Title: Supperconductivity in t1-t2-J1-J2 model on Honeycomb lattice

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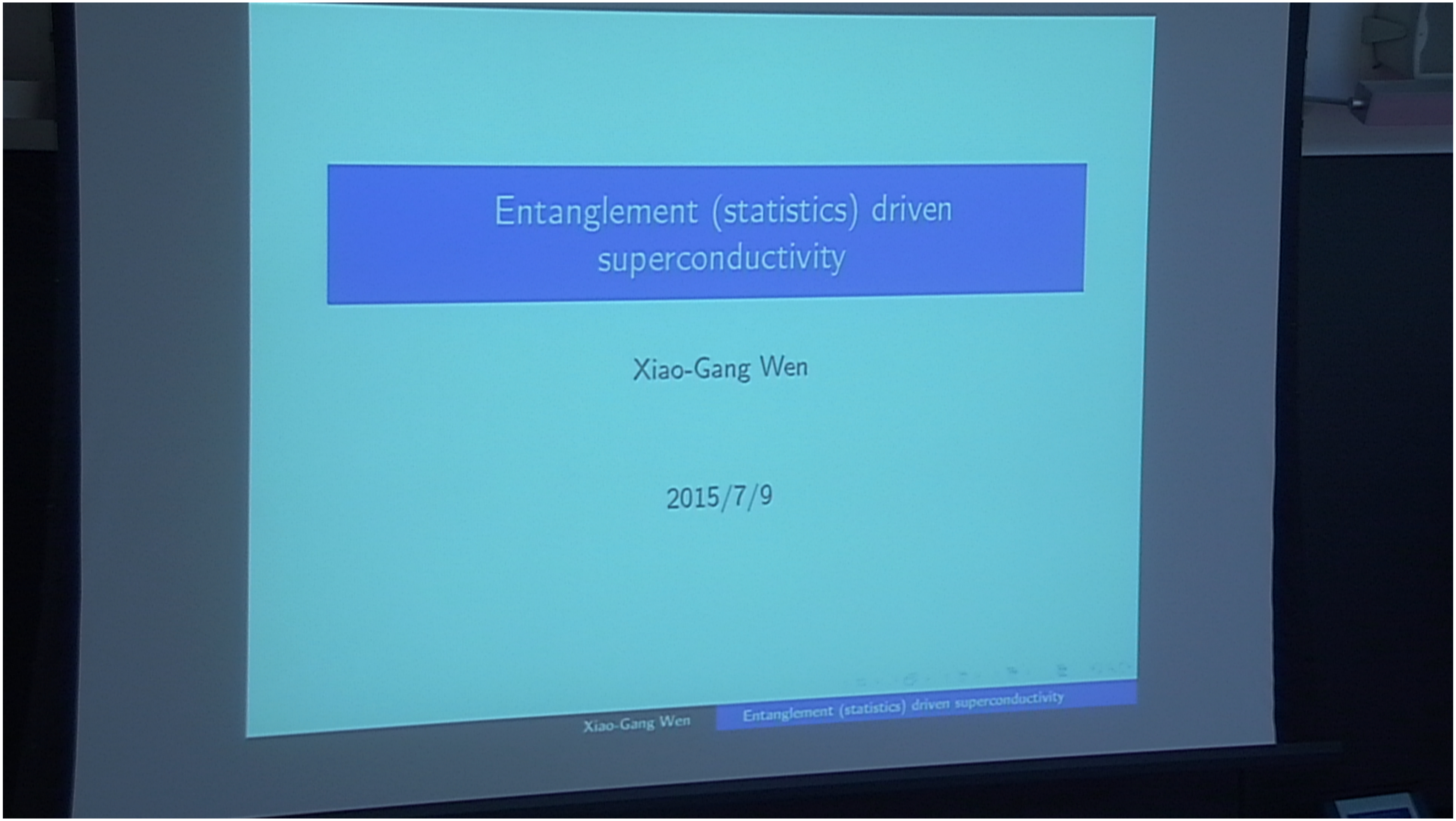
URL: http://pirsa.org/15070064

Abstract: We studied t1-t2-J1-J2 model on Honeycomb lattice at finite doping. We find that when t_1 is very small, the t1-t2-J1-J2 model on Honeycomb lattice may be in a supperconducting phase. Such a supperconducting phase is not driven by the pairing, but by entanglement.


