

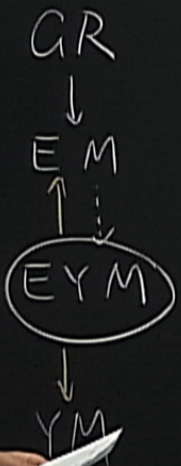
Title: Amplitudes: Applications

Date: Jul 17, 2015 09:00 AM

URL: <http://pirsa.org/15070056>

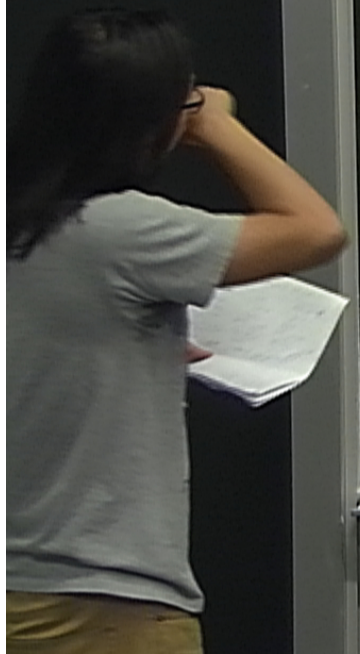
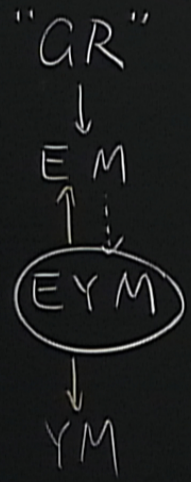
Abstract:

Some theories with simple scattering-E₂ formula



CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER
OR THE BOARDER

Some theories with simple scattering-E₂ formula



CAUTION
DO NOT TOUCH THE BOARD
OR THE MARKERS ON THE BOARD OR THE BOARD
OR THE MARKERS ON THE BOARD

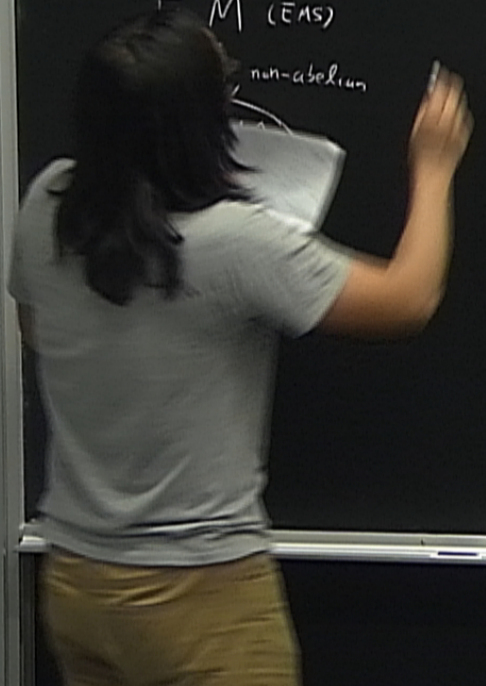
Some theories with simple scattering-E_l formula

"GR"

↓ dim red

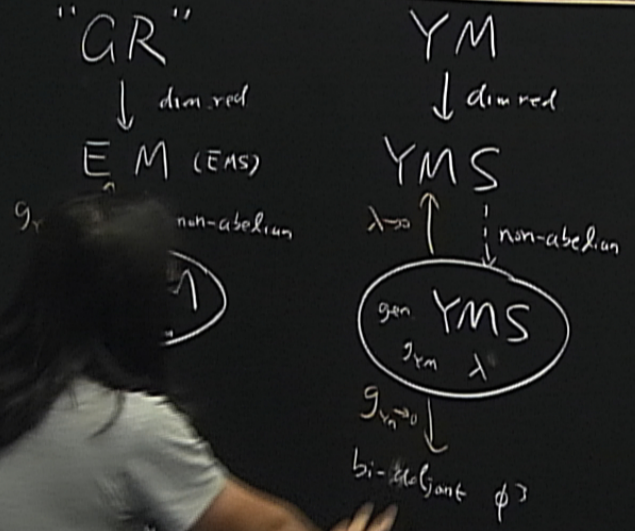
⊂ M (EAS)

non-abelian



CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER'S HANDS
OR THE BOARDER'S HANDS
OR THE BOARDER'S HANDS

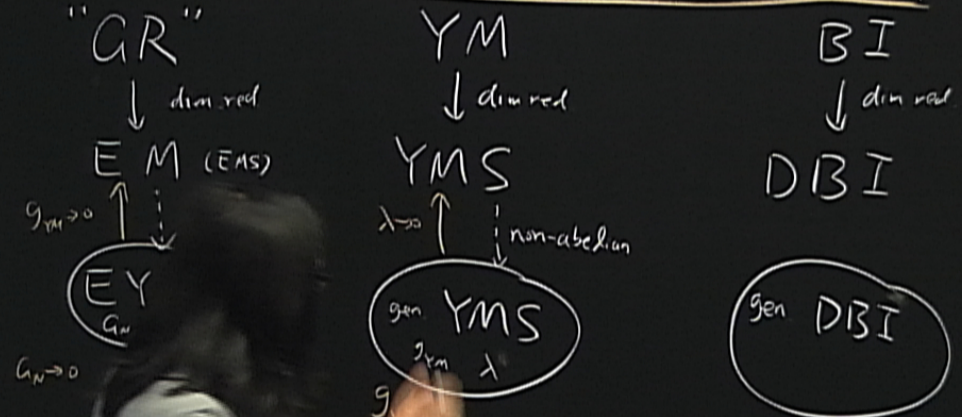
Some theories with simple scattering-E_L formula



$$+ \frac{\lambda}{3!} f \bar{f} \phi \phi \phi$$

$u(u) \times u(\bar{u})$

Some theories with simple scattering-E_L formula

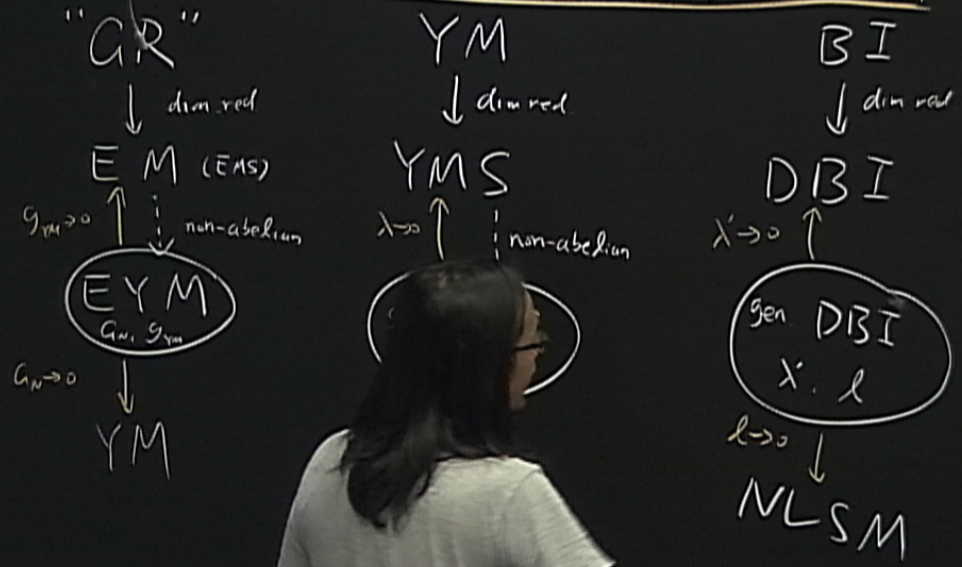


$$+ \frac{1}{3!} f \bar{f} \phi \phi \phi$$

\uparrow \uparrow
 $u(\vec{u}) \times u(\vec{u})$

(Pf'A)

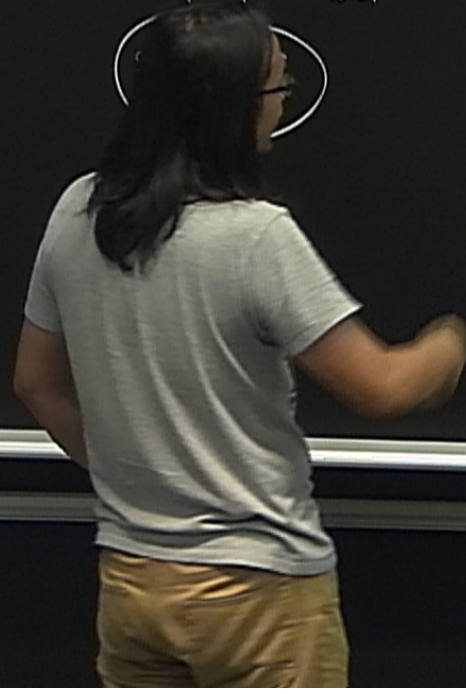
Some theories with simple scattering-E_l formula

