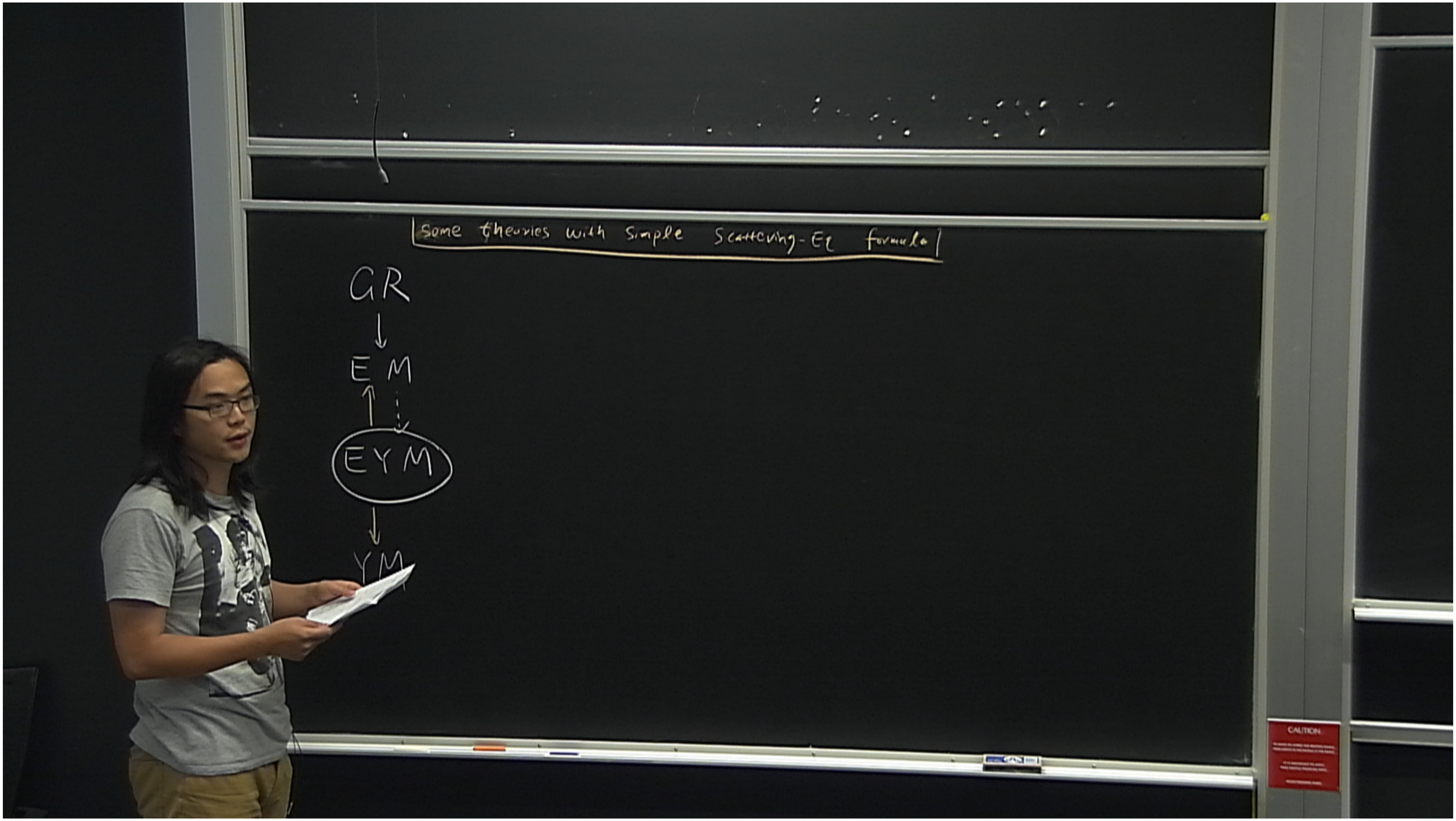
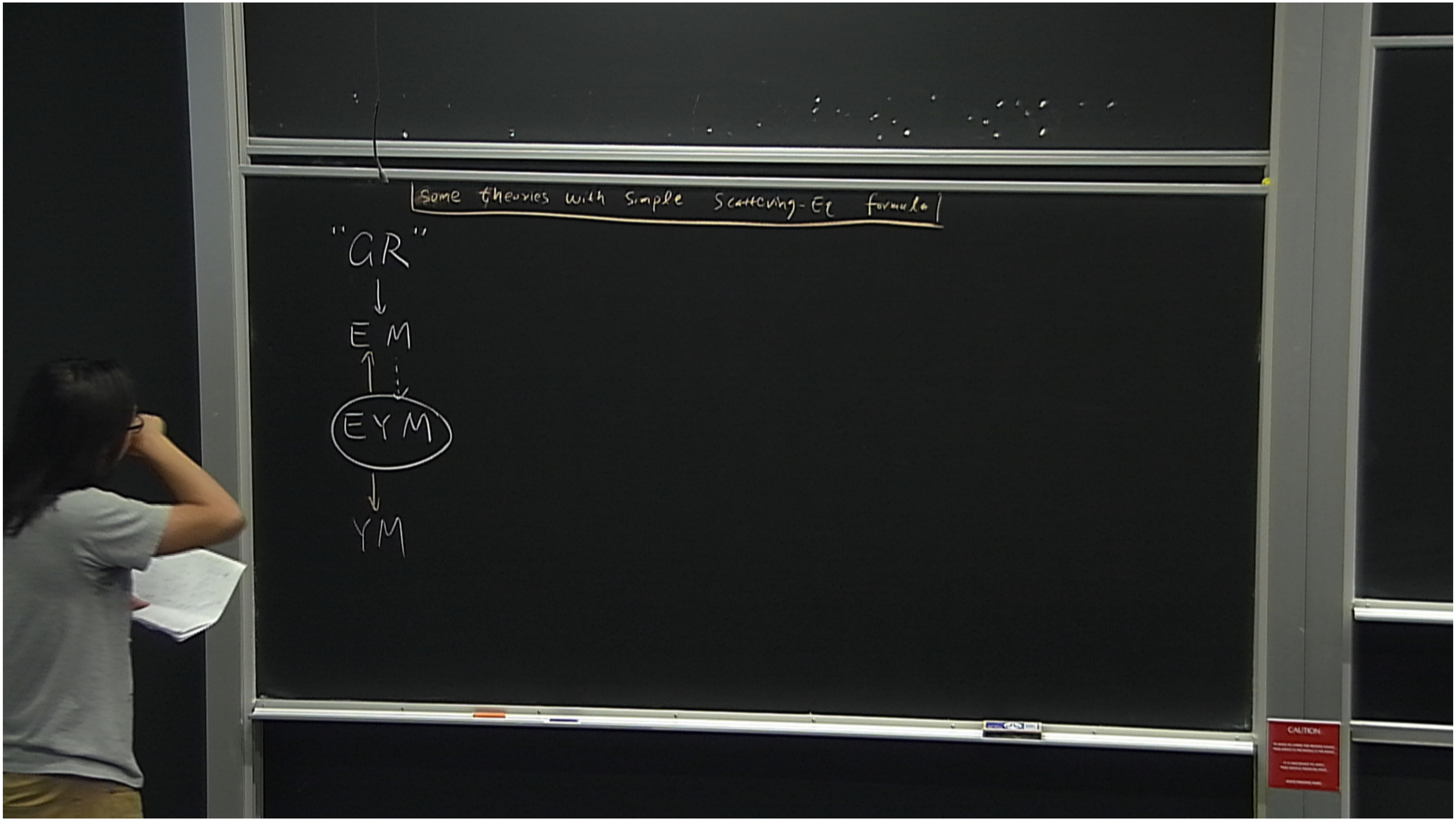
Title: Amplitudes: Applications