Entanglement (statistics) driven
superconductivity
Xiao-Gang Wen
2015/7/9
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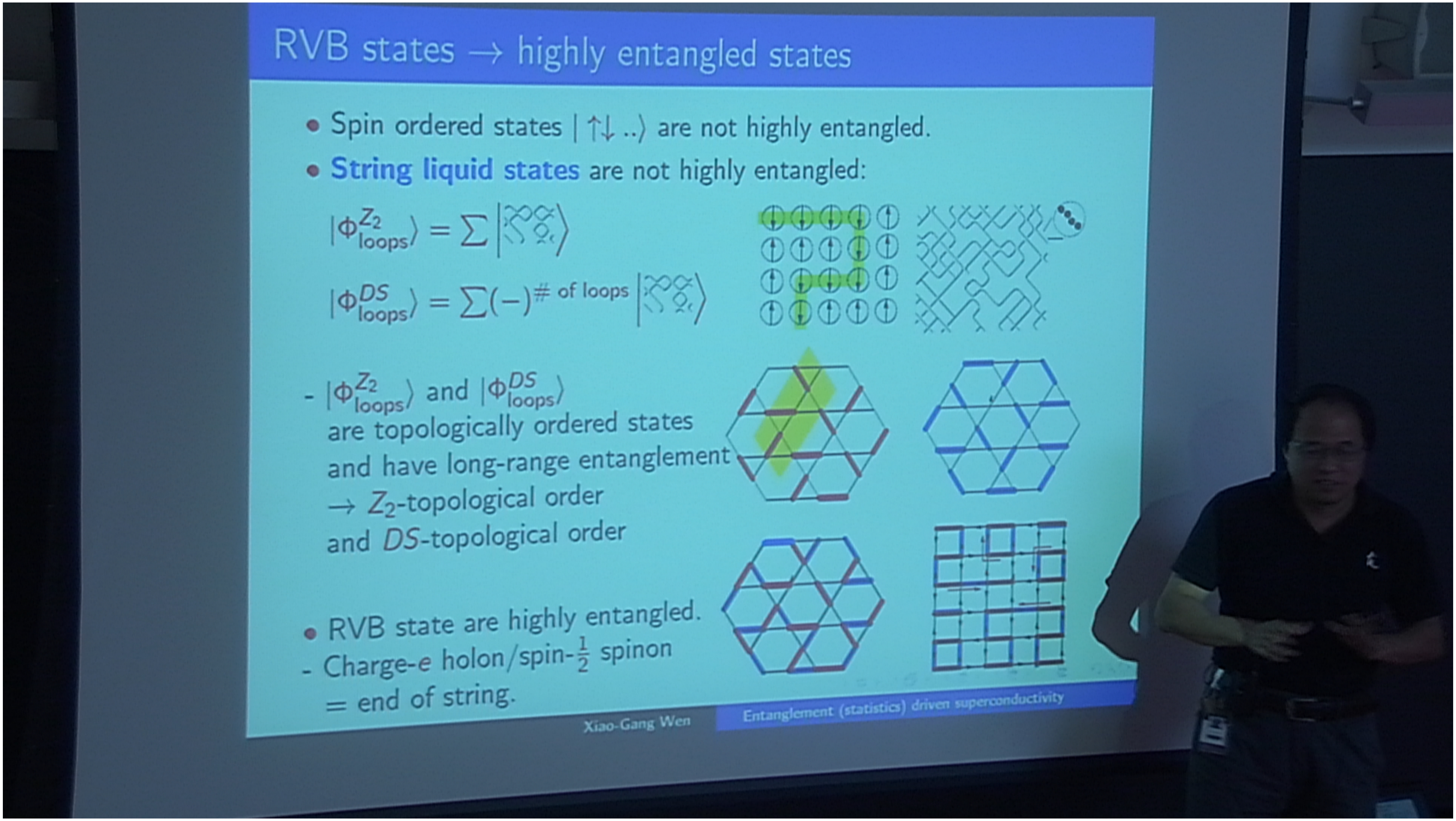
RVB states → highly entangled states
• Spin ordered states $|\uparrow\downarrow ..\rangle$ are not highly entangled.
• String liquid states are not highly entangled:
$|\Phi^{Z_2}_{\text{loops}}\rangle = \sum |\ \rangle$
$|\Phi^{DS}_{\text{loops}}\rangle = \sum (-)^{\#\text{ of loops}} |\ \rangle$
- $|\Phi^{Z_2}_{\text{loops}}\rangle$ and $|\Phi^{DS}_{\text{loops}}\rangle$
are topologically ordered states
and have long-range entanglement
→ Z_2-topological order
and DS-topological order
• RVB state are highly entangled.
- Charge-e holon/spin-$\frac{1}{2}$ spinon
= end of string.
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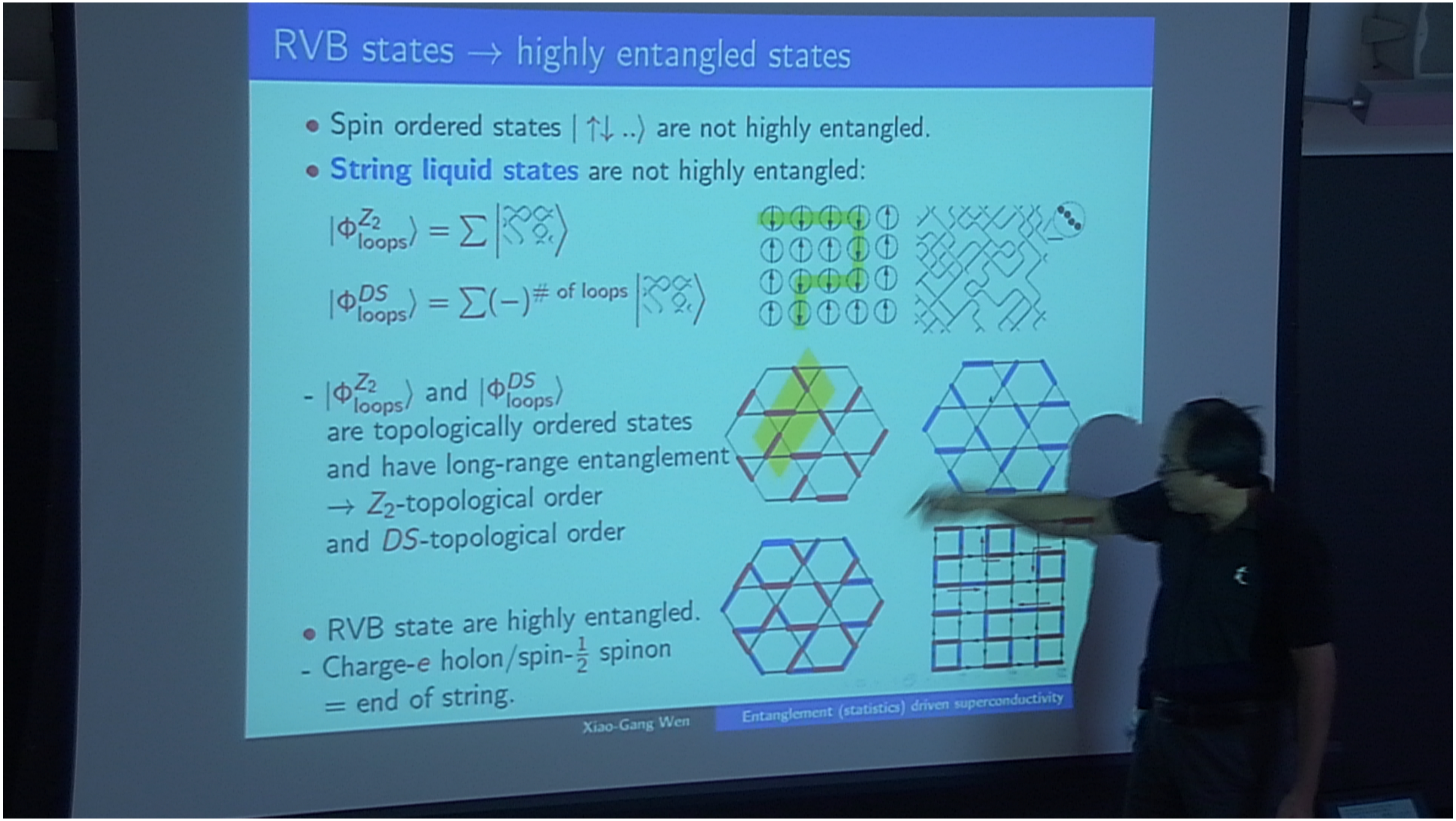
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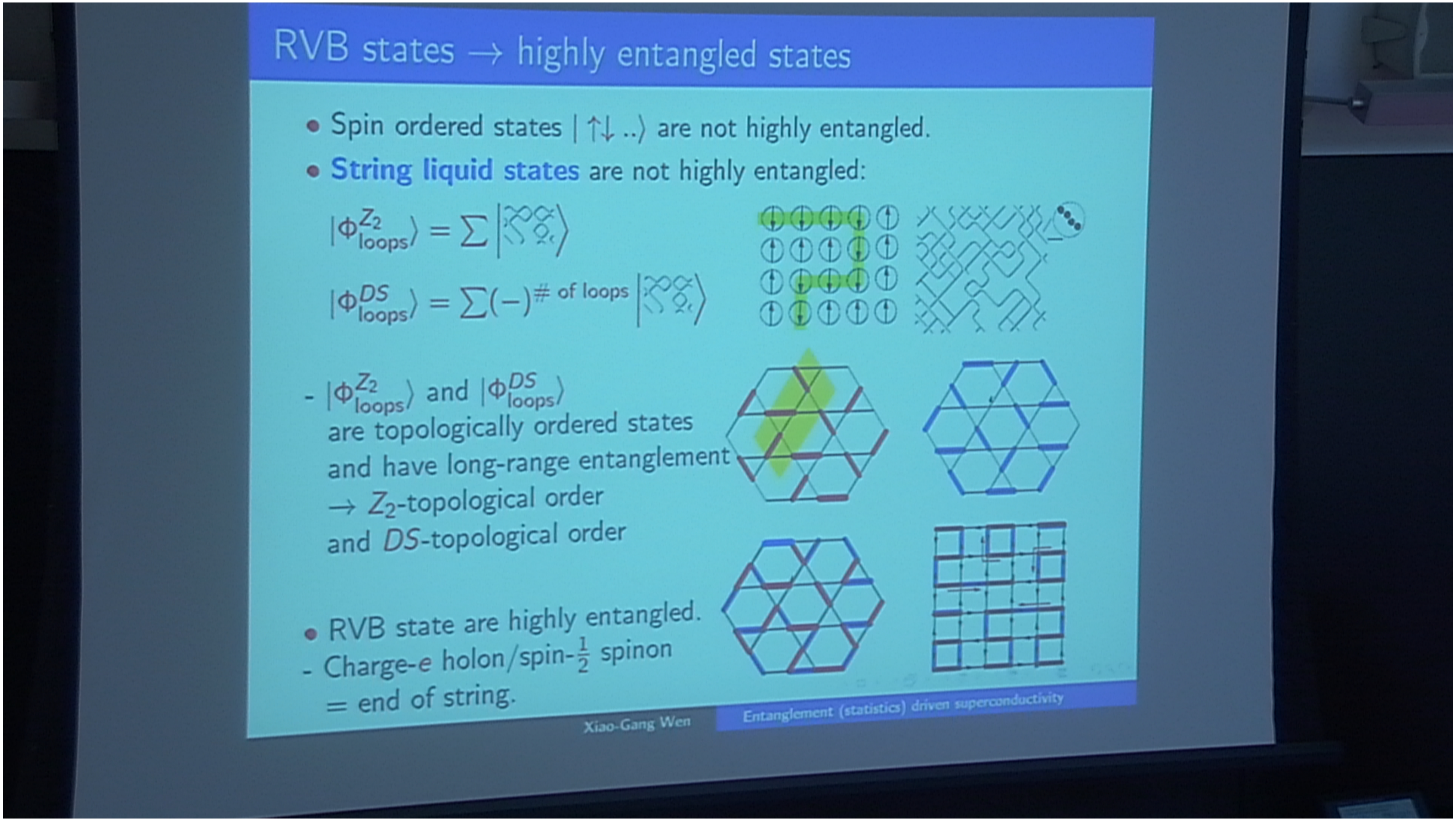
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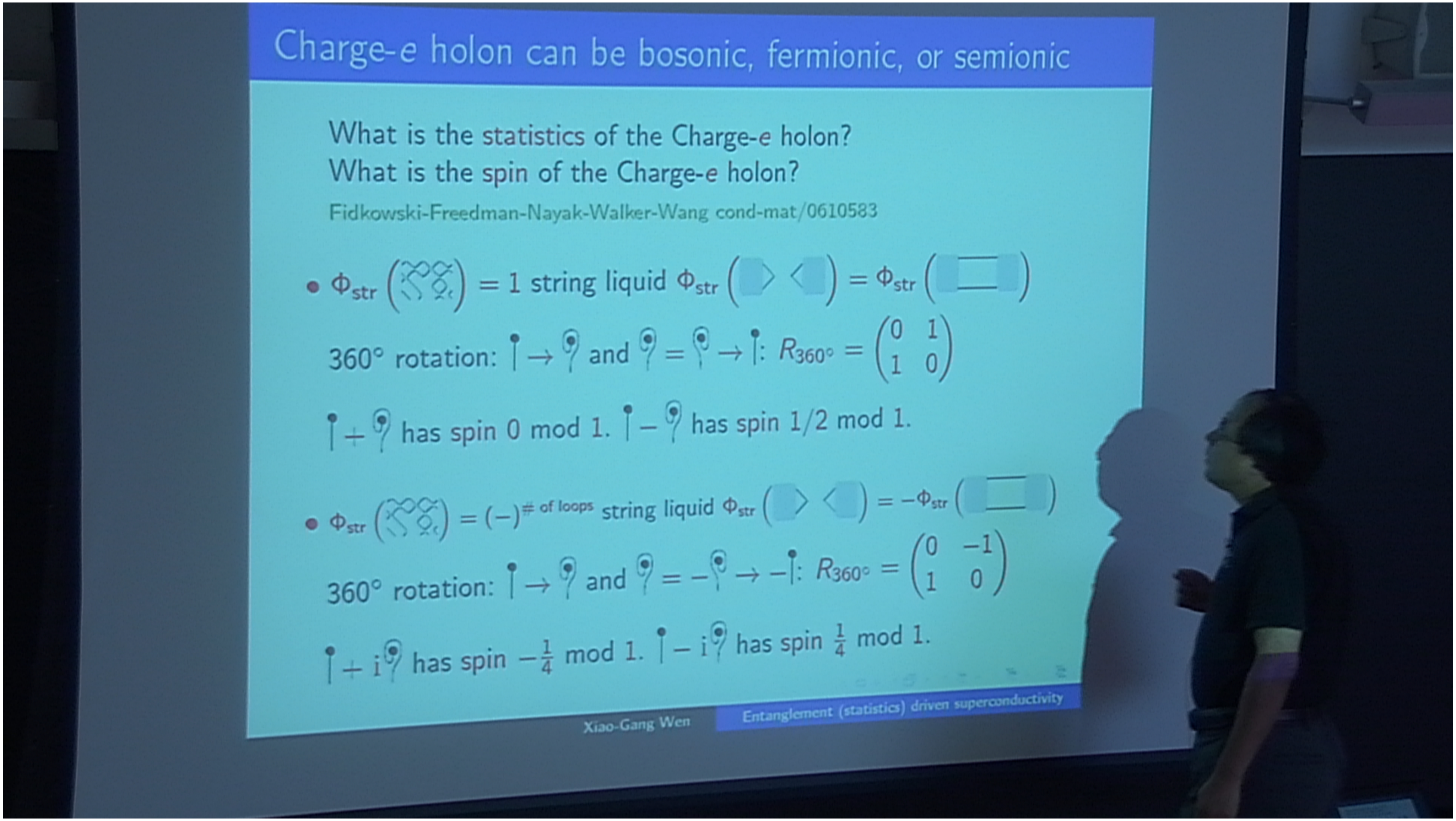
Charge-e holon can be bosonic, fermionic, or semionic
What is the statistics of the Charge-e holon?
What is the spin of the Charge-e holon?
Fidkowski-Freedman-Nayak-Walker-Wang cond-mat/0610583
Φstr = 1 string liquid Φstr = Φstr
360° rotation: and : R360° = (0 1; 1 0)
has spin 0 mod 1. has spin 1/2 mod 1.
Φstr = (−)# of loops string liquid Φstr = −Φstr
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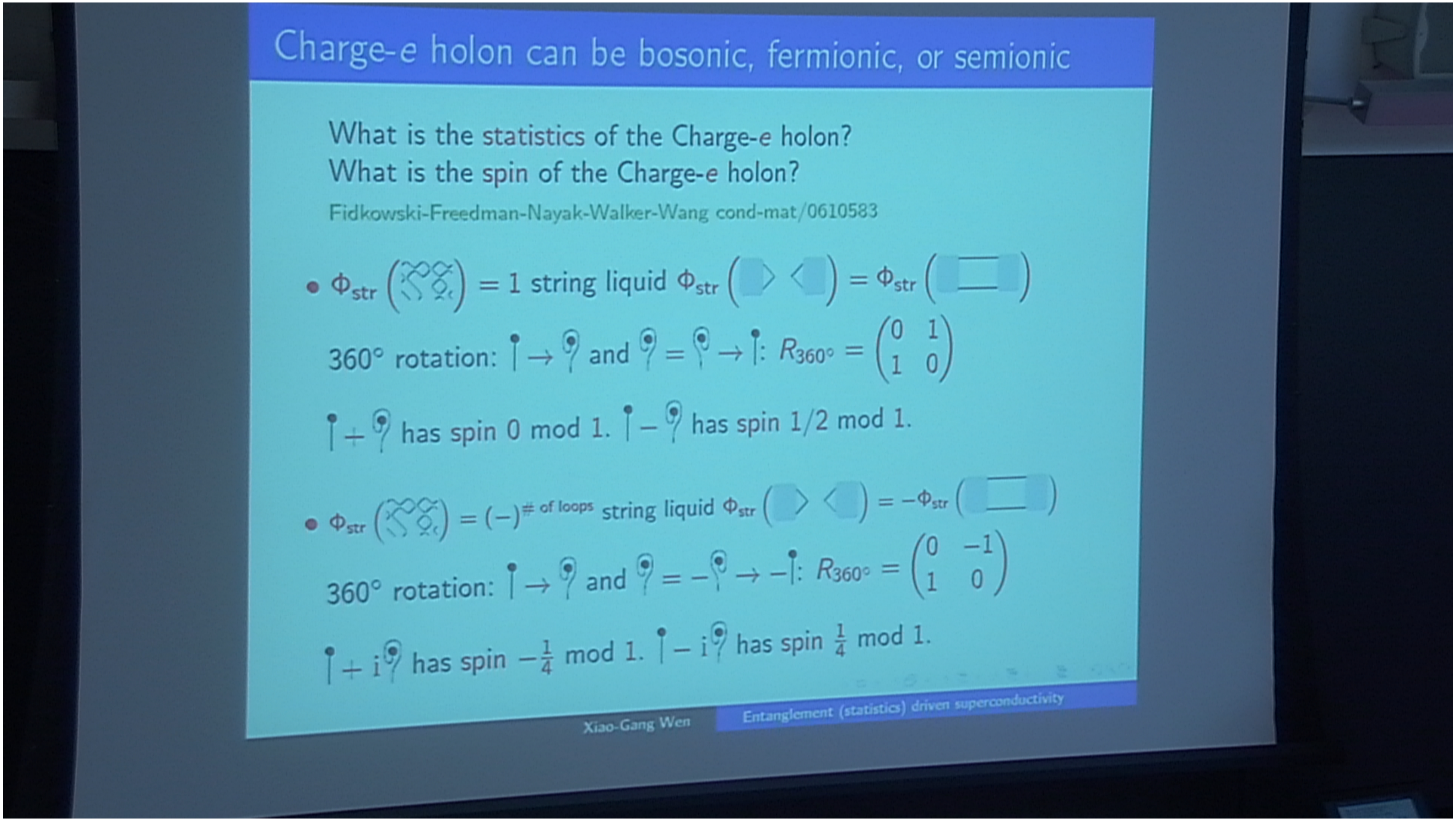
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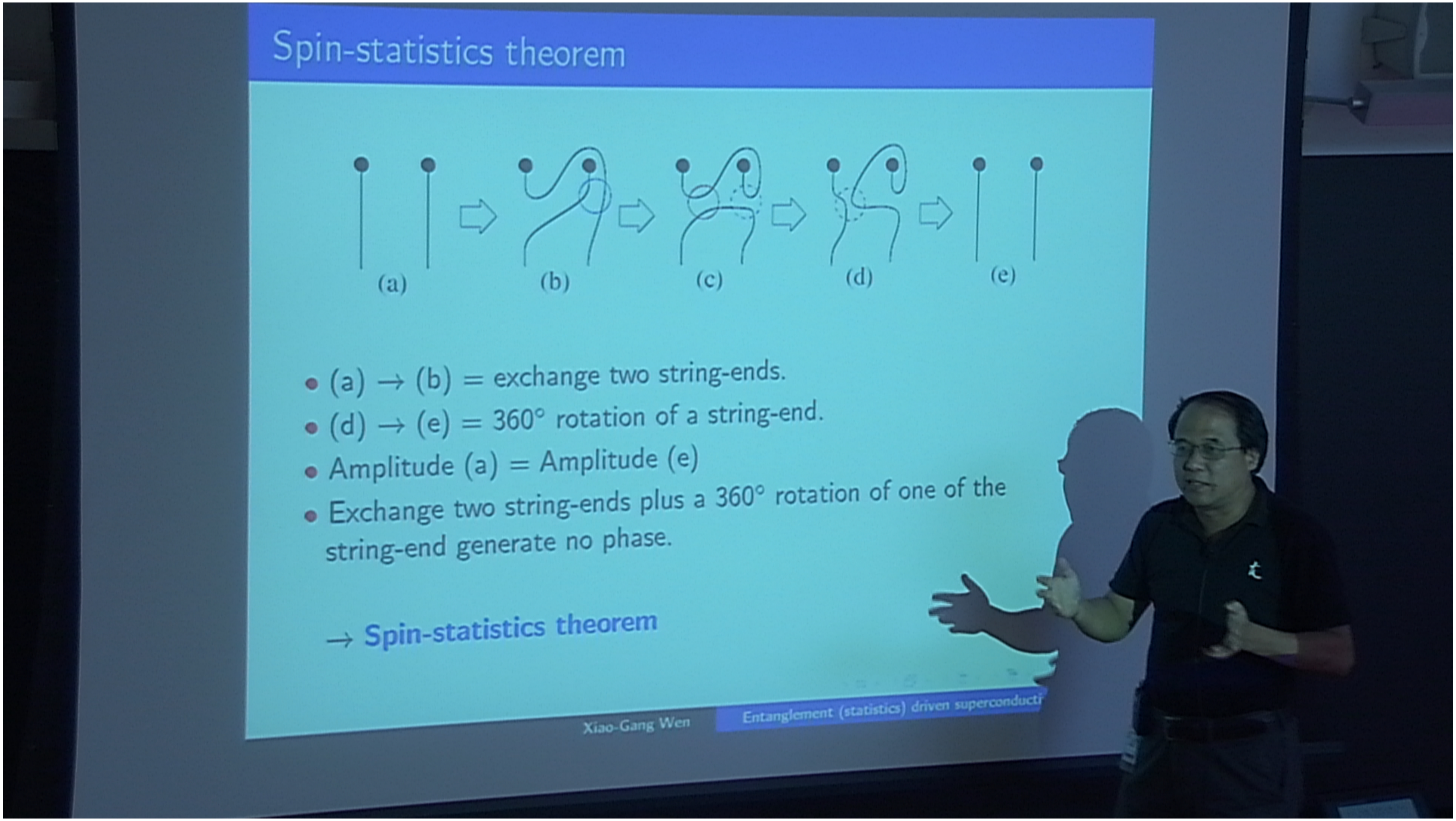
Spin-statistics theorem
(a)
(b)
(c)
(d)
(e)
• (a) → (b) = exchange two string-ends.
• (d) → (e) = 360° rotation of a string-end.
• Amplitude (a) = Amplitude (e)
• Exchange two string-ends plus a 360° rotation of one of the string-end generate no phase.
→ Spin-statistics theorem
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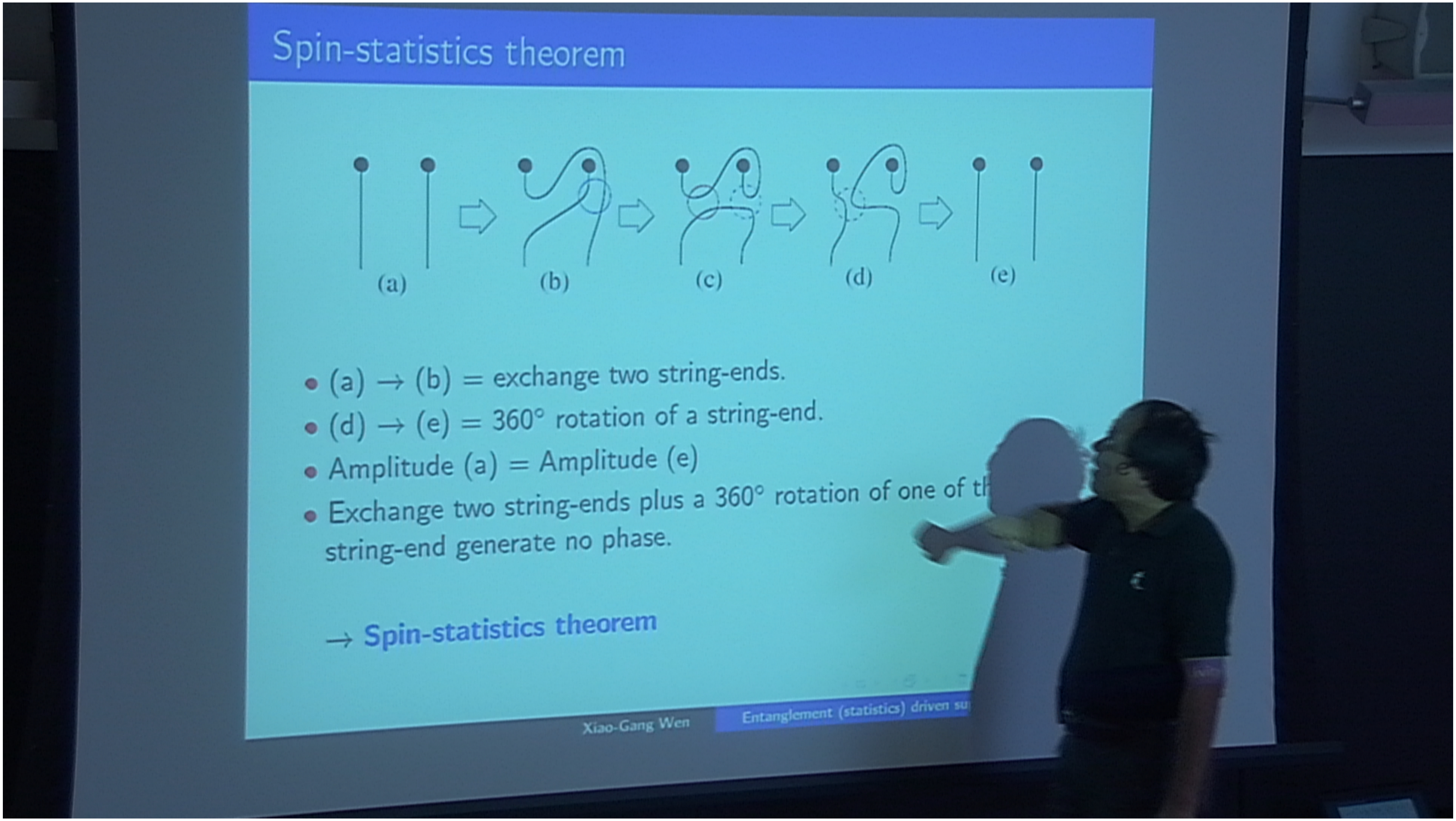
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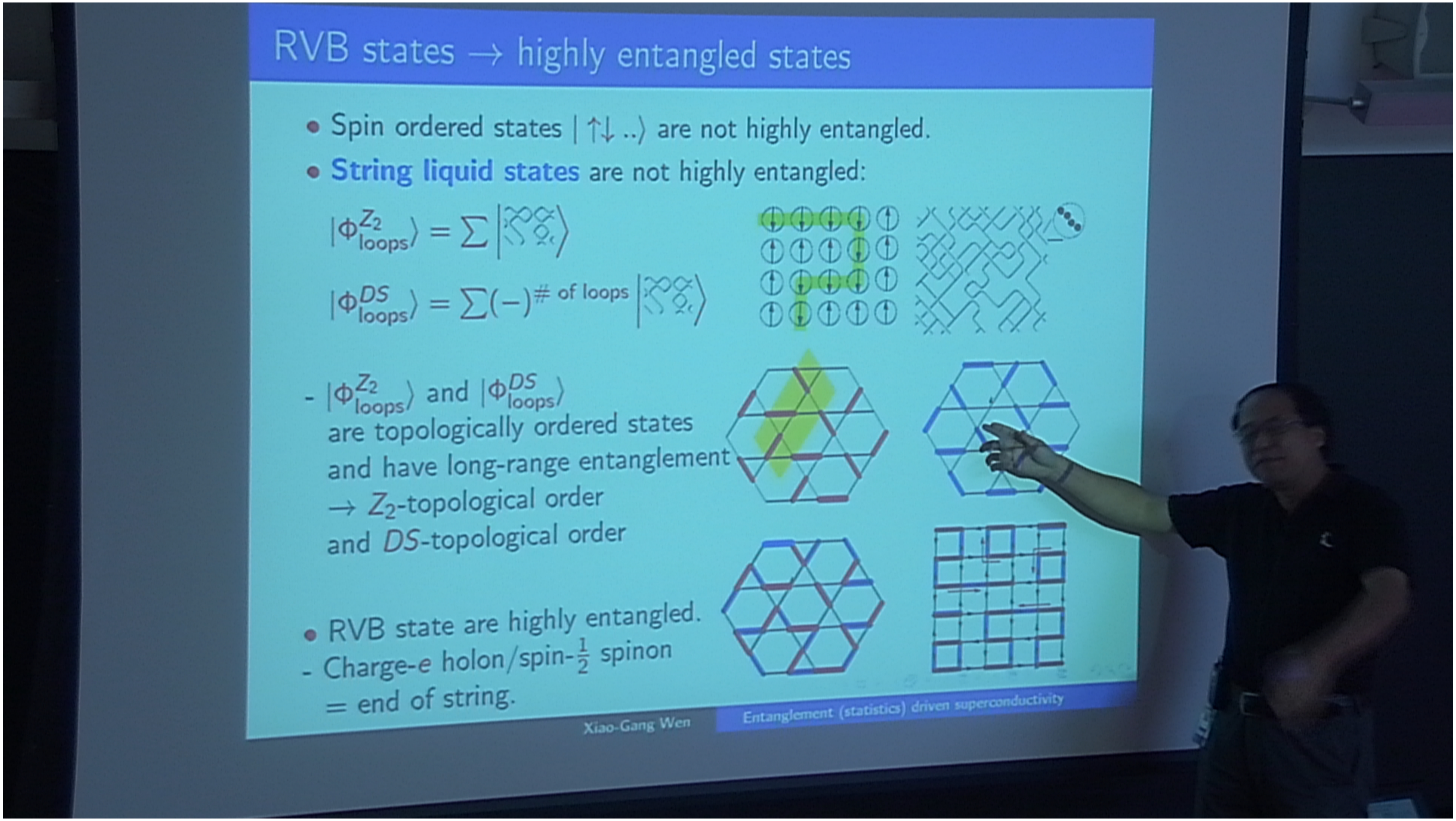
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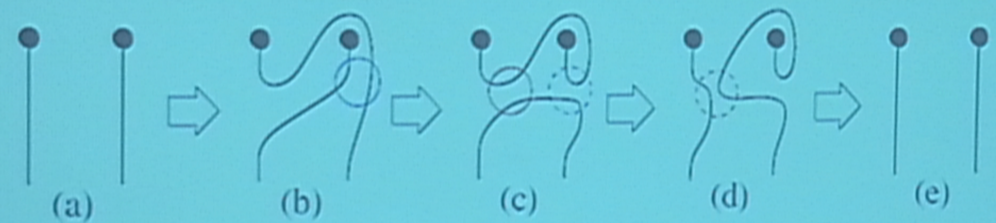
Spin-statistics theorem



