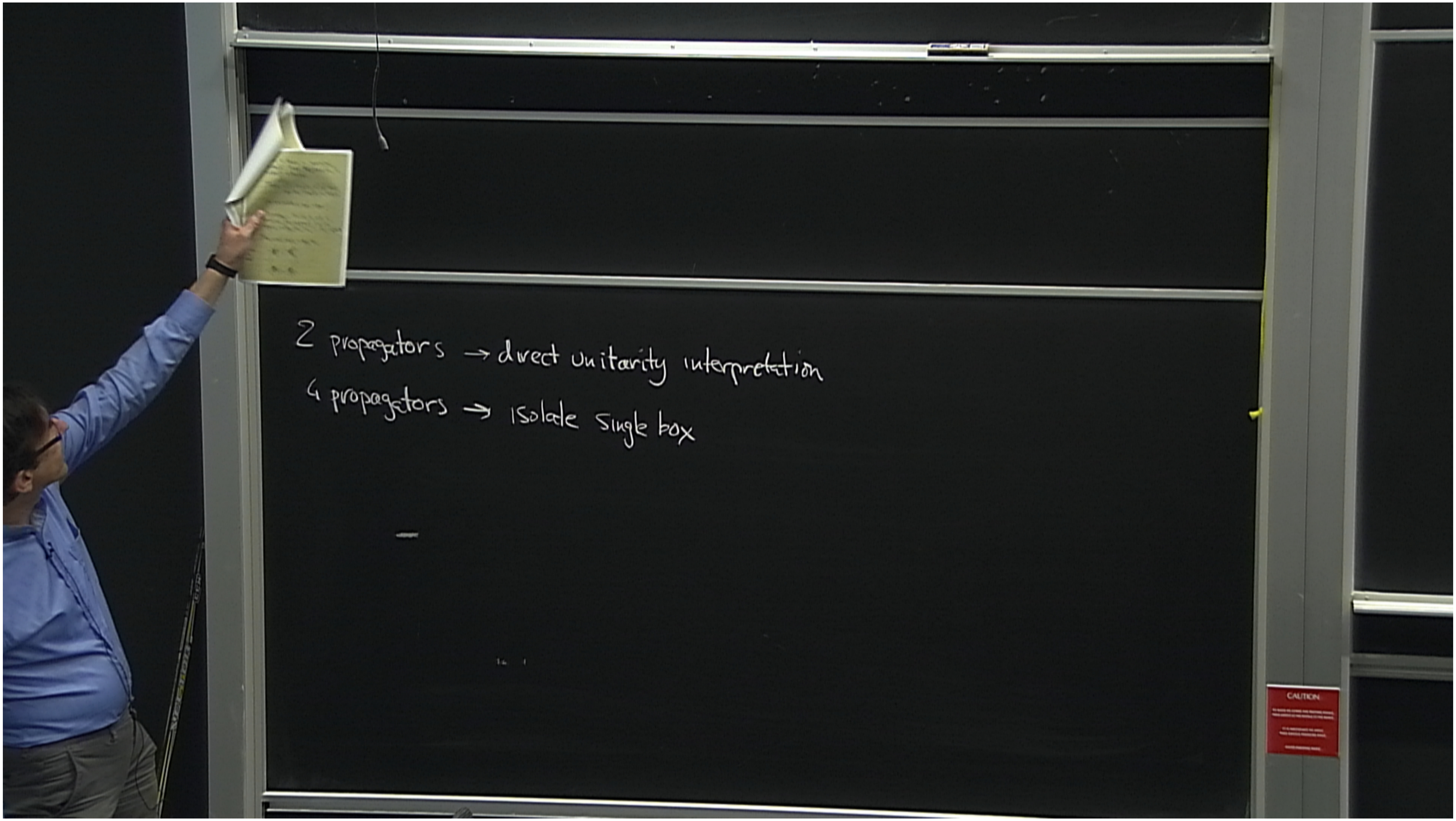
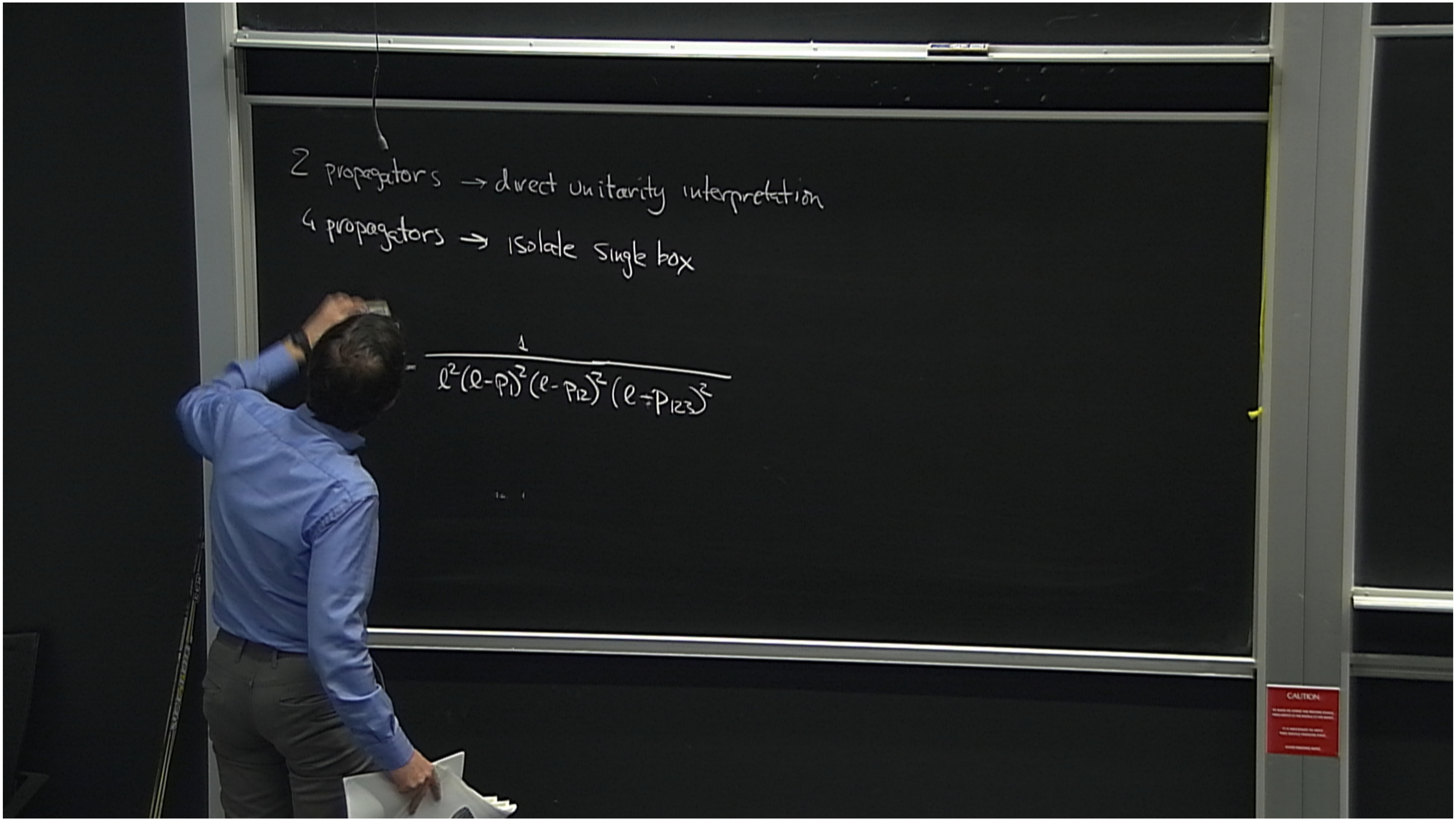
Title: Amplitudes: Generalized Unitarity

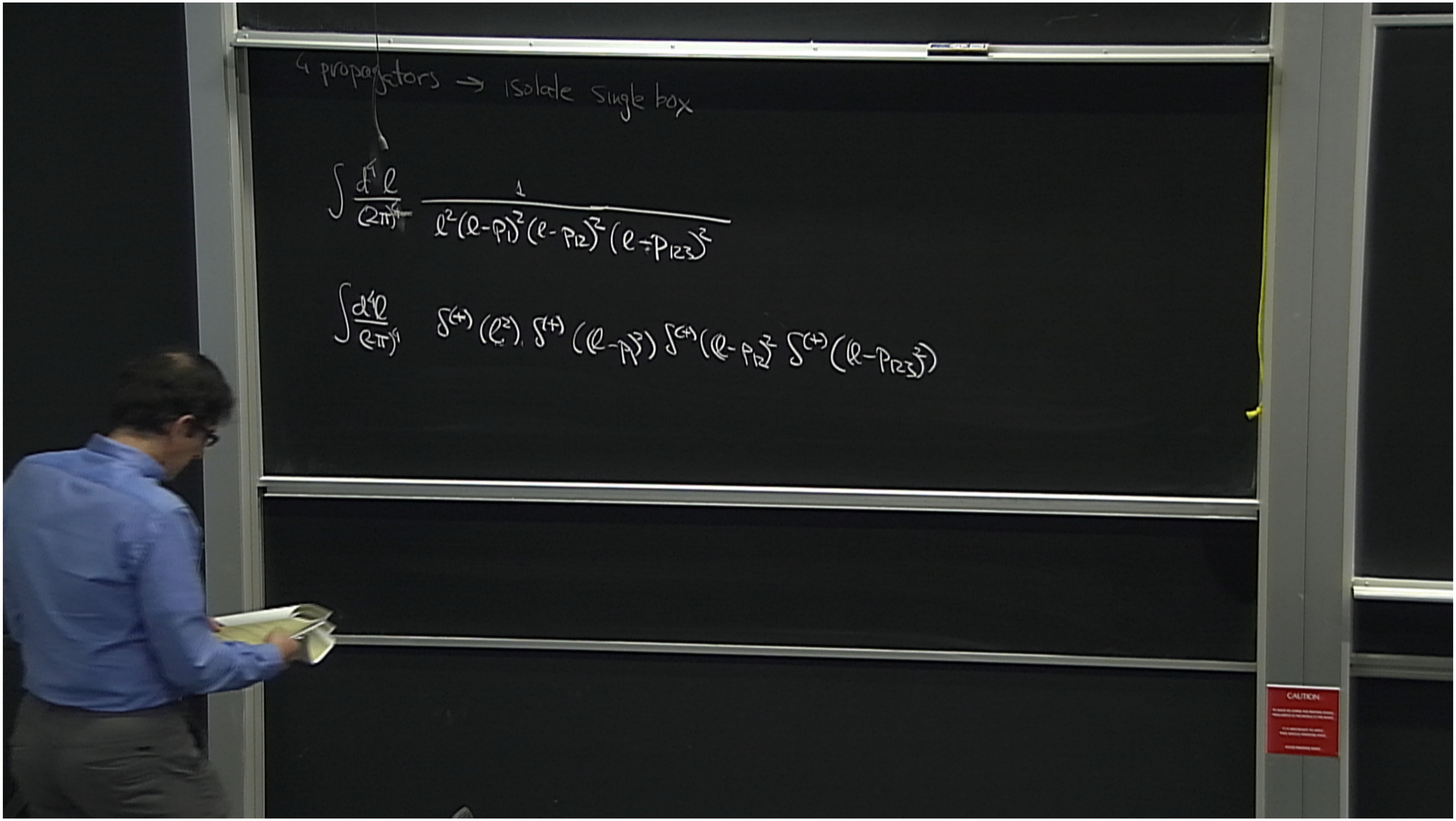
Date: Jul 17, 2015 02:30 PM

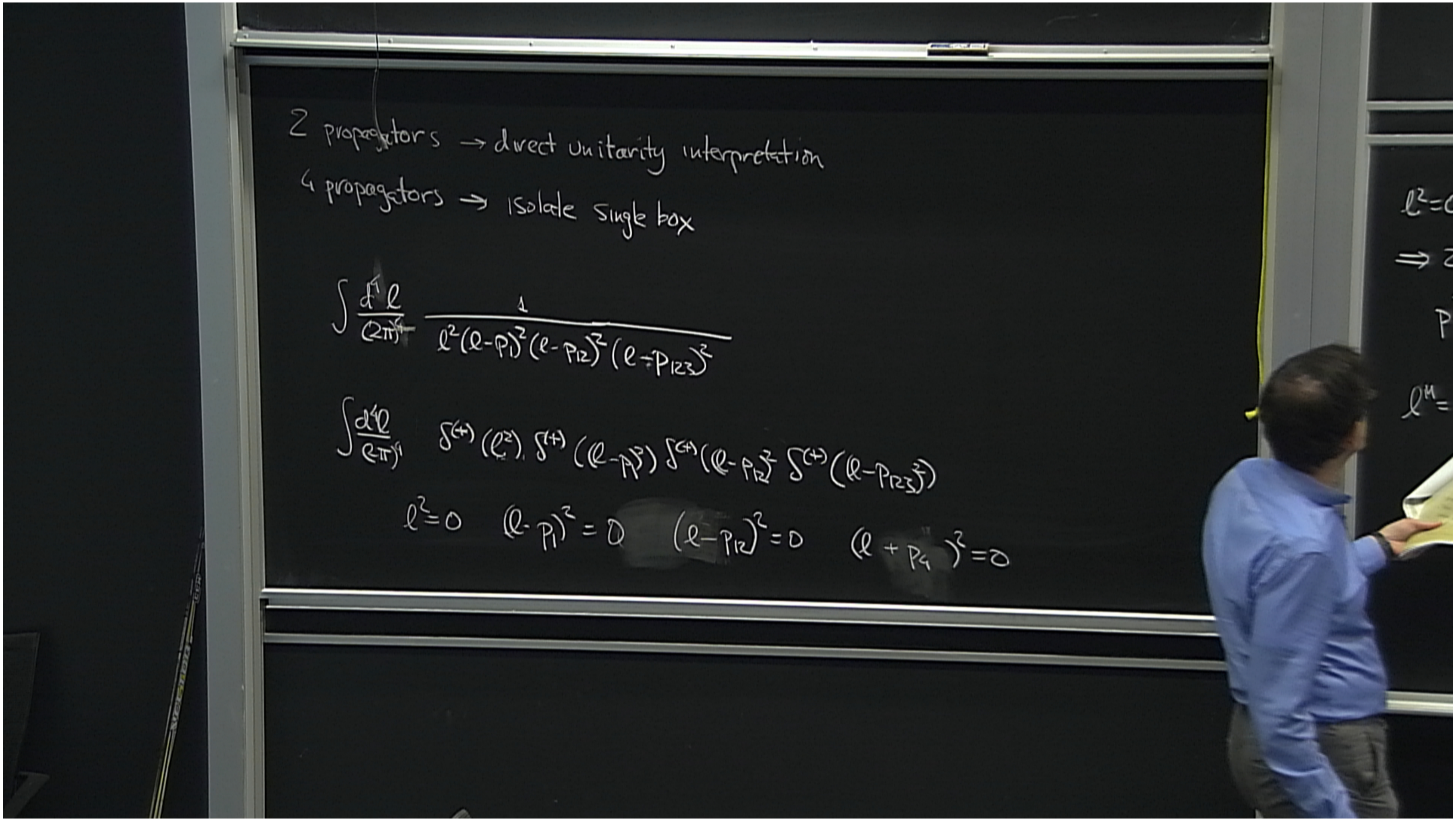
URL: http://pirsa.org/15070058

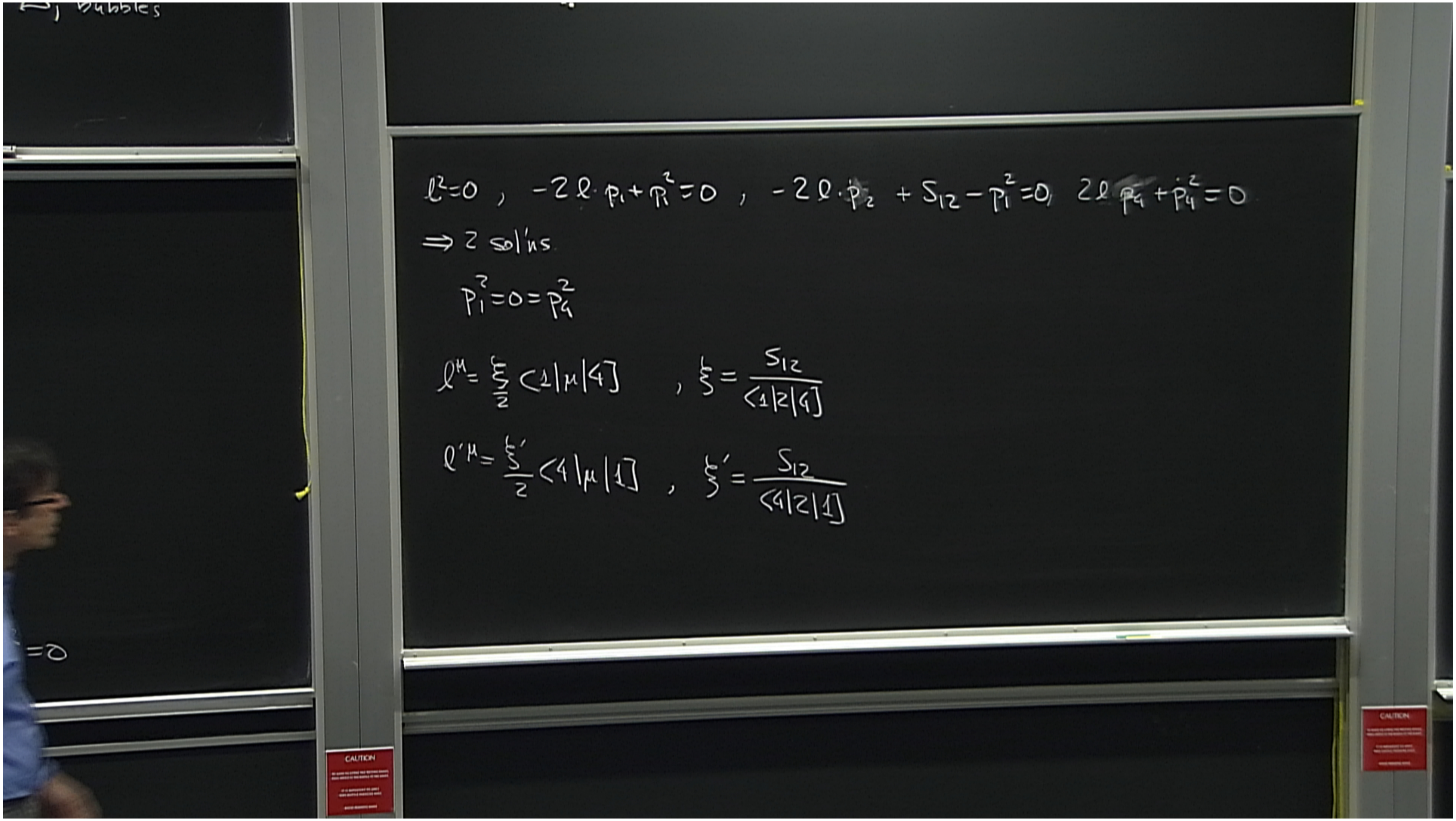
Abstract:


2 propagators → direct unitarity interpretation
4 propagators → isolate single box


2 propagators → direct unitarity interpretation
4 propagators → isolate single box
$= \frac{1}{\ell^2(\ell-p_1)^2(\ell-p_{12})^2(\ell-p_{123})^2}$


4 propagators → isolate single box
$\int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 (\ell - p_1)^2 (\ell - p_{12})^2 (\ell - p_{123})^2}$
$\int \frac{d^4 \ell}{(2\pi)^4} \delta^{(+)}(\ell^2)\, \delta^{(+)}((\ell - p_1)^2)\, \delta^{(+)}((\ell - p_{12})^2)\, \delta^{(+)}((\ell - p_{123})^2)$


2 propagators → direct unitarity interpretation
4 propagators → isolate single box
$\int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 (\ell - p_1)^2 (\ell - p_{12})^2 (\ell - p_{123})^2}$
$\int \frac{d^4 \ell}{(2\pi)^4} \delta^{(+)}(\ell^2)\, \delta^{(+)}((\ell - p_1)^2)\, \delta^{(+)}((\ell - p_{12})^2)\, \delta^{(+)}((\ell - p_{123})^2)$
$\ell^2 = 0 \quad (\ell - p_1)^2 = 0 \quad (\ell - p_{12})^2 = 0 \quad (\ell + p_4)^2 = 0$


Bubbles
$\ell^2=0$, $-2\ell\cdot p_1+p_1^2=0$, $-2\ell\cdot p_2+s_{12}-p_1^2=0$, $2\ell\cdot p_4+p_4^2=0$
$\Rightarrow$ 2 sol'ns
$p_1^2=0=p_4^2$
$\ell^\mu=\frac{\xi}{2}\langle 1|\mu|4]$, $\xi=\frac{s_{12}}{\langle 1|2|4]}$
$\ell'^\mu=\frac{\xi'}{2}\langle 4|\mu|1]$, $\xi'=\frac{s_{12}}{\langle 4|2|1]}$
$=0$

$$\ell^2=0\,,\quad -2\ell\cdot p_1+p_1^2=0\,,\quad -2\ell\cdot p_2+s_{12}-p_1^2=0\,,\quad 2\ell\cdot p_4+p_4^2=0$$

