Title: A tensor product state approach to spin-1/2 square J1-J2 Heisenberg model: spin liquid vs. deconfined quantum criticality

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Abstract: The ground state phase diagram of spin-1/2 J1-J2 antiferromagnetic Heisenberg model on square lattice around the maximally frustrated regime (J2~0.5J1) has been debated for decades. I will discuss some progresses on this old problem based on recent numerical results from density matrix renormalization group(DMRG) and tensor product states algorithms.

A tensor product state approach to spin-1/2 square J1-J2 Heisenberg model:

evidence for deconfined quantum criticality

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Perimeter Institute

Collaborator:

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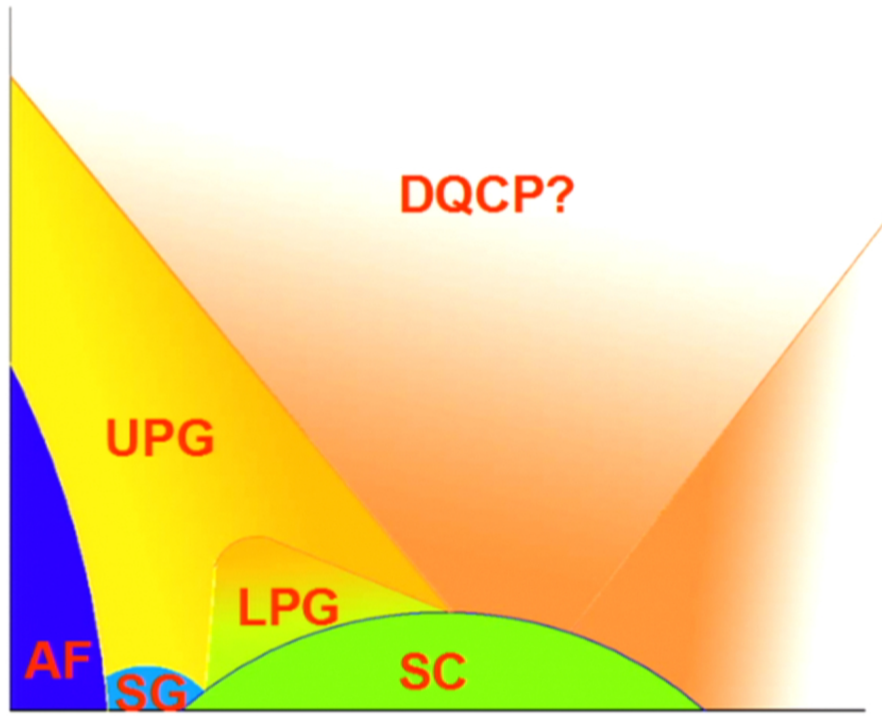
Prof. Xiao-Gang Wen (MIT)
Prof. F. Verstraete (U. of Vienna)

Deconfined Quantum critical point (QCP) in cuprates?

Subir Sachdev, Science 288, 475, (2000)



DQCP in spin models

Theory: beyond Landau's paradim

$$\mathcal{L}_z = \sum_{a=1}^{N} |\left(\partial_\mu - i a_\mu\right) z_a|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa\right)^2$$

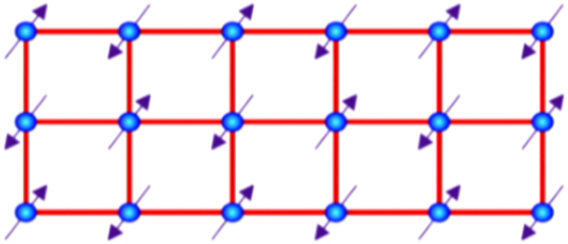
$$Q = \frac{1}{4\pi}\int d^2 r\, \hat{n}\cdot\partial_x \hat{n}\times\partial_y \hat{n},$$

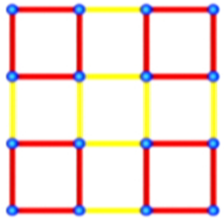
On square lattice, Q always changes by four, thus the instanton effect becomes dangerous irrelevant!

T. Senthil, Ashvin Vishwanath, Leon Balents, Subir Sachdev, M. P. A. Fisher, Science 303, 1490 (2004).

Numerical: J-Q model(simulated by using QMC)

$$H = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - Q\sum_{\langle ijkl\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j - \tfrac{1}{4})(\mathbf{S}_k\cdot\mathbf{S}_l - \tfrac{1}{4})$$





Anders W. Sandvik, Phys. Rev. Lett. 98, 227202 (2007)

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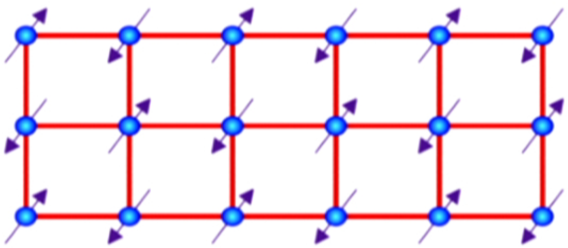
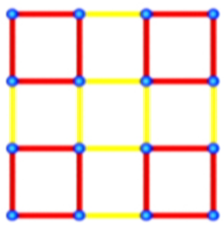


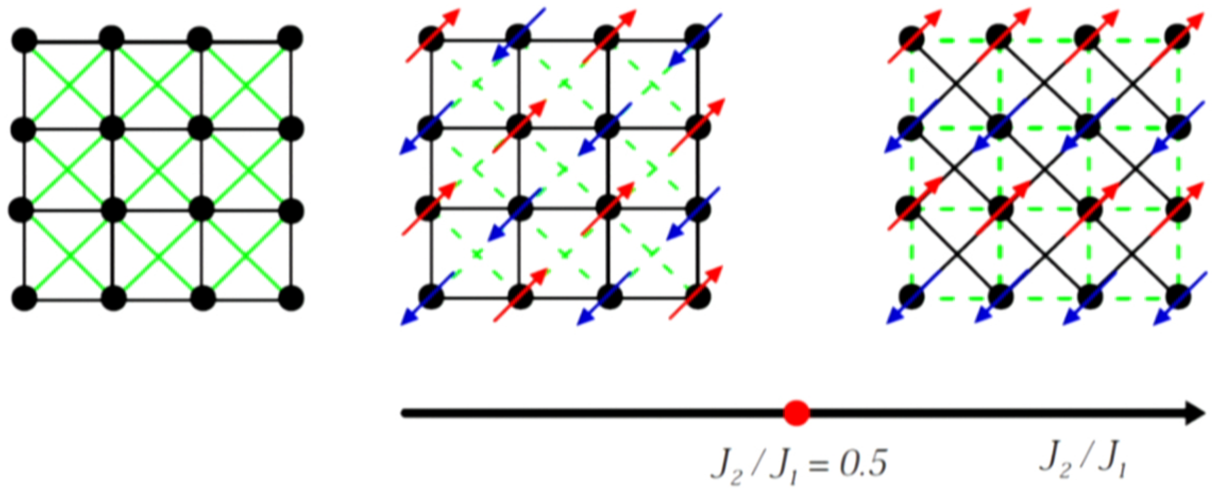


Anders W. Sandvik, Phys. Rev. Lett. 98, 227202 (2007)

Relevant spin model for cuprates

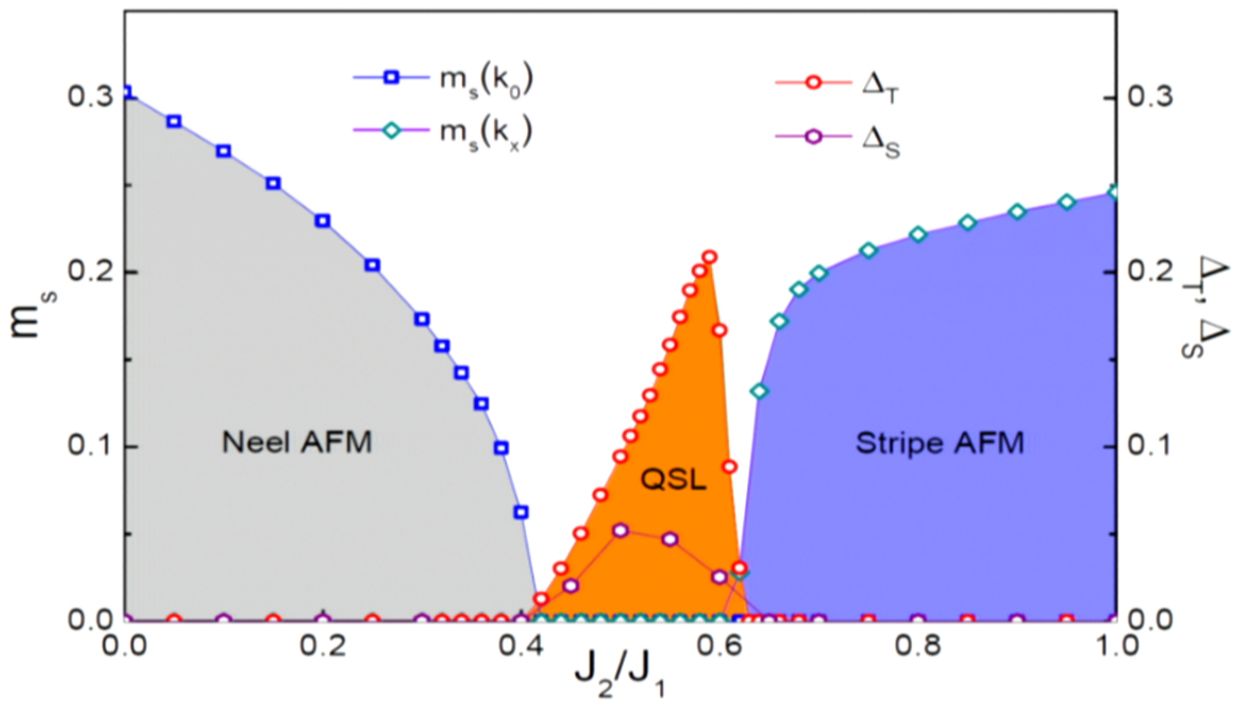
J1-J2 model: $$H = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Landau Paradigm: a meanfield phase diagram