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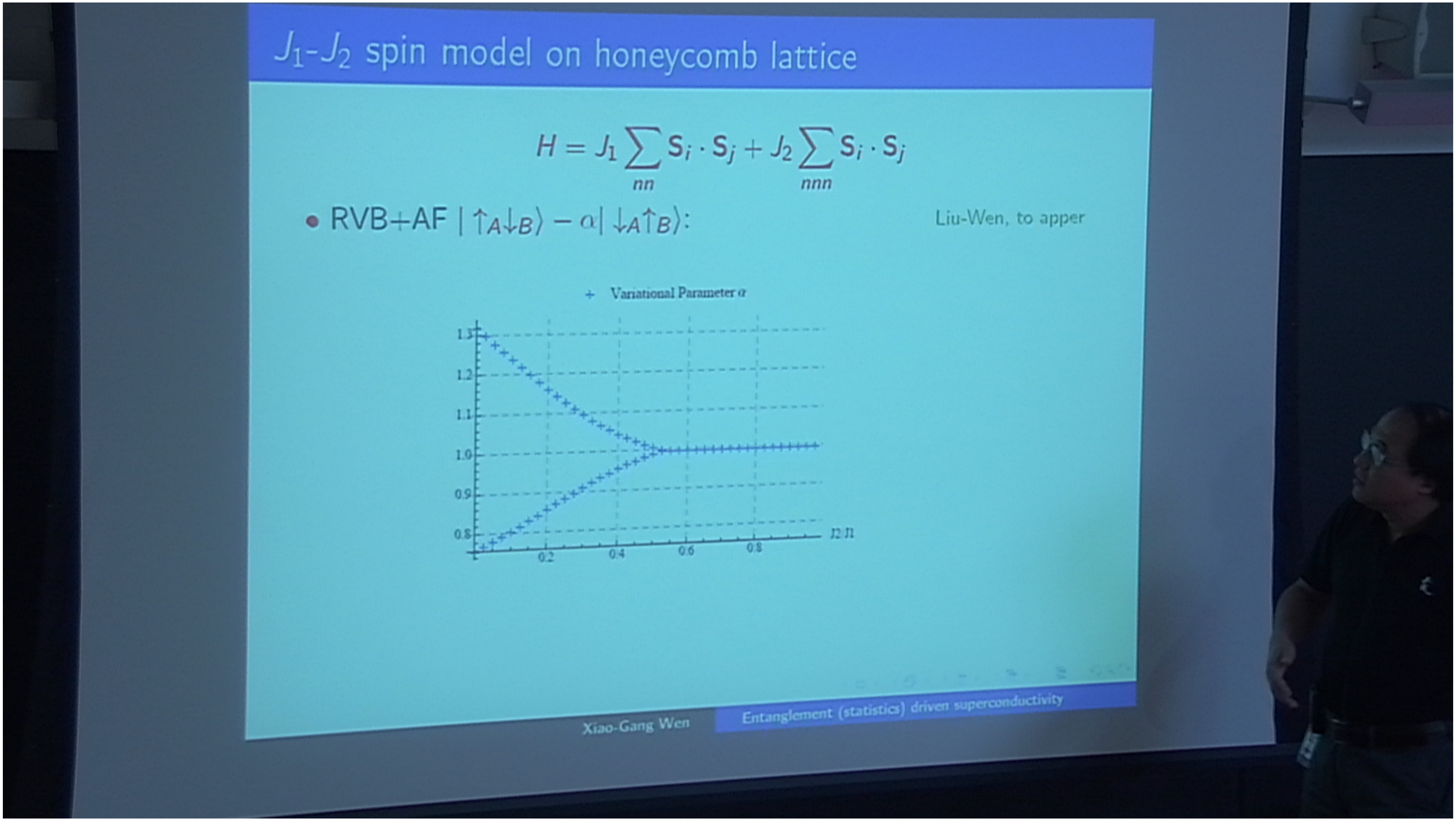
→ **Spin-statistics theorem**


J_1-J_2 spin model on honeycomb lattice
$H = J_1 \sum_{nn} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{nnn} \mathbf{S}_i \cdot \mathbf{S}_j$
• RVB+AF $|\uparrow_A \downarrow_B\rangle - \alpha |\downarrow_A \uparrow_B\rangle$:
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Variational Parameter α
1.3
1.2
1.1
1.0
0.9
0.8
0.2
0.4
0.6
0.8
J2/J1
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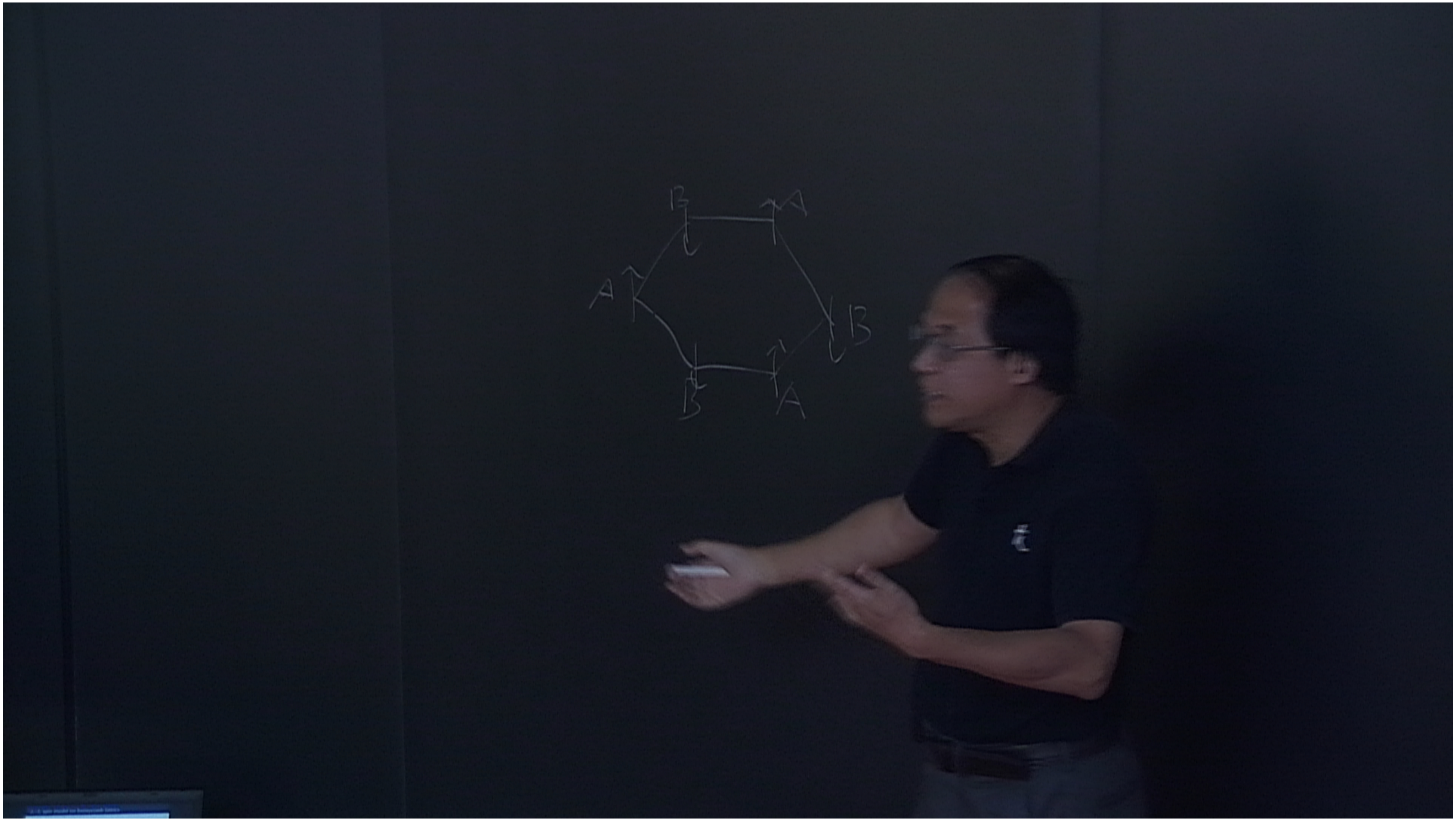
B
A
A
B
B
A

t_1-t_2-J_1-J_2 model on honeycomb lattice

- Slave-particle: Unphysical Hilbert space generated by $f_\alpha|_{\alpha=\uparrow,\downarrow}, b$
  Physical states: $|\downarrow\rangle = f_\downarrow^\dagger|0_f 0_b\rangle, |\uparrow\rangle = f_\uparrow^\dagger|0_f 0_b\rangle, |hole\rangle = b^\dagger|0_f 0_b\rangle$
  Physical operators: $c_{i\alpha} = f_{i\alpha} b_i^\dagger$, $\mathbf{S}_i = \frac{1}{2} f_i^\dagger \sigma f_i$

$$H_{tJ} = -\frac{J_1}{2}\sum_{nn} f_{i\alpha}^\dagger f_{i\alpha} f_{j\beta}^\dagger f_{j\beta} - \frac{J_2}{2}\sum_{nnn} f_{i\alpha}^\dagger f_{i\alpha} f_{j\beta}^\dagger f_{j\beta}$$
$$- t_1 \sum_{nn} f_{i\alpha}^\dagger f_{j\alpha} b_i b_j^\dagger - t_2 \sum_{nnn} f_{i\alpha}^\dagger f_{j\alpha} b_i b_j^\dagger$$

- Mean-field theory approach

$$H_{\text{mean}} = \sum_{\langle ij\rangle} (-\chi_{ij}^* f_{i\alpha}^\dagger f_{j\alpha} + \Delta_{ij} f_{i\alpha}^\dagger f_{j\beta}^\dagger \epsilon_{\alpha\beta} + h.c.) + \sum_i f_i^\dagger [a_0 + (-)^i \delta\, \sigma^z] f_i$$
$$b = \sqrt{x}, \quad f_i^\dagger f_i = 1 - x, \quad \to \quad |\Phi_{\text{mean}}(\chi_{ij}, \Delta_{ij}, \delta)\rangle$$

- Minimize $\langle \Phi_{\text{mean}}(\chi_{ij}, \Delta_{ij}, \delta)|H_{tJ}|\Phi_{\text{mean}}(\chi_{ij}, \Delta_{ij}, \delta)\rangle$ to get $\chi_{ij}, \Delta_{ij}, \delta \to$ Mean-field phase diagram.

