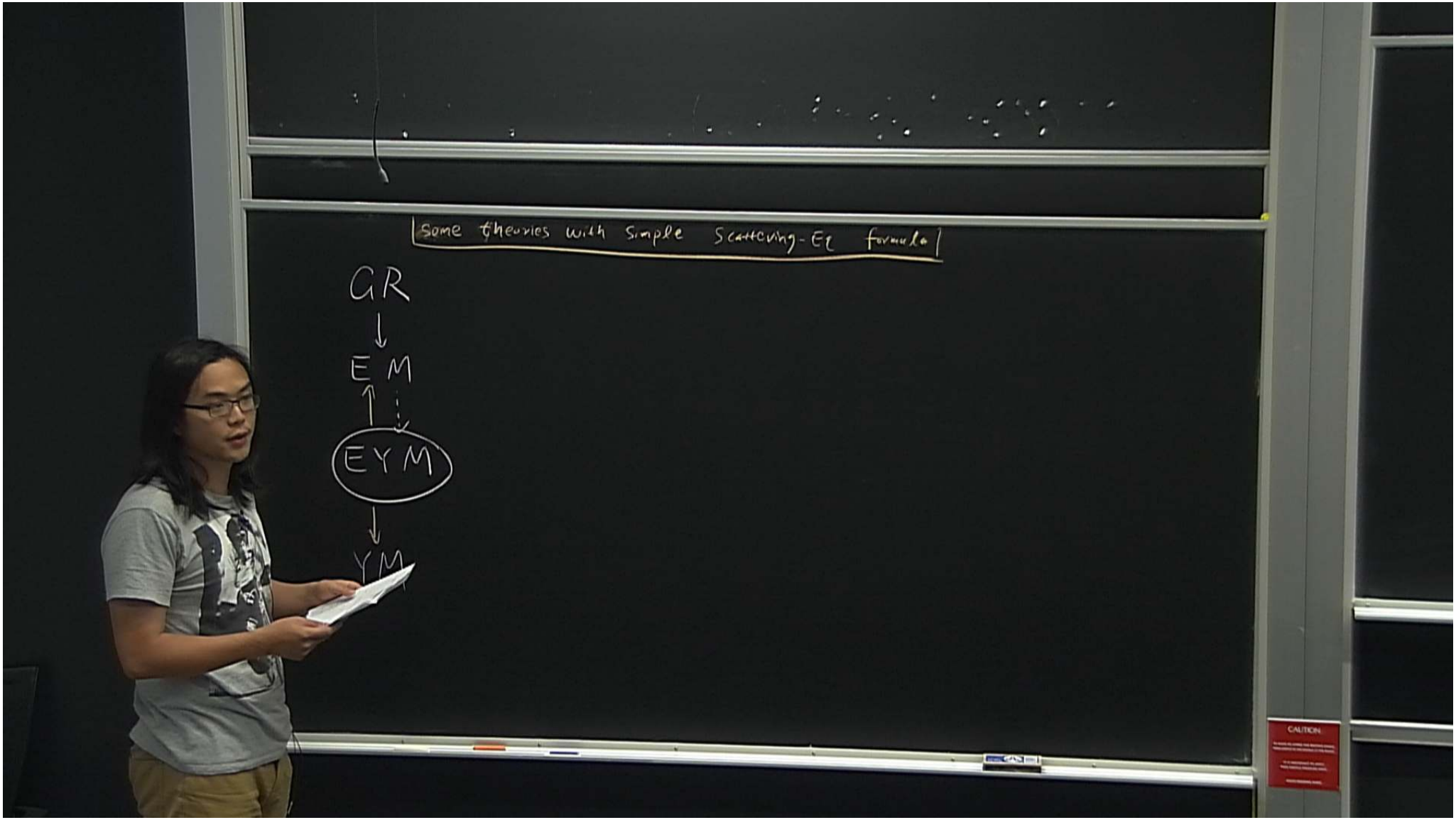
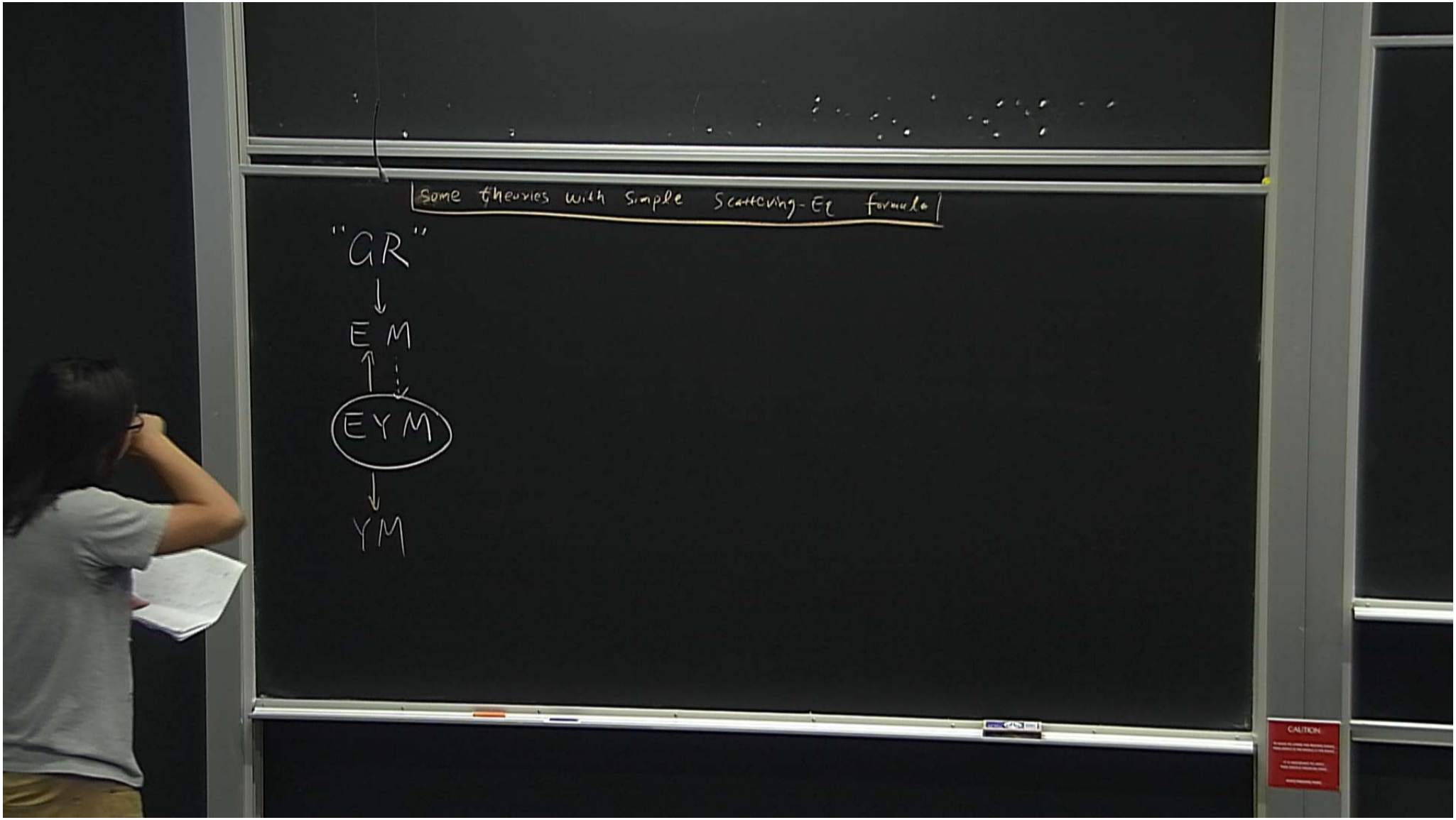
Title: Amplitudes: Applications

Date: Jul 17, 2015 09:00 AM

URL: http://pirsa.org/15070056

Abstract:


