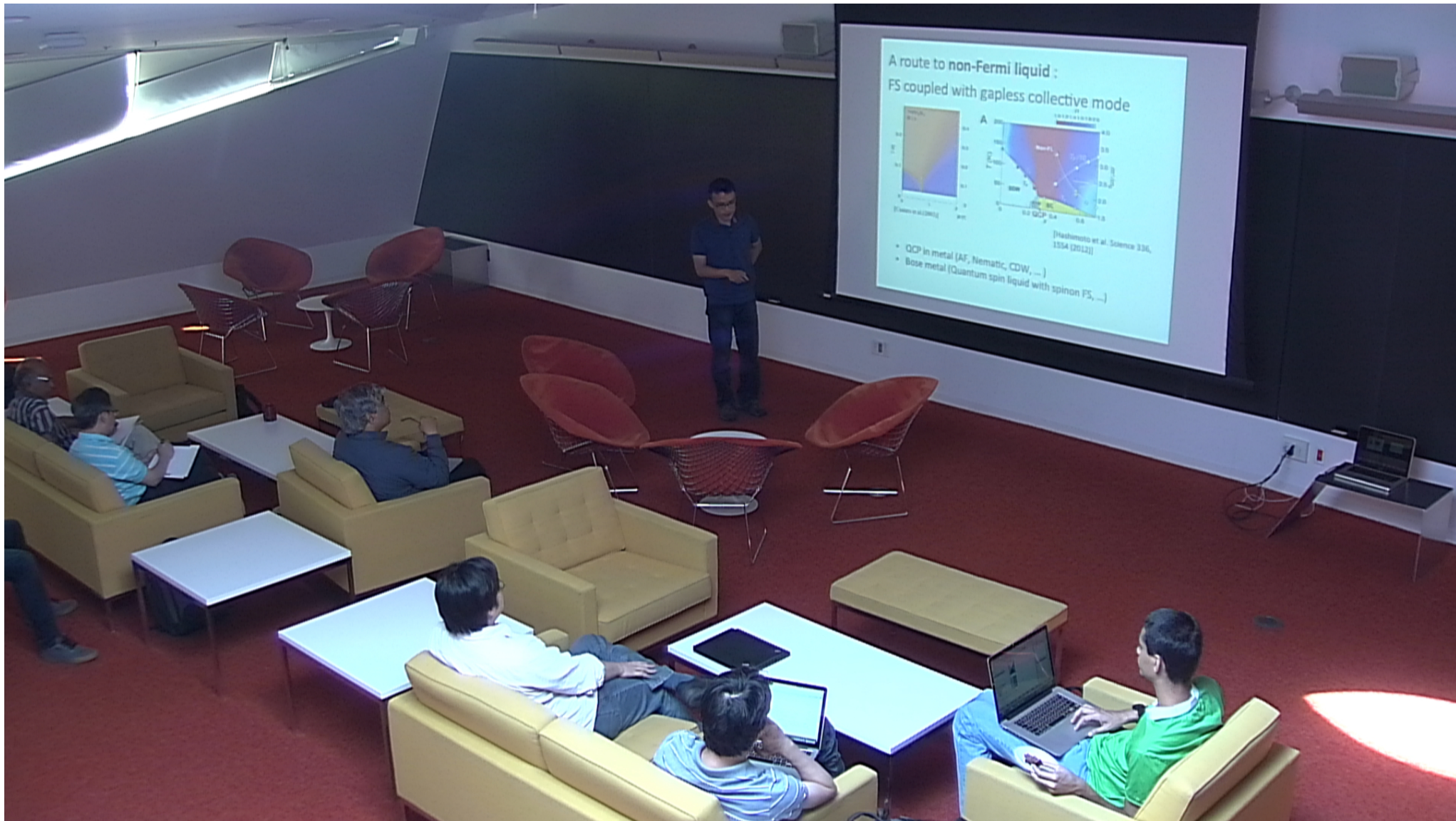


Title: Anisotropic Non-Fermi Liquids

Date: Jul 10, 2015 10:00 AM

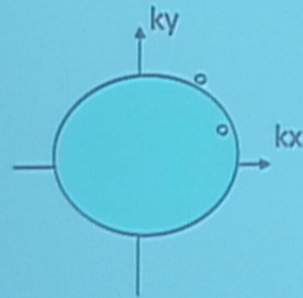
URL: <http://pirsa.org/15070061>

Abstract: We study non-Fermi liquids that arise at the quantum critical points associated with the spin and charge density wave transitions in metals with the C_2 symmetry. We use the dimensional regularization scheme, where a one-dimensional Fermi surface is embedded in $3 + \epsilon$ dimensional momentum space. In three dimensions, marginal Fermi liquids arise at the spin and charge density wave critical points. Below three dimensions, a perturbative anisotropic non-Fermi liquid is realized at the spin density wave critical point, where not only time but also different spatial coordinates develop distinct anomalous dimensions. On the other hand, the perturbative expansion breaks down at the charge density wave critical point immediately below three dimensions.

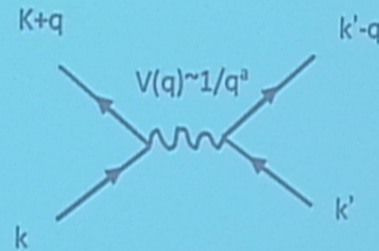




Long-range force destroys coherent quasiparticle



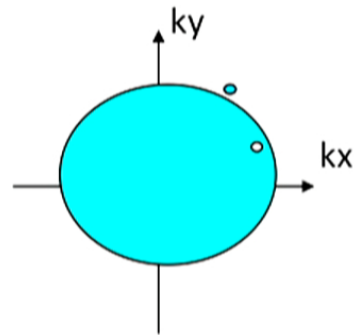
Fermi surface
+ gapless boson



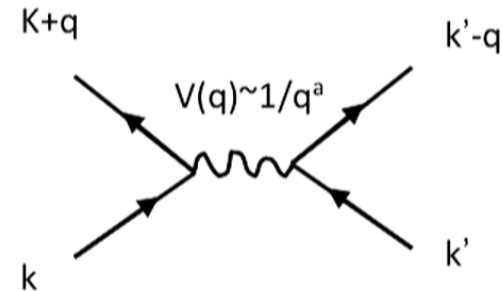
Non-forward scatterings are enhanced by long-range interactions
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No quasiparticle description