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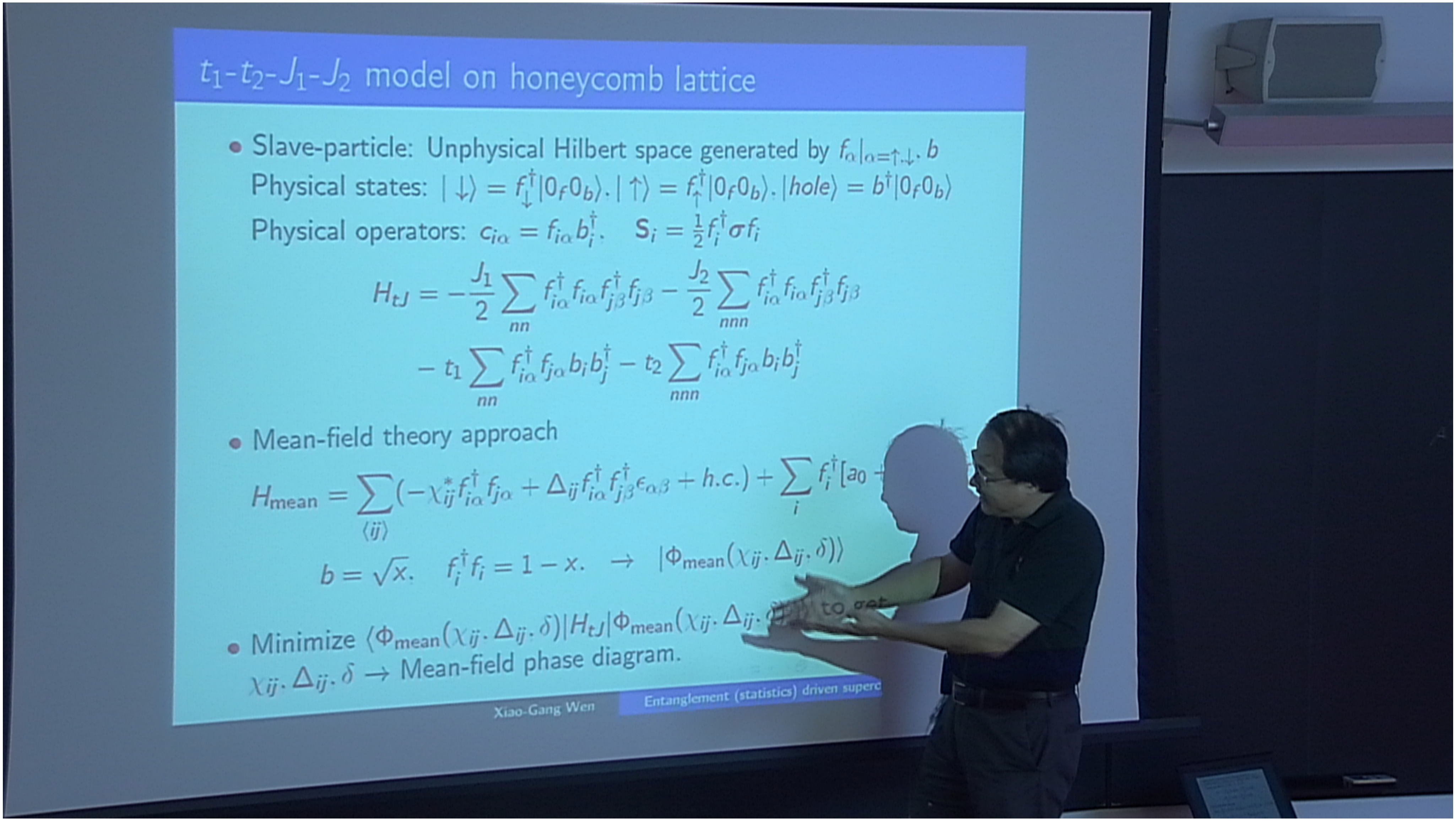
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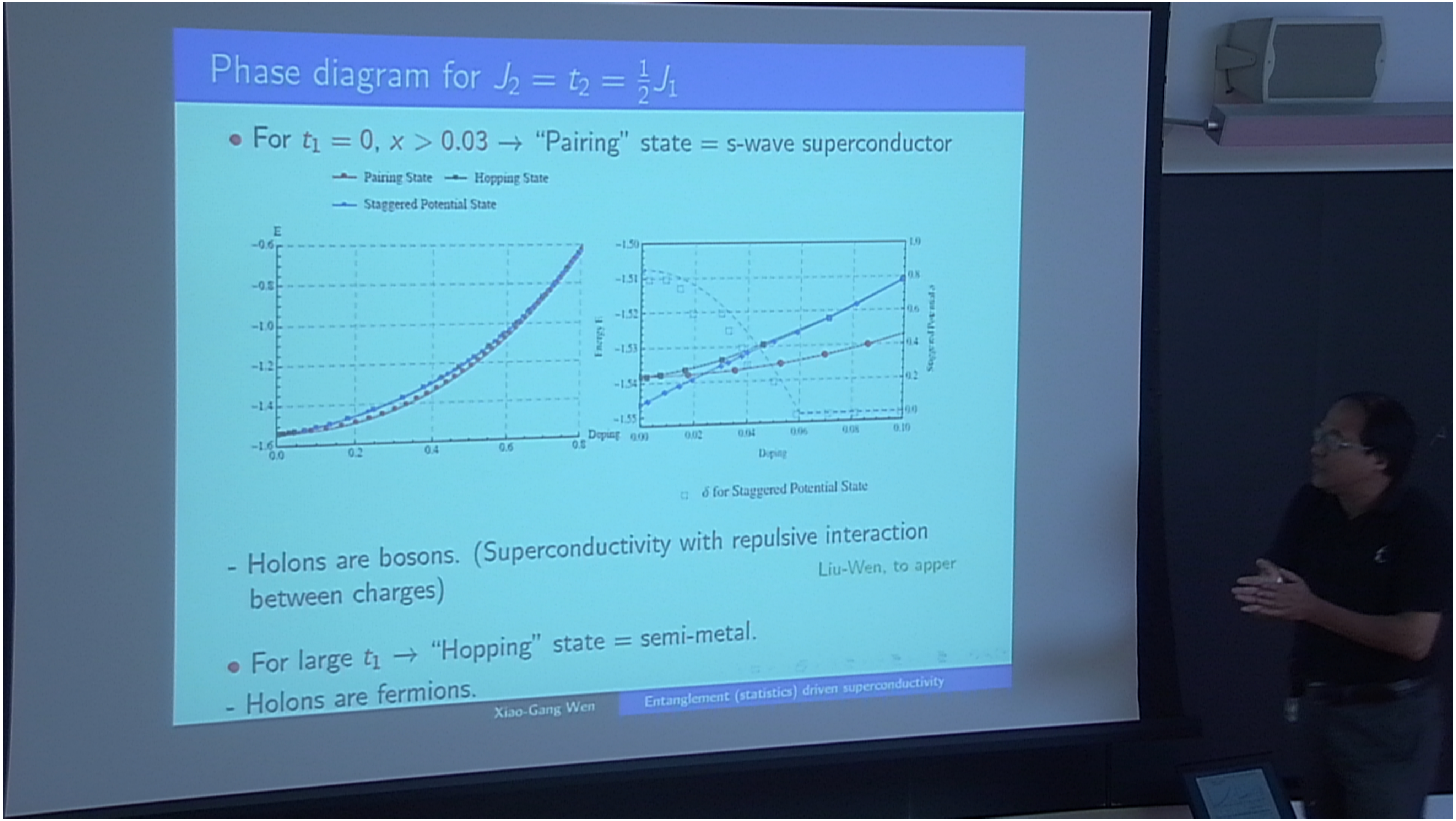
Phase diagram for $J_2 = t_2 = \frac{1}{2}J_1$
For $t_1 = 0$, $x > 0.03 \to$ "Pairing" state = s-wave superconductor
Pairing State
Hopping State
Staggered Potential State
E
Doping
Energy E
Staggered Potentials
δ for Staggered Potential State
- Holons are bosons. (Superconductivity with repulsive interaction between charges)
Liu-Wen, to apper
For large $t_1 \to$ "Hopping" state = semi-metal.
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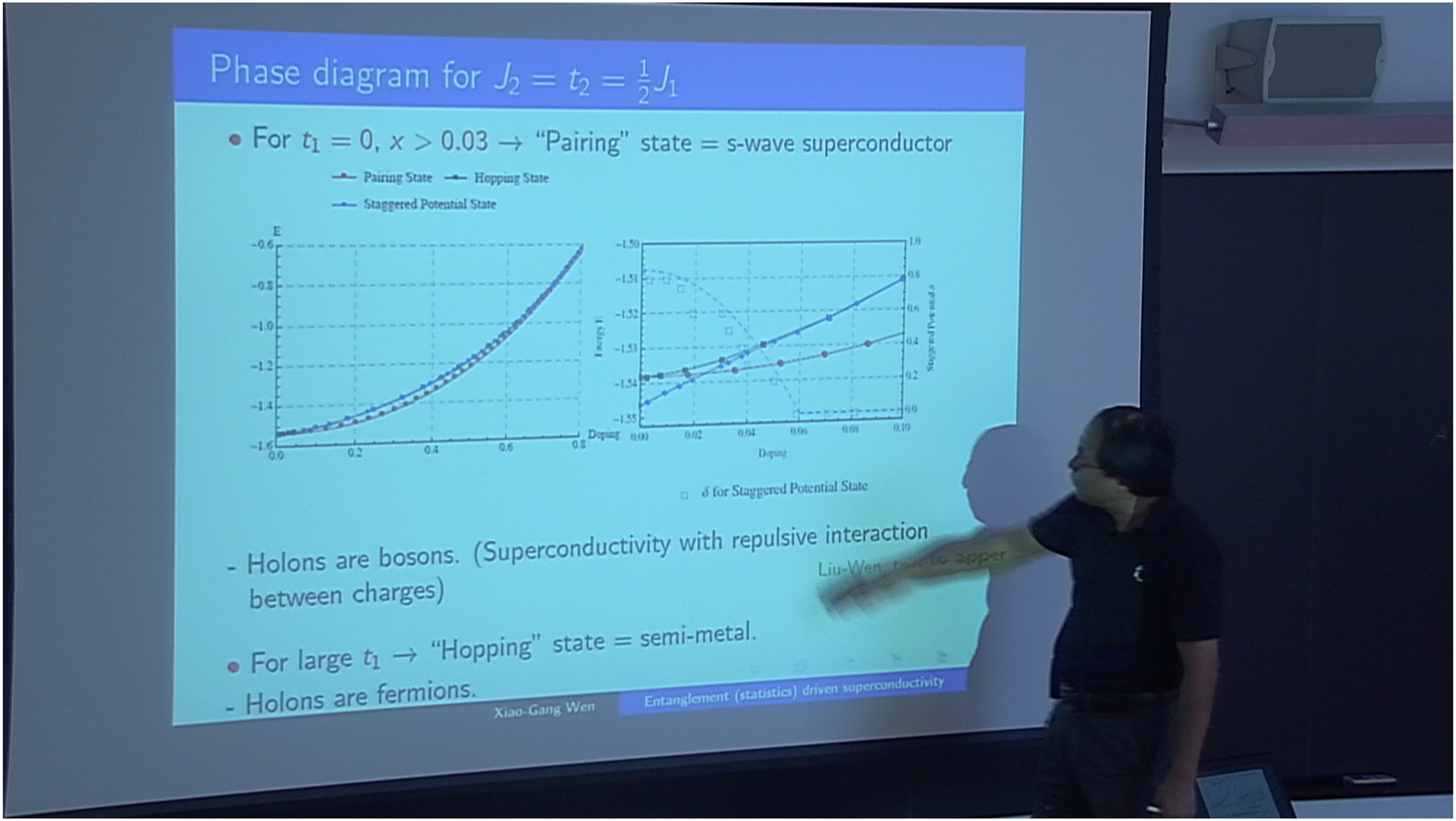

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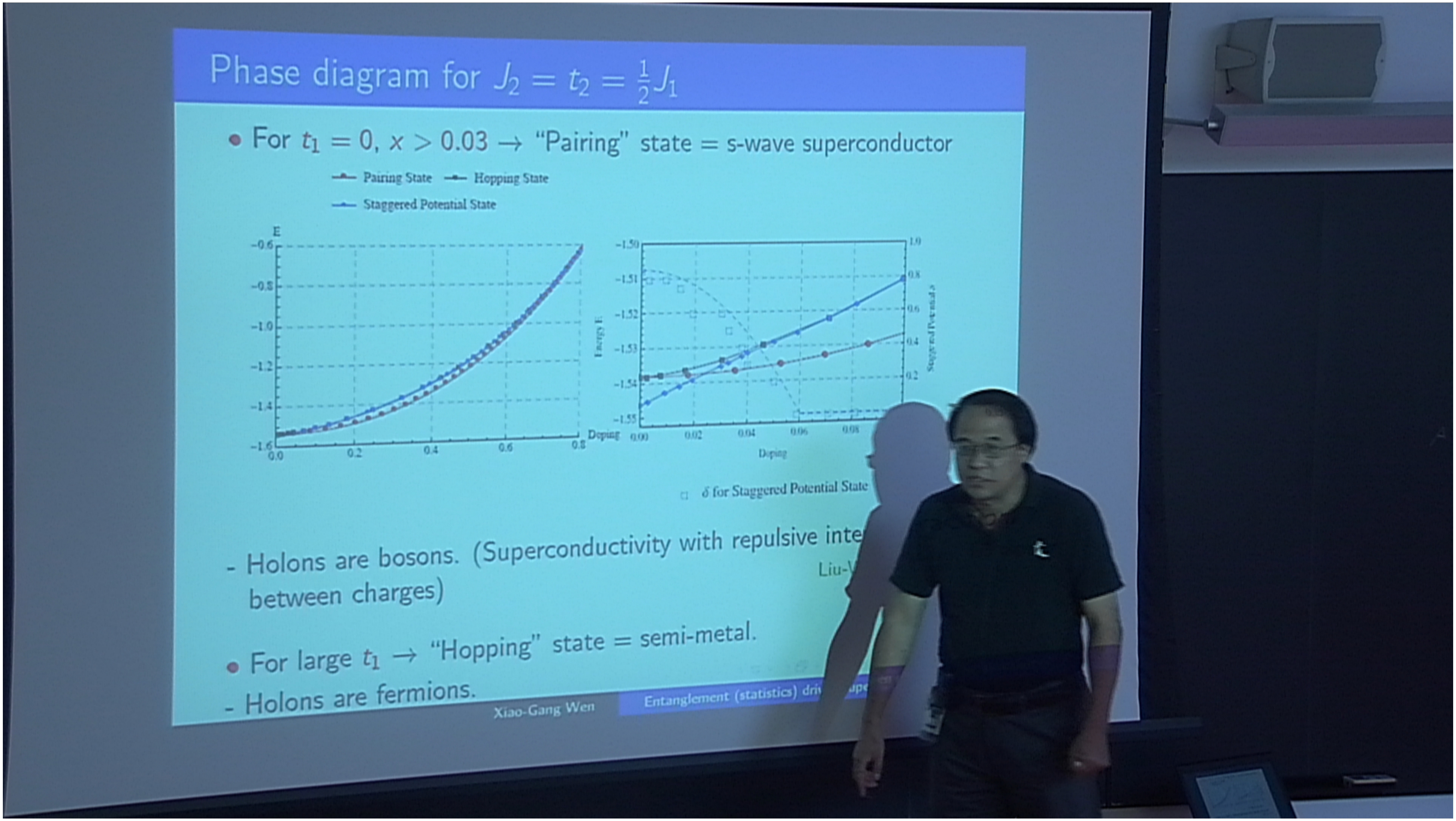
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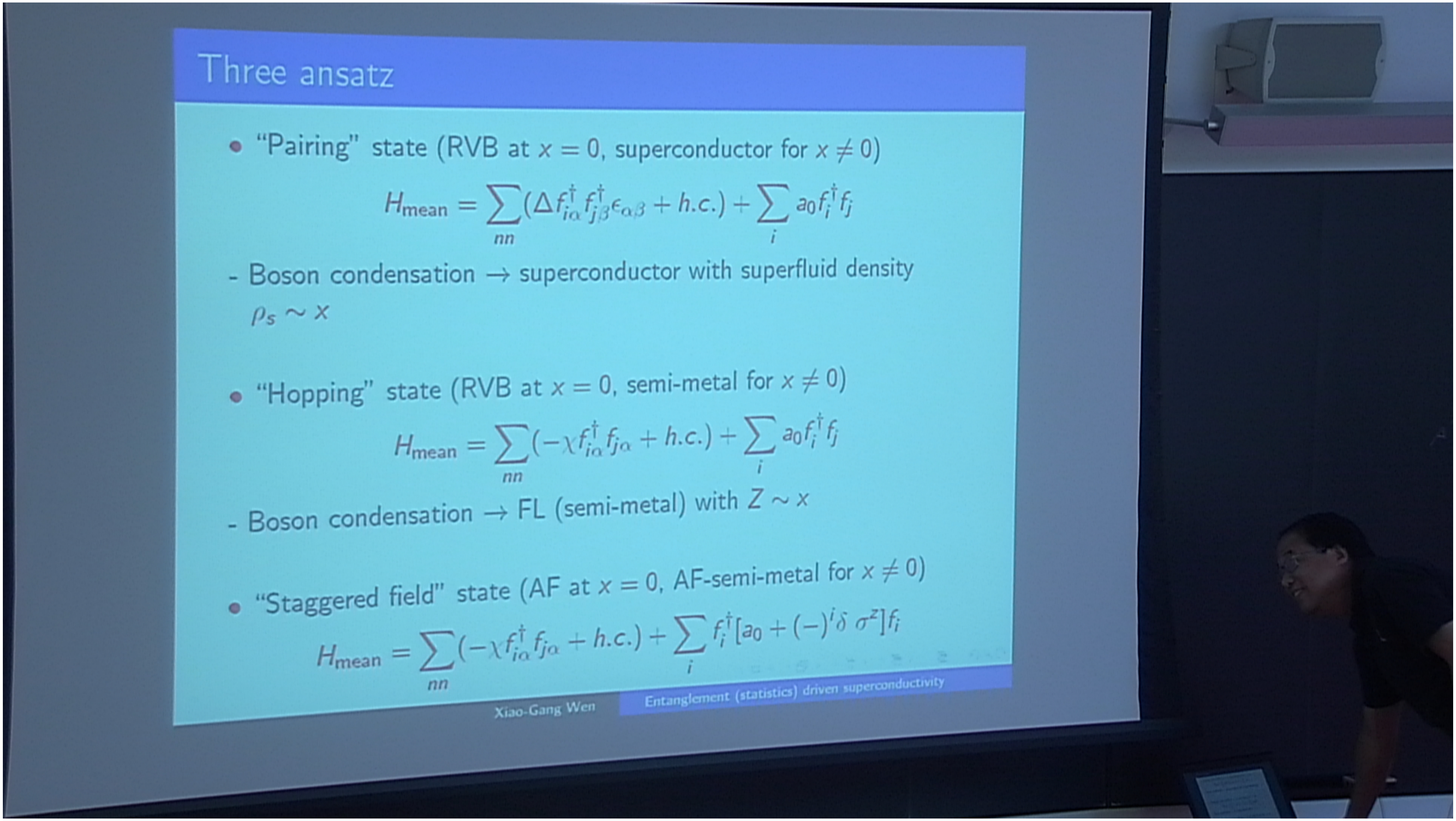

Three ansatz
• "Pairing" state (RVB at $x = 0$, superconductor for $x \neq 0$)
$H_{\text{mean}} = \sum_{nn} (\Delta f^\dagger_{i\alpha} f^\dagger_{j\beta} \epsilon_{\alpha\beta} + h.c.) + \sum_i a_0 f^\dagger_i f_i$
- Boson condensation → superconductor with superfluid density $\rho_s \sim x$
• "Hopping" state (RVB at $x = 0$, semi-metal for $x \neq 0$)
$H_{\text{mean}} = \sum_{nn} (-\chi f^\dagger_{i\alpha} f_{j\alpha} + h.c.) + \sum_i a_0 f^\dagger_i f_i$
- Boson condensation → FL (semi-metal) with $Z \sim x$
• "Staggered field" state (AF at $x = 0$, AF-semi-metal for $x \neq 0$)
$H_{\text{mean}} = \sum_{nn} (-\chi f^\dagger_{i\alpha} f_{j\alpha} + h.c.) + \sum_i f^\dagger_i [a_0 + (-)^i \delta\, \sigma^z] f_i$
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$$- t_1\sum_{nn} f_{i\alpha}^\dagger f_{j\alpha} b_i b_j^\dagger - t_2\sum_{nnn} f_{i\alpha}^\dagger f_{j\alpha} b_i b_j^\dagger$$

