

Title: Amplitudes: Soft Theorems

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Abstract:



Review of Lecture II

Scattering Eqs. $\bar{E}_a = \sum_{\substack{b=1 \\ b \neq a}}^n \frac{S_{ab}}{z_a - z_b} = 0 \quad a=1, 2, \dots, n$

$$\begin{aligned} \mathcal{M}_n^{\text{tree}}(k_1, k_2, \dots, k_n, \dots) &= \int \boxed{\prod_{a=1}^n dz_a \prod_{a=1}^n \delta(\bar{E}_a)} I_n(\{z, k, \dots\}) \\ &\equiv d\mu_n \\ &= \sum_{\substack{\{z^*\} \in \text{Solutions} \\ (n-3)!}} \frac{I_n(\{z^*, k, \dots\})}{J(\{z^*\})} \end{aligned}$$

$$C_n = \left(\frac{\text{Tr}(T^{I_1} T^{I_2} \dots T^{I_n})}{z_{12} z_{23} \dots z_{n-1,n} z_{n1}} + \text{Perm} \right) \frac{1}{(n-1)!} = \sum_{\{z\} \in \text{solutions}} \frac{I_n(\{z^*, K, \dots\})}{J(\{z^*\})} \equiv d\mu_n$$

$$M_n^{\text{adjoint } \phi^3} = \int d\mu_n C_n \bar{C}_n$$

$$M_n^{\text{YM}} = \int d\mu_n C_n \bar{C}_n$$

$$\bar{\Psi} = \begin{pmatrix} A & -C^T \\ C & B \end{pmatrix}$$

$$\text{Pf } \bar{\Psi}(\{z \in K\})$$

$$A_{ab} = \begin{cases} \frac{K_a K_b}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$B_{ab} = \begin{cases} \frac{E_a E_b}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$C_{ab} = \begin{cases} \frac{E_a F_b}{z_a - z_b} & a \neq b \\ -\sum_{c \neq a} C_{ac} & a = b \end{cases}$$

$$C_n = \left(\frac{\text{Tr}(T^{I_1} T^{I_2} \dots T^{I_n})}{z_{12} z_{23} \dots z_{n-1,n} z_{n1}} + \text{Perm} \right) \frac{1}{(n-1)!} = \sum_{\{z\} \in \text{solutions}} \frac{I_n(\{z^*, \kappa, \dots\})}{J(\{z^*\})} \equiv d\mu_n$$

$$M_n^{\text{adj}} \phi^3 = \int d\mu_n C_n \bar{C}_n$$

$$M_n^{\text{YM}} = \int d\mu_n C_n \bar{\Psi} \quad \text{Pf}' \bar{\Psi} (\{z \in K\})$$

$$\bar{\Psi} = \begin{pmatrix} A & -C^T \\ \bar{C} & B \end{pmatrix}$$

$$\zeta_a^{\mu\nu} = \epsilon_a^\mu \otimes \epsilon_a^\nu \quad (\text{symmetric \& traceless})$$