Long-range force destroys coherent quasiparticle



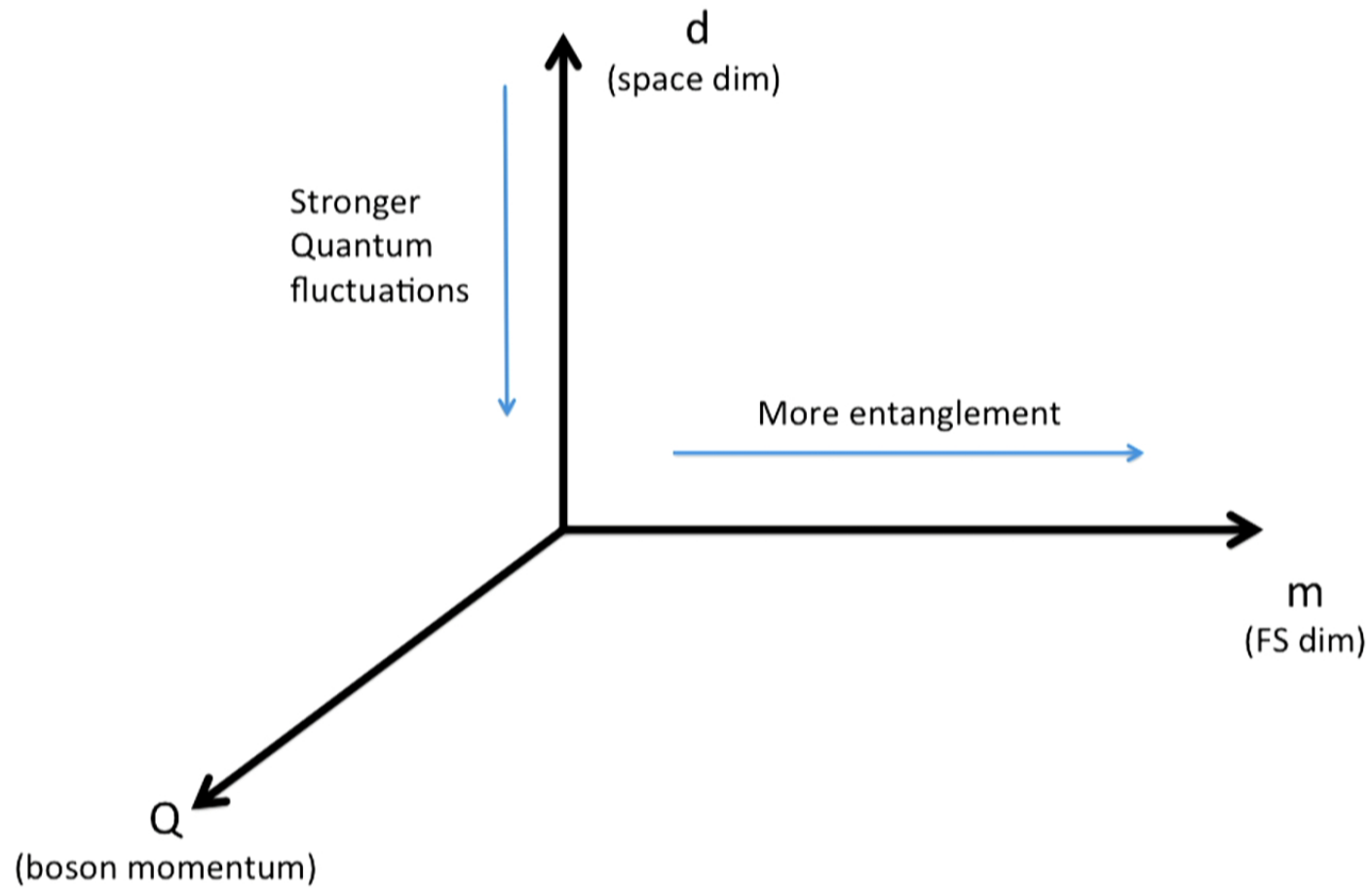
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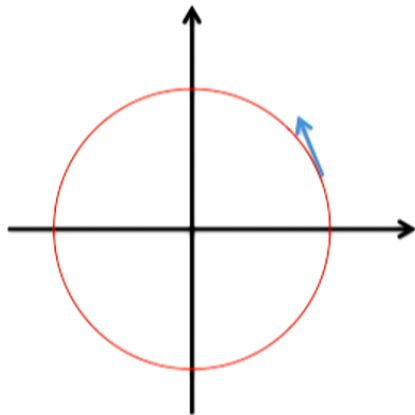
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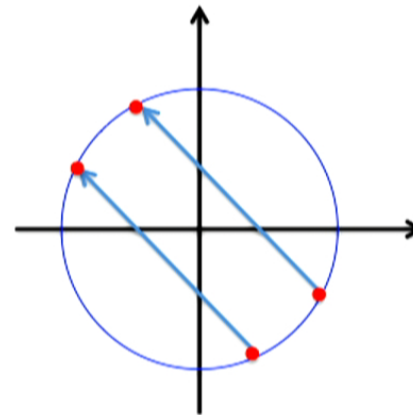
A classification of NFL



Holy Grail : NFL in $d=2$

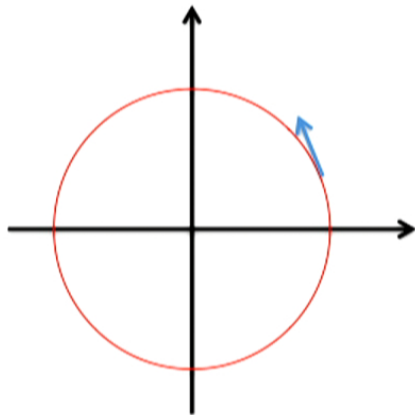


$Q=0$
Nematic, ferromagnetic QCP
Spin liquids with emergent gauge boson

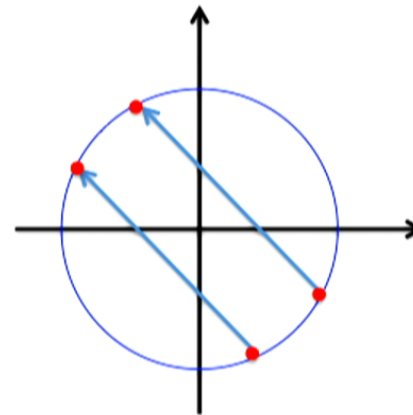


$Q \neq 0$
Spin & CDW QCP

Holy Grail : NFL in $d=2$



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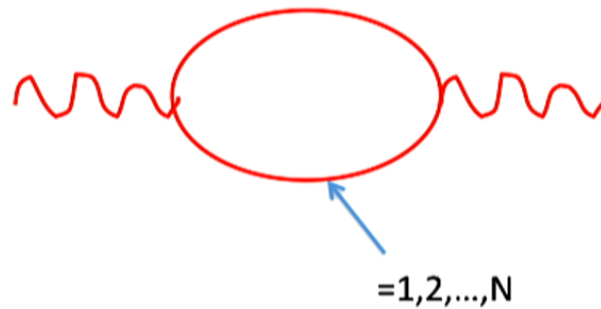


$Q \neq 0$
Spin & CDW QCP

Non-Fermi liquids in 2+1D

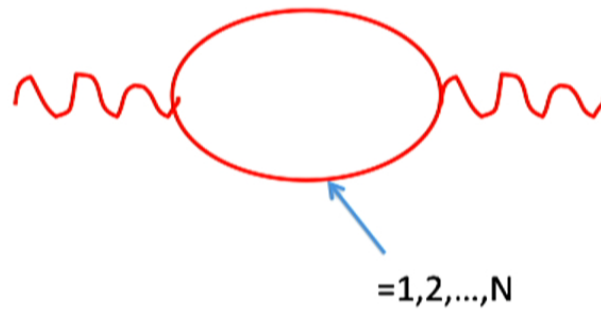
- Coupling between fermion and boson become strong
- How to tame quantum fluctuations?

1/N expansion



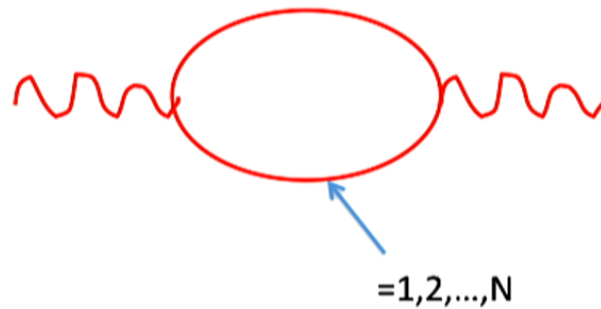
- In the large N limit (fermion flavors), boson drags a large
- Boson gets dressed heavily with fermion clouds, and fluctuations of boson are suppressed