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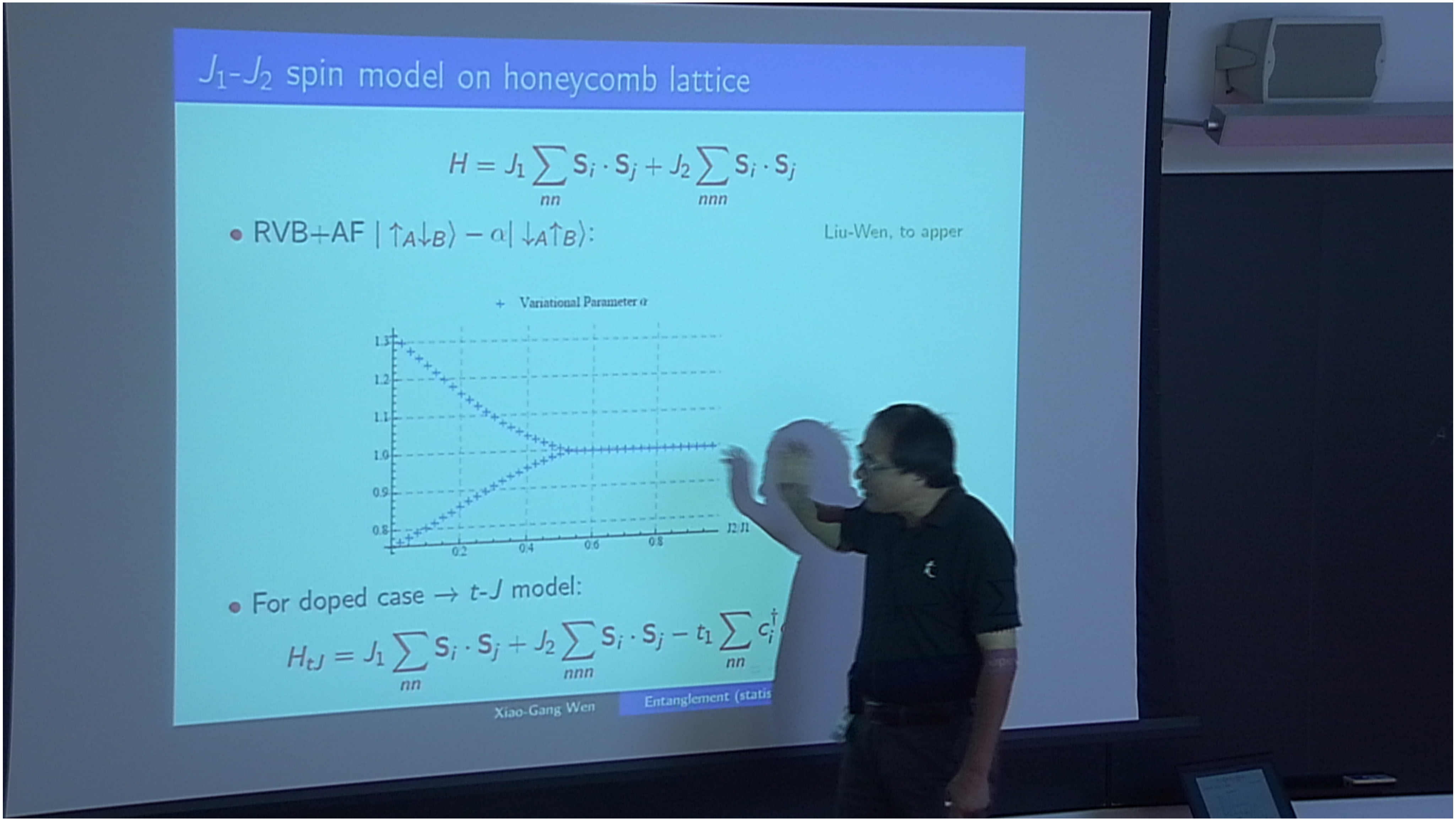
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+ Variational Parameter α
1.3
1.2
1.1
1.0
0.9
0.8
0.2
0.4
0.6
0.8
J2/J1
• For doped case → t-J model:
$H_{tJ} = J_1 \sum_{nn} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{nnn} \mathbf{S}_i \cdot \mathbf{S}_j - t_1 \sum_{nn} c_i^\dagger$
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Entanglement (statis

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- Boson condensation $\to$ superconductor with superfluid density $\rho_s \sim x$

- "Hopping" state (RVB at $x=0$, semi-metal for $x \neq 0$)

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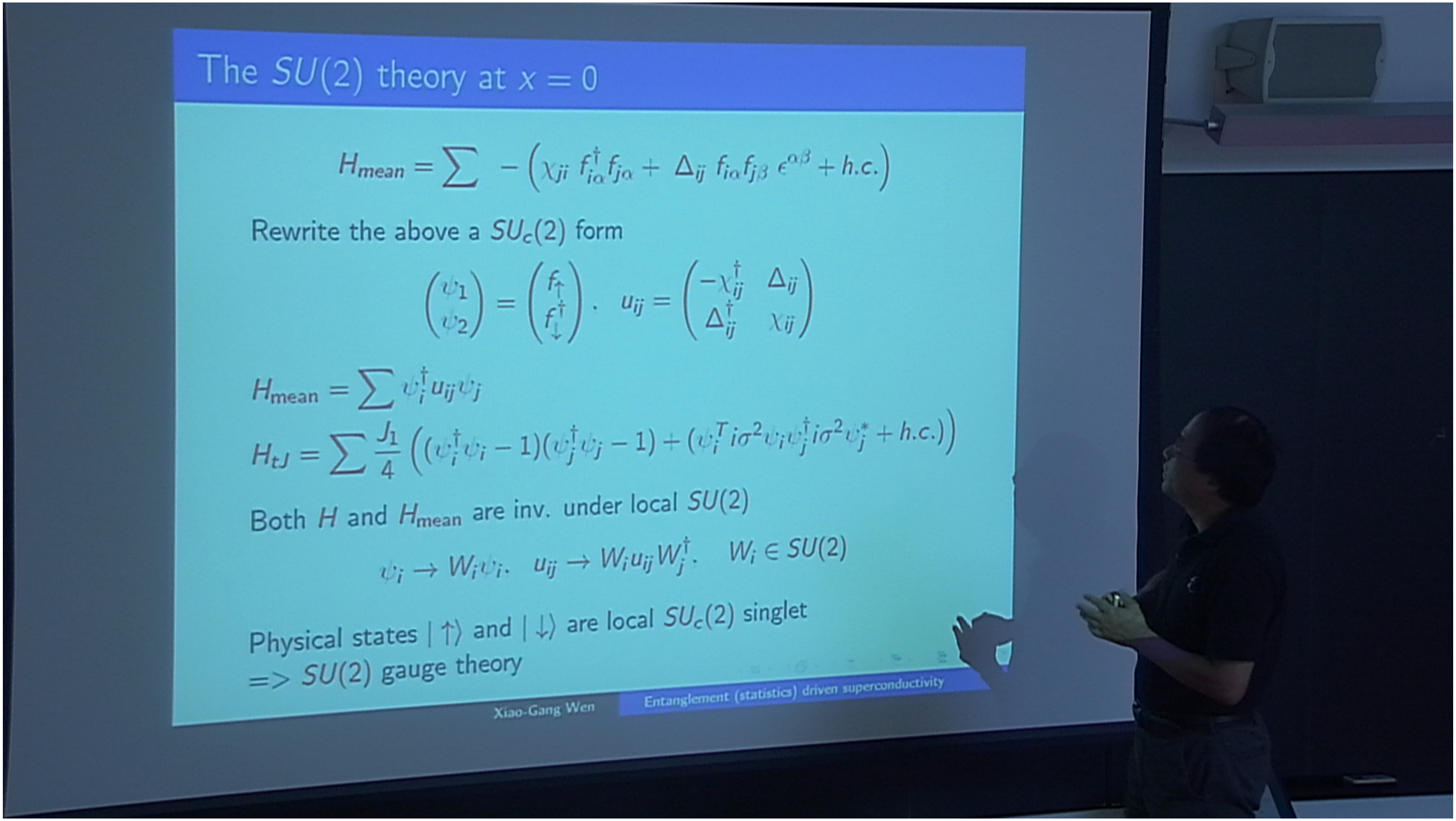
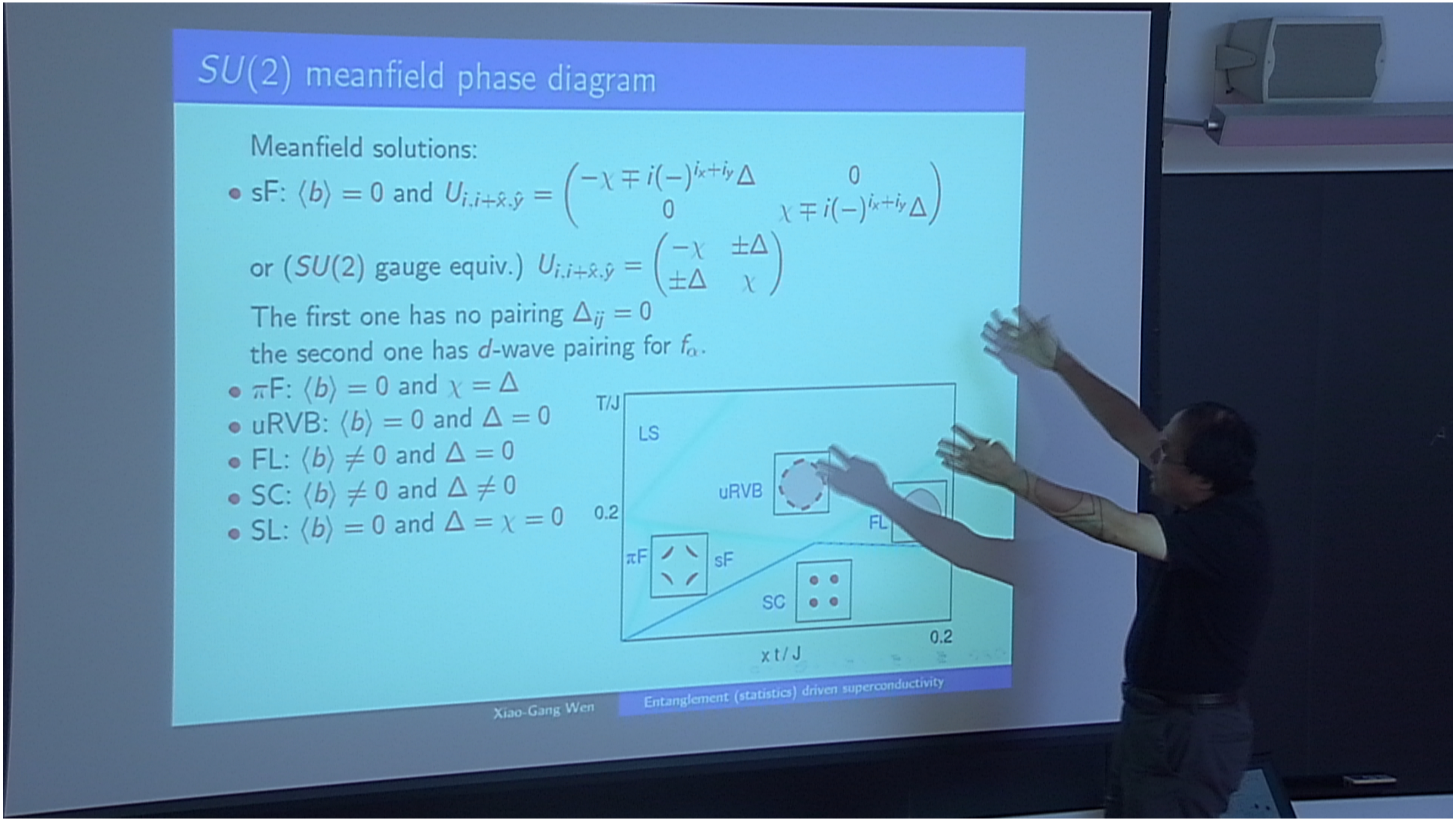
- "Staggered field" state (AF at $x=0$, AF-semi-metal for $x \neq 0$)

$$H_{\text{mean}} = \sum_{nn} (-\chi f^\dagger_{i\alpha} f_{j\alpha} + h.c.) + \sum_i f^\dagger_i [a_0 + (-)^i \delta\, \sigma^z] f_i$$


The $SU(2)$ theory at $x = 0$
$H_{mean} = \sum -\left(\chi_{ji}\, f_{i\alpha}^\dagger f_{j\alpha} + \Delta_{ij}\, f_{i\alpha} f_{j\beta}\, \epsilon^{\alpha\beta} + h.c.\right)$
Rewrite the above a $SU_c(2)$ form
$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix} f_\uparrow \\ f_\downarrow^\dagger\end{pmatrix}, \quad u_{ij} = \begin{pmatrix} -\chi_{ij}^\dagger & \Delta_{ij} \\ \Delta_{ij}^\dagger & \chi_{ij}\end{pmatrix}$
$H_{mean} = \sum \psi_i^\dagger u_{ij} \psi_j$
$H_{tJ} = \sum \frac{J_1}{4}\left((\psi_i^\dagger \psi_i - 1)(\psi_j^\dagger \psi_j - 1) + (\psi_i^T i\sigma^2 \psi_i \psi_j^\dagger i\sigma^2 \psi_j^* + h.c.)\right)$
Both H and H_{mean} are inv. under local $SU(2)$
$\psi_i \to W_i \psi_i, \quad u_{ij} \to W_i u_{ij} W_j^\dagger, \quad W_i \in SU(2)$
Physical states $|\uparrow\rangle$ and $|\downarrow\rangle$ are local $SU_c(2)$ singlet
=> $SU(2)$ gauge theory
Xiao-Gang Wen
Entanglement (statistics) driven superconductivity


$SU(2)$ meanfield phase diagram
Meanfield solutions:
• sF: $\langle b \rangle = 0$ and $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi \mp i(-)^{i_x+i_y}\Delta & 0 \\ 0 & \chi \mp i(-)^{i_x+i_y}\Delta \end{pmatrix}$
or ($SU(2)$ gauge equiv.) $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi & \pm\Delta \\ \pm\Delta & \chi \end{pmatrix}$
The first one has no pairing $\Delta_{ij} = 0$
the second one has d-wave pairing for f_α.
• πF: $\langle b \rangle = 0$ and $\chi = \Delta$
• uRVB: $\langle b \rangle = 0$ and $\Delta = 0$
• FL: $\langle b \rangle \neq 0$ and $\Delta = 0$
• SC: $\langle b \rangle \neq 0$ and $\Delta \neq 0$
• SL: $\langle b \rangle = 0$ and $\Delta = \chi = 0$
T/J
LS
uRVB
0.2
FL
πF
sF
SC
0.2
x t / J
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The $SU(2)$ theory at $x \neq 0$ — Wen-Lee cond-mat/9506065

Unphysical Hilbert space generated by $SU_c(2)$ doublets $\psi_a, b_a|_{a=1,2}$

- Physical states are local $SU_c(2)$ singlet.
  Only three physical states per site:

$$|\downarrow\rangle = |0\rangle, \quad |\uparrow\rangle = \psi_1^\dagger \psi_2^\dagger |0\rangle, \quad |hole\rangle = \frac{1}{\sqrt{2}} b^\dagger i\sigma^2 \psi^* |0\rangle$$

- Physical operators

$$\mathbf{S}_i = \frac{1}{2} f_i^\dagger \sigma f_i, \quad c_{\uparrow i} = \frac{1}{\sqrt{2}} b_i^\dagger \psi_i, \quad c_{\downarrow i} = \frac{1}{\sqrt{2}} b_i^\dagger i\sigma^2 \psi_i^*,$$

and the t-J Hamiltonian

$$H_{tJ} = \sum \frac{J_1}{4} \Big((\psi_i^\dagger \psi_i - 1)(\psi_j^\dagger \psi_j - 1) + (\psi_i^T i\sigma^2 \psi_i \psi_j^\dagger i\sigma^2 \psi_j^* + h.c.) \Big)$$
$$- t_1 \sum (c_{\alpha i}^\dagger c_{\alpha j} + h.c.)$$

are inv. under local $SU_c(2)$ (commute with $\hat{T}_i^l = \psi_i^\dagger \tau^l \psi_i + b_i^\dagger \tau^l b_i$).

$SU_c(2)$ meanfield Hamiltonian for spin liquids:

$$H_{\text{mean}} = -\sum [\psi_i^\dagger u_{ij} \psi_j + (b_i^\dagger v_{ij} b_j + \text{h.c.})]$$

$\to |\Psi_{\text{mean}}(u_{ij}, v_{ij})\rangle \to$ phase diagram.