$\Rightarrow$ 2 sol'ns

$$p_1^2=0=p_4^2$$

$$\ell^\mu=\frac{\xi}{2}\langle 1|\mu|4]\,,\quad \xi=\frac{s_{12}}{\langle 1|2|4]}$$

$$\ell'^\mu=\frac{\xi'}{2}\langle 4|\mu|1]\,,\quad \xi'=\frac{s_{12}}{\langle 4|2|1]}$$

ℓ^μ, ℓ'^μ complex

…, bubbles

$$\ell^2=0\,,\quad -2\ell\cdot p_1+p_1^2=0\,,\quad -2\ell\cdot p_2+s_{12}-p_1^2=0\,,\quad 2\ell\cdot p_4+p_4^2=0$$

$\Rightarrow$ 2 sol'ns

$$p_1^2=0=p_4^2$$

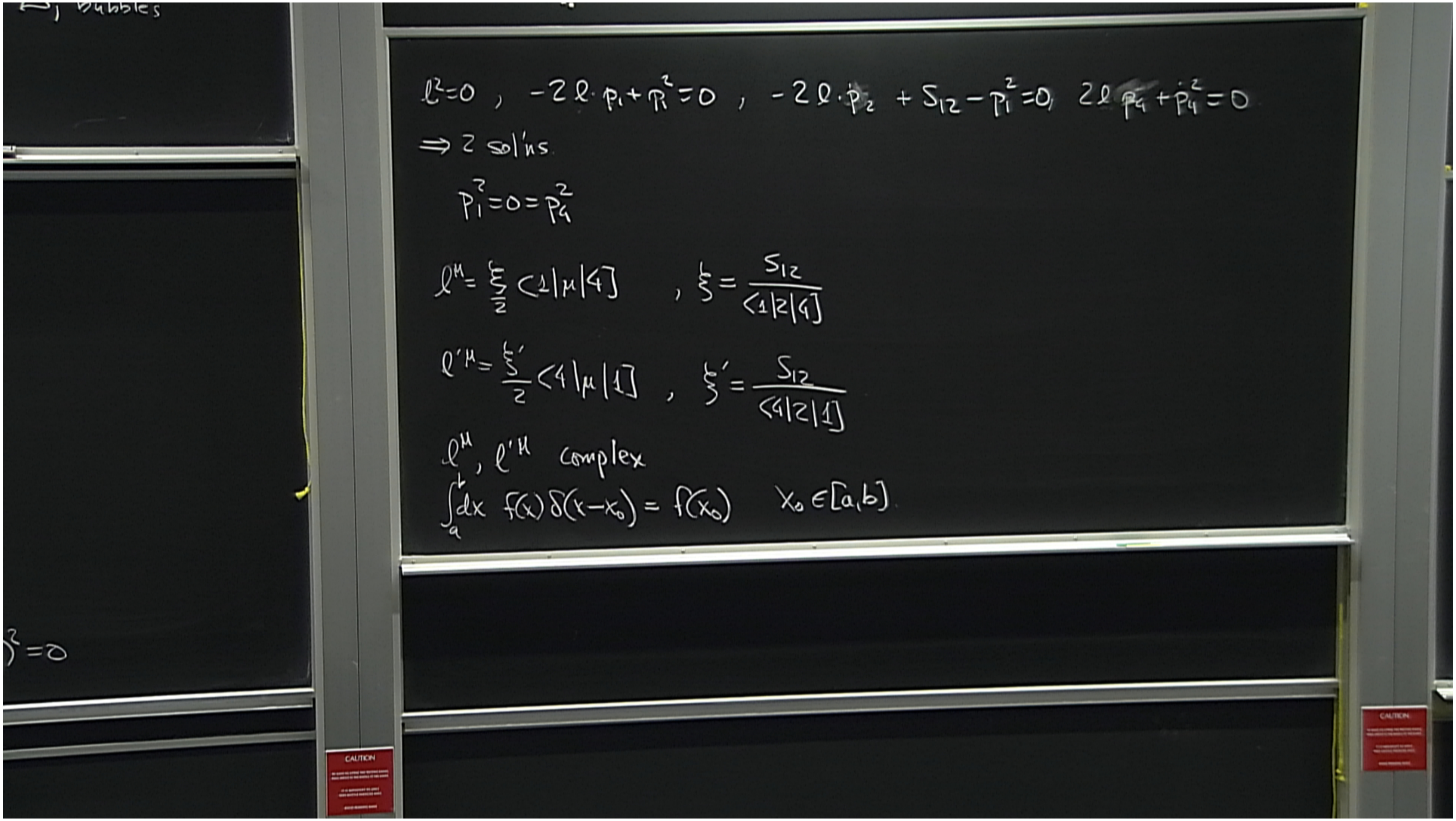
$$\ell^\mu=\frac{\xi}{2}\langle 1|\mu|4]\,,\quad \xi=\frac{s_{12}}{\langle 1|2|4]}$$

$$\ell'^\mu=\frac{\xi'}{2}\langle 4|\mu|1]\,,\quad \xi'=\frac{s_{12}}{\langle 4|2|1]}$$

ℓ^μ, ℓ'^μ complex

$$\int_a^b dx\, f(x)\,\delta(x-x_0)=f(x_0)\qquad x_0\in[a,b]$$

$)^2=0$



$\ell^2=0$, $-2\ell\cdot p_1+p_1^2=0$, $-2\ell\cdot p_2+s_{12}-p_1^2=0$, $2\ell\cdot p_4+p_4^2=0$
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ℓ^μ, ℓ'^μ complex
$\int_a^b dx\, f(x)\delta(x-x_0)=f(x_0)$ $x_0\in[a,b]$

bubbles

complex

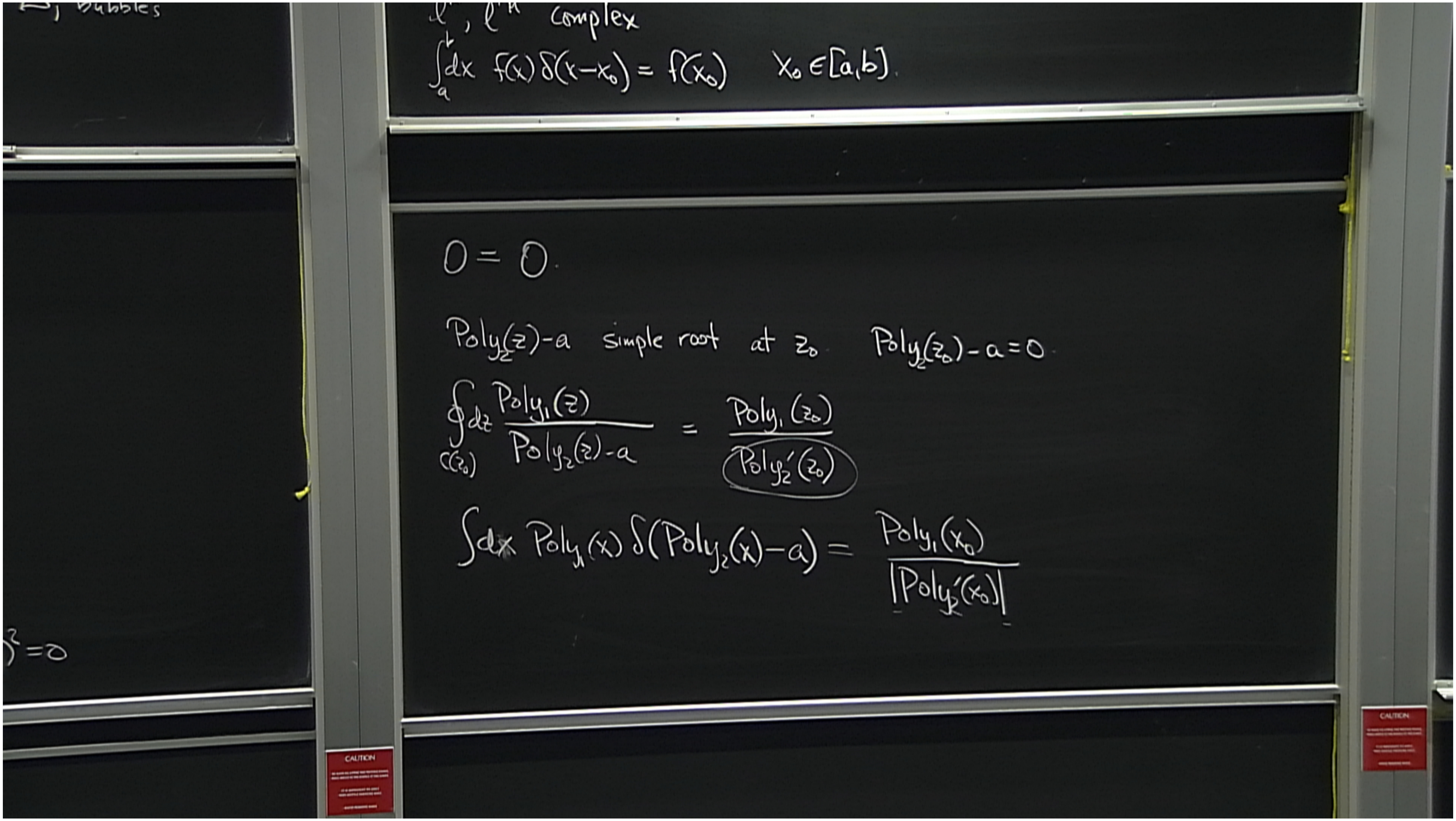
$$\int_a^b dx\, f(x)\,\delta(x-x_0) = f(x_0) \quad x_0 \in [a,b].$$

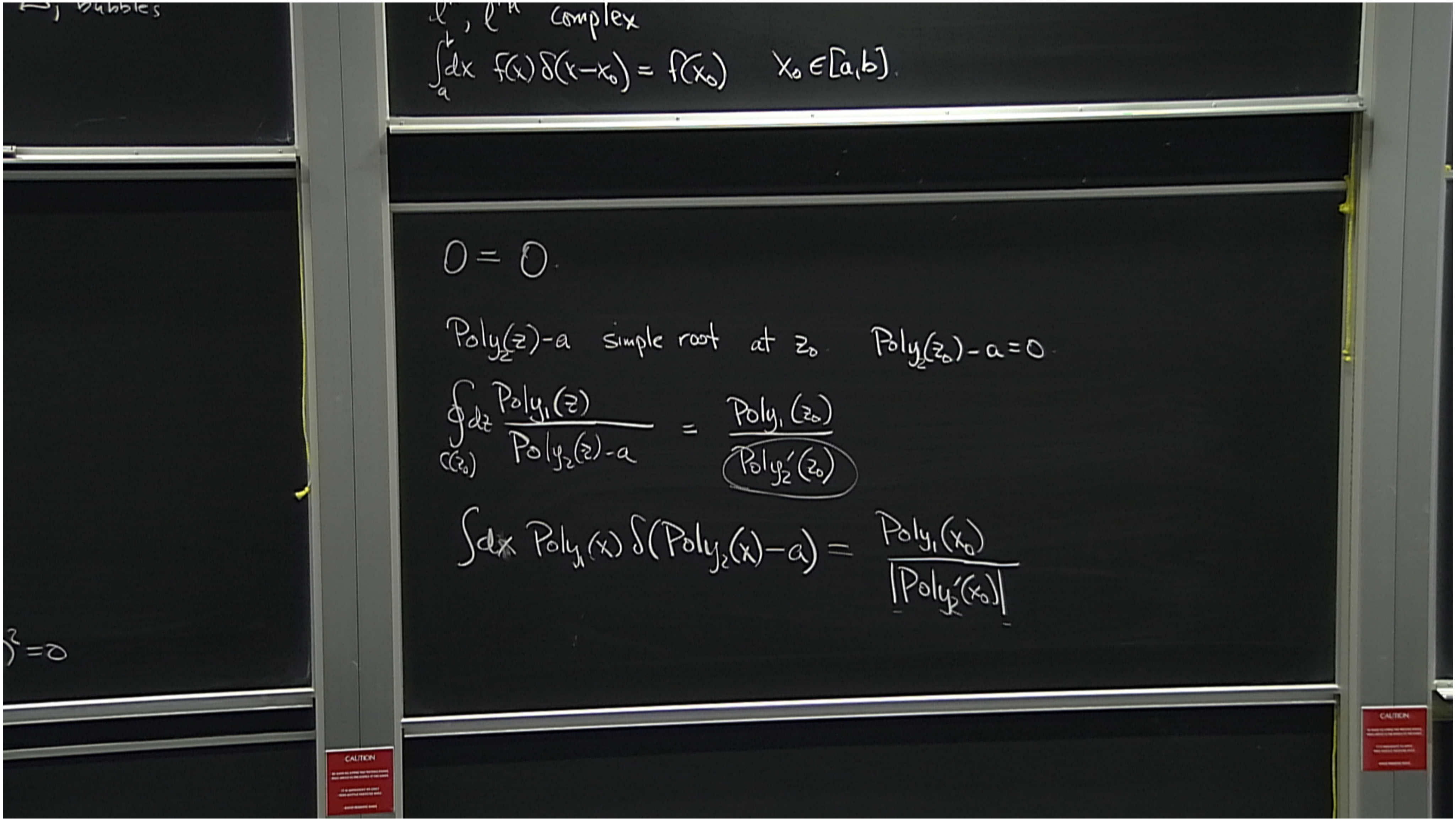
$$0 = 0.$$

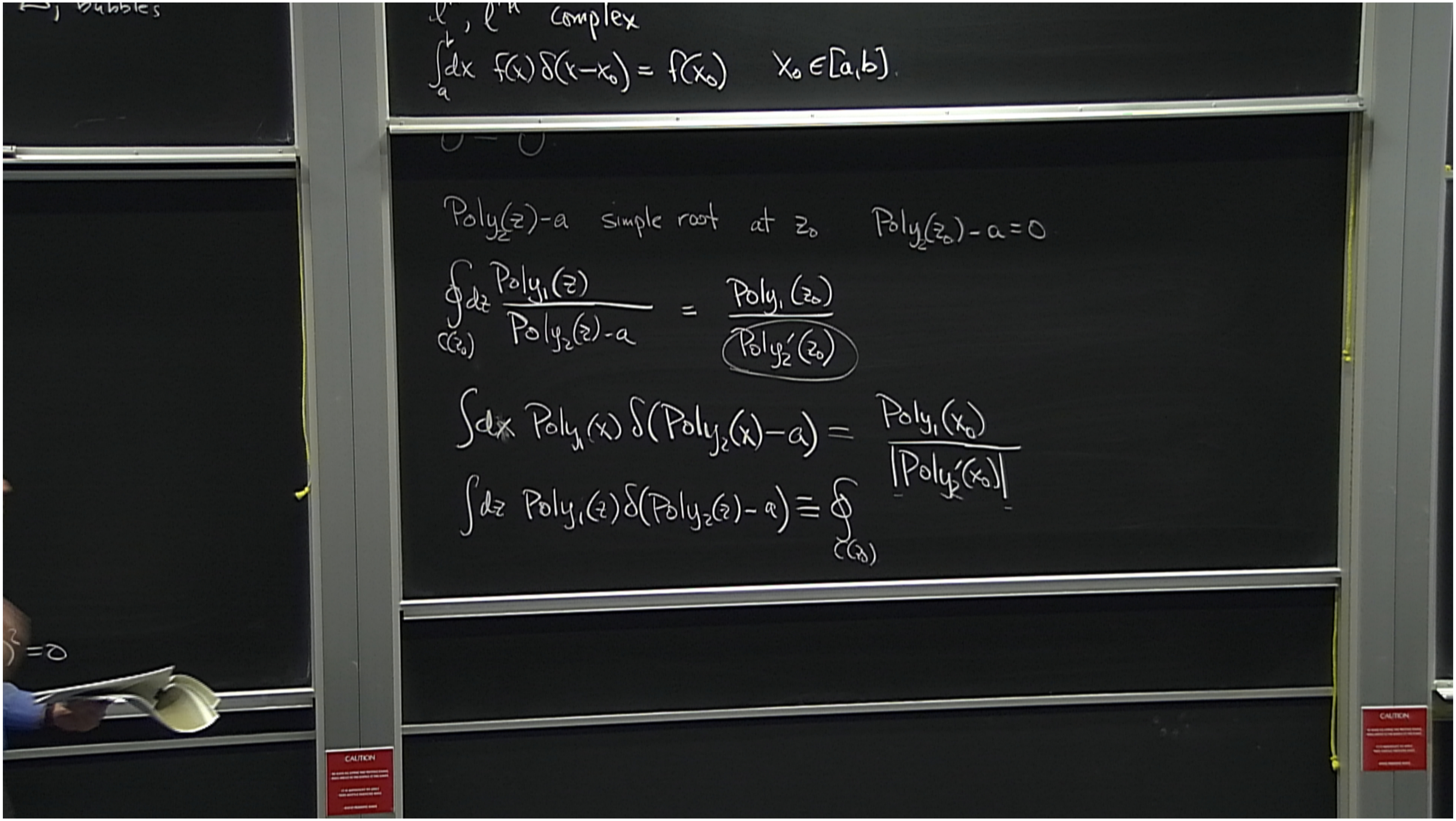
$Poly_2(z) - a$ simple root at z_0. $Poly_2(z_0) - a = 0$.

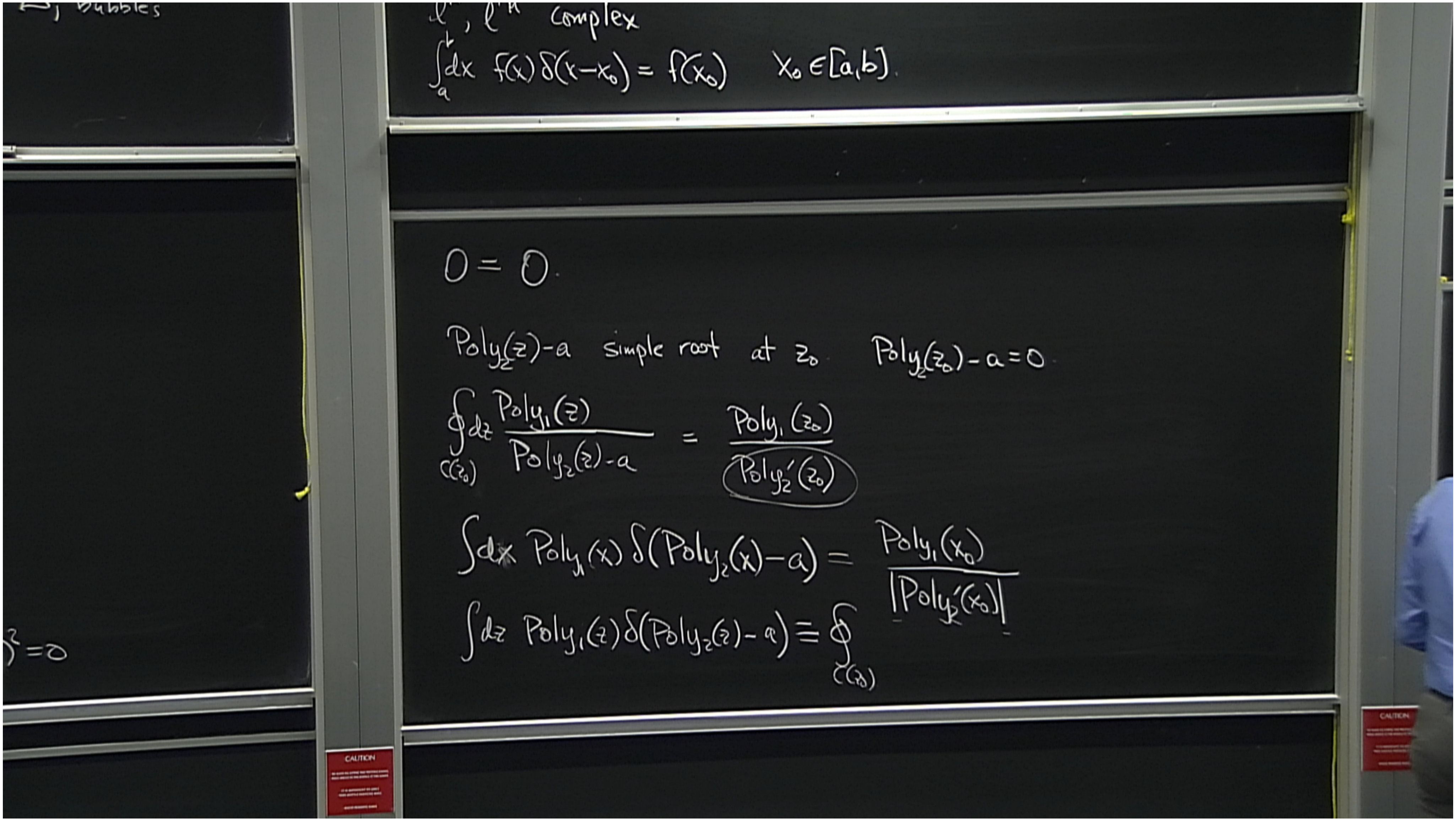
$$\oint_{C(z_0)} dz\, \frac{Poly_1(z)}{Poly_2(z) - a} = \frac{Poly_1(z_0)}{Poly_2'(z_0)}$$