1/N expansion



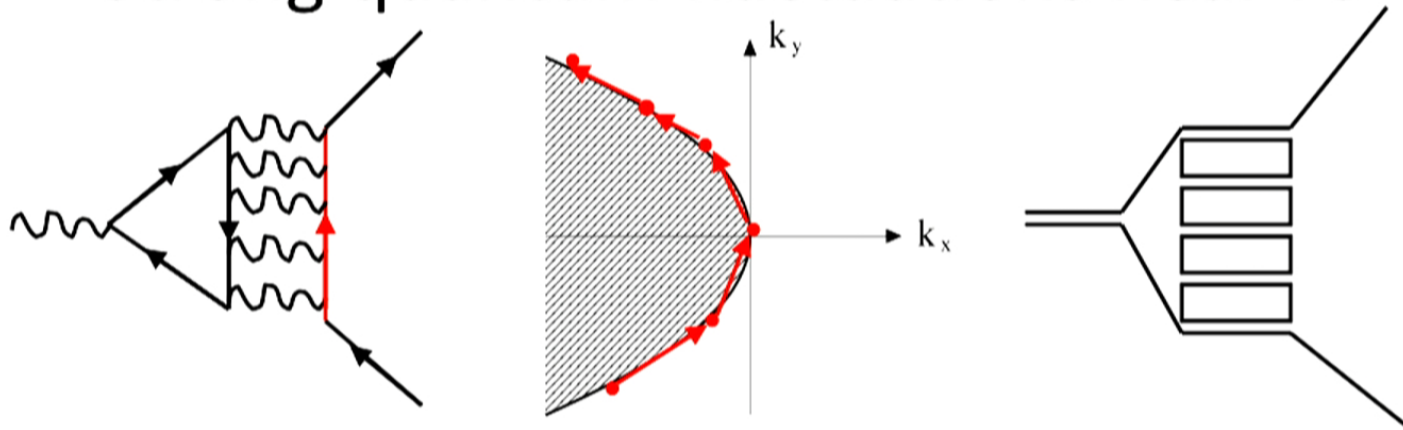
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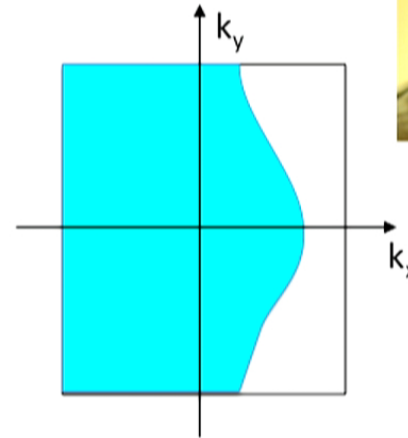
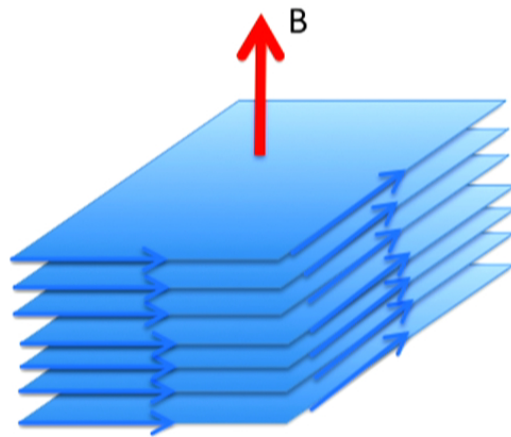
- In the large N limit (fermion flavors), boson drags a large
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Strong quantum fluctuations near FS



- Quantum fluctuations are enhanced by abundant gapless modes near the FS
- Quantum fluctuations are not tamed even in the large N limit [SL(09); Metlitski and Sachdev(10)]
- The nature of the NFL in the large N limit remains to be understood, except for the chiral NFL

Chiral NFL

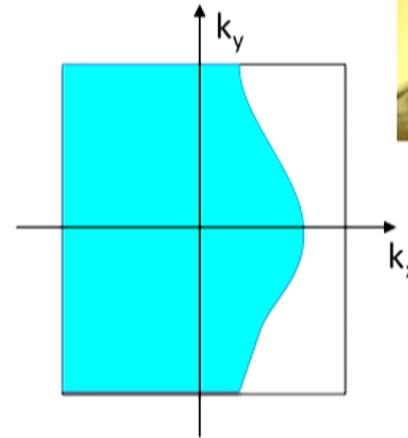
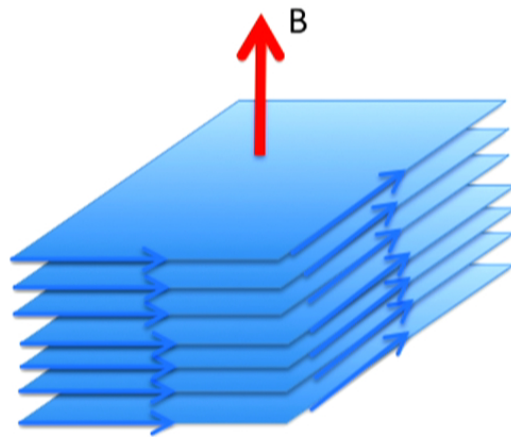


- One-loop scaling is exact due to the **holomorphic structure**
- **Exact Scaling form** of the Green's function :

$$G^{-1}(k) = (k_x + k_y^2)g(|\omega|^{2/3}/(k_x + k_y^2))$$

[Sur, SL (13)]

Chiral NFL



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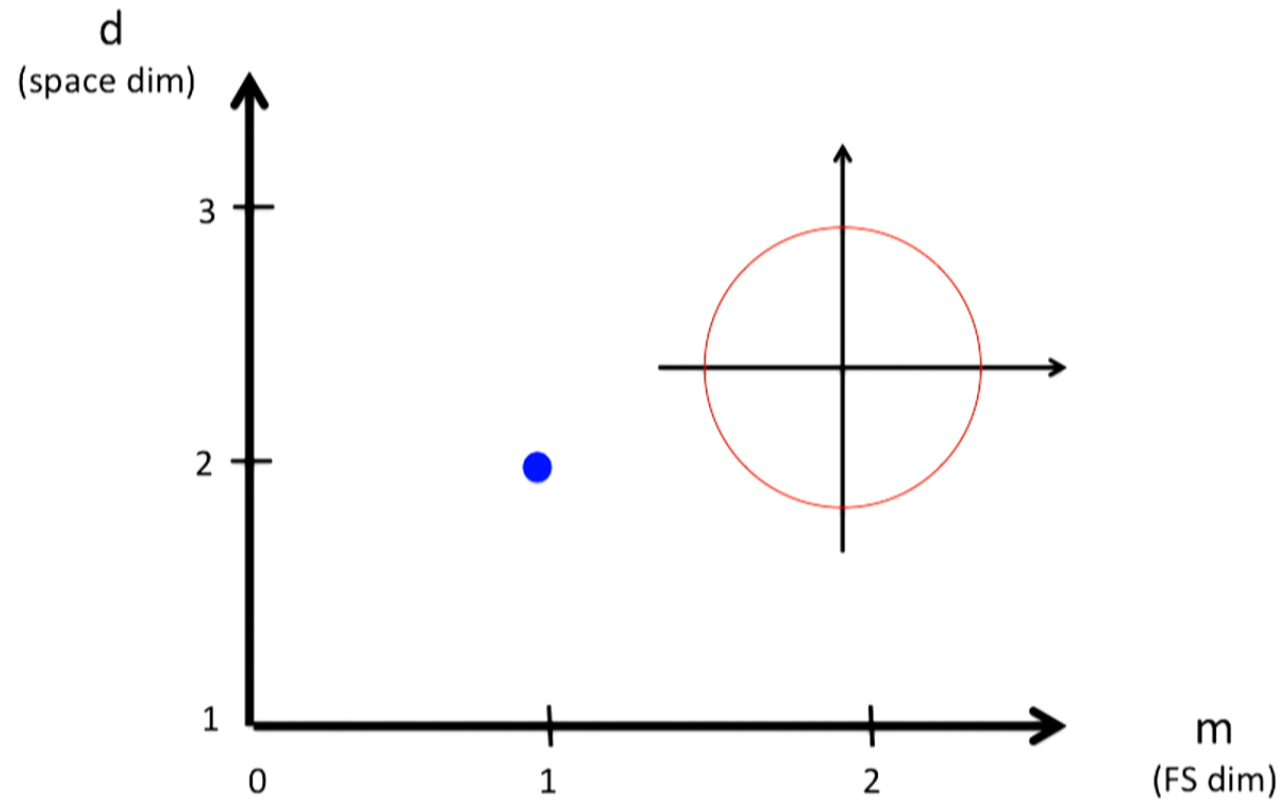
Controlled NFL in non-chiral cases