$$M_n^{\text{GR}} = \int d\mu_n \det' \bar{\Psi}$$

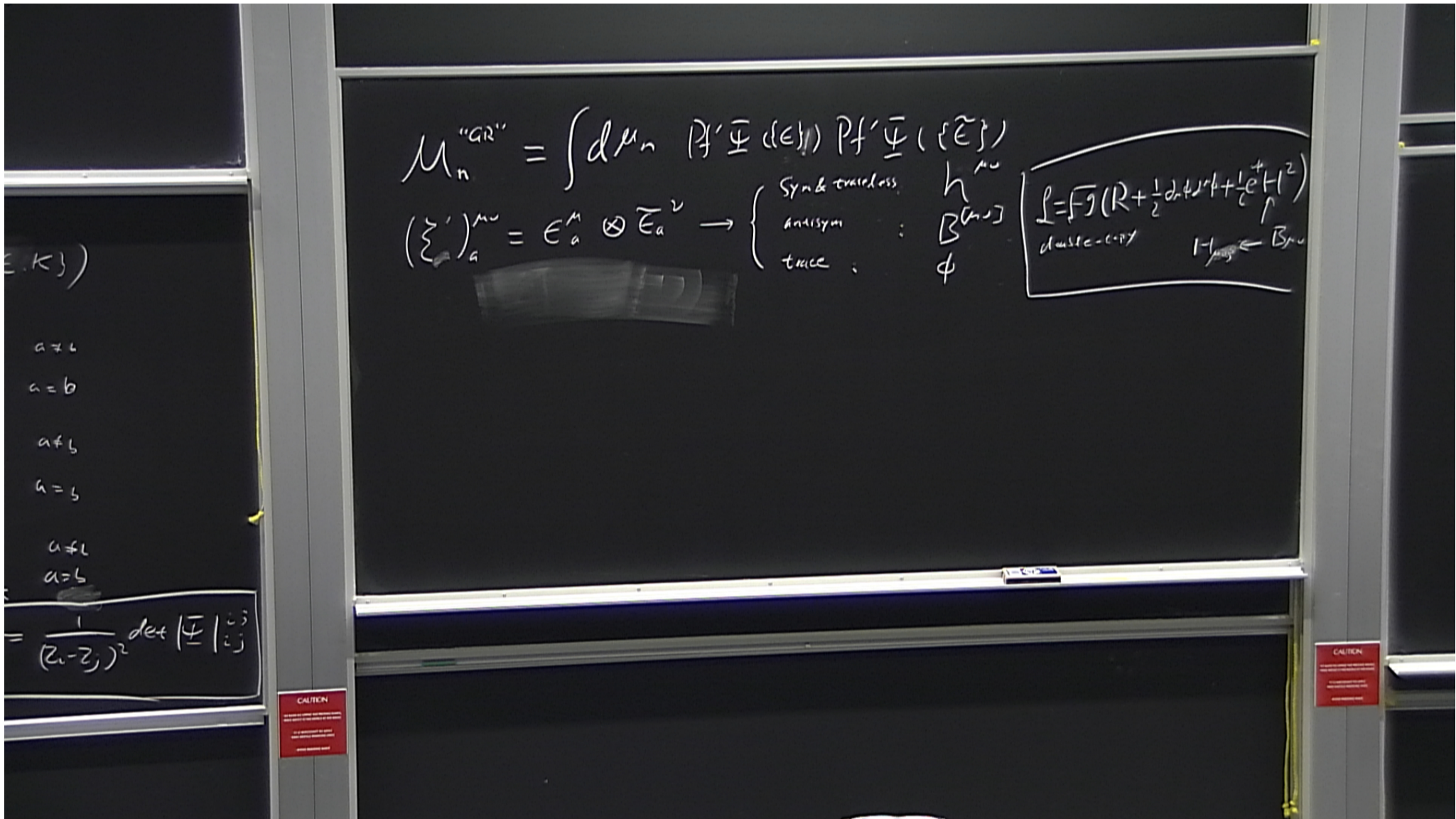
$$A_{ab} = \begin{cases} \frac{\kappa_a \kappa_b}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$B_{ab} = \begin{cases} \frac{\epsilon_a \epsilon_b}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$C_{ab} = \begin{cases} \frac{\epsilon_a \kappa_b}{z_a - z_b} & a \neq b \\ -\sum_{c \neq a} C_{ac} & a = b \end{cases}$$

$$\det' \bar{\Psi} = (\text{Pf}' \bar{\Psi})^2 = \frac{1}{(z_1 - z_2)^2} \det \begin{pmatrix} \bar{\Psi} & \begin{smallmatrix} z_1 \\ z_2 \end{smallmatrix} \\ \begin{smallmatrix} z_1 \\ z_2 \end{smallmatrix} & I_2 \end{pmatrix}$$

$$M_n^{\text{GR}}$$



$$\mathcal{M}_n^{\text{"GR"}} = \int d\mu_n \text{Pf}' \Psi(\{e\}) \text{Pf}' \bar{\Psi}(\{\tilde{e}\})$$

$$(\Sigma')^\mu_a = e^\mu_a \otimes \tilde{e}^\nu_a \rightarrow \begin{cases} \text{Sym \& traceless} & h^{\mu\nu} \\ \text{antisym} & B^{[\mu\nu]} \\ \text{trace} & \phi \end{cases}$$

$$\boxed{L = F(R + \frac{1}{2} d\tau^2 + \frac{1}{\epsilon} \epsilon^{\mu\nu} \epsilon^2)}$$

dual to copy $\nearrow B_{\mu\nu}$

$$e \cdot k \rangle$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$= \frac{1}{(z_i - z_j)^2} \det \left| \bar{\Psi} \right|_{ij}^{ij}$$

$$\mathcal{M}_n^{\text{"GR"}} = \int d\mu_n \, Pf' \bar{\Psi}(\{e\}) Pf' \bar{\Psi}(\{\tilde{e}\})$$

$$(\xi')^{\mu\nu}_a = \epsilon^\mu_a \otimes \tilde{\epsilon}^\nu_a \rightarrow \begin{cases} \text{Sym \& traceless} & h^{\mu\nu} \\ \text{antisym} & B^{\mu\nu} \\ \text{trace} & \phi \end{cases}$$