Some theories with simple Scattering-Eq formula
GR
E M
EYM
YM
CAUTION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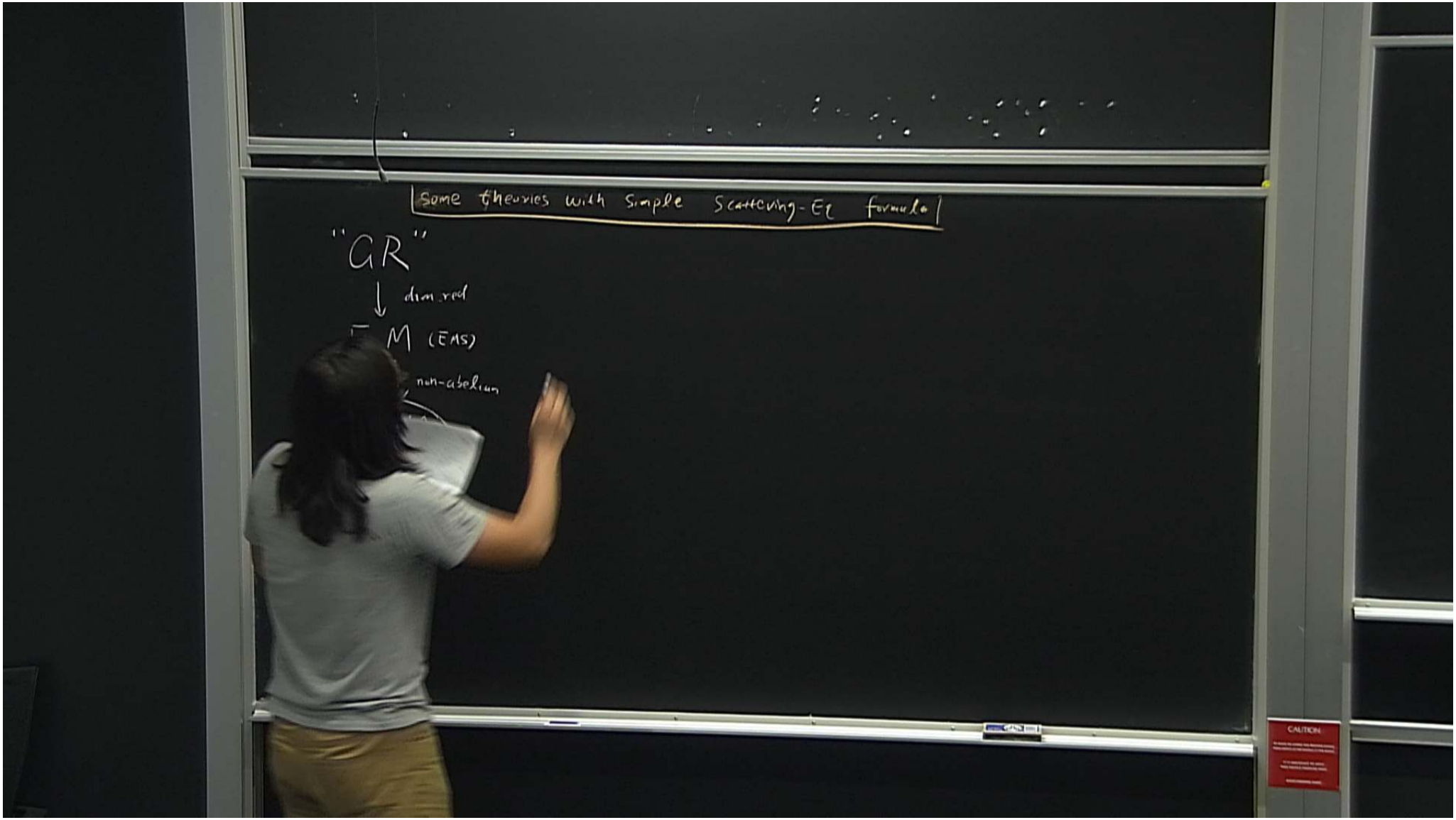
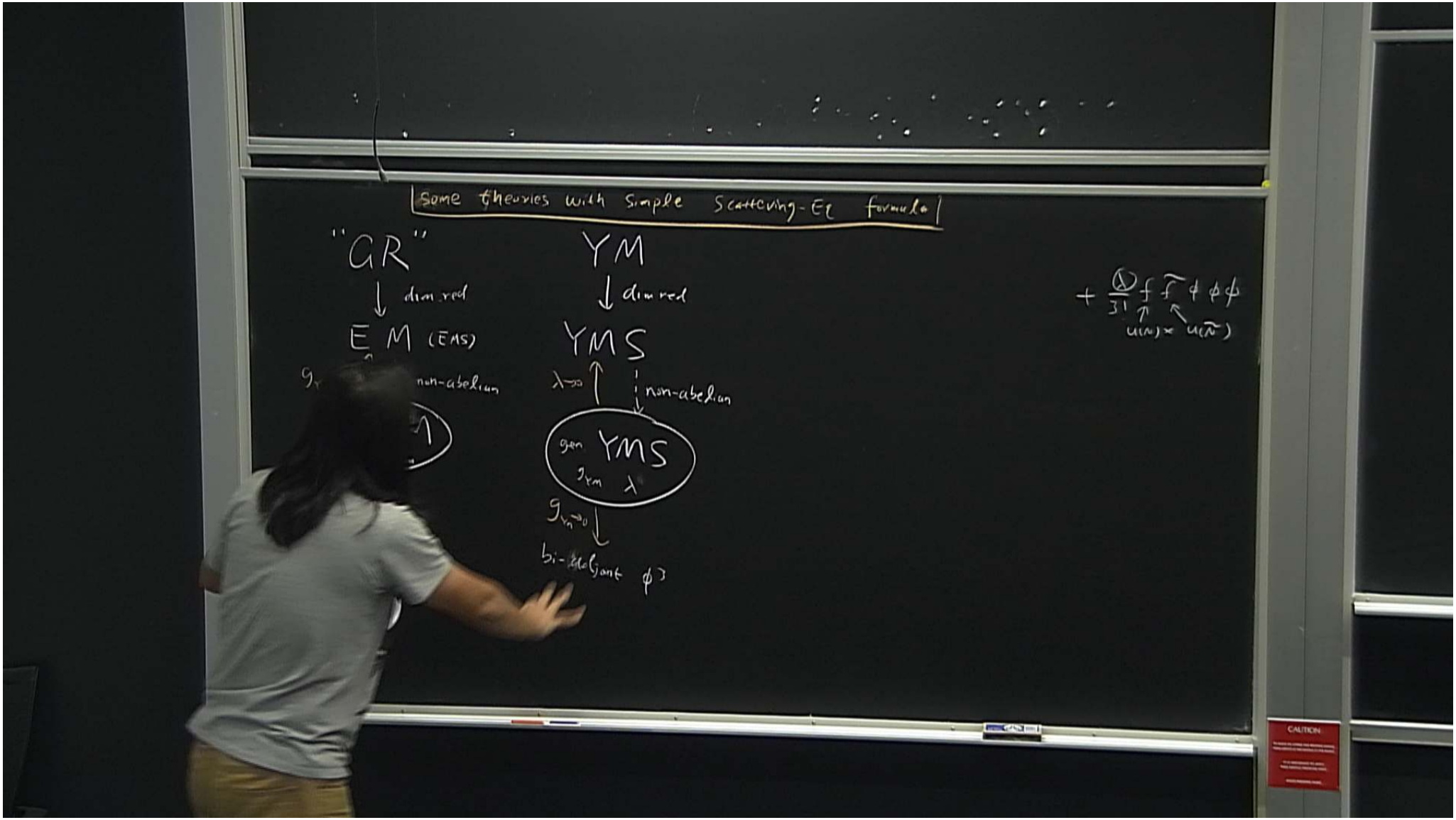
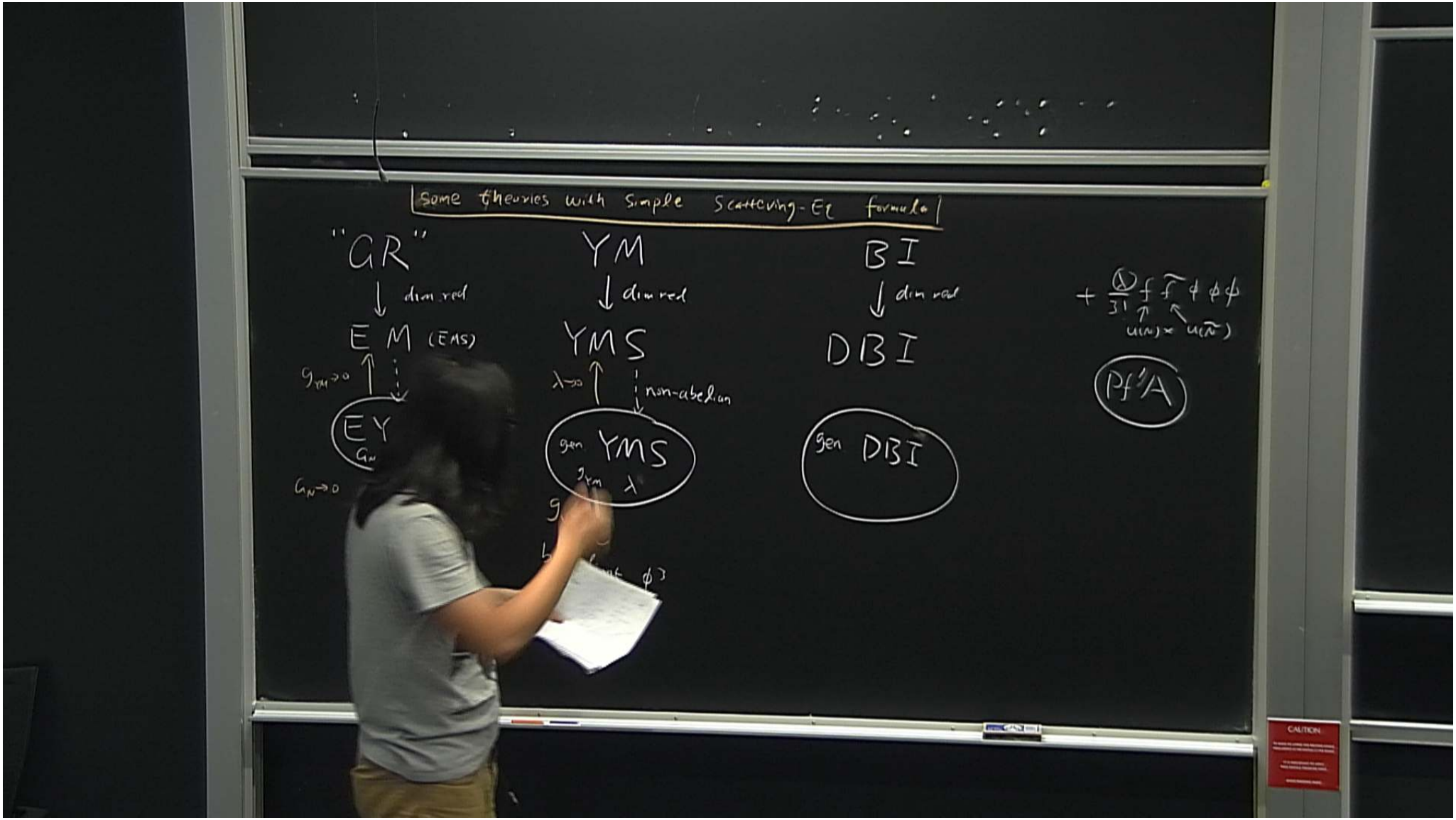
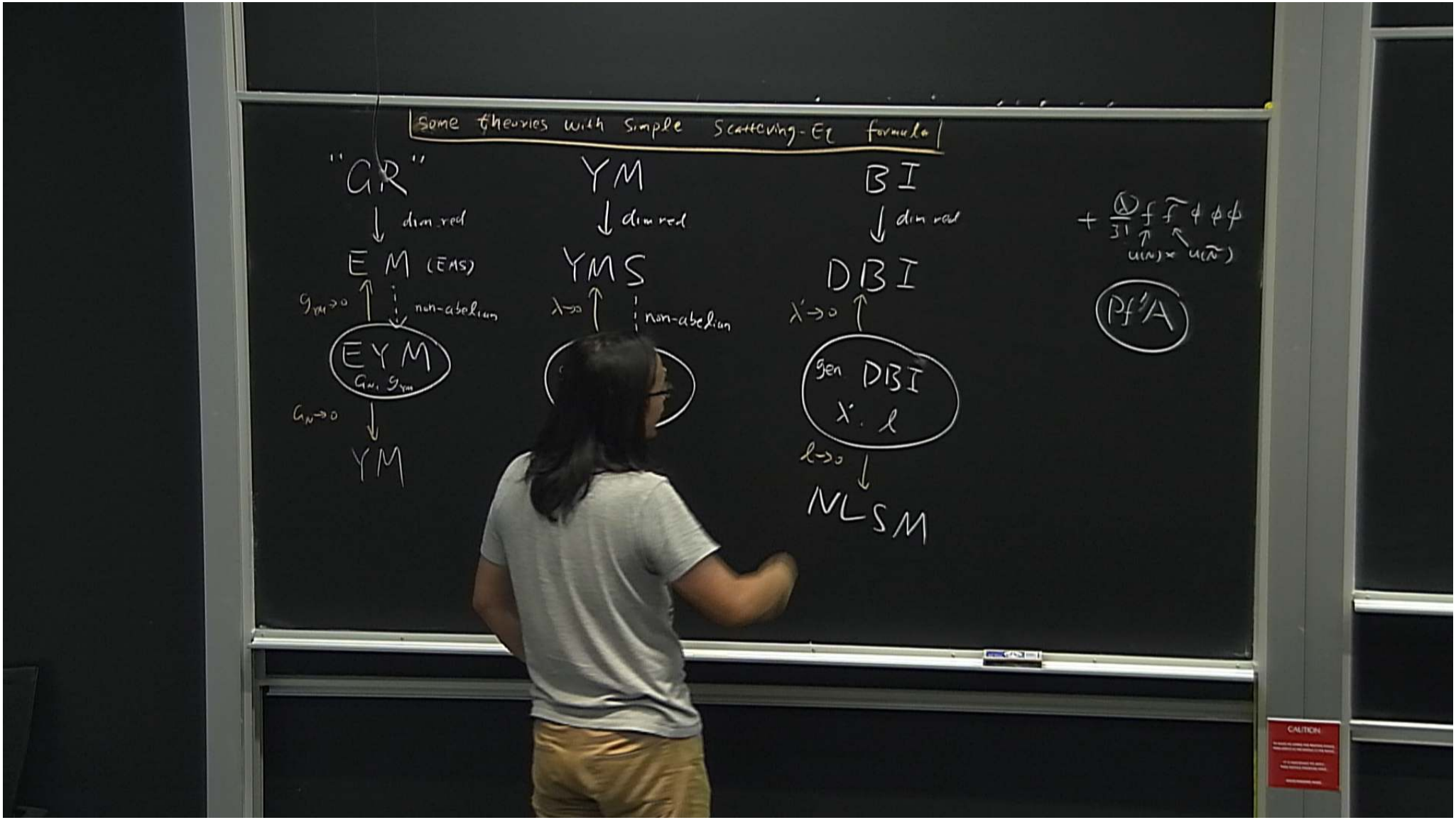
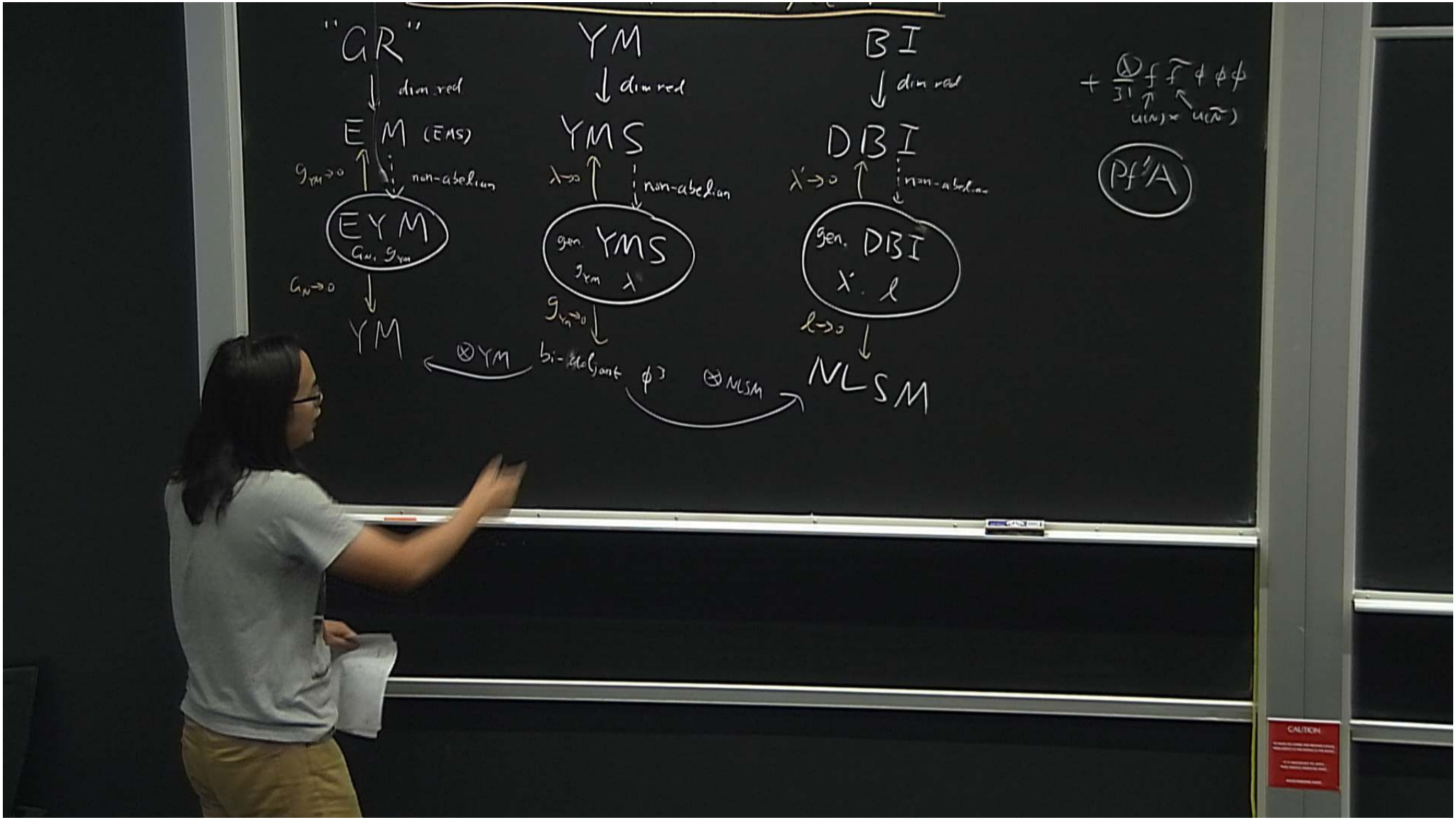
"GR"
E M
EYM
YM

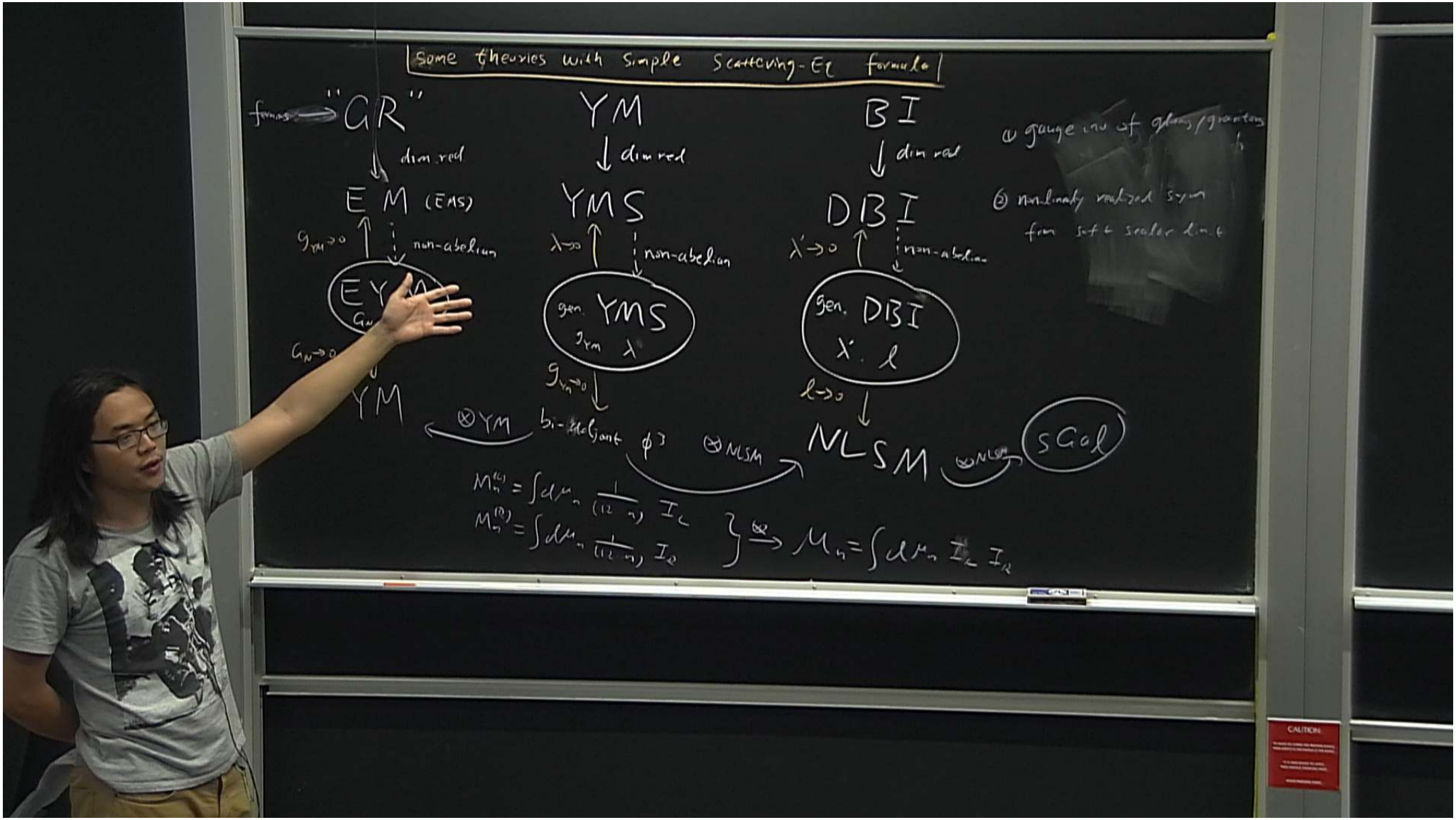

Some theories with simple Scattering-Eq formula
"GR"
dim_red
M (EMS)
non-abelian
CAUTION