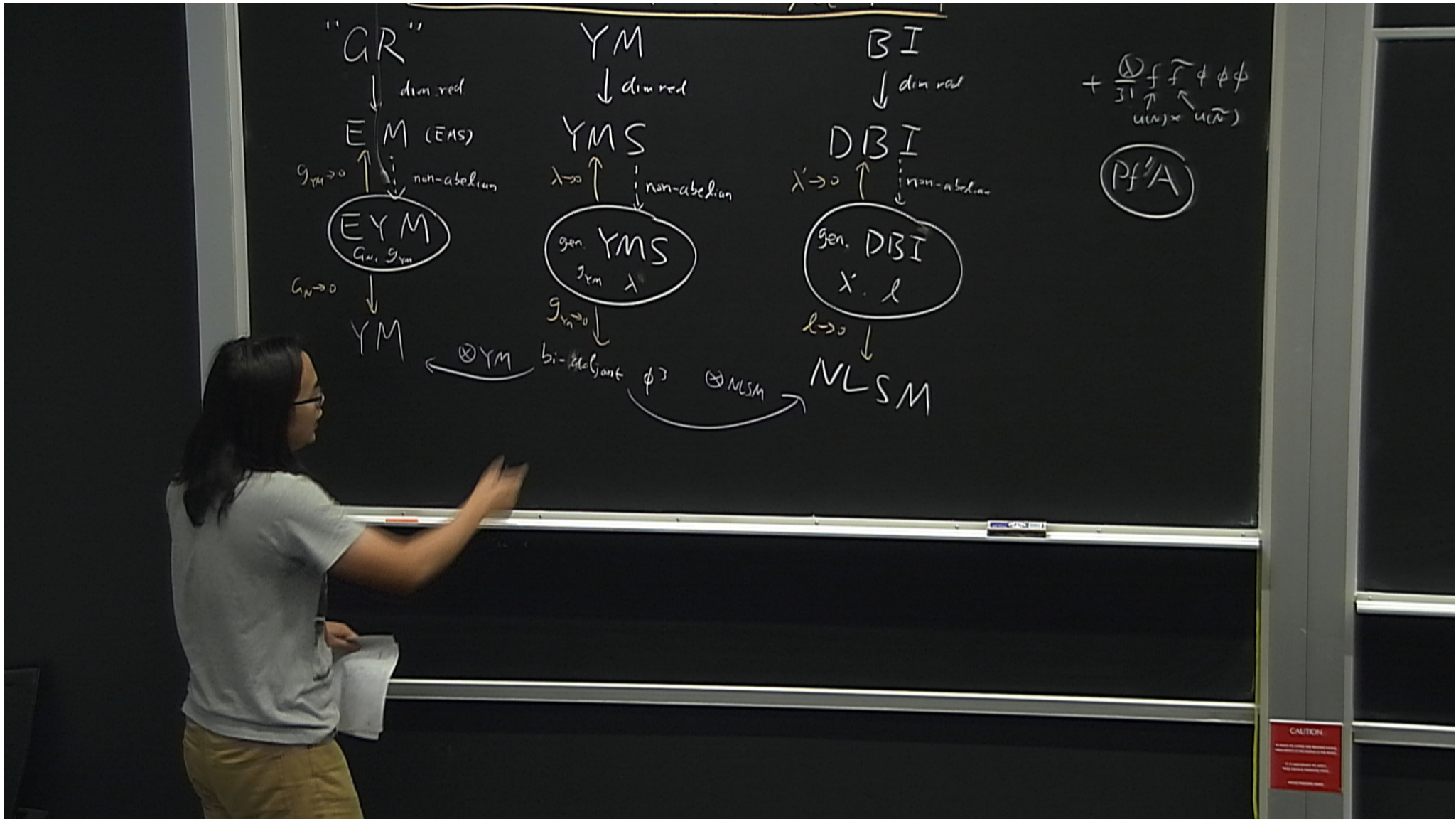


$$+ \frac{1}{3!} \vec{f} \vec{f} \phi \phi \phi$$

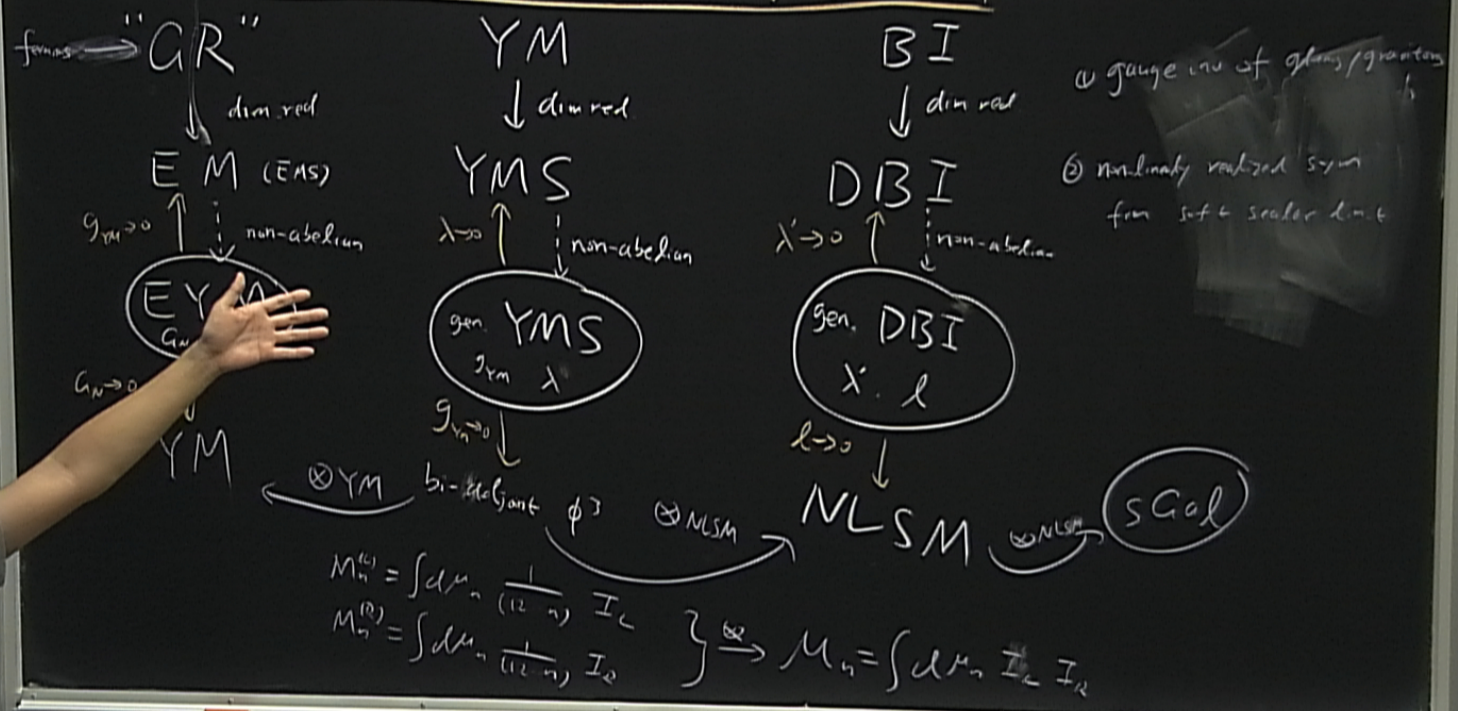
\uparrow
 $u(\vec{n}) = u(\vec{n})$

(Pf'A)



