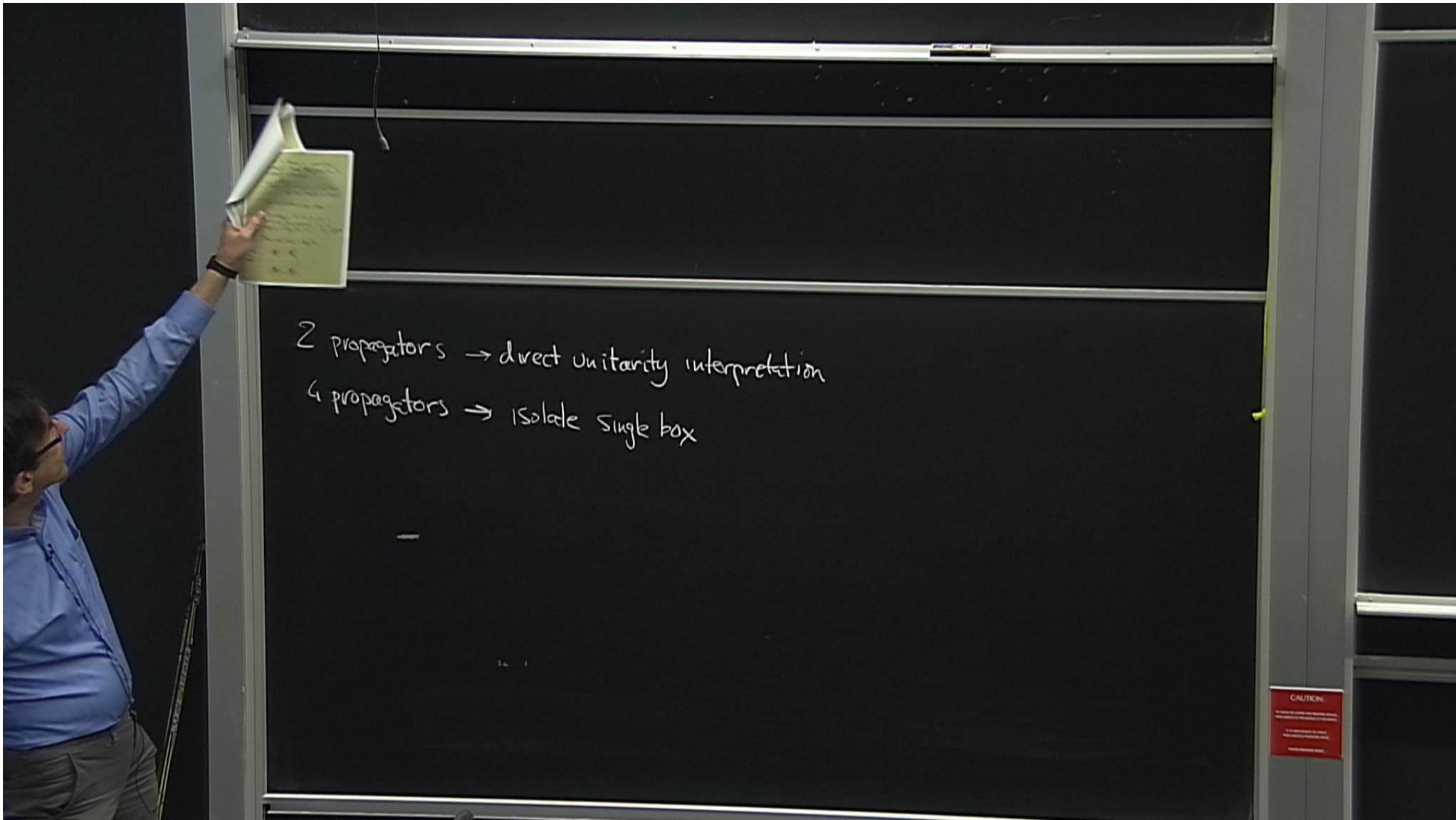


Title: Amplitudes: Generalized Unitarity

Date: Jul 17, 2015 02:30 PM

URL: <http://pirsa.org/15070058>

Abstract:



2 propagators \rightarrow direct unitarity interpretation

4 propagators \rightarrow isolate single box

$$\frac{1}{q^2(q-p_1)^2(q-p_{12})^2(q-p_{123})^2}$$

4 propagators \rightarrow isolate single box

$$\int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 (\ell-p_1)^2 (\ell-p_{12})^2 (\ell-p_{123})^2}$$

$$\int \frac{d^4 \ell}{(2\pi)^4} \delta^{(+)}(\ell^2) \delta^{(+)}(\ell-p_1) \delta^{(+)}(\ell-p_{12}) \delta^{(+)}(\ell-p_{123})$$

2 propagators \rightarrow direct unitarity interpretation

4 propagators \rightarrow isolate single box

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 (l-p_1)^2 (l-p_{12})^2 (l+p_{123})^2}$$

$$\int \frac{d^4 l}{(2\pi)^4} \delta^{(+)}(l^2) \delta^{(+)}(l-p_1) \delta^{(+)}(l-p_{12}) \delta^{(+)}(l-p_{123})$$

$$l^2=0 \quad (l-p_1)^2=0 \quad (l-p_{12})^2=0 \quad (l+p_{123})^2=0$$

$$L^2=0, \quad -2L \cdot p_1 + p_1^2=0, \quad -2L \cdot p_2 + S_{12} - p_1^2=0, \quad 2L \cdot p_4 + p_4^2=0$$

\Rightarrow 2 sol's

$$p_1^2=0=p_4^2$$

$$L^M = \frac{\xi}{2} \langle 1 | \mu | 4 \rangle, \quad \xi = \frac{S_{12}}{\langle 1 | 2 | 4 \rangle}$$

$$L'^M = \frac{\xi'}{2} \langle 4 | \mu | 1 \rangle, \quad \xi' = \frac{S_{12}}{\langle 4 | 2 | 1 \rangle}$$

$$L^2 = 0, \quad -2L \cdot p_1 + p_1^2 = 0, \quad -2L \cdot p_2 + S_{12} - p_1^2 = 0, \quad 2L \cdot p_4 + p_4^2 = 0$$

$\Rightarrow 2$ solns

$$p_1^2 = 0 = p_4^2$$

$$L^M = \frac{\xi}{2} \langle 1 | \mu | 4 \rangle, \quad \xi = \frac{S_{12}}{\langle 1 | 2 | 4 \rangle}$$

$$L'^M = \frac{\xi'}{2} \langle 4 | \mu | 1 \rangle, \quad \xi' = \frac{S_{12}}{\langle 4 | 2 | 1 \rangle}$$

L^M, L'^M complex

$$l^2 = 0, \quad -2l \cdot p_1 + p_1^2 = 0, \quad -2l \cdot p_2 + S_{12} - p_1^2 = 0, \quad 2l \cdot p_4 + p_4^2 = 0$$

$\Rightarrow 2$ sol'ns

$$p_1^2 = 0 = p_4^2$$

$$l^\mu = \frac{S_{12}}{2} \langle 1 | \mu | 4 \rangle, \quad \xi = \frac{S_{12}}{\langle 1 | 2 | 4 \rangle}$$

$$l'^\mu = \frac{S_{12}}{2} \langle 4 | \mu | 1 \rangle, \quad \xi' = \frac{S_{12}}{\langle 4 | 2 | 1 \rangle}$$

l^μ, l'^μ complex

$$\int_a^b dx f(x) \delta(x - x_0) = f(x_0) \quad x_0 \in [a, b]$$

$$L^2=0, \quad -2L \cdot p_1 + p_1^2=0, \quad -2L \cdot p_2 + S_{12} - p_1^2=0, \quad 2L \cdot p_4 + p_4^2=0$$