Some theories with simple Scattering-Eq formula
"GR"
YM
dim red
dim red
EM (EMS)
YMS
non-abelian
λ→∞
non-abelian
gen YMS
g_{YM} λ
$g_{YM}\to 0$
bi-adjoint ϕ3
$+\frac{1}{3!}f\tilde{f}\phi\phi\phi$
U(N) × U(Ñ)
CAUTION


Some theories with simple Scattering-Eq formula
"GR"
YM
BI
dim red
dim red
dim red
EM (EMS)
YMS
DBI
EY
gen YMS
gen DBI
non-abelian
Pf'A


Some theories with simple Scattering-Eq formula
"GR"
YM
BI
dim red
dim red
dim red
EM (EMS)
YMS
DBI
non-abelian
non-abelian
EYM
gen DBI
NLSM
YM
Pf'A


"GR"
YM
BI
dim red
dim red
dim red
EM (EMS)
YMS
DBI
non-abelian
non-abelian
non-abelian
EYM
gen. YMS
gen. DBI
YM
NLSM
⊗YM
⊗NLSM
Pf'A


Some theories with simple Scattering-Eq formula
"GR"
YM
BI
dim red
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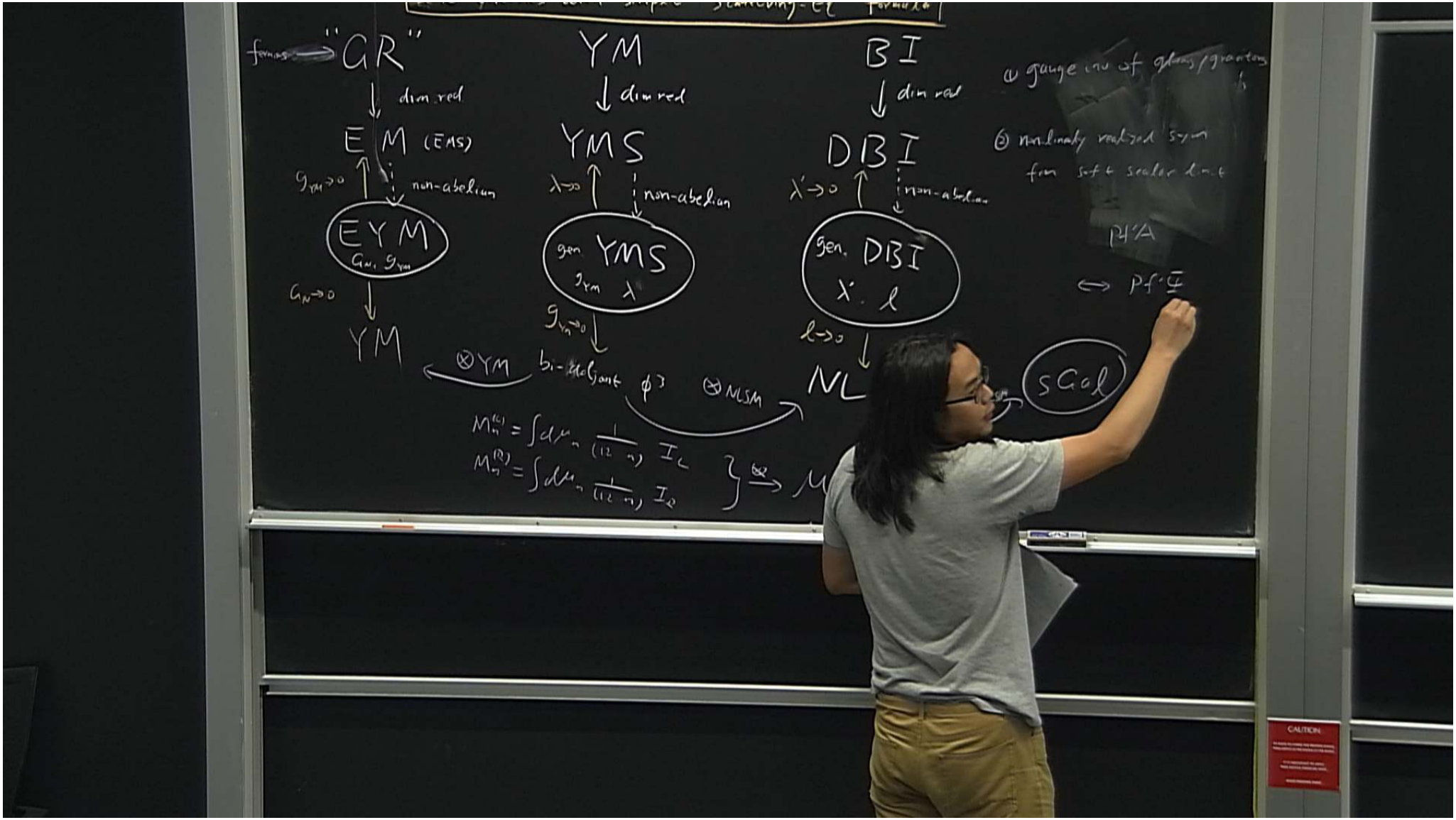
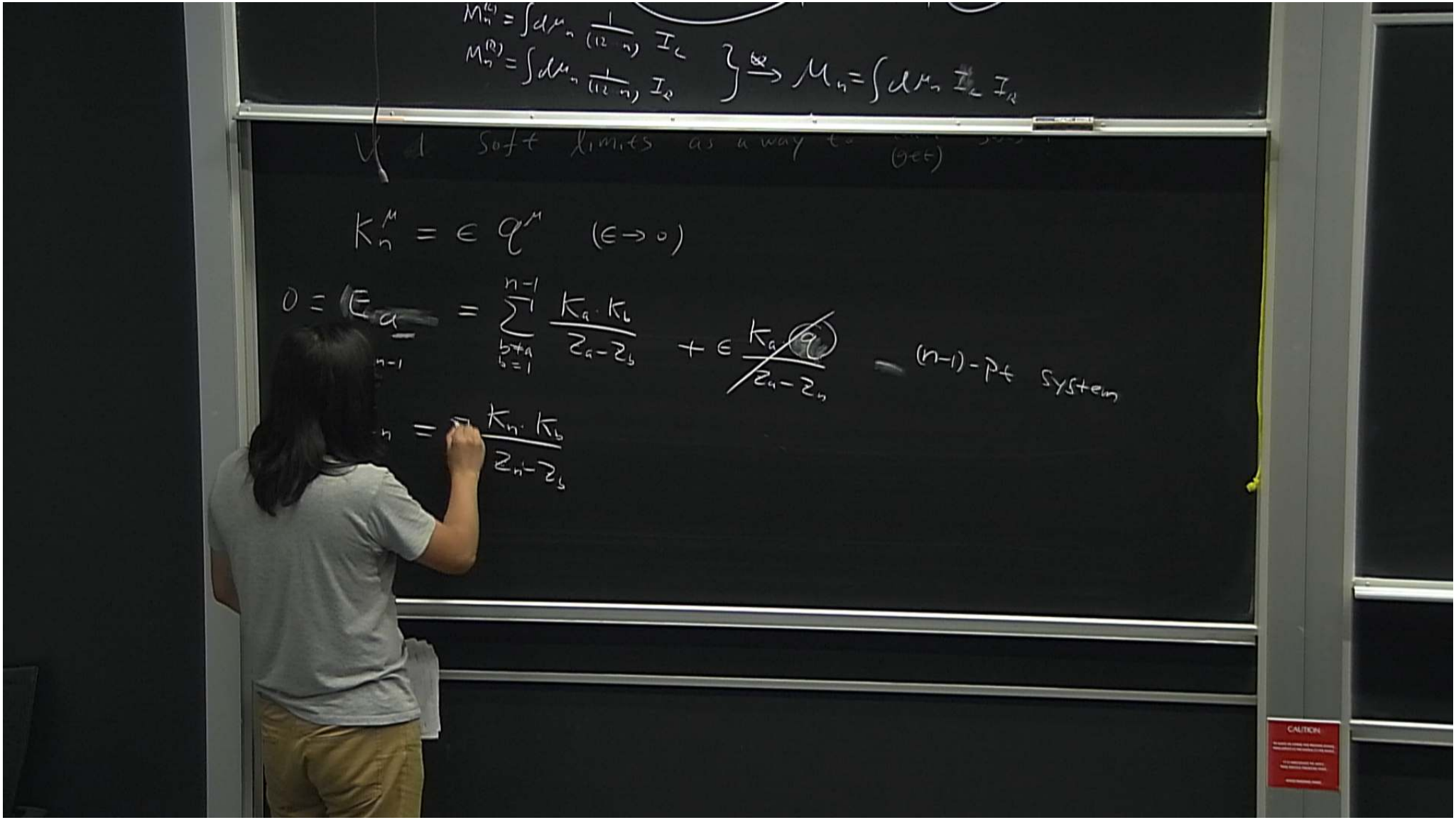
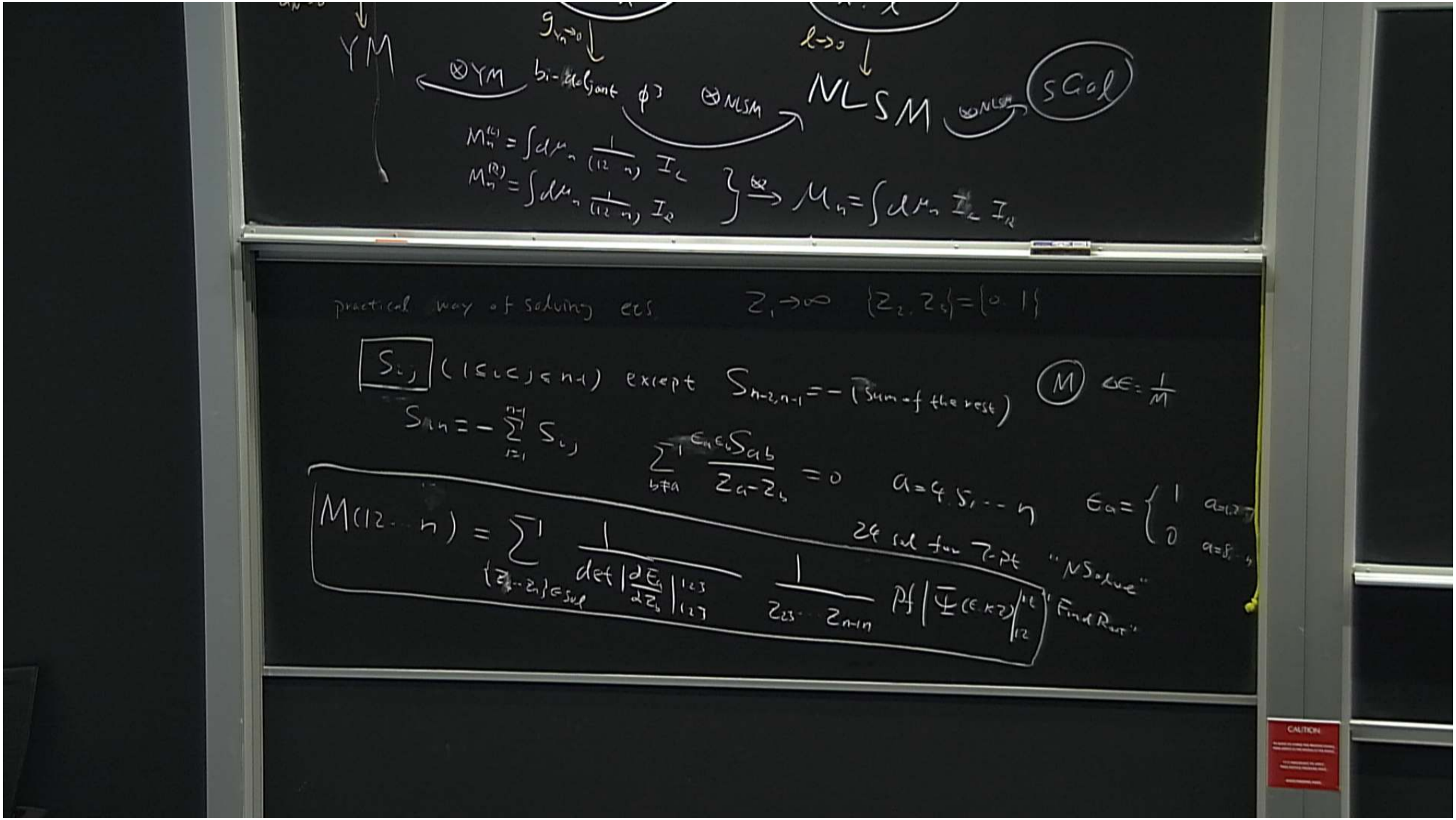
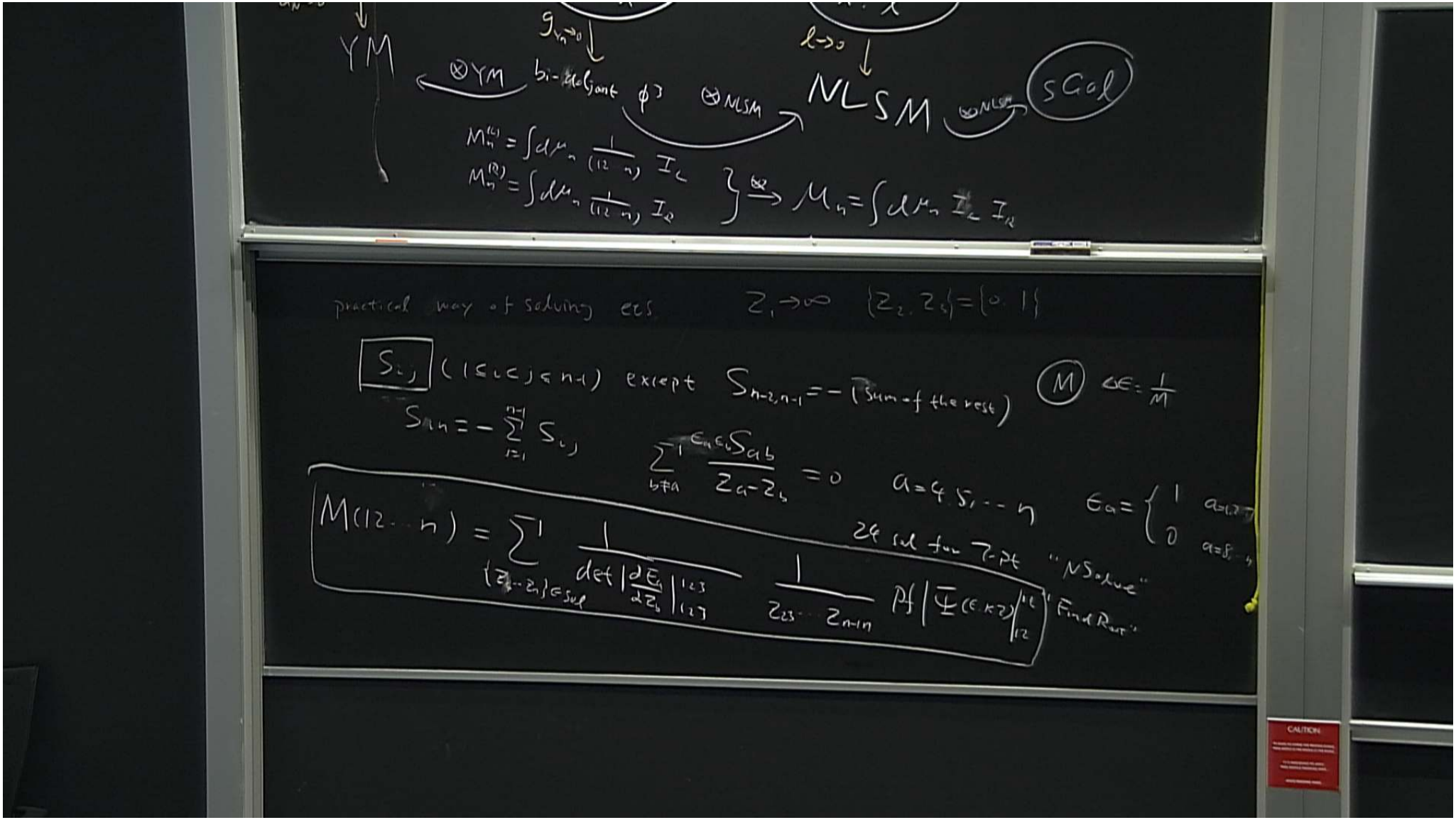
dim red
EM (EMS)
YMS
DBI
non-abelian
non-abelian
non-abelian
gen. YMS
gen. DBI
YM
NLSM
sGal
⊗YM
bi-adjoint
ϕ³
⊗NLSM
⊗NLSM
gauge inv of gluons/gravitons
non-linearly realized sym
from soft + scalar limit
CAUTION


