightarrow\infty}\frac{N}{2Q};\,\nu_{CF}=\lim_{N\rightarrow\infty}\frac{N}{2q}$$

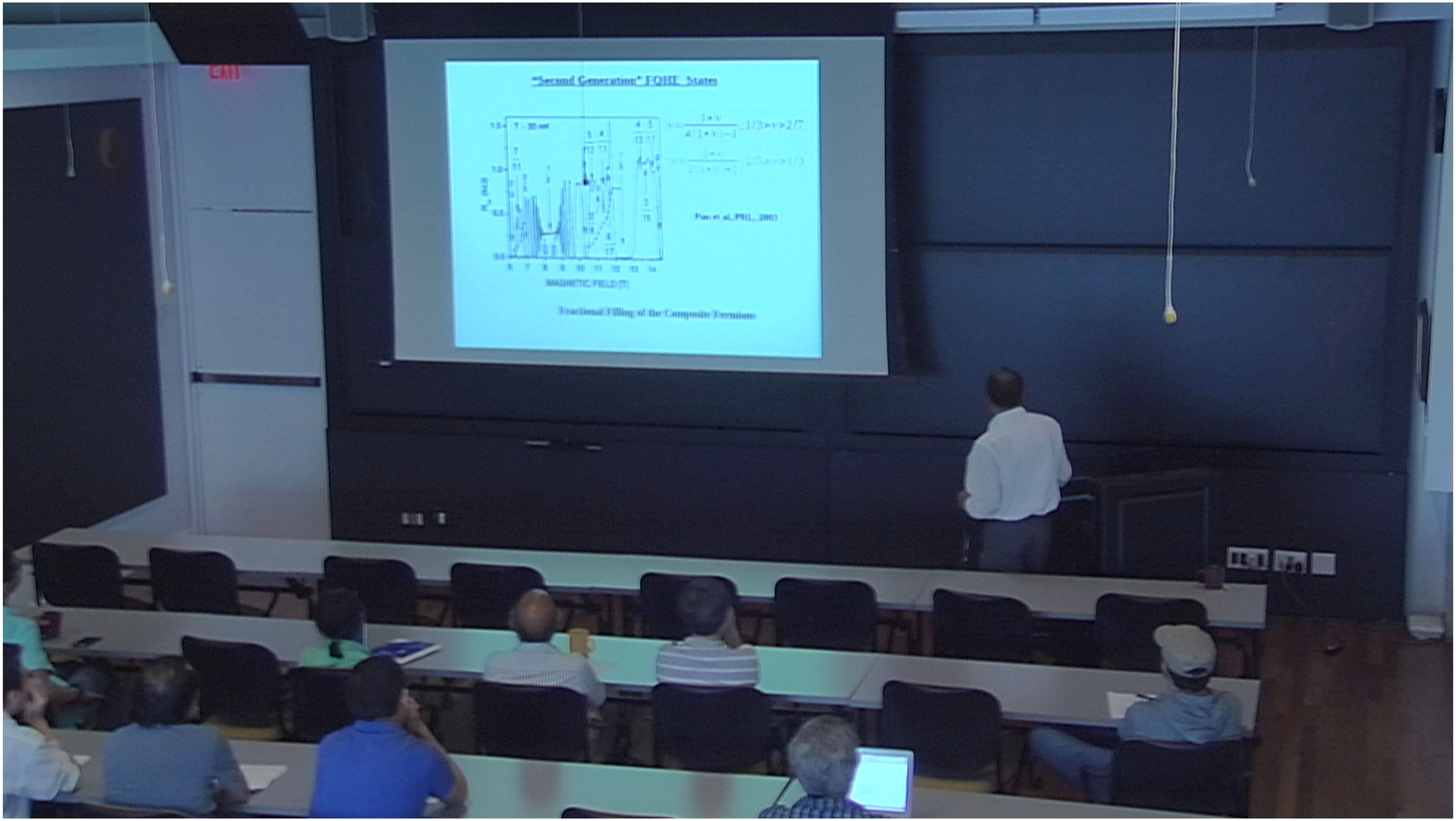
Shift: $2Q=\frac{N}{\nu}-S$

$$\nu=\frac{\nu_{CF}}{2\nu_{CF}+1}$$

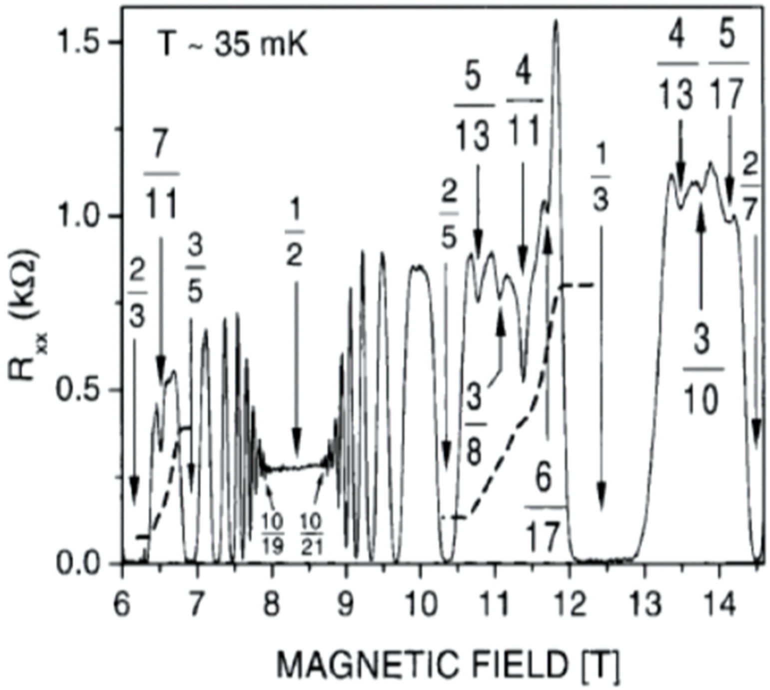
"Shift" determines *topologically distinct* Fractional quantum Hall states.