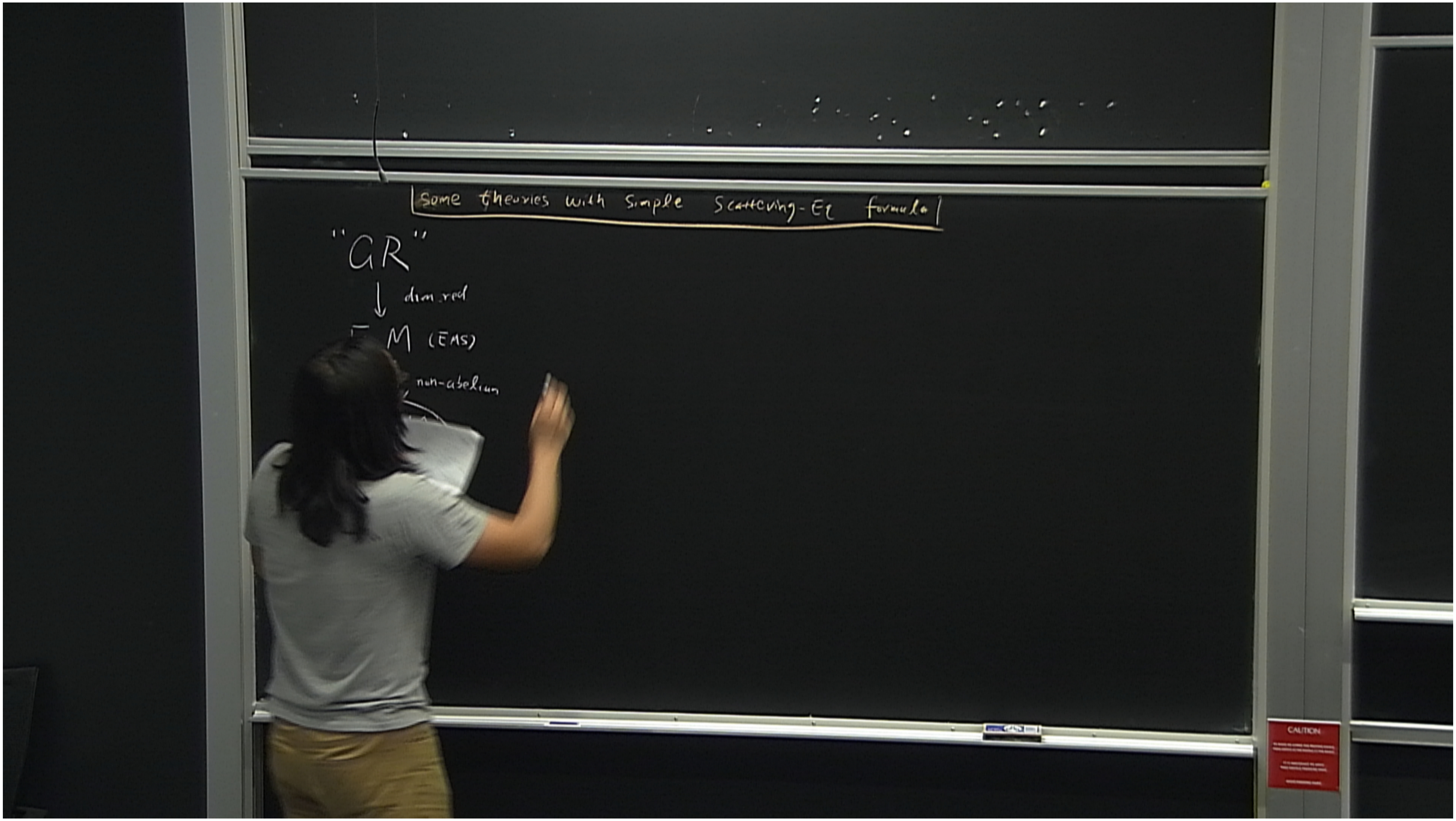
Date: Jul 17, 2015 09:00 AM

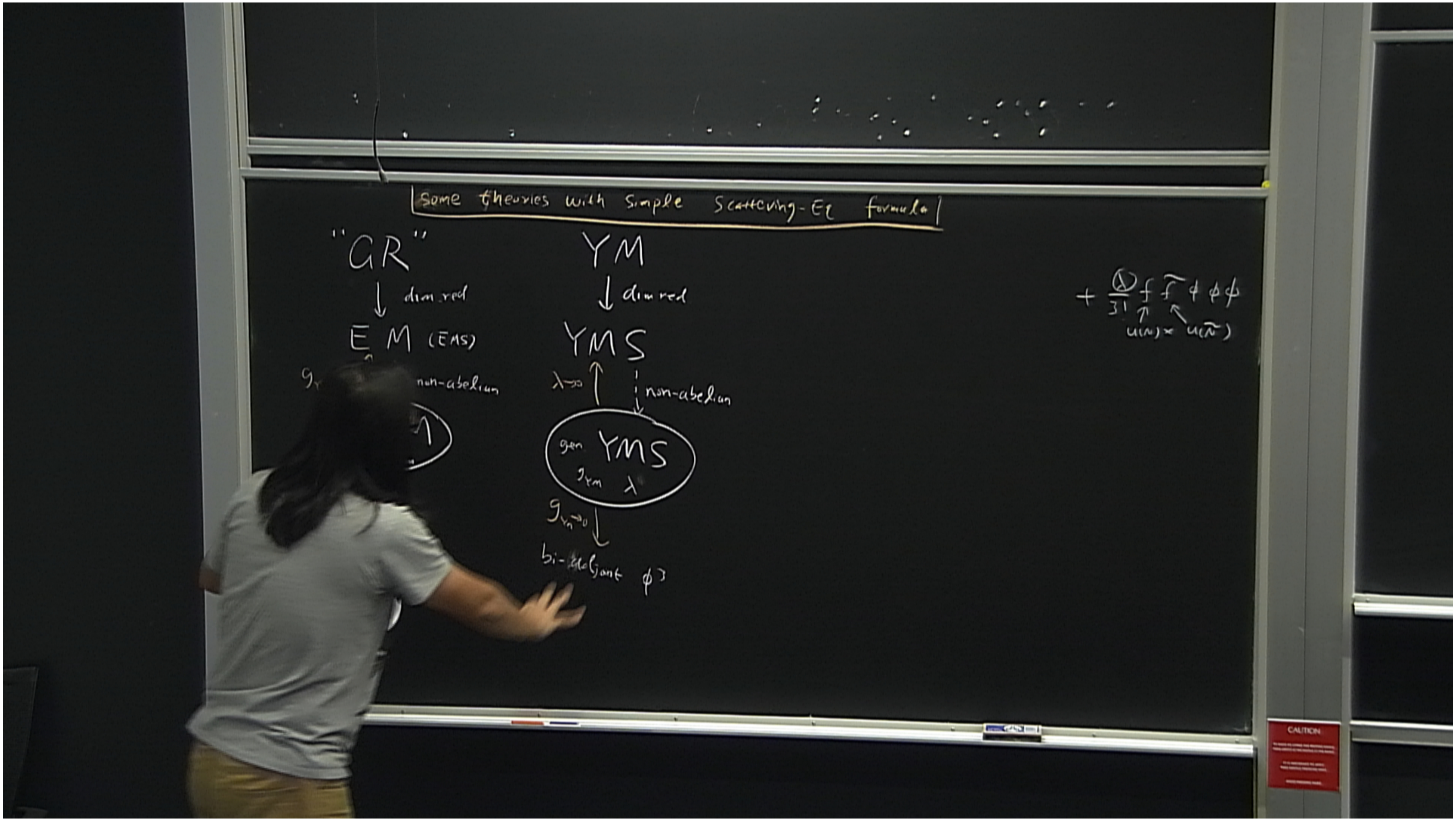
URL: http://pirsa.org/15070056

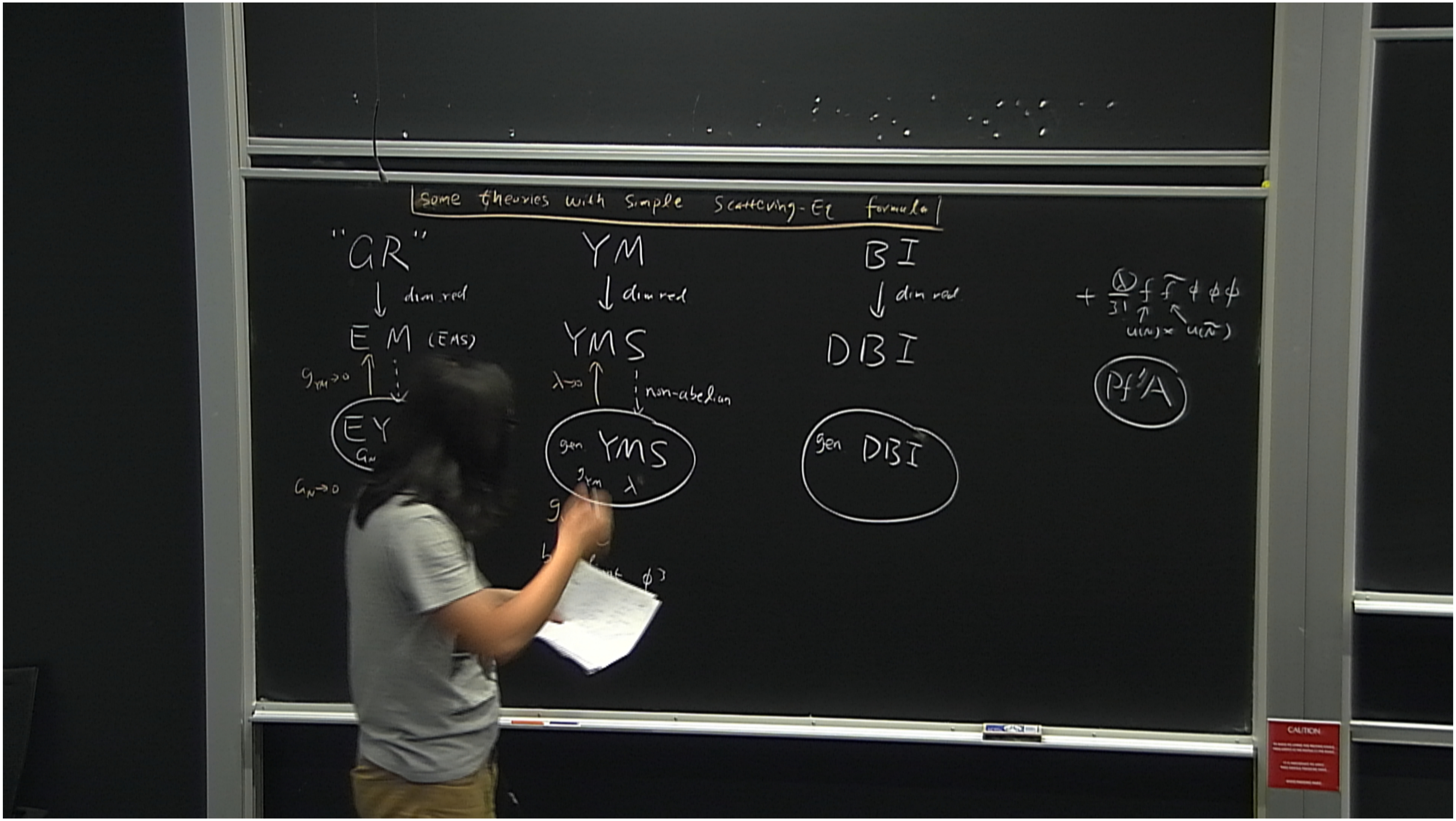
Abstract:


Some theories with simple Scattering-Eq formula
GR
EM
EYM
YM

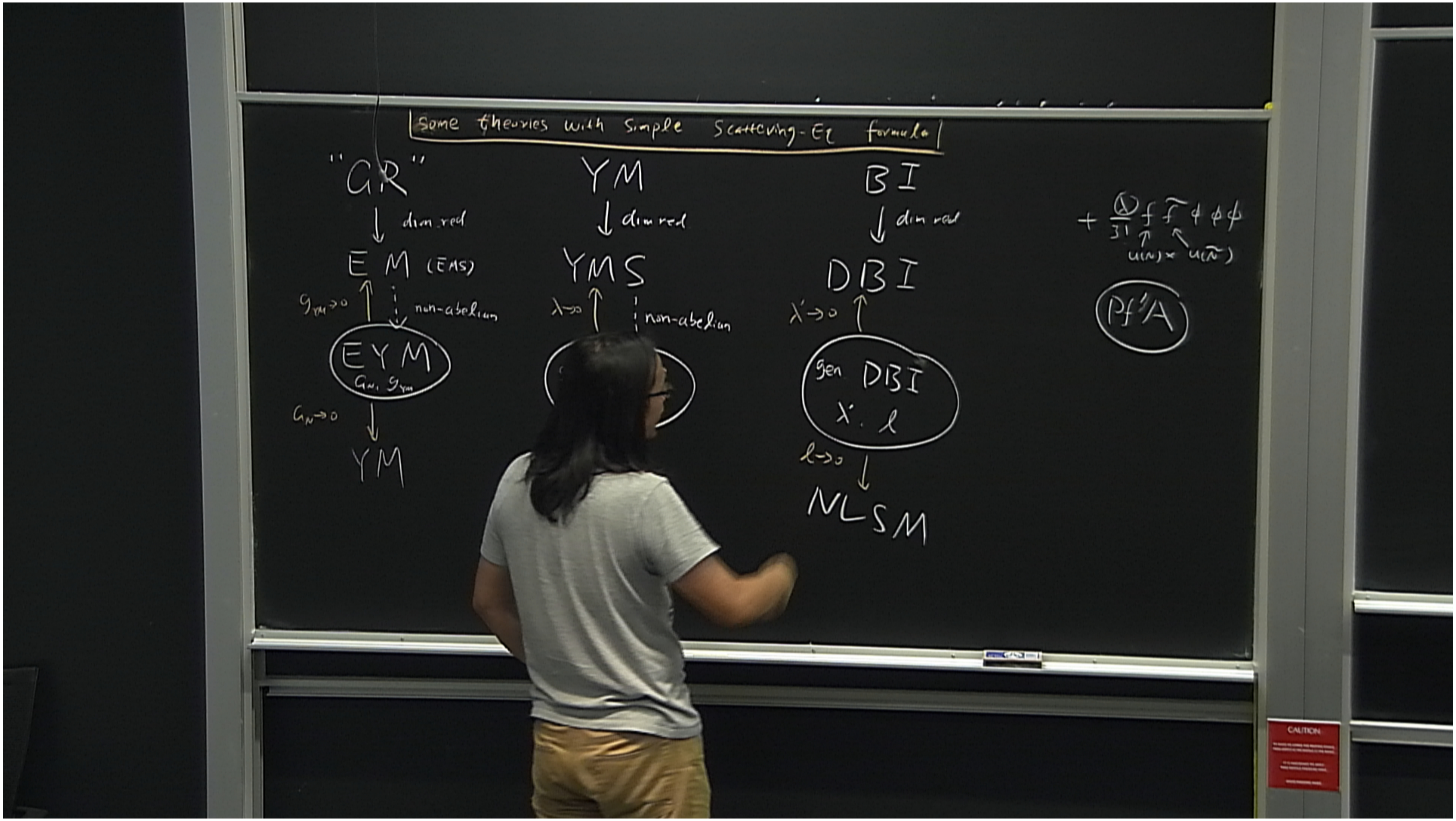

Some theories with simple Scattering-Eq formula
"GR"
EM
EYM
YM


Some theories with simple Scattering-Eq formula
"GR"
dim.red
M (EMS)
non-abelian

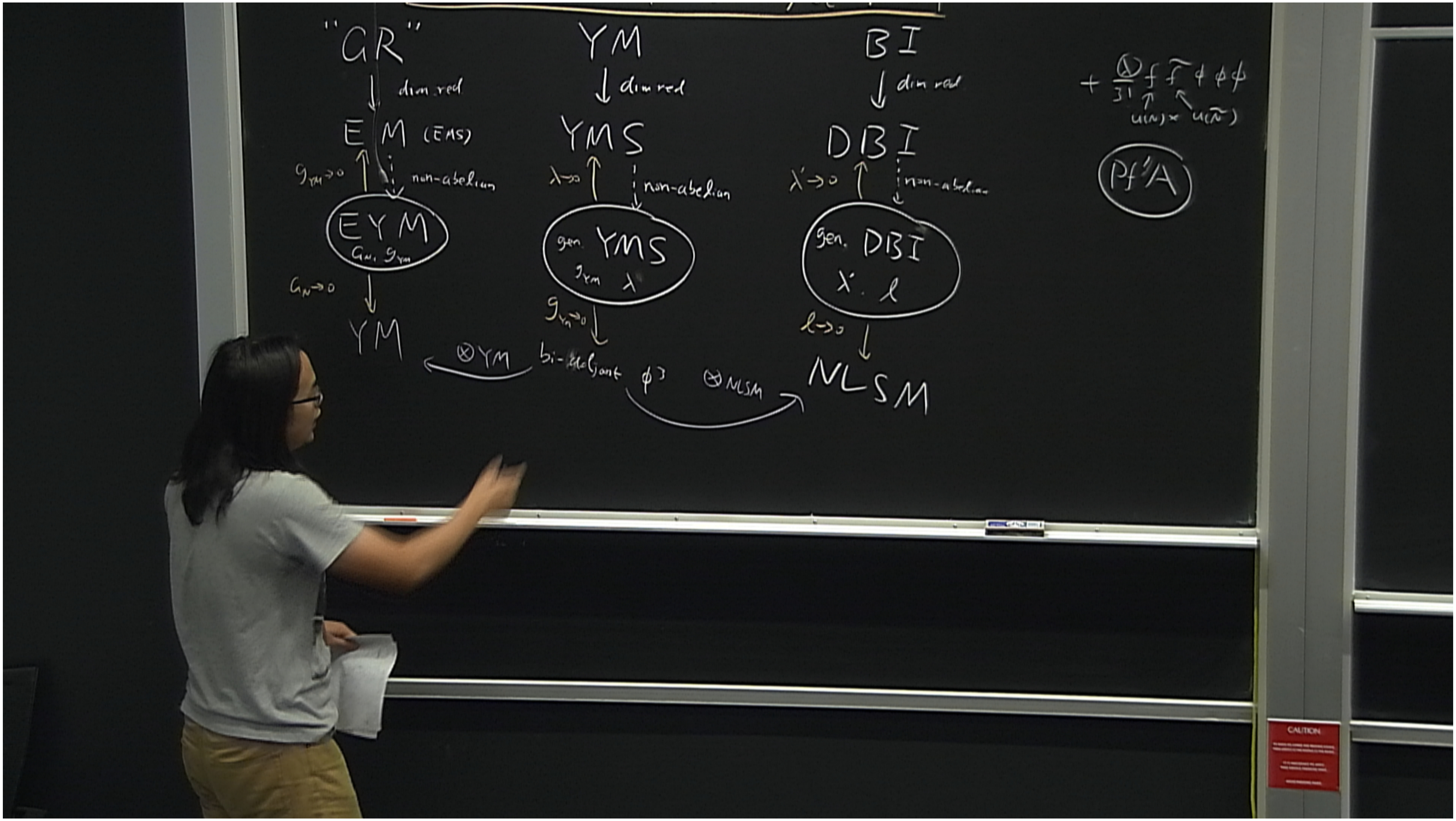

Some theories with simple Scattering-Eq formula
"GR"
dim red
EM (EMS)
non-abelian
YM
dim red
YMS
non-abelian
gen YMS
bi-adjoint φ³