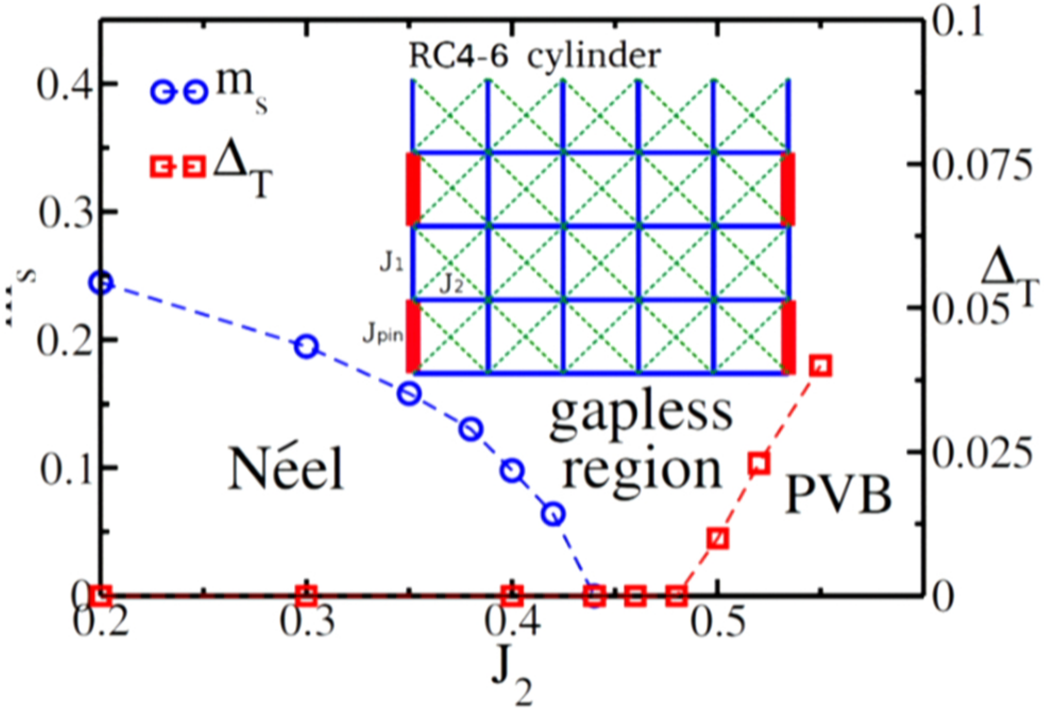
Recent progress

A recent work by using density matrix renormalization group(DMRG) algorithm claims a Z2 spin liquid



Hong-Chen Jiang, Hong Yao, Leon Balents, Phys. Rev. B 86, 024424 (2012)

Another work by using SU(2) symmetry DMRG algorithm suggests a different result



Shou-Shu Gong, Wei Zhu, D. N. Sheng, Olexei I. Motrunich, Matthew P. A. Fisher
Phys. Rev. Lett. 113. 027201 (2014)

Landau paradigm: meanfield approach

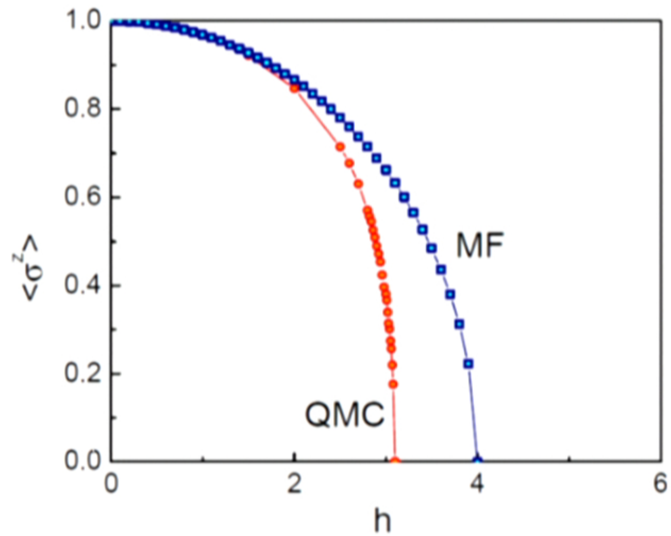
- The key concept is to find an ideal trial wave function, e.g., for a spin ½ system:

$$|\Psi_{trial}\rangle = \otimes \left(u^{\uparrow}|\uparrow\rangle_i + u^{\downarrow}|\downarrow\rangle_i\right)$$

- After minimizing the energy, we can find various symmetry ordered phases.

$$H = -\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x$$

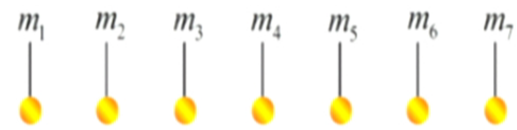
$$\begin{aligned} h^c_{MF} &= 4 \\ \beta^c_{MF} &= 0.5 \\ \langle\sigma_z\rangle &\propto |h-h_c|^{\beta} \end{aligned}$$



Tensor Product State approach

Mean-field states: $\uparrow \longrightarrow u^{\uparrow}; \quad \downarrow \longrightarrow u^{\downarrow}$

$\Psi(\{m_i\}) = u^{m_1} u^{m_2} u^{m_3} u^{m_4} \cdots; \quad m_i = \uparrow, \downarrow$



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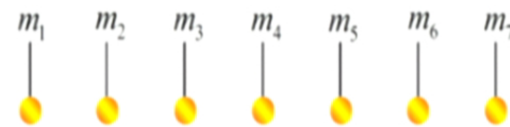
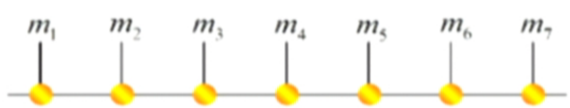
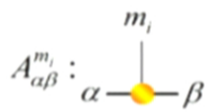


$$\Psi(\{m_i\}) = u^{m_1} u^{m_2} u^{m_3} u^{m_4} \cdots; \quad m_i = \uparrow, \downarrow$$

MPS/DMRG(the most powerful method in 1D):

$$\Psi(\{m_i\}) = \mathrm{Tr}\left[A^{m_1} A^{m_2} A^{m_3} A^{m_4} \cdots\right]; \quad m_i = \uparrow, \downarrow \quad \uparrow \rightarrow A^{\uparrow}; \quad \downarrow \rightarrow A^{\downarrow}$$





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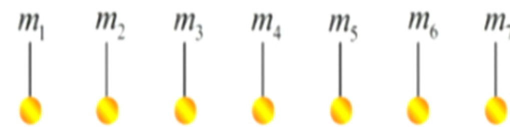

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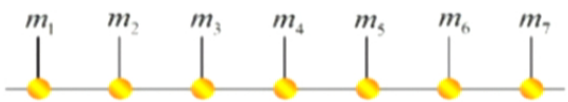
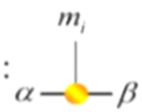


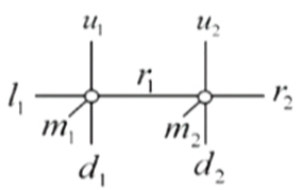
$A^{m_i}_{\alpha\beta}$:

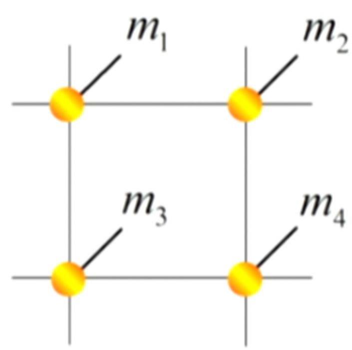


TPS: $\uparrow \longrightarrow T^{\uparrow}_{lrud}; \quad \downarrow \longrightarrow T^{\downarrow}_{lrud}$ (F. Verstraete and J. I. First 2004)