$\nu=\frac{1}{2}$;

S	Correlation
2	Gapless, Fermi liquid of CFs
3	Gapped paired state, Pfaffian
-1	Gapped paired state, Anti-Pfaffian


"Second Generation" FQHE States
MAGNETIC FIELD (T)
Pan et al, PRL, 2003
Fractional Filling of the Composite Fermions

“Second Generation” FQHE States



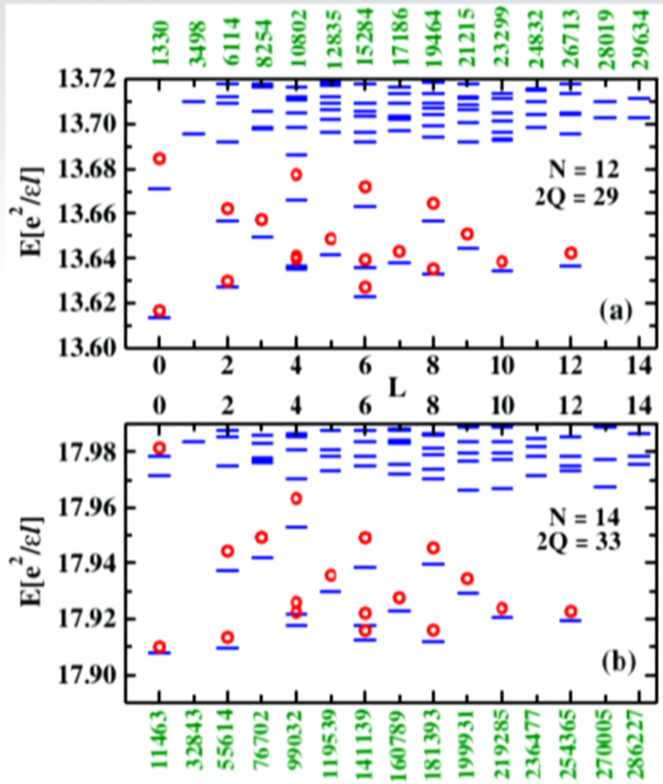
$$\nu = \frac{1+\bar{\nu}}{4(1+\bar{\nu})-1}; 1/3 > \nu > 2/7$$

$$\nu = \frac{1+\bar{\nu}}{2(1+\bar{\nu})+1}; 2/5 > \nu > 1/3$$

Pan et al, PRL, 2003

Fractional Filling of the Composite Fermions

CF diagonalization (CFD)



Comparison between exact spectra (blue dashes) and CFD spectra (red circles). The CFD energies deviate from exact ones by only ~ 0.05% for lowest mode.

SSM & Jain, PRB, 2002

$$\nu = \frac{\nu^{CF}}{2\nu^{CF}+1}$$

↓

$$\nu^{CF} = 1+\bar{\nu}$$

↓

$$\Phi^{\alpha}_{\bar{\nu}}$$

↓

$$\Phi^{\alpha}_{\nu^{CF}}$$

↓

$$\Psi^{\alpha}_{\nu} = P_{LLL} \prod_{i<j} (u_i v_j - v_i u_j)^{2p} \Phi^{\alpha}_{\nu^{CF}}$$

↓

orthogonalize

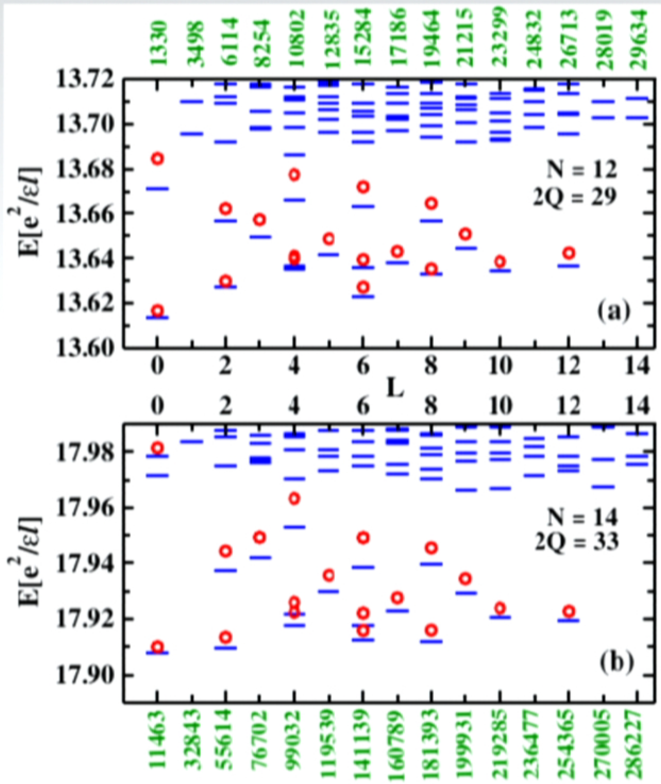
↓

evaluate the Hamiltonian matrix by Metropolis Monte Carlo method

↓

diagonalize

CF diagonalization (CFD)



Comparison between exact spectra (blue dashes) and CFD spectra (red circles). The CFD energies deviate from exact ones by only ~ 0.05% for lowest mode.

SSM & Jain, PRB, 2002

$$\nu = \frac{\nu^{CF}}{2\nu^{CF}+1}$$

↓

$$\nu^{CF} = 1+\bar{\nu}$$

↓

$$\Phi^{\alpha}_{\bar{\nu}}$$

↓

$$\Phi^{\alpha}_{\nu^{CF}}$$

↓

$$\Psi^{\alpha}_{\nu} = P_{LLL} \prod_{i<j} (u_i v_j - v_i u_j)^{2p} \Phi^{\alpha}_{\nu^{CF}}$$