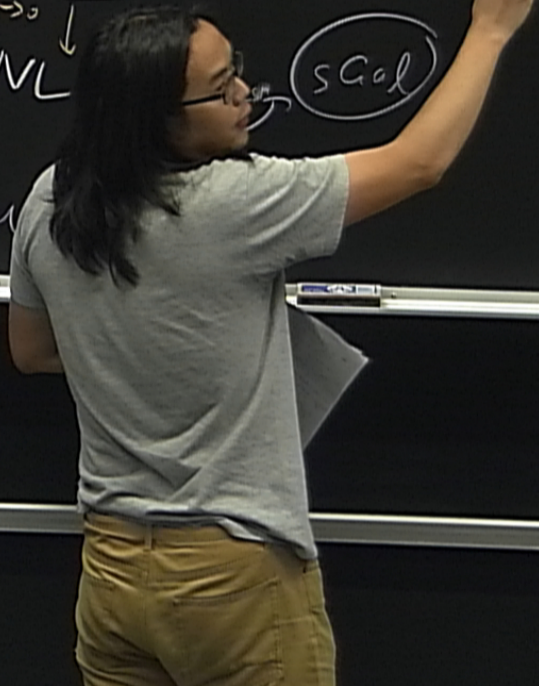
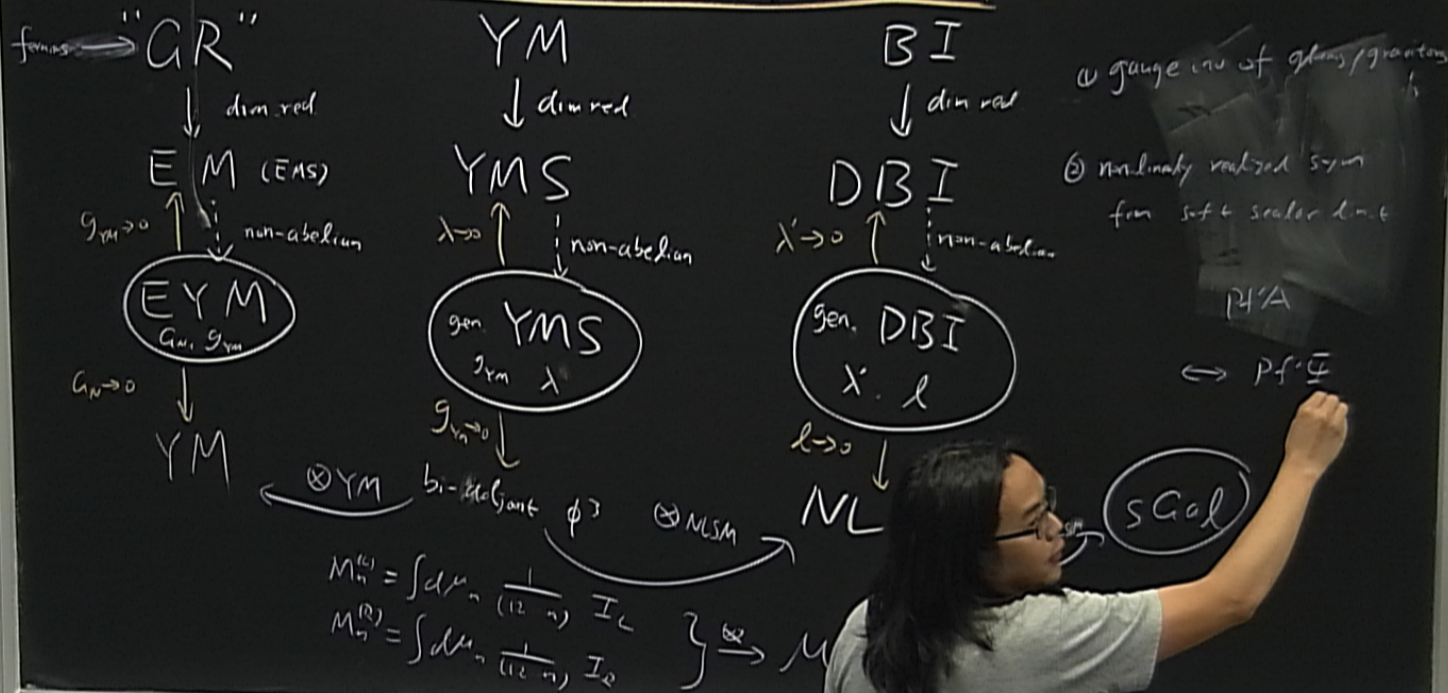


Some theories with simple scattering-E_{pl} formula



① gauge inv of gluing gravitons
 ② non-linearly realized sym from soft scalar limit





$$M_n^{(L)} = \int d\mu_n \frac{1}{(12-n)} I_L$$

$$M_n^{(R)} = \int d\mu_n \frac{1}{(12-n)} I_R$$

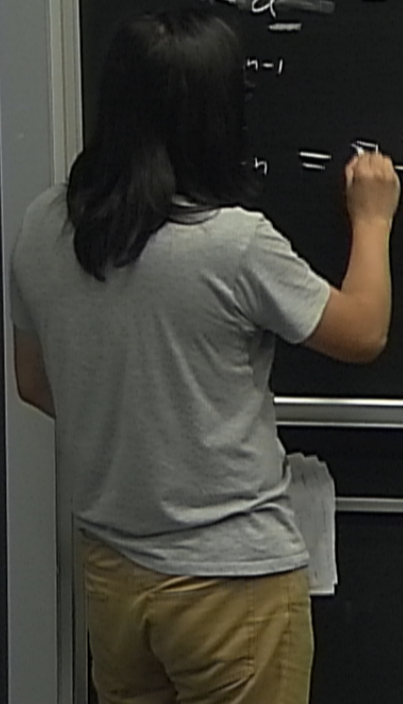
$$\} \rightarrow M_n = \int d\mu_n I_L I_R$$

Soft limits as a way to (get)

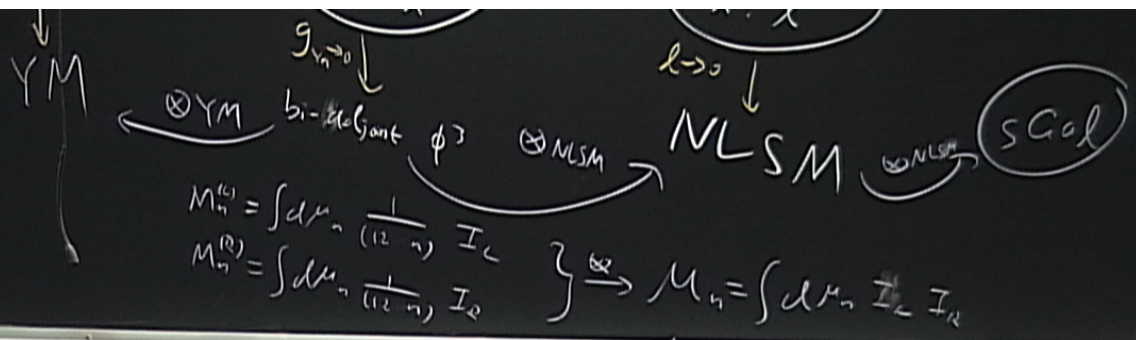
$$K_n^M = \epsilon q^M \quad (\epsilon \rightarrow 0)$$

$$0 = \sum_{a=1}^{n-1} \epsilon \frac{K_a \cdot K_b}{z_a - z_b} + \epsilon \frac{K_n \cdot q}{z_n - z_n} \quad (n-1)\text{-pt system}$$

$$= \frac{K_n \cdot K_b}{z_n - z_b}$$



CAUTION
DO NOT TOUCH THE BOARD
OR THE SURROUNDING AREA
UNLESS YOU ARE INSTRUCTED TO DO SO



practical way of solving eqs $z_1 \rightarrow \infty$ $\{z_2, z_3\} = \{0, 1\}$

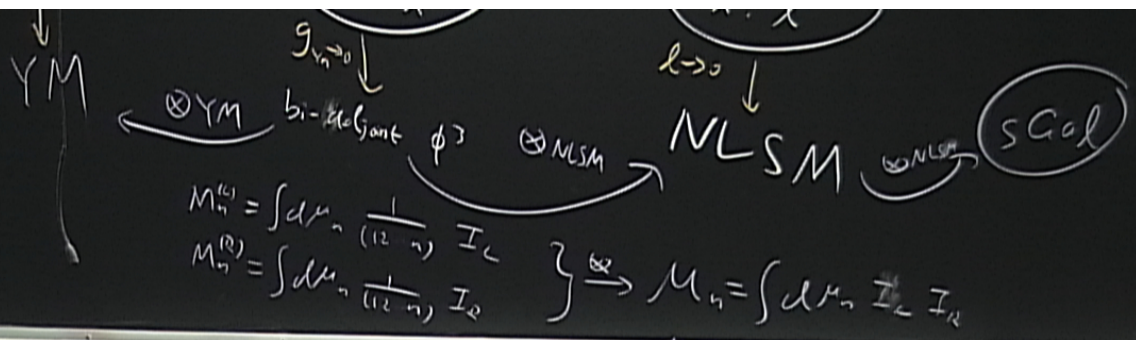
S_{ij} ($1 \leq i < j \leq n-1$) except $S_{n-2, n-1} = -(\text{sum of the rest})$ $(M) \epsilon = \frac{1}{M}$
 $S_{in} = -\sum_{j=1}^{n-1} S_{ij}$

$$\sum_{b \neq a} \frac{\epsilon_a \epsilon_b S_{ab}}{z_a - z_b} = 0 \quad a = 4, 5, \dots, n \quad \epsilon_a = \begin{cases} 1 & a=1, 2, \dots \\ 0 & a=3, \dots, n \end{cases}$$