$)^2 = 0$


complex
$\int_a^b dx\, f(x)\,\delta(x-x_0) = f(x_0) \quad x_0 \in [a,b]$
$0 = 0$
$\mathrm{Poly}_2(z)-a$ simple root at z_0 $\mathrm{Poly}_2(z_0)-a=0$
$\oint_{C(z_0)} dz \frac{\mathrm{Poly}_1(z)}{\mathrm{Poly}_2(z)-a} = \frac{\mathrm{Poly}_1(z_0)}{\mathrm{Poly}_2'(z_0)}$
$\int dx\, \mathrm{Poly}_1(x)\,\delta(\mathrm{Poly}_2(x)-a) = \frac{\mathrm{Poly}_1(x_0)}{|\mathrm{Poly}_2'(x_0)|}$


bubbles
complex
$\int_a^b dx\, f(x)\,\delta(x-x_0) = f(x_0) \quad x_0 \in [a,b]$
$0 = 0$
$Poly_2(z) - a$ simple root at z_0 $\quad Poly_2(z_0) - a = 0$
$\oint_{C(z_0)} dz \frac{Poly_1(z)}{Poly_2(z) - a} = \frac{Poly_1(z_0)}{Poly_2'(z_0)}$
$\int dx\, Poly_1(x)\, \delta(Poly_2(x) - a) = \frac{Poly_1(x_0)}{|Poly_2'(x_0)|}$
$=0$


complex
$\int_a^b dx\, f(x)\,\delta(x-x_0) = f(x_0) \quad x_0 \in [a,b]$
$Poly_2(z)-a$ simple root at z_0 $\quad Poly_2(z_0)-a=0$
$\oint_{C(z_0)} dz \frac{Poly_1(z)}{Poly_2(z)-a} = \frac{Poly_1(z_0)}{Poly_2'(z_0)}$
$\int dx\, Poly_1(x)\,\delta(Poly_2(x)-a) = \frac{Poly_1(x_0)}{|Poly_2'(x_0)|}$
$\int dz\, Poly_1(z)\,\delta(Poly_2(z)-a) \equiv \oint_{C(z_0)}$
CAUTION
CAUTION


bubbles
complex
$\int_a^b dx\, f(x)\,\delta(x-x_0) = f(x_0) \quad x_0 \in [a,b]$
$)^2 = 0$
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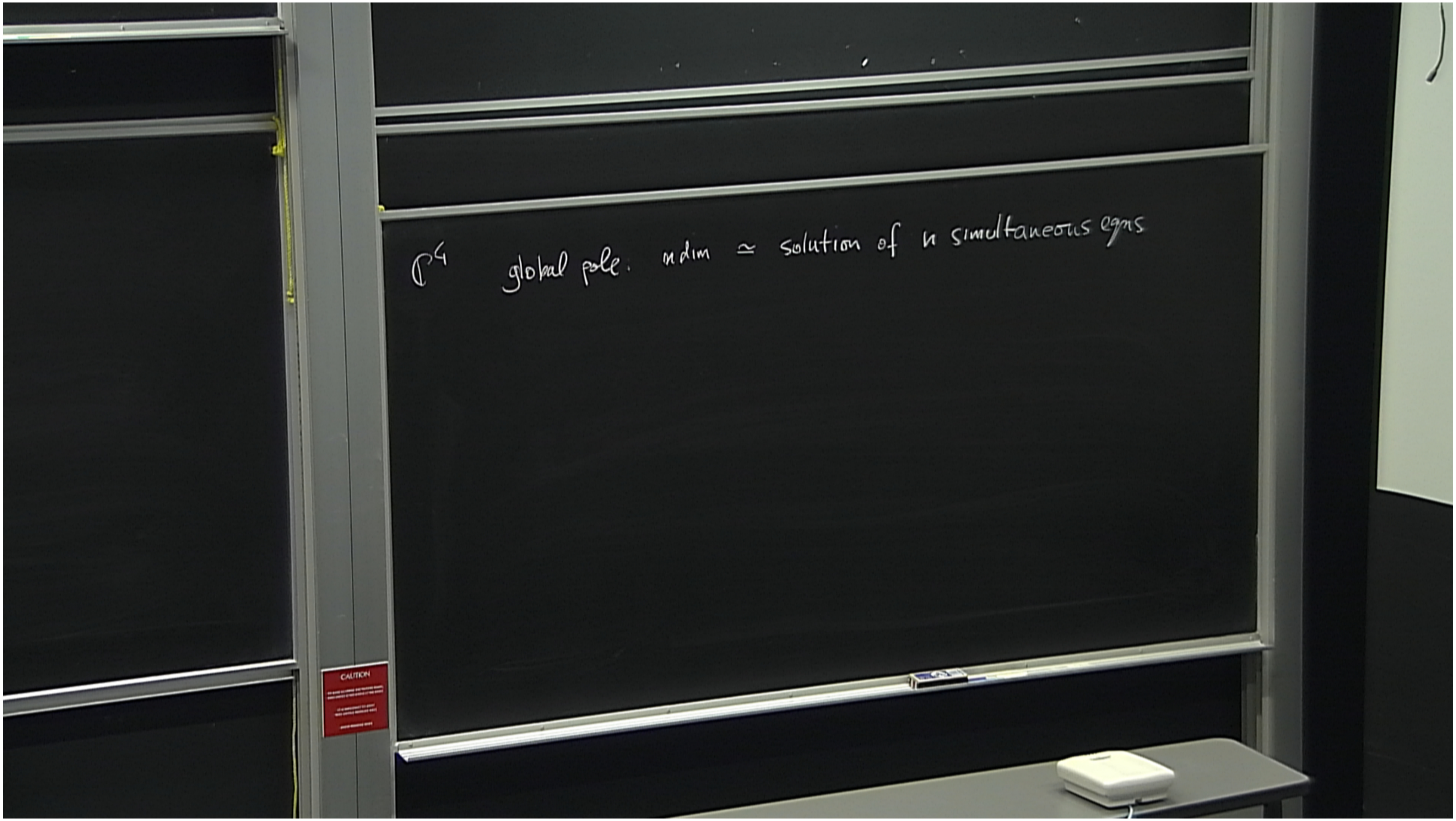
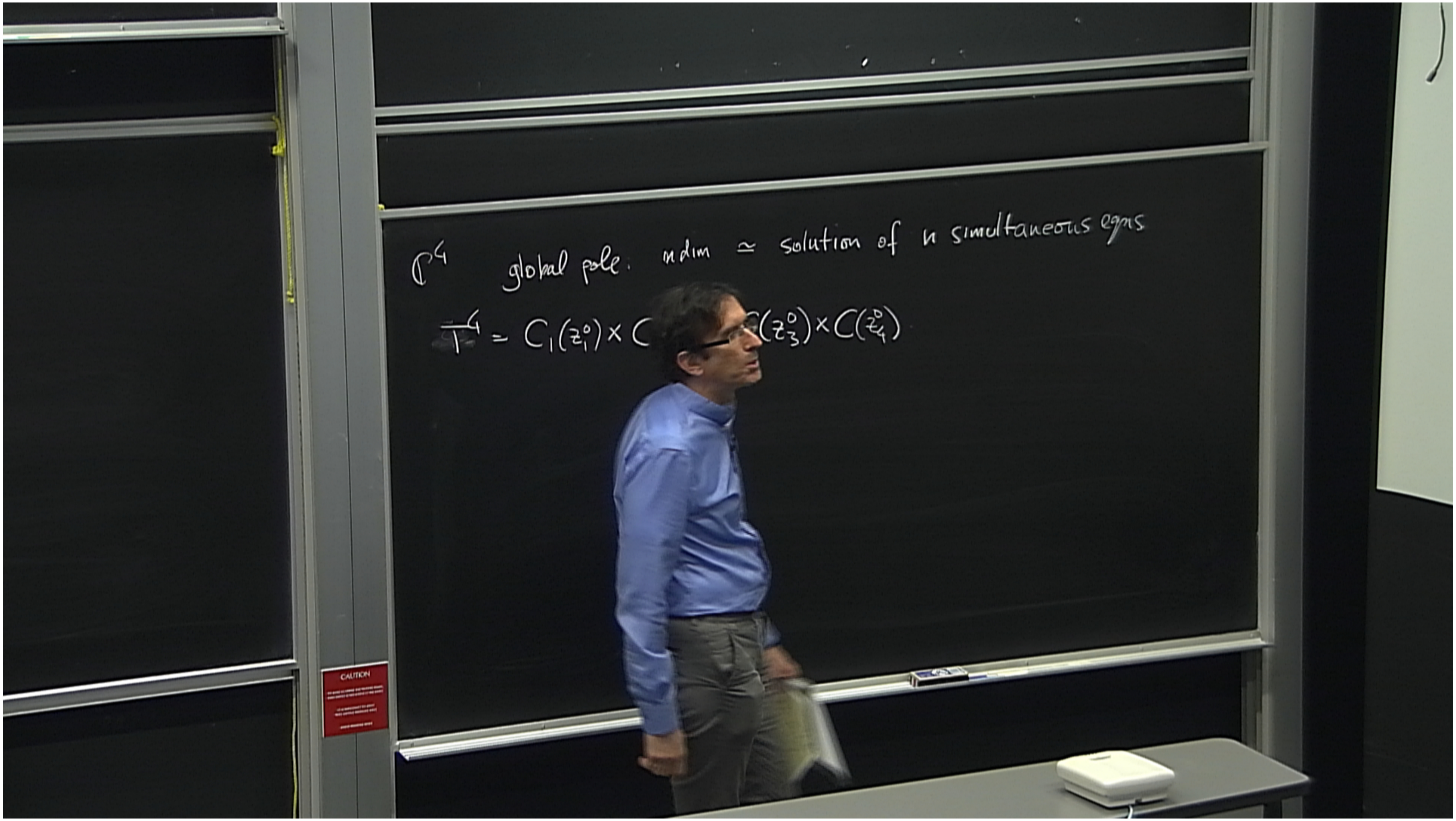
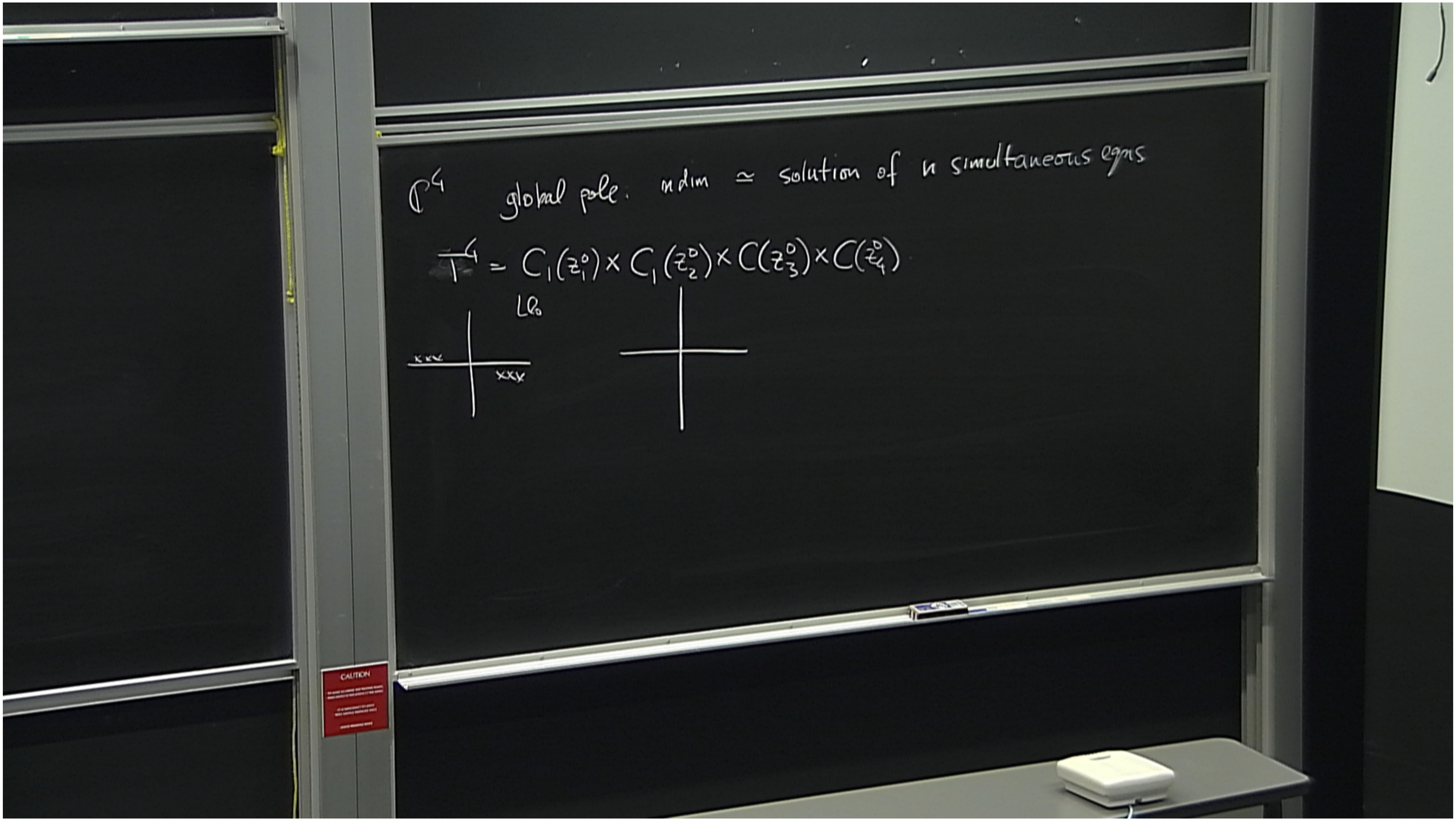
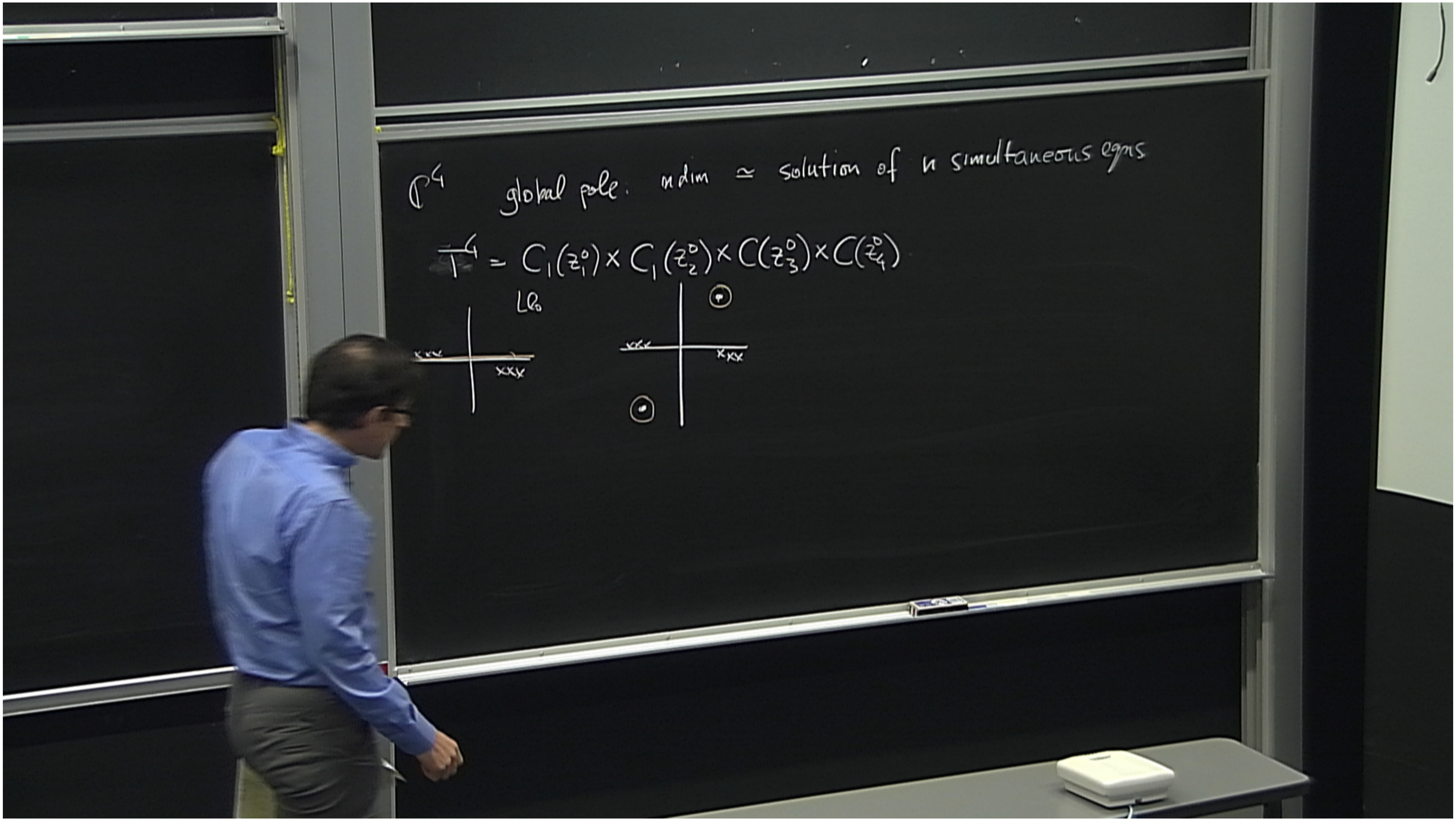
CAUTION