↓

orthogonalize

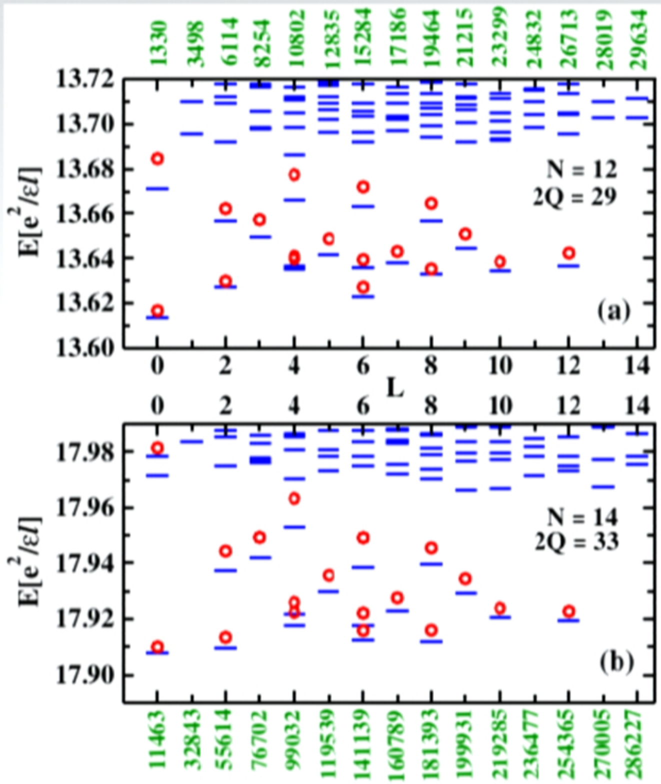
↓

evaluate the Hamiltonian matrix by Metropolis Monte Carlo method

↓

diagonalize

CF diagonalization (CFD)



Comparison between exact spectra (blue dashes) and CFD spectra (red circles). The CFD energies deviate from exact ones by only ~ 0.05% for lowest mode.

SSM & Jain, PRB, 2002

$$\nu = \frac{\nu^{CF}}{2\nu^{CF}+1}$$

↓

$$\nu^{CF} = 1+\bar{\nu}$$

↓

$$\Phi^{\alpha}_{\bar{\nu}}$$

↓

$$\Phi^{\alpha}_{\nu^{CF}}$$

↓

$$\Psi^{\alpha}_{\nu} = P_{LLL} \prod_{i<j} (u_i v_j - v_i u_j)^{2p} \Phi^{\alpha}_{\nu^{CF}}$$

↓

orthogonalize

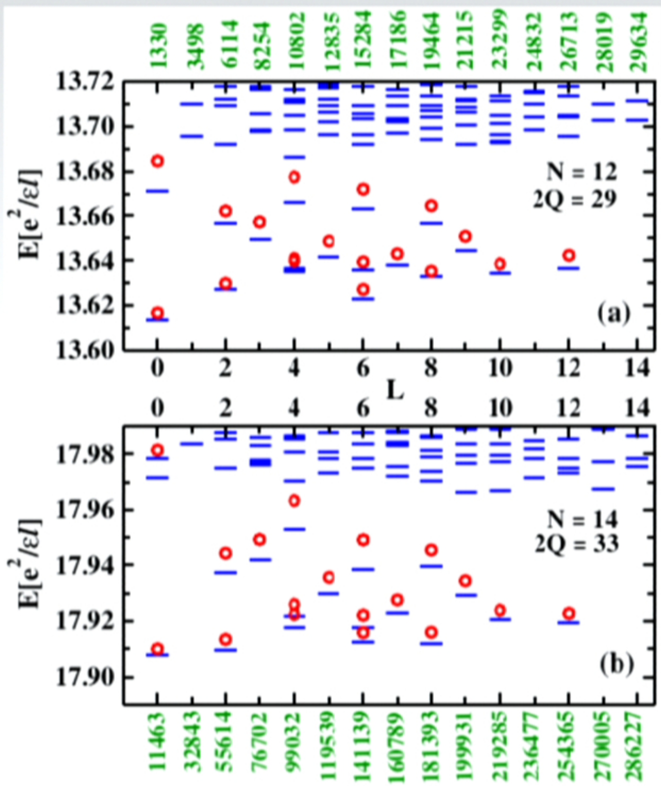
↓

evaluate the Hamiltonian matrix by Metropolis Monte Carlo method

↓

diagonalize

CF diagonalization (CFD)



Comparison between exact spectra (blue dashes) and CFD spectra (red circles). The CFD energies deviate from exact ones by only ~ 0.05% for lowest mode.

SSM & Jain, PRB, 2002

$$\nu = \frac{\nu^{CF}}{2\nu^{CF}+1}$$

↓

$$\nu^{CF} = 1+\bar{\nu}$$

↓

$$\Phi^{\alpha}_{\bar{\nu}}$$

↓

$$\Phi^{\alpha}_{\nu^{CF}}$$

↓

$$\Psi^{\alpha}_{\nu} = P_{LLL} \prod_{i<j} (u_i v_j - v_i u_j)^{2p} \Phi^{\alpha}_{\nu^{CF}}$$

↓

orthogonalize

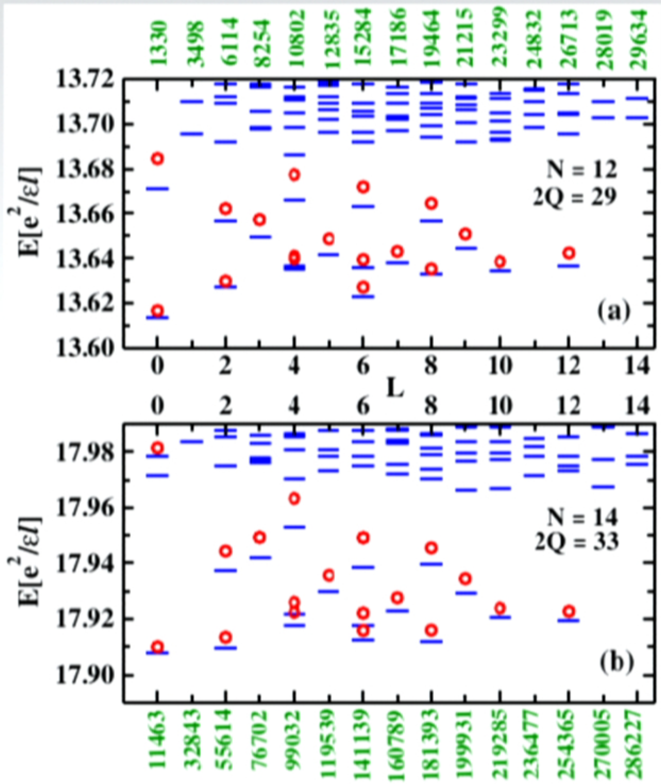
↓

evaluate the Hamiltonian matrix by Metropolis Monte Carlo method

↓

diagonalize

CF diagonalization (CFD)



Comparison between exact spectra (blue dashes) and CFD spectra (red circles). The CFD energies deviate from exact ones by only ~ 0.05% for lowest mode.