2ⁿ sol for 7-pt "NSolve" "FindRoot"

$$M(1z \dots n) = \sum_{(z_1, \dots, z_n) \in \text{sol}} \frac{1}{\det \left| \frac{\partial E_i}{\partial z_j} \right|_{1 \leq i, j \leq n}} \frac{1}{z_{13} \dots z_{n-1n}} \text{Pf} \left[\overline{\Psi}(\epsilon, k, z) \right]_{12}^{1n}$$





practical way of solving eqs $z_1 \rightarrow \infty$ $\{z_2, z_3\} = \{0, 1\}$

S_{ij} ($1 \leq i, j \leq n-1$) except $S_{n-2, n-1} = -(\text{sum of the rest})$ $(M) \epsilon = \frac{1}{M}$
 $S_{in} = -\sum_{j=1}^{n-1} S_{ij}$

$$\sum_{b \neq a} \frac{\epsilon_a \epsilon_b S_{ab}}{z_a - z_b} = 0 \quad a = 4, 5, \dots, n$$

2nd set for 7-pt "NSolve"

$$\epsilon_a = \begin{cases} 1 & a=1, 2, 3 \\ 0 & a=4, \dots, n \end{cases}$$

$$M(12 \dots n) = \sum_{\{z_1, \dots, z_n\} \in \text{sol}} \frac{1}{\det \left| \frac{\partial E_a}{\partial z_i} \right|_{1 \leq a \leq n-1, 1 \leq i \leq n}}$$

Find Root



$\sum_{k=1}^{n-1} (q_k z_k) z_{k+1}^{n-1}$
 (1) gauge inv of glans/grantans
 (2) non-linearly realized sym
 from soft + scalar dim
 PIA
 $\Leftrightarrow P \cdot \bar{\psi}$
 $\text{NLSM} \rightarrow \text{SCol}$
 I_L, I_R

2. Soft theorems (Weinberg)

Universal soft behavior $\left\{ \begin{array}{l} \text{equivalence principle} \\ \text{conservation of charges} \\ \text{Emission } S > 2 \end{array} \right.$

$$\left(\sum_{b=1}^{n-1} (g_{ab} z_b) \right) z_n^{n-3}$$

ω gauge inv of grav/gravities

ⓐ non-linearly realized sym for soft scalar limit

(P/A)

↔ P/A

(S/Gal)

ω NLSM

I_L, I_R

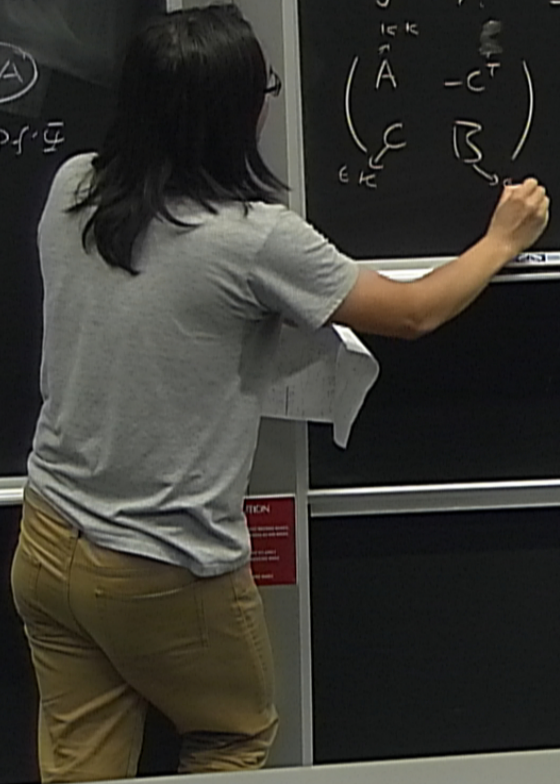
$K_n^m = e g^m$ (n-pt) eqs → (n-1) pt eqs + one more for z_n

$$\mathcal{M}_n = \int dz_n \delta \left(\sum_{b=1}^{n-1} \frac{K_n^b k_b}{z_n - z_b} \right) \int d\mathcal{M}_{n-1} I_n$$

gravity

$$I_n = \det' \overline{\Psi}_n$$

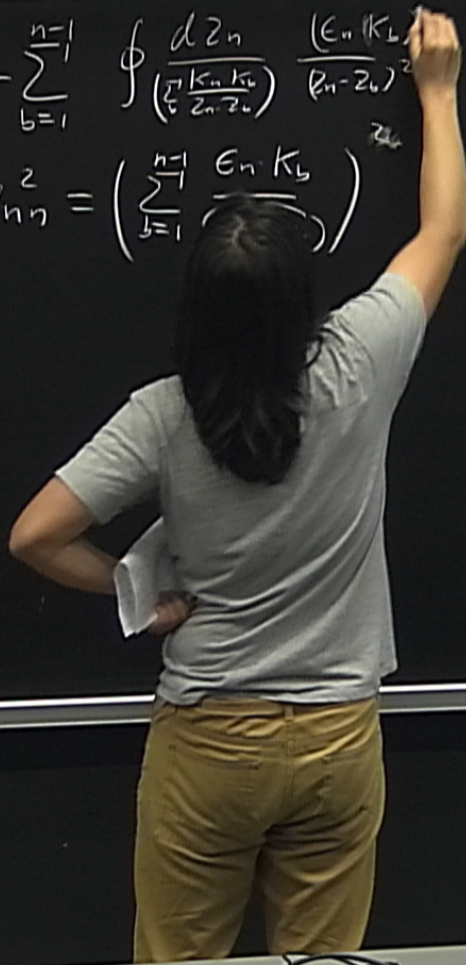
$$\begin{pmatrix} A & -C^T \\ C & B \end{pmatrix}$$



$\left(\sum_{b=1}^{n-1} (g_{ab} z_b) \right) z_{n-1}$
 gauge inv of glans/granting
 non-linearly realized sym
 for soft scalar dim 4
 P/A
 \leftrightarrow P-f- $\bar{\Psi}$
 SCof
 I_L, I_R

$$M_n^{GR} = - \sum_{b=1}^{n-1} \oint \frac{dz_n}{\left(\sum_{b=1}^{n-1} \frac{K_n K_b}{z_n z_b} \right)} \frac{(E_n K_b)}{(z_n - z_b)^2}$$

$$C_{nn}^z = \left(\sum_{b=1}^{n-1} \frac{E_n K_b}{\left(\sum_{b=1}^{n-1} \frac{K_n K_b}{z_n z_b} \right)} \right) z_n$$



CAUTION

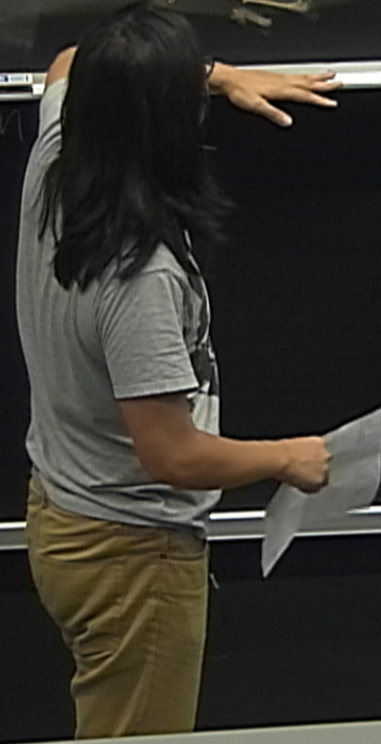
CAUTION

$\left(\sum_{b=1}^{n-1} (g_{ab} z_b) \right) z_{n-1}$
 gauge inv of grav/gravity
 non-linearly realized sym
 for soft scalar limit
 P/A
 \leftrightarrow P-f- $\bar{\Psi}$
 S Gal
 I_L, I_R

$$\mathcal{M}_n = \int dz_n \left(\frac{K_n K_1}{z_n - z_1} \right) \underline{d\mathcal{M}_{n-1}} I_n$$

gravity $I_n = \det' \bar{\Psi}_n \rightarrow C_{nn}^2 \det' \bar{\Psi}_{n-1}$

$$\mathcal{M}_n^{GR} = \int \frac{dz_n C_{nn}^2}{\left(\frac{K_n K_1}{z_n - z_1} \right)} \left(\int d\mathcal{M}_{n-1} \frac{\det' \bar{\Psi}_{n-1}}{I_{n-1}} \right) = \mathcal{M}_{n-1}^{GR}$$