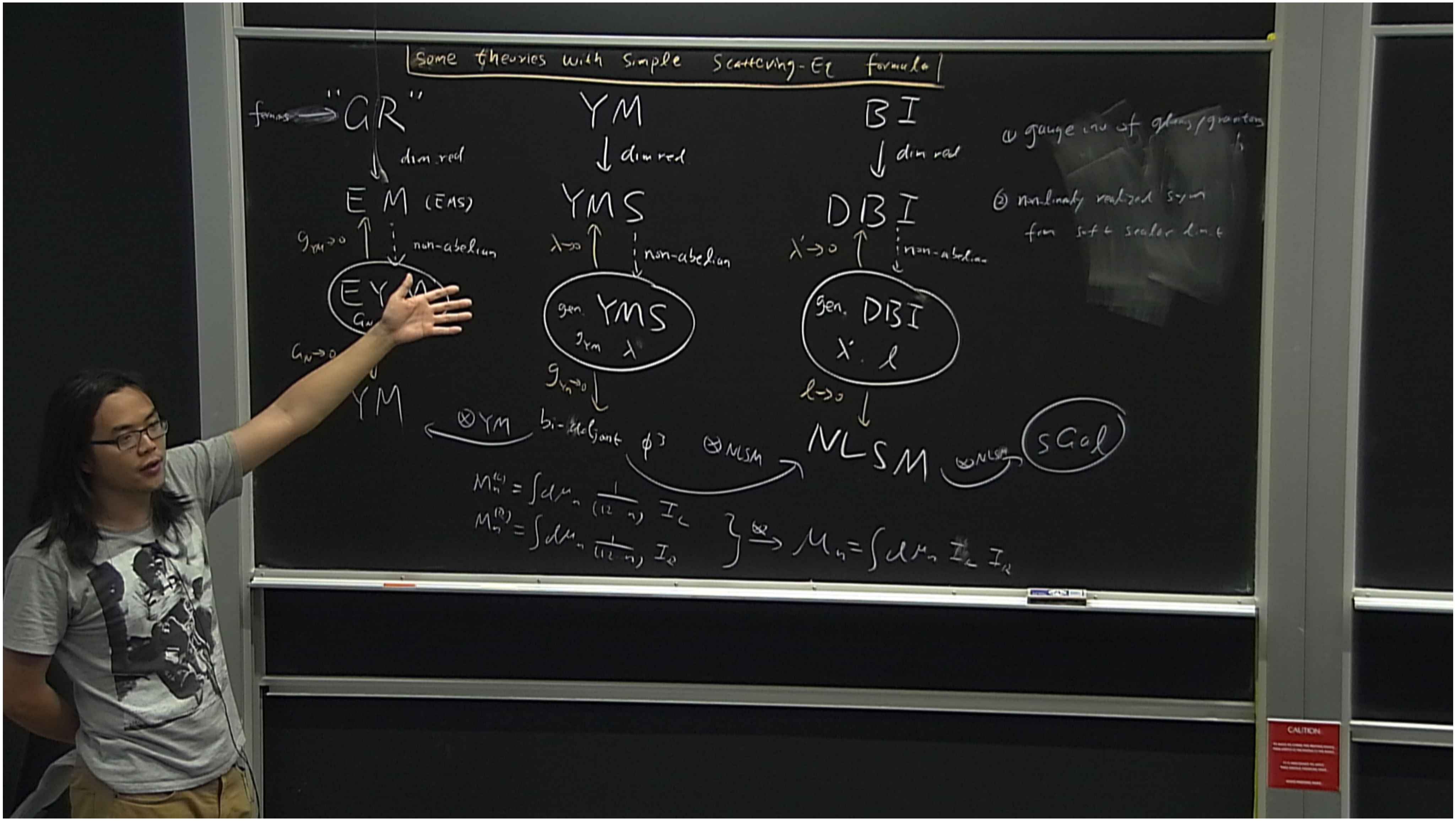


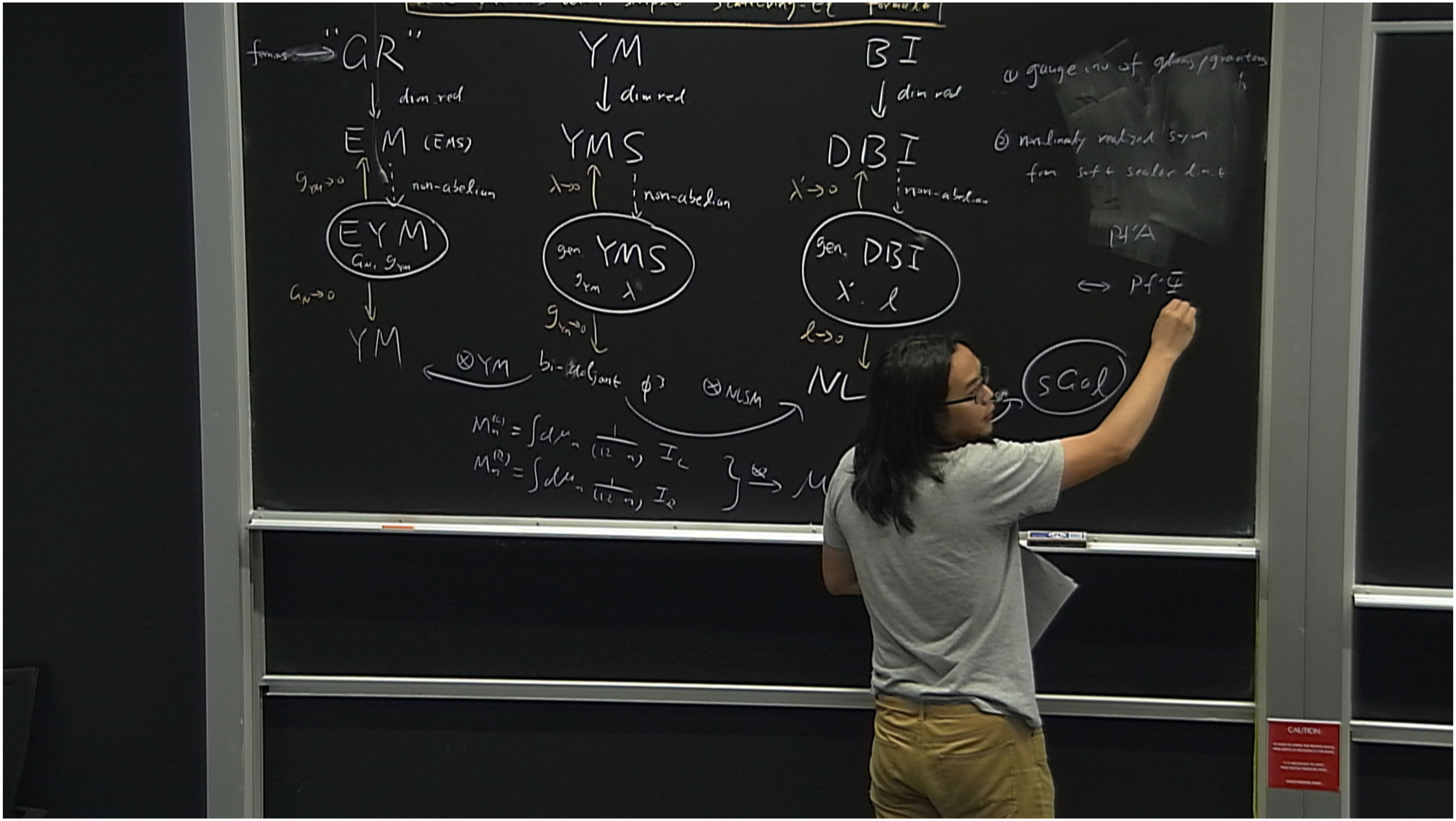
Some theories with simple Scattering-Eq formula
"GR"
dim red
EM (EMS)
$g_{YM}\to 0$
EY
$G_N \to 0$
YM
dim red
YMS
$\lambda \to 0$
non-abelian
gen YMS
g_{YM} λ
BI
dim red
DBI
gen DBI
Pf'A

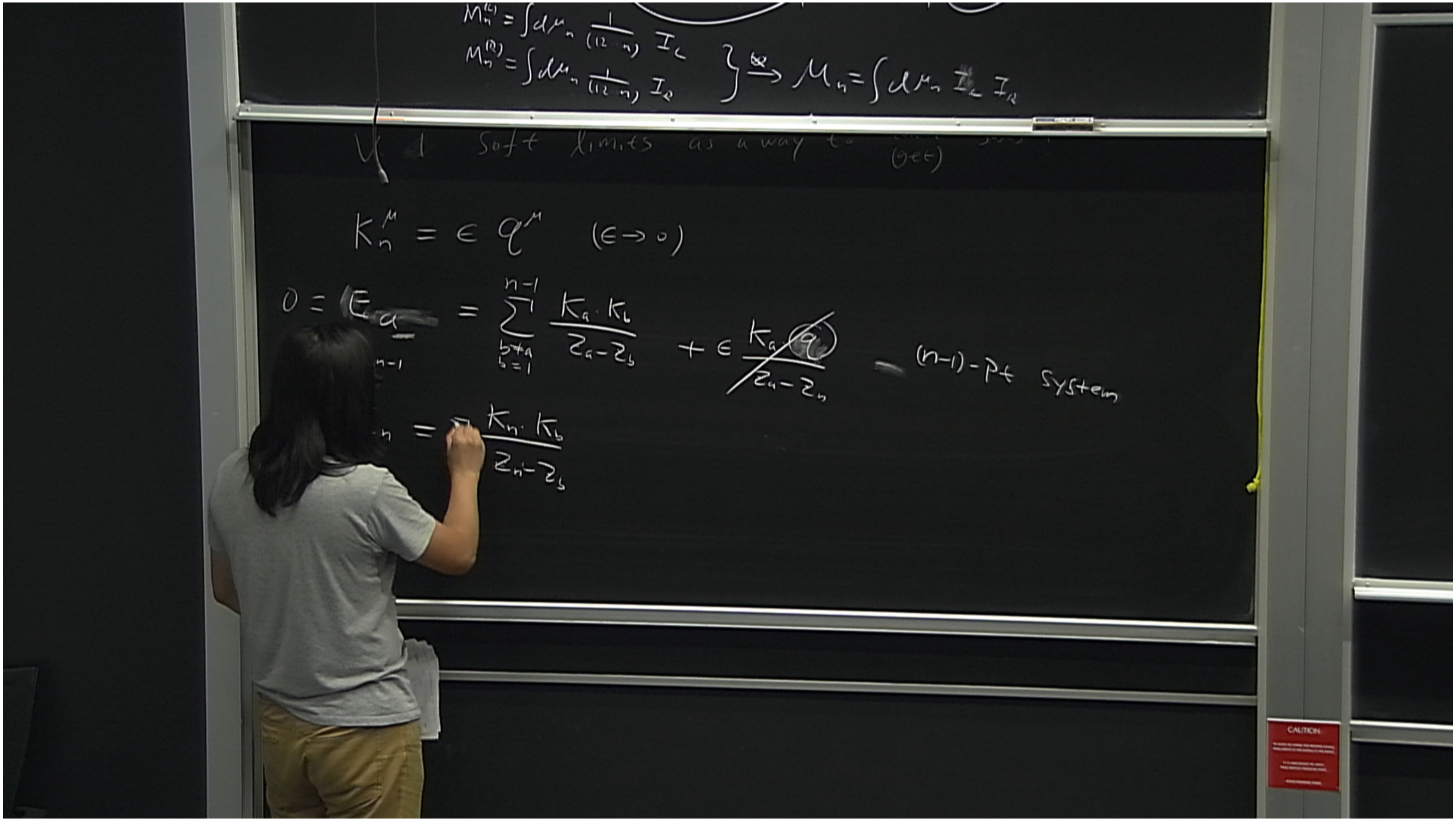


Some theories with Simple Scattering-Eq formula
"GR"
dim red
EM (EMS)
non-abelian
EYM
YM
YM
dim red
YMS
non-abelian
BI
dim red
DBI
gen DBI
NLSM
Pf'A


"GR"
YM
BI
dim red
EM (EMS)
YMS
DBI
non-abelian
EYM
gen. YMS
gen. DBI
YM
⊗YM
bi-adjoint ϕ^3
⊗NLSM
NLSM
Pf'A


Some theories with simple Scattering-Eq formula
"GR"
YM
BI
dim red
EM (EMS)
YMS
DBI
non-abelian
gen. YMS
gen. DBI
YM
bi-adjoint ϕ^3
⊗YM
⊗NLSM
NLSM
sGal
① gauge inv of gluons/gravitons
② non-linearly realized sym from soft scalar limit
$M_n^{(L)} = \int d\mu_n \frac{1}{(12 \cdots n)} I_L$
$M_n^{(R)} = \int d\mu_n \frac{1}{(12 \cdots n)} I_R$
$\xrightarrow{\otimes} M_n = \int d\mu_n I_L I_R$