$\Rightarrow 2$ sol'ns

$$p_1^2=0=p_4^2$$

$$L^M = \frac{S}{2} \langle 1 | \mu | 4 \rangle, \quad \xi = \frac{S_{12}}{\langle 1 | 2 | 4 \rangle}$$

$$L'^M = \frac{S'}{2} \langle 4 | \mu | 1 \rangle, \quad \xi' = \frac{S_{12}}{\langle 4 | 2 | 1 \rangle}$$

L^M, L'^M complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

\mathbb{C}, \mathbb{C}^n complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

$$0 = 0.$$

$\text{Poly}_2(z) - a$ simple root at z_0 $\text{Poly}_2(z_0) - a = 0$

$$\oint_{\mathcal{C}(z_0)} dz \frac{\text{Poly}_1(z)}{\text{Poly}_2(z) - a} = \frac{\text{Poly}_1(z_0)}{\text{Poly}_2'(z_0)}$$

\mathbb{C}, \mathbb{C}^n complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

$$0 = 0$$

$\text{Poly}_2(z) - a$ simple root at z_0 $\text{Poly}_2(z_0) - a = 0$

$$\oint_{\mathcal{C}(z_0)} dz \frac{\text{Poly}_1(z)}{\text{Poly}_2(z) - a} = \frac{\text{Poly}_1(z_0)}{\text{Poly}_2'(z_0)}$$

$$\int dx \text{Poly}_1(x) \delta(\text{Poly}_2(x) - a) = \frac{\text{Poly}_1(x_0)}{|\text{Poly}_2'(x_0)|}$$

\mathbb{C}, \mathbb{C}^n complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

$$0 = 0$$

$\text{Poly}_2(z) - a$ simple root at z_0 $\text{Poly}_2(z_0) - a = 0$

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\mathbb{C}, \mathbb{C}^n complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

$\text{Poly}_2(z) - a$ simple root at z_0 $\text{Poly}_2(z_0) - a = 0$

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$$\int dz \text{Poly}_1(z) \delta(\text{Poly}_2(z) - a) = \oint_{C(z_0)}$$

\mathbb{C}, \mathbb{C}^n complex

$$\int_a^b dx f(x) \delta(x-x_0) = f(x_0) \quad x_0 \in [a, b]$$

$$0 = 0$$

$\text{Poly}_2(z) - a$ simple root at z_0 $\text{Poly}_2(z_0) - a = 0$

$$\oint_{C(z_0)} dz \frac{\text{Poly}_1(z)}{\text{Poly}_2(z) - a} = \frac{\text{Poly}_1(z_0)}{\text{Poly}_2'(z_0)}$$

$$\int dx \text{Poly}_1(x) \delta(\text{Poly}_2(x) - a) = \frac{\text{Poly}_1(x_0)}{|\text{Poly}_2'(x_0)|}$$

$$\int dz \text{Poly}_1(z) \delta(\text{Poly}_2(z) - a) = \oint_{C(z_0)}$$

↳ global pole: n dim \approx solution of n simultaneous eqns

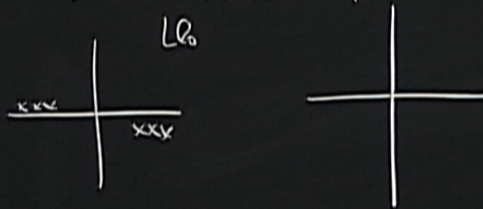
CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
IF A PERSONNEL IS IN THE BOARDER
PLEASE REPORT TO THE STAFF

\mathbb{P}^n global pole: n dim \simeq solution of n simultaneous eqns

$$\mathbb{P}^n = C_1(z_1^0) \times C_2(z_2^0) \times C_3(z_3^0) \times C_4(z_4^0)$$

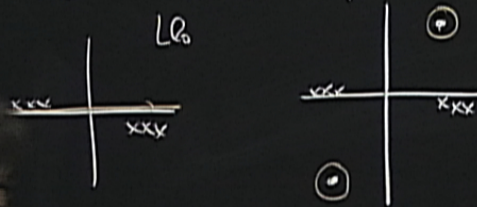
\mathbb{P}^1 global pole. $n \dim \approx$ solution of n simultaneous eqns

$$\Gamma = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$$



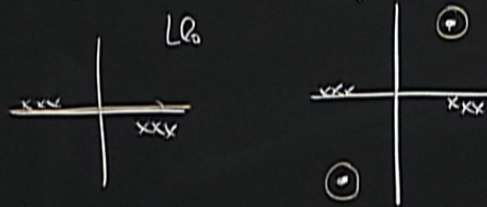
\mathbb{P}^4 global pole. $n \dim \approx$ solution of n simultaneous eqns

$$\frac{1}{T} = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$$



\mathbb{P}^4 global pole. $n \text{ dim} \approx$ solution of n simultaneous eqns

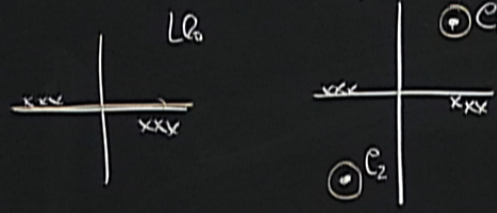
$$\frac{1}{T^4} = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$$



CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER
OR THE BOARDER

P^4 global pole. $n \text{ dim} \approx$ solution of n simultaneous eqns

$$\overline{F} = C_1(z_1^0) \times C_1(z_2^0) \times C(z_3^0) \times C(z_4^0)$$

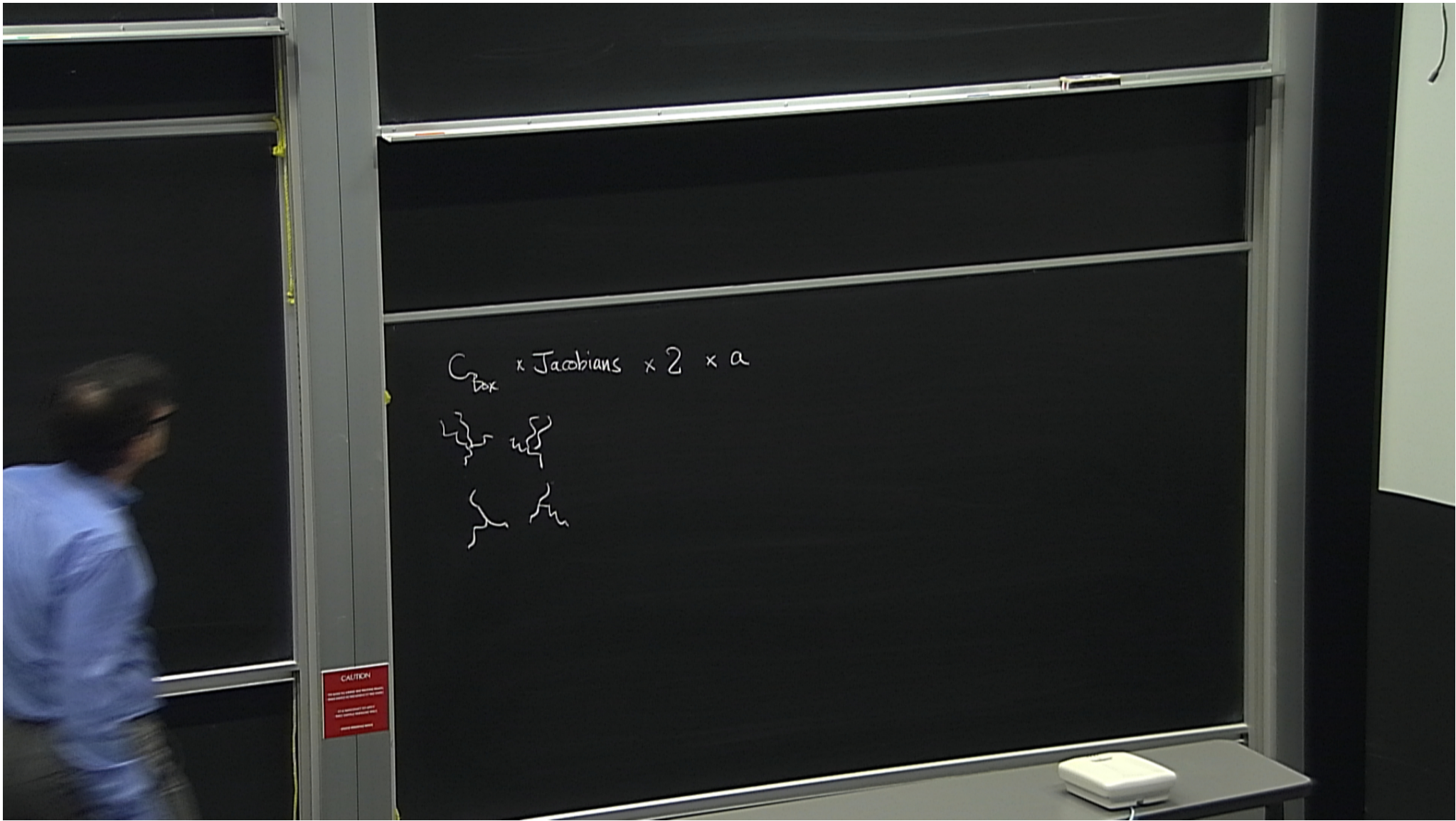


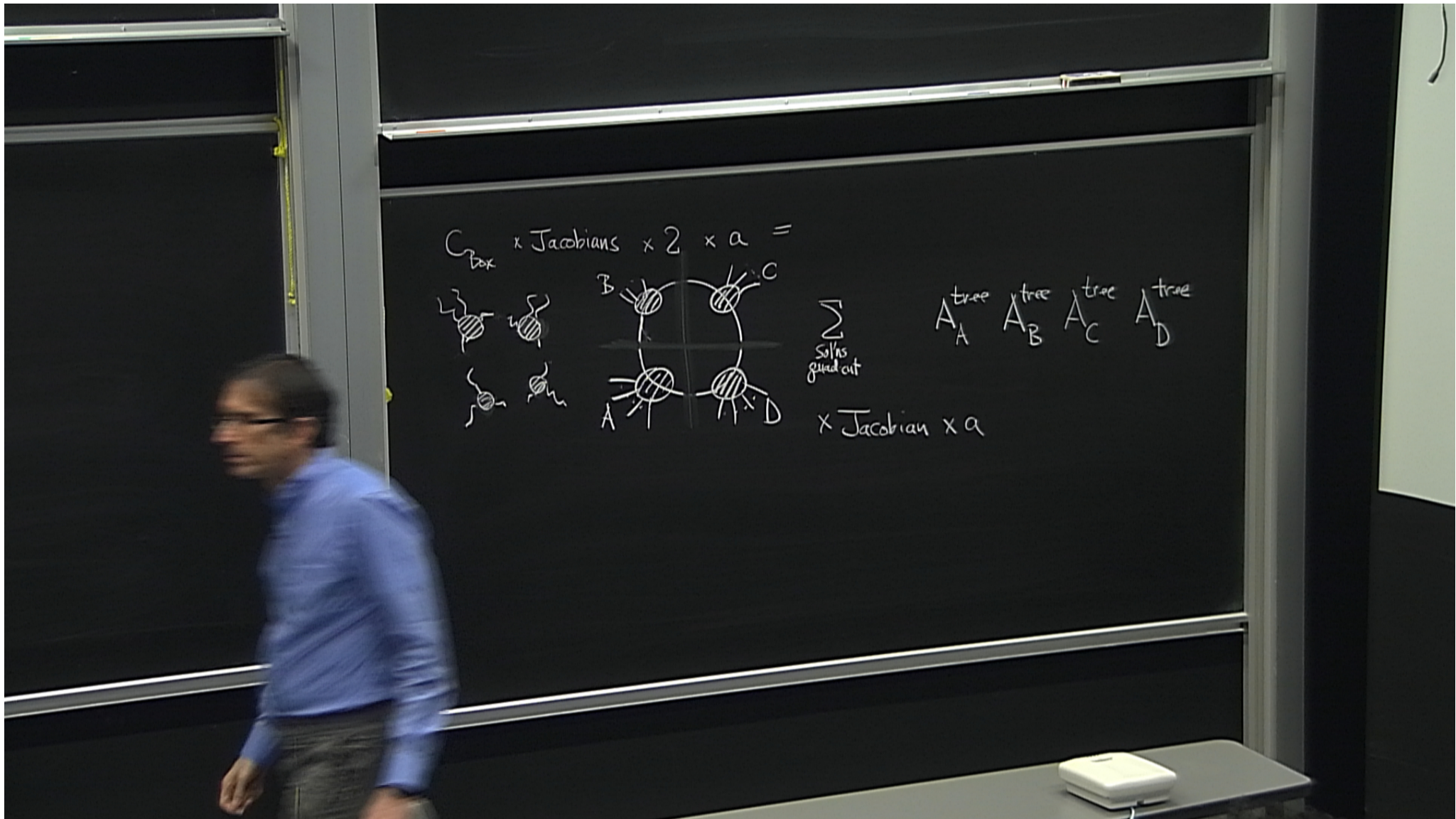
$$C = a_1 C_1 + a_2 C_2$$

$$\int \frac{E(s, p_1, p_2)}{s^2 (s-p_1)^2 (s-p_2)^2 (s+p_1)^2} ds = f(k_1, \dots, -a_2)$$

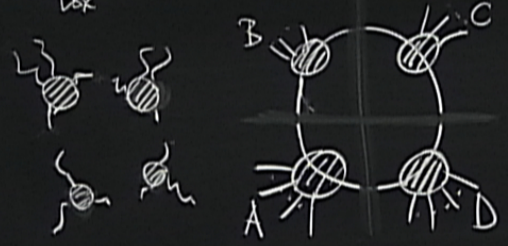
$$I_4 [E(s, p_1, p_2, p_4)] = 0$$

CAUTION
 This equipment contains high voltage
 components. Do not touch any
 internal components.
 Please refer to the manual.