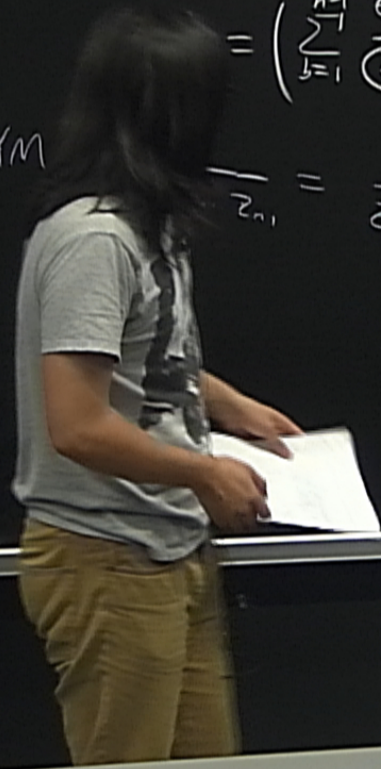
$\left(\sum_{b=1}^{n-1} (q_{n-b} z_b) \right) z_{n-1}$
 gauge inv of gluing/grants
 non-linearly realized sym
 for soft scalar limit
 P/A
 \leftrightarrow P-f- $\bar{\Psi}$
 SCol
 I_L, I_R

$$\mathcal{M}_n^{GR} = - \sum_{b=1}^{n-1} \oint \frac{dz_n}{z_n} \frac{(E_n K_b)^2}{(z_n - z_b)^2} \mathcal{M}_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(E_n K_b)^2}{K_n K_b} \right) \mathcal{M}_{n-1}^{GR}$$

$$= \left(\sum_{b=1}^{n-1} \frac{E_n K_b}{z_n - z_b} \right) S_n^{GR}$$

YM

$$\frac{1}{z_n} = \frac{1}{z_1 z_2 \dots z_{n-1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1} \right)$$



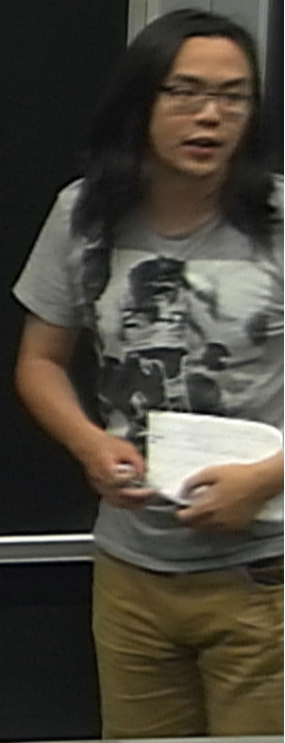
$\left(\sum_{b=1}^{n-1} (q_{n-b}) z_b \right) z_{n-1}$
 gauge inv of glans/grantons
 non-linearly realized sym
 for soft scalar limit
 P/A
 $\Leftrightarrow P \cdot \bar{\Phi}$
 SCol
 I_L, I_R

$$\mathcal{M}_n^{GR} = - \sum_{b=1}^{n-1} \oint \frac{dz_n}{\left(\sum_{b=1}^{n-1} \frac{K_n \cdot K_b}{z_n - z_b} \right) (z_n - z_b)^2} \mathcal{M}_{n-1}^{GR} = \underbrace{\left(\sum_{b=1}^{n-1} \frac{(E_n \cdot K_b)^2}{K_n \cdot K_b} \right)}_{S_n^{GR}} \mathcal{M}_{n-1}^{GR}$$

$$C_{nn}^z = \left(\sum_{b=1}^{n-1} \frac{E_n \cdot K_b}{z_n - z_b} \right)$$

$$YM_{\text{gluon}} \frac{1}{z_{12} z_{23} \dots z_{n-1}} = \frac{1}{z_{12} z_{23} \dots z_{n-1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1} \right)$$

Ex. $\mathcal{O}_1, \mathcal{O}_2 \Rightarrow$ Soft gluon theorem



$(\sum_{b=1}^{n-1} (q_{k_b} z_b) z_{b+1}^{-1})$
 gauge inv of gluing/granted
 non-linearly realized sym
 from soft scalar dim
 (P/A)
 $\leftrightarrow P \cdot \bar{\Phi}$
 (SCol)
 I_L, I_R

$$M_n^{GR} = - \sum_{b=1}^{n-1} \oint \frac{dz_n}{(z_n - z_b)} \frac{(E_n - K_L)^2}{(z_n - z_b)^2} M_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(E_n - K_L)^2}{K_n K_L} \right) M_{n-1}^{GR}$$

$$C_{n\eta}^z = \left(\sum_{b=1}^{n-1} \frac{1}{z_n - z_b} \right)$$

$$YM_{\text{soft}} = \frac{1}{z_{12} z_{23} \dots z_{n1}}$$

Ex. $\mathbb{C}, \mathbb{C} \Rightarrow \text{Soft}$
 $\left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1} \right)$

$S_n^{GR} \quad K_L \rightarrow E_n$
 $0(\epsilon^1) \leftarrow \text{NLSM} \quad (P/A)^2 \sim 0(\epsilon^2)$
 $0(\epsilon^2) \leftarrow \text{DBI} \quad (P/A)^3 \sim 0(\epsilon^3)$
 $0(\epsilon^3) \leftarrow \text{SCol} \quad (P/A)^4 \sim 0(\epsilon^4)$



CAUTION

CAUTION

$$\left(\sum_{b=1}^{n-1} (q_k z_b) z_b^{-2} \right)$$

① gauge inv of gluing/granted

② non-linearly realized sym for soft scalar dim

(P/A)

↔ P/A

(SCol)

NLSM

I_L, I_R

$$M_n^{GR} = - \sum_{b=1}^{n-1} \oint \frac{dz_n}{z_n - z_b} \frac{(E_n K_b)^2}{(z_n - z_b)^2} M_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(E_n K_b)^2}{K_n K_b} \right) M_{n-1}^{GR}$$