"GR"
YM
BI
dim red
dim red
dim red
EM (EMS)
YMS
DBI
non-abelian
non-abelian
non-abelian
EYM
gen. YMS
gen. DBI
YM
NL
sGal
⊗YM
⊗NLSM
bi-adjoint
Pf·Ψ


$M_n^{(1)} = \int d\mu_n \frac{1}{(12\ldots n)} I_L$
$M_n^{(2)} = \int d\mu_n \frac{1}{(12\ldots n)} I_R$
$\Rightarrow M_n = \int d\mu_n I_L I_R$
soft limits as a way to
$K_n^\mu = \epsilon q^\mu \quad (\epsilon \to 0)$
$0 = E_q = \sum_{\substack{b=1 \\ b \neq a}}^{n-1} \frac{K_a \cdot K_b}{z_a - z_b} + \epsilon \frac{K_a \cdot q}{z_a - z_n}$
$(n-1)$-pt system
$= \frac{K_n \cdot K_b}{z_n - z_b}$
CAUTION


YM
NLSM
scal
bi-adjoint ϕ^3
$M_n^{(L)} = \int d\mu_n \frac{1}{(12\cdots n)} I_L$
$M_n^{(R)} = \int d\mu_n \frac{1}{(12\cdots n)} I_R$
$\Rightarrow M_n = \int d\mu_n I_L I_R$
practical way of solving eqs
$z_1 \to \infty$ $\{z_2, z_3\} = \{0, 1\}$
S_{ij} $(1 \le i < j \le n-1)$ except $S_{n-2,n-1} = -$ (sum of the rest)
$S_{in} = -\sum_{i=1}^{n-1} S_{ij}$
$\sum_{b \neq a}^{'} \frac{\epsilon_a \epsilon_b S_{ab}}{z_a - z_b} = 0$ $a = 4, 5, \cdots n$
$\epsilon_a = \begin{cases} 1 & a = 2, 3 \\ 0 & a = 5, \cdots n \end{cases}$
$M(12\cdots n) = \sum' \frac{1}{\det \left| \frac{\partial E_i}{\partial z_j} \right|_{123}^{123}} \frac{1}{z_{23} \cdots z_{n-1n}} \mathrm{Pf} \left| \Psi(\epsilon, k, z) \right|_{12}^{12}$
$\{z_4 \cdots z_n\} \in$ sol
24 sol for 7-pt
"NSolve"
"FindRoot"


YM
NLSM
scal
practical way of solving eqs
$z_1 \to \infty$ $\{z_2, z_3\} = \{0, 1\}$
S_{ij} $(1 \le i < j \le n-1)$ except $S_{n-2,n-1} = -$ (sum of the rest)
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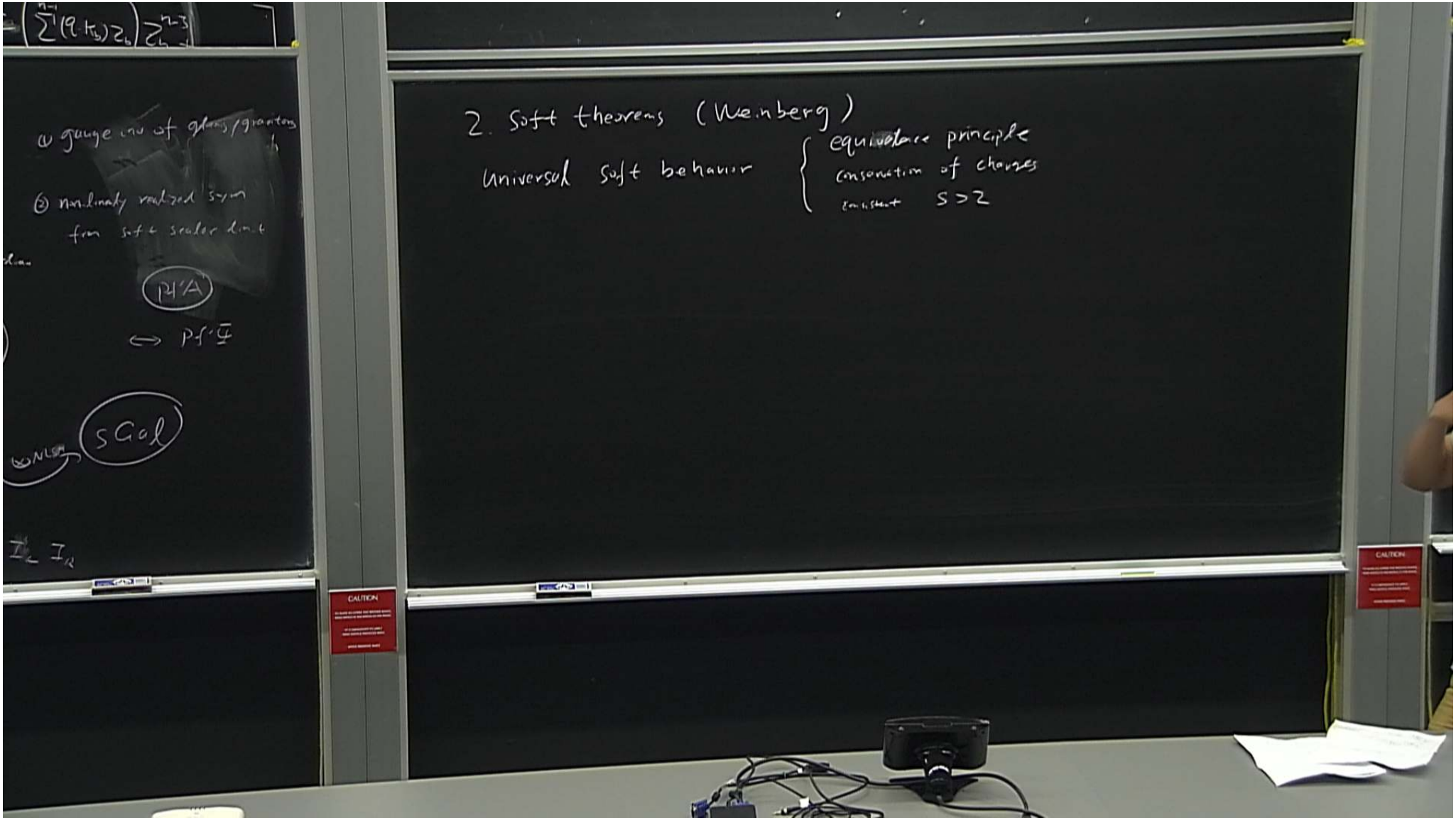
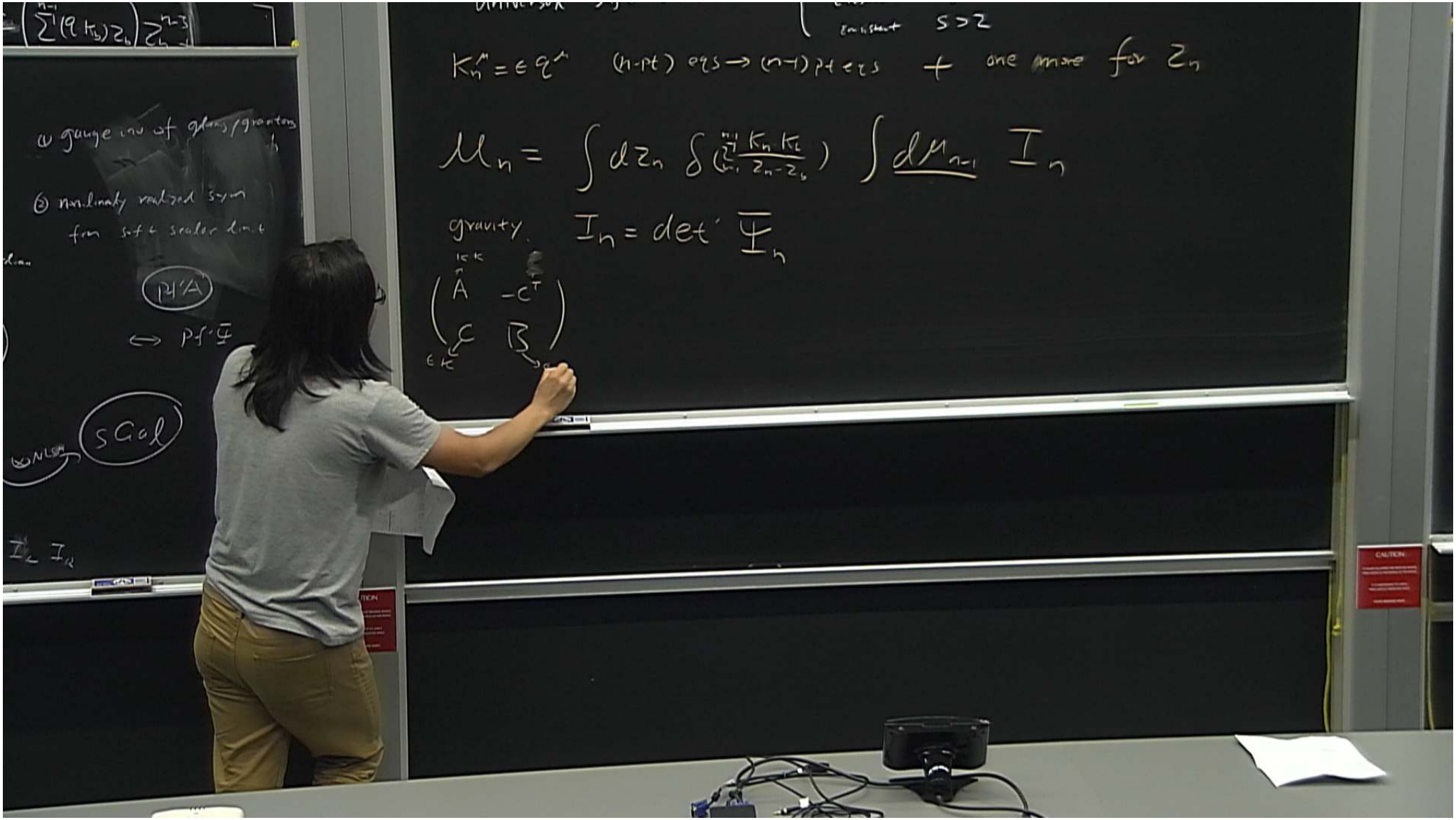
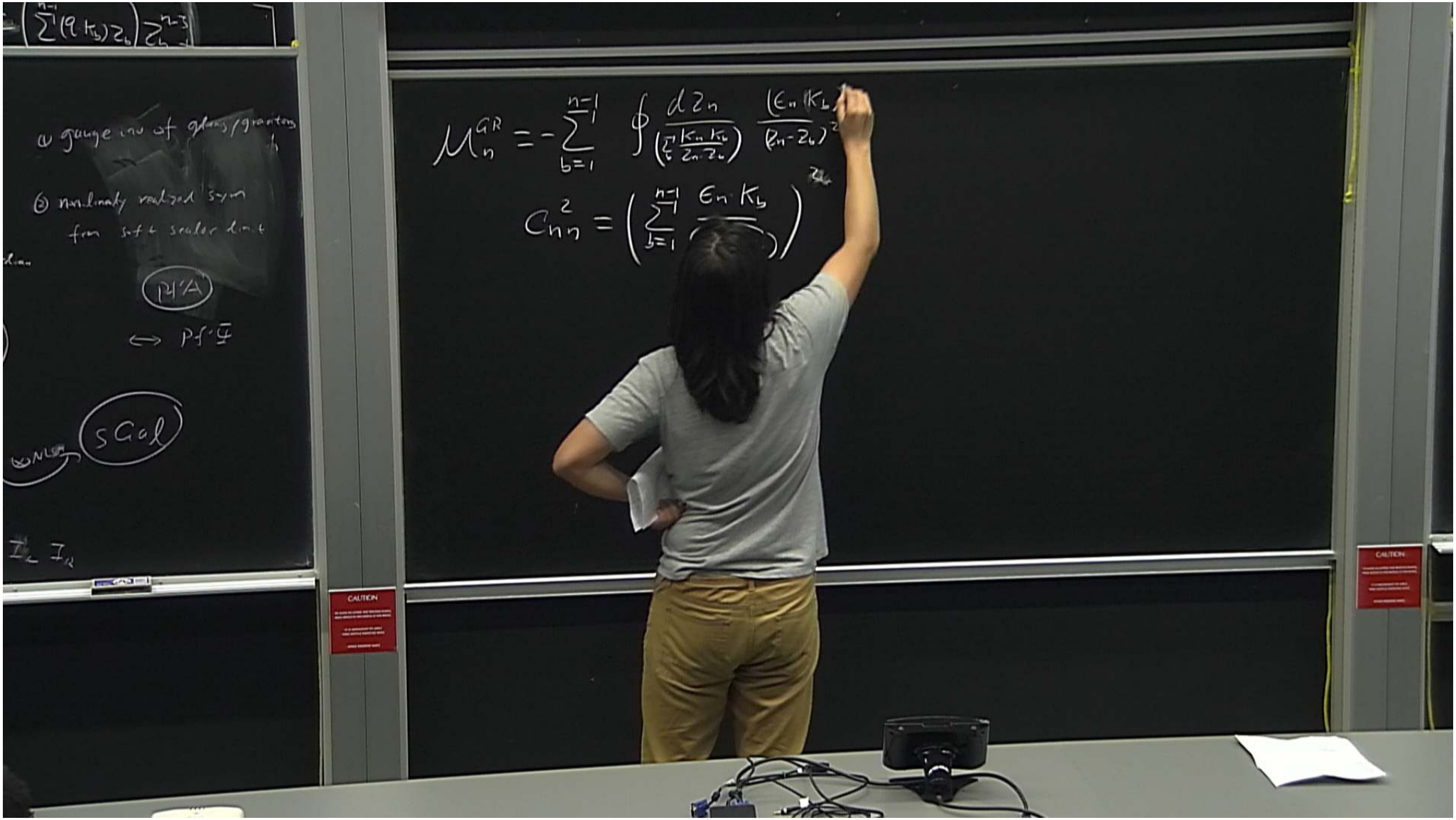
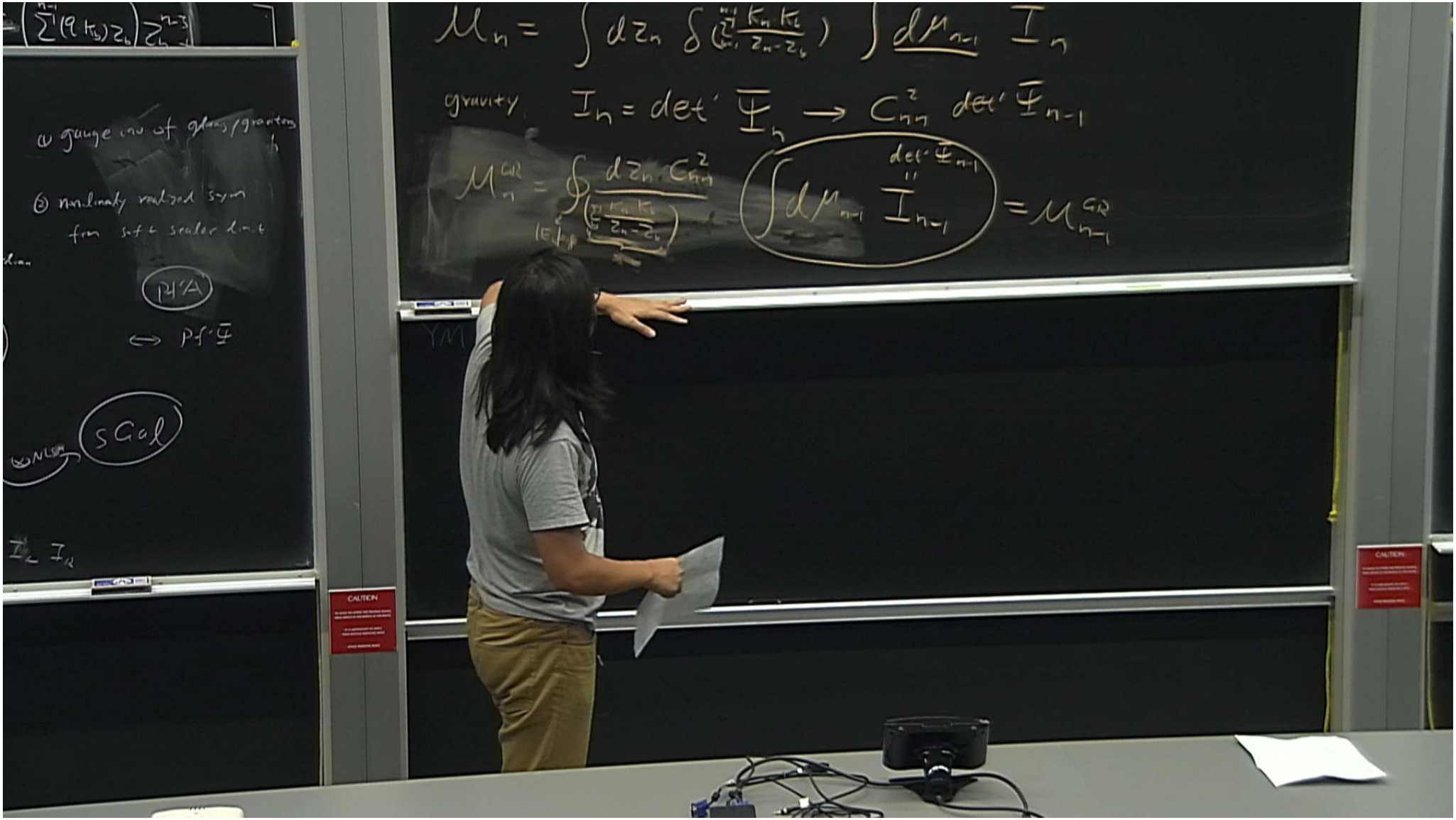
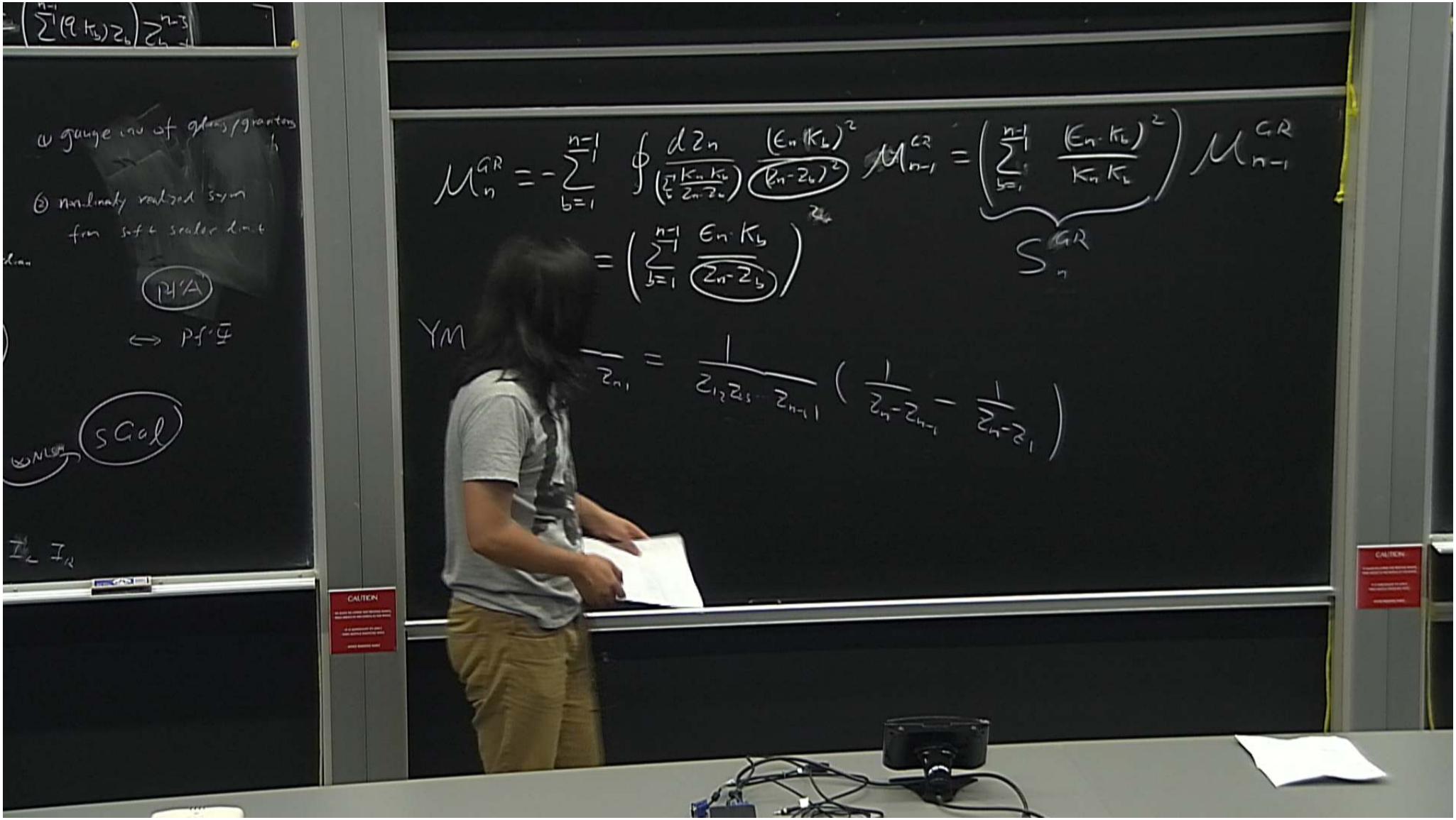
$M(12\cdots n) = \sum' \frac{1}{\det \left| \frac{\partial E_i}{\partial z_b} \right|_{123}^{127}} \frac{1}{z_{23} \cdots z_{n-1n}} \mathrm{Pf} \left| \Psi(\epsilon, k, z) \right|_{12}^{12}$
24 sol for 7-pt
"NSolve"
"Find Root"