$$C_{\text{box}} \times \text{Jacobians} \times 2 \times a =$$

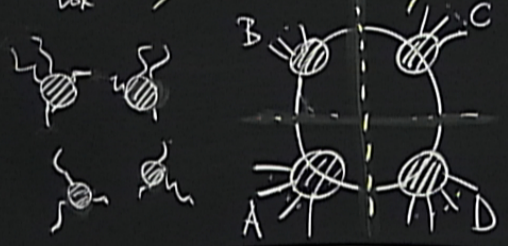


\sum
solns
quad cut

$$A_A^{\text{tree}} \quad A_B^{\text{tree}} \quad A_C^{\text{tree}} \quad A_D^{\text{tree}}$$

$$\times \text{Jacobian} \times a$$

$$C_{\text{Box}} \times \text{Jacobian} \times 2 \times d =$$



$$\sum_{\text{solns}} \sum_{\text{helicity}} A_A^{\text{tree}} A_B^{\text{tree}} A_C^{\text{tree}} A_D^{\text{tree}}$$

grad out species

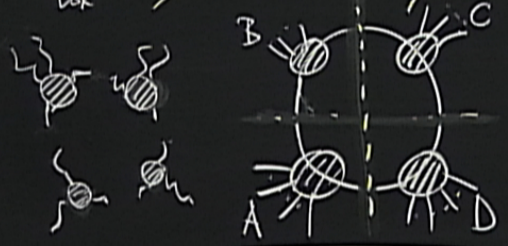
~~$\times \text{Jacobian} \times d$~~

$$C_{\text{Box}} = \frac{1}{2} \sum_{\text{solns}} \sum_{\text{helicity}} \prod A^{\text{tree}}$$

BCF

CAUTION
 Do not touch the screen or the screen frame.
 Do not touch the screen or the screen frame.
 Do not touch the screen or the screen frame.

$$C_{\text{Box}} \times \text{Jacobian} \times 2 \times d =$$



$$\sum_{\text{solns}} \sum_{\text{helicity species}} A_A^{\text{tree}} A_B^{\text{tree}} A_C^{\text{tree}} A_D^{\text{tree}}$$

~~$\times \text{Jacobian} \times d$~~

$$C_{\text{Box}} = \frac{1}{2} \sum_{\text{solns}} \sum_{\text{helicity species}} \prod A^{\text{tree}}$$

BCF

CAUTION
 This equipment is for use only in the laboratory.
 Do not use it in the classroom.
 Please return it to the lab.

$$A_n^{\text{1-loop}}(\{\lambda_i, \tilde{\lambda}_i, h_i = \pm, a_i\})$$

$$= \sum_{J=0, \frac{1}{2}, 1} n_J \sum_{c=1}^{\lfloor \frac{n}{2} \rfloor + 1} \sum_{\sigma \in S_n / S_{n_c}} Gr_{n;c}(\sigma) A_{n;c}(\sigma)$$

$$A_{n;c}(1) = \sum_{\text{perms } p} A_{n;c}(p)$$

↳ leaves trace structure invariant

Adjoint $Gr_{n;1}(1) = N_c \text{Tr}(T^{a_1} \dots T^{a_n})$

$$Gr_{n;c>1}(1) = \text{Tr}(T^{a_1} \dots T^{a_{c-1}}) \text{Tr}(T^{a_c} \dots T^{a_n})$$