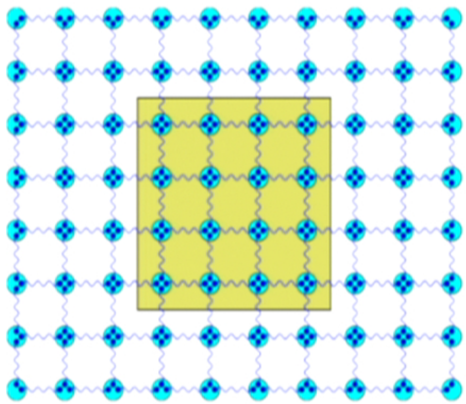
$T^{m_1}_{l_1r_1u_1d_1}$:

$\sum_{r_1} T^{m_1}_{l_1r_1u_1d_1} T^{m_2}_{r_1r_2u_2d_2}$:





Properties of TPS:



- Entangelment entropy satisfies area law

$$S(\rho_L) = \alpha L$$ (F. Verstraete *etal.*)

$$|\Psi_0\rangle = \prod_{link} |I\rangle \quad |I\rangle = \sum_{l=1}^{D} |ll\rangle$$

$$|\Psi_{TPS}\rangle = \prod_i P_i |\Psi_0\rangle \quad P_i = T^{m_i}_{lrud} |m_i\rangle\langle lrud|$$

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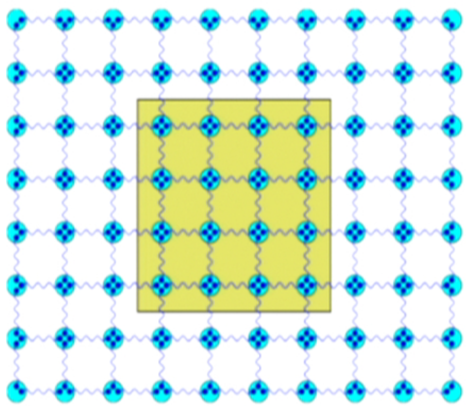

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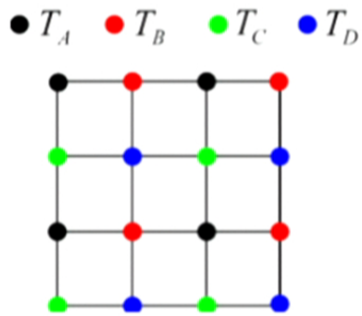
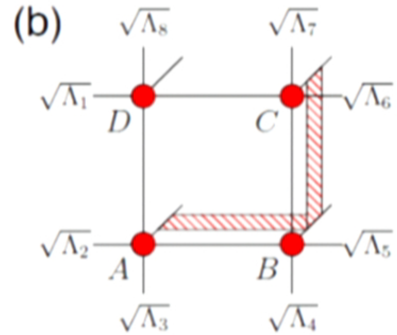
- TPS faithfully represent non-chiral topologically ordered states

(Z.C. Gu, *etal.*, PRB, 2008, O. Buerschaper, *etal.*, PRB, 2008)

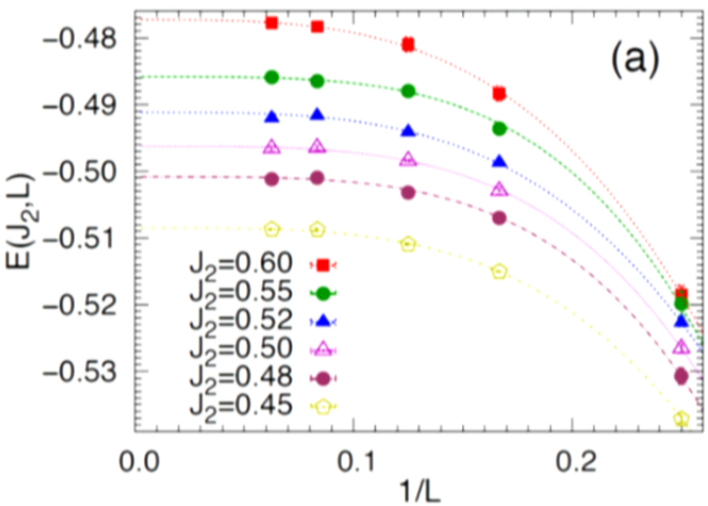
Ground state energy

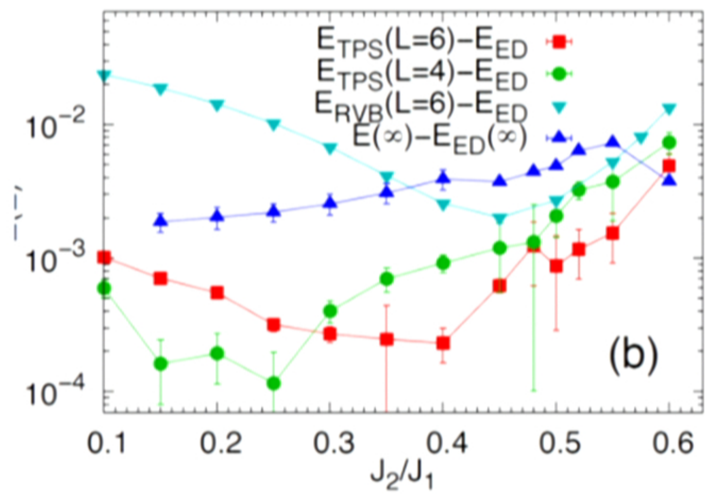


- We use an imaginary time evolution algorithm to find the variational ground state.



$$|\Psi_{GS}\rangle = \lim_{\tau \to \infty} e^{-\tau H} |\Psi_0\rangle = \lim_{N \to \infty} e^{-N\delta\tau H} |\Psi_0\rangle$$

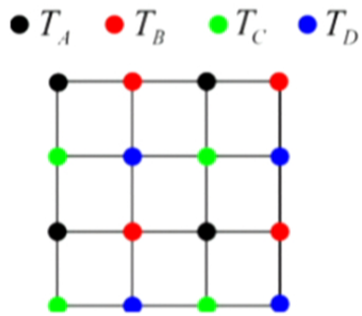




- D=9

The TPS energy is lower than the best VMC energy!

Ground state energy



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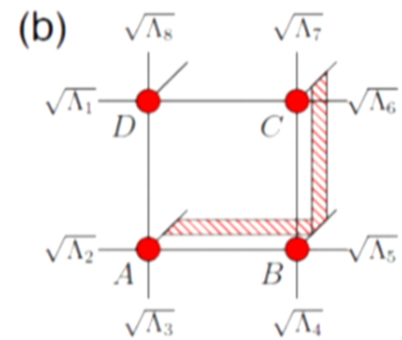

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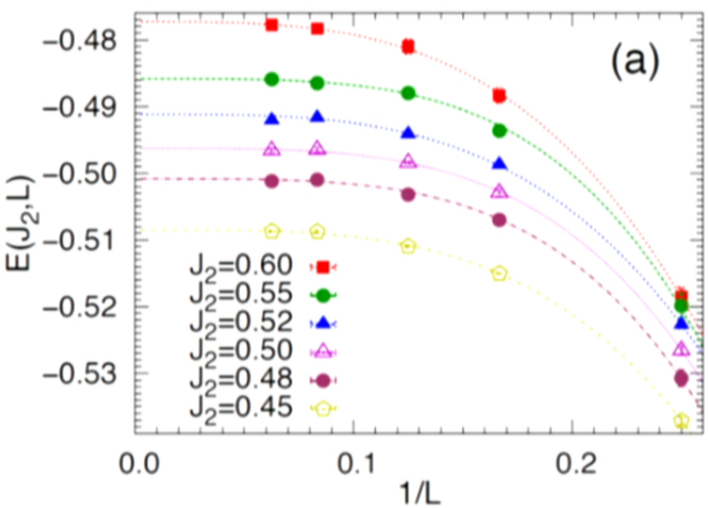

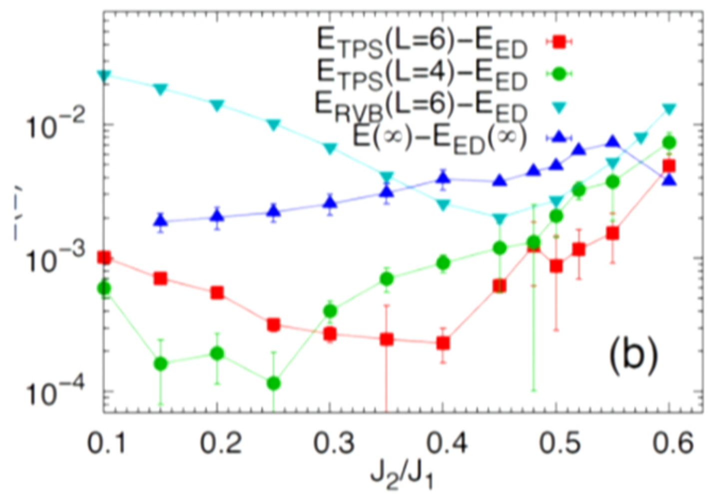

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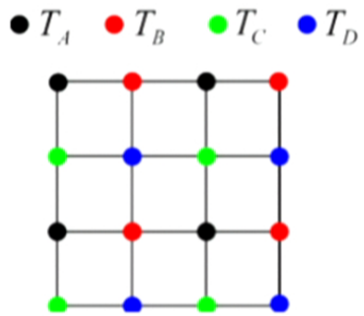