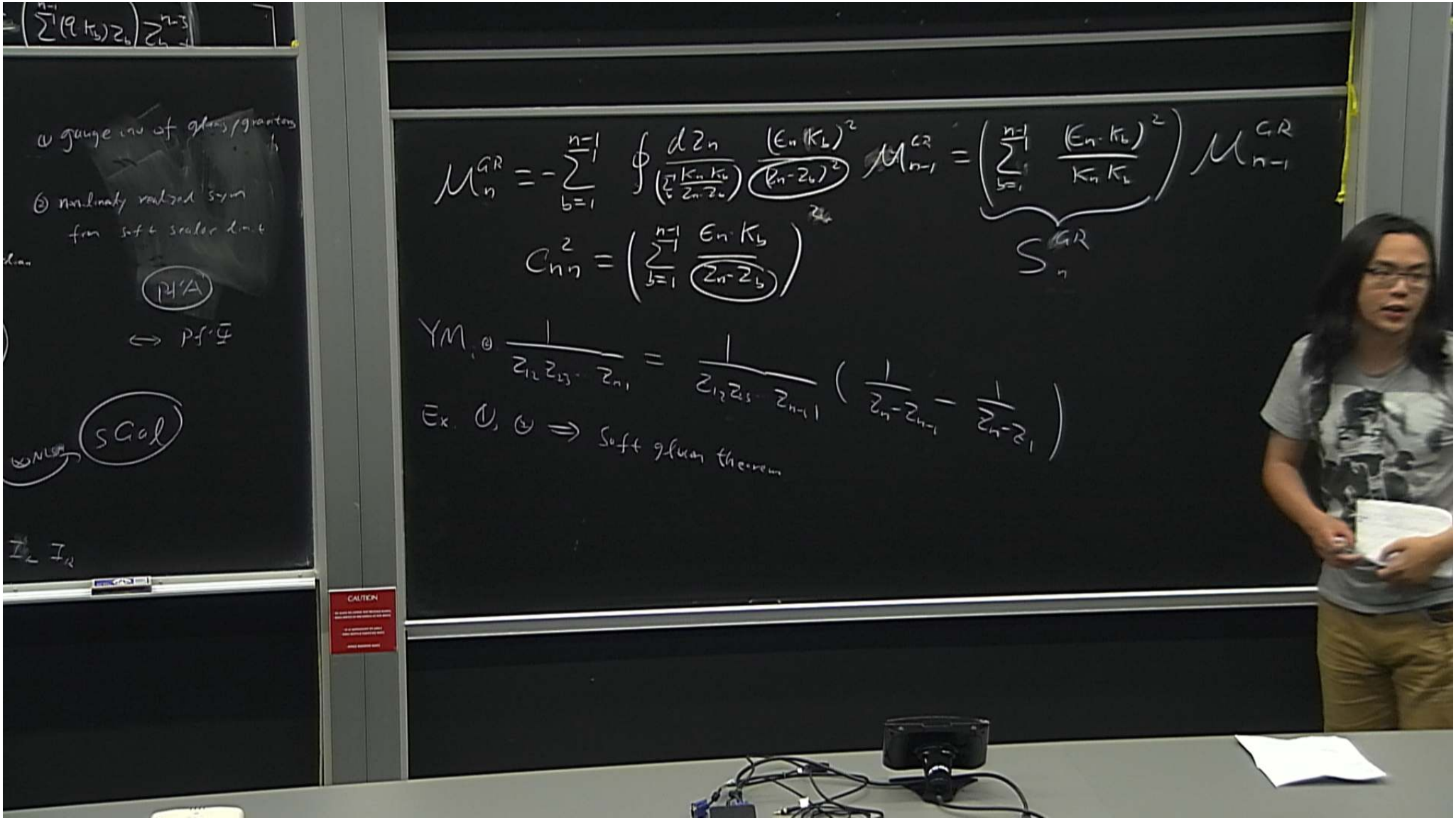

2. Soft theorems (Weinberg)
Universal Soft behavior
equivalence principle
conservation of charges
consistent S ≥ 2
gauge inv of gluons/gravitons
non-linearly realized sym
from soft scalar limit
susy
Pf. Th
sGal
NLSM
CAUTION

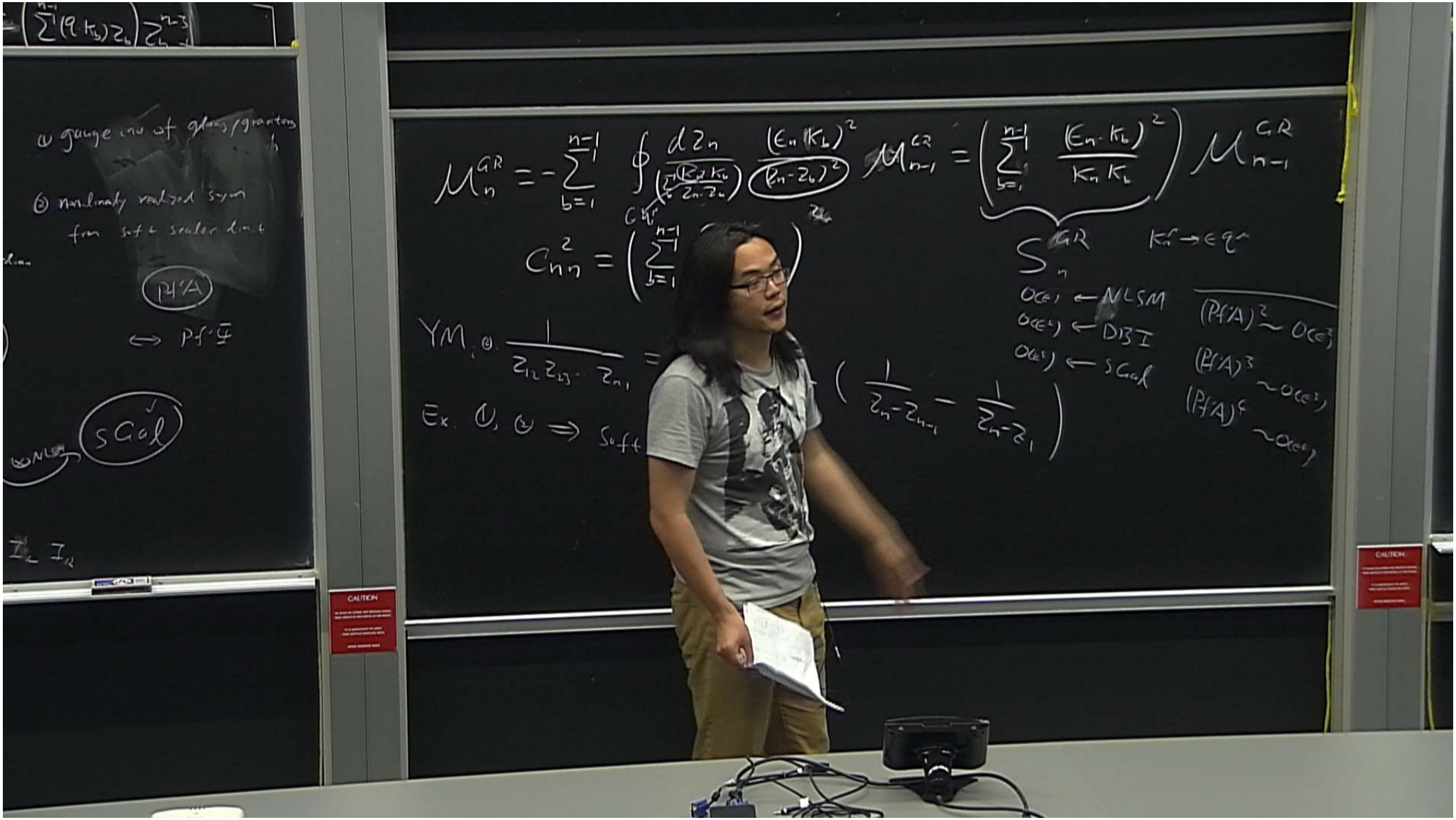

gauge inv of gluons/gravitons
nonlinearly realized sym
from soft & scaler limit
sCal
gravity
det' Ψ_n