Fundamental $Gr_{n,1} = \text{Tr}(T^{a_1} \dots T^{a_n})$



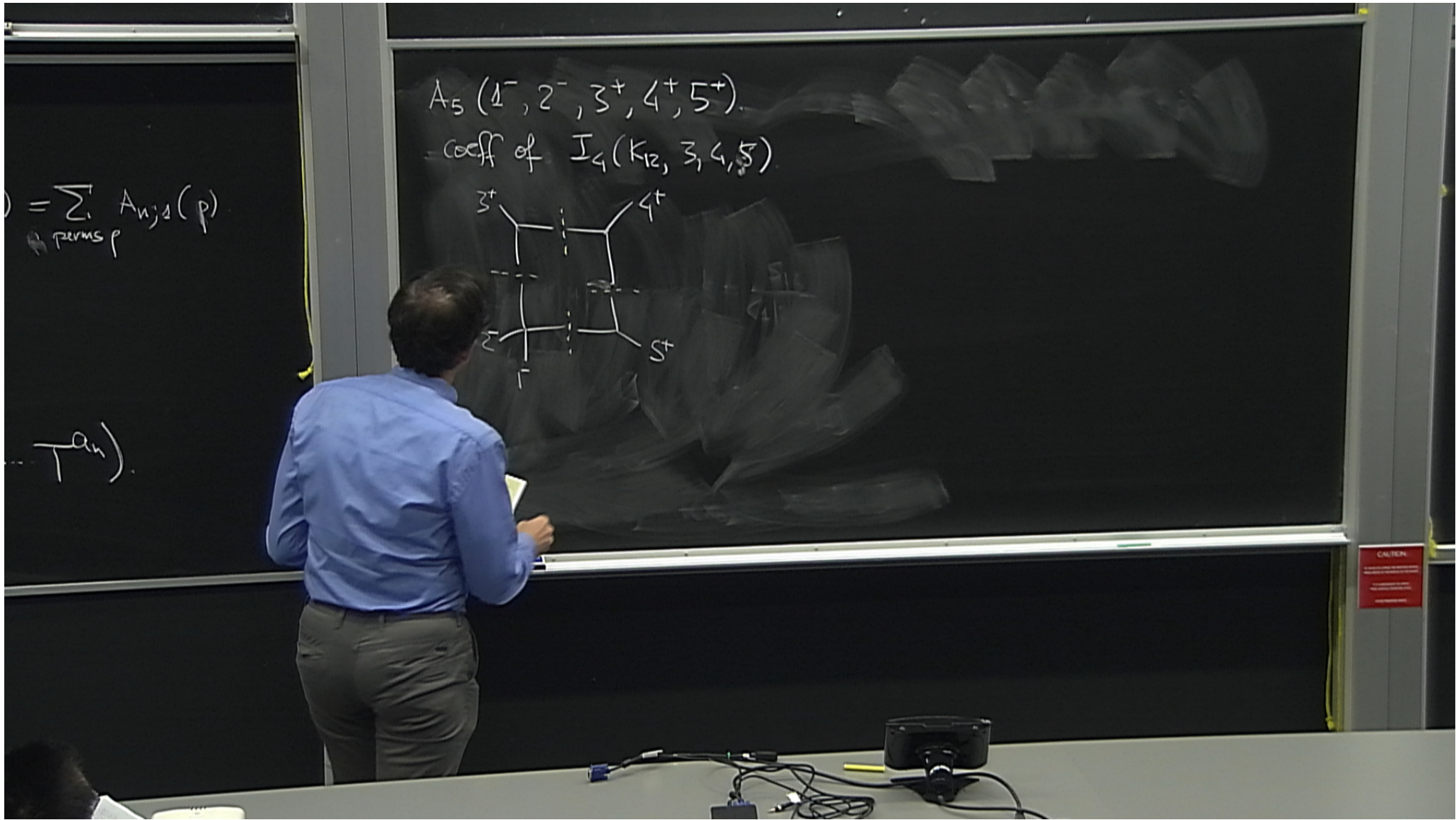
$$0 = 0.$$

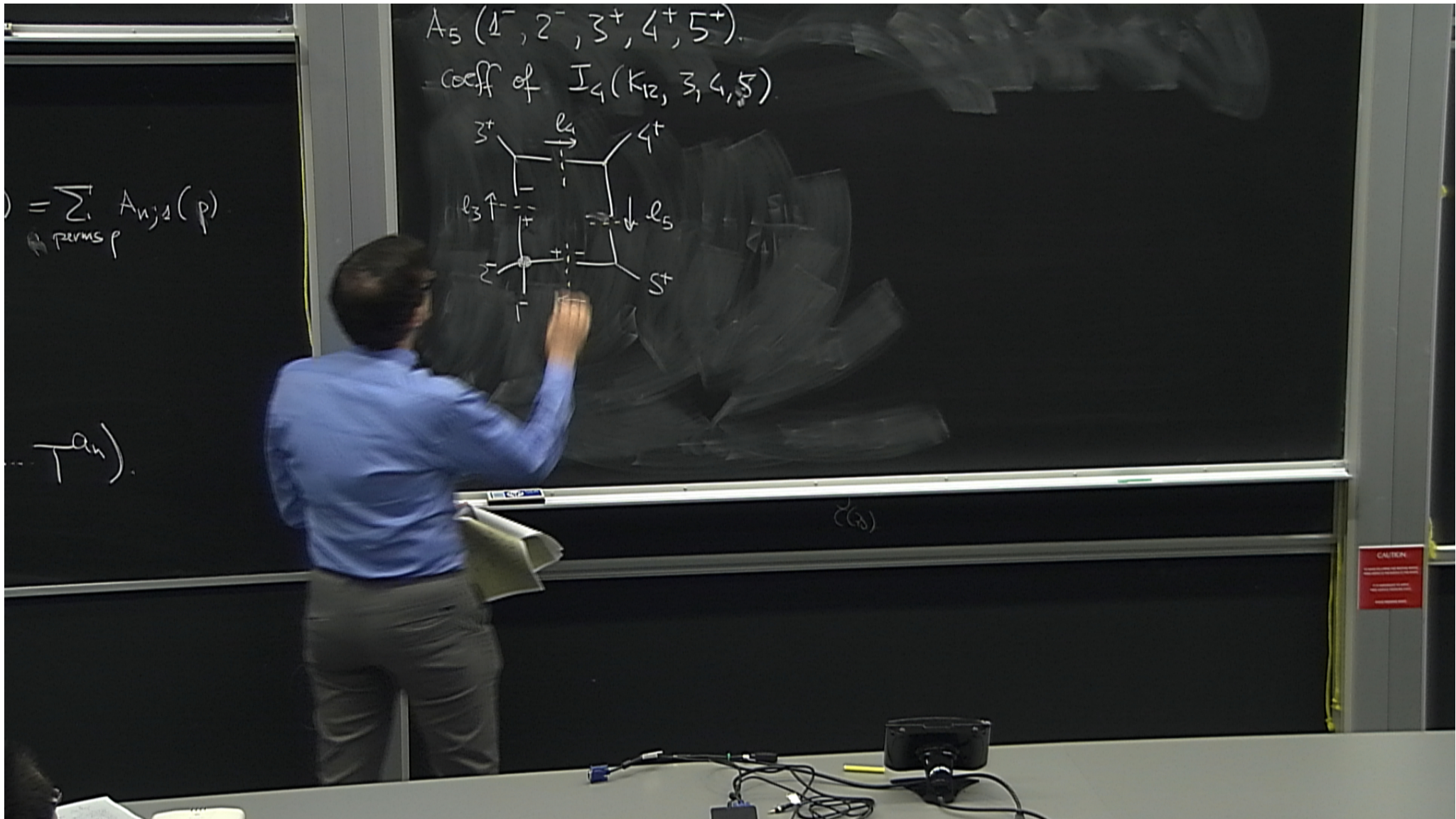
$$A_5 (1^-, 2^-, 3^+, 4^+, 5^+)$$

coeff of I_4

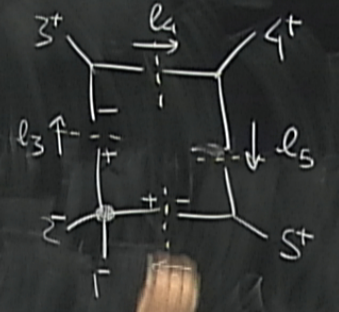
$$) = \sum_{\text{perms } p} A_{n_j s}(p)$$

$Tan)$





$A_5(1^-, 2^-, 3^+, 4^+, 5^+)$
 coeff of $I_4(K_{12}, 3, 4, 5)$



$= \sum_{\text{perms } p} A_{n_j s}(p)$

$T(n)$

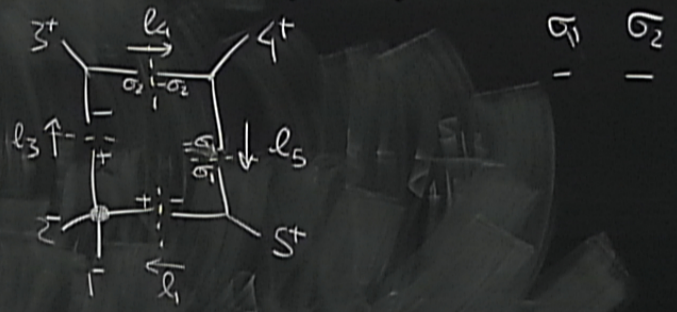
$$= \sum_{\text{perms } p} A_{n_j, s}(p)$$

Tan)

$$0 = 0$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+)$$

coeff of $I_4(K_{12}, 3, 4, 5)$



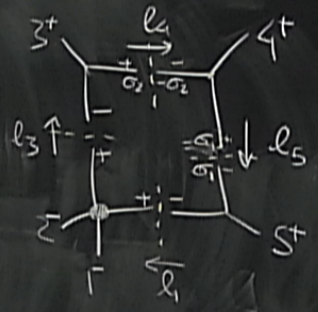
$$= \sum_{\text{perms } p} A_{n_j s}(p)$$

Tan)

$$0 = 0$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+)$$

coeff of $I_4(k_{12}, 3, 4, 5)$



$$\begin{matrix} \sigma_1 & \sigma_2 \\ - & - \\ - & + \end{matrix}$$

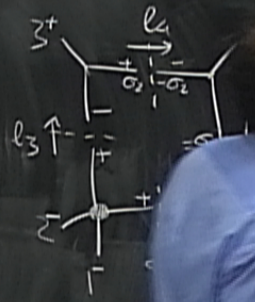
$$\begin{aligned} &\rightarrow 0 \quad \lambda_3 \& \lambda_4 \\ \langle l_4 4 \rangle = 0 &\Rightarrow \lambda_{l_4} \& \lambda_4 \\ \langle l_4 3 \rangle = 0 &\Rightarrow \lambda_{l_4} \& \lambda_3 \end{aligned}$$

$$= \sum_{\text{perms } p} A_{n_j s}(p)$$

$$0 = 0$$

$$A_5 (1^-, 2^-, 3^+, 4^+, 5^+)$$

coeff of $I_4(K_{12}, 3, 4, 5)$



$$\begin{matrix} \sigma_1 & \sigma_2 \\ - & - \\ - & + \end{matrix}$$

$$\begin{aligned} &\rightarrow 0 && \lambda_3 \text{ \& } \lambda_4 \\ \langle l_4 4 \rangle = 0 &\Rightarrow \lambda_{l_4} \text{ \& } \lambda_4 \\ \langle l_4 3 \rangle = 0 &\Rightarrow \lambda_{l_4} \text{ \& } \lambda_3 \end{aligned}$$

Tan)