- Dynamical tuning (modified boson dispersion $\sim |k|^{1+\epsilon}$) [Nayak, Wilczek (94)]
 - $1/N, \epsilon$ double expansion [Mross, McGreevy, Liu, Senthil (10)]
 - SC instability [Metlitski, Mross, Sachdev, Senthil (14)]
 - All symmetries kept
 - Locality is lost
- Perturbative NFL based on dim. Reg.
 - Ising-nematic [Dalidovich, SL (13)]
 - SDW [Sur, SL (14)]
 - Locality is kept
 - Some symmetries are broken

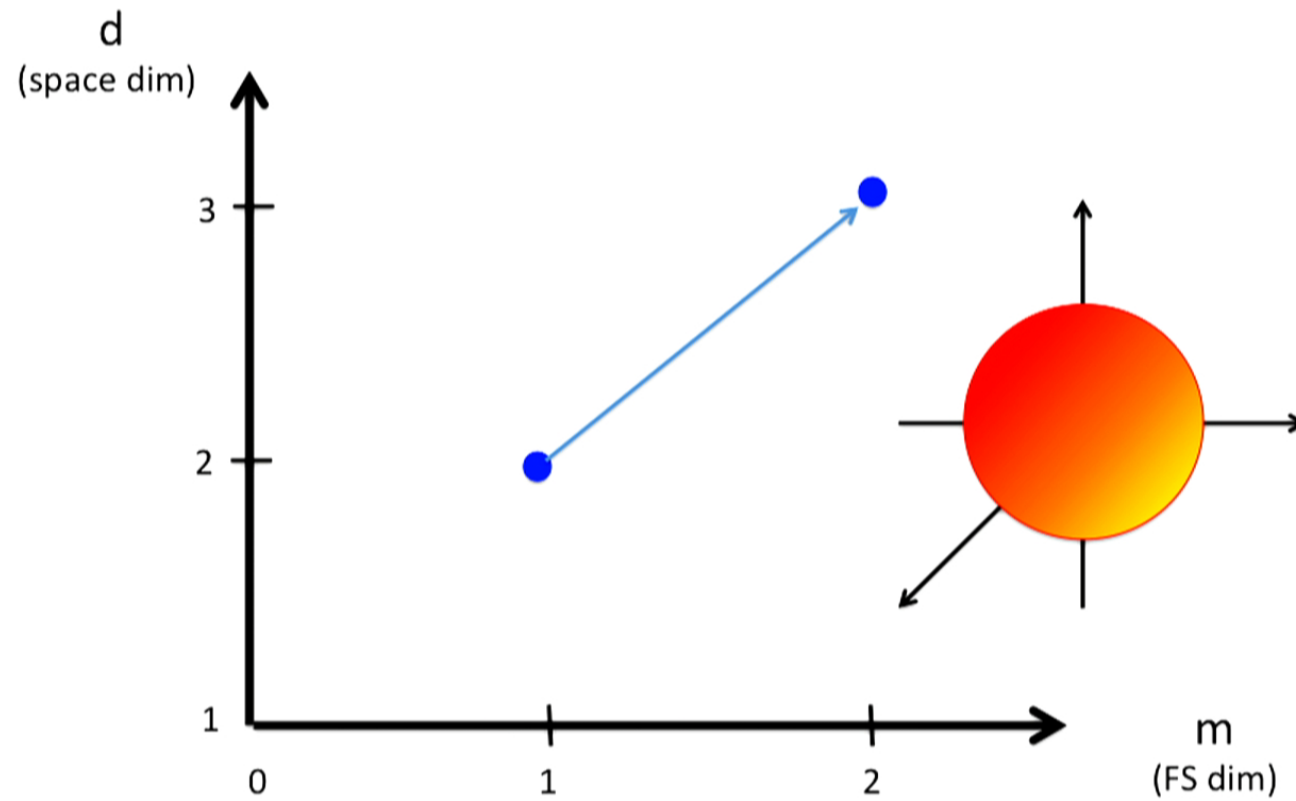
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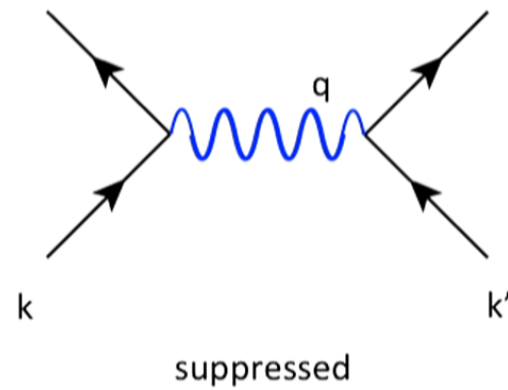
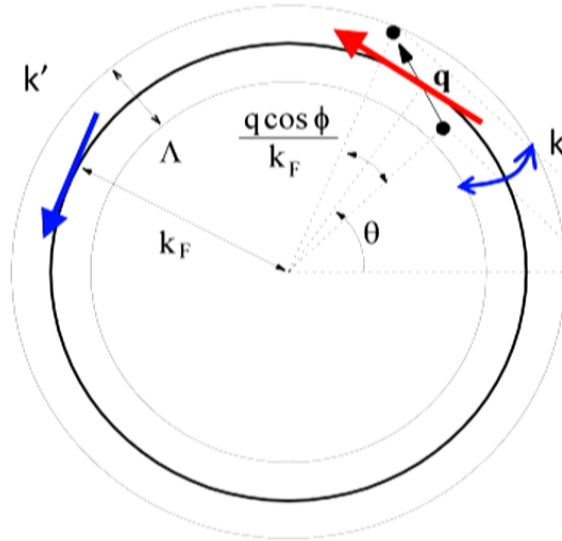
Tuning dimensions



Fix d-m

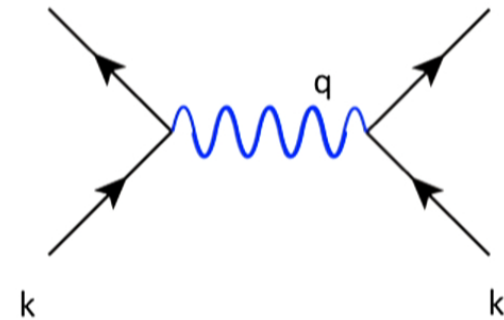
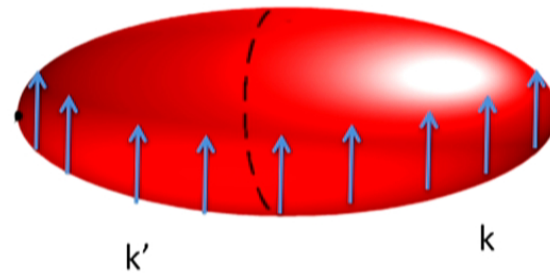


UV insensitivity for $m=1$



- Electron spectral function at a momentum can be obtained without the information about the entire Fermi surface