SSM & Jain, PRB, 2002

$$\nu = \frac{\nu^{CF}}{2\nu^{CF}+1}$$

↓

$$\nu^{CF} = 1+\bar{\nu}$$

↓

$$\Phi^{\alpha}_{\bar{\nu}}$$

↓

$$\Phi^{\alpha}_{\nu^{CF}}$$

↓

$$\Psi^{\alpha}_{\nu} = P_{LLL} \prod_{i<j} (u_i v_j - v_i u_j)^{2p} \Phi^{\alpha}_{\nu^{CF}}$$

↓

orthogonalize

↓

evaluate the Hamiltonian matrix by Metropolis Monte Carlo method

↓

diagonalize

Test of conventional wisdom

$$2Q=\nu^{-1}(N-1)-(3-\nu^{-1})(\lambda+2);\quad 2/5>\nu>2/7$$

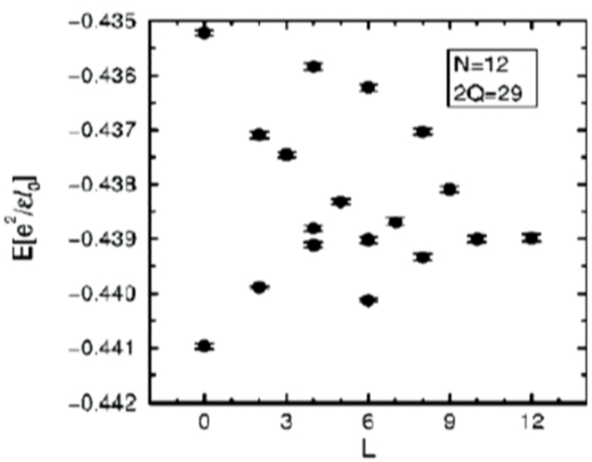
$$\nu=\frac{1+\bar{\nu}}{2(1+\bar{\nu})+1};2/5>\nu>1/3$$

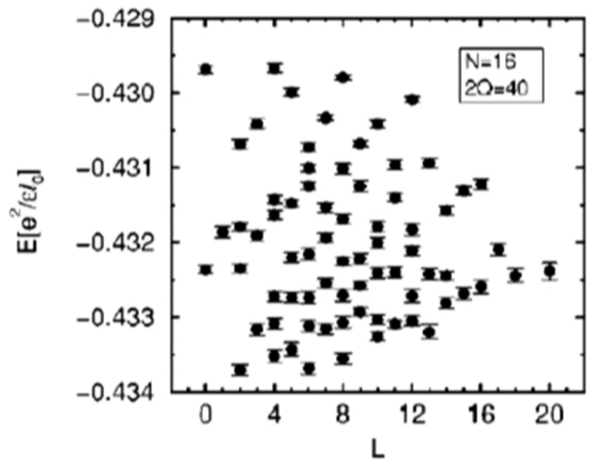
$\bar{\nu}$	ν
1/3	4/11
2/3	5/13
2/5	7/19
3/7	10/27
1/5	6/17
1/2	3/10

$$\nu=\frac{1+\bar{\nu}}{4(1+\bar{\nu})-1};1/3>\nu>2/7$$

$\bar{\nu}$	ν
1/3	4/13
2/3	5/17
1/2	3/10

Conventional: $\lambda=3,\bar{\nu}=1/3$





So conventional mechanism does not give rise to FQHE state at the filling factor 4/11.

Test of conventional wisdom

$$2Q=\nu^{-1}(N-1)-(3-\nu^{-1})(\lambda+2); \quad 2/5>\nu>2/7$$

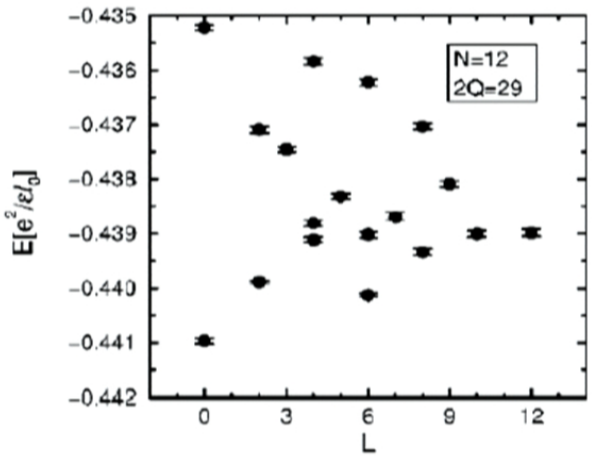
$$\nu=\frac{1+\bar{\nu}}{2(1+\bar{\nu})+1}; 2/5>\nu>1/3$$

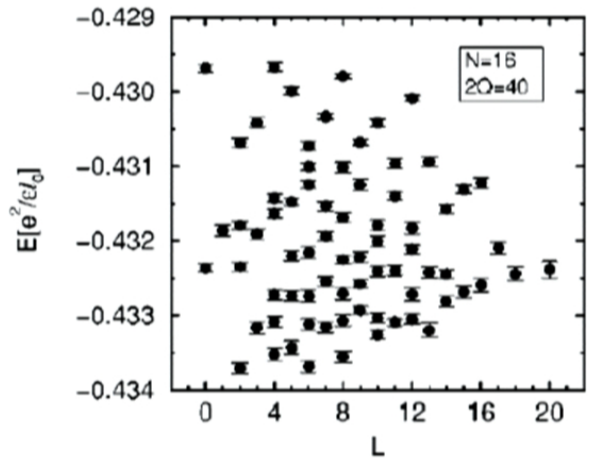
$\bar{\nu}$	ν
1/3	4/11
2/3	5/13
2/5	7/19
3/7	10/27
1/5	6/17
1/2	3/10