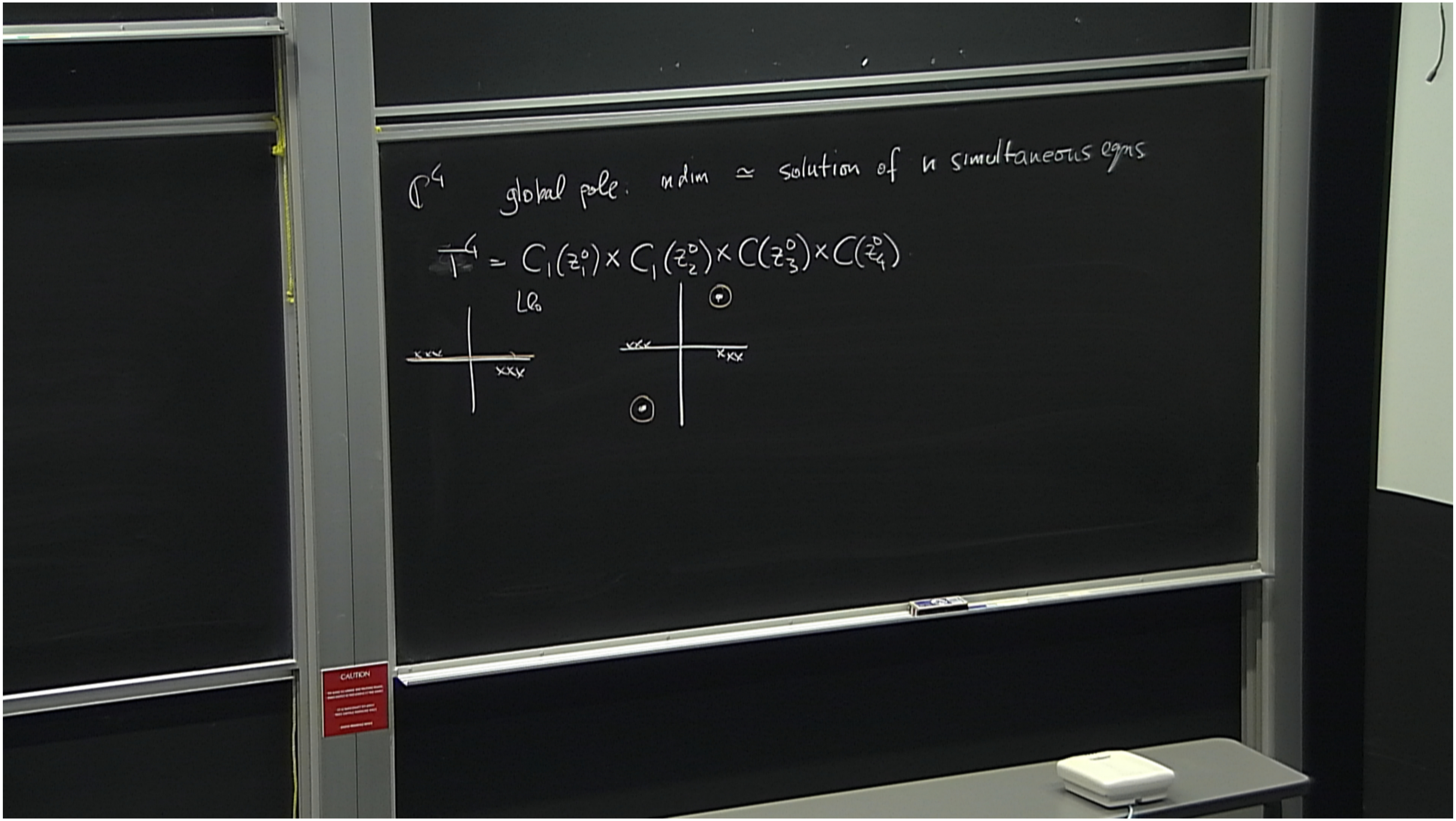


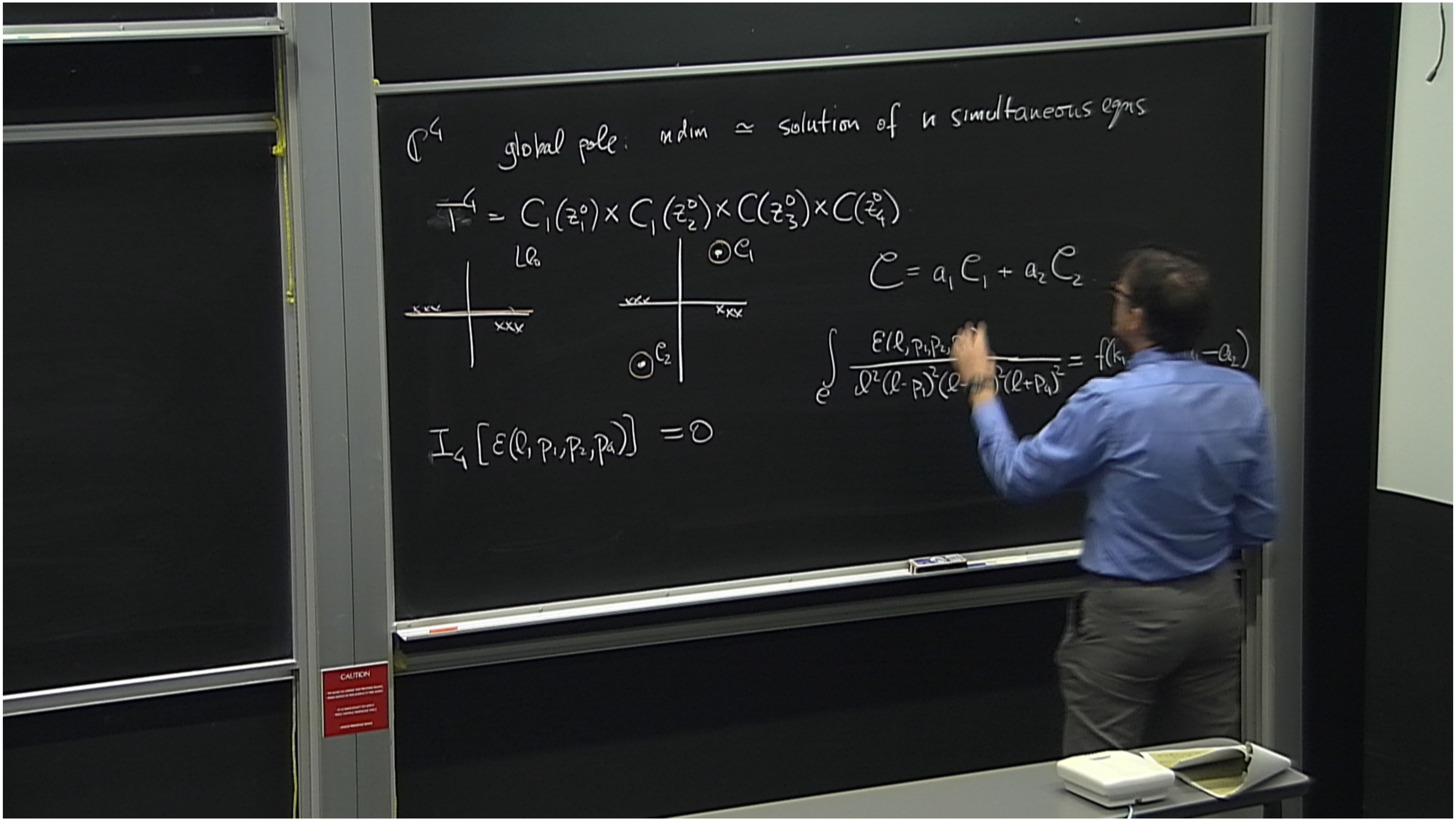
ℙ⁴ global pole. n dim ≃ solution of n simultaneous eqns


global pole. n dim ≃ solution of n simultaneous eqns
CAUTION

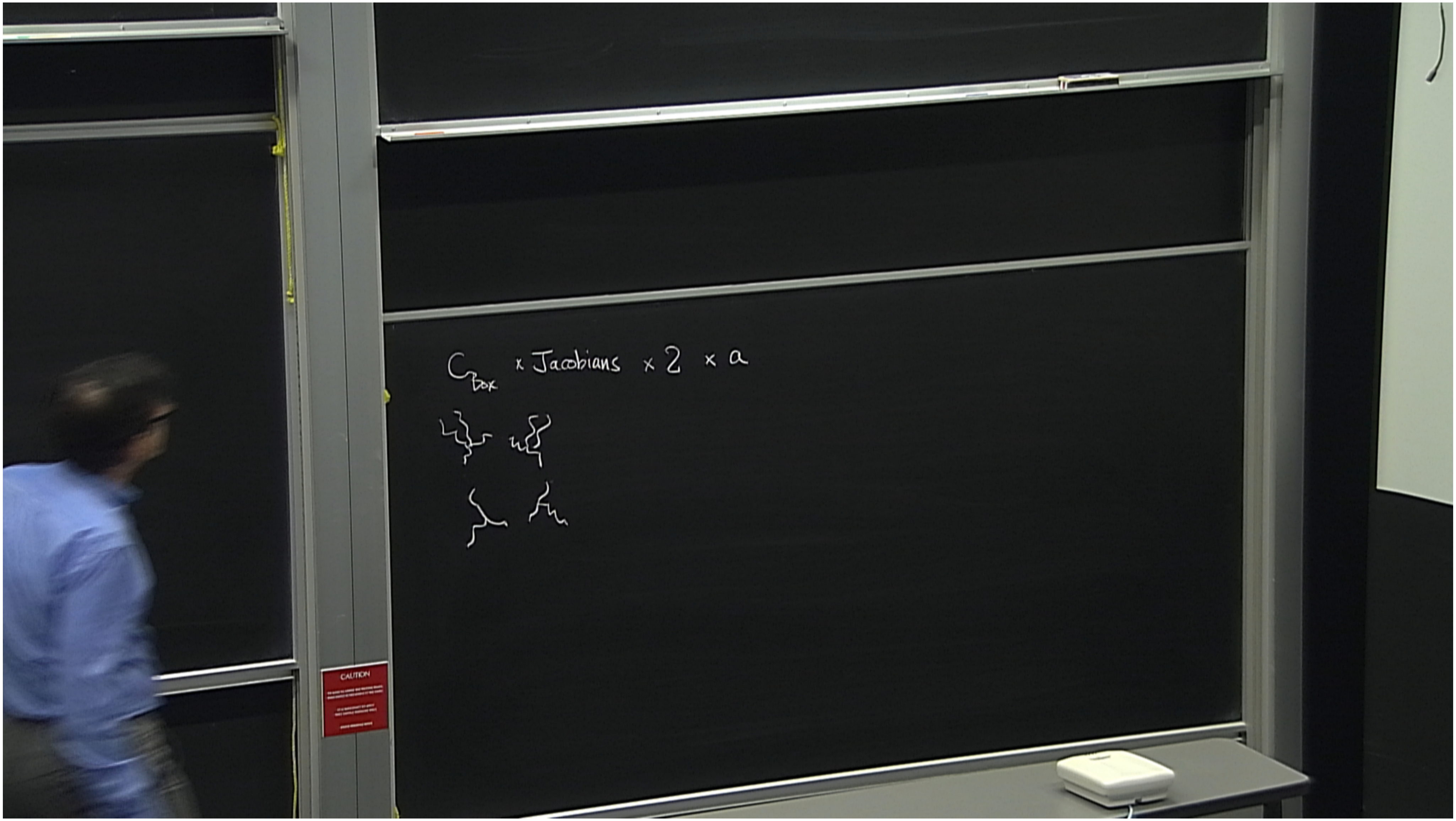

$\mathbb{P}^4$ global pole: n dim ≃ solution of n simultaneous eqns
$T^4 = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$
LP_0
xxx
xxx

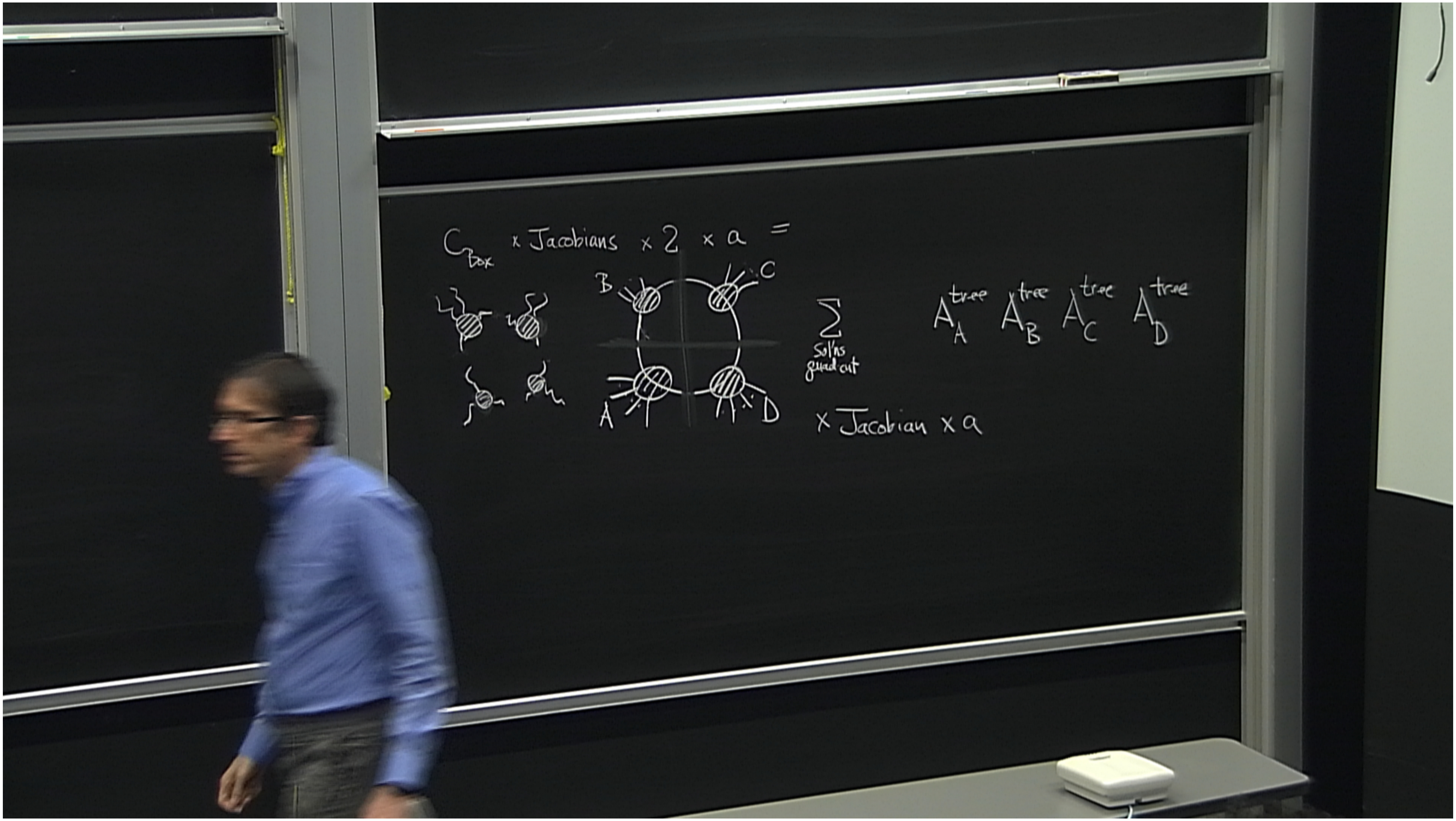

P^4 global pole: n dim ≃ solution of n simultaneous eqns
T^4 = C_1(z_1^0) × C_1(z_2^0) × C(z_3^0) × C(z_4^0)
xxx
xxx
xxx
xxx

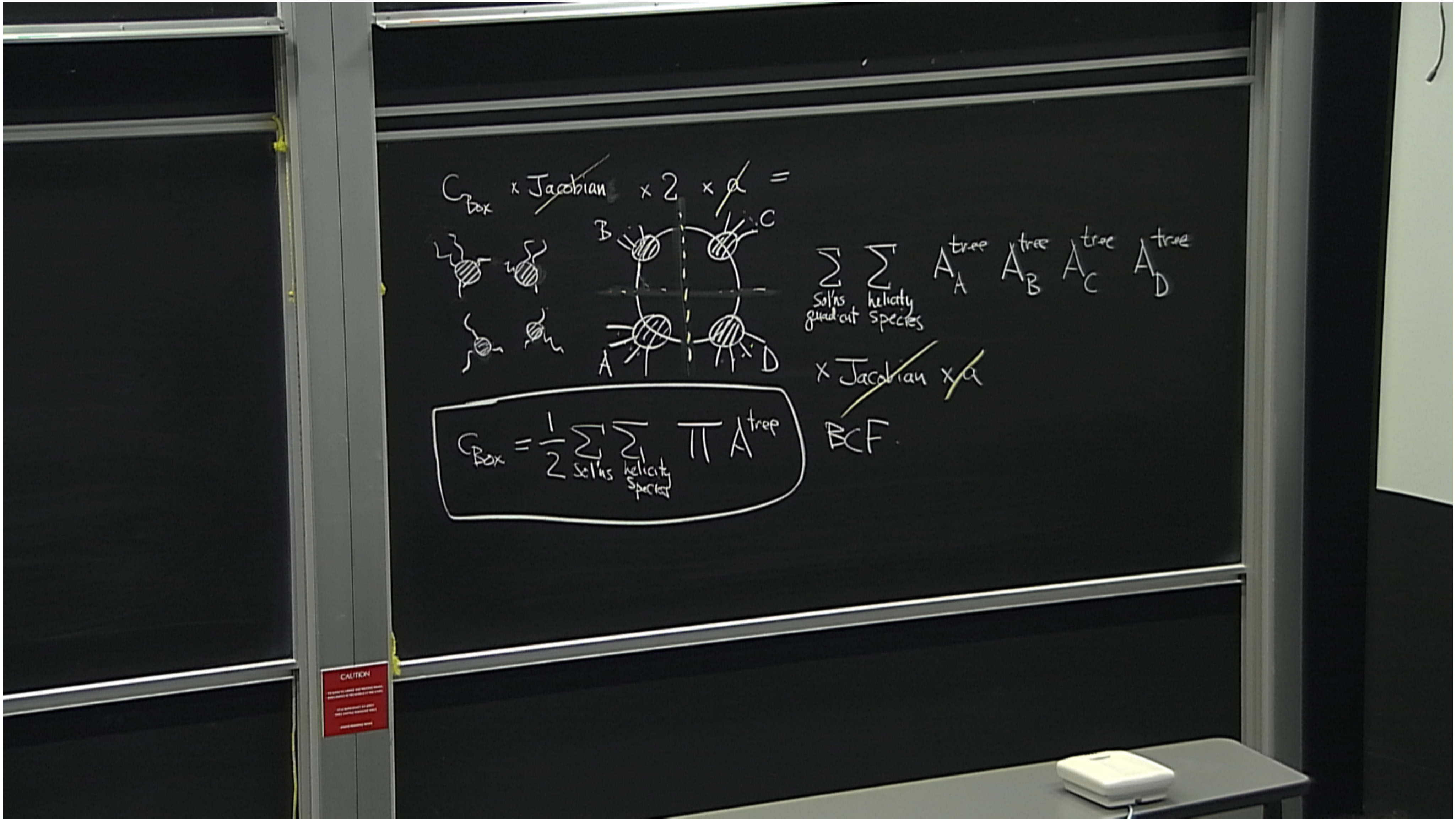

P^4 global pole: n dim ≃ solution of n simultaneous eqns
$T^4 = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$
LO

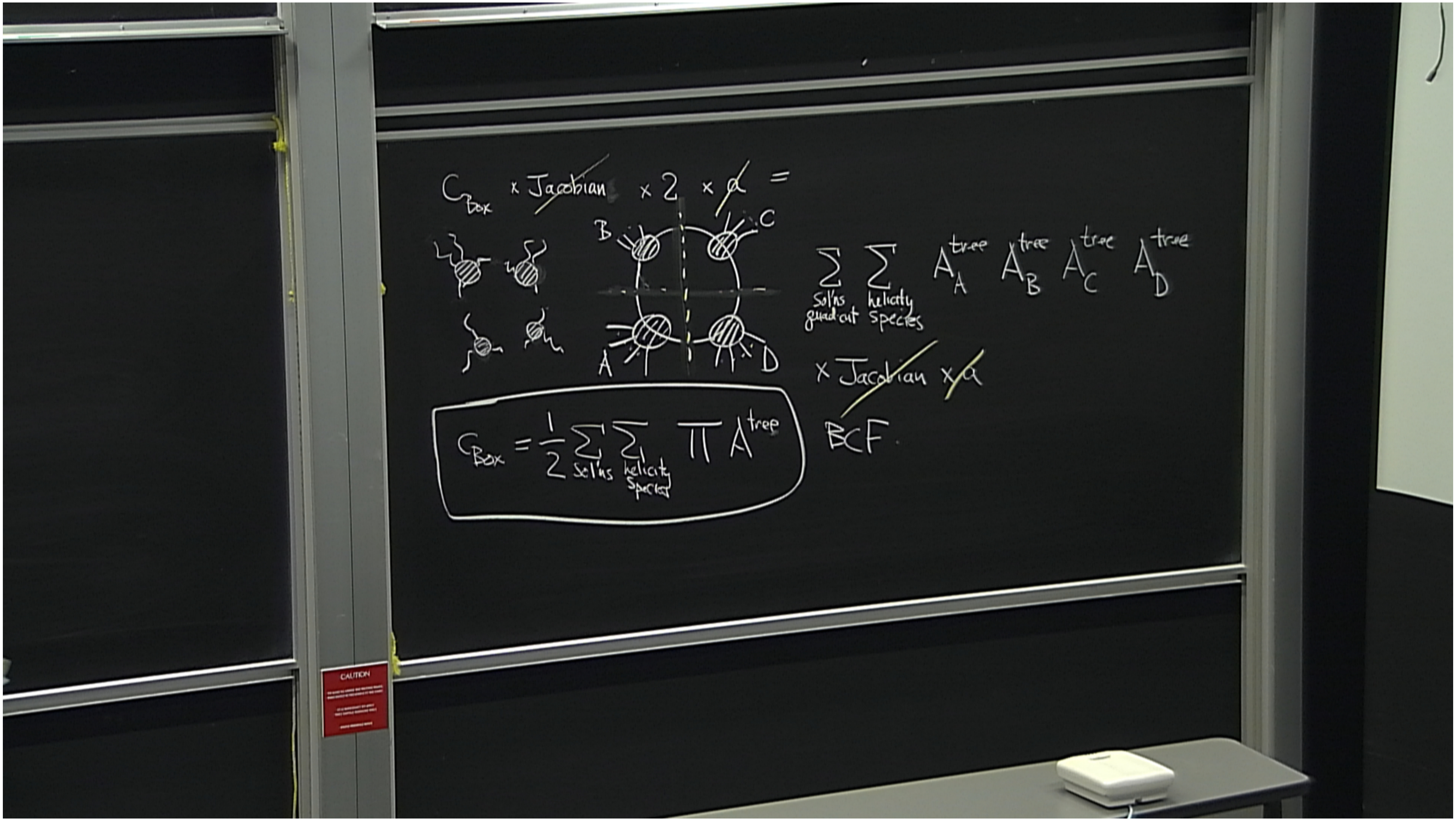


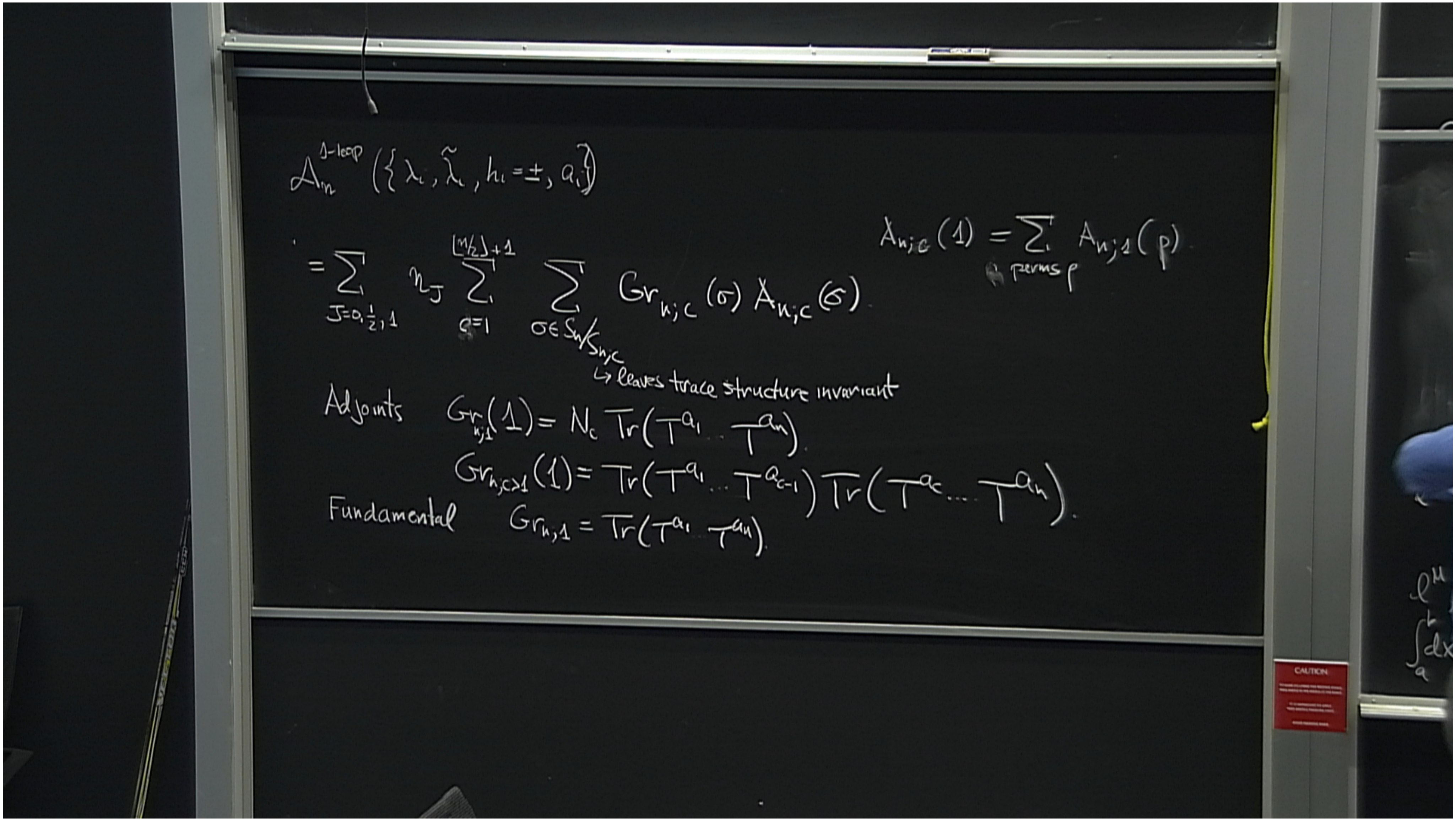
global pole: n dim ≃ solution of n simultaneous eqns
$T^4 = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$
$\mathcal{C} = a_1 \mathcal{C}_1 + a_2 \mathcal{C}_2$
$I_4[\varepsilon(l, p_1, p_2, p_4)] = 0$