UV/IR mixing for $m > 1$

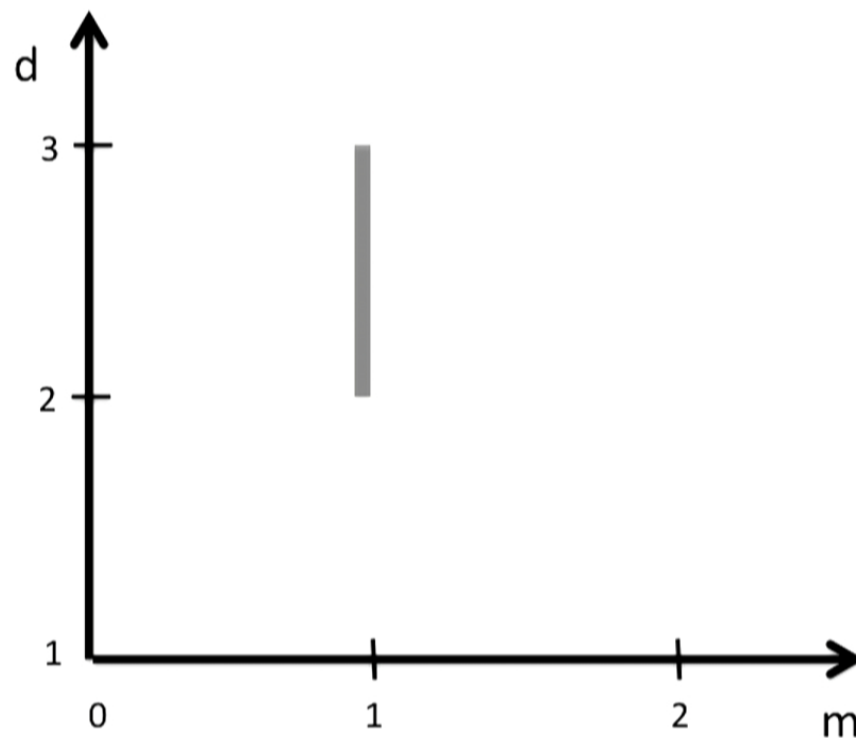


Not suppressed

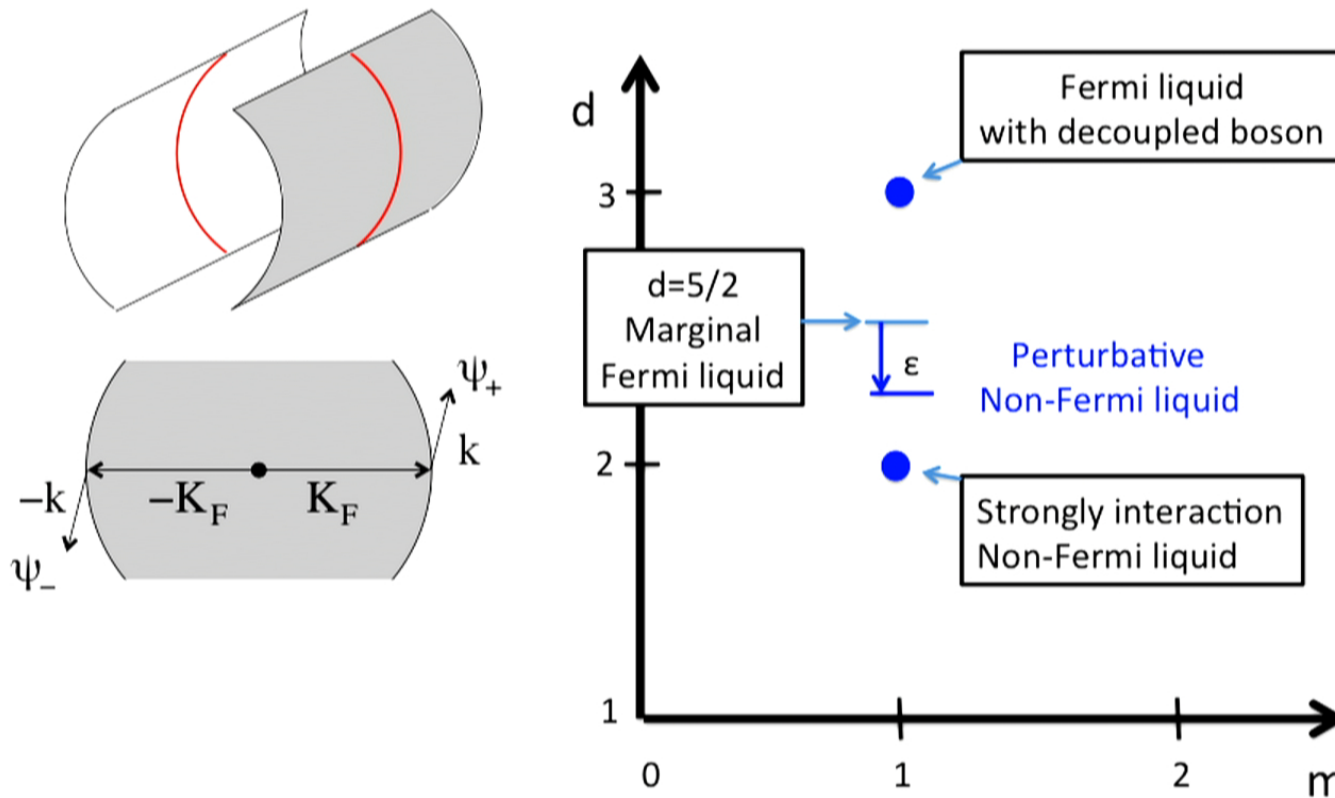
- Modes with large momenta singularly affect low energy physics in the large k_F limit
- Electron spectral function at a momentum is sensitive to the entire Fermi surface
- Low energy effective theory can not be specified without specifying the size/shape of FS

[I. Mandal, SL (14)]

Theories with $m > 1$ are qualitatively different from theories with $m = 1$ ($d = 2$)

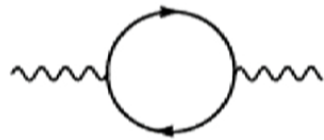


A continuous interpolation between 2d Fermi surface to 3d p-wave SC



Expansion in $e^{4/3}$ instead of e^2

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920 \left(\frac{3}{2} - \epsilon \right) \frac{e^{7/3}}{N} + 0.01073 \left(\frac{3}{2} - \epsilon \right) \frac{e^{11/3}}{N^2}$$



$$D_1(k) = \frac{1}{\boxed{|\vec{K}|^2 + k_x^2} + k_y^2 + \beta_d e^2 \frac{|\vec{K}|^{d-1}}{|k_y|}}$$

irrelevant

$$\vec{K} = k_0, k_1, \dots, k_{d-2}$$

Landau damping, which is generated by interaction, dominates over the bare kinetic term