$$G_R = \gamma M^2 / \psi^3$$

$$\boxed{\begin{aligned} \mathcal{L} &= F^2 \left(R + \frac{1}{2} D_\mu \psi D^\mu \psi + \frac{1}{6} \psi^2 \right) \\ \text{double-copy } \uparrow ? \\ \mathcal{L}_{YM} &= \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad 1 - \gamma_{\mu\nu} \leftarrow B_{\mu\nu} \end{aligned}}$$

$$\mathcal{M}_n^{\text{Spin-5}} = \int d\mu_n (e_n)^{2-5} (Pf' \bar{\Psi})^5 \rightarrow \text{no-local}$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$= \frac{1}{(z_i - z_j)^2} \det \left[\bar{\Psi} \right]_{i,j}^{\begin{smallmatrix} 1,2 \\ 3 \end{smallmatrix}}$$

$$(E_a) \prod_n (z, k, \dots)$$

$$\{ \dots \}$$

$$n=5$$

$$a \neq b$$

$$a=b$$

$$= \frac{1}{(z_i - z_j)^2} \det \left(\frac{\partial^2 \Phi}{\partial z_i \partial z_j} \right)$$

Theorem given $\mathcal{M}_n = \int d\mu_n^{(2)} I_L^{(2)} I_R^{(2)} = \left(\sum_{\alpha, \beta=1}^{n-3} \frac{I_L^{(\alpha)} I_R^{(\beta)}}{J^{(\alpha, \beta)}} \right)$

introduce $L^\alpha(z), R^\beta(z)$ $\alpha, \beta = 1, 2, \dots, (n-3)!$

relation among a.m.p.s with $(n-3)!$ -dim vectors

$$L(1, 2, \dots, n) = R(1, 2, \dots, n) = \frac{1}{z_1 z_2 \dots z_n}$$

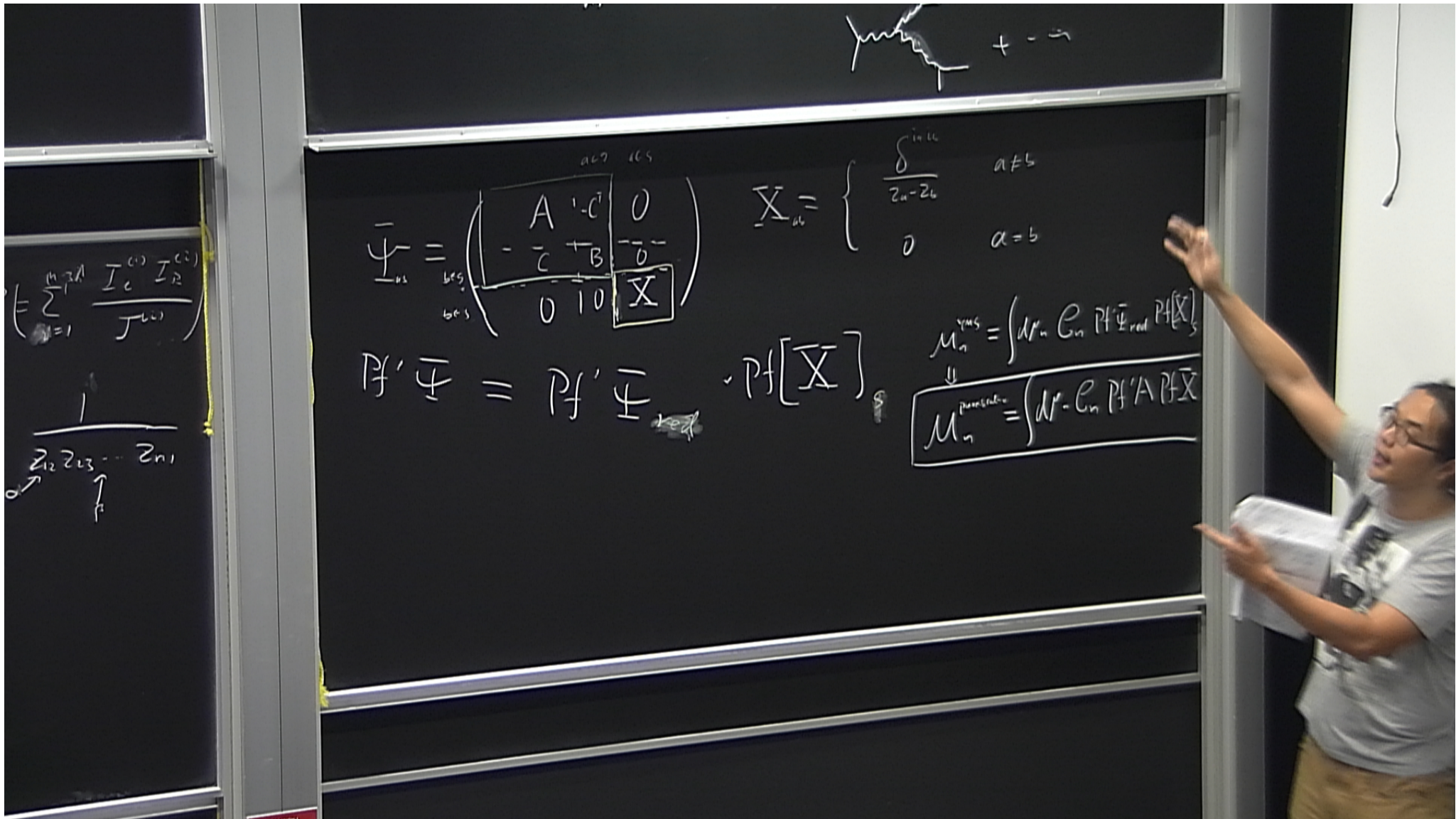
$$\mathcal{M}_n = \sum_{\alpha, \beta=1}^{(n-3)!} \boxed{M_n^L(\alpha)} \bar{m}_{n(\alpha|\beta)} \boxed{M_n^R(\beta)}$$