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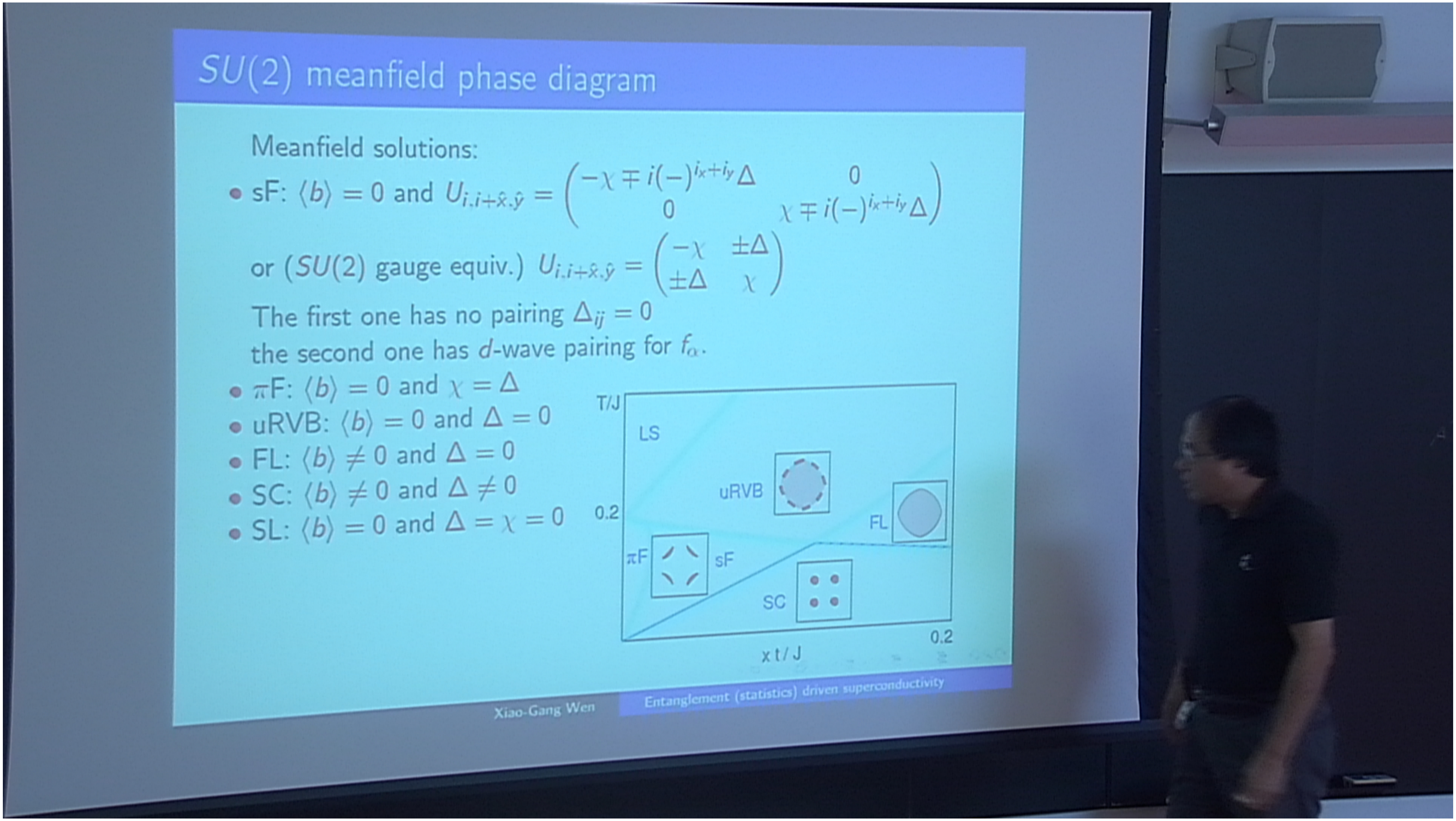
$SU(2)$ meanfield phase diagram
Meanfield solutions:
• sF: $\langle b \rangle = 0$ and $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi \mp i(-)^{i_x+i_y}\Delta & 0 \\ 0 & \chi \mp i(-)^{i_x+i_y}\Delta \end{pmatrix}$
or ($SU(2)$ gauge equiv.) $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi & \pm\Delta \\ \pm\Delta & \chi \end{pmatrix}$
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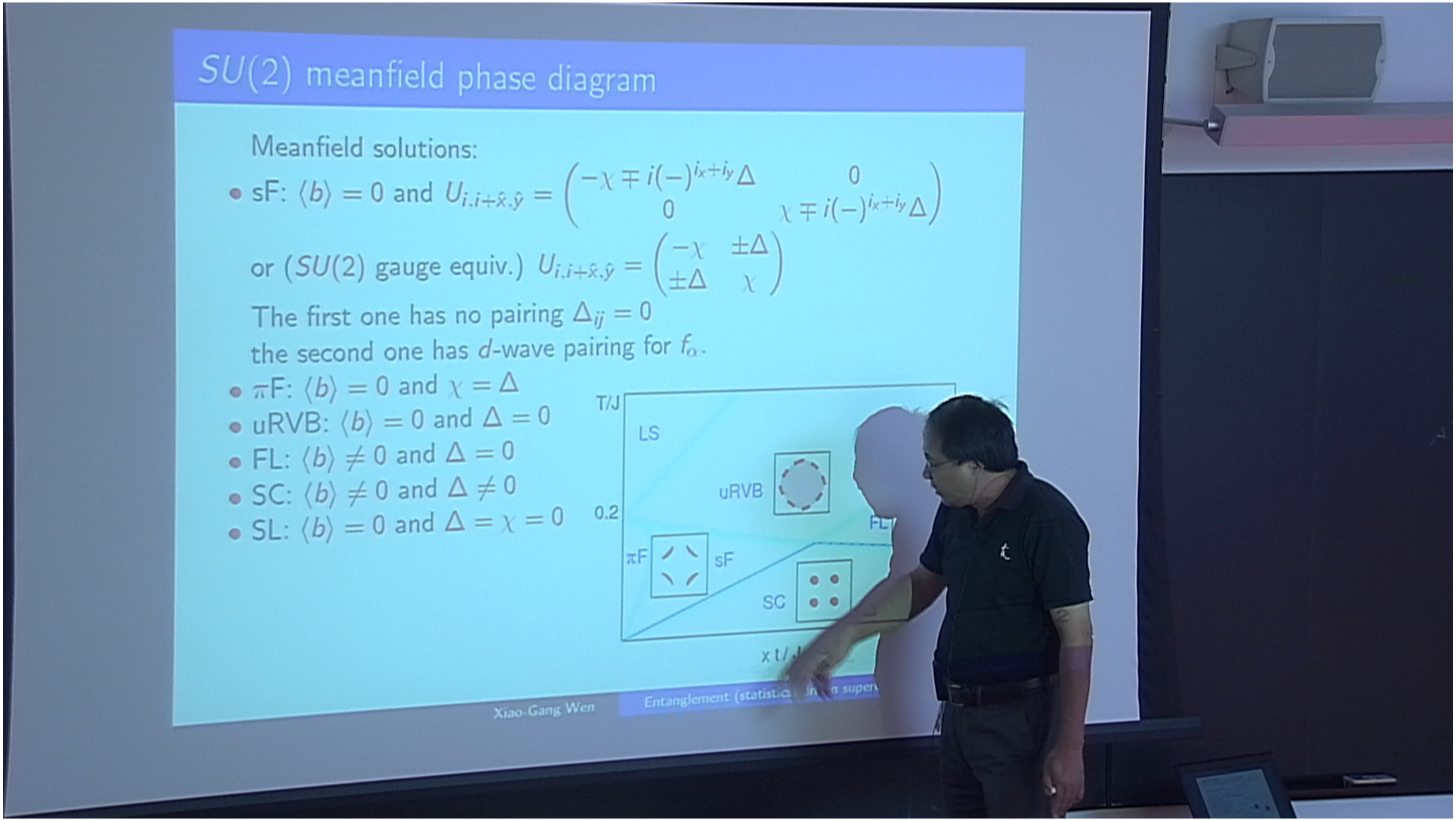
• FL: $\langle b \rangle \neq 0$ and $\Delta = 0$
• SC: $\langle b \rangle \neq 0$ and $\Delta \neq 0$
• SL: $\langle b \rangle = 0$ and $\Delta = \chi = 0$
T/J
LS
uRVB
0.2
FL
πF
sF
SC
0.2
x t / J
Xiao-Gang Wen
Entanglement (statistics) driven superconductivity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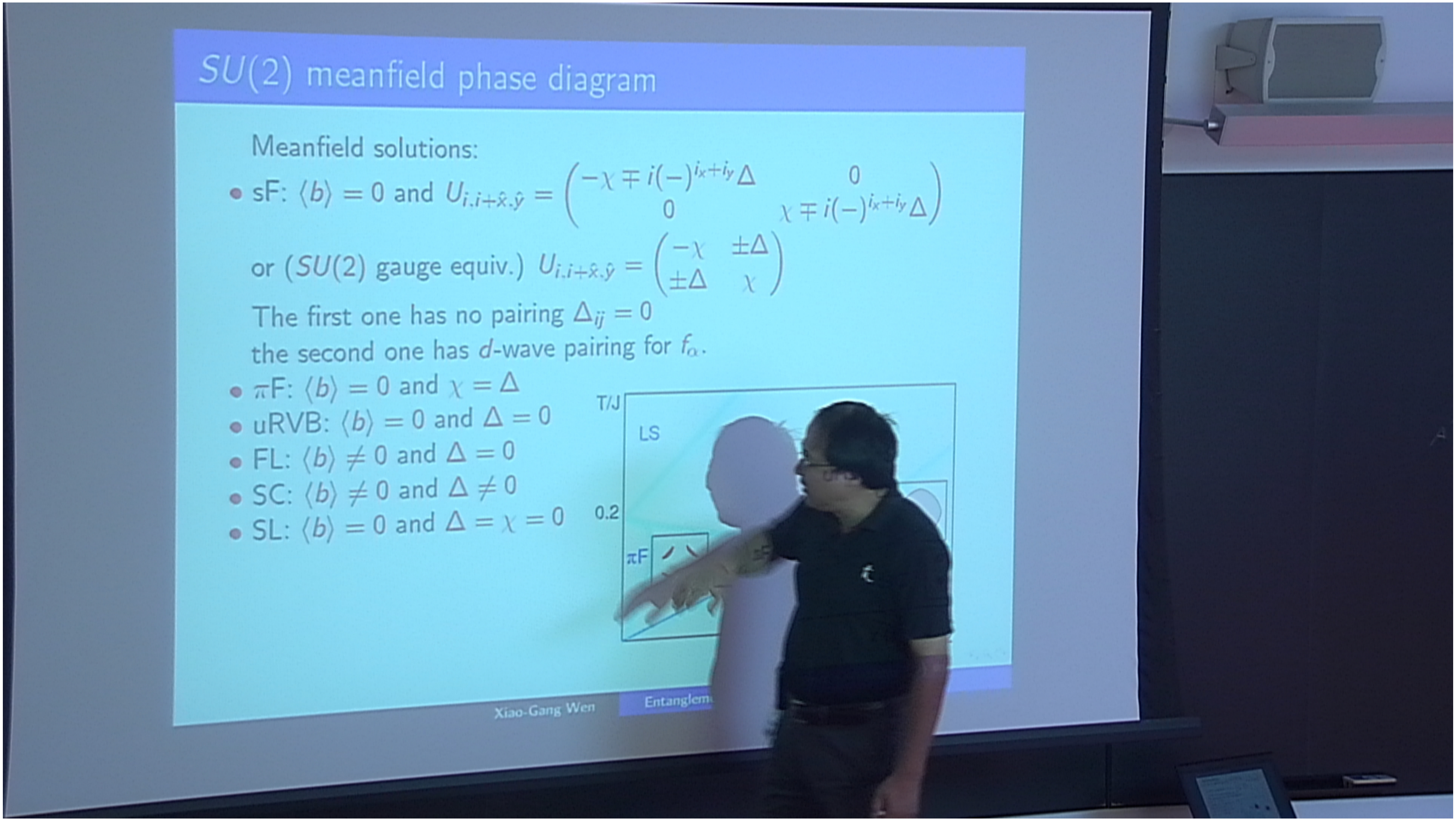
• FL: $\langle b\rangle \neq 0$ and $\Delta = 0$
• SC: $\langle b\rangle \neq 0$ and $\Delta \neq 0$
• SL: $\langle b\rangle = 0$ and $\Delta = \chi = 0$
T/J
LS
uRVB
0.2
πF
sF
SC
Xiao-Gang Wen


$SU(2)$ meanfield phase diagram
Meanfield solutions:
sF: $\langle b \rangle = 0$ and $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi \mp i(-)^{i_x+i_y}\Delta & 0 \\ 0 & \chi \mp i(-)^{i_x+i_y}\Delta \end{pmatrix}$
or ($SU(2)$ gauge equiv.) $U_{i,i+\hat{x},\hat{y}} = \begin{pmatrix} -\chi & \pm\Delta \\ \pm\Delta & \chi \end{pmatrix}$
The first one has no pairing $\Delta_{ij} = 0$
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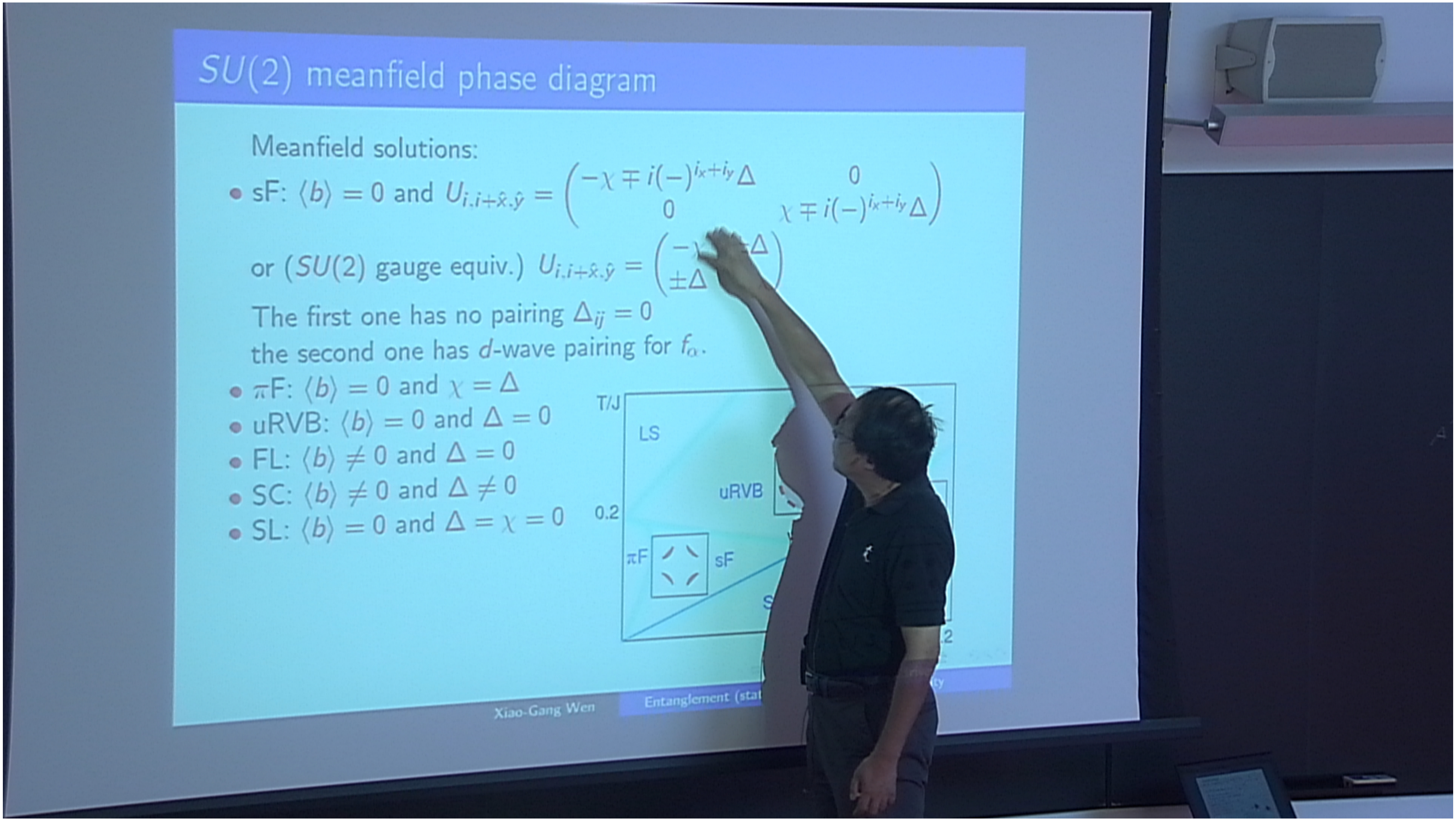
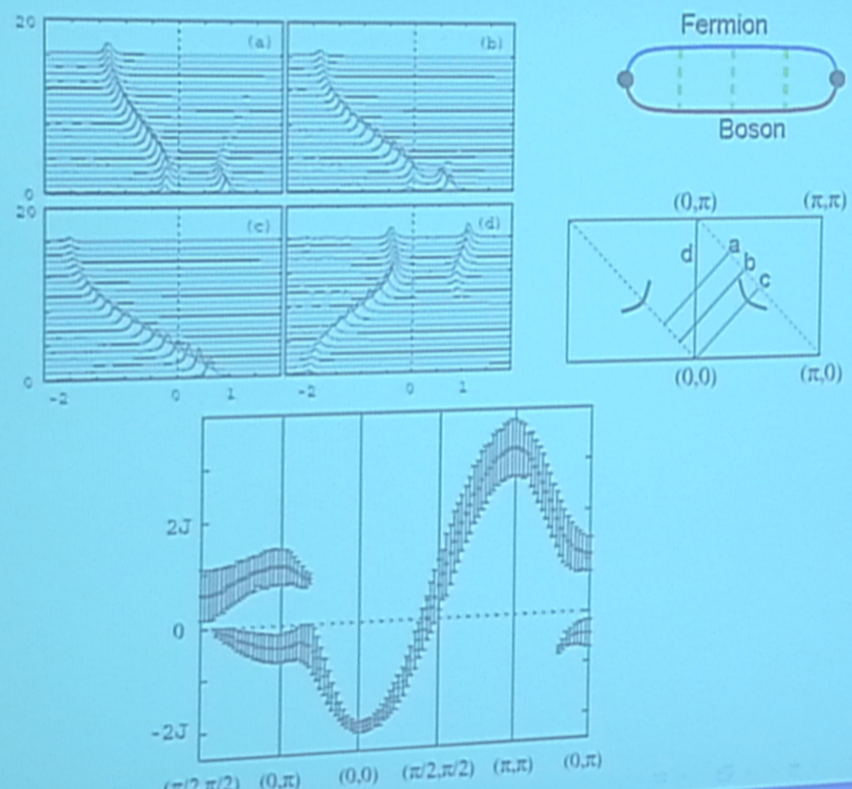
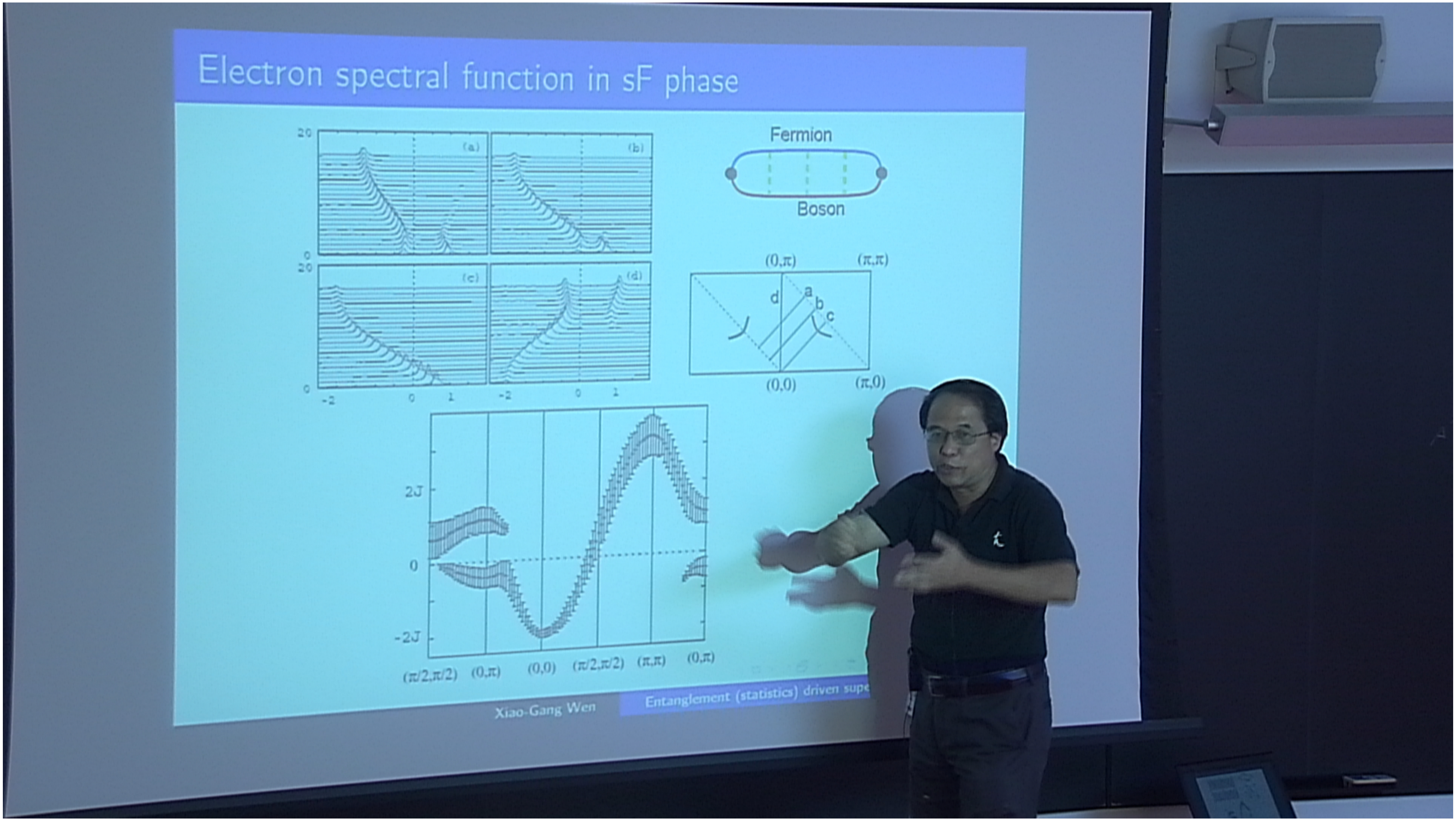
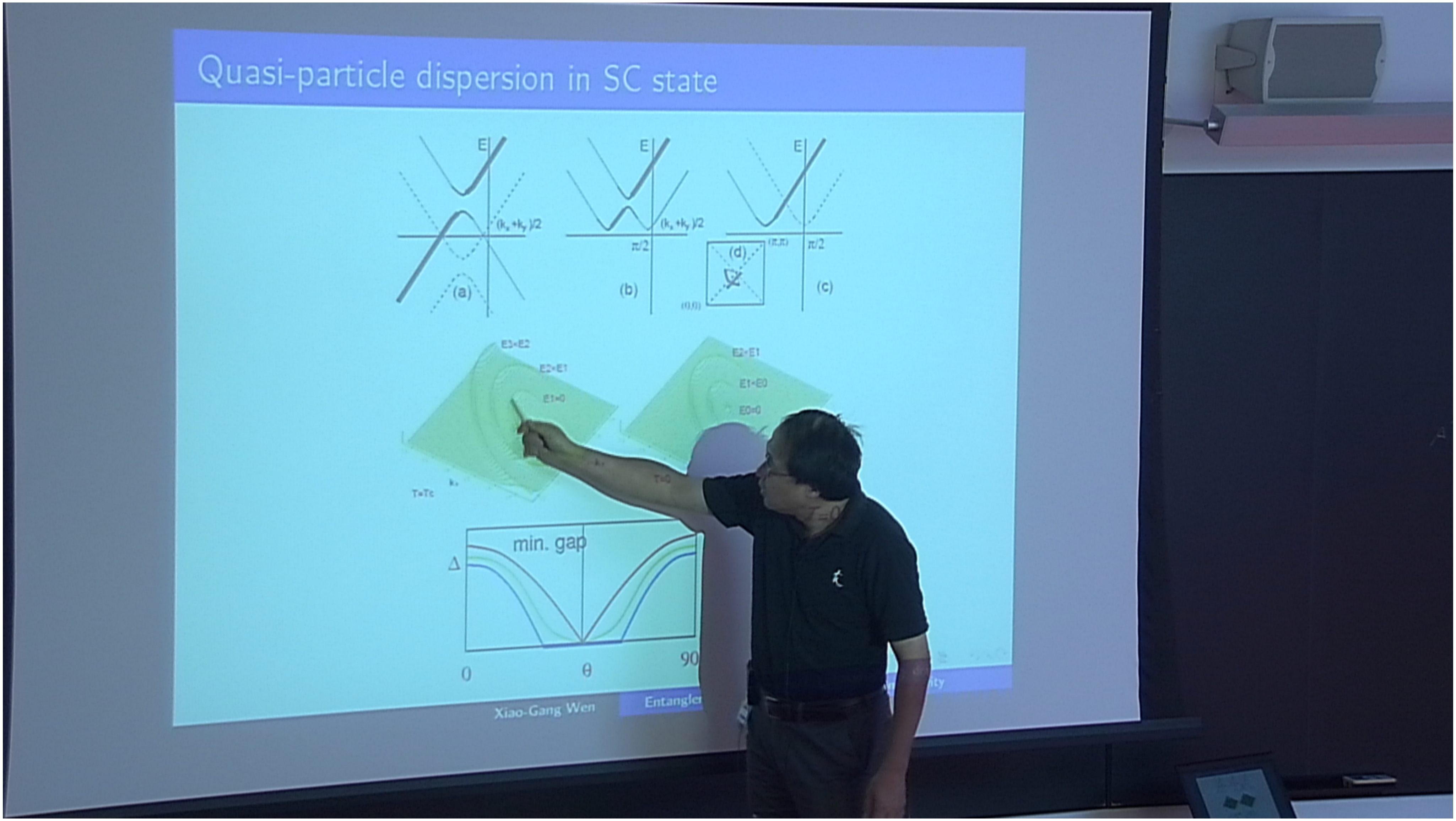
πF: $\langle b \rangle = 0$ and $\chi = \Delta$
uRVB: $\langle b \rangle = 0$ and $\Delta = 0$
FL: $\langle b \rangle \neq 0$ and $\Delta = 0$
SC: $\langle b \rangle \neq 0$ and $\Delta \neq 0$
SL: $\langle b \rangle = 0$ and $\Delta = \chi = 0$
T/J
LS
0.2
πF
Xiao-Gang Wen