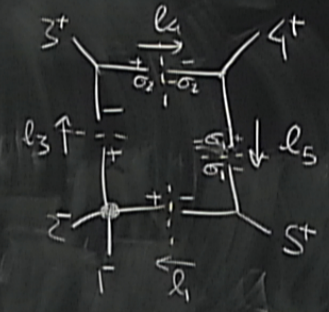
$$= \sum_{\text{perms } p} A_{n_j s}(p)$$

Tan)

$$0 = 0$$

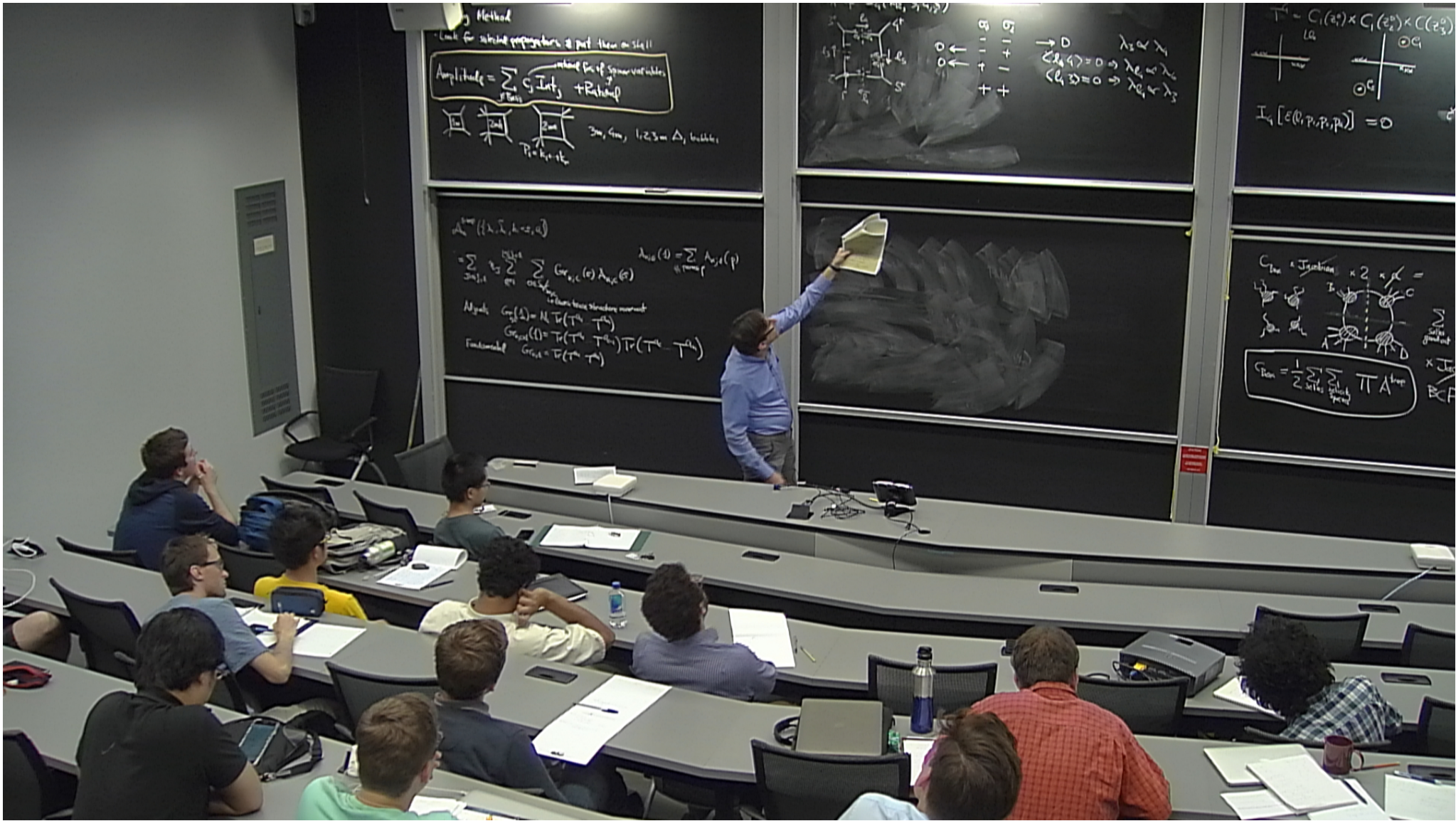
$$A_5(1^-, 2^-, 3^+, 4^+, 5^+)$$

coeff of $I_4(k_{12}, 3, 4, 5)$



	σ_1	σ_2
0 ←	-	-
0 ←	+	+
	+	+

→ 0 $\lambda_3 \& \lambda_4$
 $\langle l_4 4 \rangle = 0 \Rightarrow \lambda_{l_4} \propto \lambda_4$
 $\langle l_4 3 \rangle = 0 \Rightarrow \lambda_{l_4} \propto \lambda_3$



$$) = \sum_{\text{perms } p} A_{n_j s}(p)$$

$$l_1 \propto \langle 3 | \mu | 4 \rangle$$

$$|l_1\rangle = |5\rangle$$

$$Q_4^2 = 0, (Q_4 + K_3)^2 = 0, (Q_4 - K_4)^2 = 0$$

$$Q_4^2 = (Q_4 - K_{45})^2 = 0 \Rightarrow \frac{\langle 45 \rangle}{2 \langle 35 \rangle} \langle 3 | \mu | 4 \rangle$$

Tan)

CAUTION

$$) = \sum_i A_{ij}(p)$$

perms p

Tan)

$$k_4 \propto (3|\mu|4]$$

$$|k_1\rangle = |5\rangle$$

$$|k_3\rangle = |3\rangle$$

$$|k_4\rangle = \frac{\langle 45 \rangle}{\langle 35 \rangle} |3\rangle$$

$$|k_5\rangle = |5\rangle$$

$$k_4^2 = 0, (k_4 + k_3)^2 = 0, (k_4 - k_4)^2 = 0$$

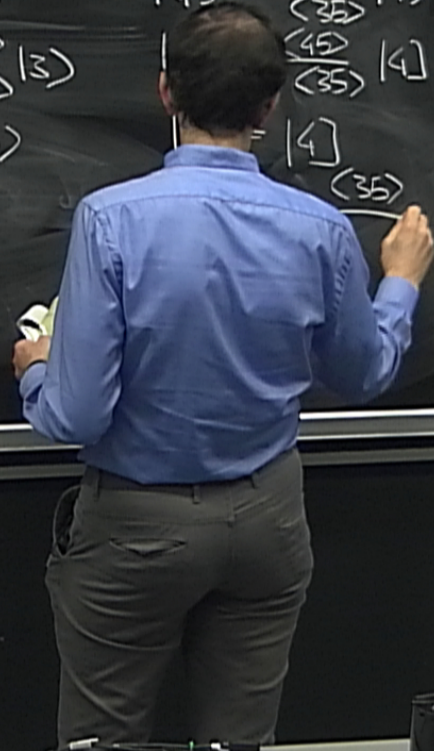
$$k_1^2 = (k_4 - k_{45})^2 = 0 \Rightarrow \frac{\langle 45 \rangle}{2\langle 35 \rangle} (3|\mu|4]$$

$$|k_1\rangle = -\frac{\langle 34 \rangle}{\langle 35 \rangle} |4\rangle - |5\rangle$$

$$= \frac{\langle 45 \rangle}{\langle 35 \rangle} |4\rangle + |3\rangle$$

$$= |4\rangle$$

(35)