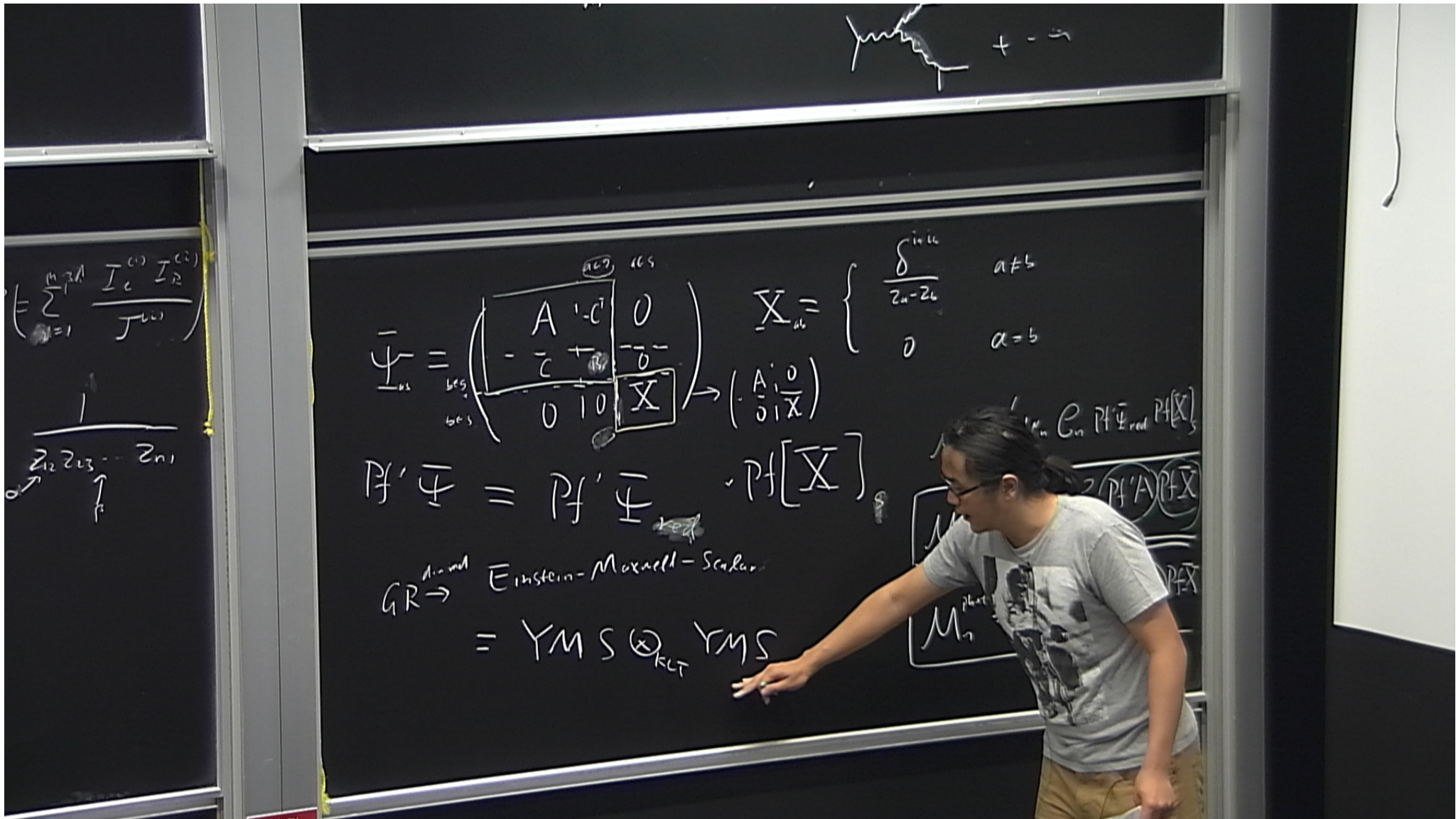
where $\bar{M}_n^L(\alpha) = \int d\mu_n I_L \cdot L(\alpha)$