$$C_{n,1}^2 = \left(\sum_{b=1}^{n-1} \frac{E_n K_b}{z_n - z_b} \right)$$

$S_n^{GR} \quad K_i \rightarrow E_i q_i$

$O(e^1) \leftarrow$ NLSM

$O(e^2) \leftarrow$ DBI

$O(e^3) \leftarrow$ SCol

$(P/A)^2 \sim O(e^3)$

$(P/A)^3 \sim O(e^4)$

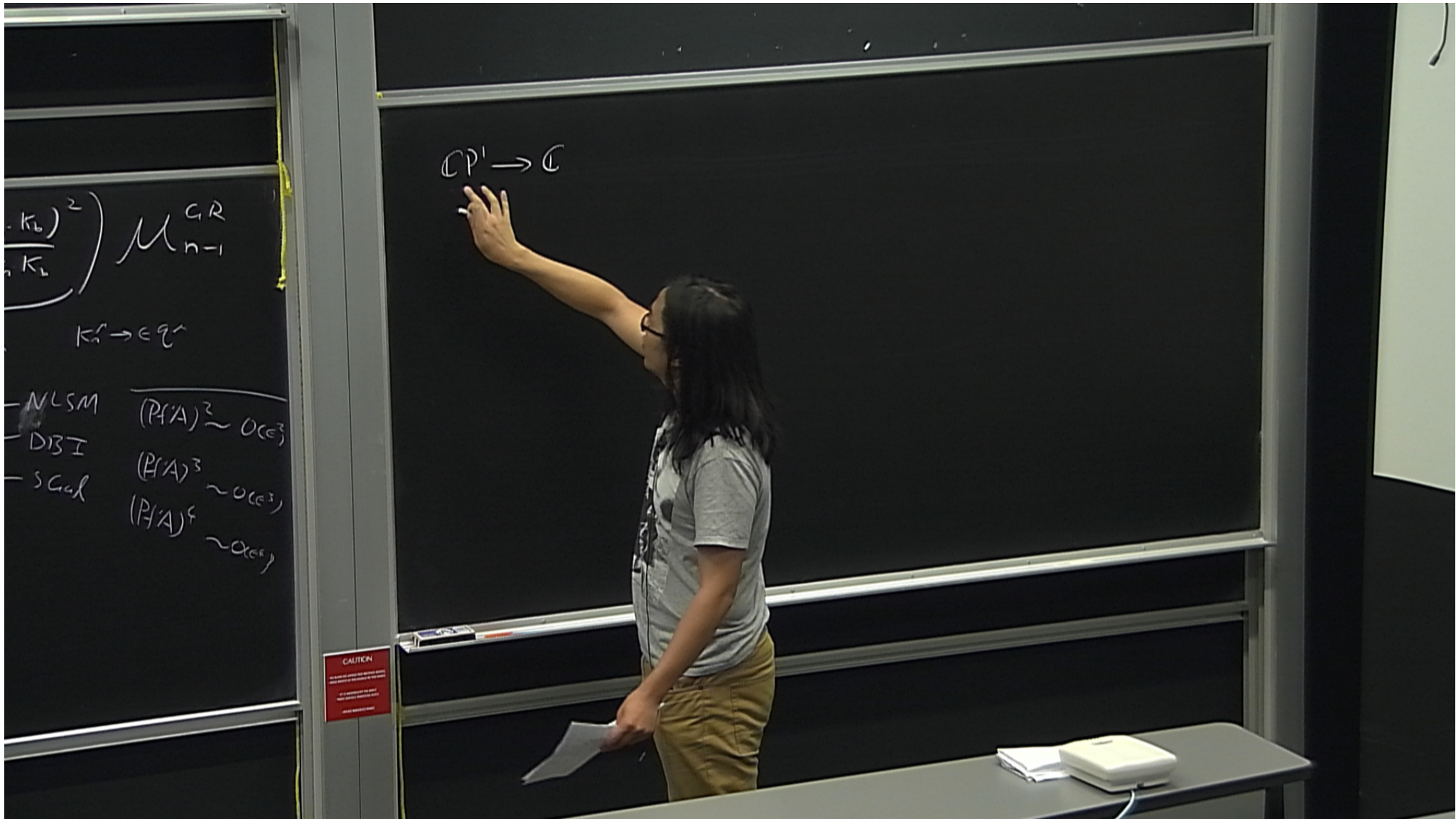
$(P/A)^4 \sim O(e^5)$

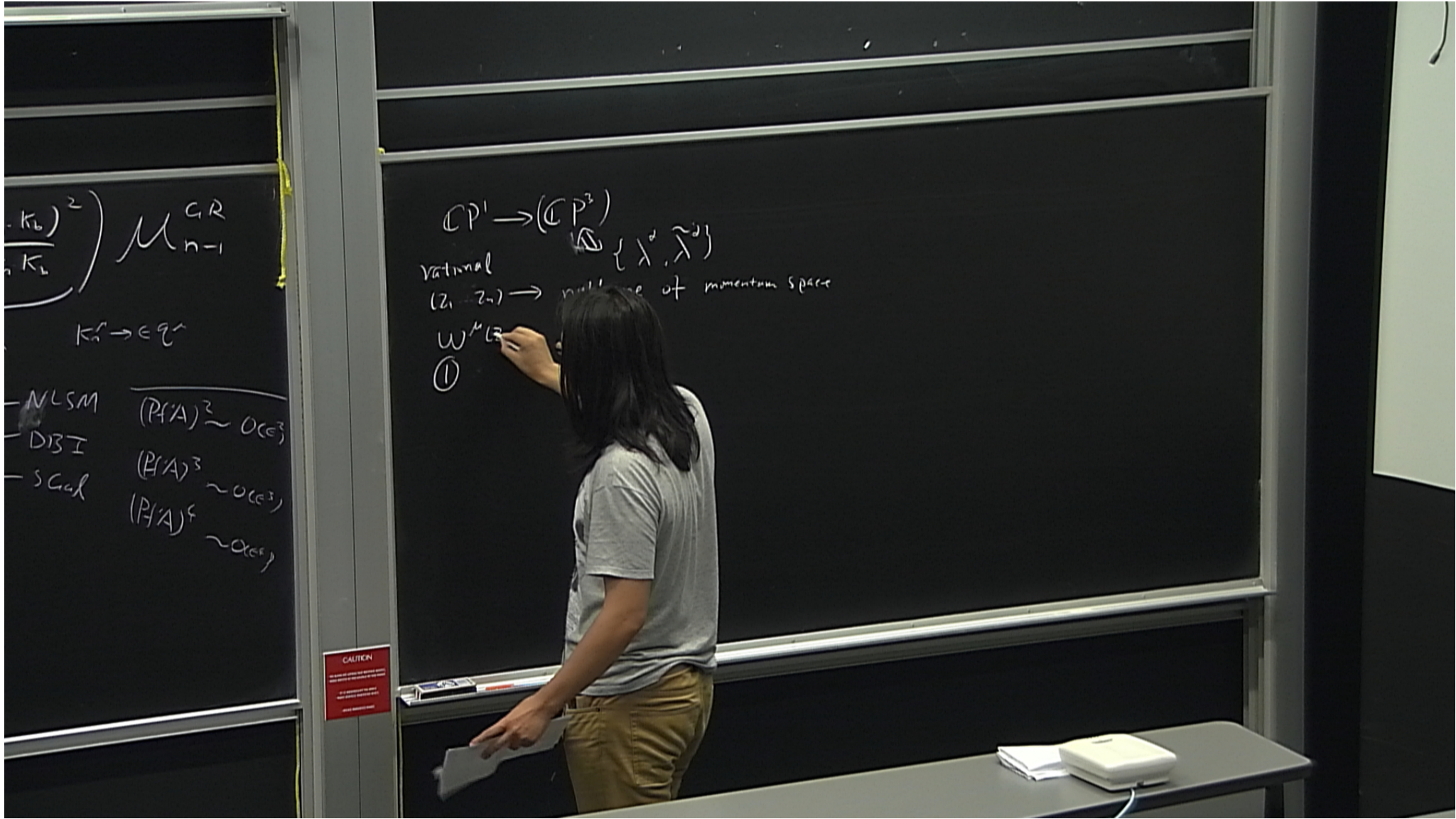
$$YM_{\partial} \frac{1}{z_{12} z_{23} \dots z_{n1}} = \frac{1}{z_{12} z_{23} \dots z_{n-1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1} \right)$$

Ex. ①, ② ⇒ soft gluon theorem

CAUTION

CAUTION



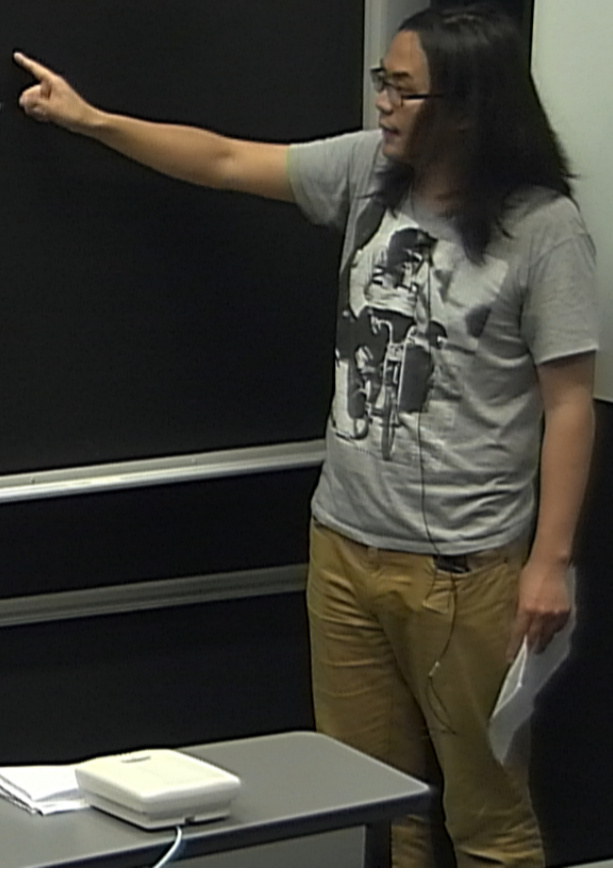


$$\left(\frac{-k_b}{k_a} \right)^2 M_{n-1}^{GR}$$

$$k_a^m \rightarrow \epsilon q^m$$

- NLSM $(P/A)^2 \sim O(\epsilon^3)$
- DBI $(P/A)^3 \sim O(\epsilon^3)$
- SGal $(P/A)^4 \sim O(\epsilon^3)$