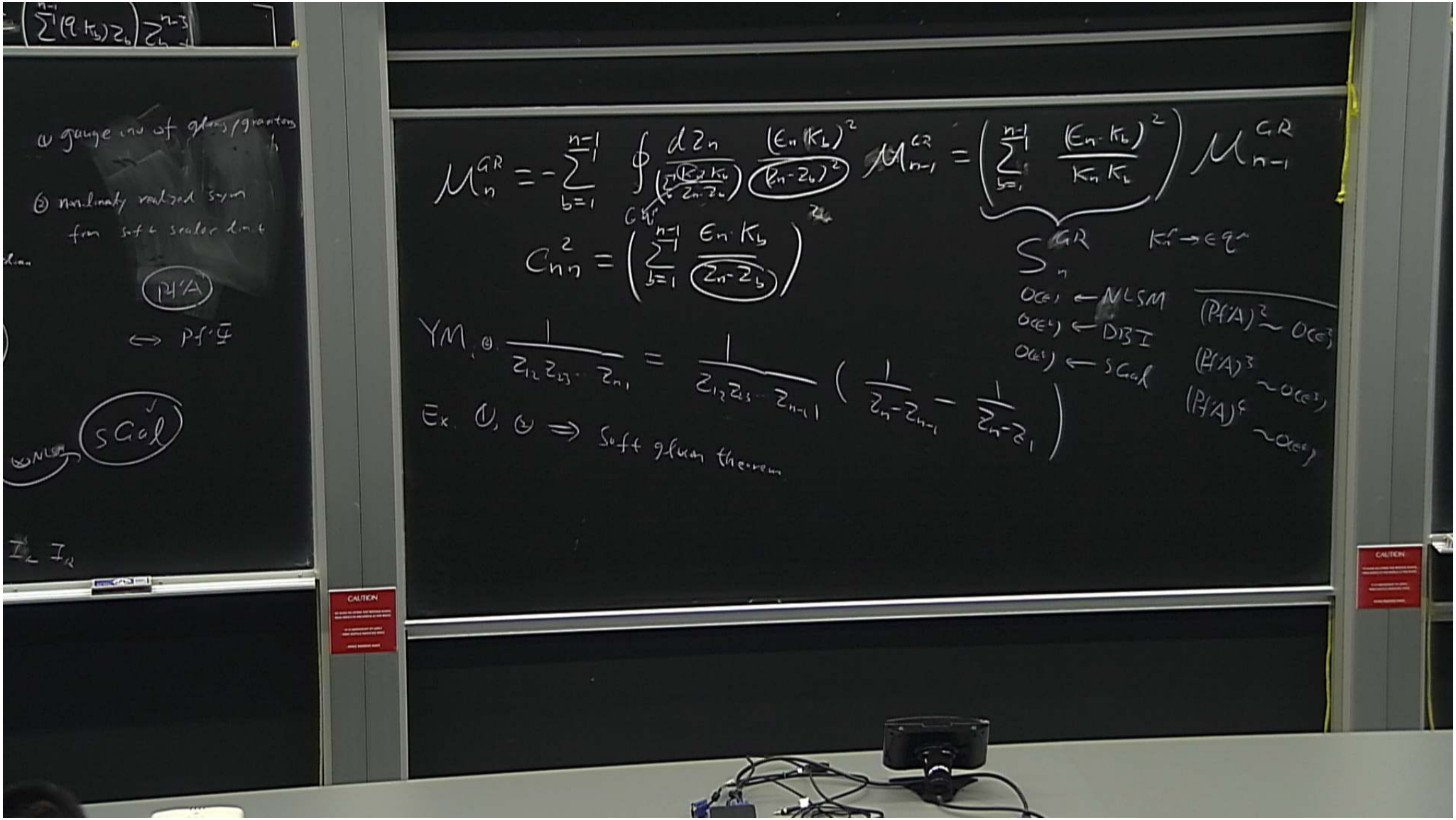

gauge inv of gluons/gravitons
nonlinearly realized sym
from soft & scaler limit
scal
CAUTION

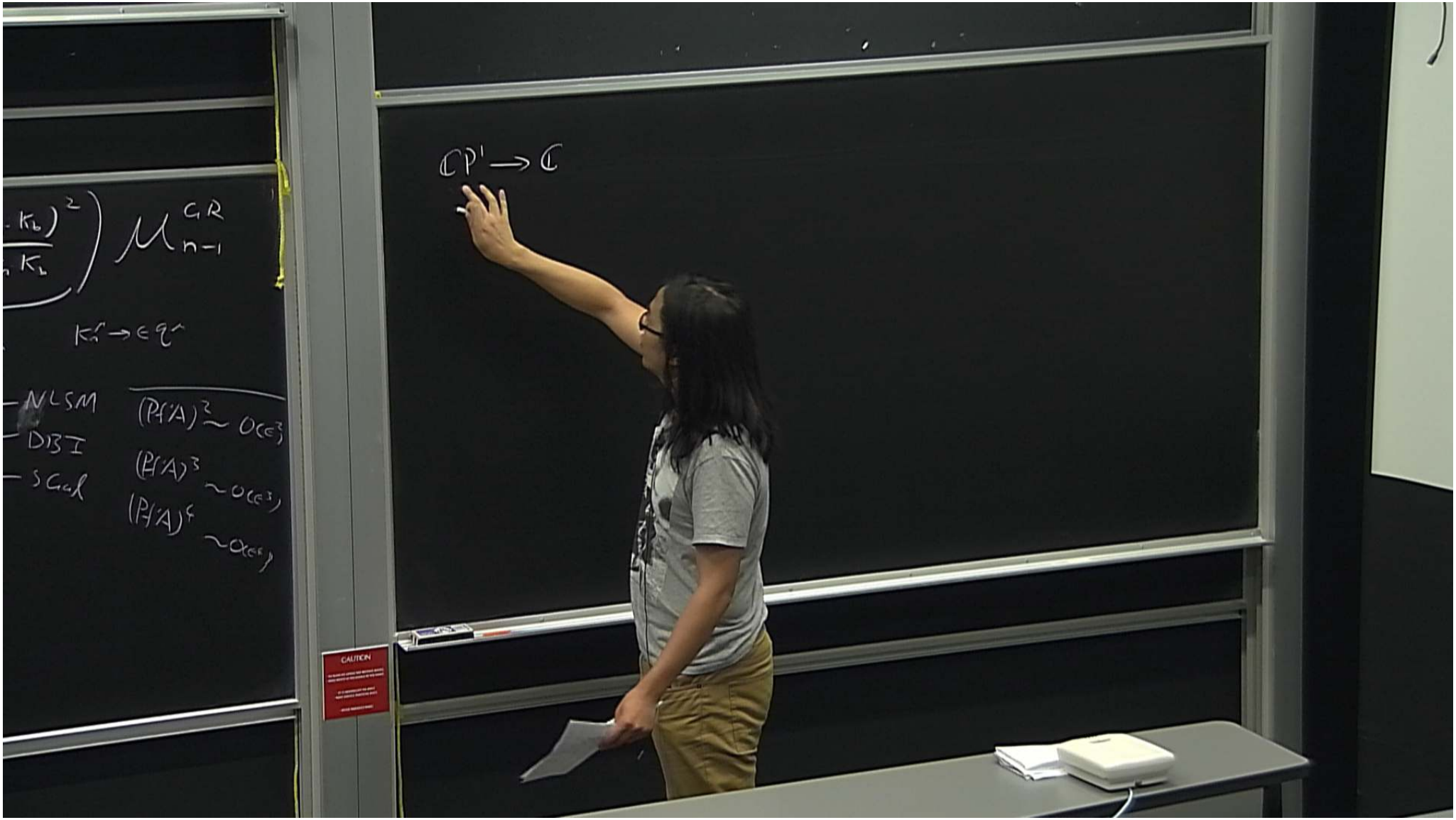

gravity
CAUTION
CAUTION

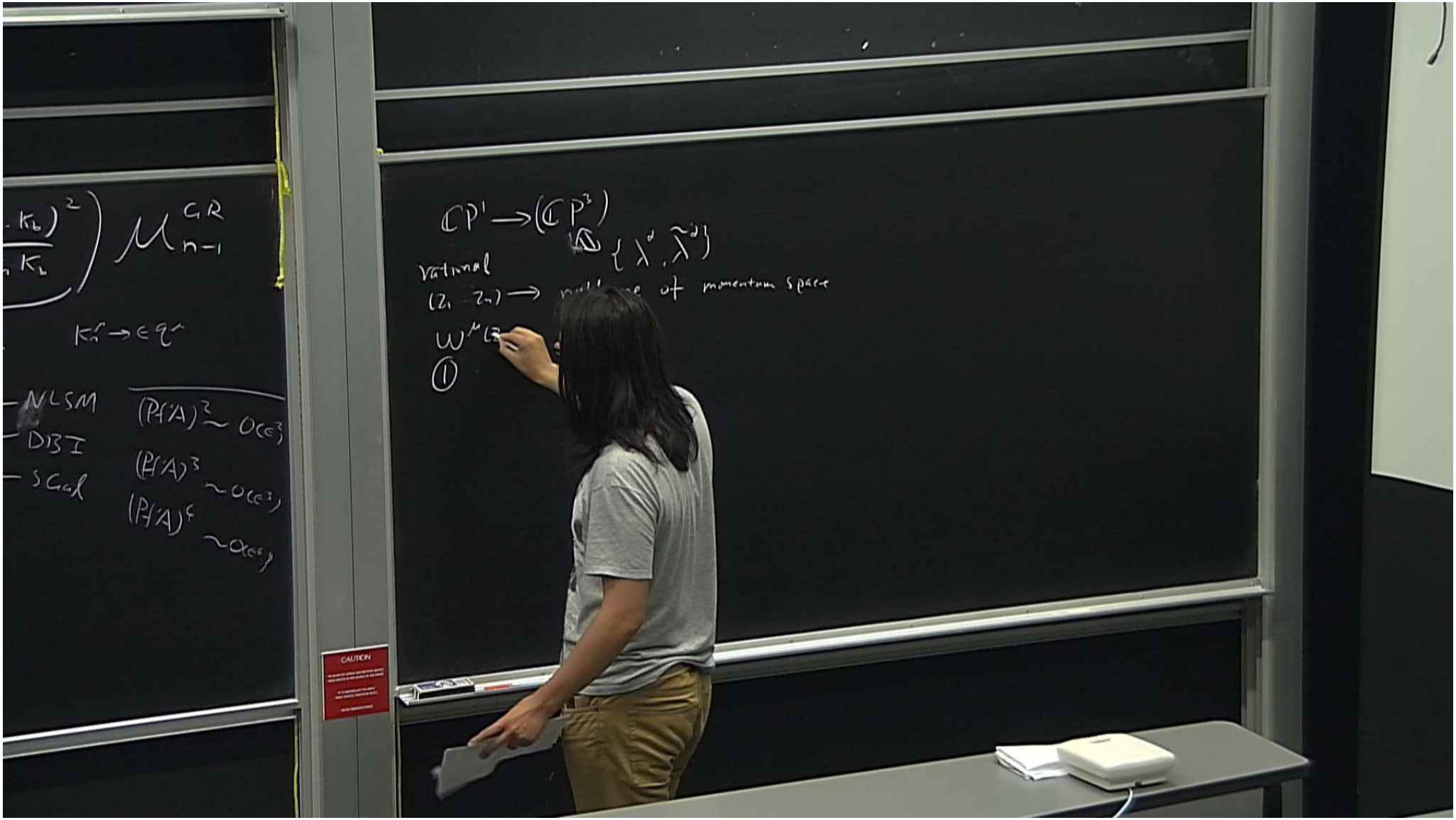

gauge inv of gluons/gravitons
nonlinearly realized sym
from soft & scaler limit
N=4 SYM
⟺ Pf. Ψ
sGal
NLSM
$\mathcal{M}_n^{GR} = -\sum_{b=1}^{n-1} \oint \frac{dz_n}{(\sum \frac{k_n k_b}{z_n - z_b})} \frac{(\epsilon_n k_b)^2}{(z_n - z_b)^2}$
$\mathcal{M}_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(\epsilon_n k_b)^2}{k_n k_b}\right) \mathcal{M}_{n-1}^{GR}$
$= \left(\sum_{b=1}^{n-1} \frac{\epsilon_n k_b}{z_n - z_b}\right)^2$
S_n^{GR}
YM
$\frac{1}{z_{n1}} = \frac{1}{z_{12} z_{23} \cdots z_{n-1,1}} \left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1}\right)$
CAUTION