$\bar{M}_n^R(\beta) = \int d\mu_n I_R \cdot R(\beta)$

$m(\alpha|\beta) = \int d\mu_n L(\alpha) R(\beta)$







$$I = \sum_{i=1}^{m-2} \frac{I_i^{(1)} I_i^{(2)}}{J^{(2)}}$$

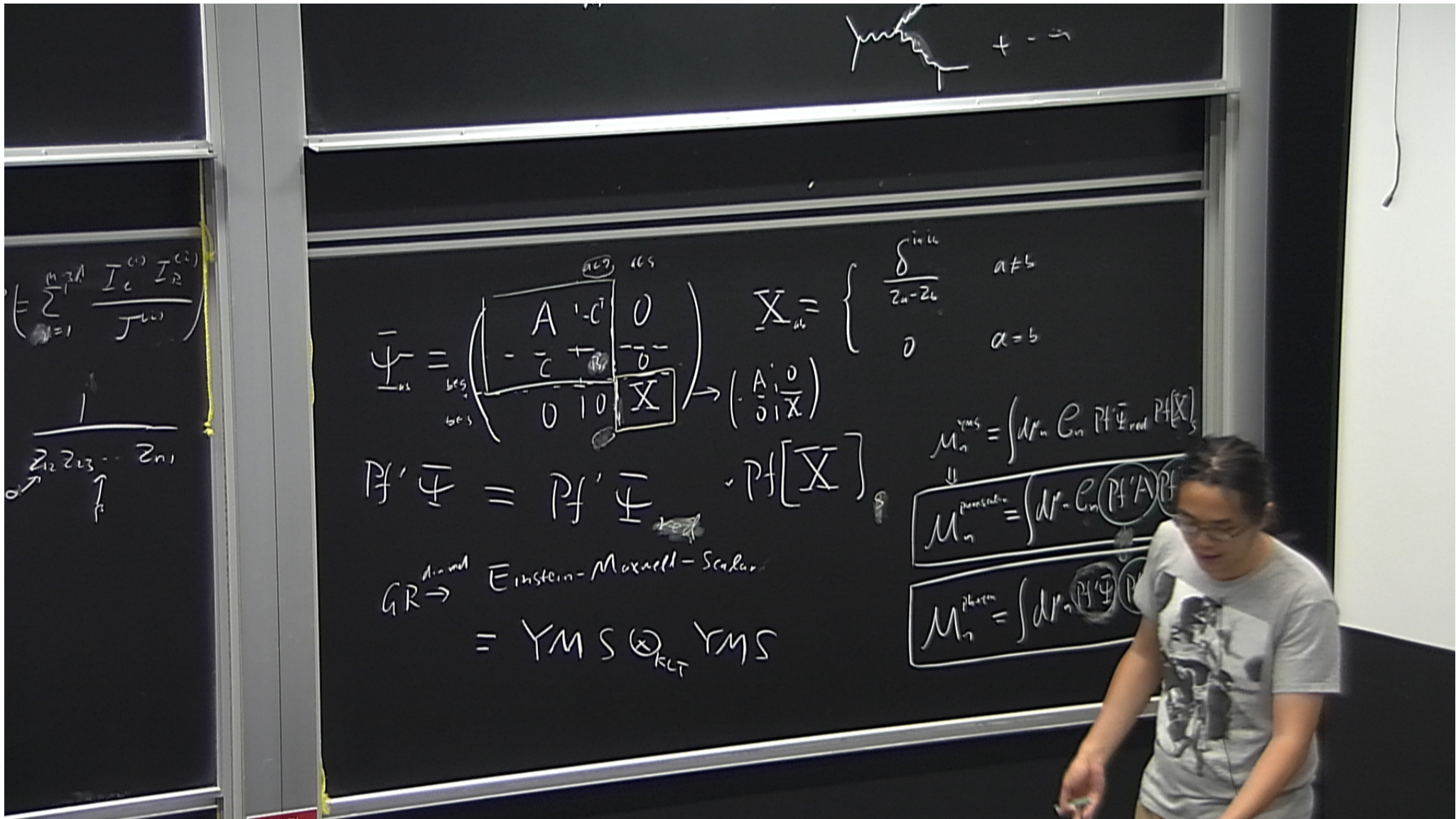
$$\frac{1}{z_{12} z_{13} \dots z_{n1}}$$