$CP^1 \rightarrow (CP^2)$
 Rational $\{ \lambda^a, \tilde{\lambda}^a \}$
 $(z_1, z_2) \rightarrow$ null line of momentum space
 $\omega^\mu(z)$
 $\oint_{|z-z_1|=\epsilon} \omega^\mu(z) dz = K_a^\mu \Rightarrow \omega^\mu(z) = \sum_{a=1}^n \frac{K_a^\mu}{z-z_a}$



$$\left(\frac{k_b}{k_a} \right)^2 M_{n-1}^{GR}$$

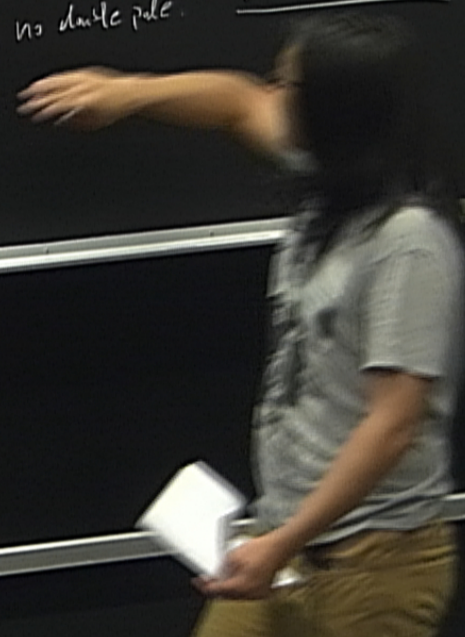
$$k_a \rightarrow \epsilon q^{\pm}$$

- NLSM $(P/A)^2 \sim O(\epsilon^3)$
- DBI $(P/A)^3 \sim O(\epsilon^3)$
- S Gal $(P/A)^4 \sim O(\epsilon^3)$

$CP \rightarrow (CP^1)$
 rational $(z_1, z_2) \rightarrow$ null cone of momentum space
 rational $W^M(z) : CP^1 \rightarrow$ Minkowski space
 $\oint \phi W^M(z) dz = K_a^M \Leftrightarrow W^M(z) = \sum_{a=1}^n \frac{K_a^M}{z-z_a}$
 $|z-z_a| = \epsilon$
 $\langle 20 \rangle = W^M(z) W^N(z) = 2 \sum_{a,b=1}^n \frac{K_a^M K_b^N}{(z-z_a)(z-z_b)}$

Note: $n = \text{pole} @ \infty$
 $\text{Res}_{z=a} W^M(z) = \sum_{a=1}^n K_a^M = 0$

no single pole $\rightarrow \sum_{b+a} \frac{K_a K_b}{z-z_a} = 0$
 $K_a^L = 0 \quad a=1, 2, \dots, n$
 no double pole



$\left(\frac{k_b}{k_a}\right)^2 M_{n-1}^{GR}$
 $k_a \rightarrow \epsilon q^{\epsilon}$
 - NLSM $(P/A)^2 \sim O(\epsilon^3)$
 - DBI $(P/A)^3 \sim O(\epsilon^3)$
 - SCaL $(P/A)^4 \sim O(\epsilon^3)$

$G = 1 + 4 + 1$
 $24 = 1 + 11 + 11 + 1$

$$M_{NUP}^{ym}(1, 2, \dots, n) = \int \frac{d^{2n} \Delta}{(GL(n)) (1, 2, \dots, n) \cdot (n, 1)} \prod_{I \in N} \delta \left(\sum_{i \in I} \lambda_i + \sum_{j \in P(I)} \frac{\lambda_j}{(I, j)} \right) \prod_{i \in N} \delta^2 \left(\lambda_i - \sum_{j \in N(I)} \frac{\lambda_j}{(I, j)} \right)$$

$N \cup P = \{1, 2, \dots, n\}$
 \downarrow
 $SL(n) \times GL(n)$

$$\delta_a = t_a \left(\frac{1}{z_a} \right)$$

$$(ab) = (\delta_a \delta_b) = t_a t_b (z_a - z_b)$$

CAUTION

$$\left(\frac{k_b}{k_a}\right)^2 M_{n-1}^{GR}$$

$k_a \rightarrow \epsilon q^a$

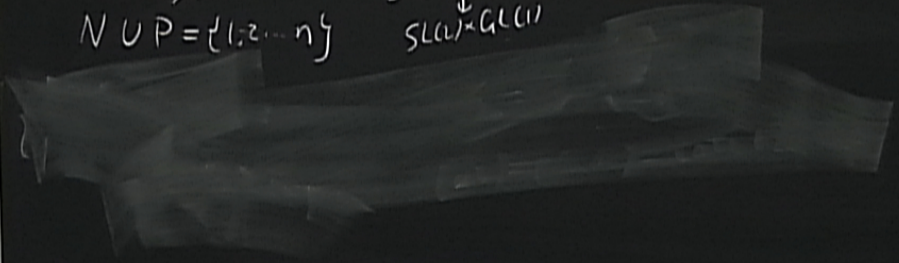
- NLSM $(P/A)^2 \sim O(\epsilon^3)$
- DBI $(P/A)^3 \sim O(\epsilon^3)$
- SGal $(P/A)^4 \sim O(\epsilon^3)$

$$24 = 1 + 1 + 1 + 1$$

Witten-Rainin-Spindlin-Vorlesch (2014)

$$M_{NUP}^{ym}(1, 2, \dots, n) = \int \frac{d^{2n} \Delta}{(GL(2n)) (1, 2, \dots, n)} \prod_{I \in N} \delta^2 \left(\tilde{\lambda}_I + \sum_{j \in P(I)} \tilde{\lambda}_j \right) \prod_{i \in P} \delta^2 \left(\lambda_i - \sum_{j \in M(I)} \lambda_j \right)$$

\downarrow
SLO(2) = GL(2)



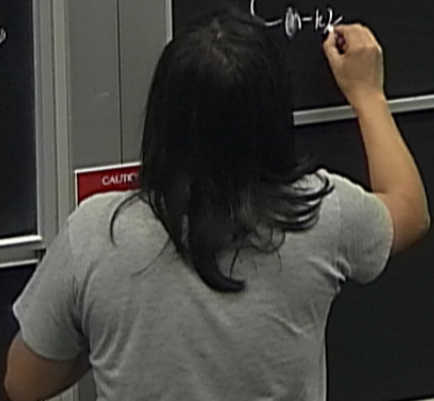
CAUTION
 Das ist ein Laser und kann Ihre Augen schaden.
 Bitte nicht in den Laserstrahl schauen.
 Bitte nicht mit dem Laser spielen.
 Bitte nicht mit dem Laser fotografieren.

$\left(\frac{k_b}{k_a}\right)^2 M_{n-1}^{GR}$
 $k_a \rightarrow \epsilon q^a$
 - NLSM $(P/A)^2 \sim O(\epsilon^3)$
 - DBI $(P/A)^3 \sim O(\epsilon^3)$
 - SGal $(P/A)^4 \sim O(\epsilon^3)$

$24 = (1+1+1+1)$
 Witten-Rosen-Spindler-Vielnich (2004)
 $M_{NUP}^{ym}(1, 2, \dots, n) = \int \frac{d^{2n} \Delta}{(GL(2n)) (1, 2, \dots, n)}$
 $NUP = \{1, 2, \dots, n\}$
 $= \{(1, k)\} = \{(1, 1, \dots, n)\}$
 $C_{k \times \eta^k} \begin{pmatrix} 1 & & & \\ & 0 & \frac{1}{(k+1)} & \dots & \frac{1}{(k+1)} \\ & & & \dots & \\ & 0 & & & \frac{1}{(k+1)} \end{pmatrix}$
 $C_{(n-k)}$