$$\nu=\frac{1+\bar{\nu}}{4(1+\bar{\nu})-1}; 1/3>\nu>2/7$$

$\bar{\nu}$	ν
1/3	4/13
2/3	5/17
1/2	3/10

Conventional: $\lambda=3, \bar{\nu}=1/3$

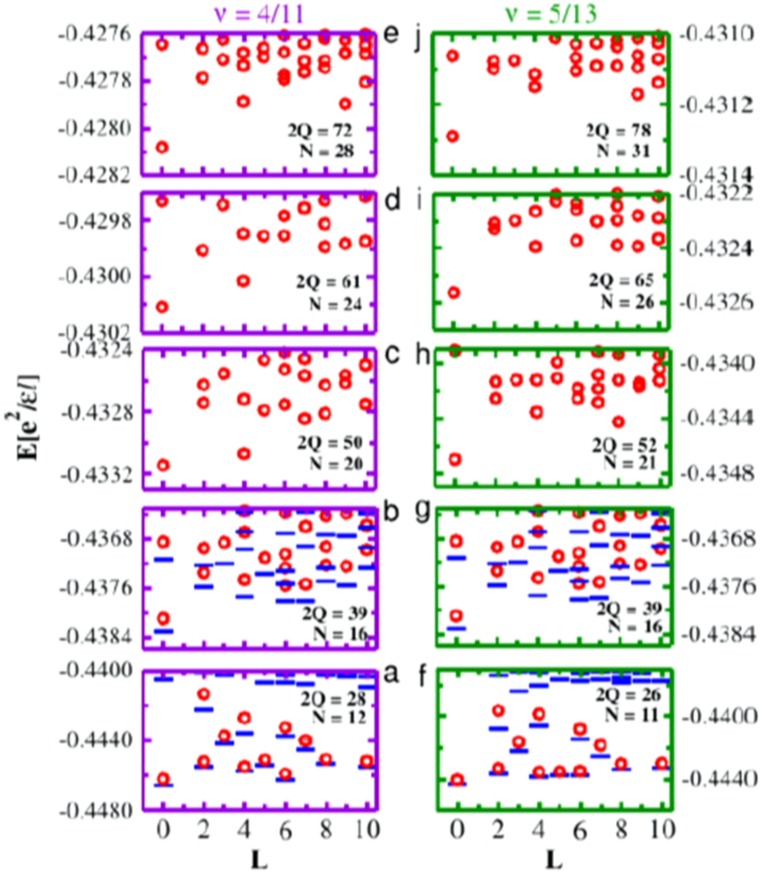




So conventional mechanism does not give rise to FQHE state at the filling factor 4/11.

CFD spectra at specific filling and shift

S. Mukherjee, SSM, Y-H Wu, A. Wojs, and J. K. Jain, PRL, 2014



$$2Q = 2q + 2(N-1)$$

$$2q = \bar{N}/\bar{\nu} - \lambda$$

$$2Q = \nu^{-1}(N-1) - (3-\nu^{-1})(\lambda+2);$$

$$\nu = 4/11, \bar{\nu} = \frac{1}{3}, \lambda = 7$$

$$\nu = 5/13, \bar{\nu} = \frac{2}{3}, \lambda = -2$$

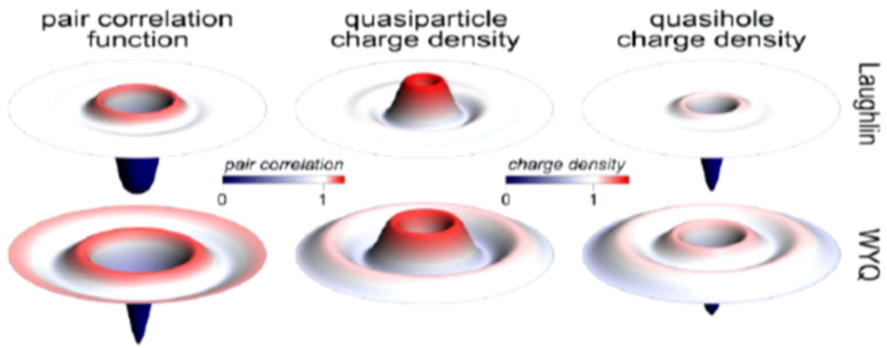
Characterization of FQHE state 4/11

Interaction of two CFs in the second ELL (when the lowest ELL is fully filled) has maximum pseudo-potential in the relative angular momentum channel 3.

$$V = \sum_m V_m |m\rangle\langle m| \qquad V_3 > V_1$$

The zero energy and incompressible ground state is possible for V_3 potential at flux $2Q = 5N - 9$

Only incompressible state is possible at flux $2Q = 3N - 7$ (Wojs, Yi, and Quinn, 2004)



We find FQHE ground state at 4/11 when the flux in second ELL is $2(q+1) = 3\bar{N} - 7$

S. Mukherjee, SSM, Y-H Wu, A. Wojs, and J. K. Jain, PRL, 2014

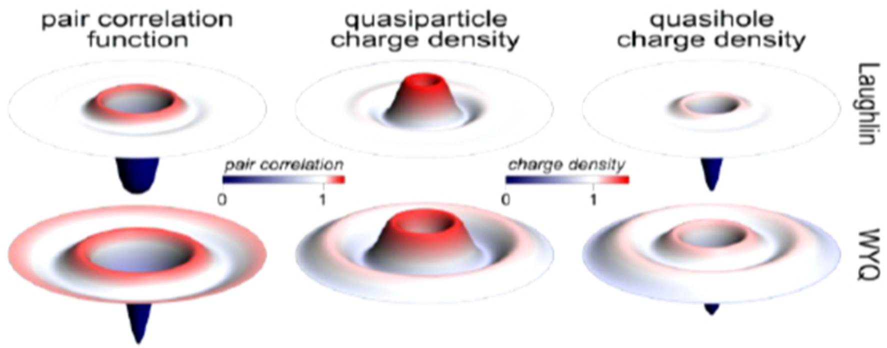
Characterization of FQHE state 4/11

Interaction of two CFs in the second ELL (when the lowest ELL is fully filled) has maximum pseudo-potential in the relative angular momentum channel 3.

$$V = \sum_m V_m |m\rangle\langle m| \qquad V_3 > V_1$$

The zero energy and incompressible ground state is possible for V_3 potential at flux $2Q = 5N - 9$

Only incompressible state is possible at flux $2Q = 3N - 7$ (Wojs, Yi, and Quinn, 2004)