$$\bar{\Psi} = \begin{pmatrix} A & -C^T & 0 \\ -C & B & -\bar{D} \\ 0 & 0 & X \end{pmatrix} \rightarrow \begin{pmatrix} A & 0 \\ 0 & X \end{pmatrix}$$

$$\bar{X}_a = \begin{cases} \frac{\delta}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$Pf' \bar{\Psi} = Pf' \bar{\Psi} \cdot Pf[X]$$

$$GR \xrightarrow{A. and} \text{Einstein-Maxwell-Scalar} \\ = YMS \otimes_{KLT} YMS$$



$$I = \sum_{i=1}^{m-2} \frac{I_c^{(1)} I_c^{(2)}}{J^{(2)}}$$

$$\frac{1}{z_{12} z_{13} \dots z_{n1}}$$

↑
f

$$\bar{\Psi} = \begin{pmatrix} A & -C^T & 0 \\ -C & 0 & 0 \\ 0 & 0 & X \end{pmatrix} \rightarrow \begin{pmatrix} A & 0 \\ 0 & X \end{pmatrix}$$

$$\bar{X}_a = \begin{cases} \frac{\delta}{z_a - z_b} & a \neq b \\ 0 & a = b \end{cases}$$

$$Pf' \bar{\Psi} = Pf' \bar{\Psi} \cdot Pf[X]$$