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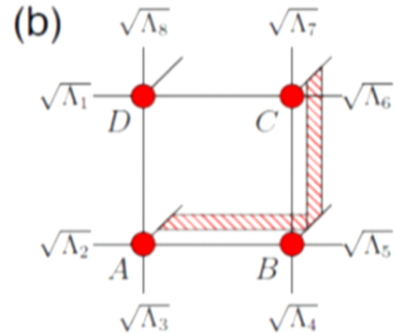

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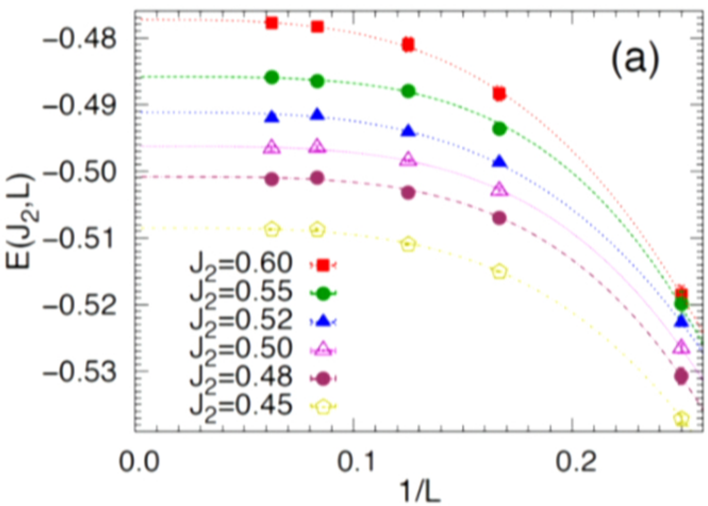

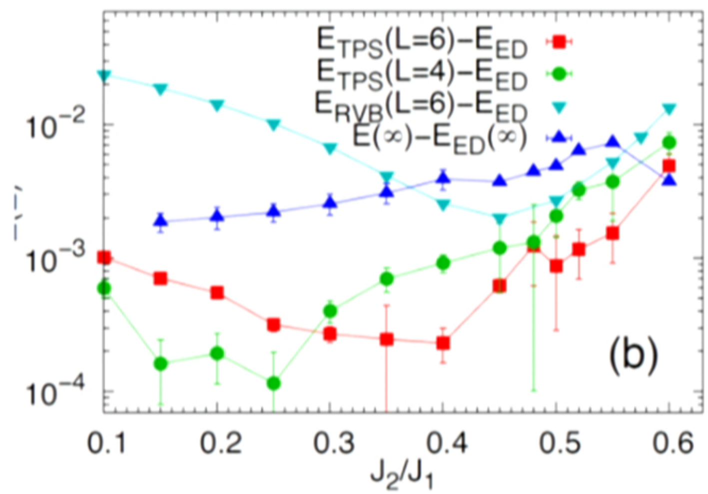

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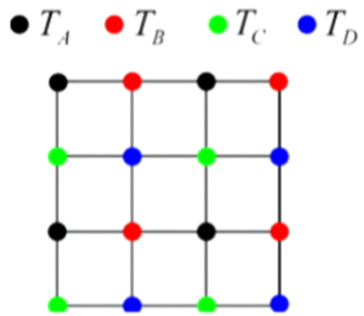

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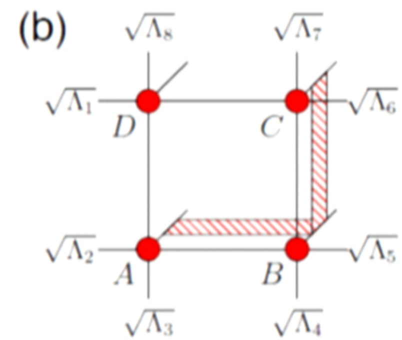

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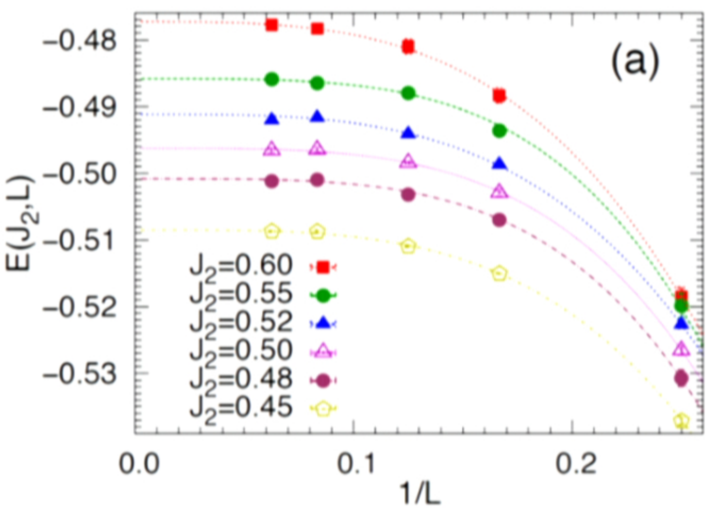

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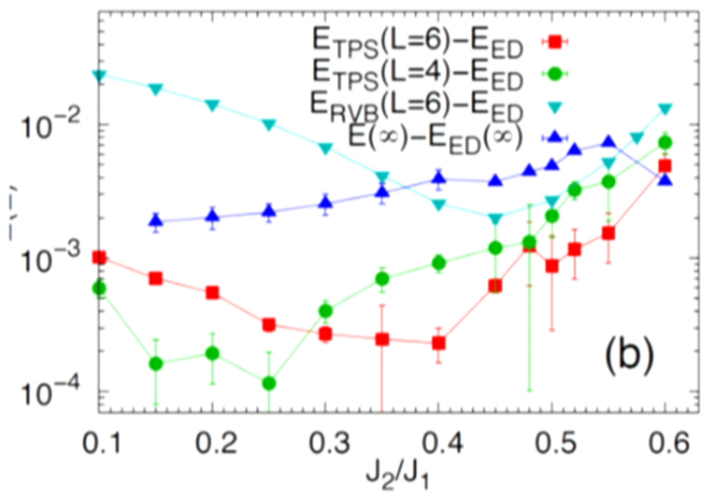

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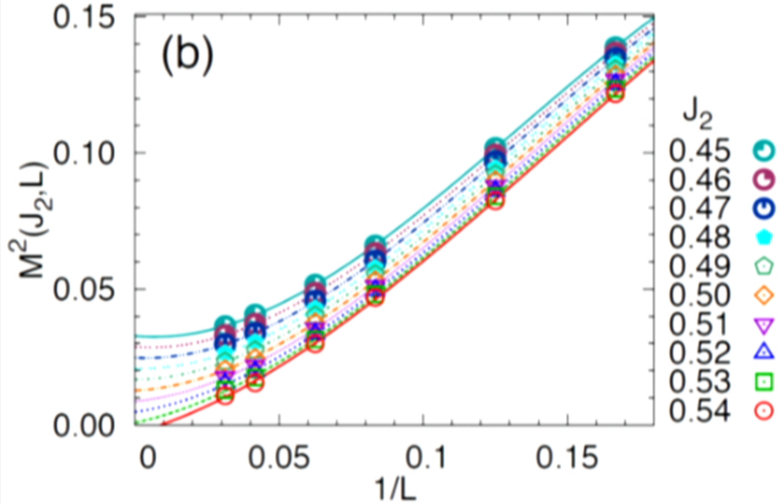
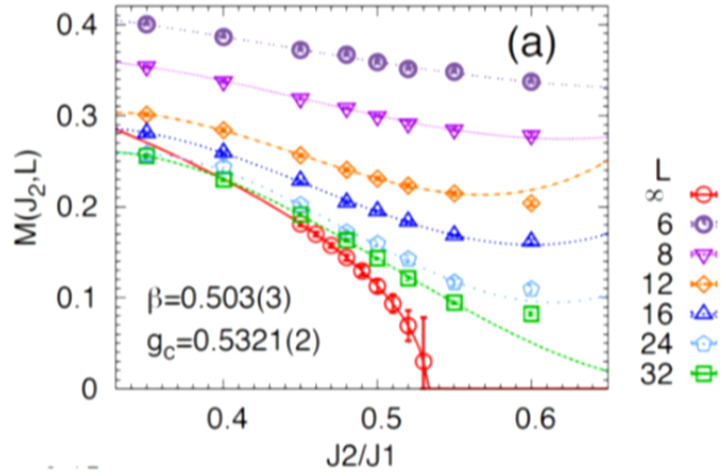
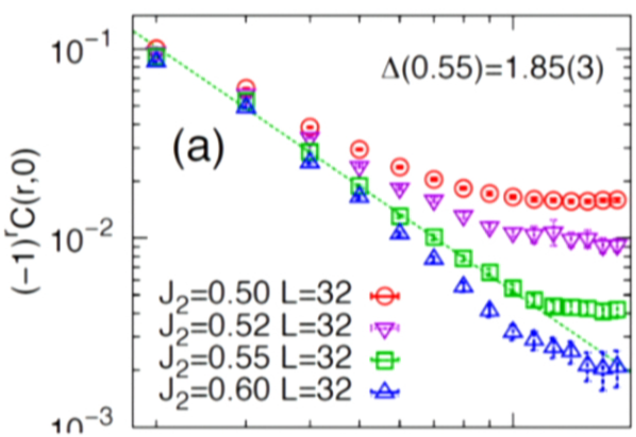
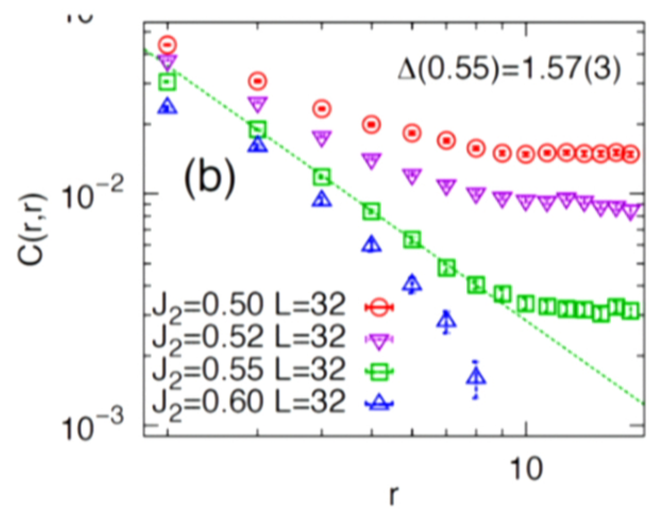