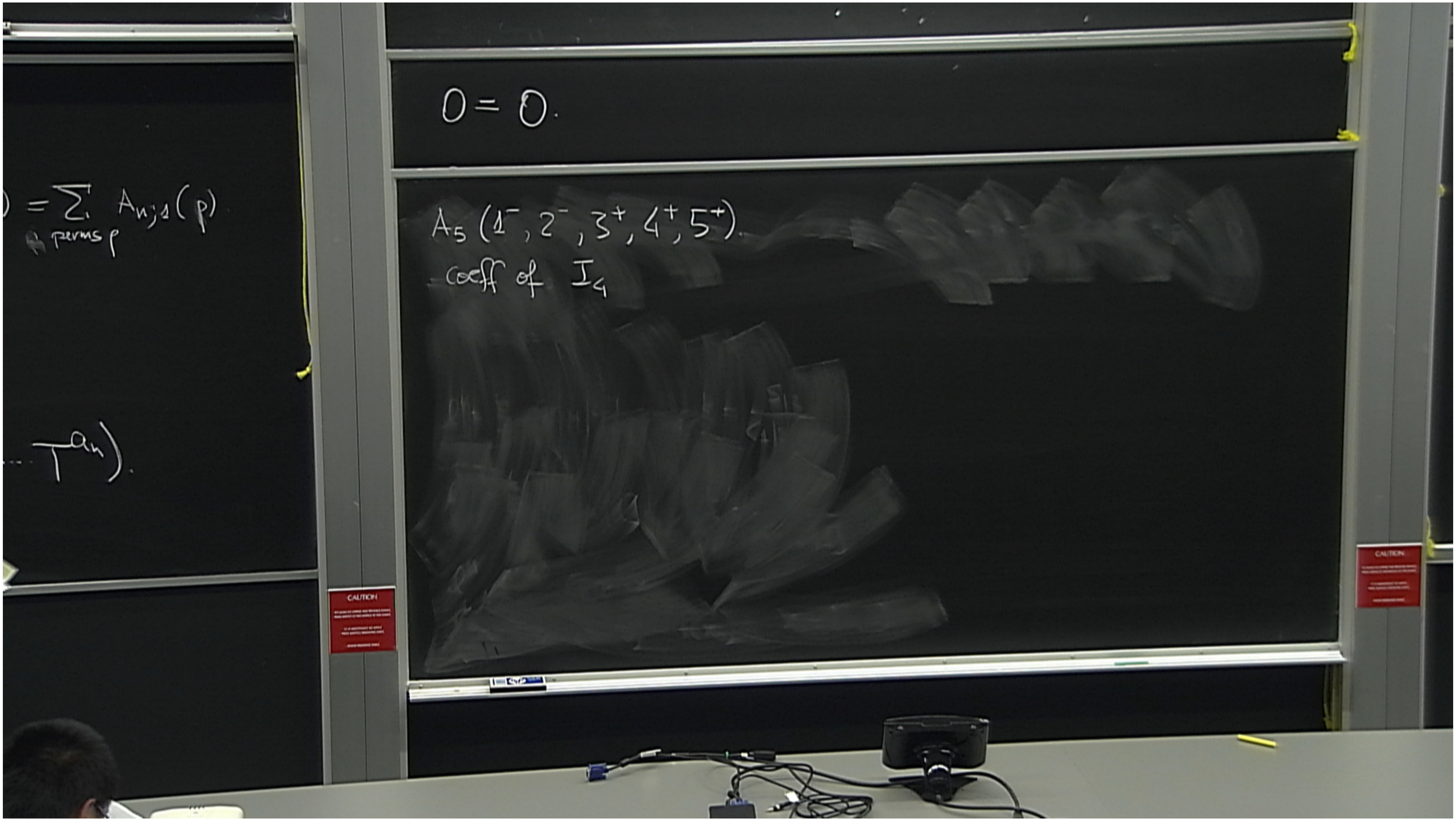

C Box x Jacobians x 2 x a
CAUTION

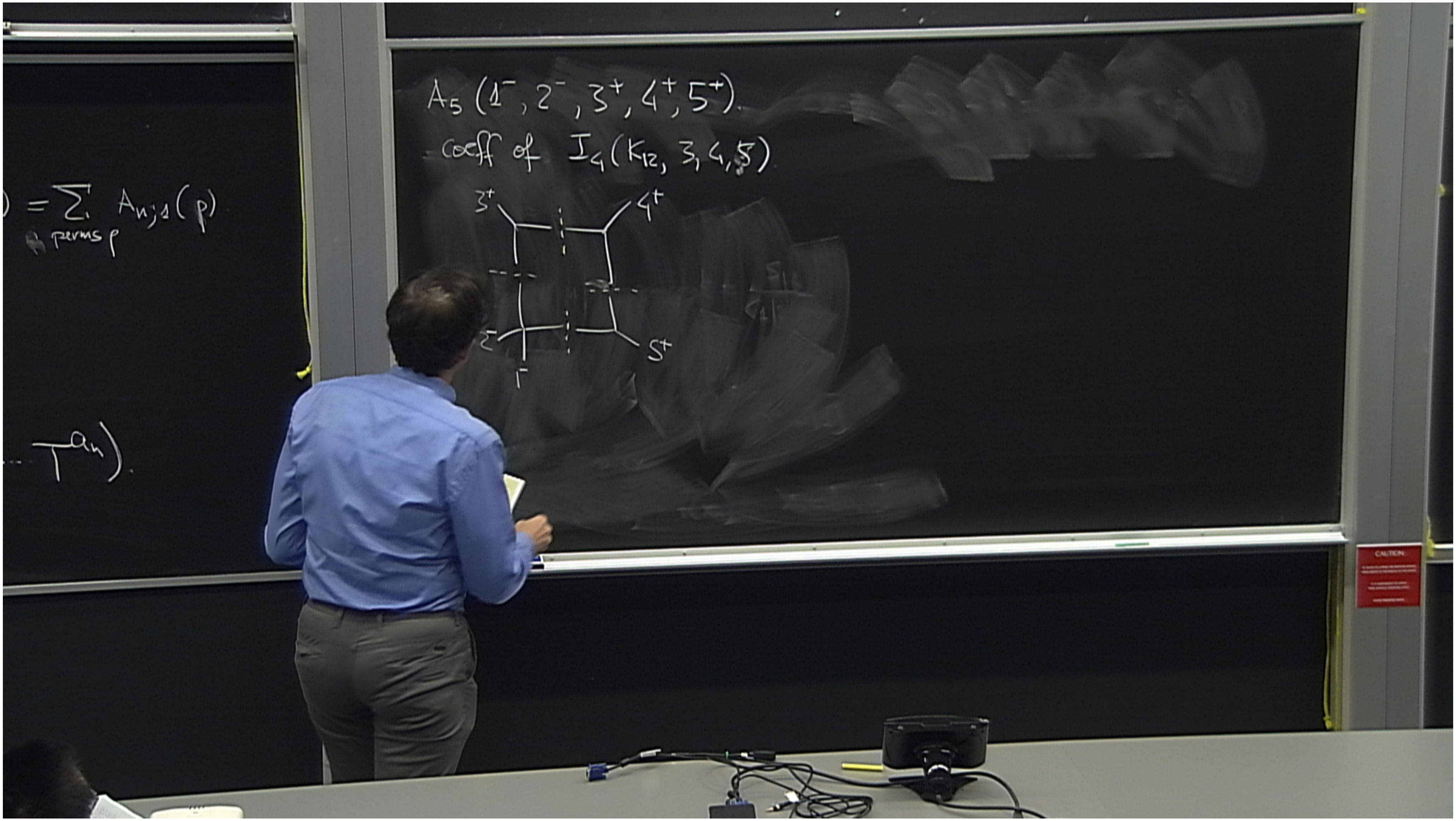

C box x Jacobians x 2 x a =
B
C
A
D
Σ
Sol'ns
quad cut
A A tree A B tree A C tree A D tree
x Jacobian x a


C Box × Jacobian × 2 × α =
B C A D
Σ Σ A A tree A B tree A C tree A D tree
sol'ns quad cut helicity species
× Jacobian × α
BCF
C Box = 1/2 Σ Σ Π A tree
sol'ns helicity species

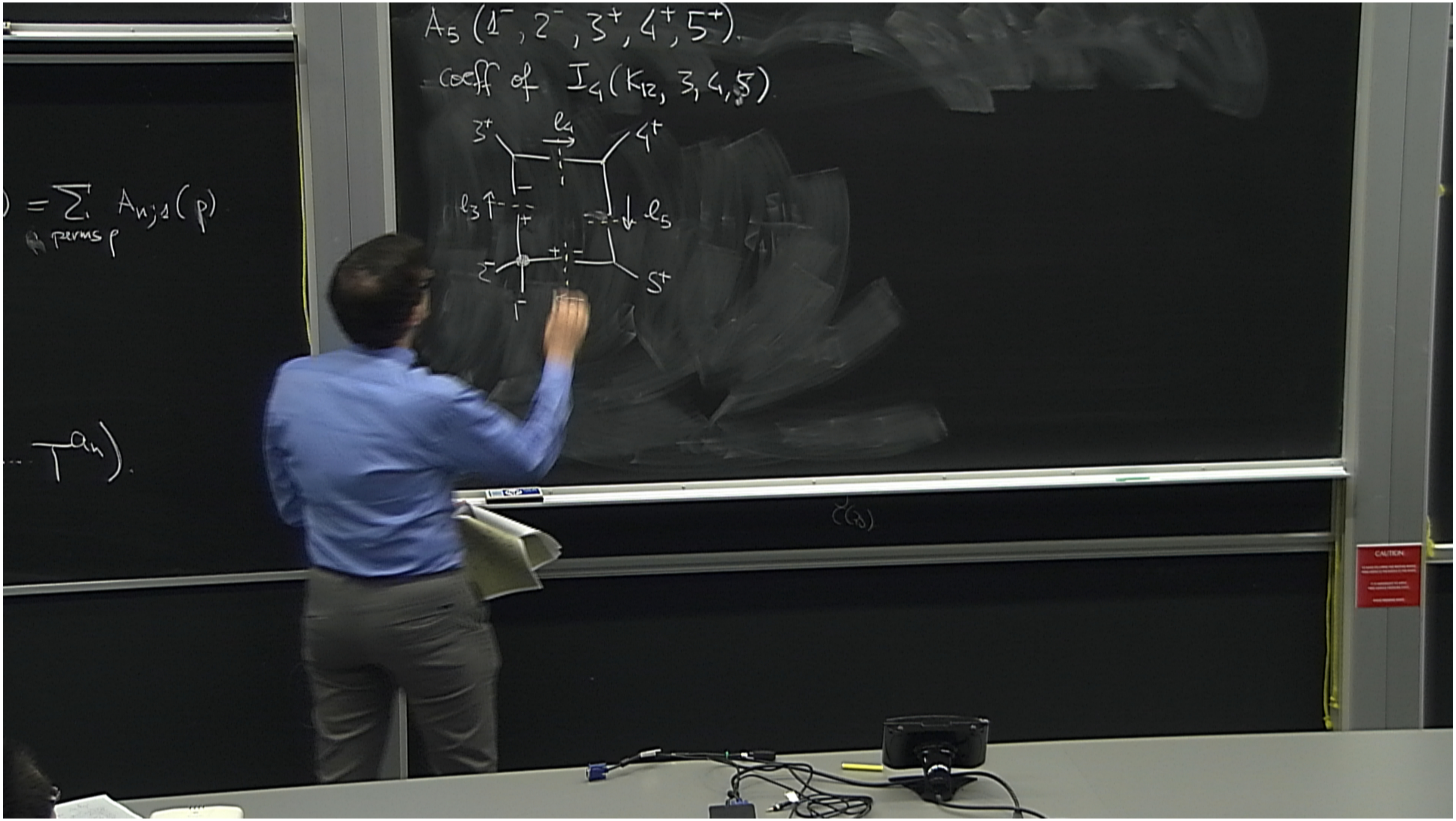

C Box × Jacobian × 2 × α =
B
C
A
D
Σ Σ A A tree A B tree A C tree A D tree
Sol'ns quad-cut
helicity species
× Jacobian × α
BCF
C Box = 1/2 Σ Σ Π A tree
Sol'ns
helicity species
CAUTION


$A_n^{1\text{-loop}}(\{\lambda_i, \tilde{\lambda}_i, h_i = \pm, a_i\})$
$= \sum_{J=0,\frac{1}{2},1} n_J \sum_{c=1}^{\lfloor n/2 \rfloor + 1} \sum_{\sigma \in S_n/S_{n;c}} Gr_{n;c}(\sigma) A_{n;c}(\sigma)$
↳ leaves trace structure invariant
$A_{n;c}(1) = \sum_{\text{perms } \rho} A_{n;1}(\rho)$
Adjoints $Gr_{n;1}(1) = N_c \, Tr(T^{a_1} \ldots T^{a_n})$
$Gr_{n;c>1}(1) = Tr(T^{a_1} \ldots T^{a_{c-1}}) \, Tr(T^{a_c} \ldots T^{a_n})$
Fundamental $Gr_{n;1} = Tr(T^{a_1} \ldots T^{a_n})$

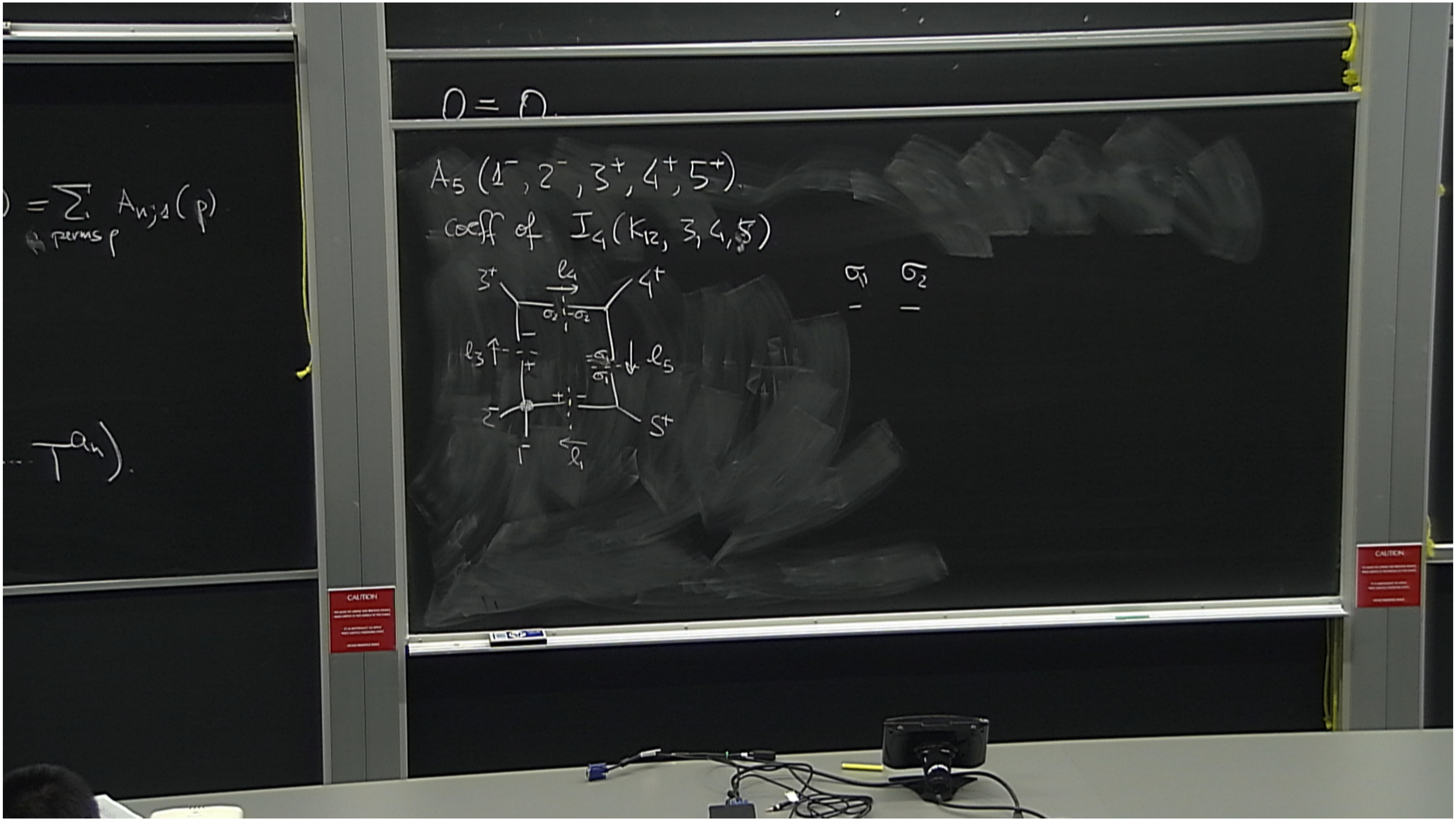

) = Σ A_{n;1}(ρ)
perms ρ
... T^{a_n}).
0 = 0.
A_5 (1^-, 2^-, 3^+, 4^+, 5^+).
coeff of I_4