GR $\xrightarrow{dim}$ Einstein-Maxwell-Scalar

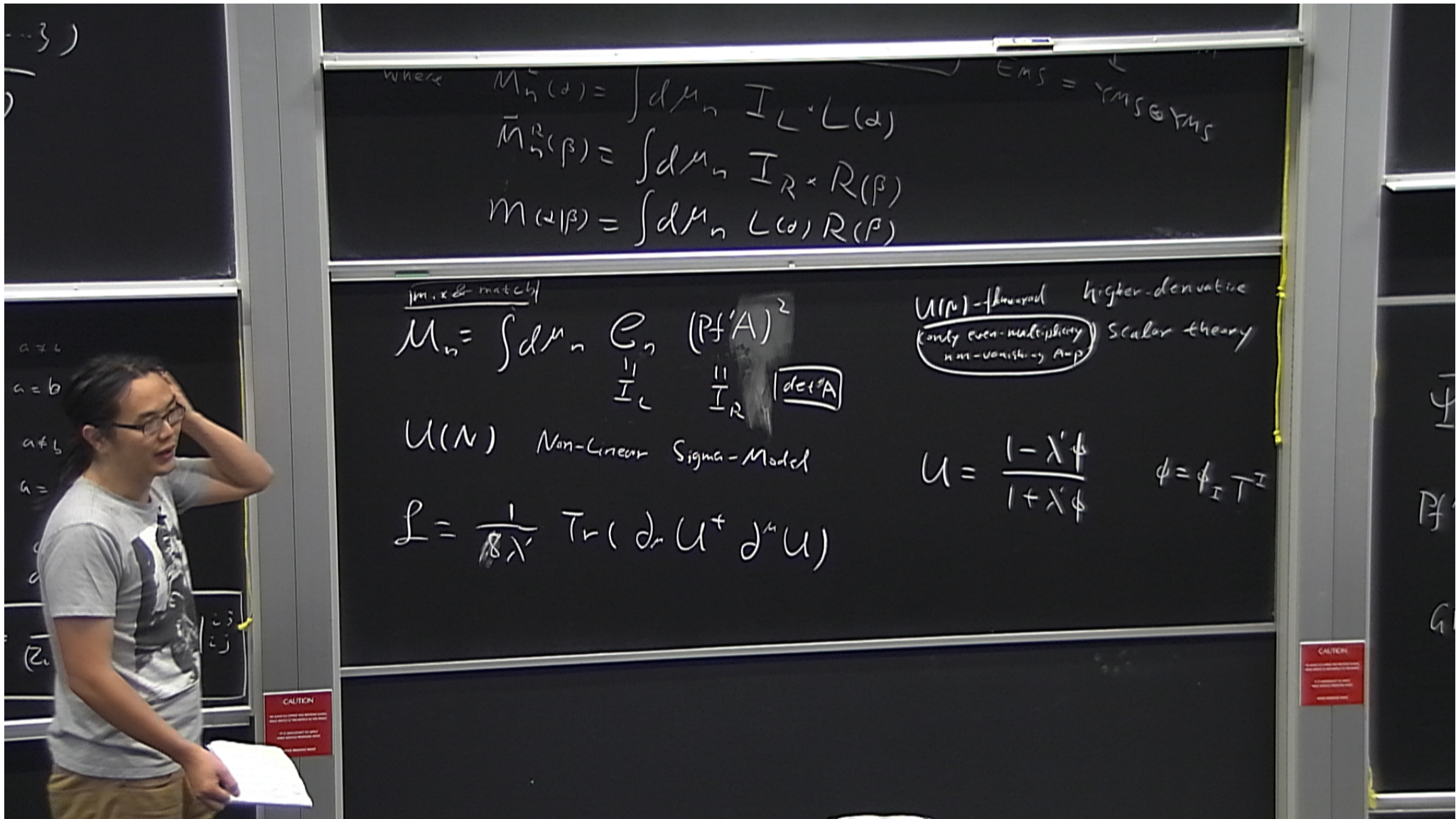
$$= YMS \otimes_{KLT} YMS$$

$$\mathcal{M}_n^{YM} = \int d^n x \, \mathcal{L}_n [Pf \bar{\Psi}, Pf X]$$

$$\Downarrow$$

$$\mathcal{M}_n^{graviton} = \int d^n x \, \mathcal{L}_n [Pf' A, Pf X]$$

$$\mathcal{M}_n^{photon} = \int d^n x \, \mathcal{L}_n [Pf \bar{\Psi}, Pf' A, Pf X]$$



where

$$M_n(\alpha) = \int d\mu_n I_L \cdot L(\alpha)$$

$$\bar{M}_n^R(\beta) = \int d\mu_n I_R \cdot R(\beta)$$

$$M(\alpha|\beta) = \int d\mu_n L(\alpha) R(\beta)$$

$$E_{MS} = \gamma_{MS} \otimes \gamma_{MS}$$

im. x dr-matrix

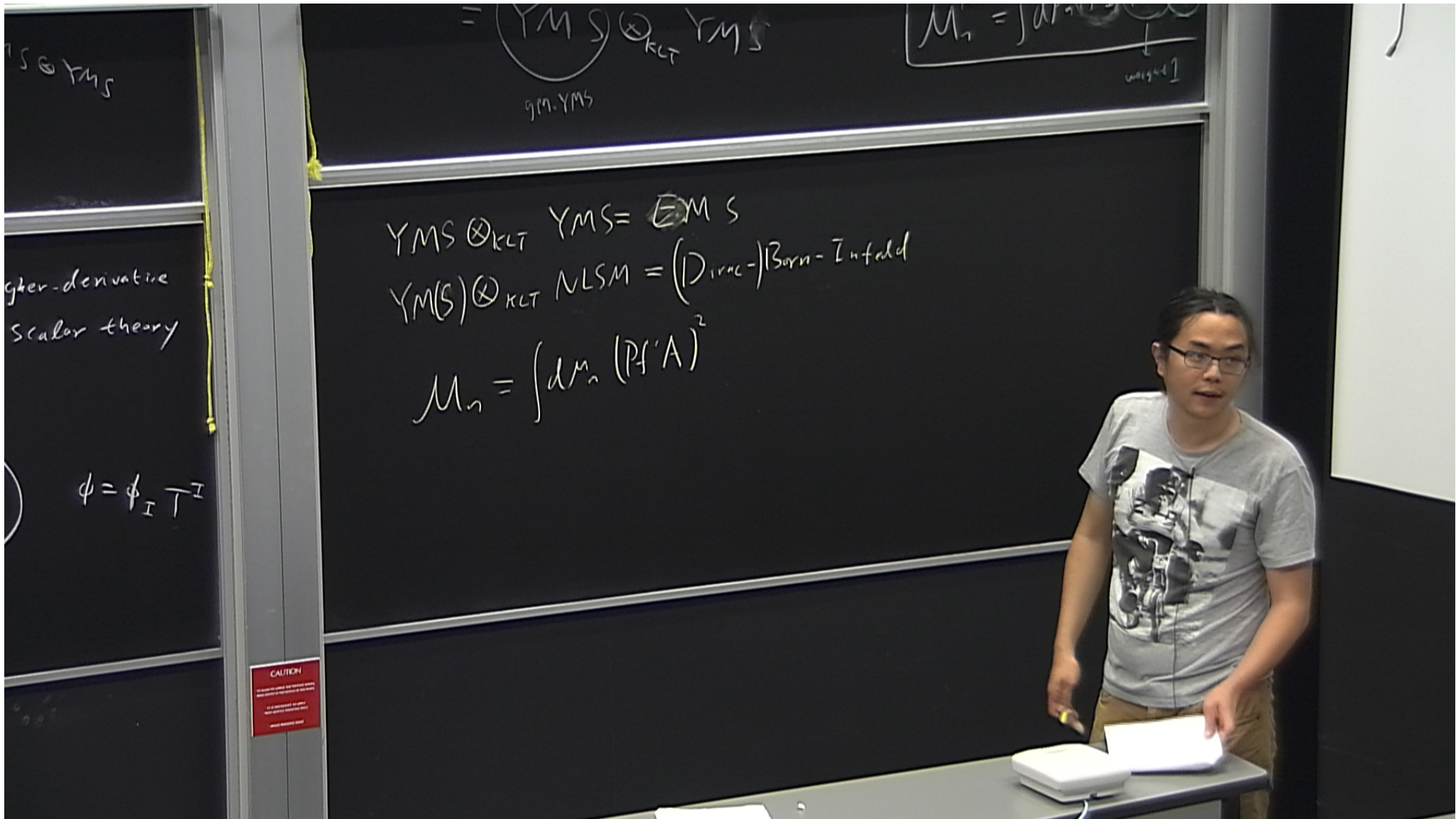
$$M_n = \int d\mu_n \begin{matrix} C_n \\ I_L \end{matrix} \begin{matrix} (Pf'A)^2 \\ I_R \end{matrix} \boxed{\det A}$$