Spin-Spin Correlation:





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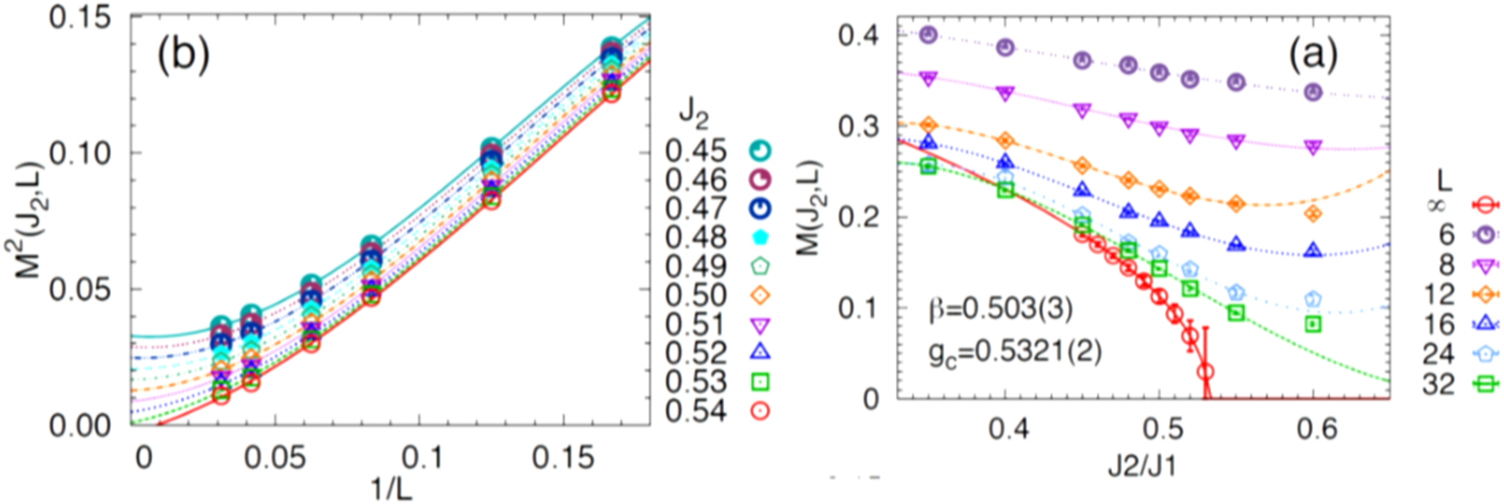

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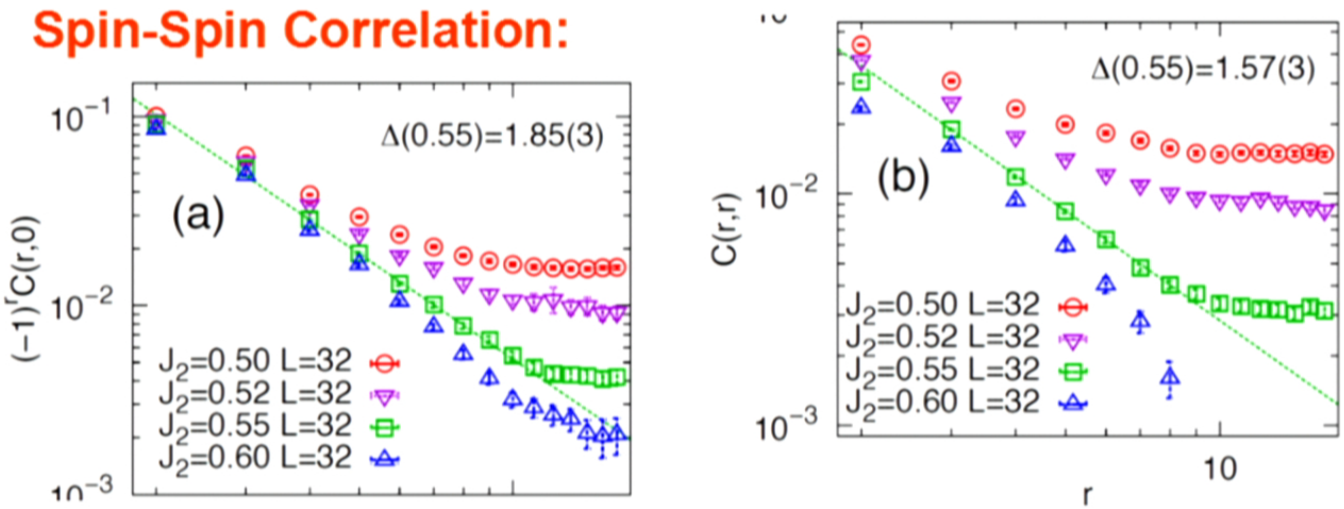

Dimer-Dimer Correlation:

$$C^*_{dx}(r,0) = C_{dx}(r,0) - C_{dx}(r-1,0), \qquad D_x(x,y) = \mathbf{S}_{(x,y)} \cdot \mathbf{S}_{(x+1,y)},$$
$$C^*_{dx}(r,r) = C_{dx}(r,r) - C_{dx}(r-1,r). \qquad D_y(x,y) = \mathbf{S}_{(x,y)} \cdot \mathbf{S}_{(x,y+1)}.$$

$$C_{dx}(r_x, r_y) = \frac{1}{N}\sum_{x,y} D_x(x,y) D_x(x+r_x, y+r_y),$$

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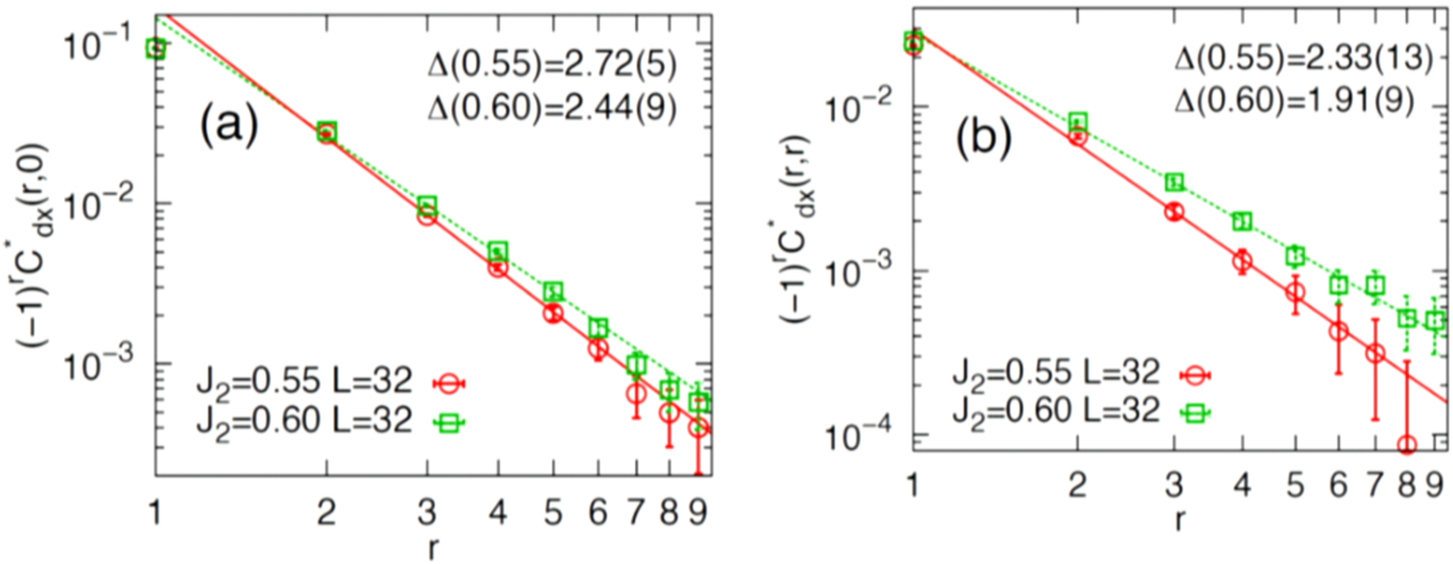
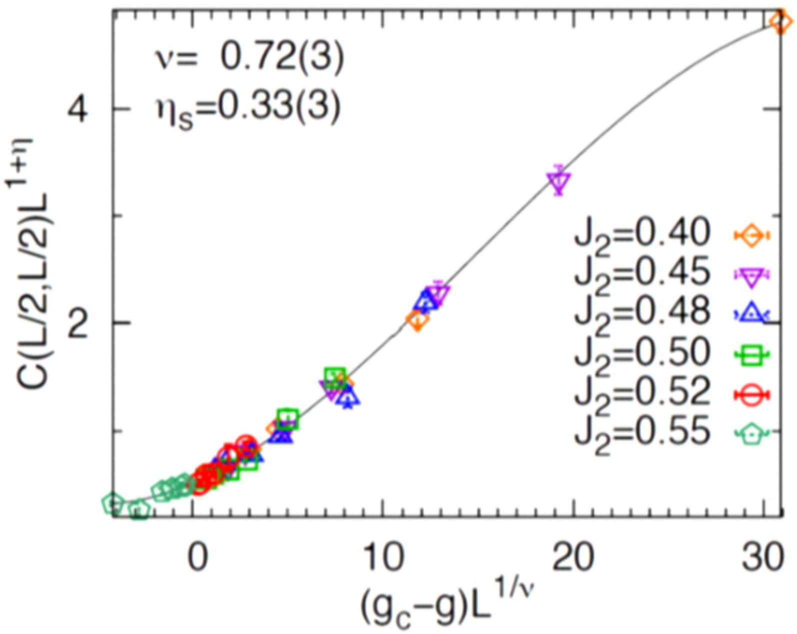