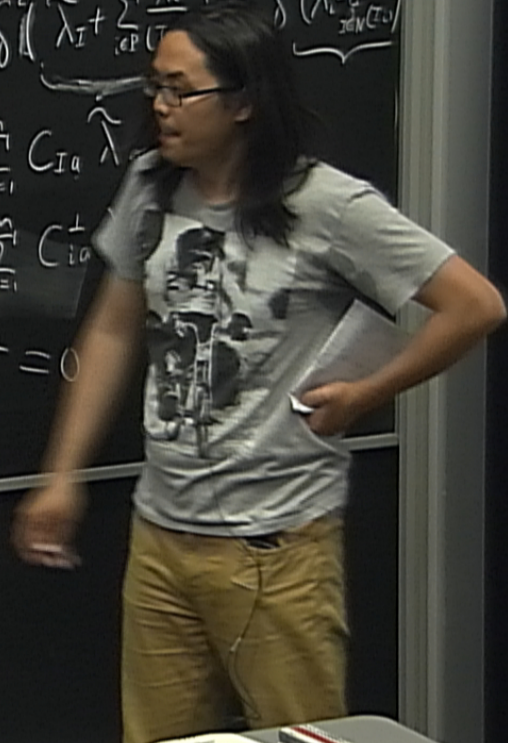
$$\prod_{I \in \mathcal{N}} \int \delta^2 \left(\tilde{\lambda}_I + \sum_{i \in P(I)} \frac{\tilde{\lambda}_i}{(I, i)} \right) \prod_{i \in P} \delta^2 \left(\lambda_i - \sum_{j \in M(I, i)} \frac{\lambda_j}{(I, j)} \right)$$

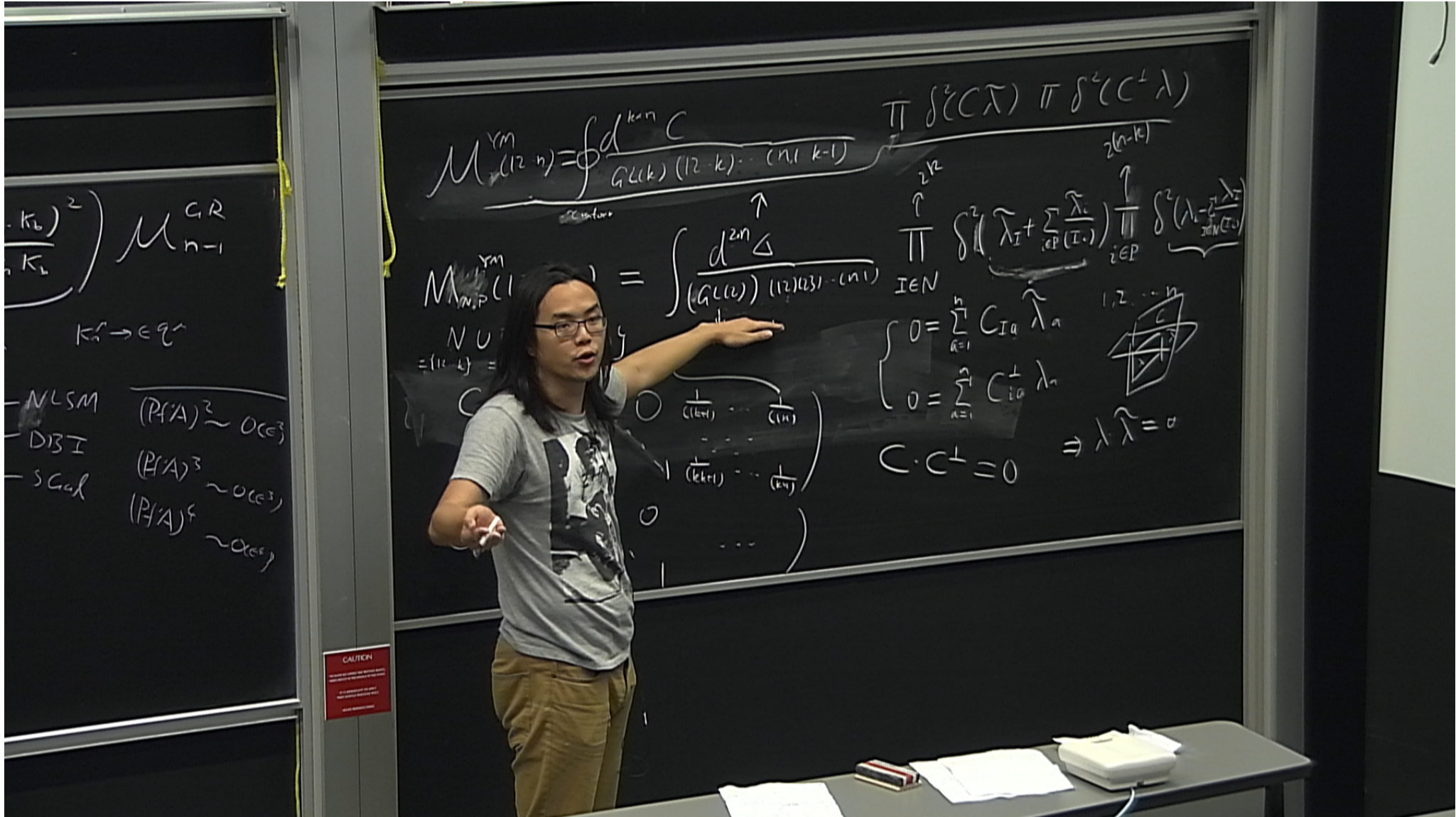
$$\begin{cases} 0 = \sum_{a=1}^n C_{Ia} \tilde{\lambda}_a \\ 0 = \sum_{a=1}^n C_{ia}^\perp \lambda_a \end{cases}$$



$\left(\frac{k_b}{k_a}\right)^2 M_{n-1}^{GR}$
 $k_n^+ \rightarrow \epsilon q^+$
 - NLSM $(P/A)^2 \sim O(\epsilon^3)$
 - DBI $(P/A)^3 \sim O(\epsilon^3)$
 - S Gal $(P/A)^4 \sim O(\epsilon^3)$

$24 = 1 + 1 + 1 + 1$
 Witten-Rosen-Spindler-Vieloch (2004)
 $M_{NUP}^{ym}(1, 2, \dots, n) = \int \frac{d^{2n} \Delta}{(GL(2n)) (1, 2, \dots, n)}$
 $NUP = \{1, 2, \dots, n\}$
 $= \{1, 2, \dots, n\}$
 $C_{k \times \bar{n} \times k} \begin{pmatrix} 1 & 0 & \frac{1}{(k+1)} & \dots & \frac{1}{(k+1)} \\ 0 & \ddots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \frac{1}{(k+1)} & \dots & \frac{1}{(k+1)} \end{pmatrix}$
 $C_{(n-k) \times \bar{n}}^\perp \begin{pmatrix} 1 & 0 & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & \dots \end{pmatrix}$
 $\prod_{I \in N} \int \tilde{\lambda}_I + \sum_{I \in P} \frac{\tilde{\lambda}_I}{\epsilon P(I)} \int \delta^2(\lambda_i - \frac{\tilde{\lambda}_i \lambda_j}{\epsilon P(I)})$
 $0 = \sum_{a=1}^n C_{Ia} \tilde{\lambda}_a$
 $0 = \sum_{a=1}^n C_{Ia}^\perp \tilde{\lambda}_a$
 $C \cdot C^\perp = 0$





$\left(\frac{k_b}{k_a}\right)^2 M_{n-1}^{GR}$
 $k_a \rightarrow e q_a$
 - NLSM $(P/A)^2 \sim O(e^3)$
 - DBI $(P/A)^3 \sim O(e^3)$
 - S Gal $(P/A)^4 \sim O(e^3)$

$$M_{(12 \dots n)}^{YM} = \int \frac{d^{kn} C}{GL(k) (12 \dots k) \dots (n1 \dots k-1)}$$

$$M_{NUP(1)}^{YM} = \int \frac{d^{2n} \Delta}{(GL(2)) (12 \dots 231) \dots (n1)}$$

$$\prod_{I \in \mathcal{N}} \delta^2 \left(\tilde{\lambda}_I + \sum_{i \in P(I)} \frac{\tilde{\lambda}_i}{i \cdot I} \right) \prod_{i \in P} \delta^2 \left(\lambda_i - \sum_{j \in N(I)} \frac{\lambda_j}{j \cdot I} \right)$$

$$\begin{cases} 0 = \sum_{a=1}^n C_{Ia} \tilde{\lambda}_a \\ 0 = \sum_{a=1}^n C_{ia}^\perp \lambda_a \end{cases}$$

$$C \cdot C^\perp = 0 \Rightarrow \lambda \cdot \tilde{\lambda} = 0$$

$\begin{pmatrix} \frac{1}{(k+1)} & & & \\ & \dots & & \\ & & \frac{1}{(k+1)} & \\ & & & \dots \end{pmatrix}$



CAUTION