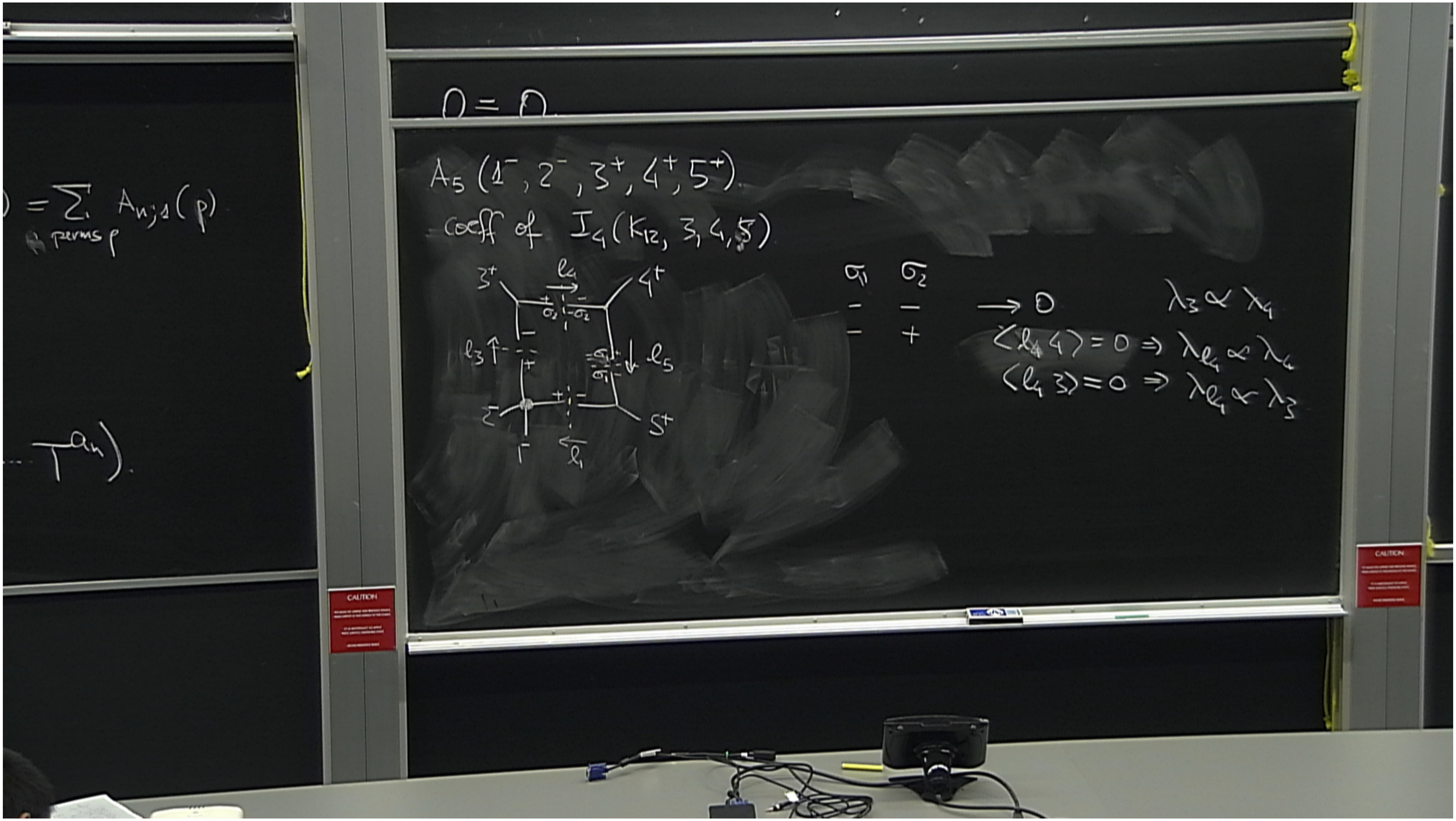


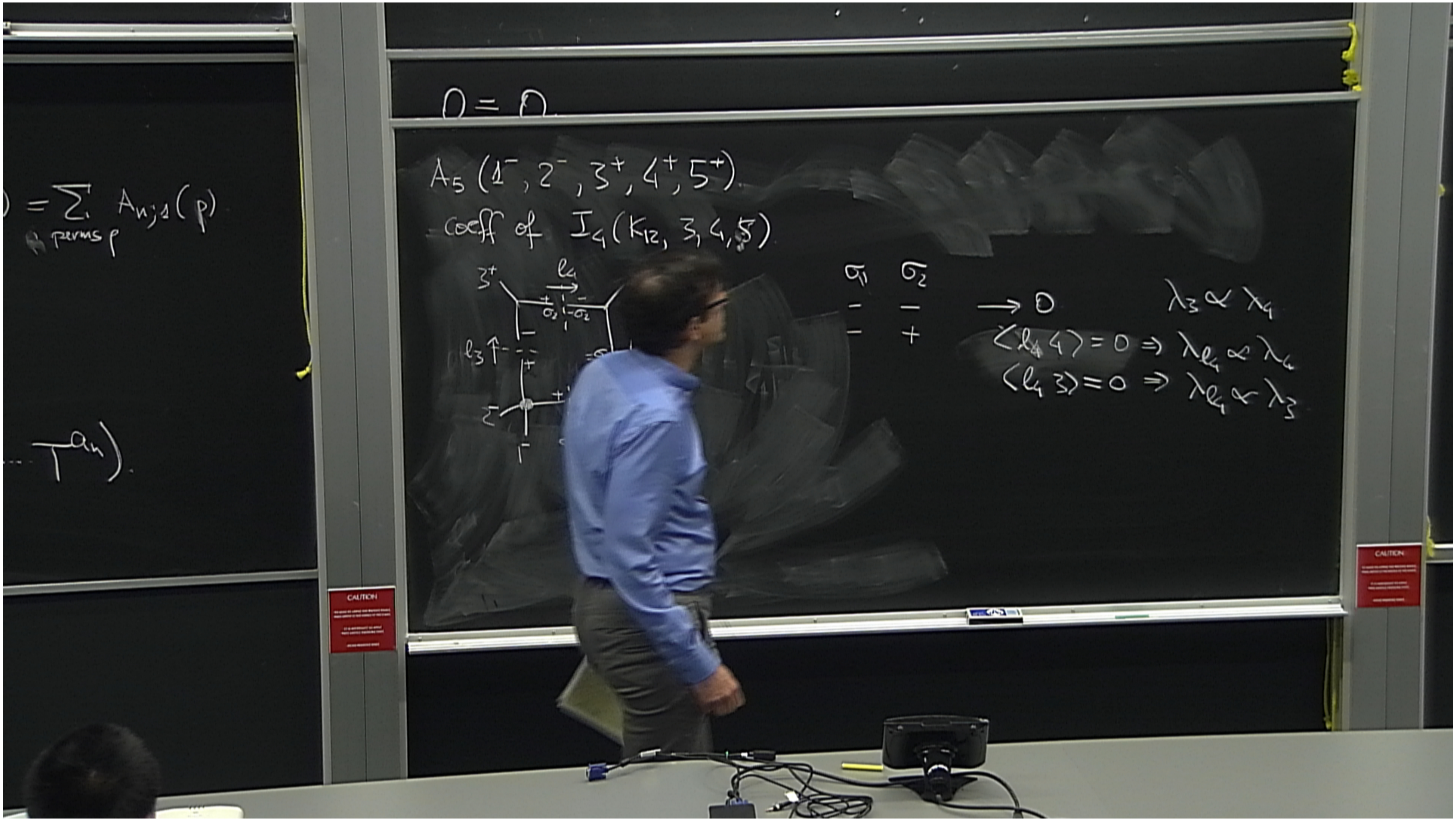
A5 (1⁻, 2⁻, 3⁺, 4⁺, 5⁺)
coeff of I4 (K12, 3, 4, 5)
3⁺
4⁺
2⁻
1⁻
5⁺
) = Σ An;1 (ρ)
perms ρ
T^an)

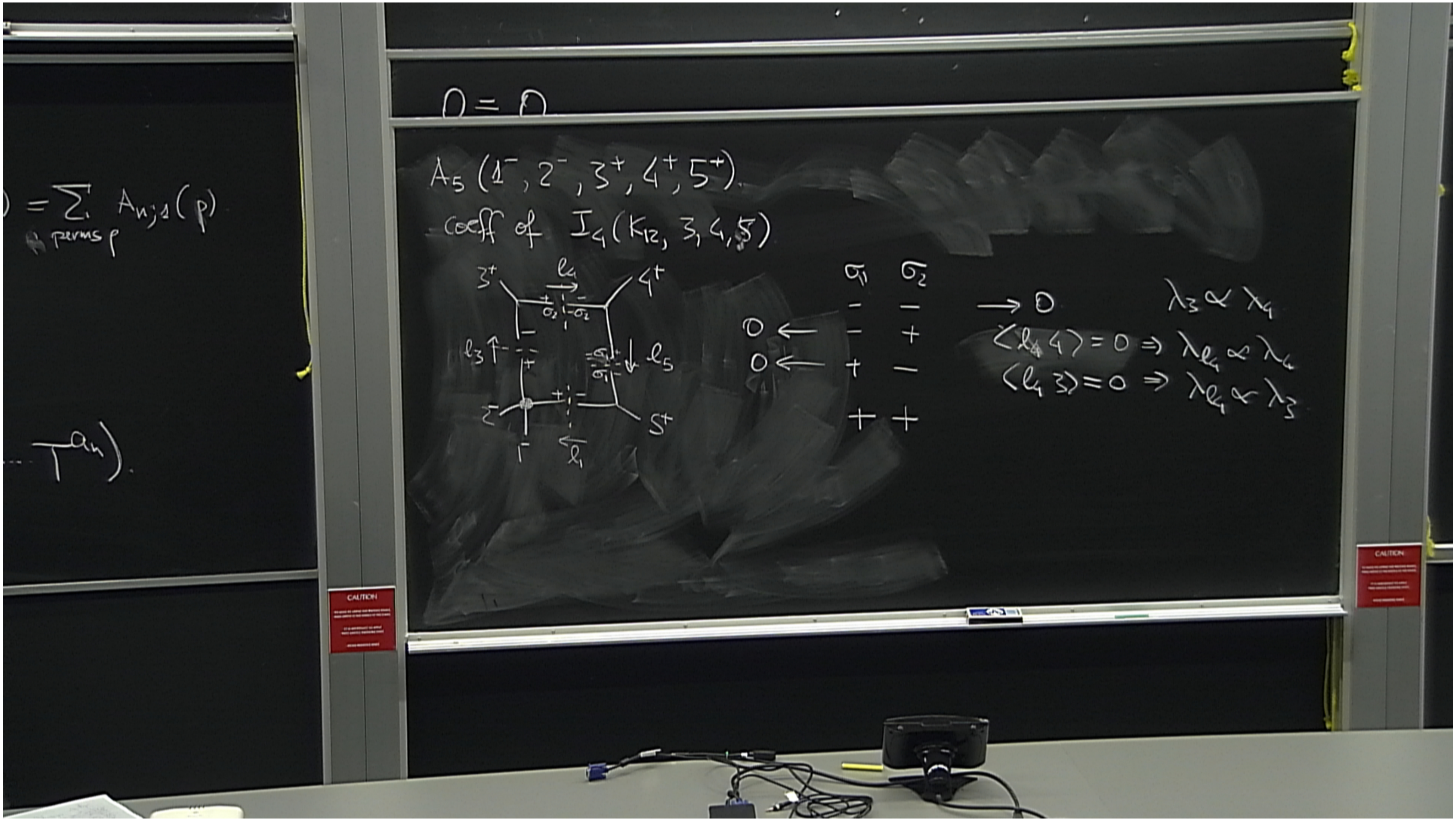


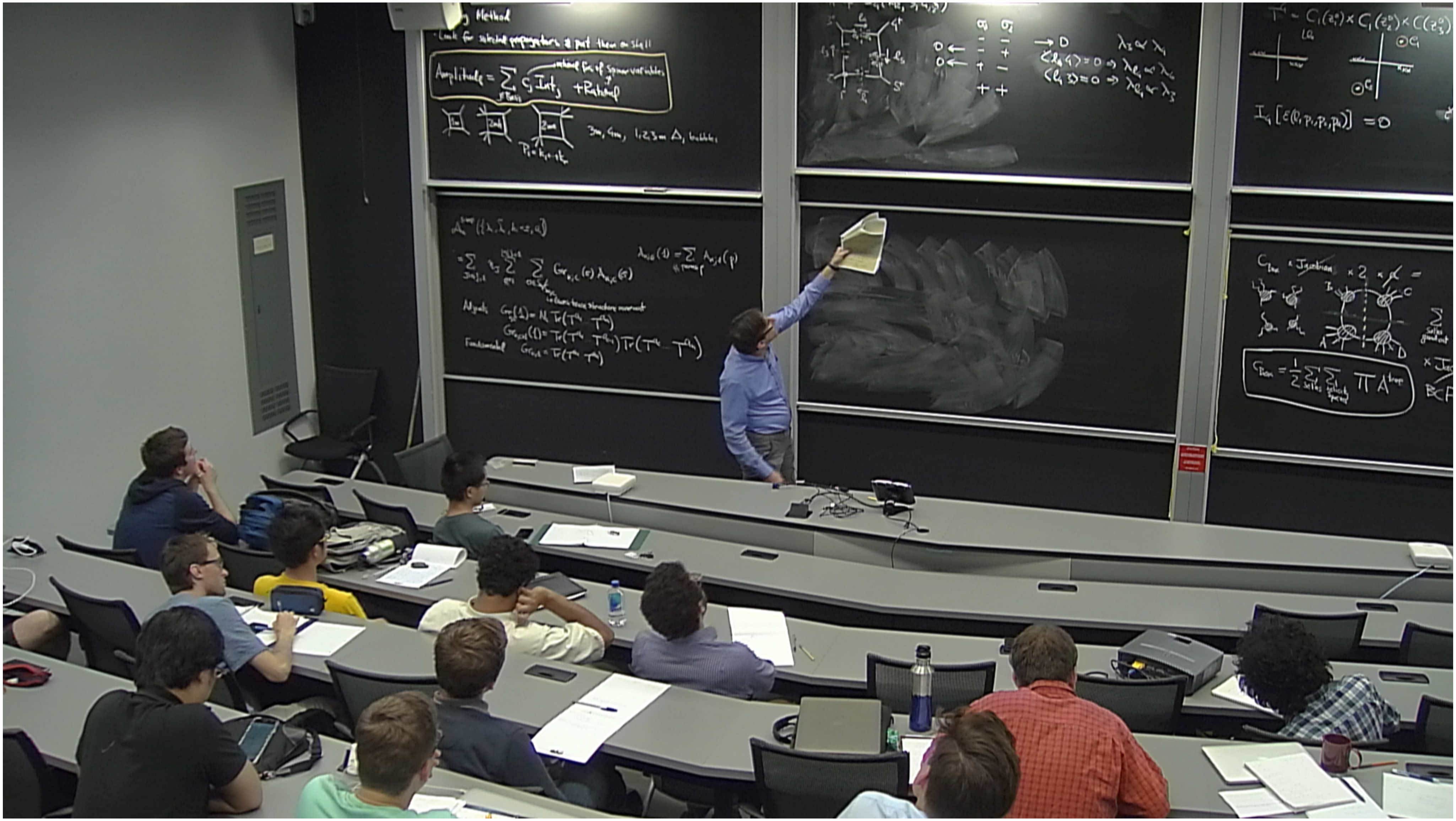
A5 (1⁻, 2⁻, 3⁺, 4⁺, 5⁺)
coeff of I4(K12, 3, 4, 5)
3⁺
l4
4⁺
l3
l5
2⁻
5⁺
1⁻
= Σ An;1(ρ)
perms ρ

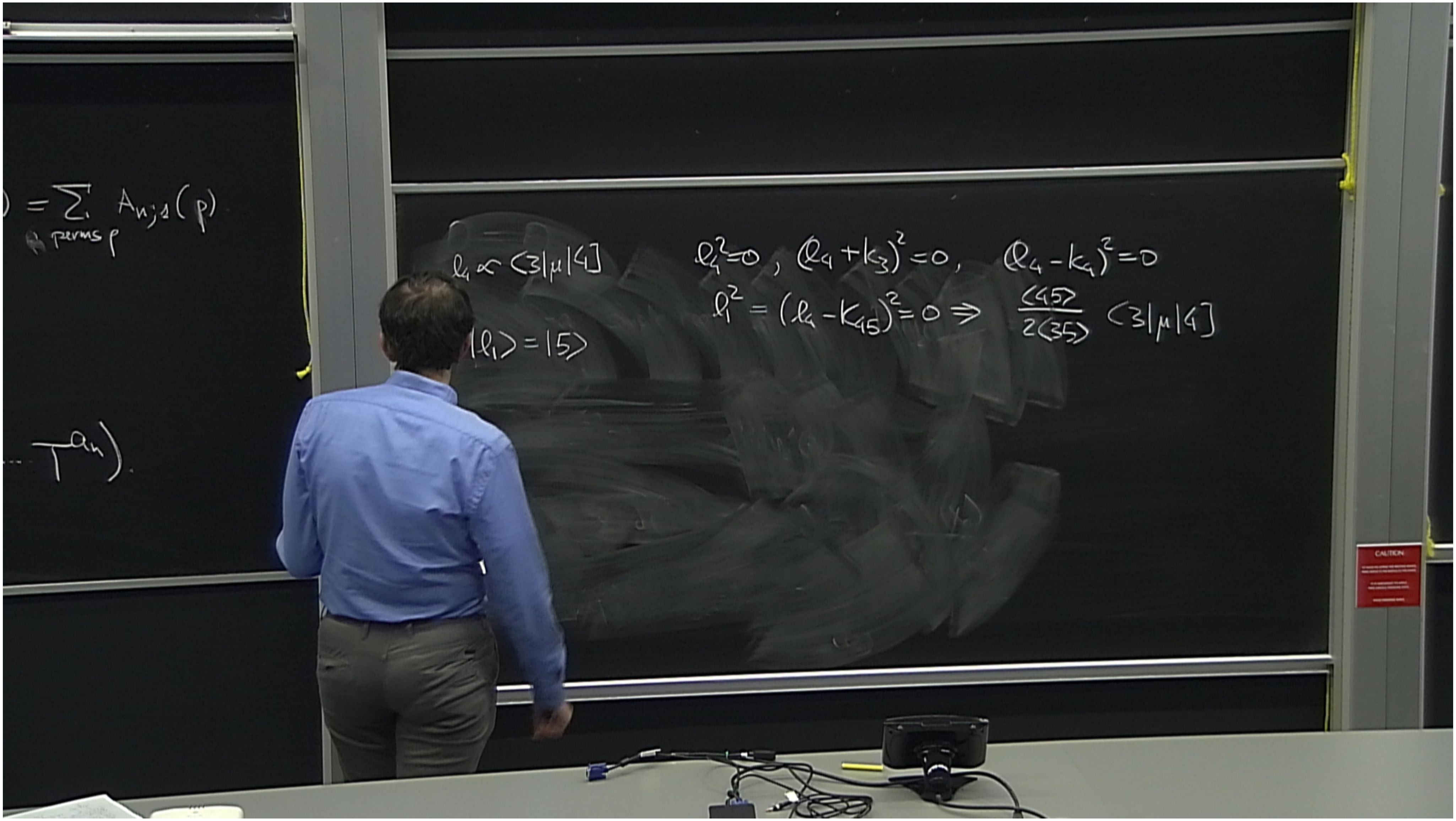

O = O
A5 (1-, 2-, 3+, 4+, 5+)
coeff of I4(K12, 3, 4, 5)
σ1 σ2
CAUTION


0 = 0
= Σ A_{n;1}(ρ)
perms ρ
A_5 (1^-, 2^-, 3^+, 4^+, 5^+)
coeff of I_4(K_12, 3, 4, 5)
σ_1 σ_2
− − → 0
− +
λ_3 ∝ λ_4
⟨ℓ_4 4⟩ = 0 ⇒ λ_{ℓ_4} ∝ λ_4
⟨ℓ_4 3⟩ = 0 ⇒ λ_{ℓ_4} ∝ λ_3
CAUTION

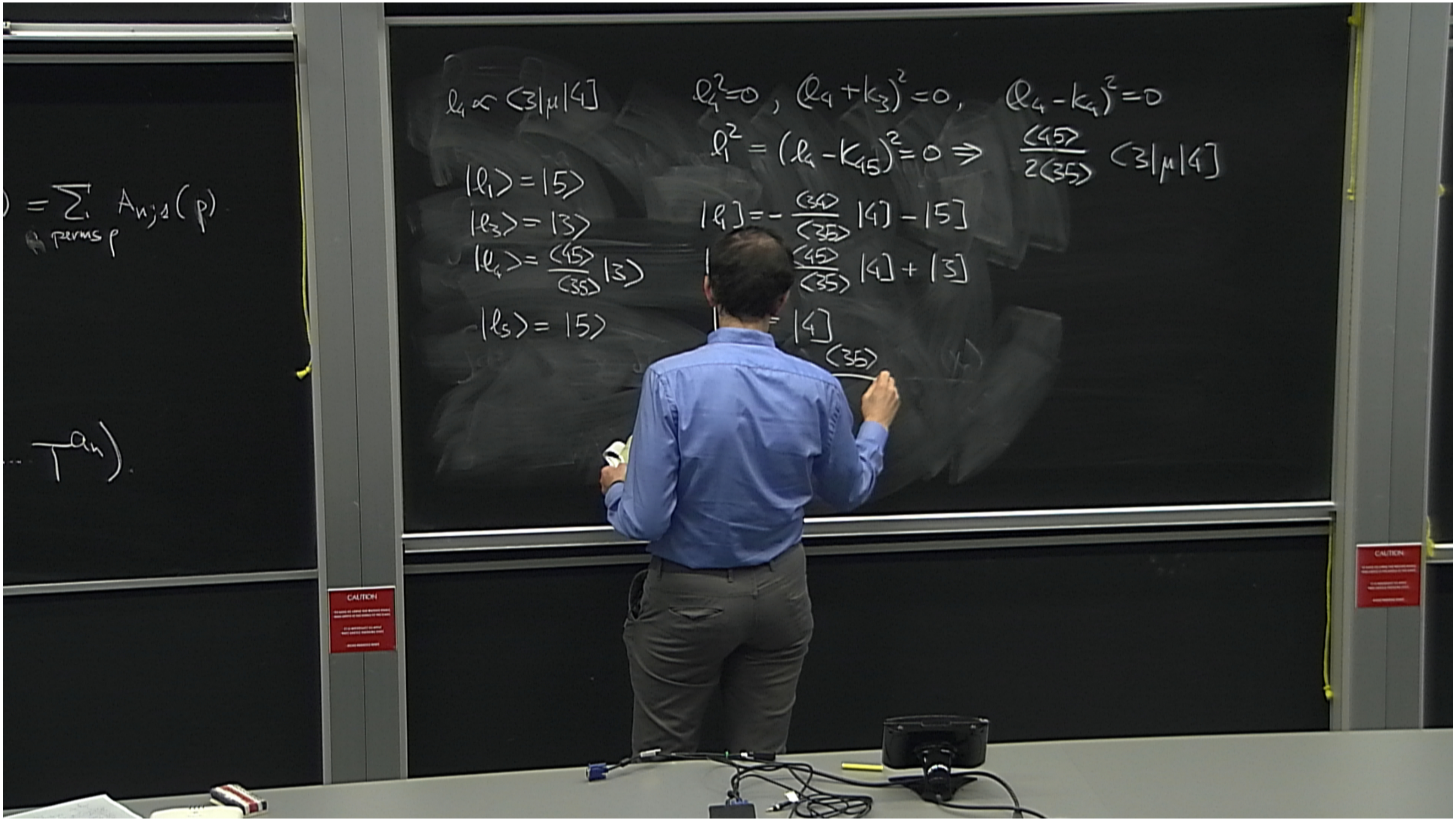

0 = 0
A5 (1−, 2−, 3+, 4+, 5+)
coeff of I4(K12, 3, 4, 5)
= Σ An;1(p)
perms p
σ1 σ2
− − → 0
− +
λ3 ∝ λ4
⟨l4 4⟩ = 0 ⇒ λl4 ∝ λ4
⟨l4 3⟩ = 0 ⇒ λl4 ∝ λ3
CAUTION