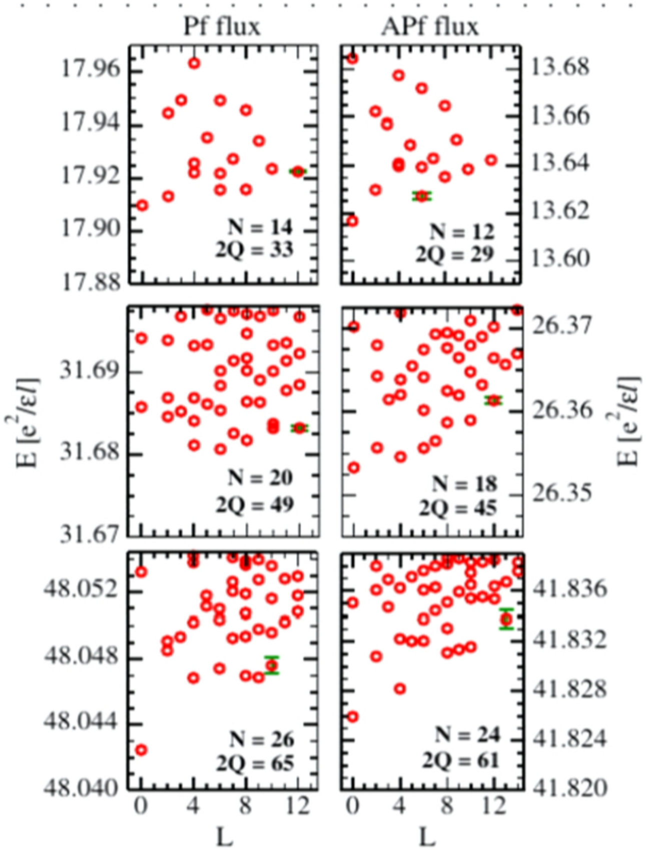


We find FQHE ground state at 4/11 when the flux in second ELL is $2(q+1) = 3\bar{N} - 7$

S. Mukherjee, SSM, Y-H Wu, A. Wojs, and J. K. Jain, PRL, 2014

Fully Polarized 3/8 state

$$2Q = 2q + 2(N-1)$$
$$2q = \bar{N}/\bar{\nu} - \lambda$$
$$2Q = \nu^{-1}(N-1) - (3-\nu^{-1})(\lambda+2);$$
$$\nu = 3/8, \bar{\nu} = \frac{1}{2}$$
$$\lambda = 3 \text{ (Pfaffian)}, -1 \text{ (Antipfaffian)}$$



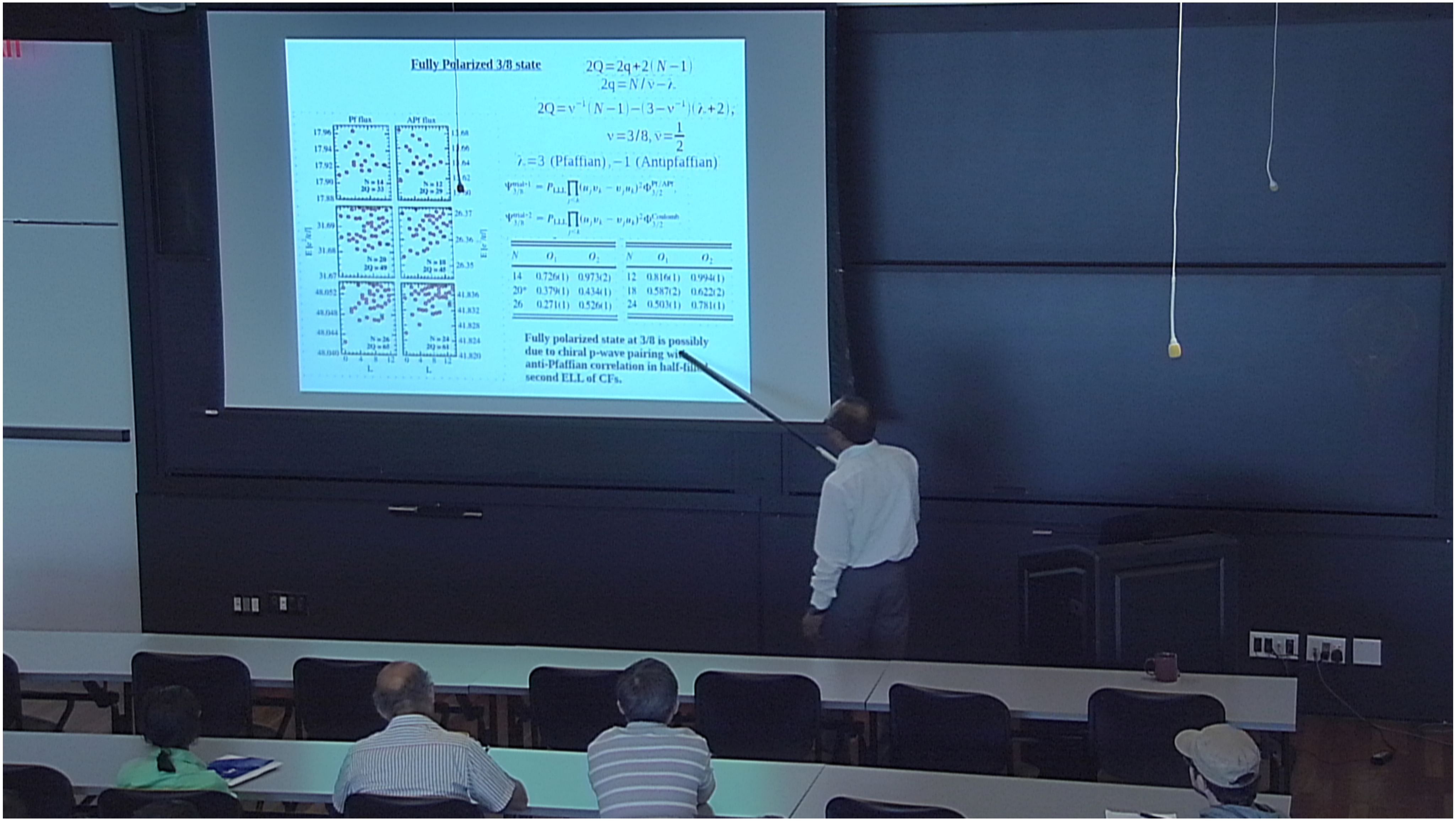
$$\Psi^{\text{trial-1}}_{3/8} = P_{\text{LLL}} \prod_{j<k} (u_j v_k - v_j u_k)^2 \Phi^{\text{Pf/APf}}_{3/2},$$

$$\Psi^{\text{trial-2}}_{3/8} = P_{\text{LLL}} \prod_{j<k} (u_j v_k - v_j u_k)^2 \Phi^{\text{Coulomb}}_{3/2}.$$

N	O_1	O_2
14	0.726(1)	0.973(2)
20*	0.379(1)	0.434(1)
26	0.271(1)	0.526(1)

N	O_1	O_2
12	0.816(1)	0.994(1)
18	0.587(2)	0.622(2)
24	0.503(1)	0.781(1)

Fully polarized state at 3/8 is possibly due to chiral p-wave pairing with anti-Pfaffian correlation in half-filled second ELL of CFs.