"GR"
dim red
EM (EMS)
non-abelian
EYM
YM
YM
dim red
YMS
non-abelian
gen. YMS
bi-adjoint ϕ^3
⊗YM
⊗NLSM
BI
dim red
DBI
non-abelian
gen. DBI
NL
sGal
① gauge inv of gluons/gravitons
② non-linearly realized sym
from soft scalar limit
Pf'A
⟺ Pf'Ψ


Soft limits as a way to (get)
K_n^μ = ε q^μ (ε → 0)
(n−1)-Pt System

YM

$g_{YM} \to 0$ $\quad$ $\ell \to 0$

$$\text{YM} \xleftarrow{\otimes \text{YM}} \text{bi-adjoint } \phi^3 \xrightarrow{\otimes \text{NLSM}} \text{NLSM} \xrightarrow{\otimes \text{NLSM}} \text{sGal}$$

$$\left.\begin{aligned} M_n^{(L)} &= \int d\mu_n \frac{1}{(12\cdots n)} I_L \\ M_n^{(R)} &= \int d\mu_n \frac{1}{(12\cdots n)} I_R \end{aligned}\right\} \xrightarrow{\otimes} M_n = \int d\mu_n I_L I_R$$

practical way of solving eqs $\quad z_1 \to \infty \quad \{z_2, z_3\} = \{0, 1\}$

$\boxed{S_{i,j}}$ $(1 \le i < j \le n-1)$ except $S_{n-2,n-1} = -(\text{sum of the rest})$ $\quad$ M $\quad \epsilon = \frac{1}{M}$

$$S_{i\,n} = -\sum_{j=1}^{n-1} S_{i,j}$$

$$\sum_{b \neq a}{}' \frac{\epsilon_a \epsilon_b S_{ab}}{z_a - z_b} = 0 \quad a = 4, 5, \ldots, n \quad \epsilon_a = \begin{cases} 1 & a = 1, 2, \ldots 7 \\ 0 & a = 8, \ldots, n \end{cases}$$

24 sol for 7-pt "NSolve" "FindRoot"

$$\boxed{M(12\cdots n) = \sum_{\{z_4 \ldots z_n\} \in \text{sol}}{}' \frac{1}{\det\left|\frac{\partial E_a}{\partial z_b}\right|^{123}_{123}} \frac{1}{z_{23} \cdots z_{n-1\,n}} \operatorname{Pf}\left|\Psi(\epsilon, k, z)\right|^{12}_{12}}$$

YM

$g_{YM} \to 0$ $\ell \to 0$

YM $\xleftarrow{\otimes \text{YM}}$ bi-adjoint ϕ^3 $\xrightarrow{\otimes \text{NLSM}}$ NLSM $\xrightarrow{\otimes \text{NLSM}}$ sGal

$$M_n^{(L)} = \int d\mu_n \frac{1}{(12\cdots n)} I_L$$
$$M_n^{(R)} = \int d\mu_n \frac{1}{(12\cdots n)} I_R \quad \Big\} \xrightarrow{\otimes} \mathcal{M}_n = \int d\mu_n I_L I_R$$

practical way of solving eqs $z_1 \to \infty$ $\{z_2, z_3\} = \{0, 1\}$

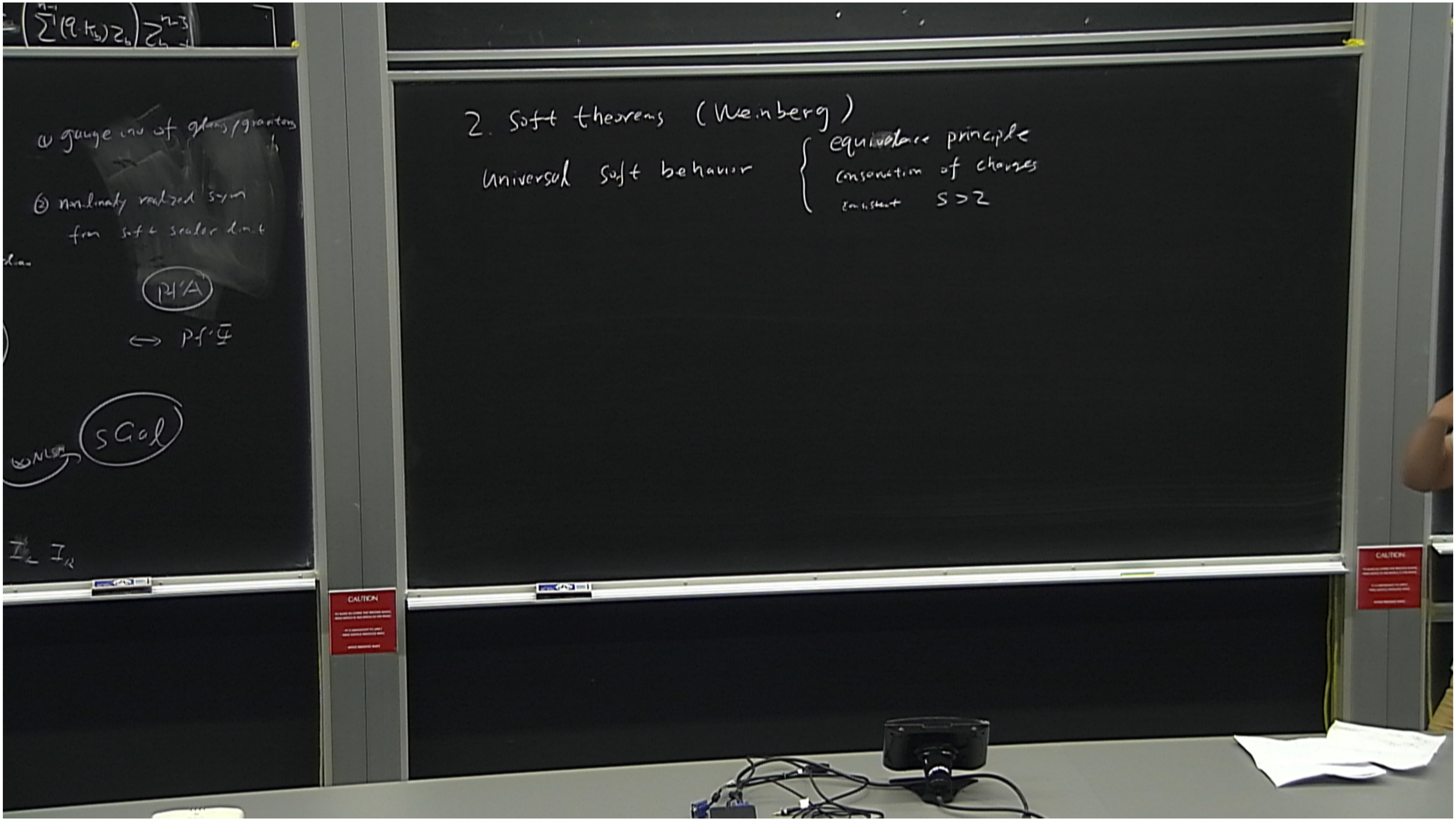
$\boxed{S_{i,j}}$ $(1 \le i < j \le n-1)$ except $S_{n-2,n-1} = -$(sum of the rest) M $\epsilon = \frac{1}{M}$

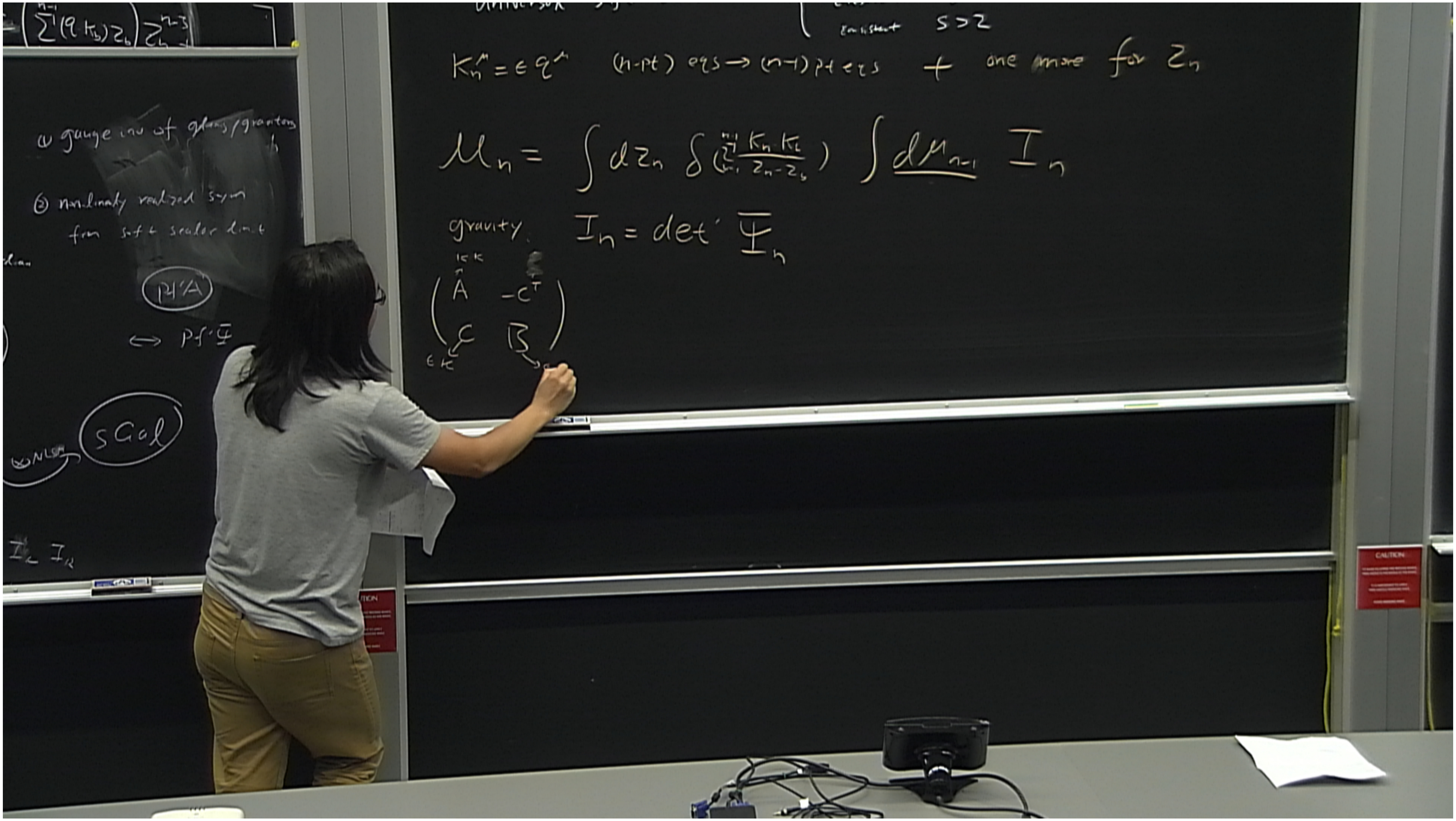
$$S_{in} = -\sum_{i=1}^{n-1} S_{ij}$$

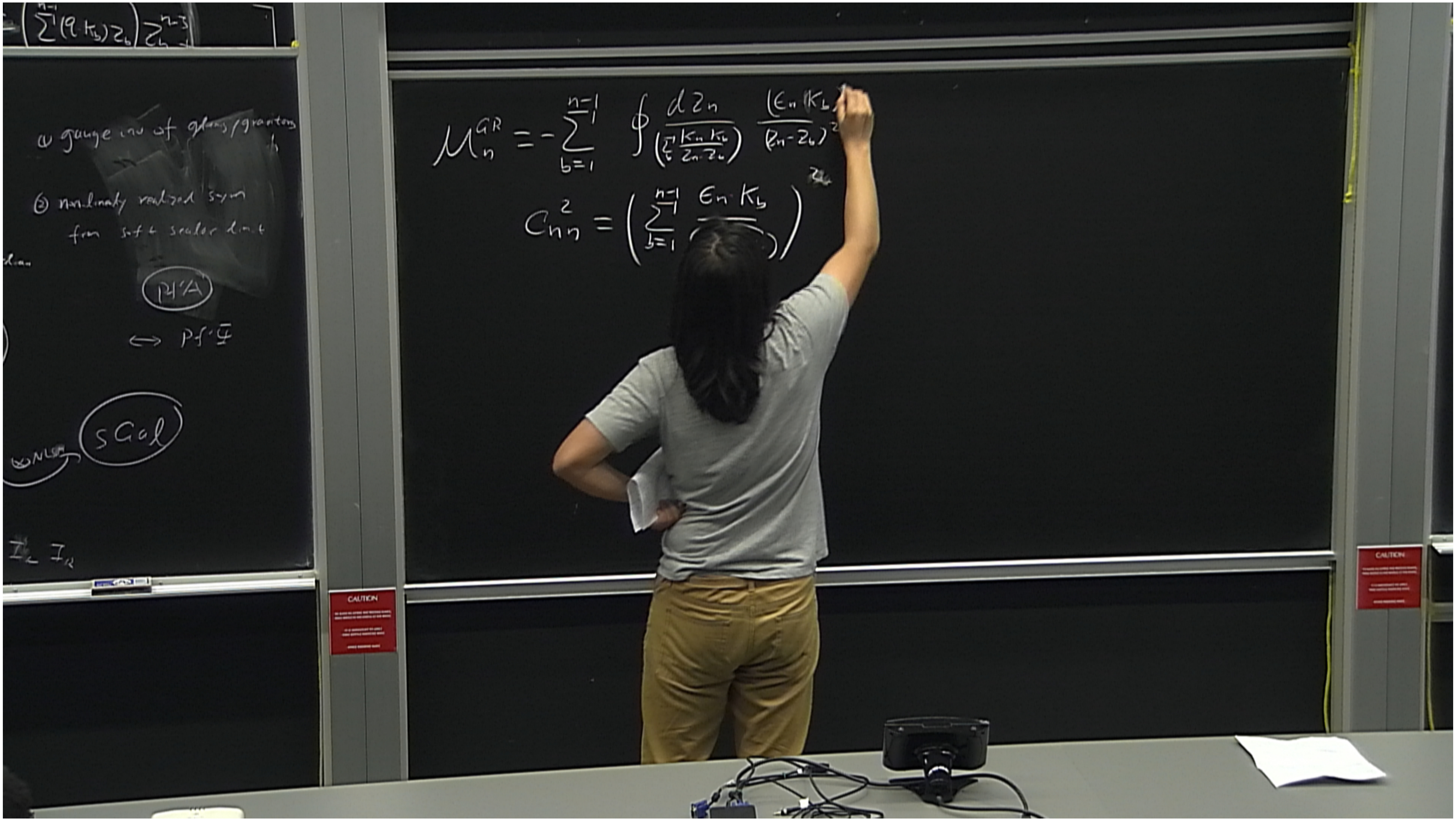
$$\sum_{b \neq a}' \frac{\epsilon_a \epsilon_b S_{ab}}{z_a - z_b} = 0 \quad a = 4, 5, \dots, n \quad \epsilon_a = \begin{cases} 1 & a = 1, 2, \dots, 7 \\ 0 & a = 8, \dots, n \end{cases}$$