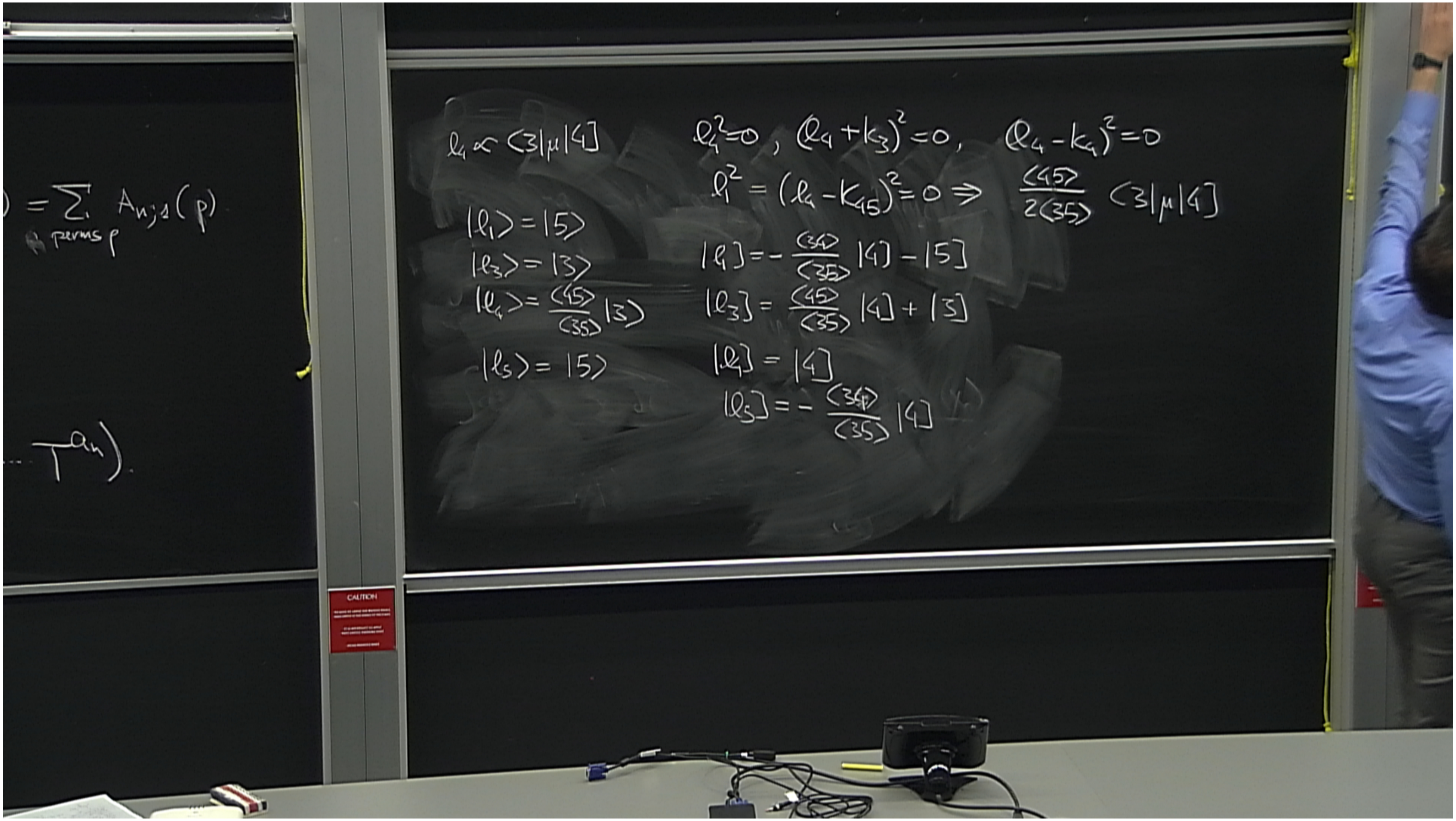

0 = 0
A5 (1−, 2−, 3+, 4+, 5+)
coeff of I4(K12, 3, 4, 5)
σ1 σ2
− − → 0
0 ← − +
0 ← + −
+ +
λ3 ∝ λ4
⟨l4 4⟩ = 0 ⇒ λl4 ∝ λ4
⟨l4 3⟩ = 0 ⇒ λl4 ∝ λ3
CAUTION



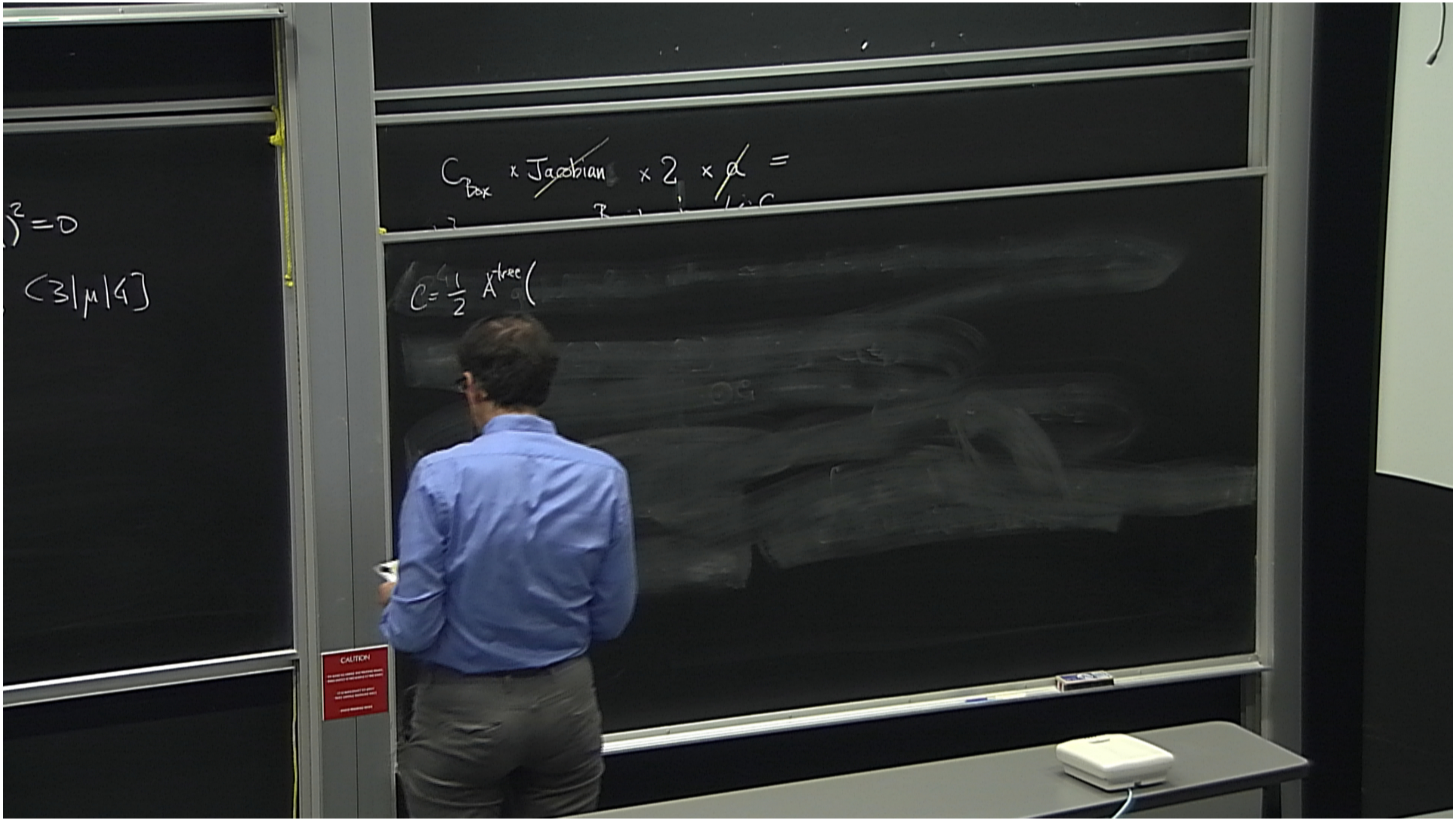


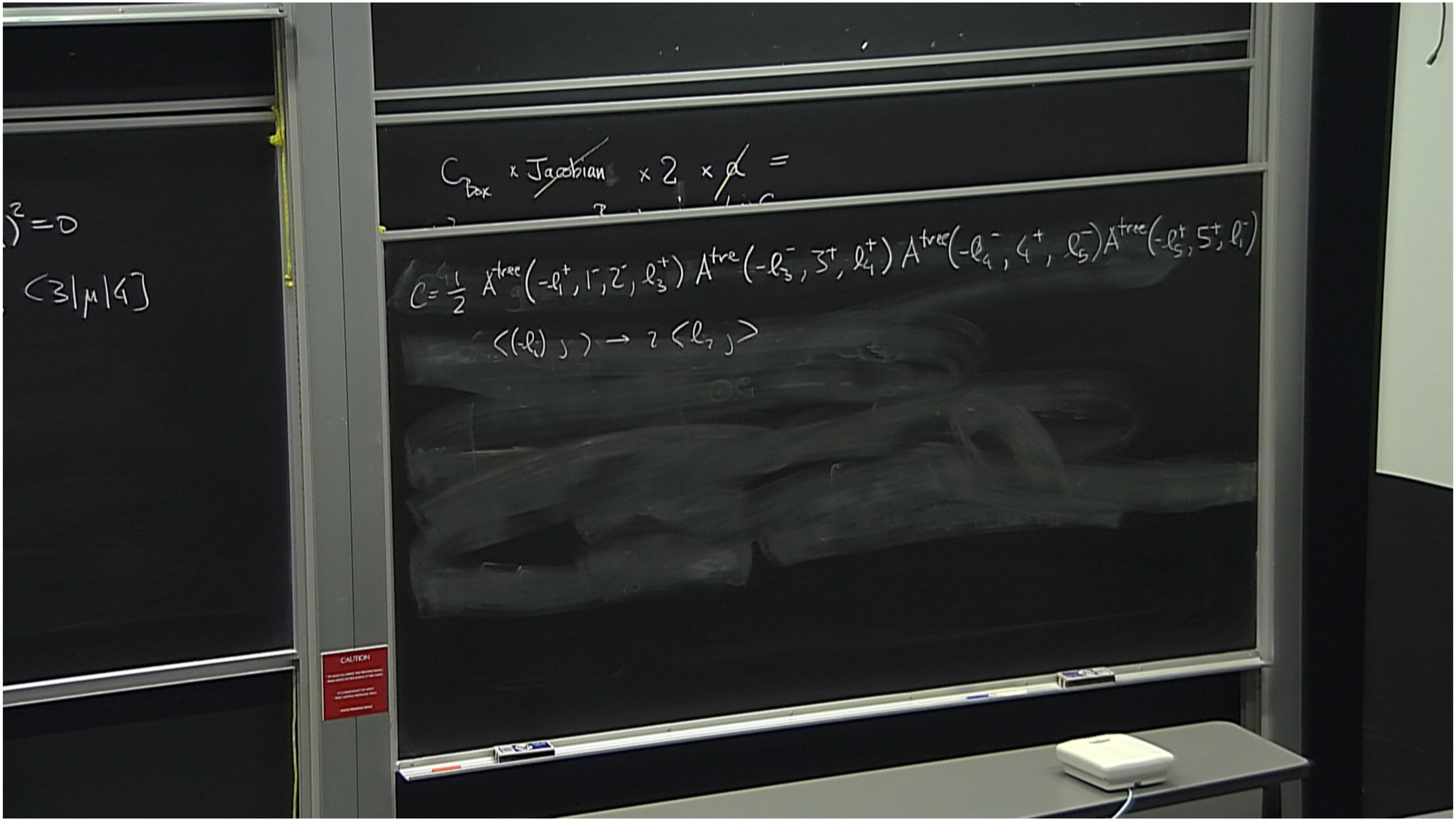
ℓ4 ∝ ⟨3|μ|4]
ℓ1² = 0 , (ℓ4 + k3)² = 0 , (ℓ4 − k4)² = 0
ℓ1² = (ℓ4 − K45)² = 0 ⇒ ⟨45⟩ / 2⟨35⟩ ⟨3|μ|4]
|ℓ1⟩ = |5⟩
CAUTION

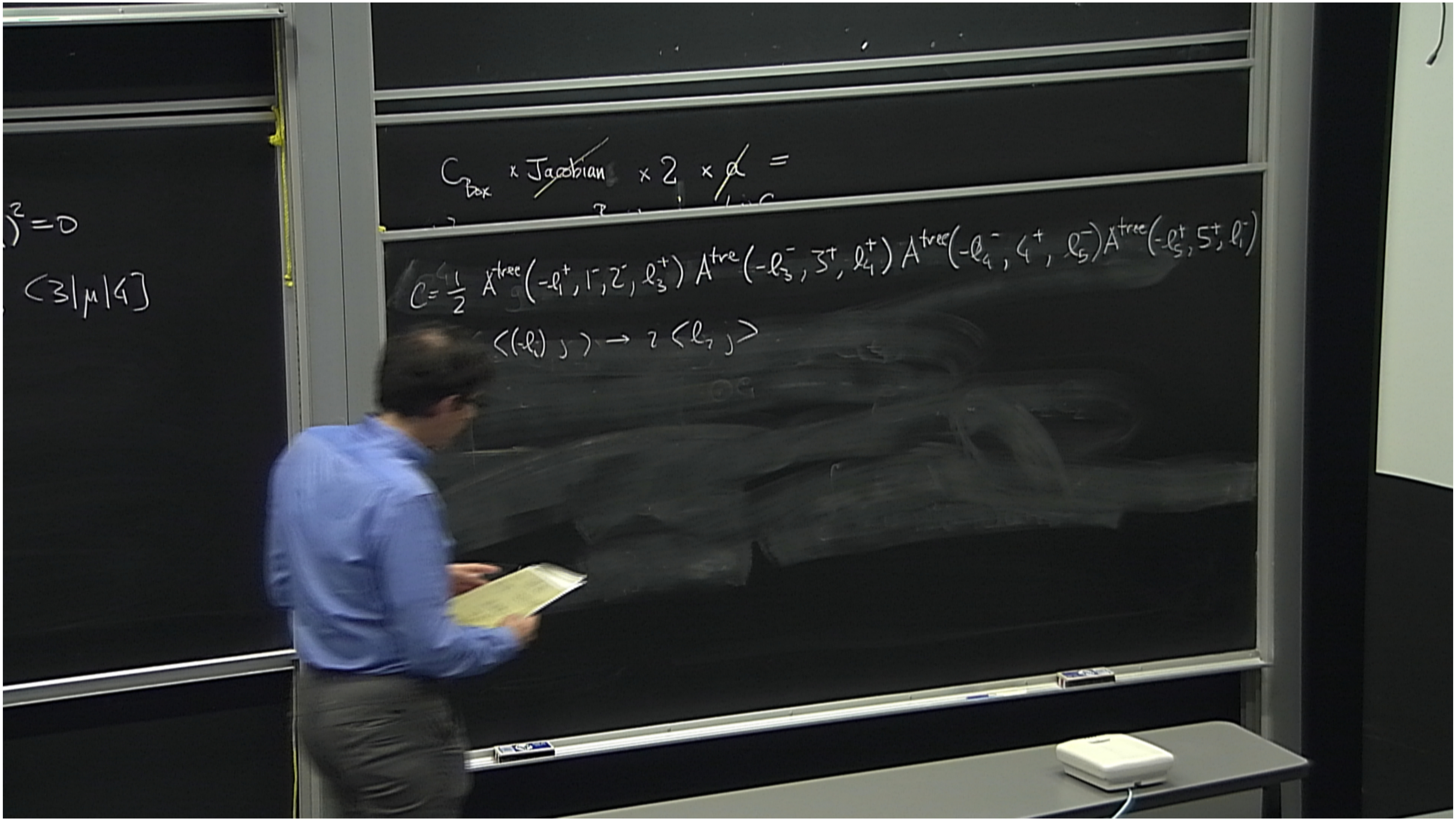

l4 ∝ ⟨3|μ|4]
l1² = 0 , (l4 + k3)² = 0 , (l4 − k4)² = 0
l1² = (l4 − K45)² = 0 ⇒ ⟨45⟩ / 2⟨35⟩ ⟨3|μ|4]
|l1⟩ = |5⟩
|l3⟩ = |3⟩
|l4⟩ = ⟨45⟩/⟨35⟩ |3⟩
|l5⟩ = |5⟩
|l1] = − ⟨34⟩/⟨35⟩ |4] − |5]
⟨45⟩/⟨35⟩ |4] + |3]
|4]
⟨35⟩
) = Σ perms ρ An;1(ρ)
T^an)