CAUTION

CAUTION

$$= \sum_i A_{ij} s(p)$$

Tan)

$$l_1 \propto \langle 3 | \mu | 4 \rangle$$

$$|l_1\rangle = |5\rangle$$

$$|l_3\rangle = |3\rangle$$

$$|l_4\rangle = \frac{\langle 45 \rangle}{\langle 35 \rangle} |3\rangle$$

$$|l_5\rangle = |5\rangle$$

$$l_1^2 = 0, (l_1 + l_3)^2 = 0, (l_4 - l_5)^2 = 0$$

$$l_1^2 = (l_1 - l_{45})^2 = 0 \Rightarrow \frac{\langle 45 \rangle}{2 \langle 35 \rangle} \langle 3 | \mu | 4 \rangle$$

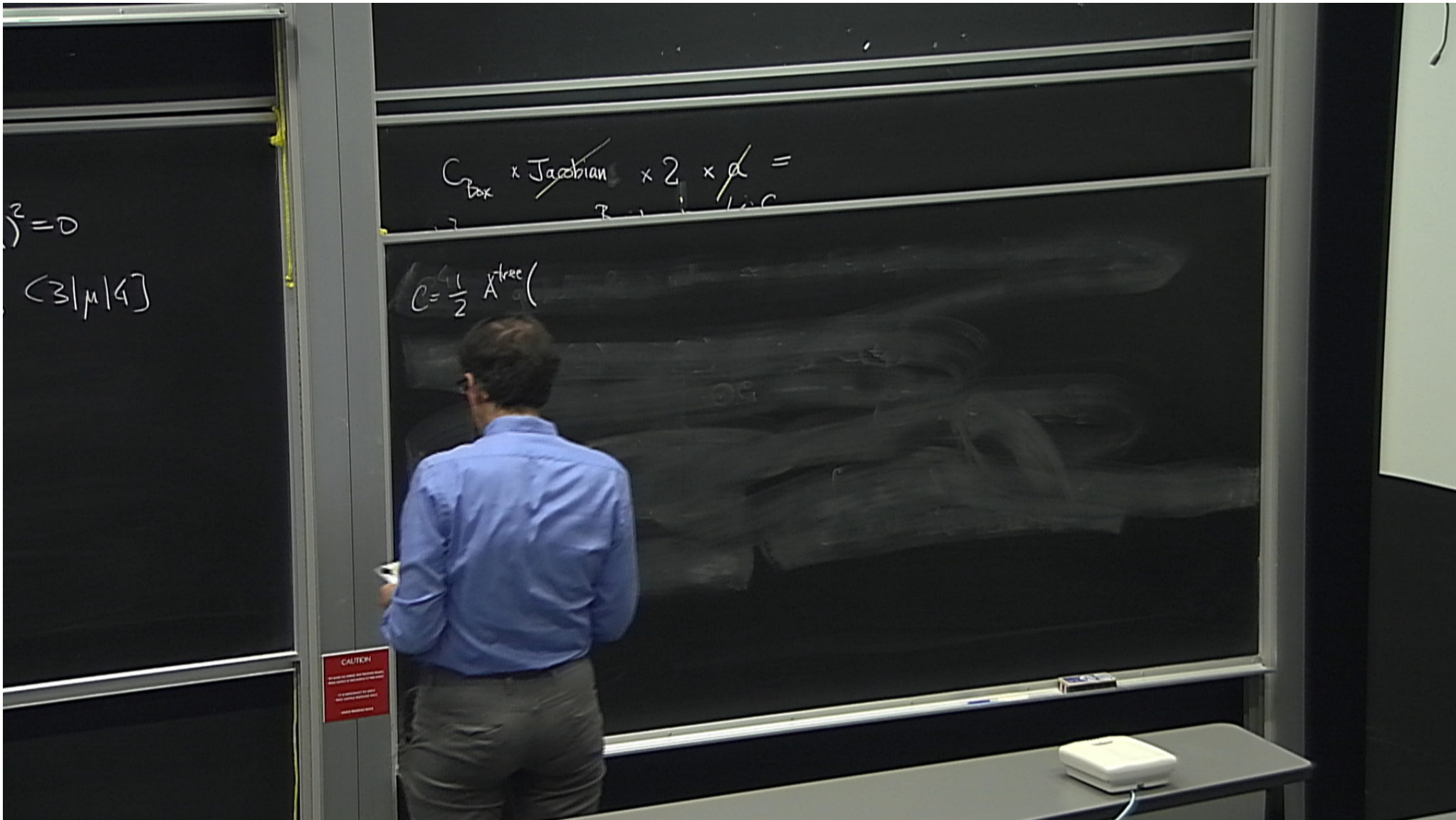
$$|l_1\rangle = -\frac{\langle 34 \rangle}{\langle 35 \rangle} |4\rangle - |5\rangle$$

$$|l_3\rangle = \frac{\langle 45 \rangle}{\langle 35 \rangle} |4\rangle + |3\rangle$$

$$|l_4\rangle = |4\rangle$$

$$|l_5\rangle = -\frac{\langle 34 \rangle}{\langle 35 \rangle} |4\rangle$$

CAUTION



$$)^2 = 0$$

$$\langle 3 | \mu | 4 \rangle$$

$$C_{\text{Box}} \times \text{Jacobian} \times 2 \times d =$$

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l)_j \rangle \rightarrow 2 \langle l_j \rangle$$