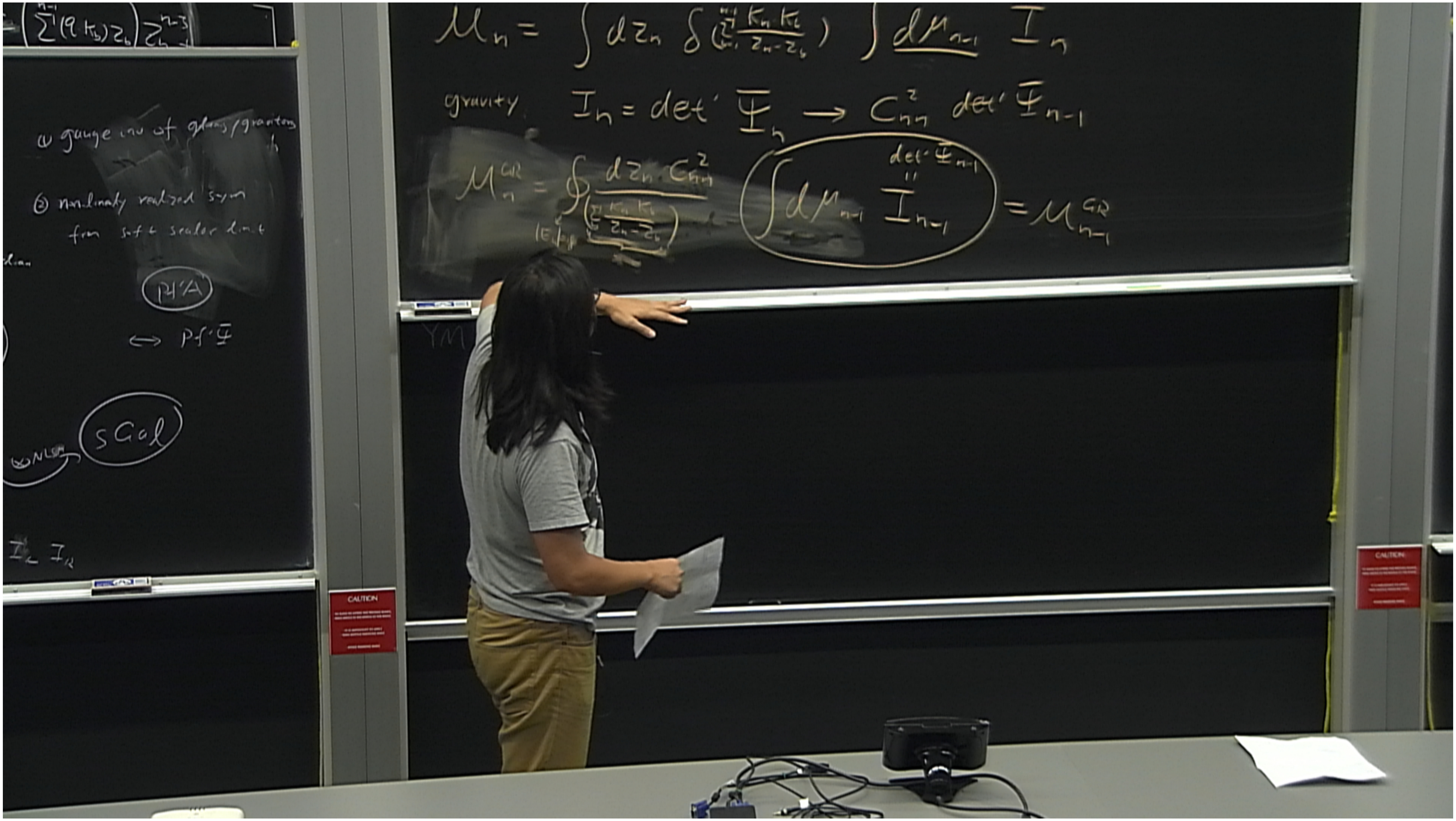
$$M(12\cdots n) = \sum_{\{z_4 \cdots z_n\} \in \text{sol}}' \frac{1}{\det' \left| \frac{\partial E_a}{\partial z_b} \right|^{123}_{123}} \frac{1}{z_{23} \cdots z_{n-1 n}} \text{Pf}' \left| \Psi(\epsilon, k, z) \right|^{12}_{12}$$

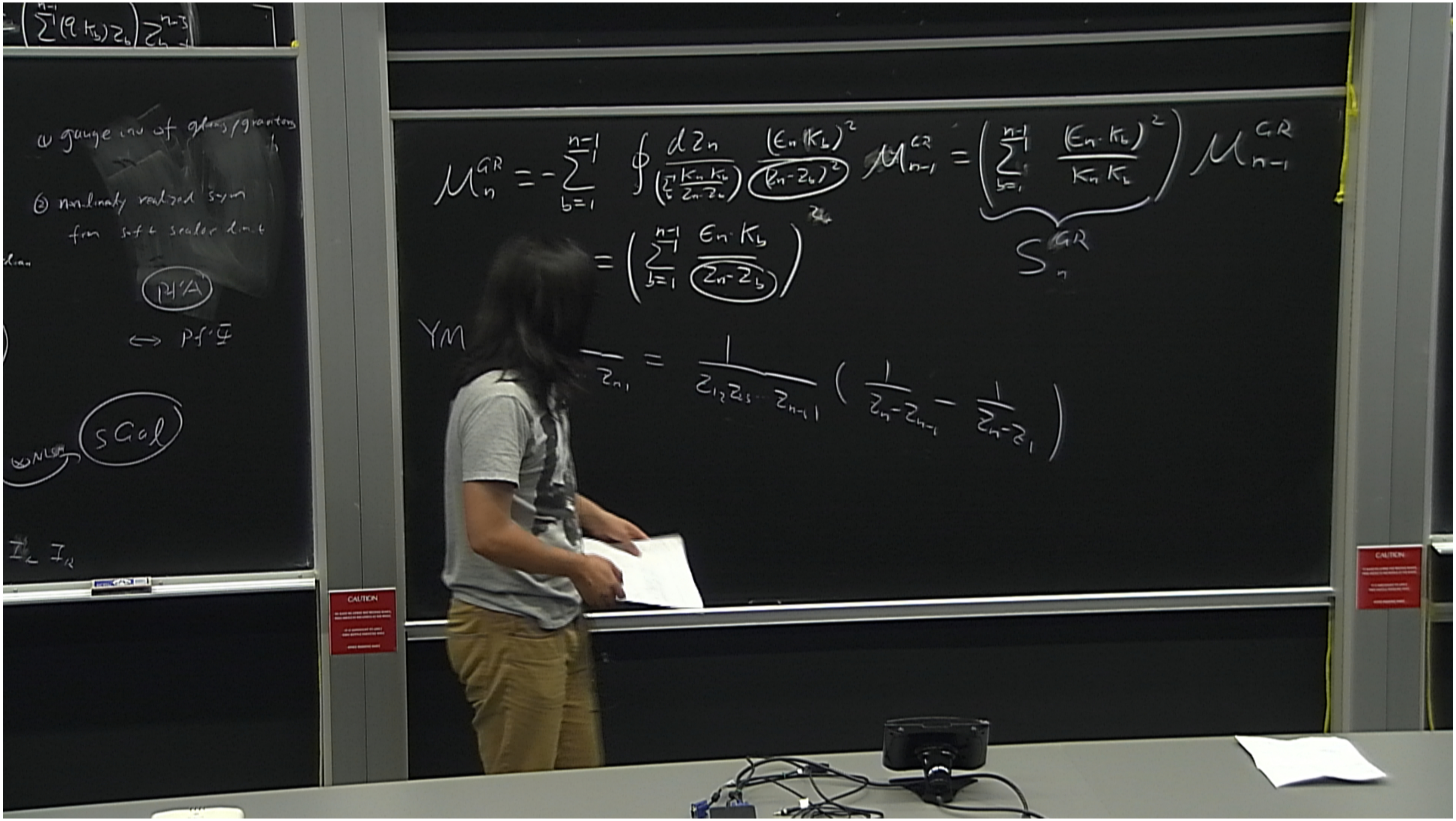
24 sol for 7-pt "NSolve" "FindRoot"

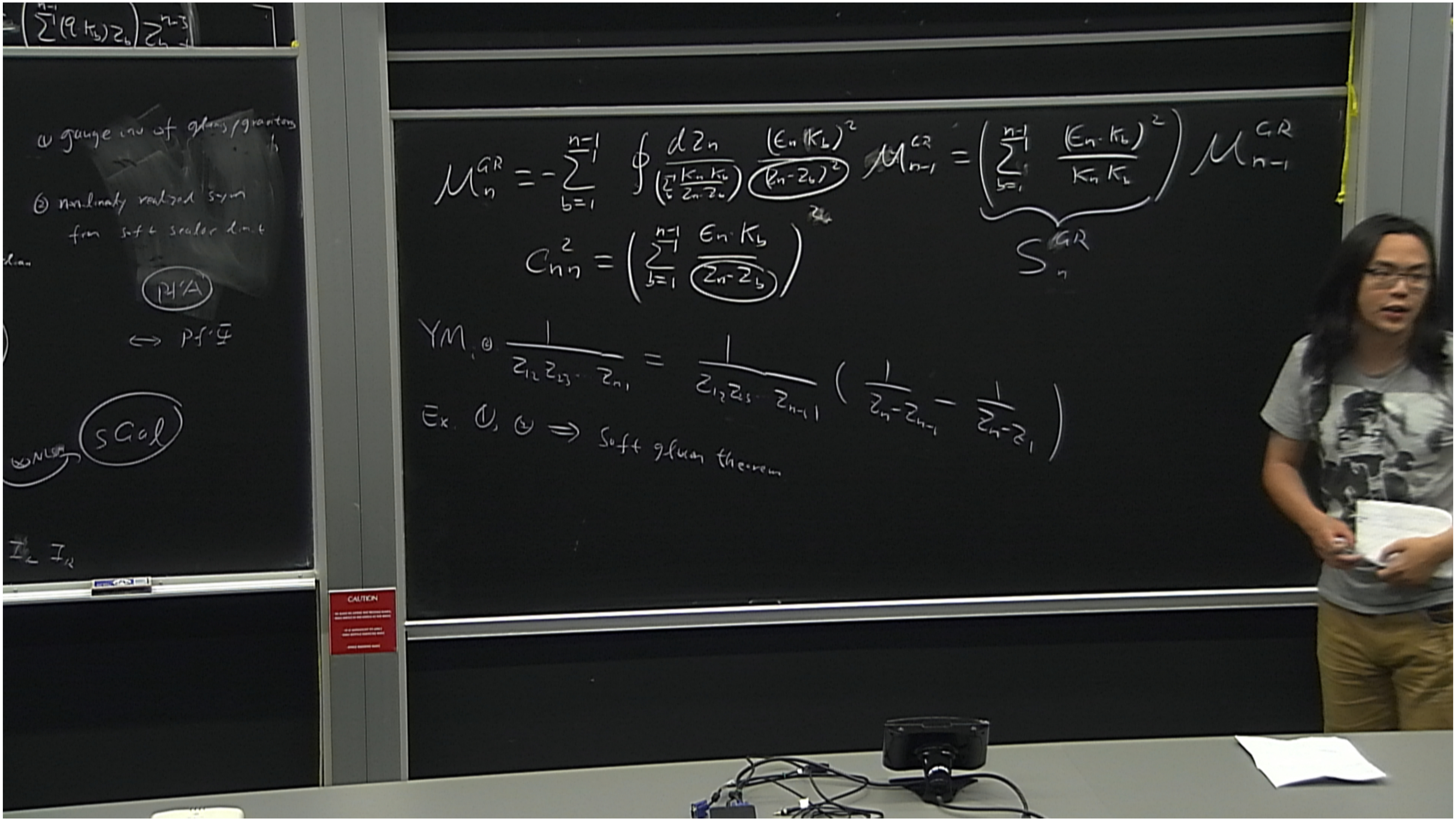

2. Soft theorems (Weinberg)
Universal soft behavior
equivalence principle
conservation of charges
consistent S > 2
① gauge inv of gluons/gravitons
② nonlinearly realized sym
from soft scalar limit
sGal