U(1)-theory higher-derivative
 (only even-multiplicity non-vanishing A-p) Scalar theory

U(N) Non-Linear Sigma-Model

$$U = \frac{1 - \lambda' \phi}{1 + \lambda' \phi} \quad \phi = \phi_I T^I$$

$$L = \frac{1}{8\lambda'} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U)$$



$3)$
 $a \neq b$
 $a = b$
 $a \neq b$
 $a = b$
 $a \neq b$
 $a = b$
 $\frac{1}{(z_i - z_j)^2} \det \begin{pmatrix} \mp & \dots \\ \dots & \dots \end{pmatrix}$

$\bar{M}_n(\alpha) = \int d\mu_n I_L \cdot L(\alpha)$
 $\bar{M}_n(\beta) = \int d\mu_n I_R \cdot R(\beta)$
 $M(\alpha|\beta) = \int d\mu_n L(\alpha) R(\beta)$
 $(NLSM) \otimes_{KLT} (NLSM)$
 $\mathcal{M}_n = \int d\mu_n (P^\dagger A)^4 \Leftrightarrow \mathcal{L}_{S_{Gal}} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \sum_{m=3}^{\infty} g_m \mathcal{L}_m$
 $= (\det A)^{2-n}$
 $\mathcal{M}_4 = S + \mathcal{L}$
 $= \phi \det(\partial^\mu \phi \partial_\mu \phi)$
 $j = 1, 2, \dots, n-1$



$$-3)$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$a \neq b$$

$$a = b$$

$$\frac{1}{(z_i - z_j)^2} \det \left| \begin{matrix} \bar{z} & 1 \\ \bar{z} & 1 \end{matrix} \right|_{i,j}^{2,3}$$

$$\bar{M}_n(\alpha) = \int d\mu_n I_L \cdot L(\alpha)$$

$$\bar{M}_n(\beta) = \int d\mu_n I_R \cdot R(\beta)$$

$$M(\alpha|\beta) = \int d\mu_n L(\alpha) R(\beta)$$

$$(NLSM) \otimes_{KLT} (NLSM)$$

$$\mathcal{M}_n = \int d\mu_n (P^\dagger A)^4 \Leftrightarrow \mathcal{L}_{S_{Cal}} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \sum_{m=3}^{\infty} \frac{1}{m!} g_m \mathcal{L}_m$$

$$= (\det A)^{2-n}$$

$$= \phi \det(\partial^\mu \phi \partial_\mu \phi)$$

$$\mathcal{M}_4 = 5 + 4$$

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 WHILE THE BOARD IS IN USE

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