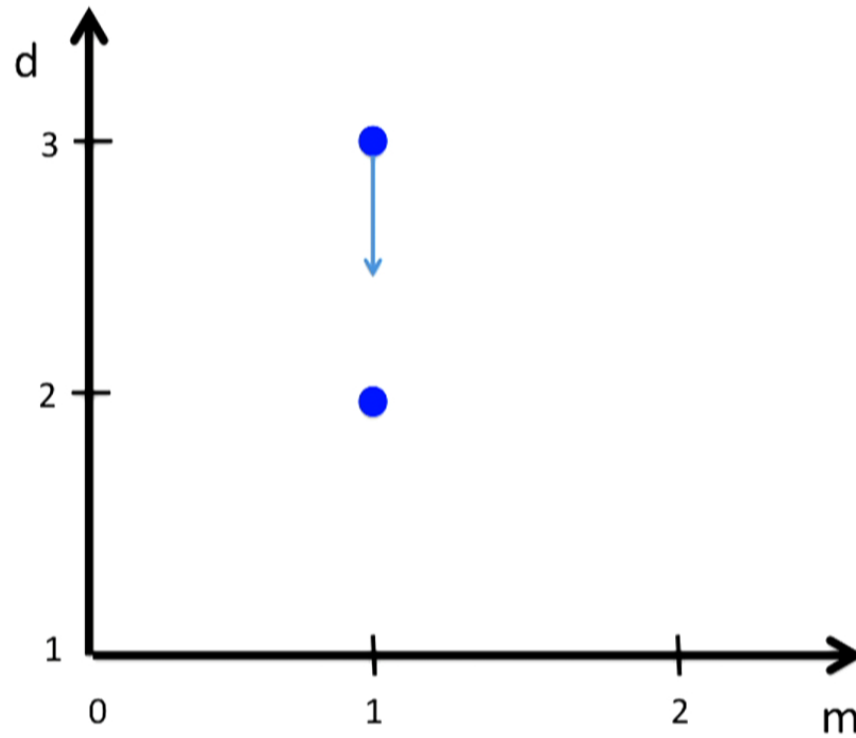
Physical properties

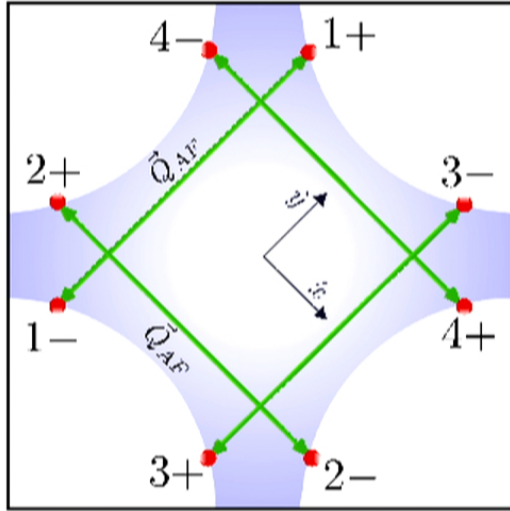
- Fermion Green fnt : $G(k) = \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}} g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right)$
- Boson Green fnt : $D(k) = \frac{1}{k_d^2} f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right)$
- Specific heat : $c \sim T^{(d-2)+\frac{1}{z}}$ $z = \frac{3}{3-2\epsilon}$
- No SC instability for small ϵ , but SC fluctuations are enhanced

Physical properties

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$$m=1, Q \neq 0$$



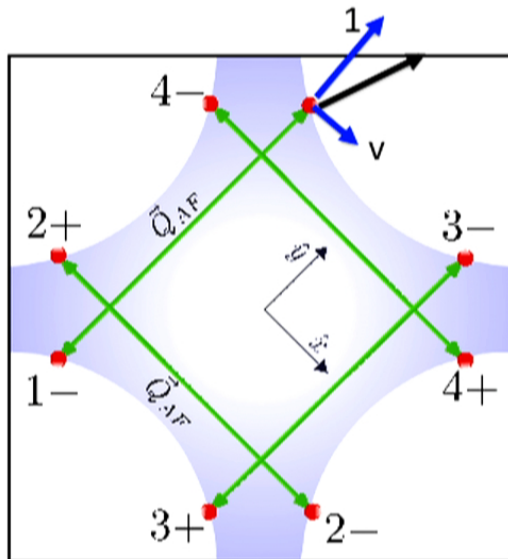


Minimal Theory for SDW in 2d

[Abanov and Chubukov]

$$\begin{aligned}
 \mathcal{S} = & \sum_{l=1}^4 \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3 k}{(2\pi)^3} \psi_{l,\sigma}^{(m)*}(k) \left[i k_0 + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\
 & + \frac{1}{2} \int \frac{d^3 q}{(2\pi)^3} [q_0^2 + c^2 |\vec{q}|^2] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\
 & + g_0 \sum_{l=1}^4 \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right] \\
 & + \frac{u_0}{4!} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \left[\vec{\Phi}(k_1 + q) \cdot \vec{\Phi}(k_2 - q) \right] \left[\vec{\Phi}(k_1) \cdot \vec{\Phi}(k_2) \right]
 \end{aligned}$$

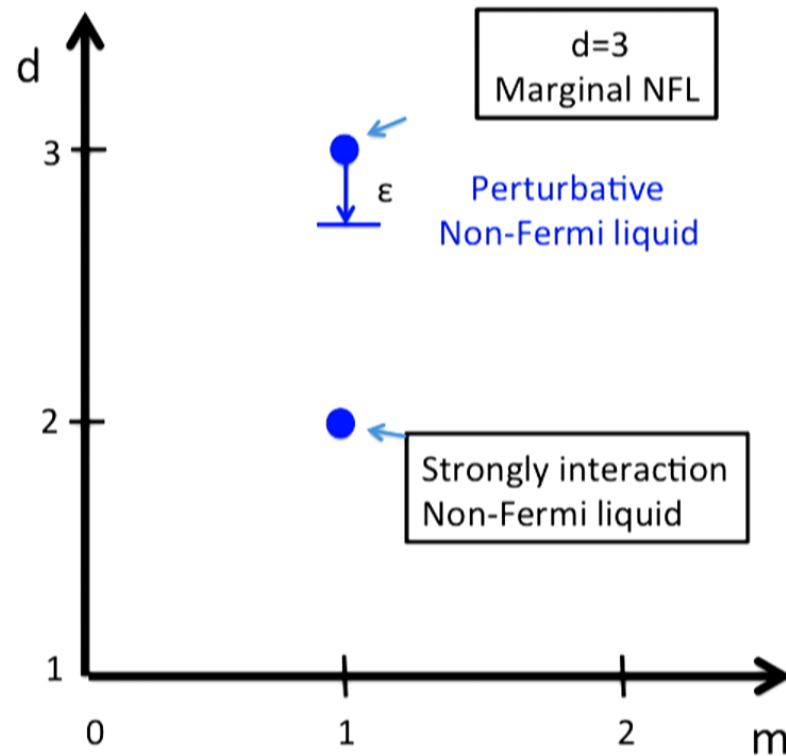
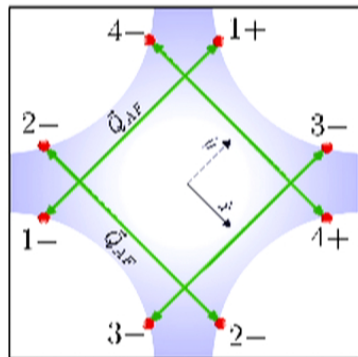
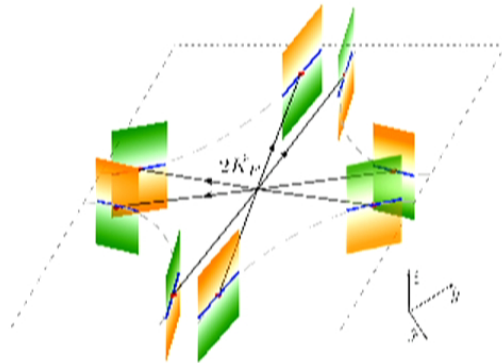
Parameters of the theory



- v : Fermi velocity
perpendicular to \vec{Q}_{AF}
- c : boson velocity
- g : Yukawa coupling
- u : quartic boson coupling

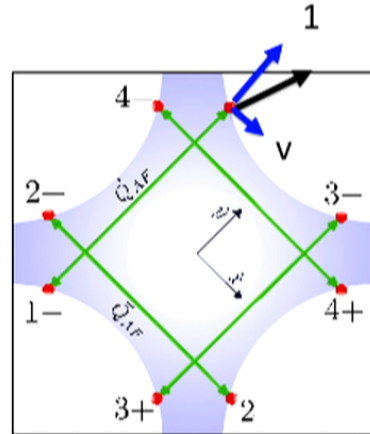
- If $v=0$, hot spots connected by \vec{Q}_{AF} are nested
- The four parameters can not be scaled away

