

Title: Cosmology & Observations: Gravitational lensing

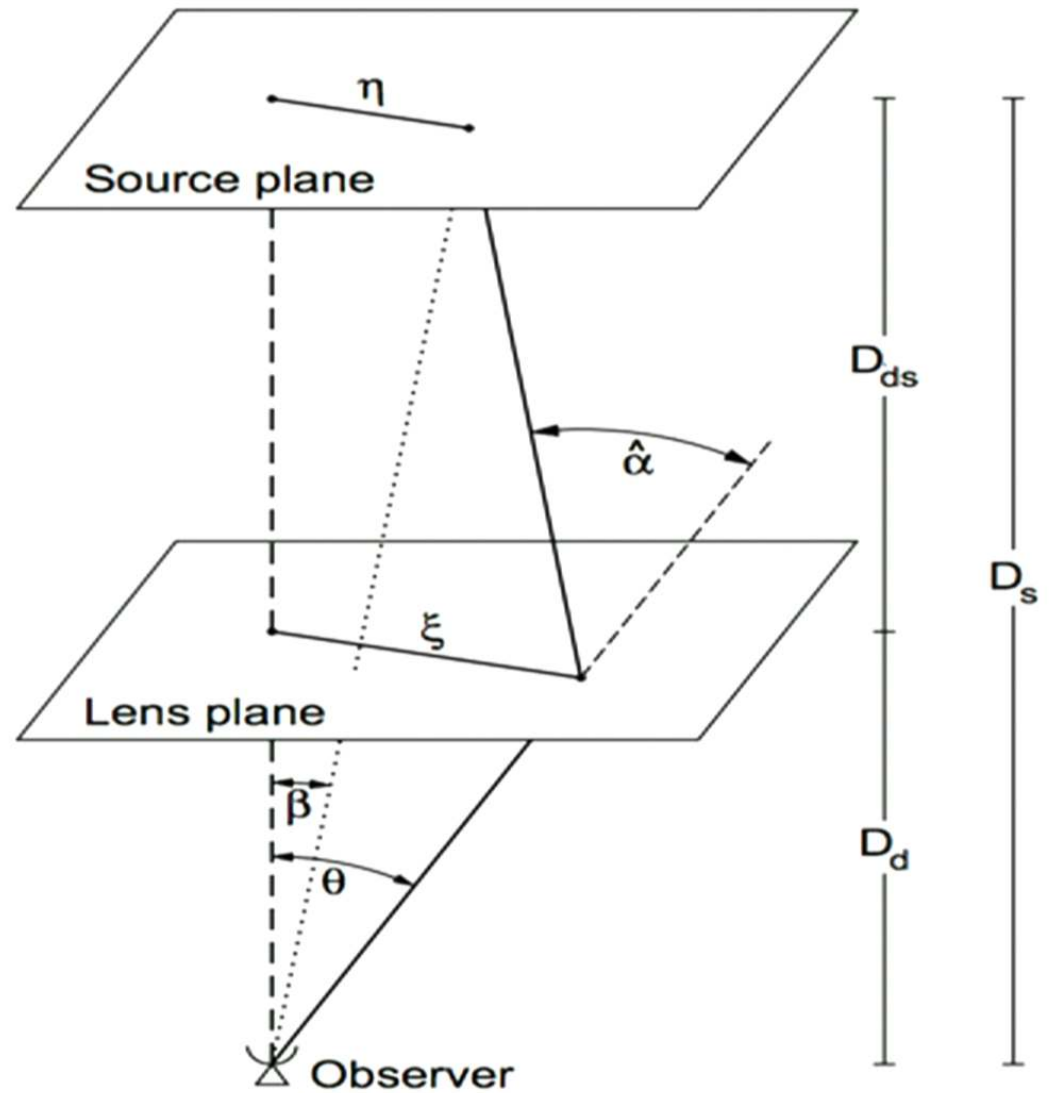