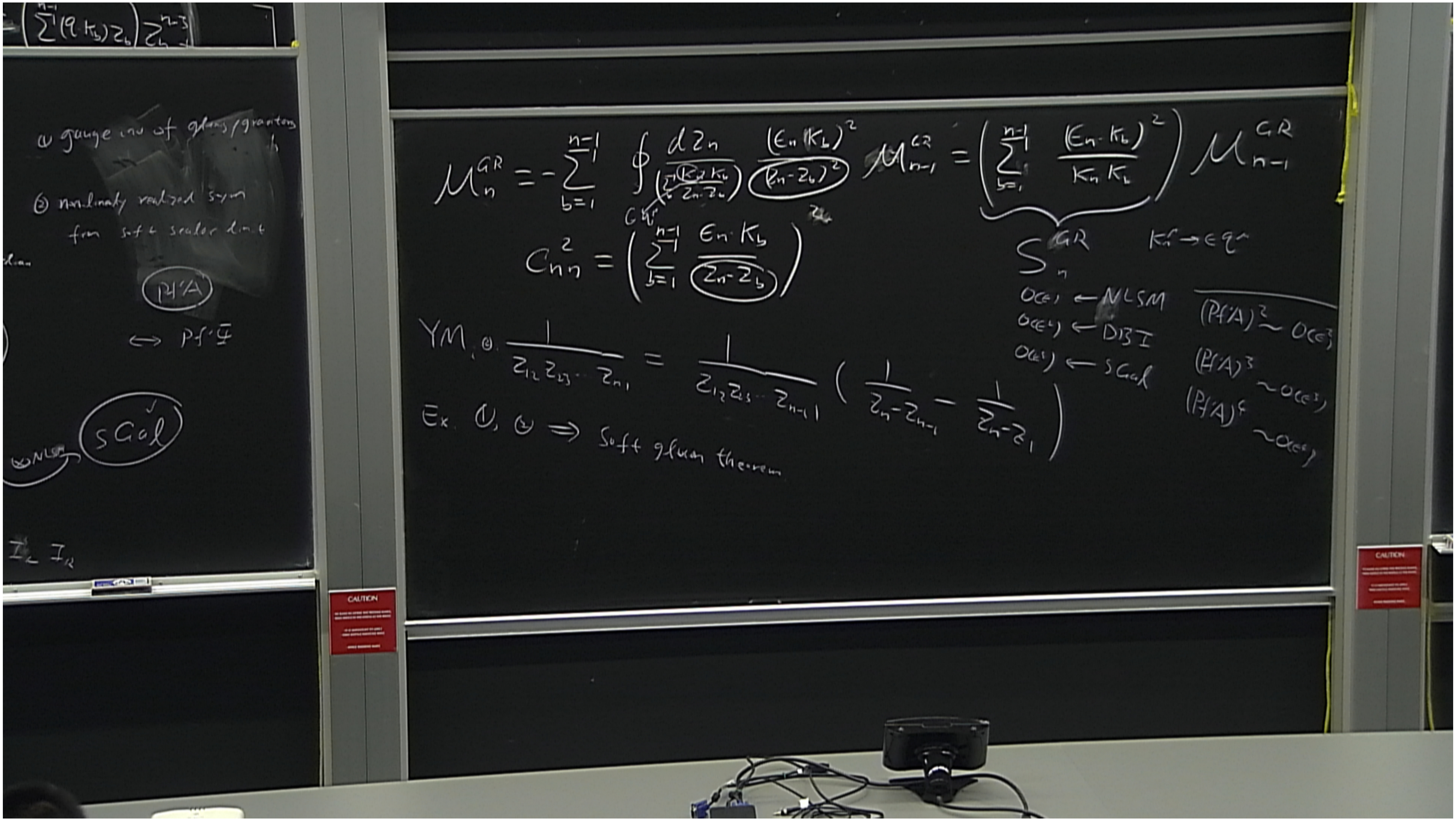

s > 2
Kn^μ = ε q^μ (n-pt) eqs → (n-1) pt eqs + one more for zn
Mn = ∫ dzn δ(Σ Kn Kb / zn - zb) ∫ dμn-1 In
gravity In = det' Ψn
① gauge inv of gluons/gravitons
② nonlinearly realized sym from soft scalar limit
sGal

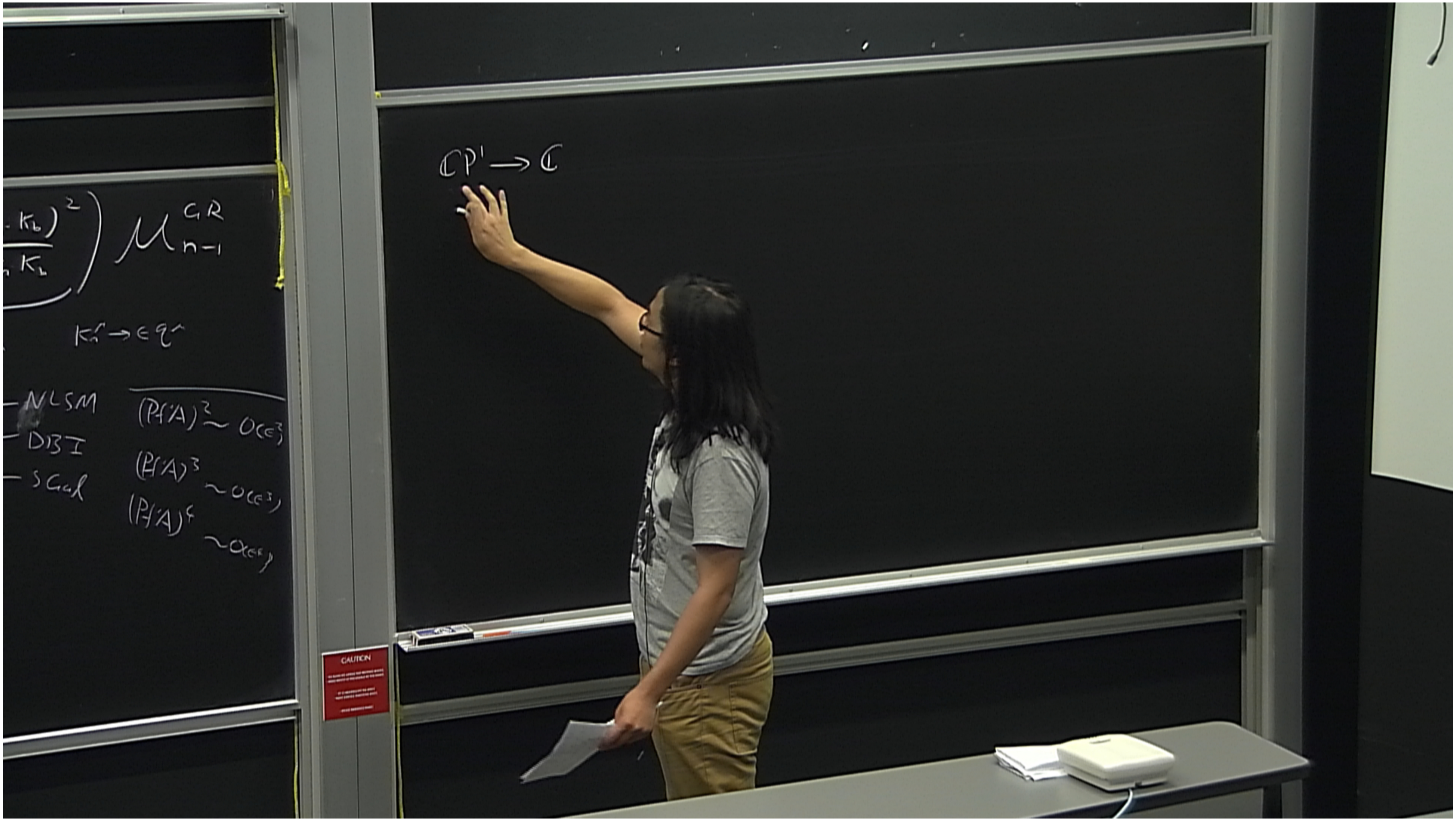

① gauge inv of gluons/gravitons
② nonlinearly realized sym
from soft scalar limit
Pf'A
⟷ Pf'Ψ
sGal
NLSM
$M_n^{GR} = -\sum_{b=1}^{n-1} \oint \frac{dz_n}{\left(\sum_b \frac{k_n \cdot k_b}{z_n - z_b}\right)} \frac{(\epsilon_n \cdot k_b)}{(z_n - z_b)^2}$
$C_{nn}^2 = \left(\sum_{b=1}^{n-1} \frac{\epsilon_n \cdot k_b}{\quad}\right)$

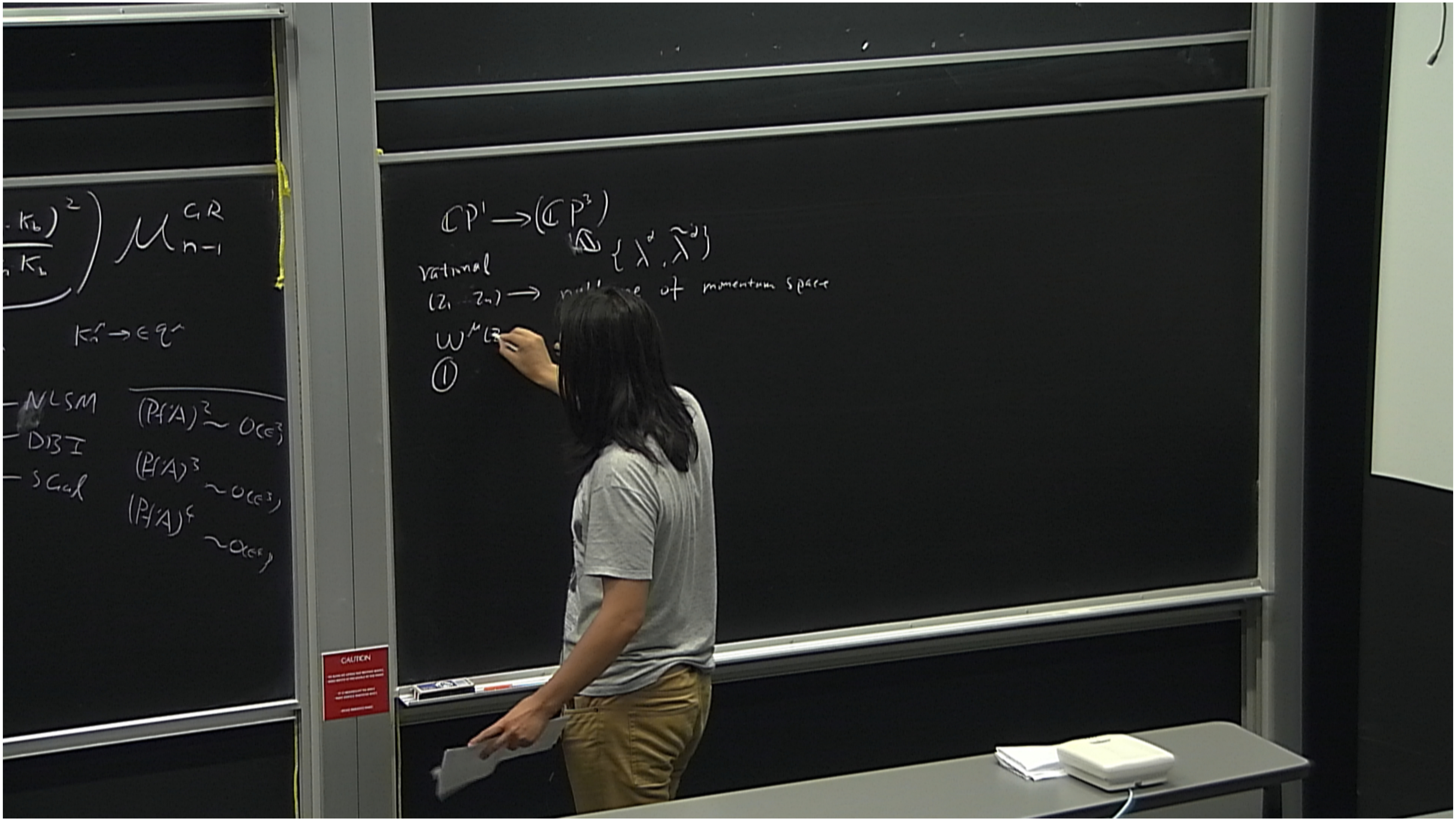

gravity
① gauge inv of gluons/gravitons
② nonlinearly realized sym from soft scalar limit
Pf'A
sGal
NLSM
CAUTION