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes



IR fixed point in 3d : exact

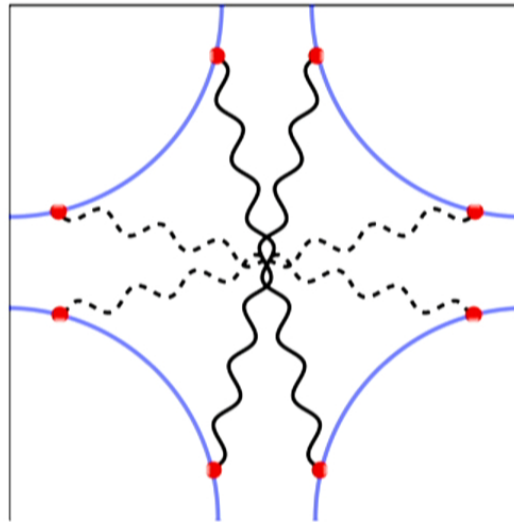
- Marginal FL with $z=1$
- FS nested FS as $v \sim 1/\log(\log(L))$
- Boson loses dispersion in the place of line nodes
- Stable Quasi-local Marginal Fermi liquid



[S. Sur, SL (14)]

Enhanced SC correlation in QLSM

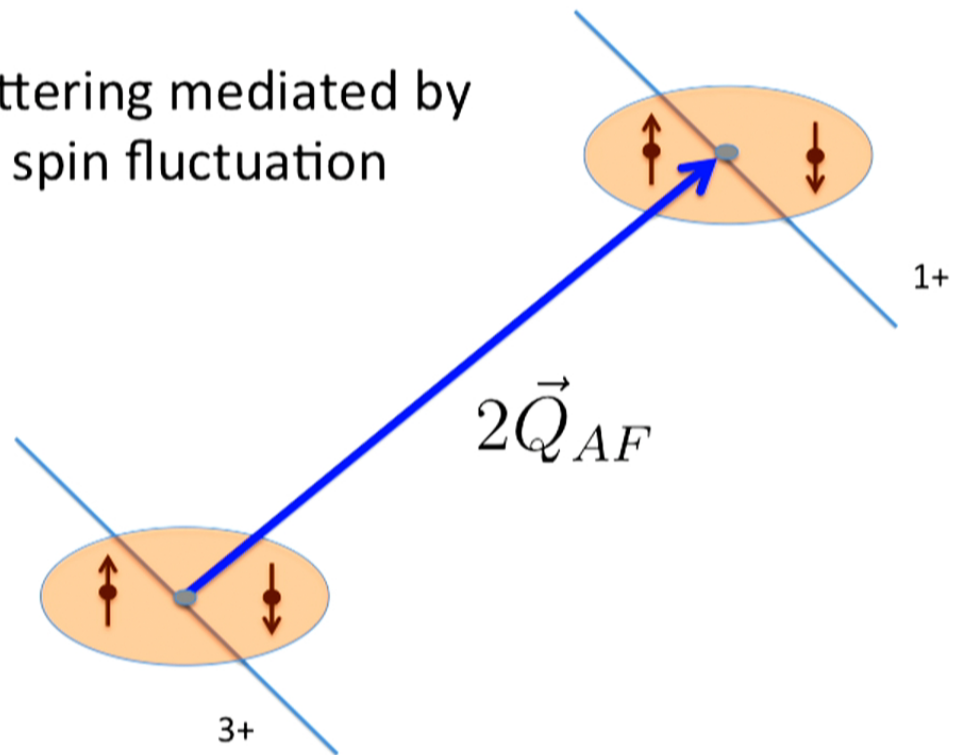
2) d-wave



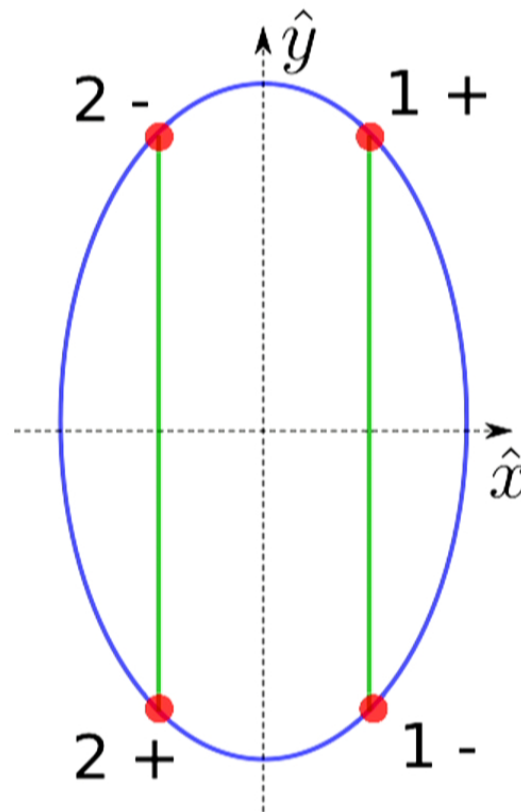
Related to the d-wave bond density wave through pseudospin rotation [Metlitski, Sachdev]

Physical mechanism for FFLO

Inter-patch scattering mediated by commensurate spin fluctuation

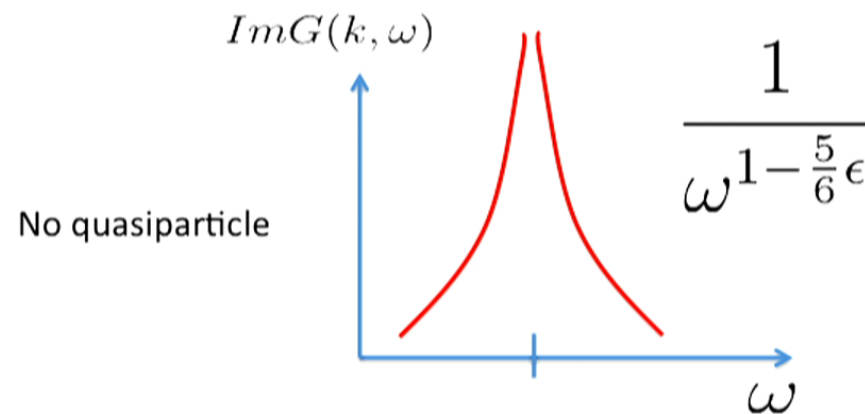


C_2 symmetric case



IR fixed point in $(3-\epsilon)d$: 1-loop

- NFL with $z > 1$
- FS nested FS as $v \sim 1/(\log(L))$
- Boson lose dispersion in the place of line nodes (not protected from higher-loop corrections)
- Stable Quasi-Local Strange Metal



[S. Sur, SL (14)]

Anisotropic scaling

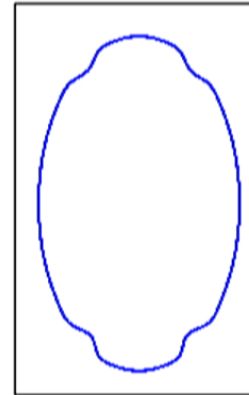
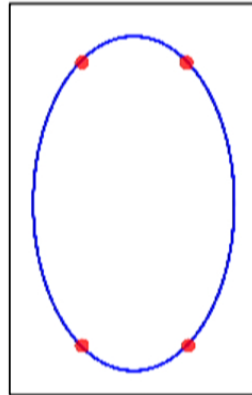
$$z_\tau = 1 + \frac{\aleph(N_c, N_f)}{2} \epsilon - 8 \left(2 + \frac{\aleph(N_c, N_f)}{N_c^2 - 1} \right) \left(\frac{2\aleph^4(N_c, N_f)}{N_c N_f} h_6(v_*) \right)^{1/3} \epsilon^{4/3},$$

$$z_x = 1 - 16 \left(\frac{2\aleph^4(N_c, N_f)}{N_c N_f} h_6(v_*) \right)^{1/3} \epsilon^{4/3},$$