Date: Jul 15, 2015 04:00 PM

URL: <http://pirsa.org/15070051>

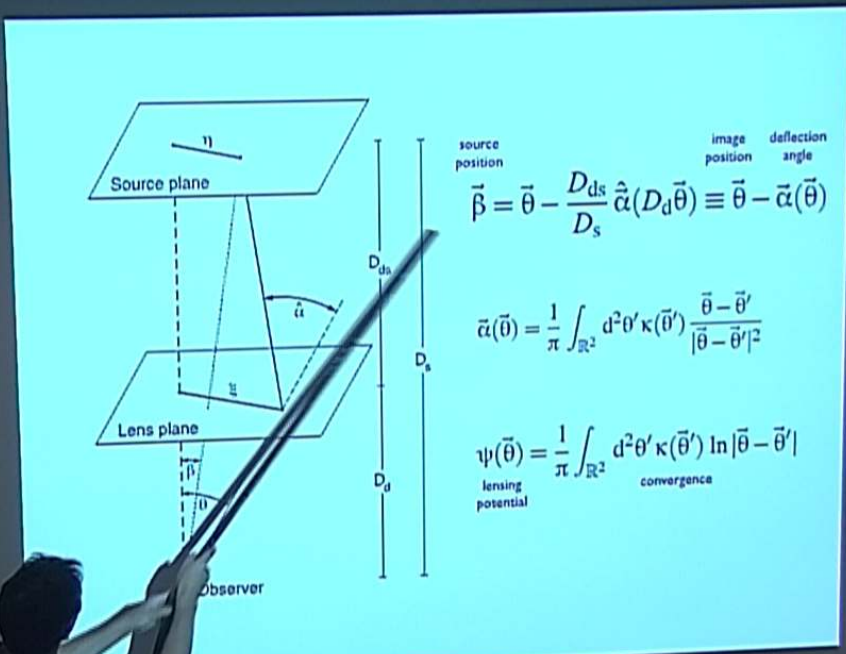
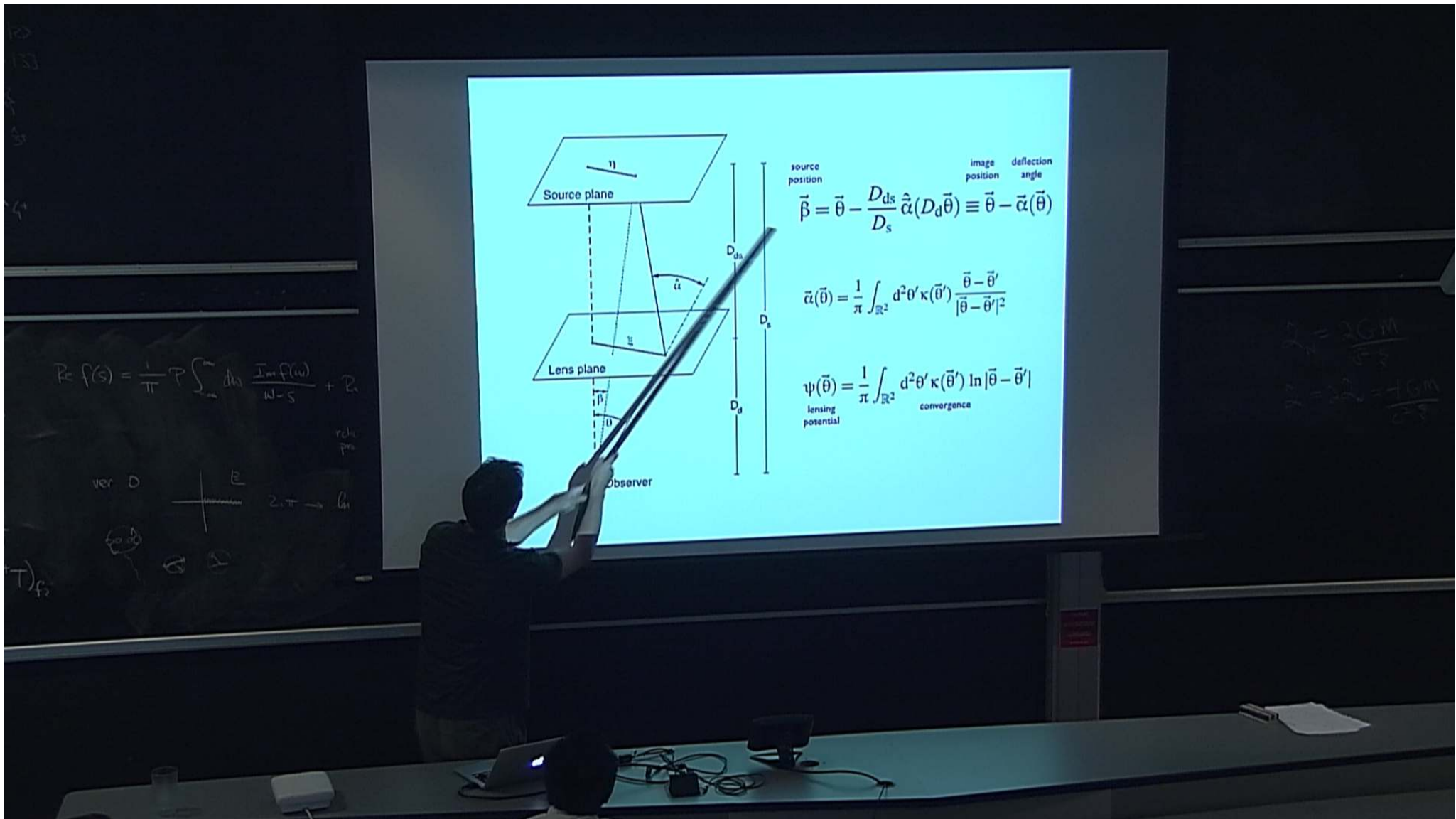
Abstract:

Gravitational lensing

$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$



Bartelmann & Schneider
<http://arxiv.org/abs/astro-ph/9912508>



source position image position deflection angle

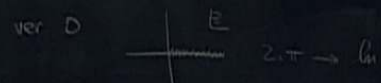
$$\vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \vec{\theta}) \equiv \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$

$$\psi(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$

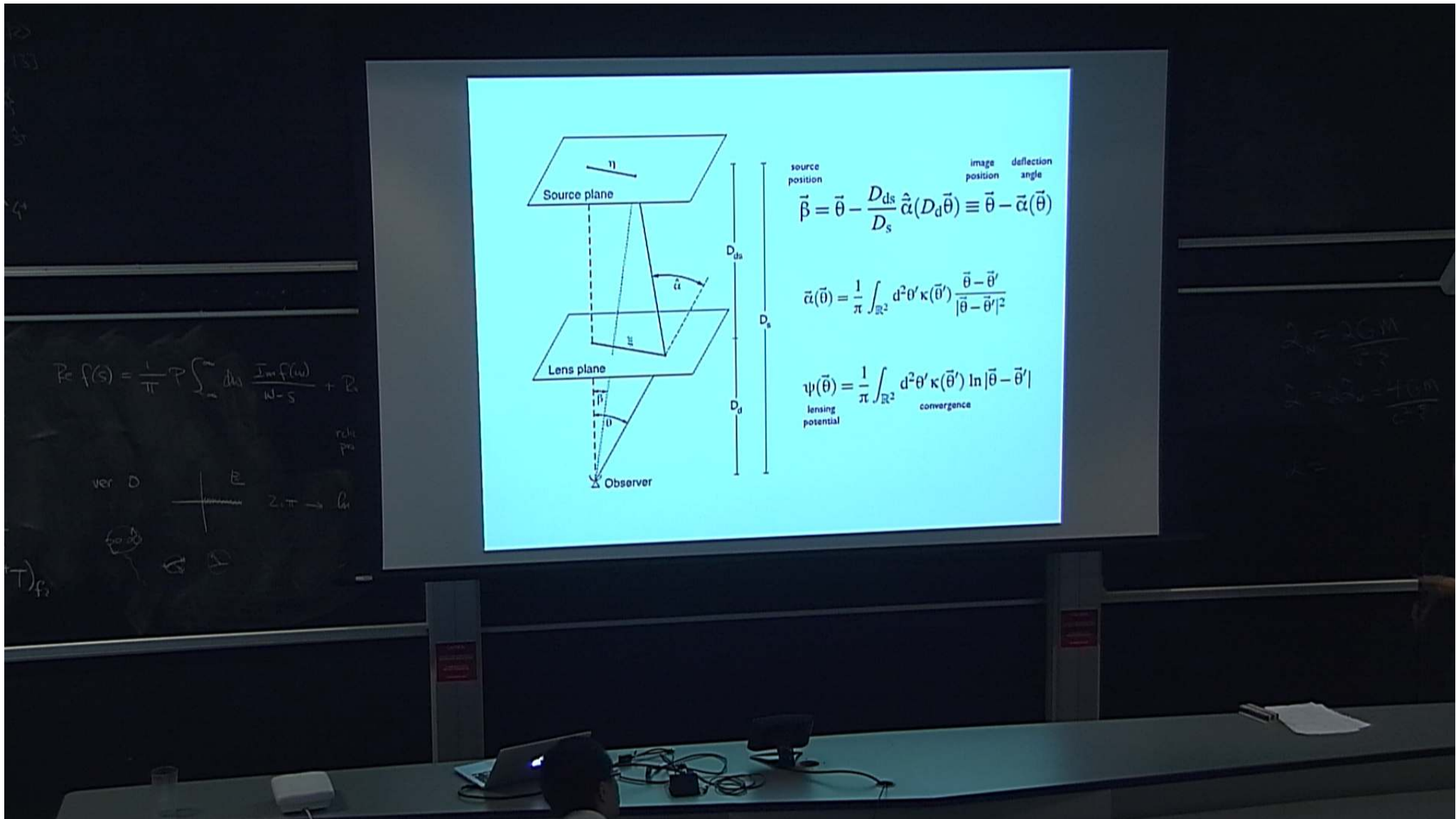
lensing potential convergence

$$\text{Re } f(s) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dx \frac{\text{Im } f(x)}{x-s} + \mathcal{P}$$



$$\Delta \varphi = \frac{2GM}{c^2 b}$$

$$\Delta \varphi = \frac{4GM}{c^2 b}$$



source position image position deflection angle

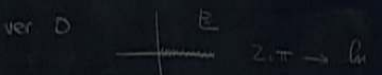
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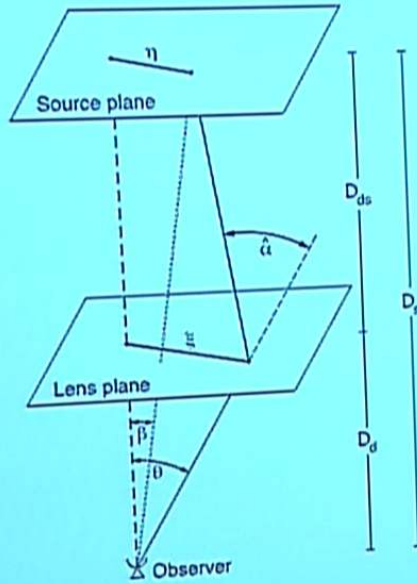
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lensing potential convergence

$$Re f(s) = \frac{1}{\pi} P \int_{-\infty}^{\infty} ds' \frac{Im f(s')}{s' - s} + P_0$$