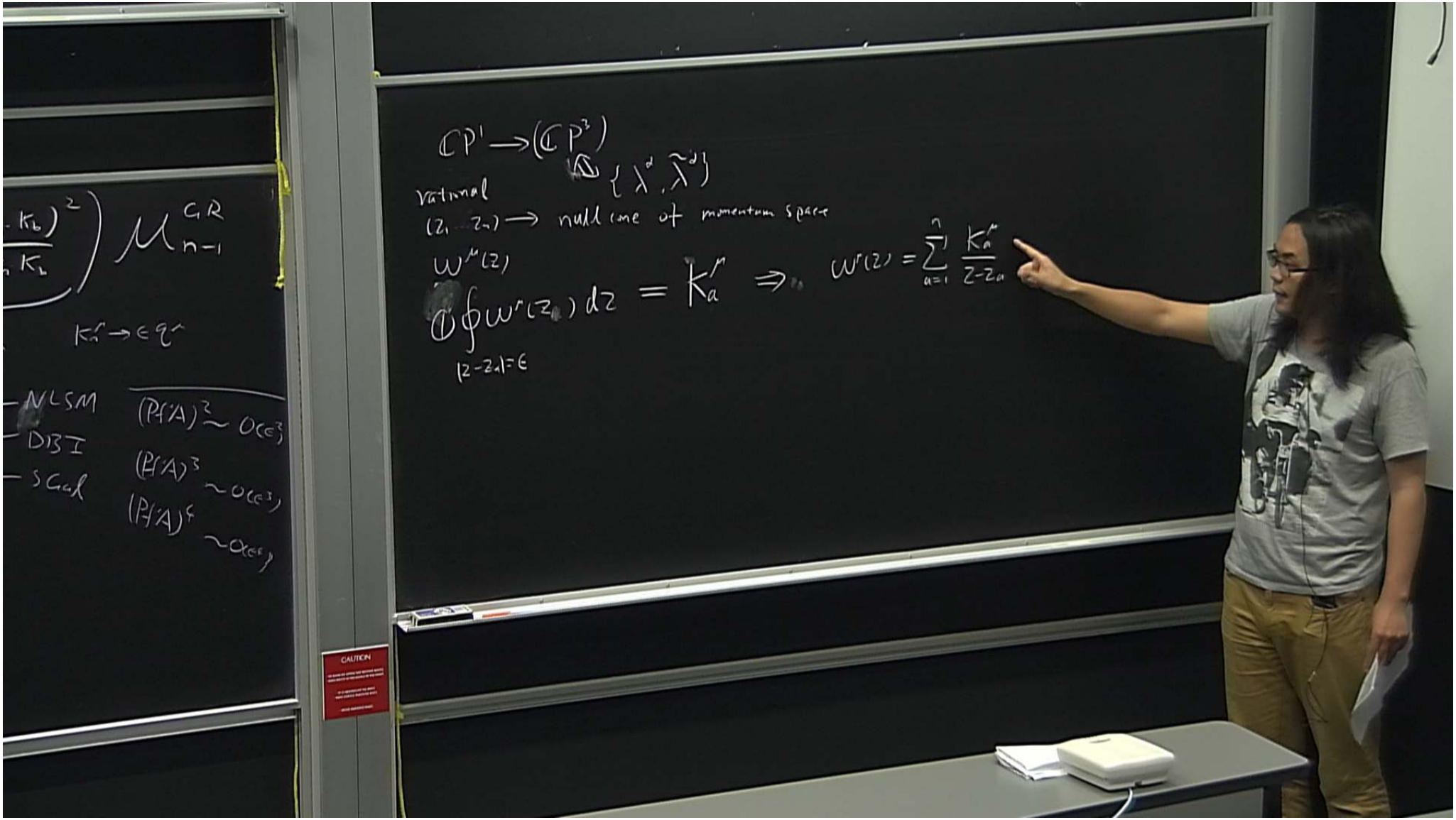

gauge inv of gluons/gravitons
Ex. ①, ② ⇒ Soft gluon theorem
CAUTION

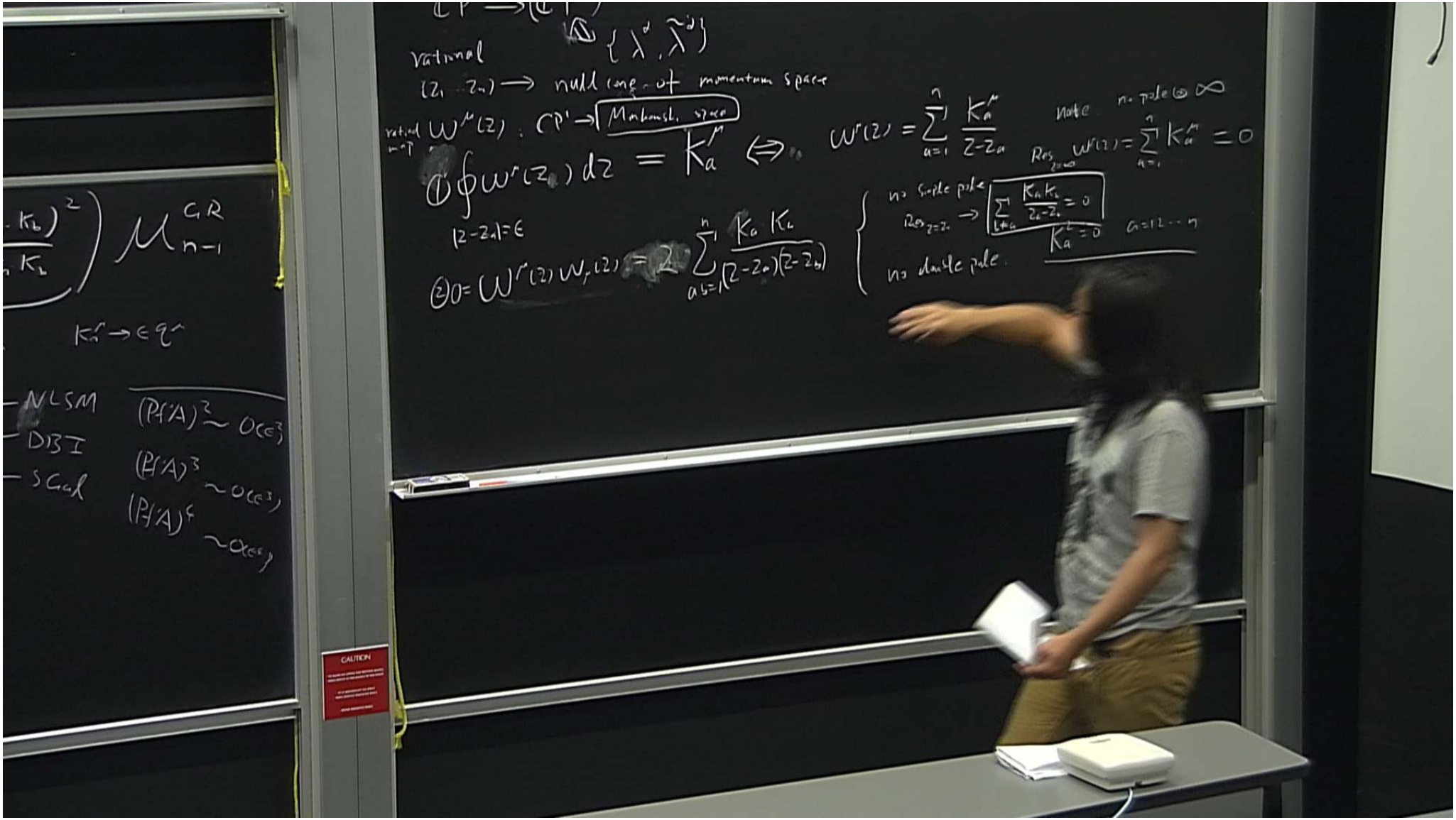

gauge inv of gluons/gravitons
nonlinearly realized sym
from soft & scaler limit
PfA
Pf·Φ
sGal
NLSM
$M_n^{GR} = -\sum_{b=1}^{n-1} \oint dz_n \frac{(\epsilon_n \cdot k_b)^2}{(k_n \cdot k_b)(z_n - z_b)^2} M_{n-1}^{GR} = \left(\sum_{b=1}^{n-1} \frac{(\epsilon_n \cdot k_b)^2}{k_n \cdot k_b}\right) M_{n-1}^{GR}$
S_n^{GR} $k_i \to \epsilon q$
$O(\epsilon^1) \leftarrow$ NLSM
$O(\epsilon^2) \leftarrow$ DBI
$O(\epsilon^3) \leftarrow$ sGal
$(Pf'A)^2 \sim O(\epsilon^3)$
$(Pf'A)^3 \sim O(\epsilon^5)$
$(Pf'A)^4 \sim O(\epsilon^6)$
$C_{nn}^2 = \left(\sum_{b=1}^{n-1}$
$YM_n \frac{1}{z_{12} z_{23} \cdots z_{n1}} =$
$\left(\frac{1}{z_n - z_{n-1}} - \frac{1}{z_n - z_1}\right)$
Ex. ①, ② ⇒ Soft
CAUTION

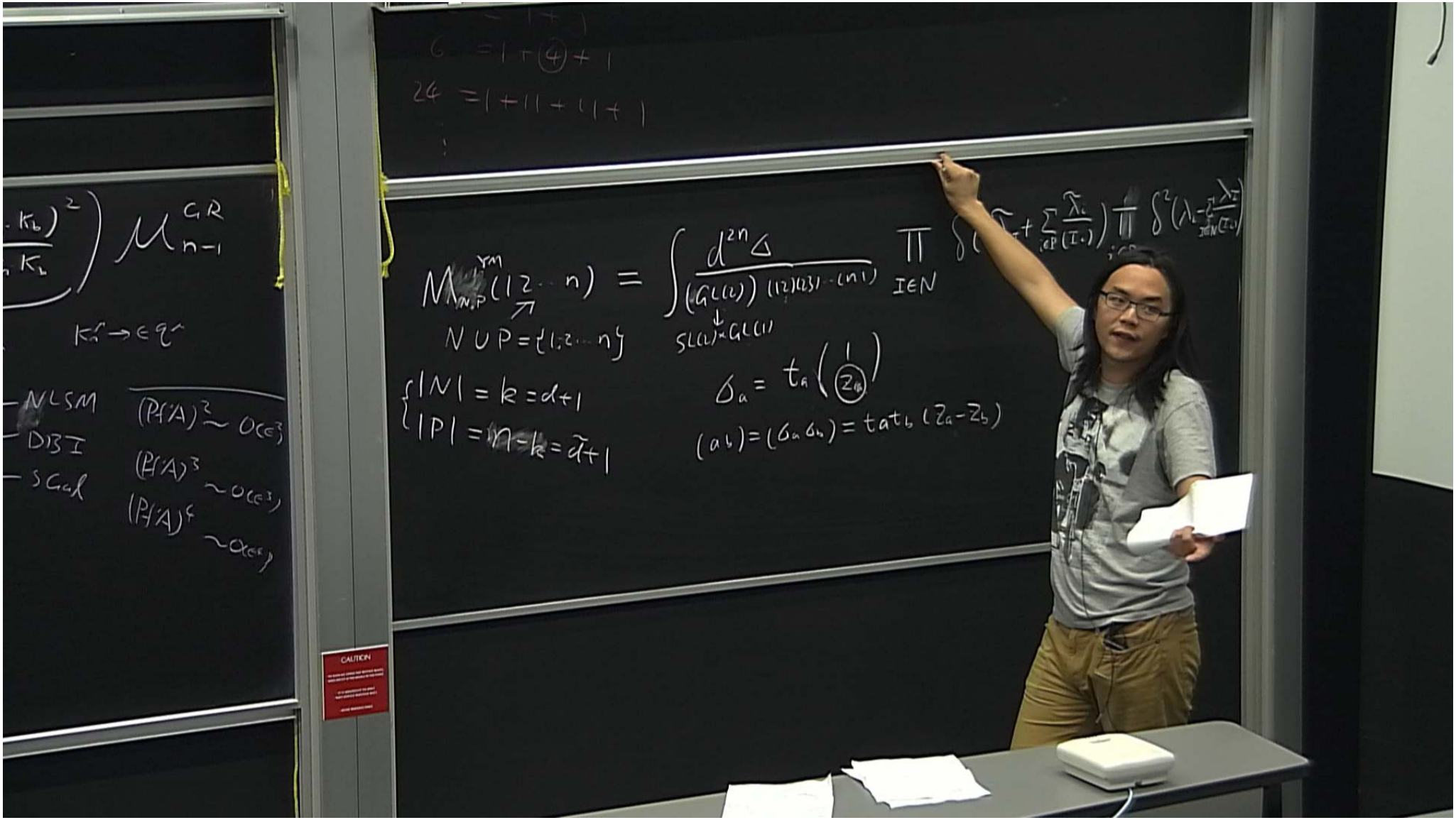

① gauge inv of gluons/gravitons
② non-linearly realized sym
from soft & scaler limit
CHY
NLSM
sGal
DBI
Ex. ①, ② ⇒ Soft gluon theorem
CAUTION

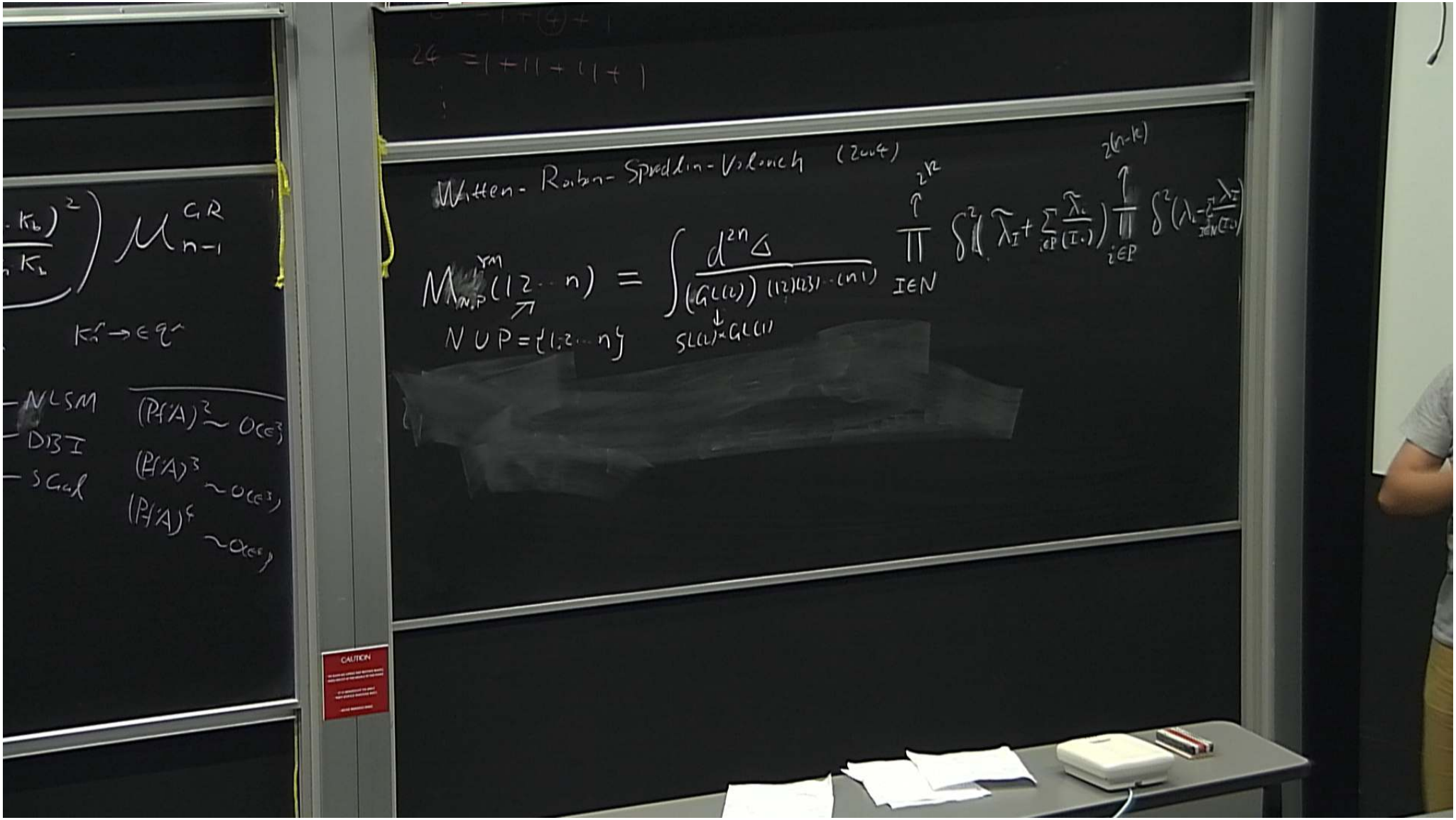

$\mathbb{C}P^1 \to \mathbb{C}$
$\mathcal{M}^{GR}_{n-1}$
NLSM
DBI
SGal
$(P\cdot A)^3 \sim O(\epsilon^3)$
$(P\cdot A)^3 \sim O(\epsilon^3)$
$(P\cdot A)^5 \sim O(\epsilon^5)$
CAUTION