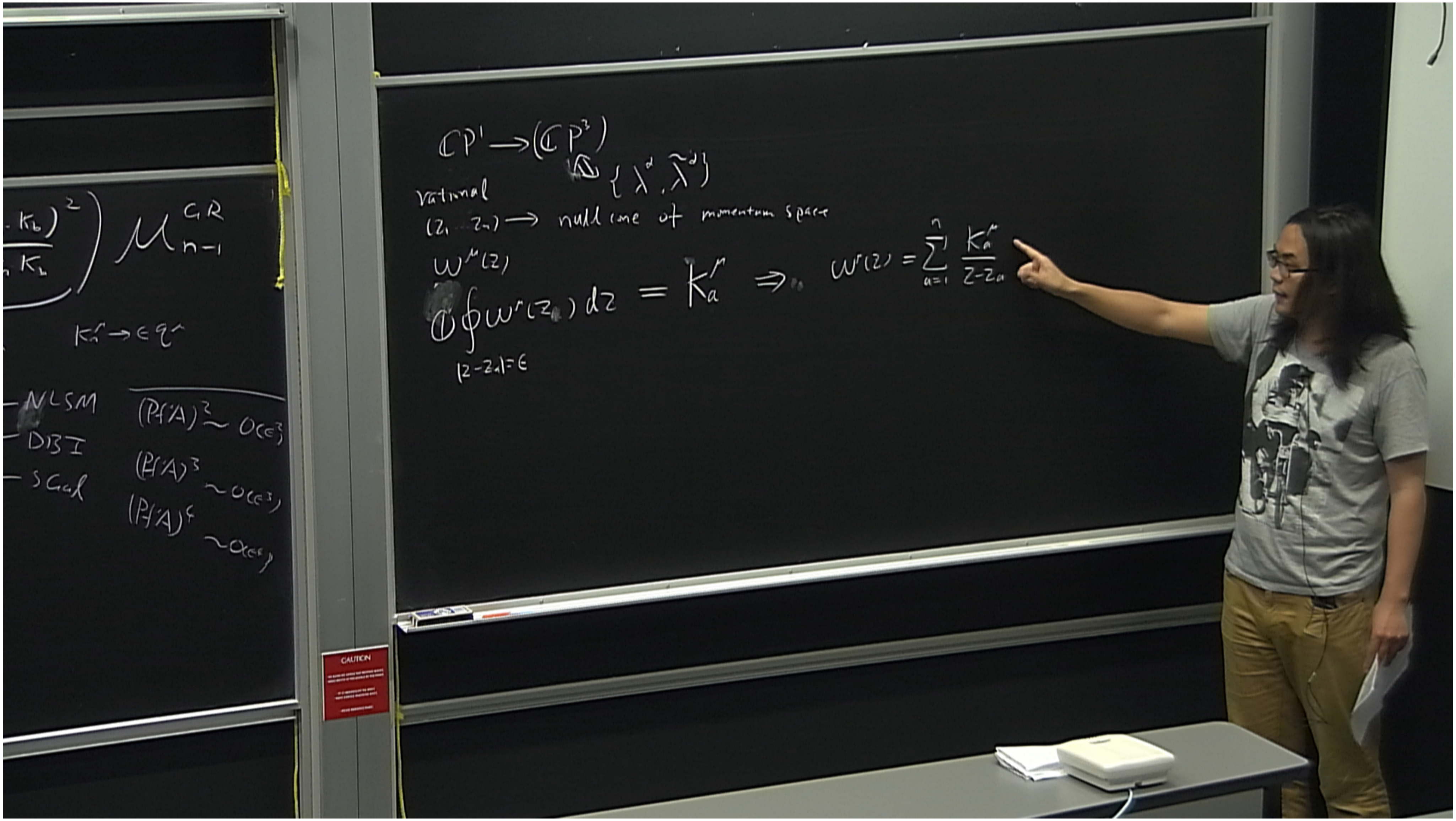

① gauge inv of gluons/gravitons
② nonlinearly realized sym from soft scalar limit
sGal
NLSM
YM

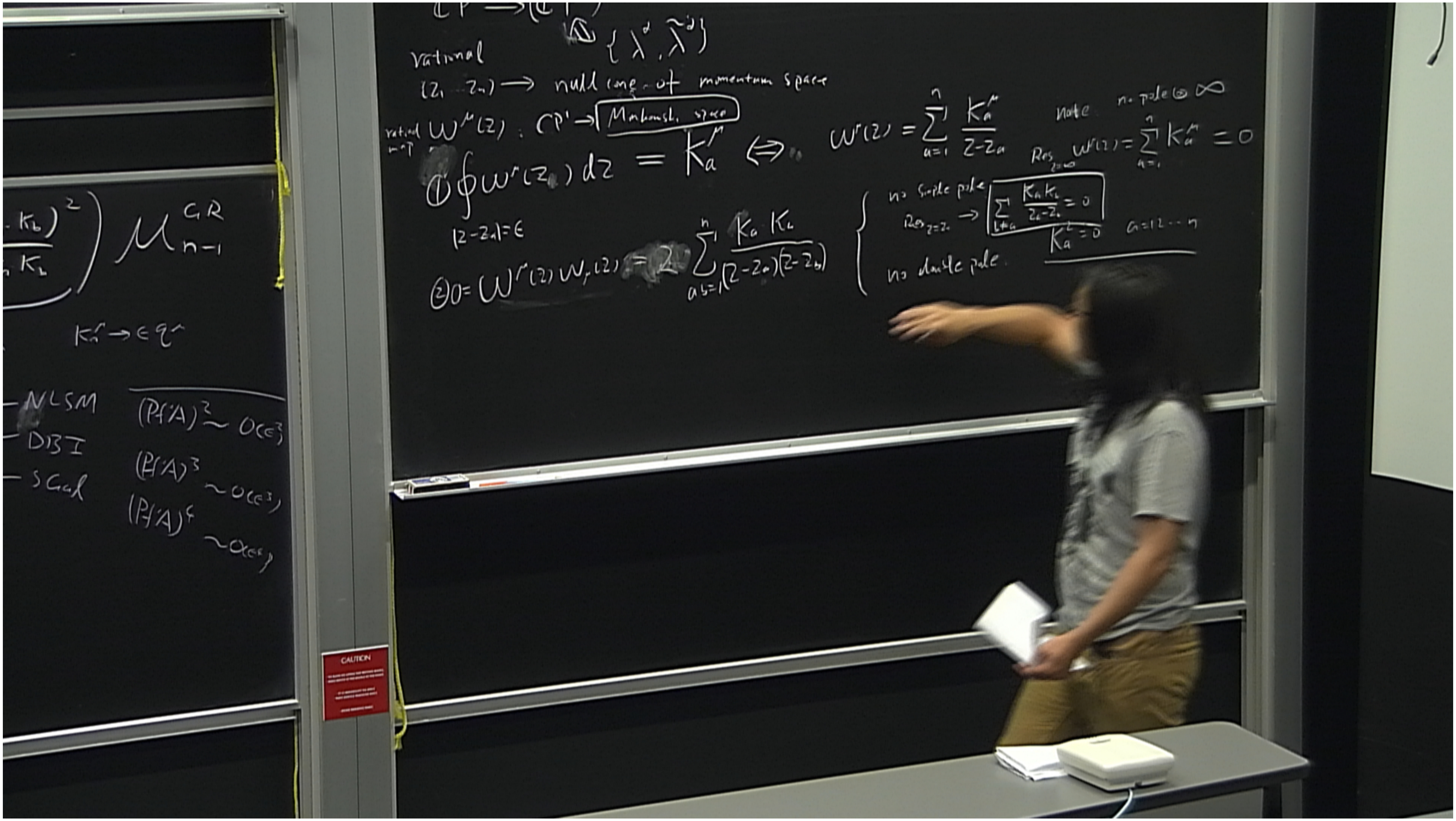

① gauge inv of gluons/gravitons
② non-linearly realized sym from soft scalar limit
Pf'A
⟺ Pf'Ψ
sGal
NLSM
$\mathcal{M}_n^{GR} = -\sum_{b=1}^{n-1} \oint \frac{dz_n}{\left(\sum \frac{k_n \cdot k_b}{z_n - z_b}\right)} \frac{(\epsilon_n \cdot k_b)^2}{(z_n - z_b)^2} \mathcal{M}_{n-1}^{GR} = \underbrace{\left(\sum_{b=1}^{n-1} \frac{(\epsilon_n \cdot k_b)^2}{k_n \cdot k_b}\right)}_{S_n^{GR}} \mathcal{M}_{n-1}^{GR}$
$C_{nn}^2 = \left(\sum_{b=1}^{n-1} \frac{\epsilon_n \cdot k_b}{z_n - z_b}\right)$
YM: $\frac{1}{z_{12} z_{23} \cdots z_{n1}} = \frac{1}{z_{12} z_{23} \cdots z_{n-1\,1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1}\right)$
Ex. ①, ② ⟹ soft gluon theorem

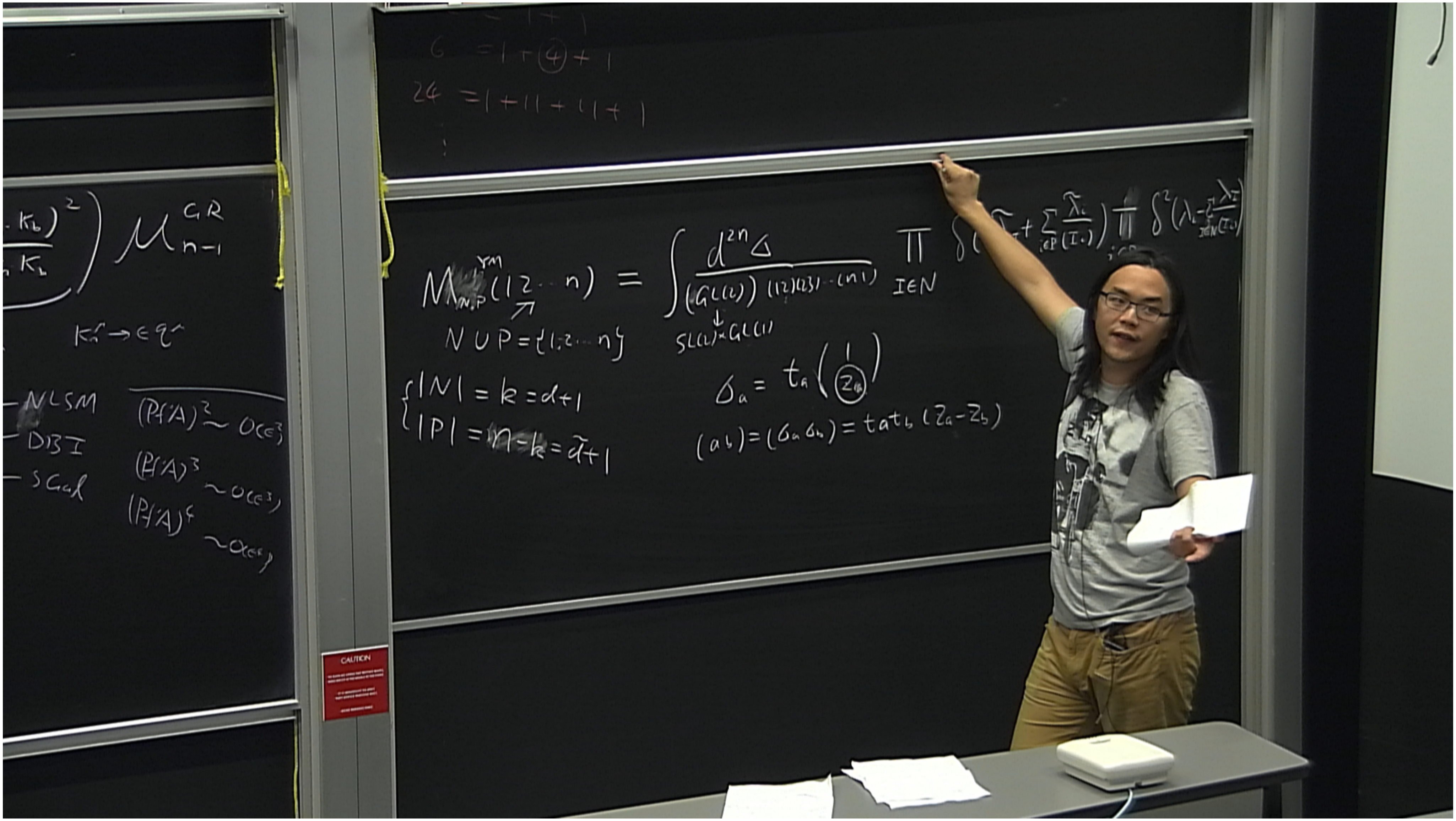

① gauge inv of gluons/gravitons
② non-linearly realized sym from soft scalar limit
Pf'A
↔ Pf'Φ
sGal
NLSM
$M_n^{GR} = -\sum_{b=1}^{n-1} \oint \frac{dz_n}{C_{nn}} \frac{(\epsilon_n \cdot k_b)^2}{(z_n - z_b)^2} M_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(\epsilon_n \cdot k_b)^2}{k_n \cdot k_b}\right) M_{n-1}^{GR}$
$C_{nn}^2 = \left(\sum_{b=1}^{n-1} \frac{\epsilon_n \cdot k_b}{z_n - z_b}\right)$
S_n^{GR}
$O(\epsilon)$ ← NLSM
$O(\epsilon^2)$ ← DBI
$O(\epsilon^3)$ ← sGal
$(Pf'A)^2 \sim O(\epsilon)$
$(Pf'A)^3 \sim O(\epsilon^2)$
$(Pf'A)^4 \sim O(\epsilon^3)$
YM ⊗ $\frac{1}{z_{12} z_{23} \cdots z_{n1}} = \frac{1}{z_{12} z_{23} \cdots z_{n-1\,1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1}\right)$
Ex. ①, ② ⟹ soft gluon theorem
CAUTION