The universal scaling function



$$C(L/2, L/2) = L^{-1-\eta} f(L^{1/\nu}(g_c - g)/g_c),$$

The critical exponents are intrinsically close to the DQCP behavior observed in other systems, e.g., J-Q model

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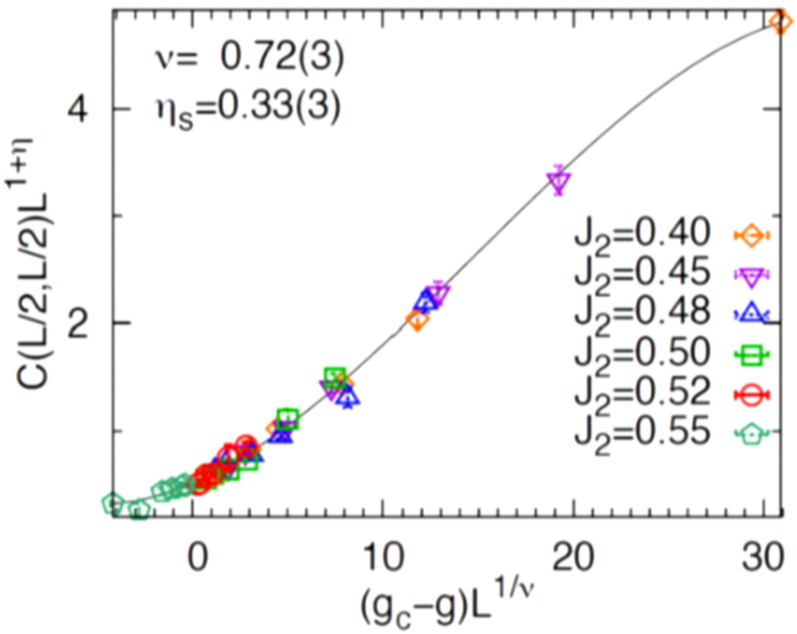

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U(1) spin liquid is unstable, a VBS order with exponentially small amplitude might develop at long wave length

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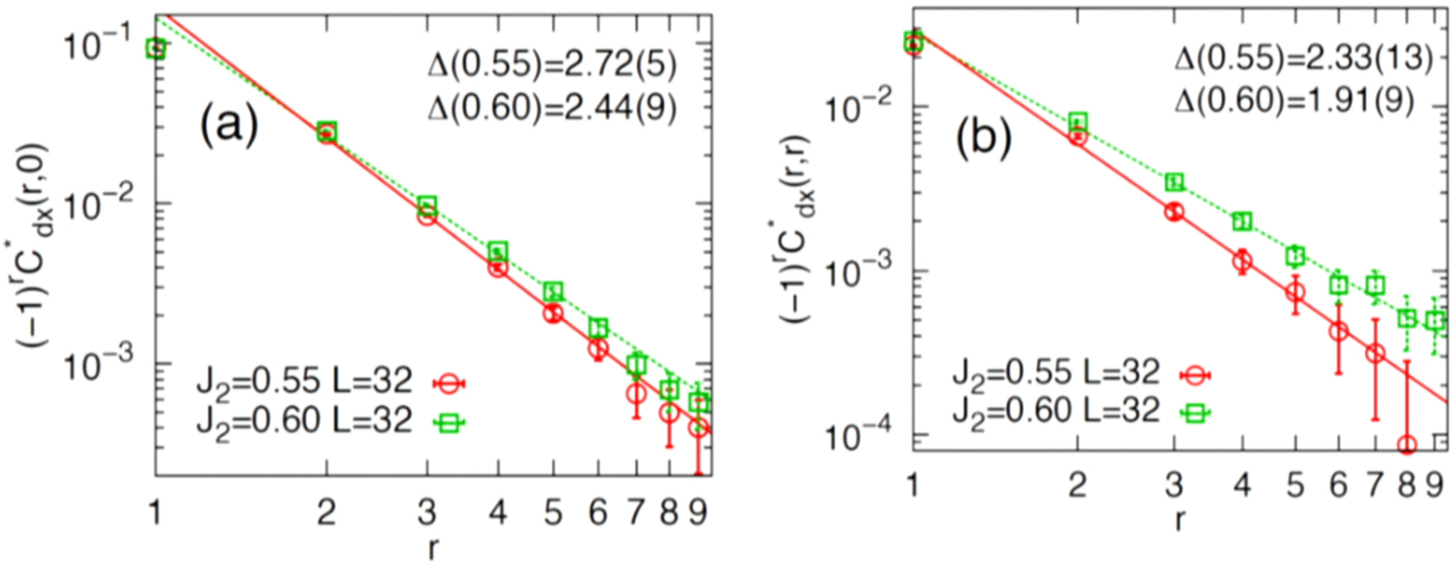
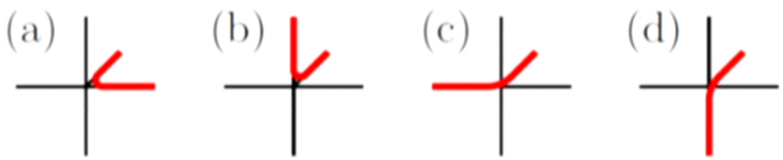


Single parameter variational approach

A D=3 TPS description of short range RVB state



$$\mathcal{P}_1 = \sum_{k=1}^{4} (|\uparrow\rangle\langle 0|_k + |\downarrow\rangle\langle 1|_k) \otimes \langle 222|_{/k}$$

$$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$$

$$|\Psi\rangle_{\mathrm{s-RVB}} = \prod_V \mathcal{P}_1 \prod_B |\mathcal{S}\rangle$$

Single parameter variational approach

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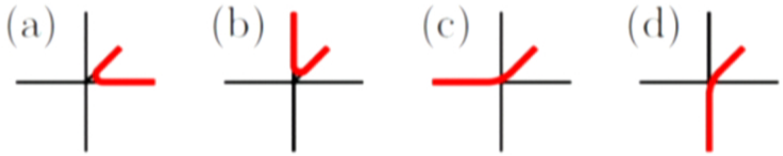

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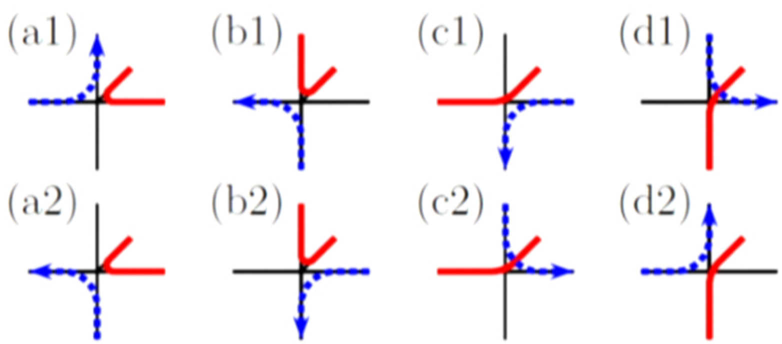
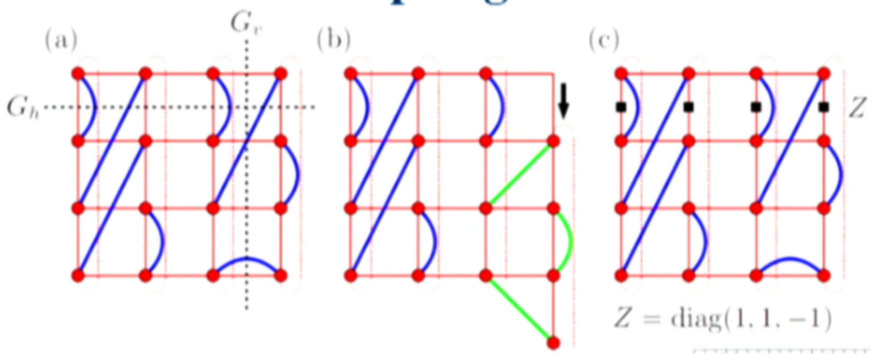
$$\mathcal{P}_2 = \sum_{i \neq j \neq k \neq l} (|\uparrow\rangle\langle 0|_i + |\downarrow\rangle\langle 1|_i) \otimes \langle 2|_j \otimes \langle \epsilon|_{kl}$$



Topological sectors and variational energy

Similar to the short range RVB state, we can define four different topological sectors.