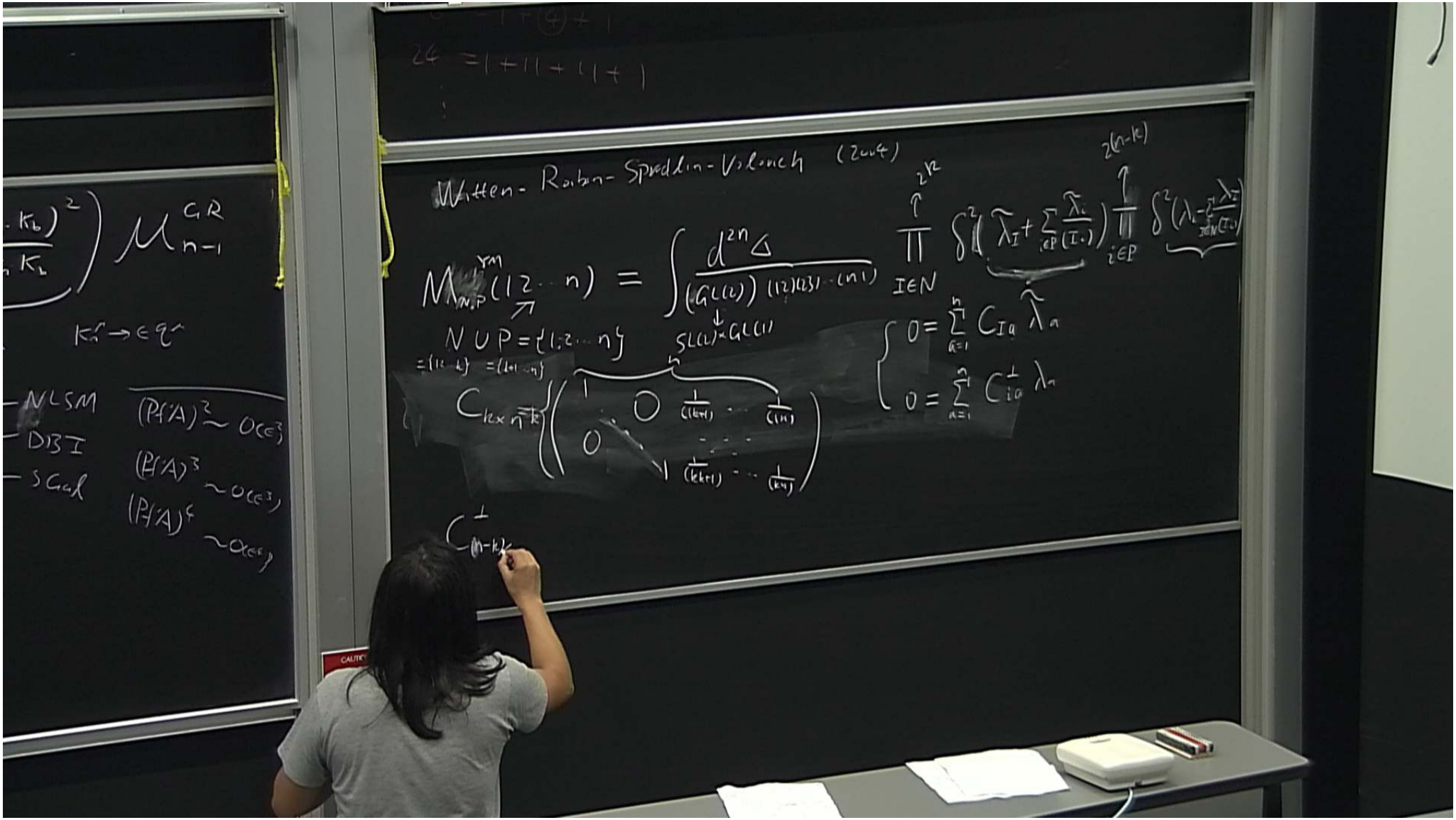

$\mathbb{CP}^1 \to (\mathbb{CP}^3)$
$\{\lambda^\alpha, \tilde{\lambda}^{\dot{\alpha}}\}$
rational
momentum space
W^μ
①
$\mathcal{M}_{n-1}^{GR}$
$K_n^\mu \to \epsilon q^\mu$
NLSM
DBI
sGal
$(P\cdot A)^3 \sim O(\epsilon^3)$
$(P\cdot A)^3 \sim O(\epsilon^3)$
$(P\cdot A)^5 \sim O(\epsilon^5)$
CAUTION

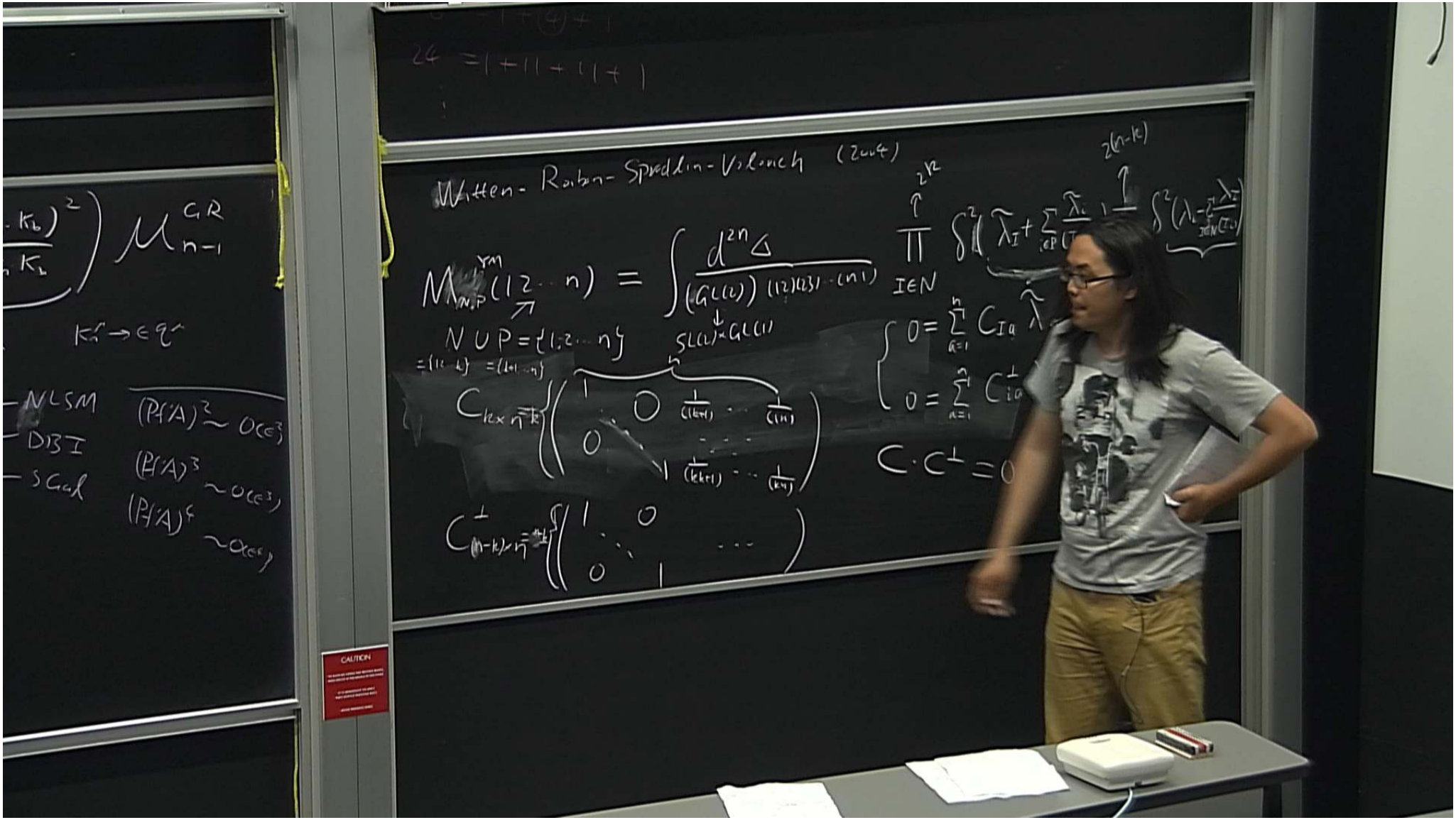

$\mathbb{CP}^1 \to (\mathbb{CP}^3)$
$\{\lambda^\alpha, \tilde{\lambda}^{\dot{\alpha}}\}$
rational
$(z_1 \ldots z_{n-1}) \to$ null line of momentum space
$W^\mu(z)$
$\oint W^\mu(z)\, dz = k_a^\mu \Rightarrow W^\mu(z) = \sum_{a=1}^{n} \frac{k_a^\mu}{z - z_a}$
$|z - z_a| = \epsilon$
$k_a^\mu \to \epsilon q^\mu$
NLSM
DBI
sGal
$(Pf'A)^2 \sim O(\epsilon^3)$
$(Pf'A)^3 \sim O(\epsilon^3)$
$(Pf'A)^5 \sim O(\epsilon^5)$
CAUTION

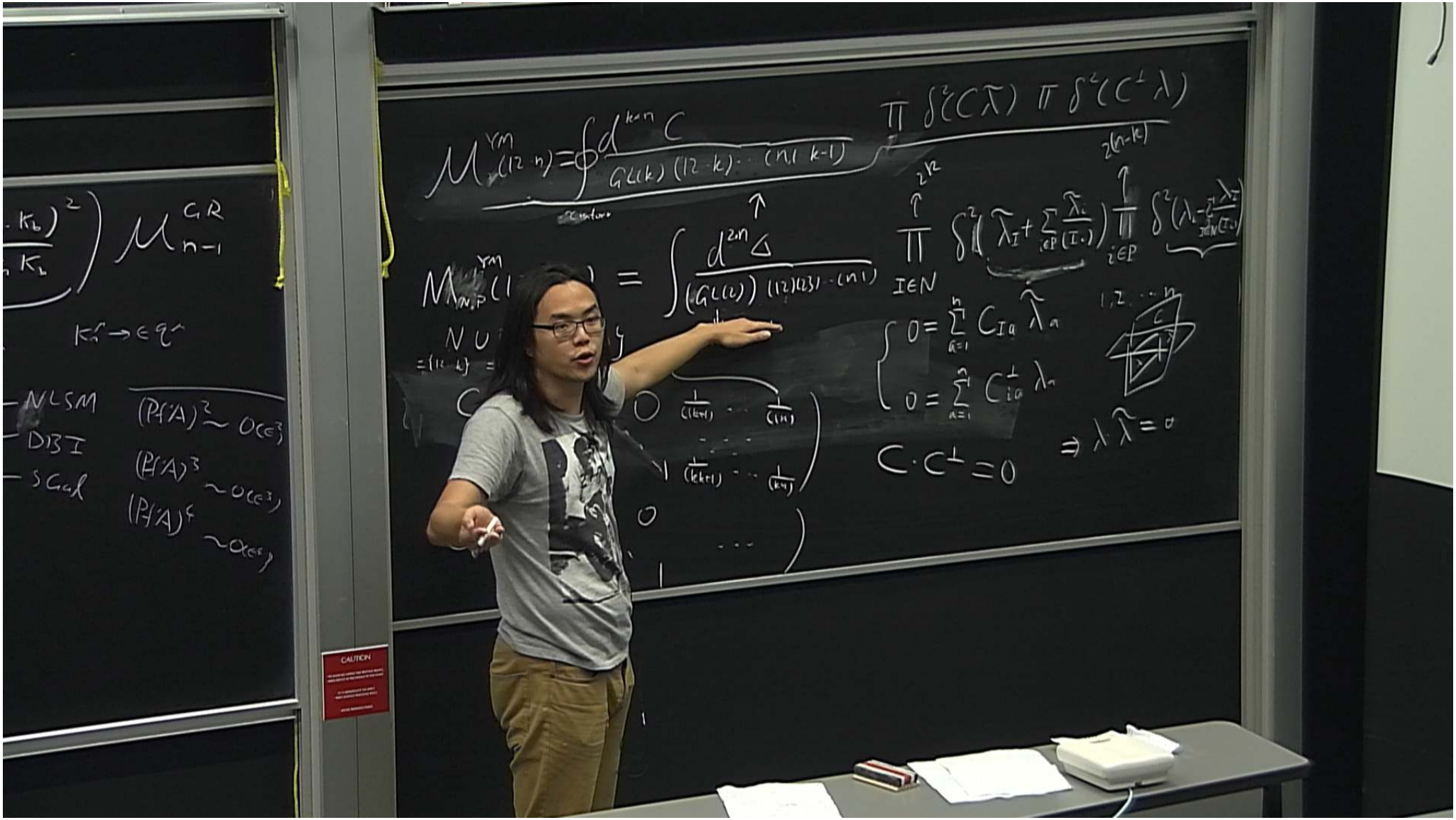

rational
$(z_1 \cdots z_n) \longrightarrow$ null cone of momentum space
rational map $W^\mu(z) : \mathbb{CP}^1 \to$ Minkowski space
$\oint W^\mu(z_0)\, dz = K_a^\mu \Leftrightarrow W^\mu(z) = \sum_{a=1}^{n} \frac{K_a^\mu}{z - z_a}$
Note no pole @ ∞
$\operatorname{Res}_{z=\infty} W^\mu(z) = \sum_{a=1}^{n} K_a^\mu = 0$
$|z - z_a| = \epsilon$
$W^\mu(z)\, W_\mu(z) = \sum_{a,b=1}^{n} \frac{K_a \cdot K_b}{(z - z_a)(z - z_b)}$
no simple pole
$\operatorname{Res}_{z=z_a} \rightarrow \sum_{b \neq a} \frac{K_a \cdot K_b}{z_a - z_b} = 0$
$K_a^\mu = 0$ $a = 1 \cdots n$
no double pole
NLSM
DBI
SCalf
$(Pf'A)^3 \sim O(\epsilon^3)$
$(Pf'A)^3 \sim O(\epsilon^3)$
$(Pf'A)^6 \sim O(\epsilon^6)$
CAUTION


$M_{N,P}^{YM}(1\,2\cdots n) = \int \frac{d^{2n}\Delta}{(GL(1))\,(12)(23)\cdots(n1)} \prod_{I\in N}$
$SL(1)\backslash GL(1)$
$N \cup P = \{1,2\cdots n\}$
$\sigma_a = t_a\left(\frac{1}{z_a}\right)$
$(ab) = (\sigma_a \sigma_b) = t_a t_b (z_a - z_b)$
$|N| = k = d+1$
$|P| = n-k = \tilde{d}+1$
$\mathcal{M}_{n-1}^{GR}$
NLSM
DBI
SCalf
$(Pf'A)^2 \sim O(\epsilon^2)$
$(Pf'A)^3 \sim O(\epsilon^3)$
$(Pf'A)^4 \sim O(\epsilon^4)$
CAUTION


Witten - Roiban - Spradlin - Volovich (2004)
$M_{N,P}^{YM}(1\,2\cdots n) = \int \frac{d^{2n}\Delta}{(GL(1))\,(12)(23)\cdots(n1)} \prod_{I\in N}^{2^{k2}} \delta^2\Big(\tilde{\lambda}_I + \sum_{i\in P(I)} \frac{\tilde{\lambda}_i}{\cdot}\Big) \prod_{i\in P}^{2(n-k)} \delta^2\Big(\lambda_i - \frac{\cdot}{\cdot}\lambda_I\Big)$
$N \cup P = \{1,2\cdots n\}$
$SL(1) \backslash GL(1)$
CAUTION


Witten - Roiban - Spradlin - Volovich (2004)
NLSM
DBI
sGal
$(Pf'A)^2 \sim O(\epsilon^3)$
$(Pf'A)^3 \sim O(\epsilon^3)$
$(Pf'A)^4 \sim O(\epsilon^4)$


Witten - Roiban - Spradlin - Volovich (2004)
NLSM
D3I
SCal
CAUTION


NLSM
D3I
SCal
CAUTION