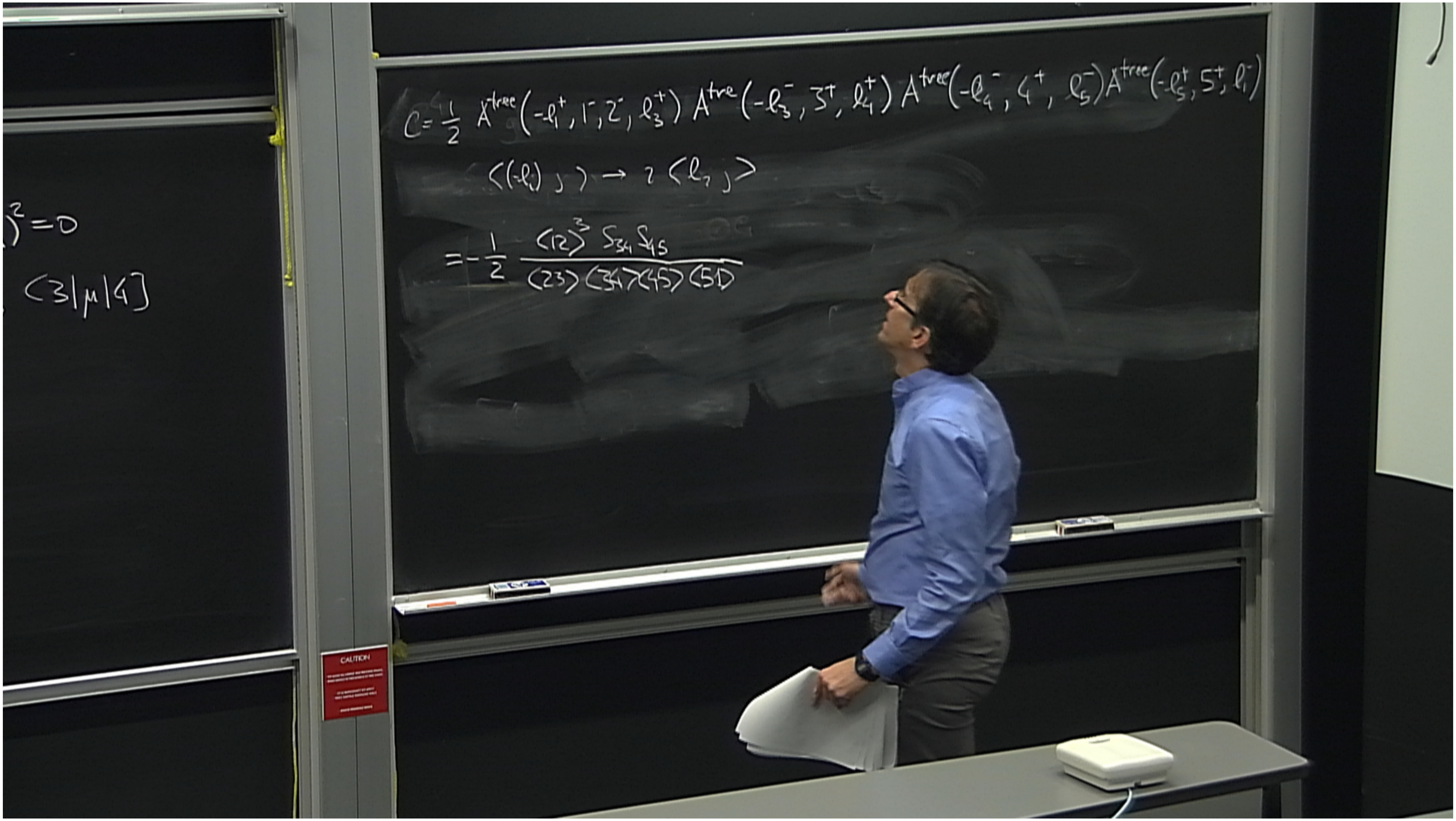


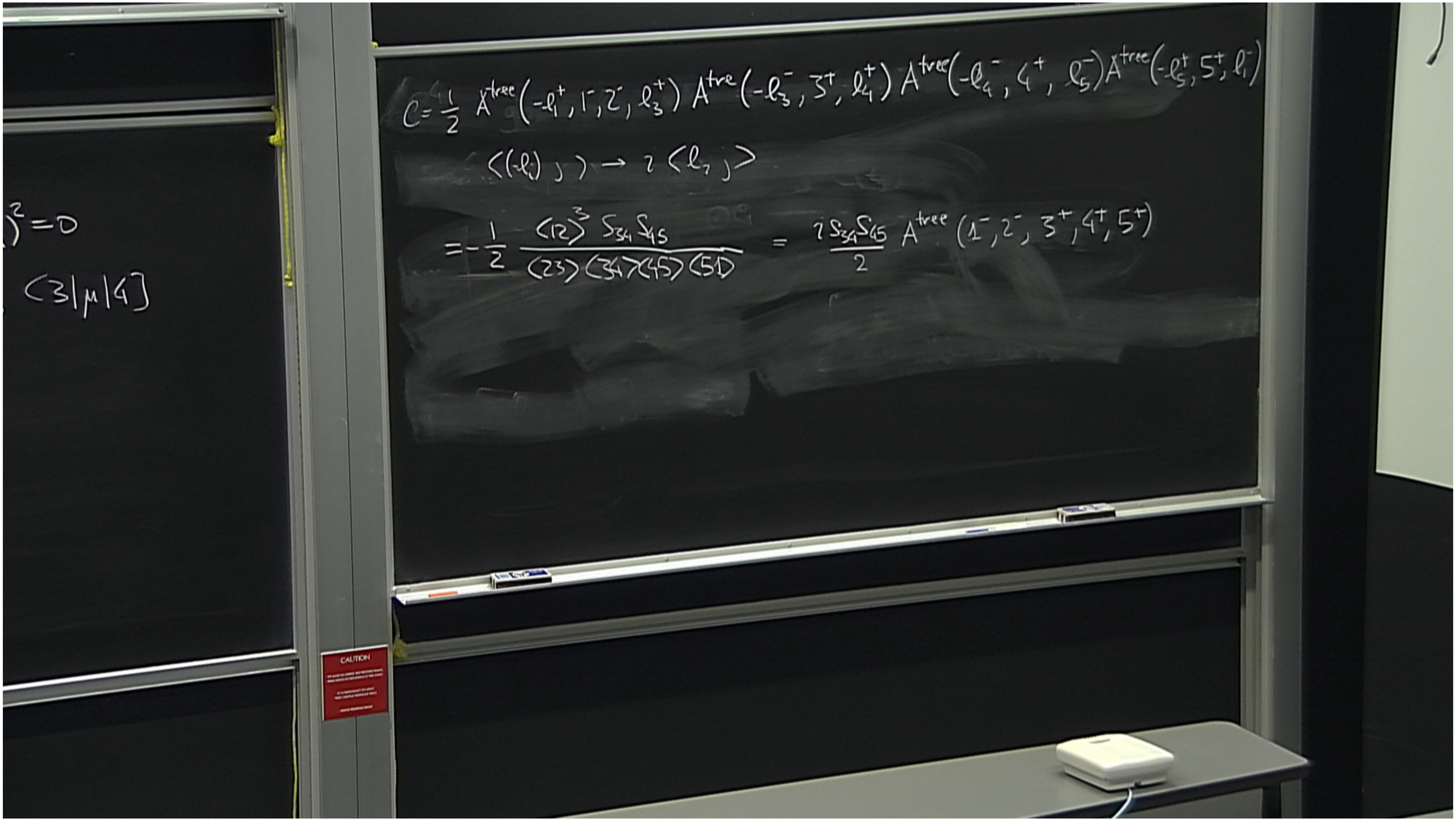
$= \sum_i A_{n;1}(p)$
perms p
$l_4 \propto \langle 3|\mu|4]$
$l_1^2=0$, $(l_4+k_3)^2=0$, $(l_4-k_4)^2=0$
$l_1^2=(l_4-K_{45})^2=0 \Rightarrow \frac{\langle 45\rangle}{2\langle 35\rangle}\langle 3|\mu|4]$
$|l_1\rangle=|5\rangle$
$|l_3\rangle=|3\rangle$
$|l_4\rangle=\frac{\langle 45\rangle}{\langle 35\rangle}|3\rangle$
$|l_5\rangle=|5\rangle$
$|l_1]=-\frac{\langle 34\rangle}{\langle 35\rangle}|4]-|5]$
$|l_3]=\frac{\langle 45\rangle}{\langle 35\rangle}|4]+|3]$
$|l_4]=|4]$
$|l_5]=-\frac{\langle 34\rangle}{\langle 35\rangle}|4]$
CAUTION

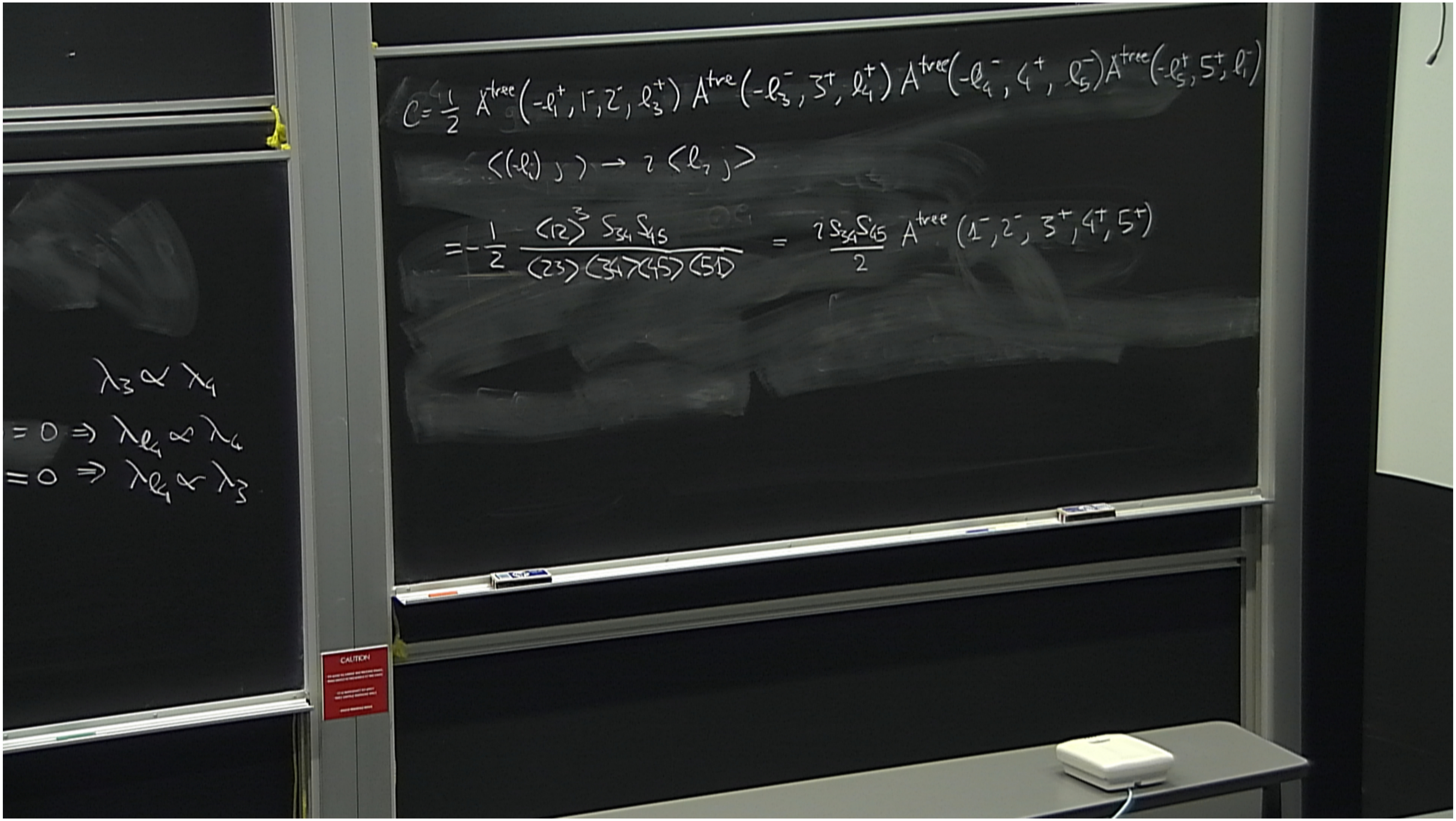

)^2=0
<3|μ|4]
C Box x Jacobian x 2 x α =
C = 1/2 A tree (

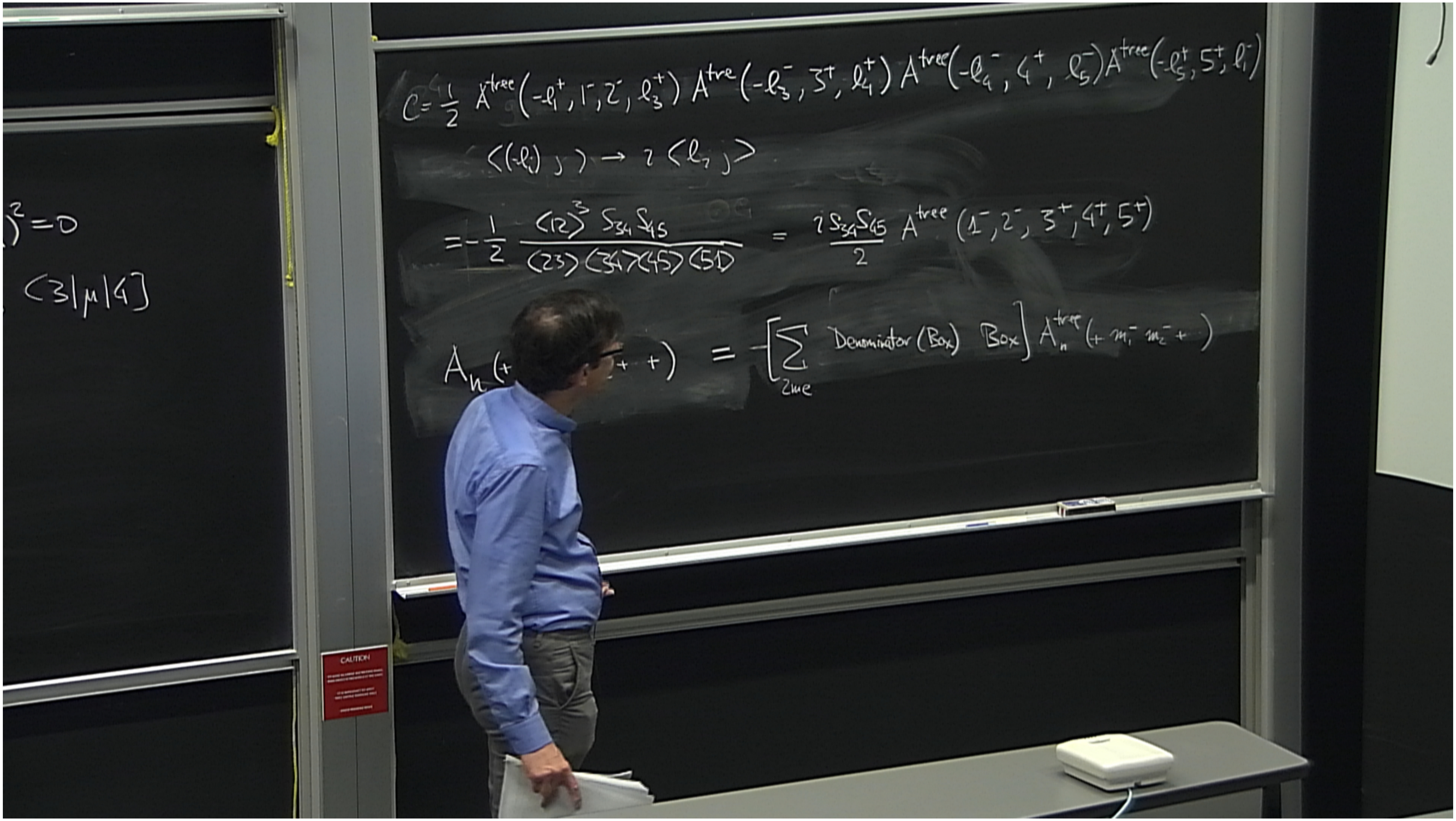

C_Box × Jacobian × 2 × α =
)^2 = 0
<3|μ|4]
C = 1/2 A^tree(-l1+, 1-, 2-, l3+) A^tre(-l3-, 3+, l4+) A^tree(-l4-, 4+, l5-) A^tree(-l5+, 5+, l1-)
<(-li) j> → i <li j>
CAUTION

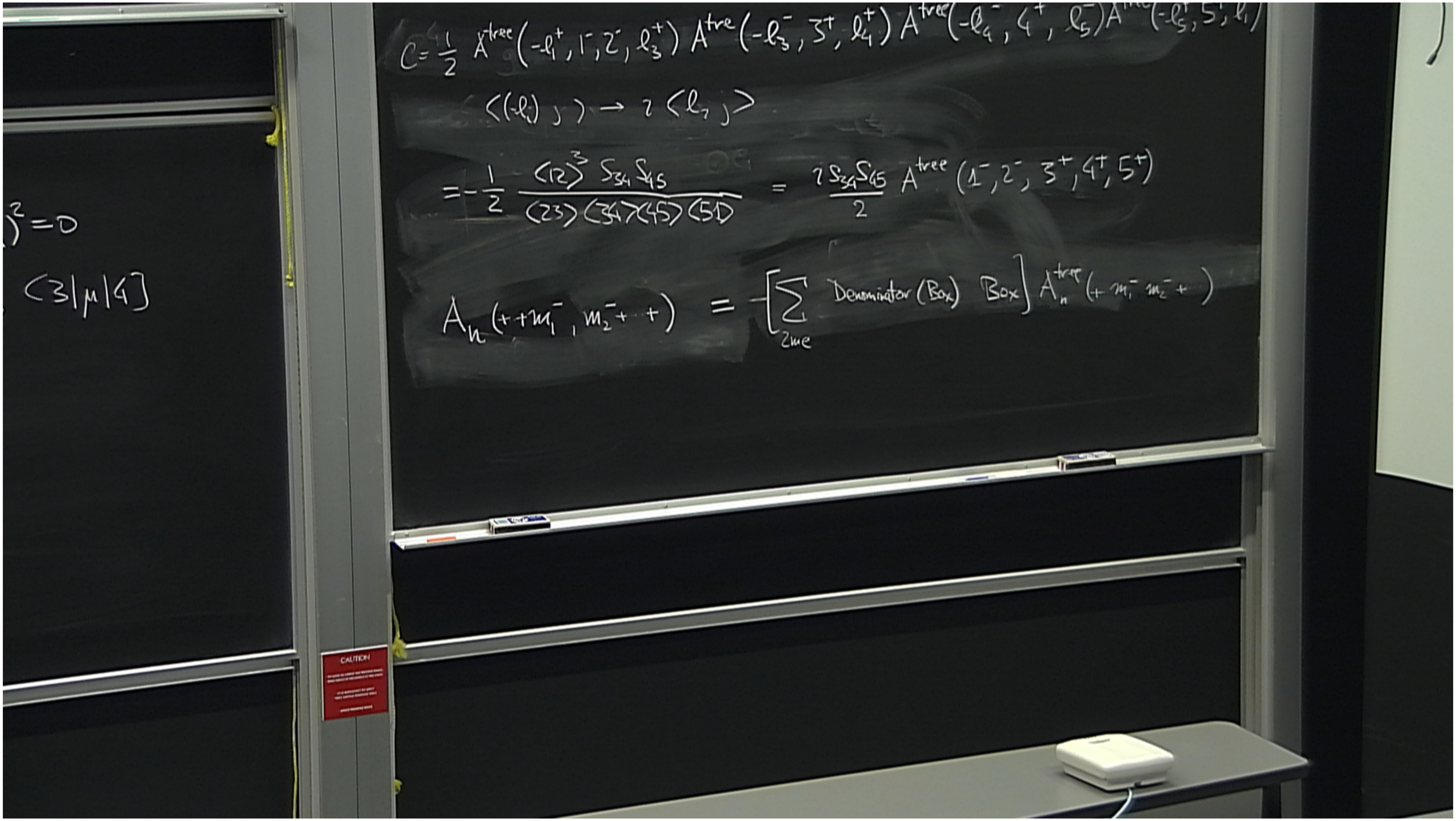

)^2 = 0
⟨3|μ|4]
C_Box × Jacobian × 2 × α =
C = 1/2 A^tree(-l1+, 1-, 2-, l3+) A^tree(-l3-, 3+, l4+) A^tree(-l4-, 4+, l5-) A^tree(-l5+, 5+, l1-)
⟨(-l1) j⟩ → i⟨l1 j⟩


$C = \frac{1}{2} A^{tree}(-\ell_1^+, 1^-, 2^-, \ell_3^+) A^{tree}(-\ell_3^-, 3^+, \ell_4^+) A^{tree}(-\ell_4^-, 4^+, \ell_5^-) A^{tree}(-\ell_5^+, 5^+, \ell_1^-)$
$\langle (-\ell_1)\, j \rangle \to i \langle \ell_1\, j \rangle$
$= -\frac{1}{2} \frac{\langle 12 \rangle^3 s_{34} s_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$
$)^2 = 0$
$\langle 3|\mu|4]$
CAUTION


$C = \frac{1}{2} A^{tree}(-\ell_1^+, 1^-, 2^-, \ell_3^+) A^{tree}(-\ell_3^-, 3^+, \ell_4^+) A^{tree}(-\ell_4^-, 4^+, \ell_5^-) A^{tree}(-\ell_5^+, 5^+, \ell_1^-)$
$\langle (-\ell_i)\, j \rangle \to i \langle \ell_i\, j \rangle$
$= -\frac{1}{2} \frac{\langle 12 \rangle^3 s_{34} s_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} = \frac{i s_{34} s_{45}}{2} A^{tree}(1^-, 2^-, 3^+, 4^+, 5^+)$
$)^2 = 0$
$\langle 3 | \mu | 4]$
CAUTION


C = 1/2 A^tree(-l1+, 1-, 2-, l3+) A^tree(-l3-, 3+, l4+) A^tree(-l4-, 4+, l5-) A^tree(-l5+, 5+, l1-)
⟨(-l1) j⟩ → i⟨l1 j⟩
= -1/2 ⟨12⟩³ s34 s45 / ⟨23⟩⟨34⟩⟨45⟩⟨51⟩ = i s34 s45 / 2 A^tree(1-, 2-, 3+, 4+, 5+)
λ3 ∝ λ4
= 0 ⇒ λl4 ∝ λ4
= 0 ⇒ λl4 ∝ λ3


C = 1/2 A^tree(-l1+, 1-, 2-, l3+) A^tre(-l3-, 3+, l4+) A^tree(-l4-, 4+, l5-) A^tree(-l5+, 5+, l1-)
⟨(-l_i) j⟩ → i ⟨l_i j⟩
= -1/2 ⟨12⟩^3 s34 s45 / ⟨23⟩⟨34⟩⟨45⟩⟨51⟩ = i s34 s45 / 2 A^tree(1-, 2-, 3+, 4+, 5+)
Denominator (Box) Box
CAUTION


C = 1/2 A^tree(-l_1^+, 1^-, 2^-, l_3^+) A^tree(-l_3^-, 3^+, l_4^+) A^tree(-l_4^-, 4^+, l_5^-) A^tree(-l_5^+, 5^+, l_1)
= -1/2 ⟨12⟩^3 s_34 s_45 / ⟨23⟩⟨34⟩⟨45⟩⟨51⟩ = i s_34 s_45 / 2 A^tree(1^-, 2^-, 3^+, 4^+, 5^+)
A_n(+ + m_1^-, m_2^- + ... +) = -[Σ Denominator (Box) Box] A_n^tree(+ m_1^- m_2^- +)
⟨3|μ|4]