Fully Polarized 3/8 state
2Q=2q+2(N−1)
2q=N/ν̄−λ
2Q=ν^−1(N−1)−(3−ν^−1)(λ+2);
ν=3/8, ν̄=1/2
λ=3 (Pfaffian), −1 (Antipfaffian)
Pf flux
APf flux
N O1 O2
14 0.726(1) 0.973(2)
20* 0.379(1) 0.434(1)
26 0.271(1) 0.526(1)
N O1 O2
12 0.816(1) 0.994(1)
18 0.587(2) 0.622(2)
24 0.503(1) 0.781(1)
Fully polarized state at 3/8 is possibly
due to chiral p-wave pairing wi
anti-Pfaffian correlation in half-fill
second ELL of CFs.

Character of "second generation" FQHE states

(1) **Electrons capture 2 or 4 quantum vortices to form CFs**

(2) **CFs fill the lowest ELL completely and the second ELL partially; CFs interact in the partially filled ELL.**

(3) **FQHE of CFs in the second ELL caused by the IQHE of higher-order CFs which capture two more vortices does not take place.**

SSM and Jain, PRB, 2002

(4) **"WYQ" correlation causing FQHE of interacting CFs or their holes in the filling factors 1/3, 2/3 gives rise to FQHE states at (4/11, 4/13) .**

Mukherjee, SSM, Wu, Wojs, and Jain, PRL, 2013

(5) **"Ant-Pfaffian" pairing correlation causing FQHE for those interacting CFs at 1/2 filling gives rise to FQHE states at 3/8.**

Mukherjee, SSM, Wojs, and Jain, PRL, 2012; Mukherjee, Jain, and SSM, PRB, 2014

(6) **"Second generation" states are topologically distinct from the prominent states.**

Character of "second generation" FQHE states

(1) **Electrons capture 2 or 4 quantum vortices to form CFs**

(2) **CFs fill the lowest ELL completely and the second ELL partially; CFs interact in the partially filled ELL.**

(3) **FQHE of CFs in the second ELL caused by the IQHE of higher-order CFs which capture two more vortices does not take place.**

SSM and Jain, PRB, 2002

(4) **"WYQ" correlation causing FQHE of interacting CFs or their holes in the filling factors 1/3, 2/3 gives rise to FQHE states at (4/11, 4/13) .**

Mukherjee, SSM, Wu, Wojs, and Jain, PRL, 2013

(5) **"Ant-Pfaffian" pairing correlation causing FQHE for those interacting CFs at 1/2 filling gives rise to FQHE states at 3/8.**

Mukherjee, SSM, Wojs, and Jain, PRL, 2012; Mukherjee, Jain, and SSM, PRB, 2014

(6) **"Second generation" states are topologically distinct from the prominent states.**

Dispersion using Single Mode Approximation

Excited State: $\Psi_k = P_{LLL} \sum_{j=1}^{N} e^{i(k_x X_j + k_y Y_j)} \Psi_0$

$$\Delta(k) = 2[\bar{S}(k)]^{-1} \int \frac{d\boldsymbol{q}}{(2\pi)^2} \sin^2\left(\frac{\boldsymbol{k}\times\boldsymbol{q}}{2} l^2\right) e^{-k^2 l^2/2} \times [v(|\boldsymbol{q}-\boldsymbol{k}|) e^{\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}/2)l^2} - v(q)]\bar{S}(q),$$

Girvin, MacDonald, and Platzamn, PRB, 1986

Energy dispersion for neutral excitations can be determined by calculating the pair-correlation function using the ground state wavefunction.

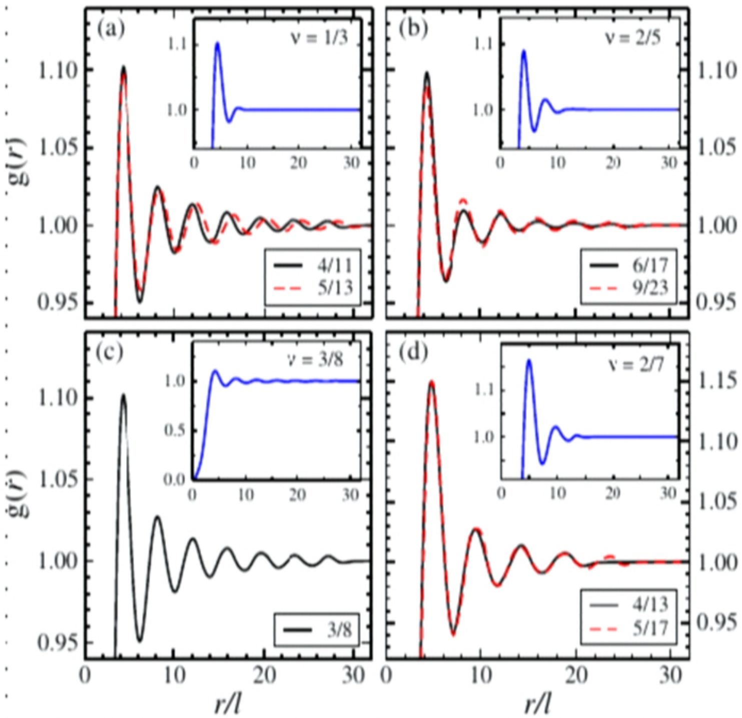
$$g(r) \sim \left\langle \sum_{i\neq j} \delta(\vec{r} - \vec{r}_i + \vec{r}_j) \right\rangle$$