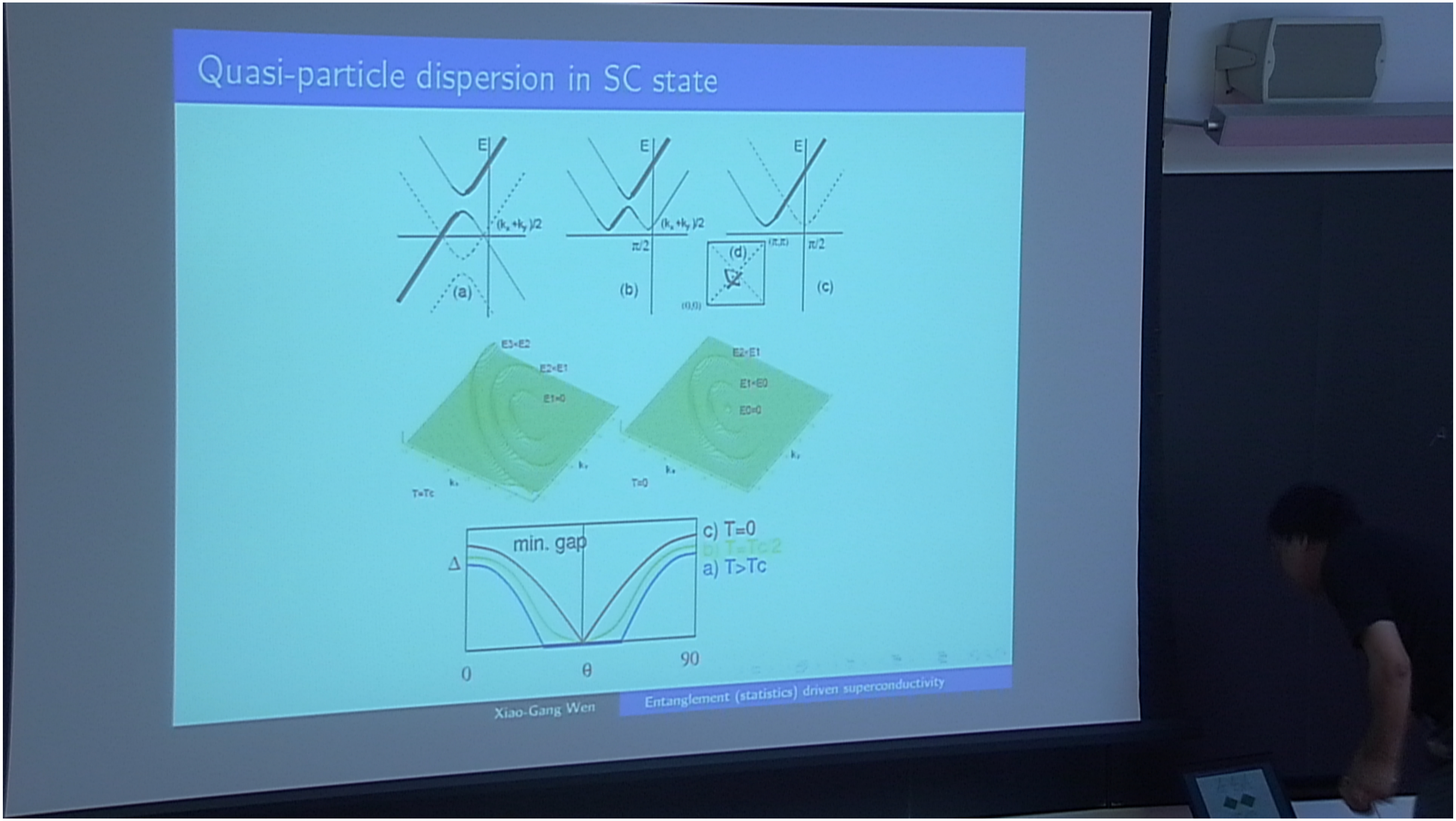

SU(2) meanfield phase diagram
Meanfield solutions:
• sF: ⟨b⟩ = 0 and U_{i,i+x̂,ŷ} = (−χ ∓ i(−)^{i_x+i_y}Δ, 0; 0, χ ∓ i(−)^{i_x+i_y}Δ)
or (SU(2) gauge equiv.) U_{i,i+x̂,ŷ} =
The first one has no pairing Δ_ij = 0
the second one has d-wave pairing for f_α.
• πF: ⟨b⟩ = 0 and χ = Δ
• uRVB: ⟨b⟩ = 0 and Δ = 0
• FL: ⟨b⟩ ≠ 0 and Δ = 0
• SC: ⟨b⟩ ≠ 0 and Δ ≠ 0
• SL: ⟨b⟩ = 0 and Δ = χ = 0
T/J
LS
uRVB
0.2
πF
sF
Xiao-Gang Wen
Entanglement (stat