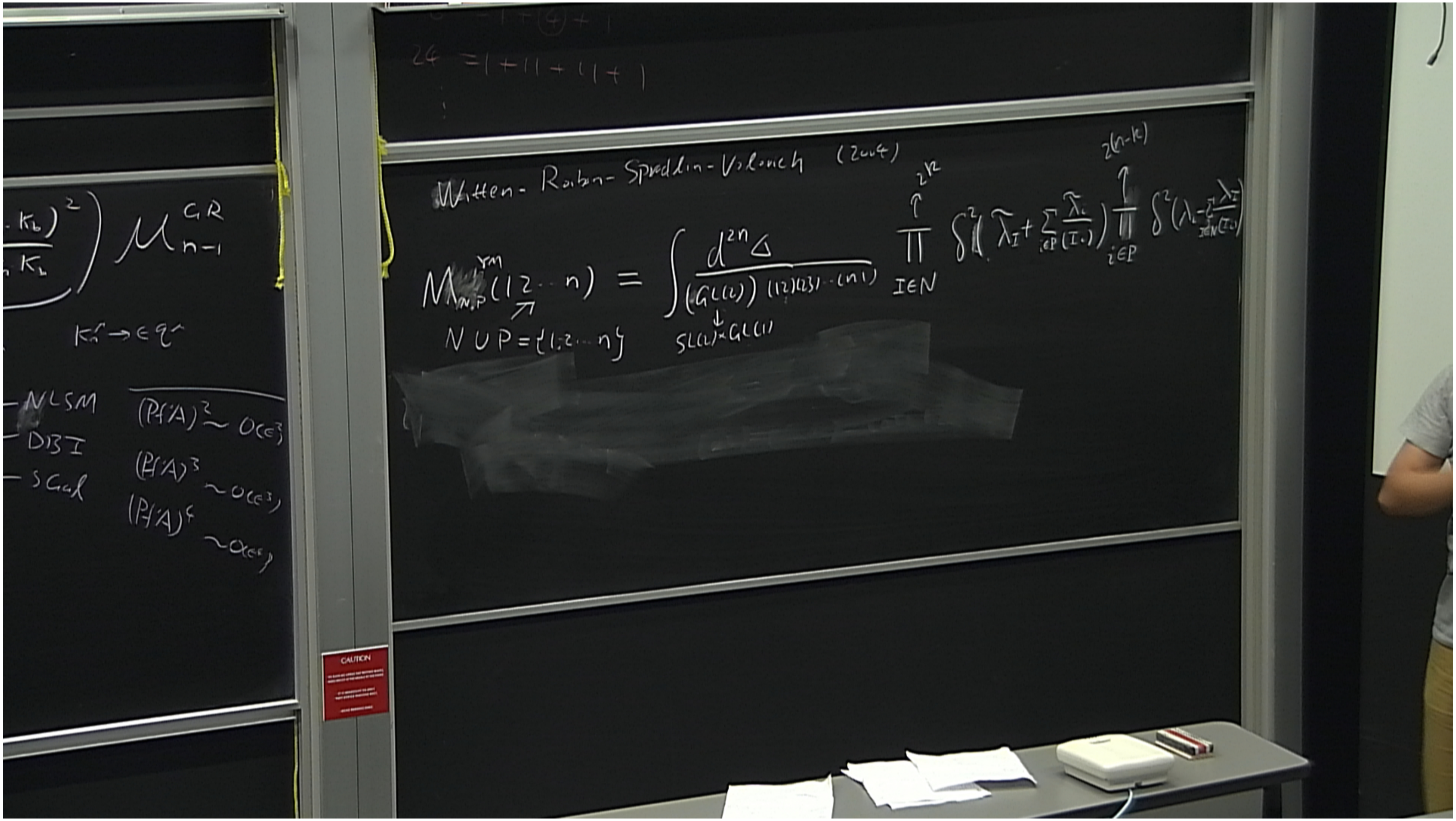

NLSM
DBI
SGal
CAUTION


CP' → (CP³)
rational
momentum space
NLSM
DBI
sGal
CAUTION

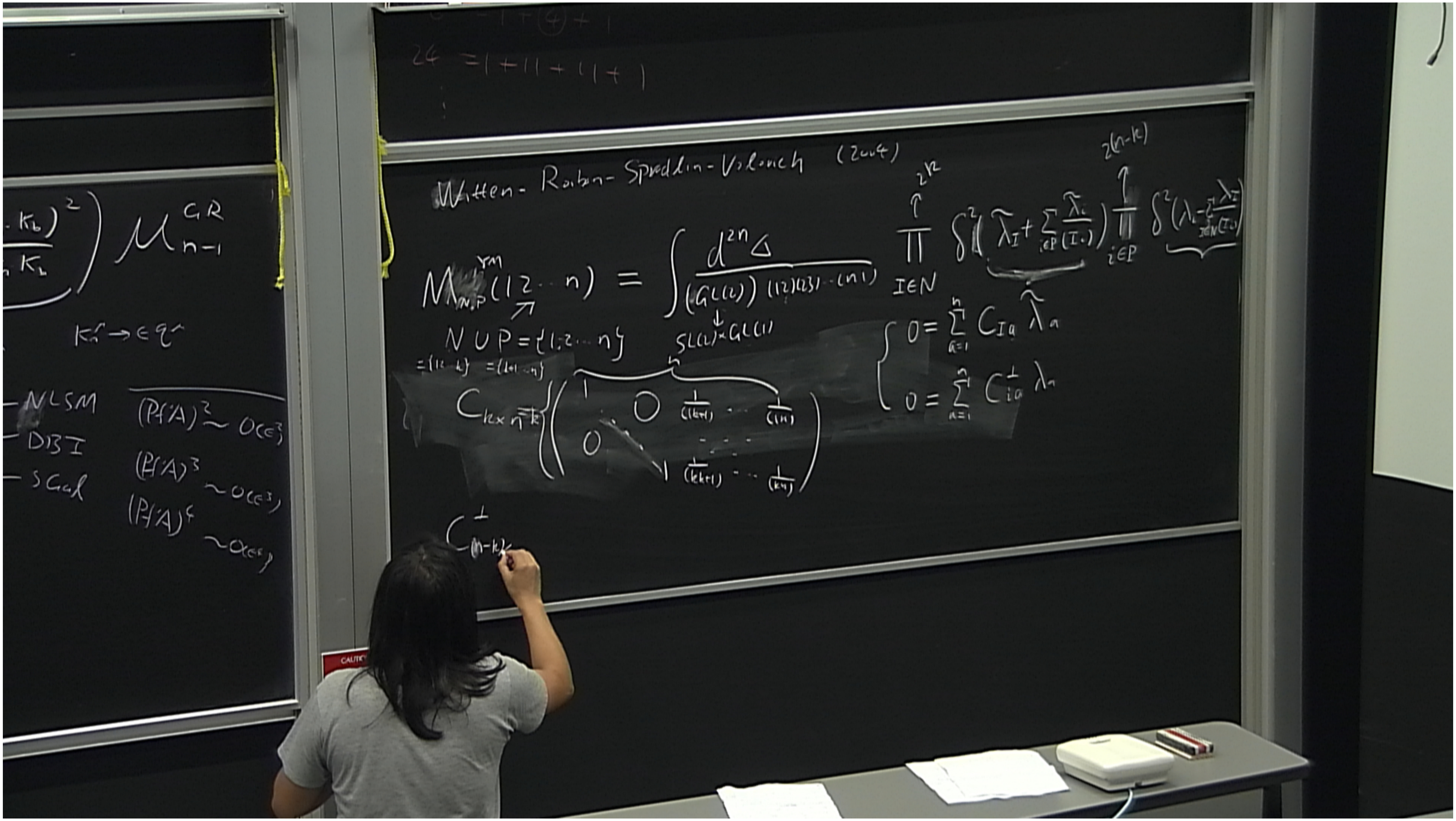

$\mathbb{CP}^1 \longrightarrow \mathbb{CP}^3$
$\{\lambda^\alpha, \tilde{\lambda}^{\dot\alpha}\}$
rational
$(z_1 \ldots z_n) \longrightarrow$ null cone of momentum space
$\omega^\mu(z)$
$\oint_{|z-z_a|=\epsilon} \omega^\mu(z)\, dz = k_a^\mu \Rightarrow \omega^\mu(z) = \sum_{a=1}^{n} \frac{k_a^\mu}{z-z_a}$
$\mathcal{M}^{GR}_{n-1}$
$k_a^\mu \to \epsilon q^\mu$
NLSM
DBI
SGal
$(\mathrm{Pf}'A)^2 \sim O(\epsilon^3)$
$(\mathrm{Pf}'A)^3 \sim O(\epsilon^3)$
$(\mathrm{Pf}'A)^4 \sim O(\epsilon^3)$

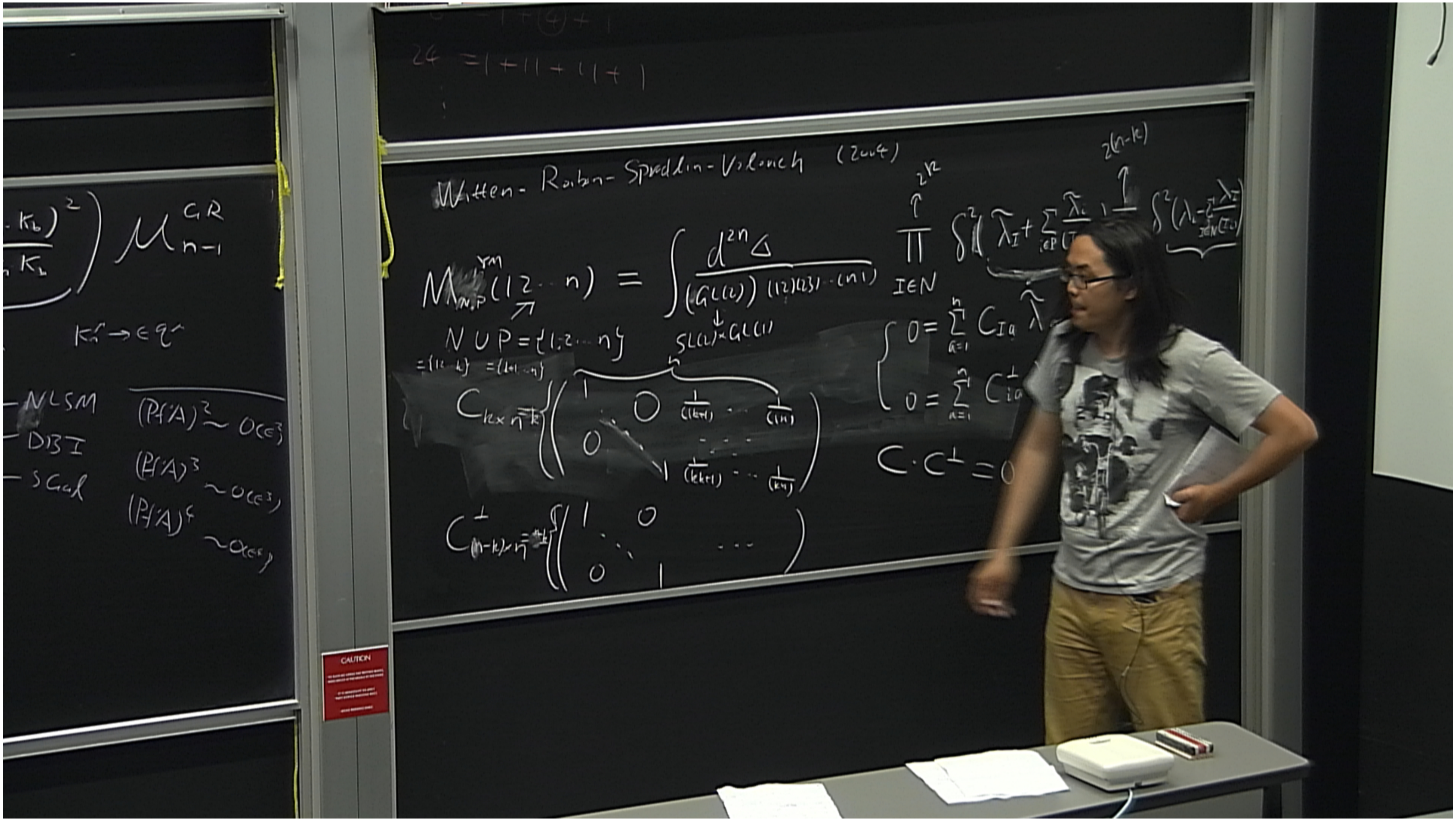



6 = 1 + 4 + 1
24 = 1 + 11 + 11 + 1
NLSM
DBI
SGal
$M_{N,P}^{YM}(12\cdots n) = \int \frac{d^{2n}\sigma}{(GL(2))\,(12)(23)\cdots(n1)} \prod_{I\in N}$
$SL(2)\times GL(1)$
$N \cup P = \{1,2\cdots n\}$
$|N| = k = d+1$
$|P| = n-k = \tilde{d}+1$
$\sigma_a = t_a \begin{pmatrix} 1 \\ z_a \end{pmatrix}$
$(ab) = (\sigma_a \sigma_b) = t_a t_b (z_a - z_b)$
M_{n-1}^{GR}



(2004)
NLSM
DBI
SGal


Witten-Roiban-Spradlin-Volovich (2004)
N ∪ P = {1, 2 … n}
SL(2) × GL(1)
NLSM
DBI
SGal


Witten-Roiban-Spradlin-Volovich (2004)
N ∪ P = {1, 2, ..., n}
C·C⊥ = 0
CAUTION