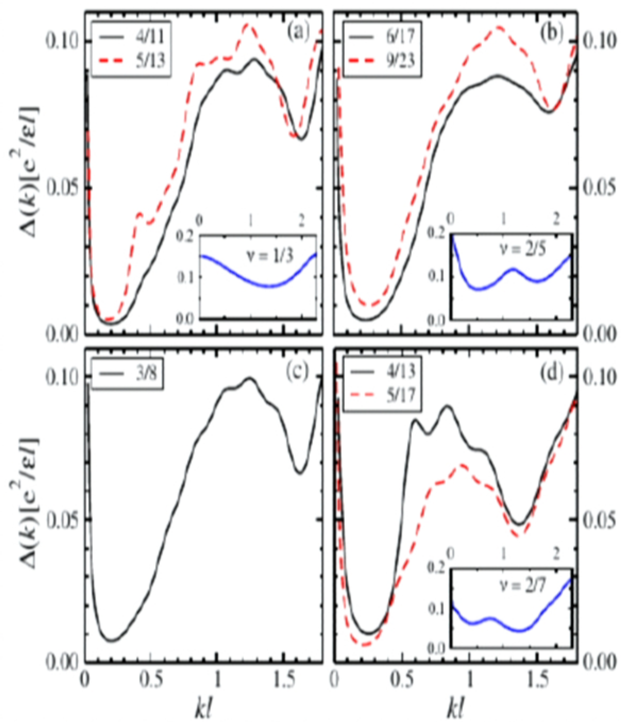
$\Psi_{\nu}^{uncon} = P_{LLL} \prod_{i<j} (z_i - z_j)^2 \Phi_{1+\bar{\nu}}^{WYQ}$ $\quad \bar{\nu} = 1/3, 2/3, \ldots$

$\Psi_{3/8}^{uncon} = P_{LLL} \prod_{i<j} (z_i - z_j)^2 \Phi_{1+1/2}^{Apf}$ $\quad \nu = 4/11, 5/13, \ldots$

Pair-correlation Function and energy dispersion

S. Mukherjee and SSM, PRL, 2015



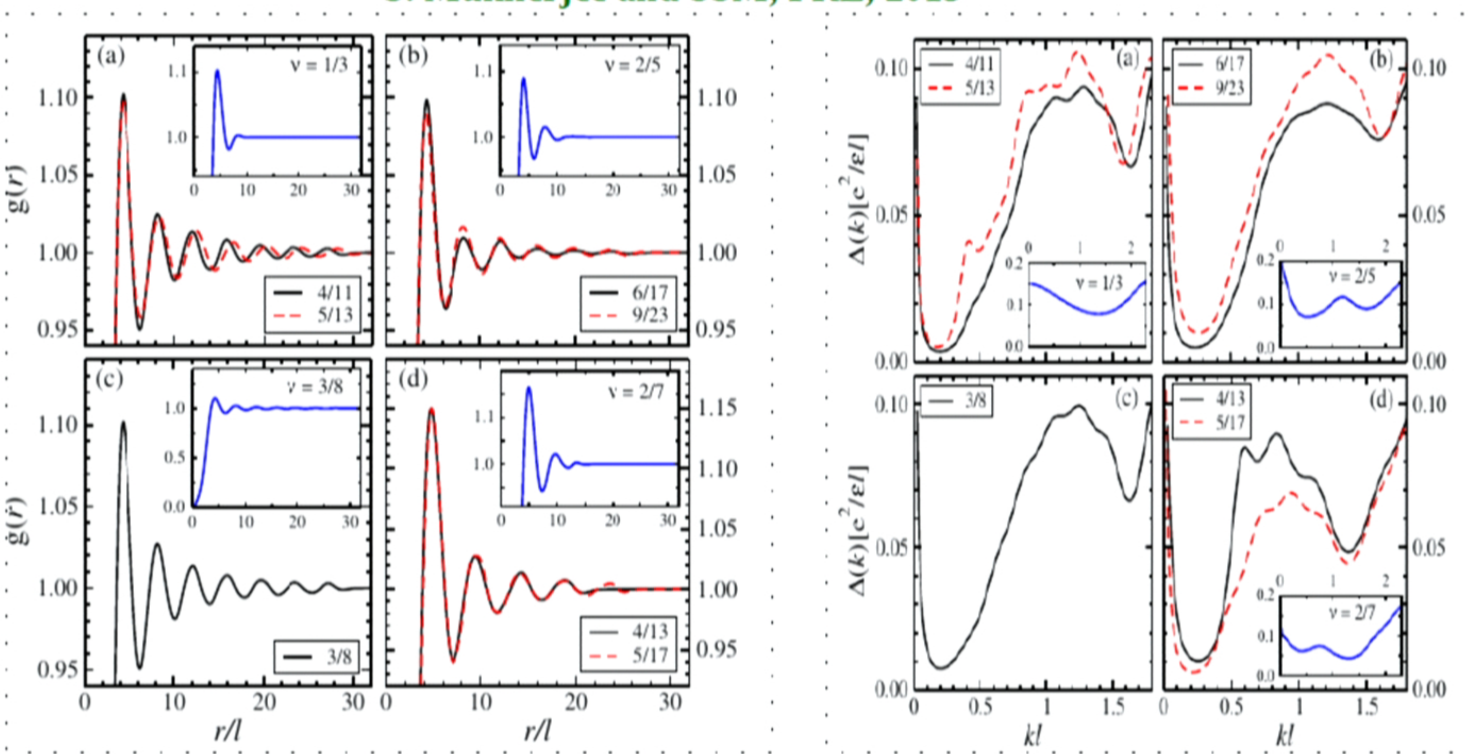


Magnetoroton minima in the energy range

$$0.004-0.011\, e^2/\epsilon l \sim 0.05-0.14\, meV$$

Pair-correlation Function and energy dispersion

S. Mukherjee and SSM, PRL, 2015



Magnetoroton minima in the energy range

$0.004-0.011\, e^2/\epsilon l \sim 0.05-0.14\, meV$

Conclusion

"Second Generation" FQHE states are novel as they emerge due to "unconventional" mechanisms of FQHE of composite fermions.

What will be the analytic forms of the wave functions that will describe FQHE in these states?

What will be the character of the edge states for these FQHE states?