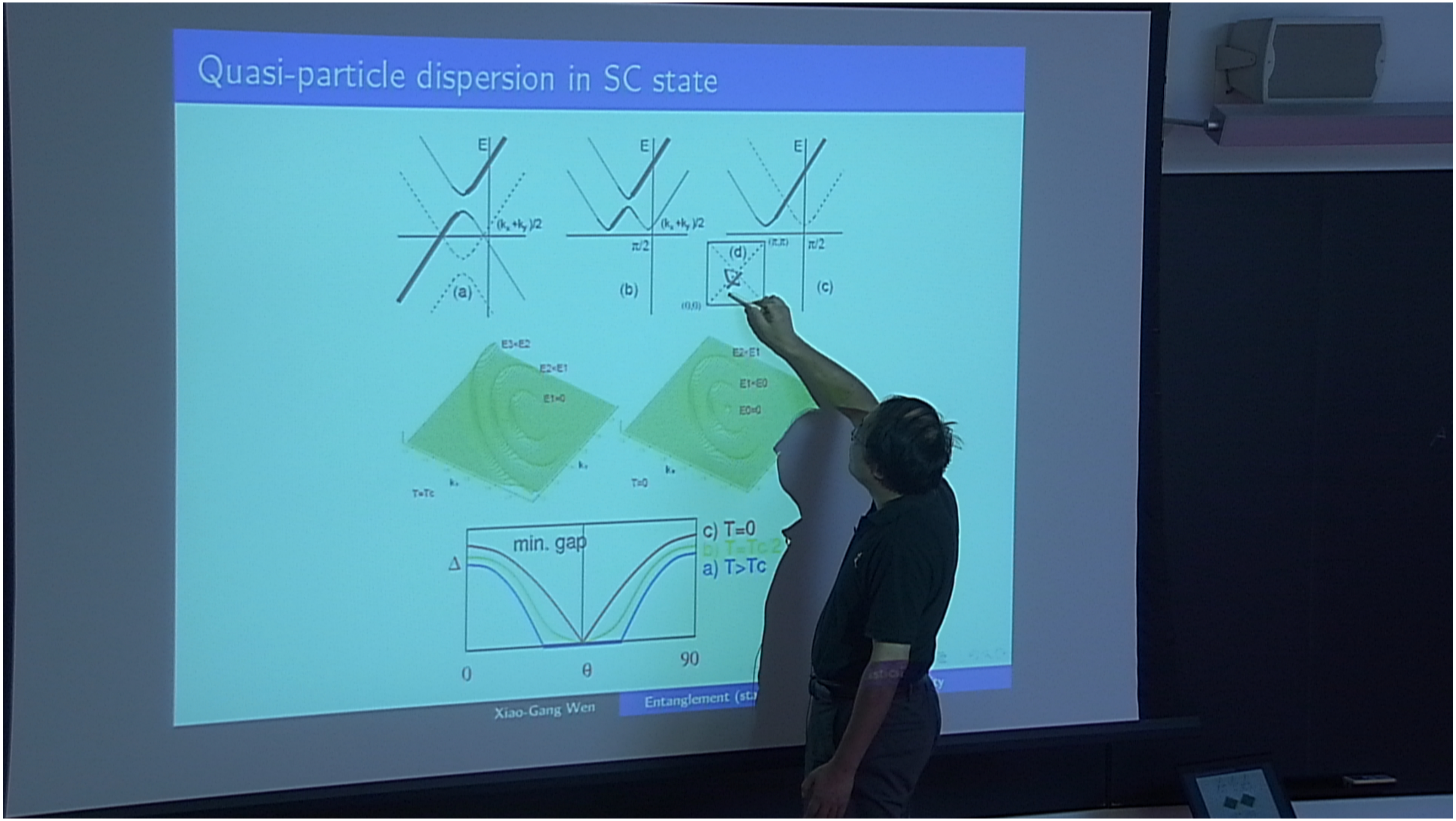
Electron spectral function in sF phase

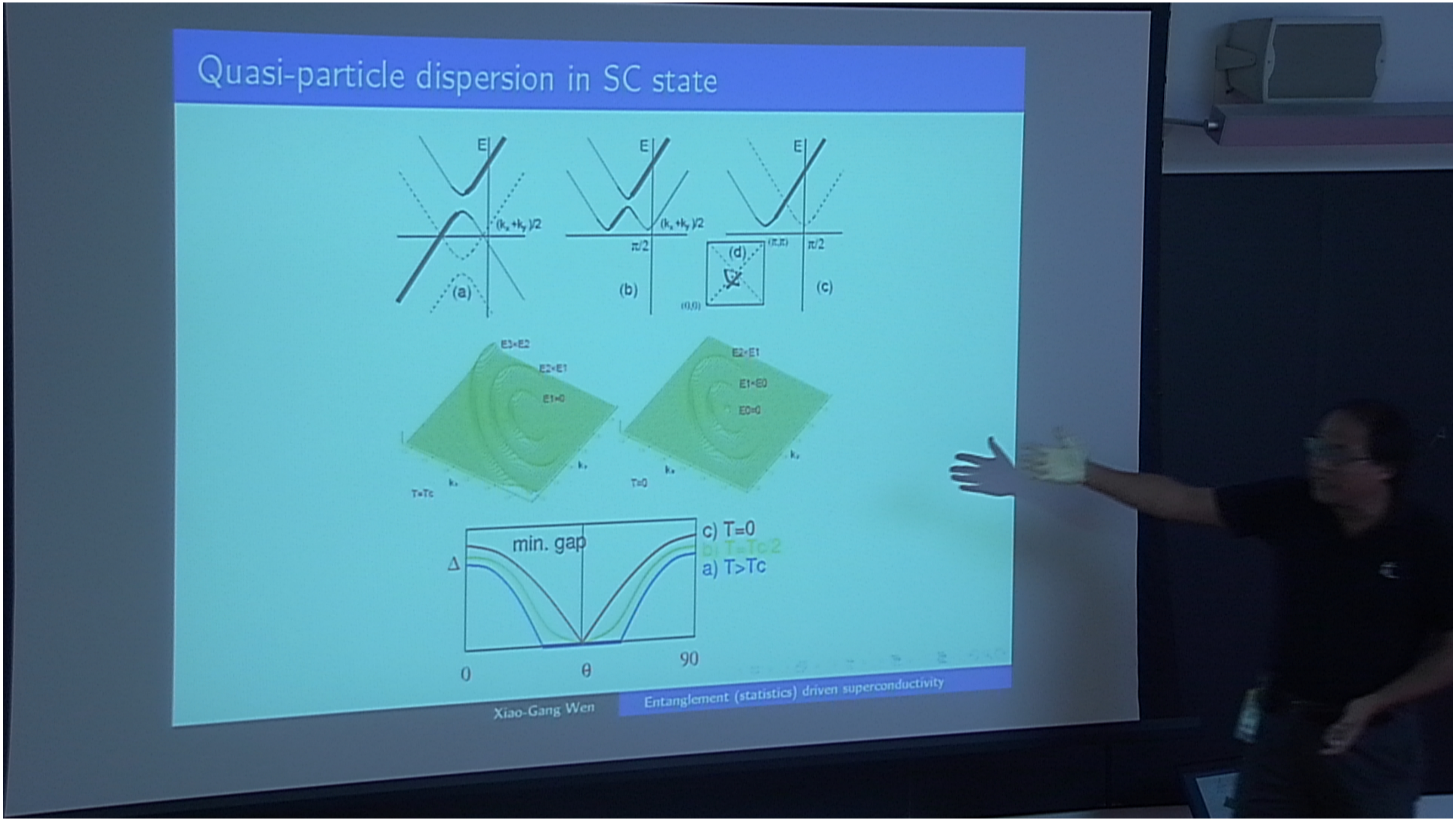


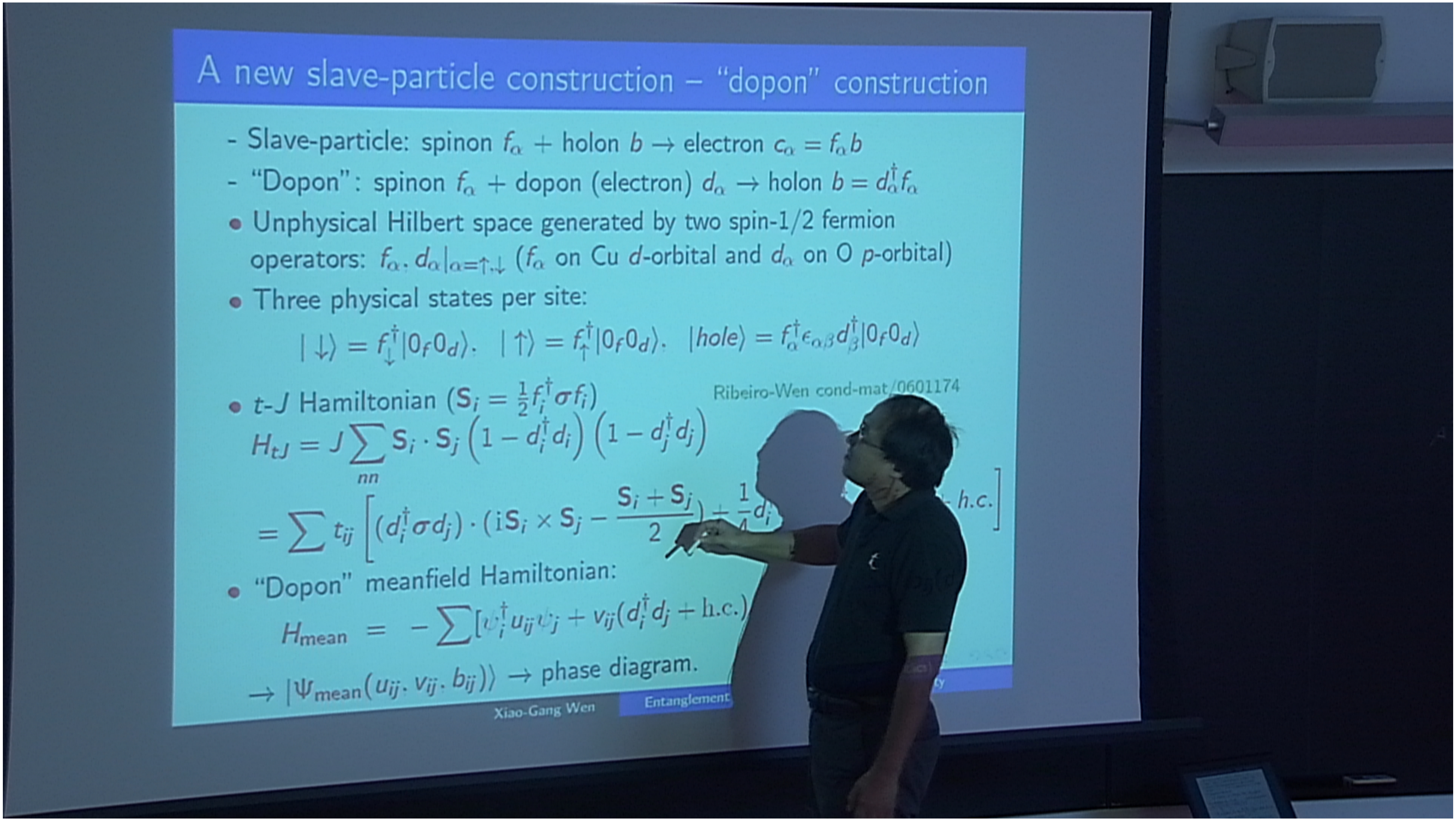

Electron spectral function in sF phase
(a)
(b)
(c)
(d)
20
0
-2
0
1
Fermion
Boson
(0,π)
(π,π)
(0,0)
(π,0)
a
b
c
d
2J
0
-2J
(π/2,π/2)
(0,π)
(0,0)
(π/2,π/2)
(π,π)
(0,π)
Xiao-Gang Wen
Entanglement (statistics) driven

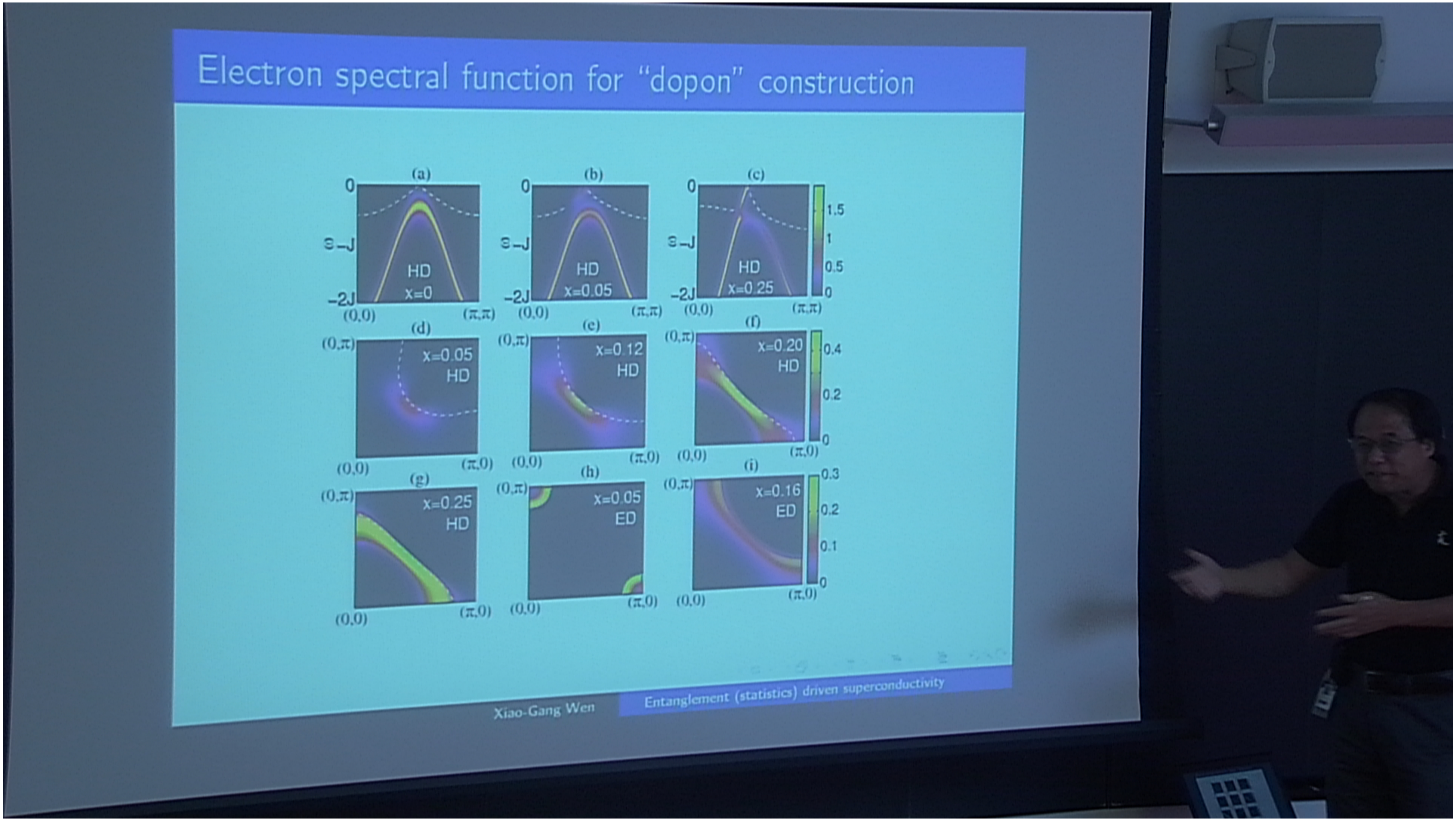

Quasi-particle dispersion in SC state
E
$(k_x+k_y)/2$
(a)
(b)
$\pi/2$
(c)
(d)
min. gap
Δ
0
θ
90
Xiao-Gang Wen

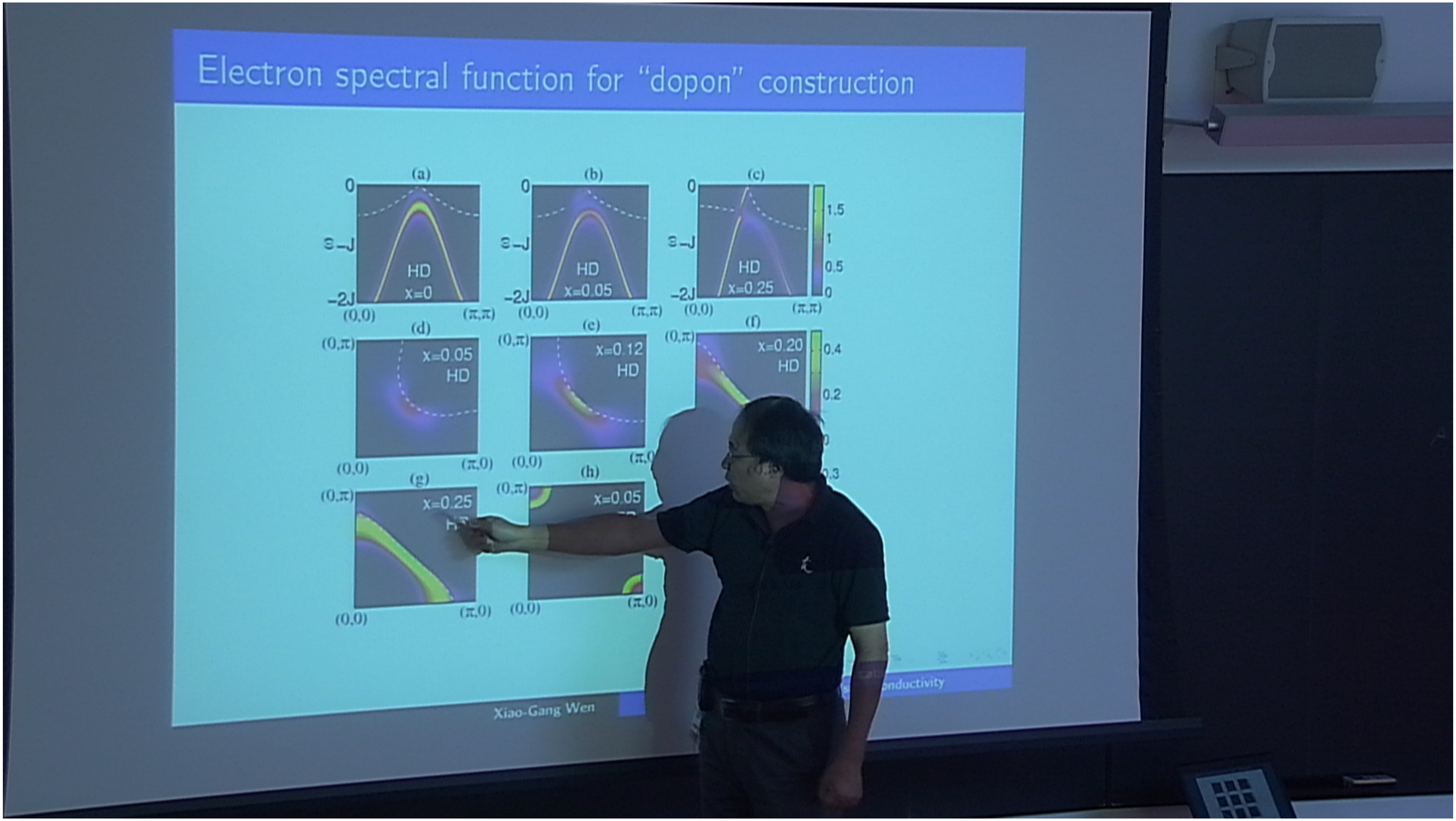

Quasi-particle dispersion in SC state
E
$(k_x+k_y)/2$
(a)
(b)
$\pi/2$
(c)
(d)
(π,π)
(0,0)
E3=E2
E2=E1
E1=0
E2=E1
E1=E0
E0=0
T=Tc
T=0
min. gap
Δ
c) T=0
b) T=Tc/2
a) T>Tc
0
θ
90
Xiao-Gang Wen
Entanglement (statistics) driven superconductivity


Quasi-particle dispersion in SC state
E
$(k_x+k_y)/2$
(a)
(b)
$\pi/2$
(d)
(c)
(π,π)
(0,0)
E3=E2
E2=E1
E1=0
E0=0
T=Tc
T=0
min. gap
c) T=0
a) T>Tc
Δ
0
θ
90
Xiao-Gang Wen


Quasi-particle dispersion in SC state
E
(k_x+k_y)/2
π/2
(a)
(b)
(c)
(d)
E3=E2
E2=E1
E1=0
T=Tc
E1=E0
E0=0
T=0
min. gap
Δ
c) T=0
b) T=Tc/2
a) T>Tc
0
θ
90
Xiao-Gang Wen
Entanglement (statistics) driven superconductivity


A new slave-particle construction – "dopon" construction
- Slave-particle: spinon f_α + holon $b \to$ electron $c_\alpha = f_\alpha b$
- "Dopon": spinon f_α + dopon (electron) $d_\alpha \to$ holon $b = d_\alpha^\dagger f_\alpha$
• Unphysical Hilbert space generated by two spin-1/2 fermion operators: $f_\alpha, d_\alpha|_{\alpha=\uparrow,\downarrow}$ (f_α on Cu d-orbital and d_α on O p-orbital)
• Three physical states per site:
$|\downarrow\rangle = f_\downarrow^\dagger|0_f 0_d\rangle, \quad |\uparrow\rangle = f_\uparrow^\dagger|0_f 0_d\rangle, \quad |hole\rangle = f_\alpha^\dagger \epsilon_{\alpha\beta} d_\beta^\dagger|0_f 0_d\rangle$
Ribeiro-Wen cond-mat/0601174
• t-J Hamiltonian ($\mathbf{S}_i = \frac{1}{2} f_i^\dagger \sigma f_i$)
$H_{tJ} = J \sum_{nn} \mathbf{S}_i \cdot \mathbf{S}_j \left(1 - d_i^\dagger d_i\right)\left(1 - d_j^\dagger d_j\right)$
$= \sum t_{ij} \left[(d_i^\dagger \sigma d_j) \cdot (\mathrm{i}\mathbf{S}_i \times \mathbf{S}_j - \frac{\mathbf{S}_i + \mathbf{S}_j}{2}) + \frac{1}{4} d_i$ ... h.c.$\right]$
• "Dopon" meanfield Hamiltonian:
$H_{\text{mean}} = -\sum [\psi_i^\dagger u_{ij} \psi_j + v_{ij}(d_i^\dagger d_j + \text{h.c.})$
$\to |\Psi_{\text{mean}}(u_{ij}, v_{ij}, b_{ij})\rangle \to$ phase diagram.
Xiao-Gang Wen
Entanglement


Electron spectral function for "dopon" construction
(a)
(b)
(c)
HD
x=0
HD
x=0.05
HD
x=0.25
(d)
(e)
(f)
x=0.05
HD
x=0.12
HD
x=0.20
HD
(g)
(h)
(i)
x=0.25
HD
x=0.05
ED
x=0.16
ED
Xiao-Gang Wen
Entanglement (statistics) driven superconductivity


Electron spectral function for "dopon" construction
(a)
(b)
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HD
x=0
HD
x=0.05
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x=0.25
(d)
(e)
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x=0.05
HD
x=0.12
HD
x=0.20
HD
(g)
(h)
x=0.25
x=0.05
Xiao-Gang Wen


Electron spectral function for "dopon" construction
HD
x=0
HD
x=0.05
HD
x=0.25
x=0.05
HD
x=0.12
HD
x=0.20
HD
x=0.25
HD
x=0.05
ED
x=0.16
ED


Electron spectral function for "dopon" construction