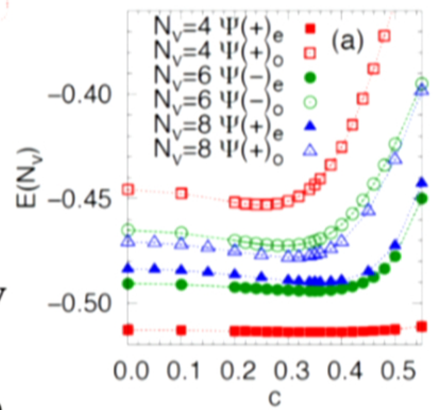


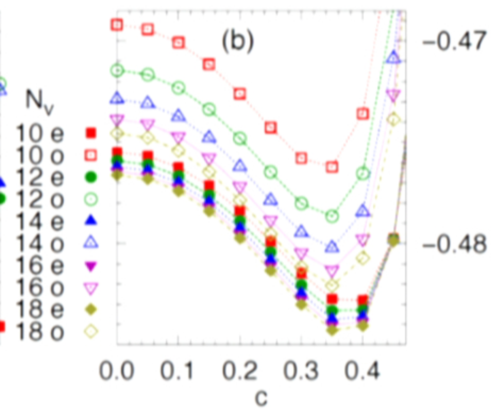
Vison sectors

$$|\Psi(\pm)\rangle \equiv |\Psi\rangle_{G_h=1} \pm |\Psi\rangle_{G_h=-1}$$

Best variational energy at c=0.35

- E=-0.4862/site on cylinder/stripe geometry
- Comparing to E=-0.494/site with D=9

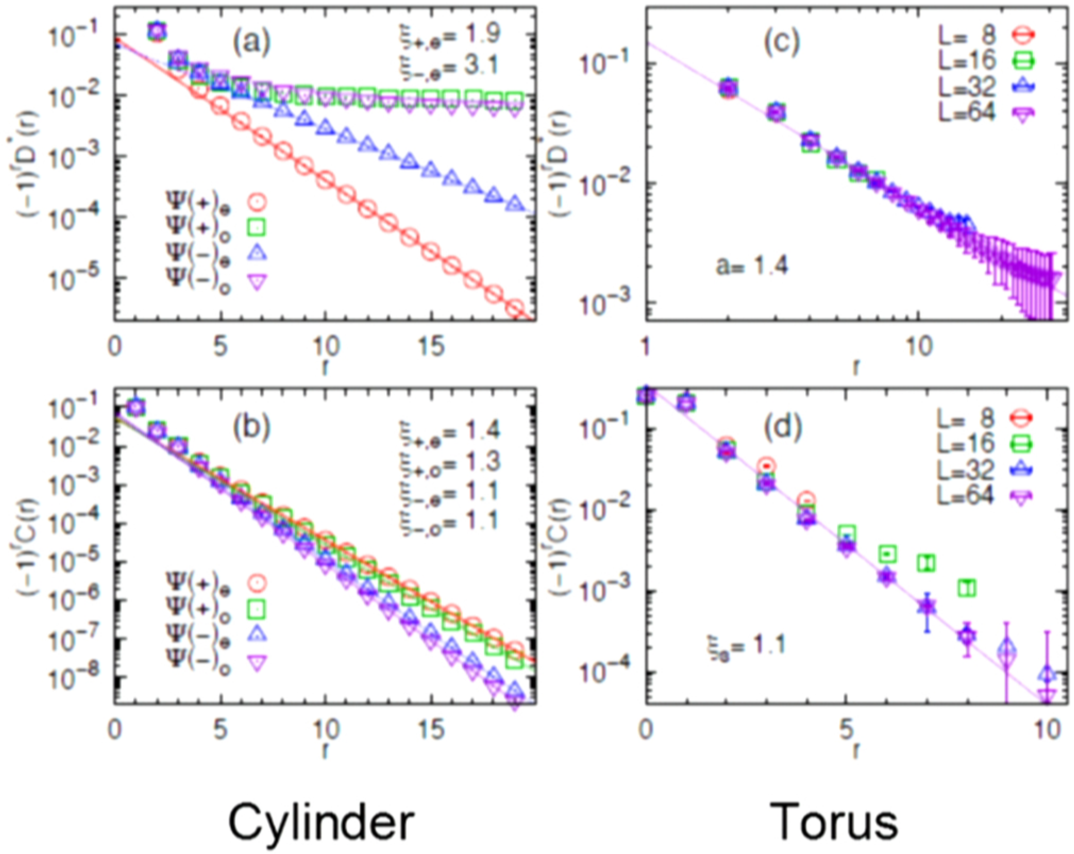




- Cylinder
- Stripe

Correlation functions

Dimer-dimer correlation shows different behaviors on cylinder and torus!(Challenges to DMRG)



Entanglement entropy

Both dimer-dimer correlation and entanglement entropy indicate gapless spin liquid behaviors!

$$S_2(L) = a_1 L - \frac{1}{2} \ln L + b_1$$

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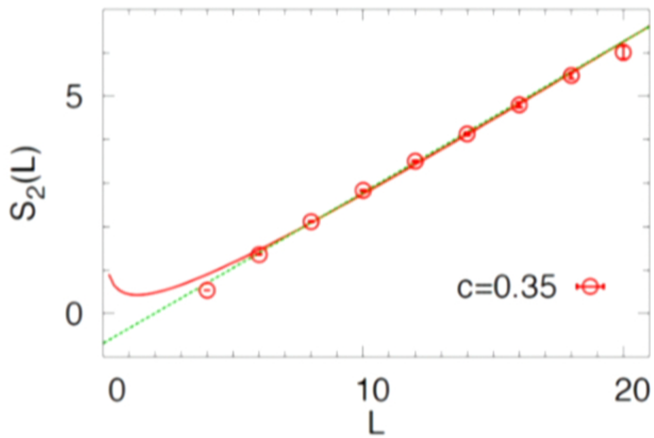
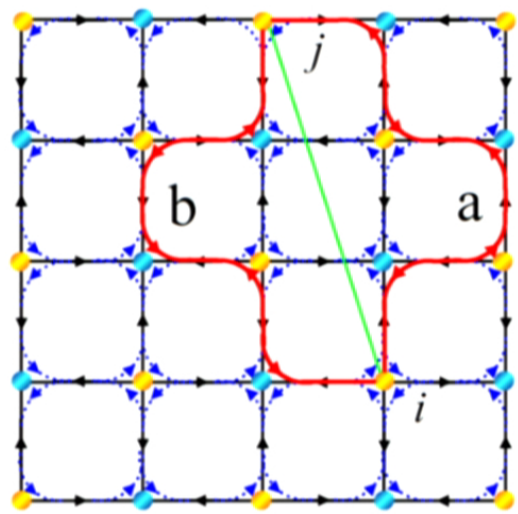
$$b_2 = -0.68(1)$$



A linear fitting leads to a negative constant close to ln2

All these results can be understood as vanishing of same sublattice pairing in our variational ansatz, which describes a U(1) spin liquid



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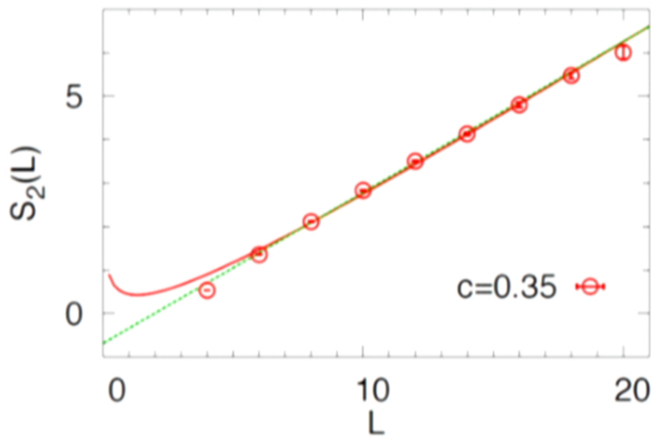