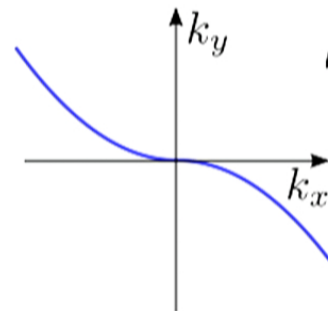
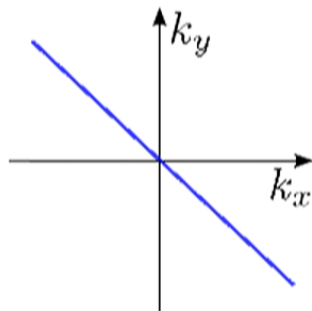
$$\tilde{\eta}_\psi = 4 \left(\frac{2\aleph^4(N_c, N_f)}{N_c N_f} h_6(v_*) \right)^{1/3} \epsilon^{4/3},$$

$$\tilde{\eta}_\phi = 16 \left(\frac{2\aleph^4(N_c, N_f)}{N_c N_f} h_6(v_*) \right)^{1/3} \epsilon^{4/3}$$

FS is deformed into a universal shape



[S. Sur, SL, to appear]



$$k_y \sim \text{sgn}(k_x) |k_x|^{1/z_x}$$

$$z_x = 1 - 16 \left(\frac{2N^4(N_c, N_f)}{N_c N_f} h_6(v_*) \right)^{1/3} \epsilon^{4/3}$$

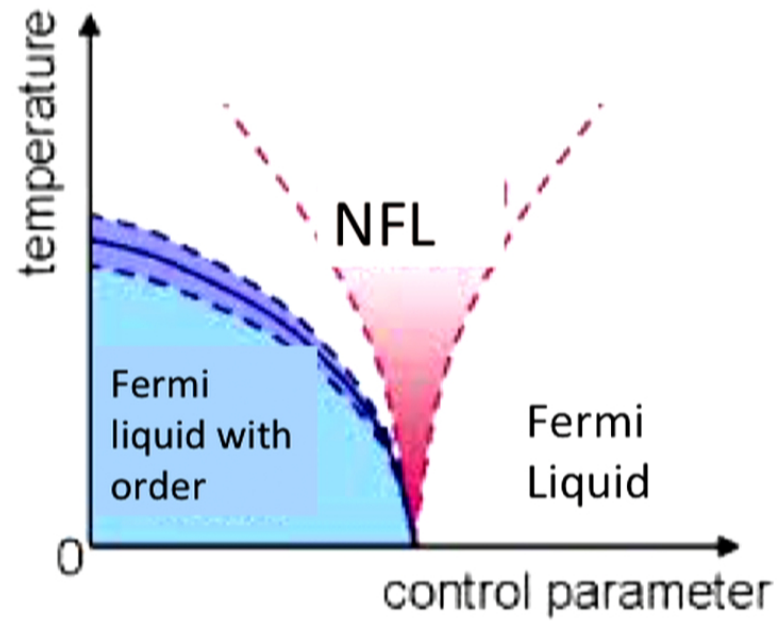
Summary

- Validity of patch theory for NFL with one-dimensional FS : absence of UV/IR mixing
- Perturbative non-Fermi liquids based on dimensional regularization with fixed FS dimension
 - $Q=0$: expansion in non-analytic powers of coupling
 - SDW with C4 : Emergent locality

Fate in 2d



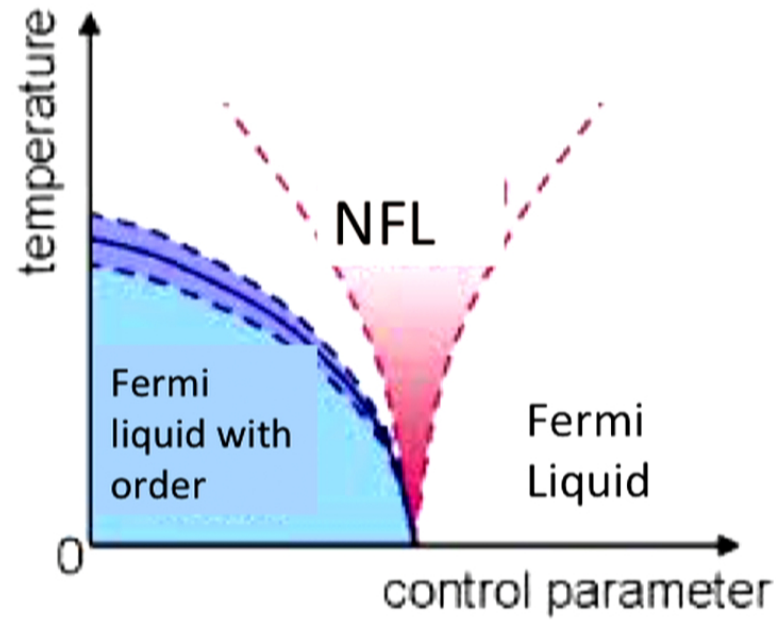
2d phase diagram



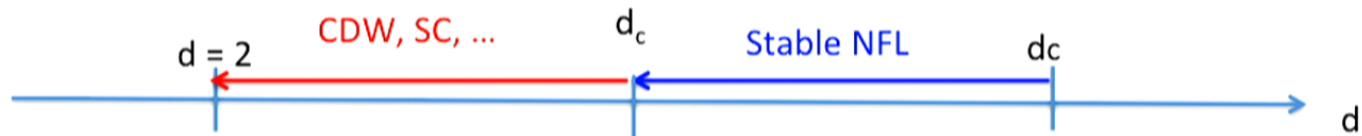
Fate in 2d



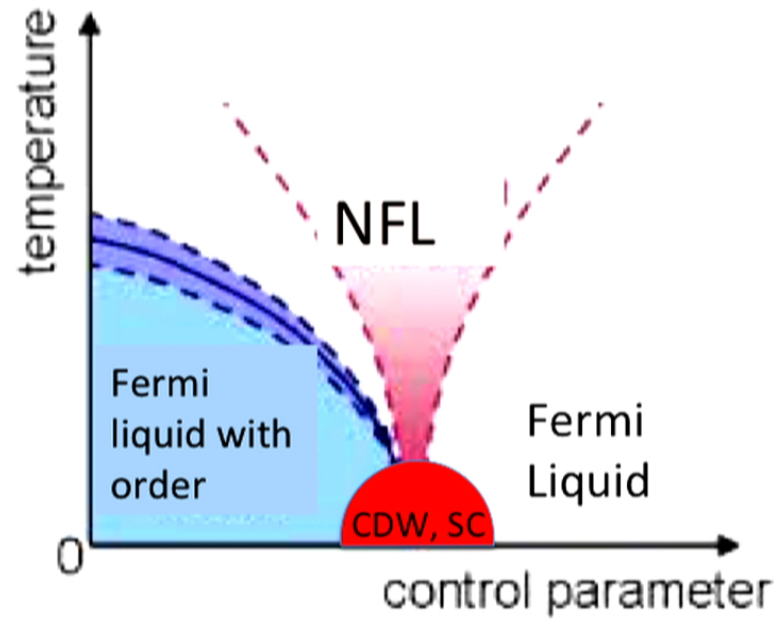
2d phase diagram



Fate in 2d



2d phase diagram



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 - SDW with C2 : Anisotropic scaling