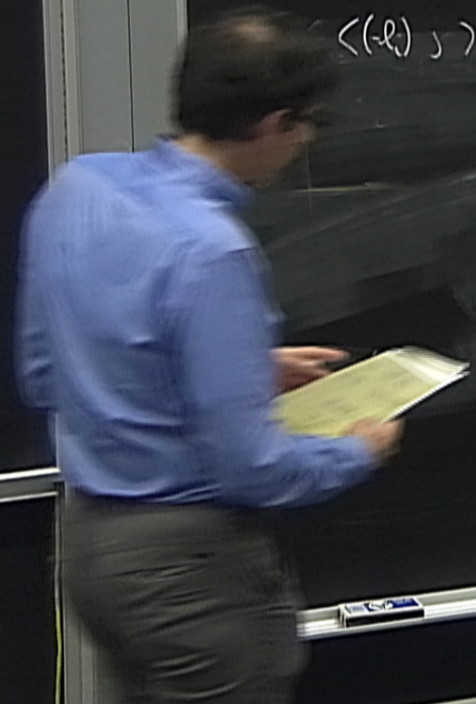
CAUTION
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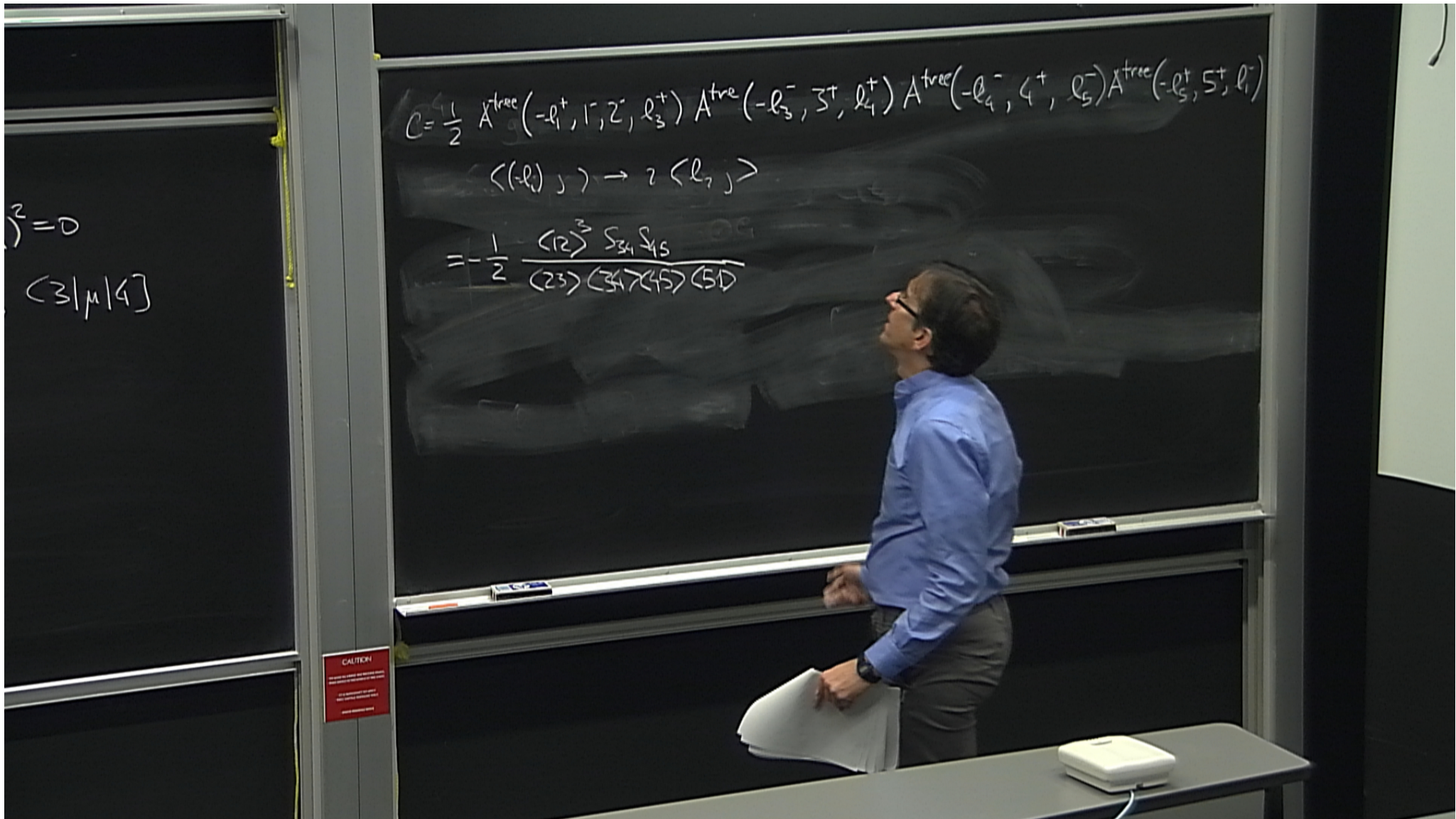
$$\begin{aligned} & \} = 0 \\ & \langle 3 | \mu | 4 \rangle \end{aligned}$$

$$C_{\text{Box}} \times \text{Jacobian} \times 2 \times d =$$

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l)_j \rangle \rightarrow 2 \langle l_j \rangle$$





$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l)_j \rangle \rightarrow 2 \langle l_{j,j} \rangle$$

$$= -\frac{1}{2} \frac{\langle 12 \rangle^3 s_{34} s_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

$$\rangle^2 = 0$$

$$\langle 3|p|4 \rangle$$

CAUTION
 WARNING: DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT.
 TO PREVENT ACCIDENTS, PLEASE WAIT UNTIL THE BOARD IS COOL.
 THANK YOU FOR YOUR PATIENCE.

$\}^2 = 0$
 $\langle 3 | \mu | 4 \rangle$

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l)_j \rangle \rightarrow 2 \langle l_j \rangle$$

$$= -\frac{1}{2} \frac{\langle 12 \rangle^3 S_{34} S_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} = \frac{2 S_{34} S_{45}}{2} A^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+)$$

CAUTION
 WARNING: DO NOT TOUCH THE BOARD
 SURFACE OR THE BOARD
 SURFACE OR THE BOARD SURFACE

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l)_j \rangle \rightarrow 2 \langle l_j \rangle$$

$$= -\frac{1}{2} \frac{\langle 12 \rangle^3 S_{34} S_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} = \frac{2 S_{34} S_{45}}{2} A^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+)$$

$$\lambda_3 \propto \lambda_4$$

$$= 0 \Rightarrow \lambda_4 \propto \lambda_5$$

$$= 0 \Rightarrow \lambda_4 \propto \lambda_3$$

CAUTION
 WARNING: DO NOT TOUCH THE BOARD
 SURFACE OR THE BOARD FRAME
 TO AVOID INJURY OR DAMAGE TO THE BOARD
 SURFACE.

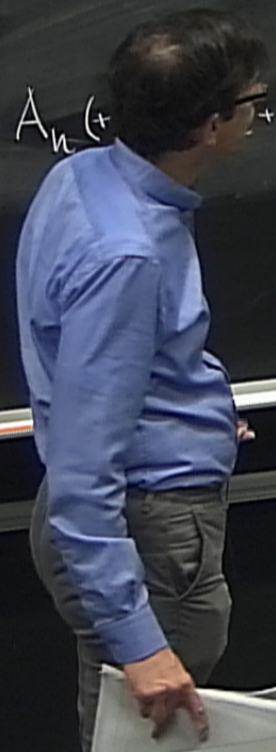
$\}^2 = 0$
 $\langle 3 | \mu | 4 \rangle$

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^+, l_1^-)$$

$$\langle (-l_i)_j \rangle \rightarrow i \langle l_j \rangle$$

$$= -\frac{1}{2} \frac{\langle 12 \rangle^3 S_{34} S_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} = \frac{1}{2} S_{34} S_{45} A^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+)$$

$$A_n(+, \dots, +) = \left[\sum_{\text{Zwe}} \text{Denominator}(\text{Box}) \text{Box} \right] A_n^{\text{tree}}(+, m_1^-, m_2^-, +)$$



CAUTION
 ATTENTION
 ATTENTION

$$)^2 = 0$$

$$\langle 3 | \mu | 4 \rangle$$

$$C = \frac{1}{2} A^{\text{tree}}(-l_1^+, 1^-, 2^-, l_3^+) A^{\text{tree}}(-l_3^-, 3^+, l_4^+) A^{\text{tree}}(-l_4^-, 4^+, l_5^-) A^{\text{tree}}(-l_5^+, 5^-, l_1^-)$$

$$\langle (-l_1)_j \rangle \rightarrow 2 \langle l_1, j \rangle$$

$$= -\frac{1}{2} \frac{\langle 12 \rangle^3 S_{34} S_{45}}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} = \frac{2 S_{34} S_{45}}{2} A^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+)$$

$$A_n(+m_1^-, m_2^+, \dots) = \left[\sum_{\text{Zwe}} \text{Denominator}(\text{Box}) \text{Box} \right] A_n^{\text{tree}}(+m_1^-, m_2^+, \dots)$$

CAUTION
 WARNING: DO NOT TOUCH THE BOARD
 WHEN IT IS BEING USED BY THE
 PROFESSOR OR TA.
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 PROFESSOR OR TA.