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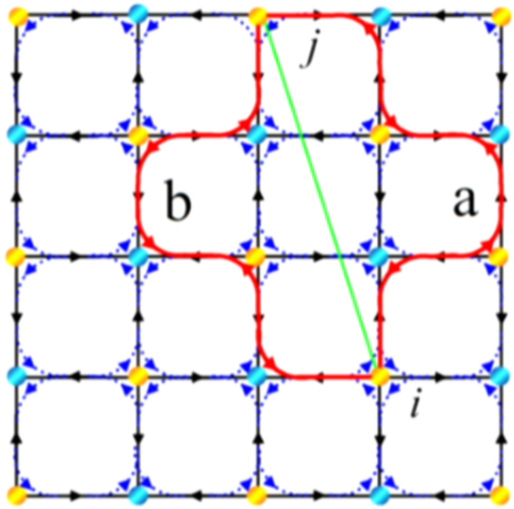

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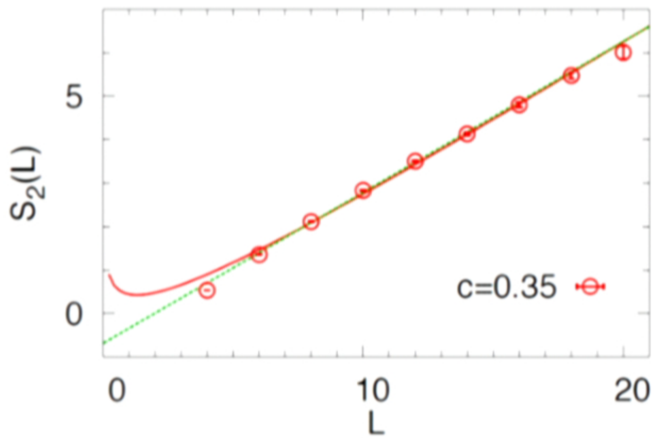

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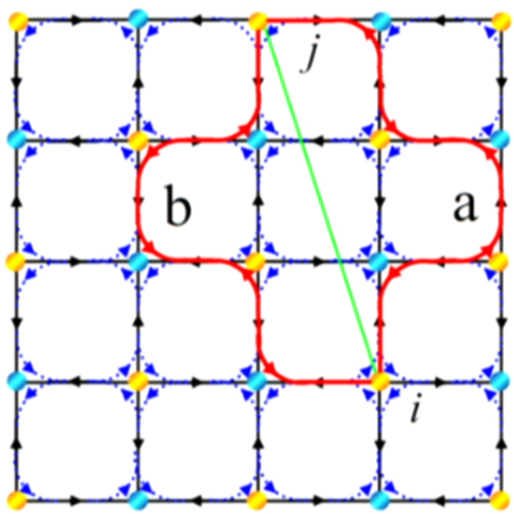

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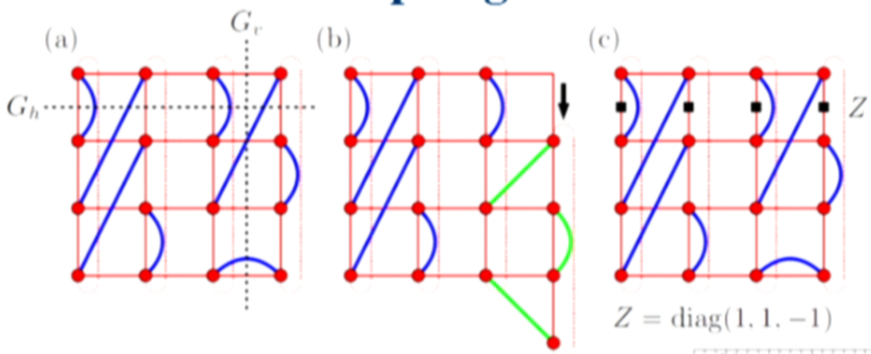

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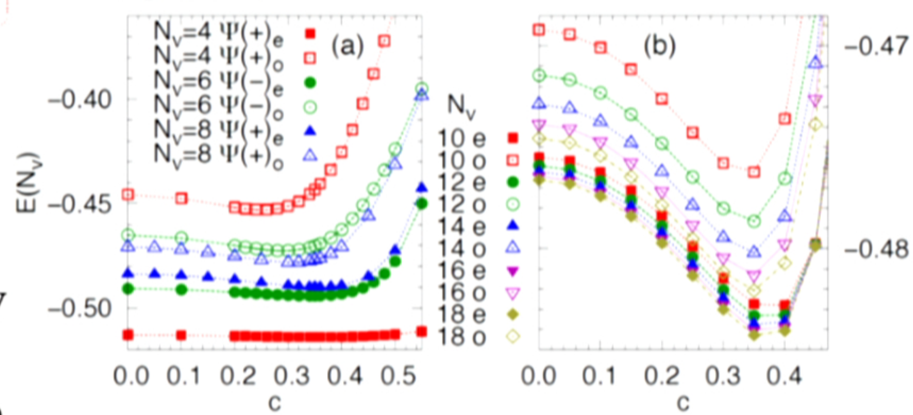

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Single parameter variational approach

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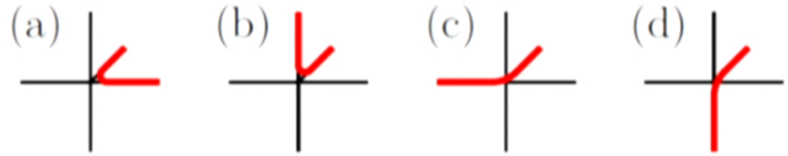

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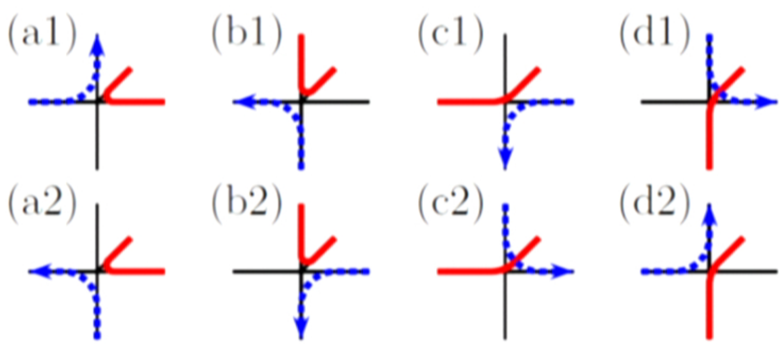


A single-parameter ansatz

$$\mathcal{P} = \mathcal{P}_1 + \epsilon\mathcal{P}_2$$

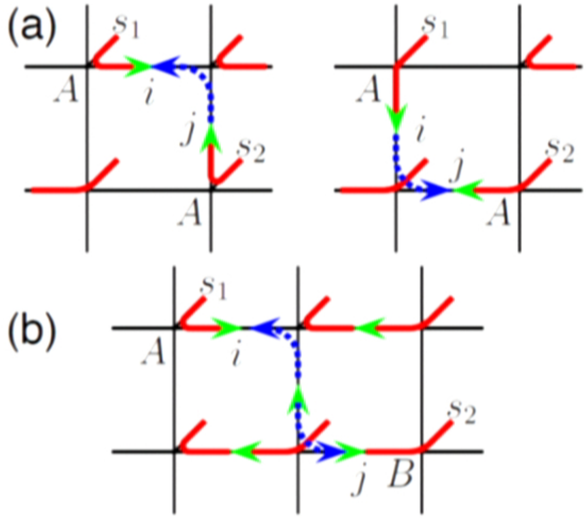
Discussions and future directions:

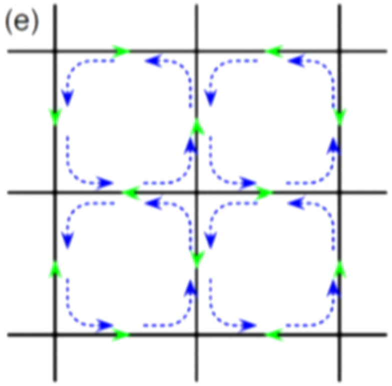
Other variational approach

- The best Schwinger VMC approach also predicts gapless U(1) spin liquid.
- The best Slave boson VMC approach predicts gapless Z2 spin liquid with a very small vison gap.
- In general, symmetric spin liquid must be gapless if it is close to AF state.

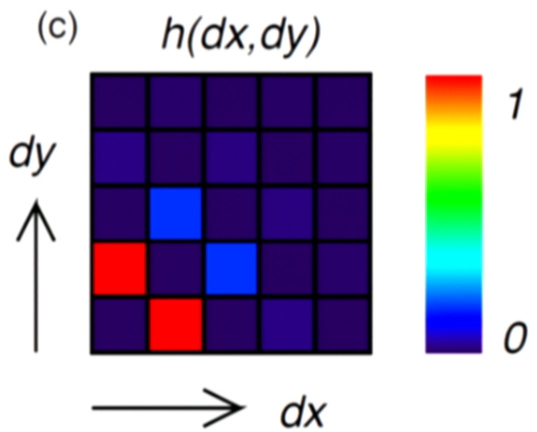
Properties of the variational state

Longer range RVB configurations can be generated through quantum teleportation





- Sign convention for bond singlets



Longer range RVB decays exponentially