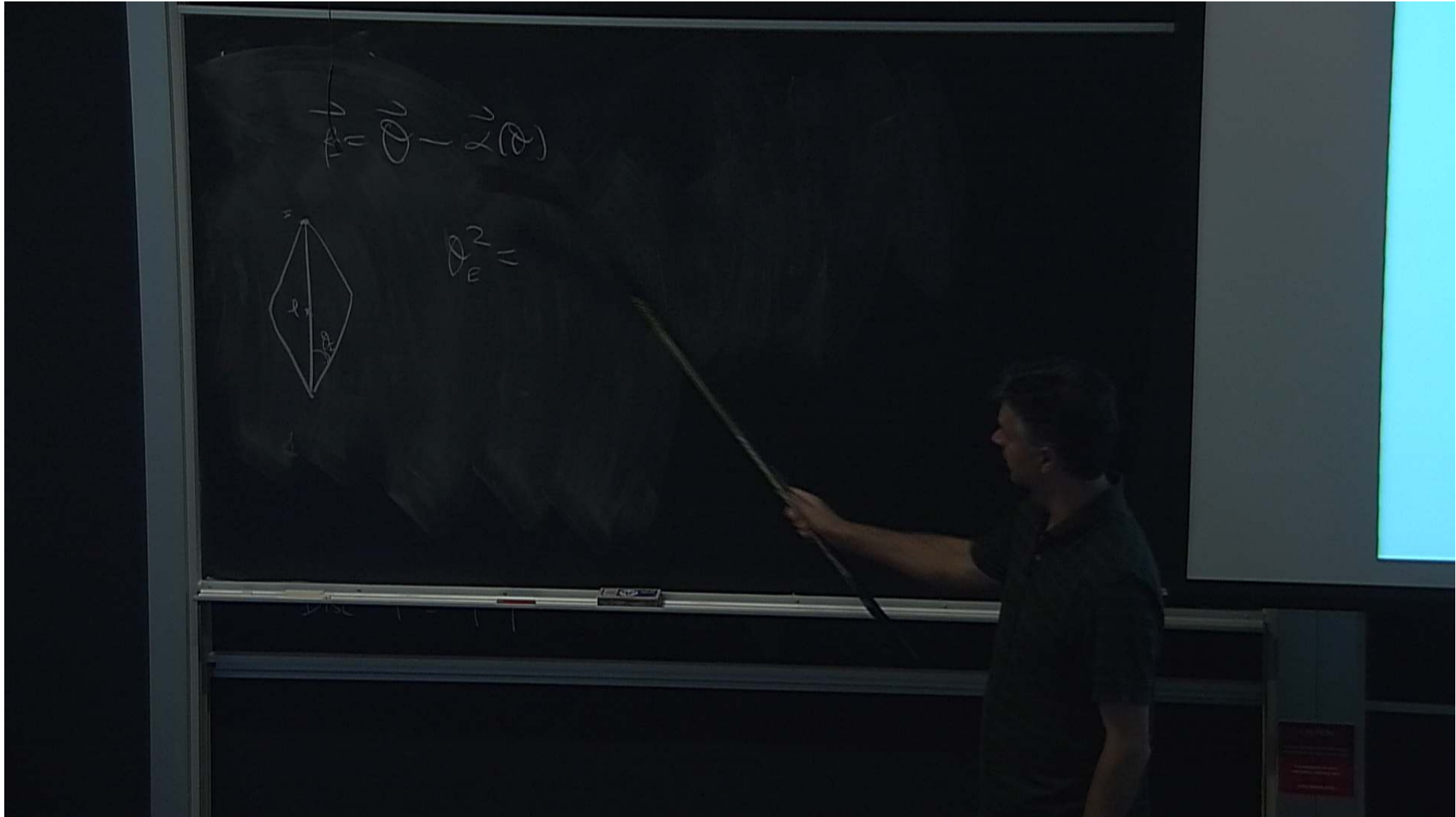
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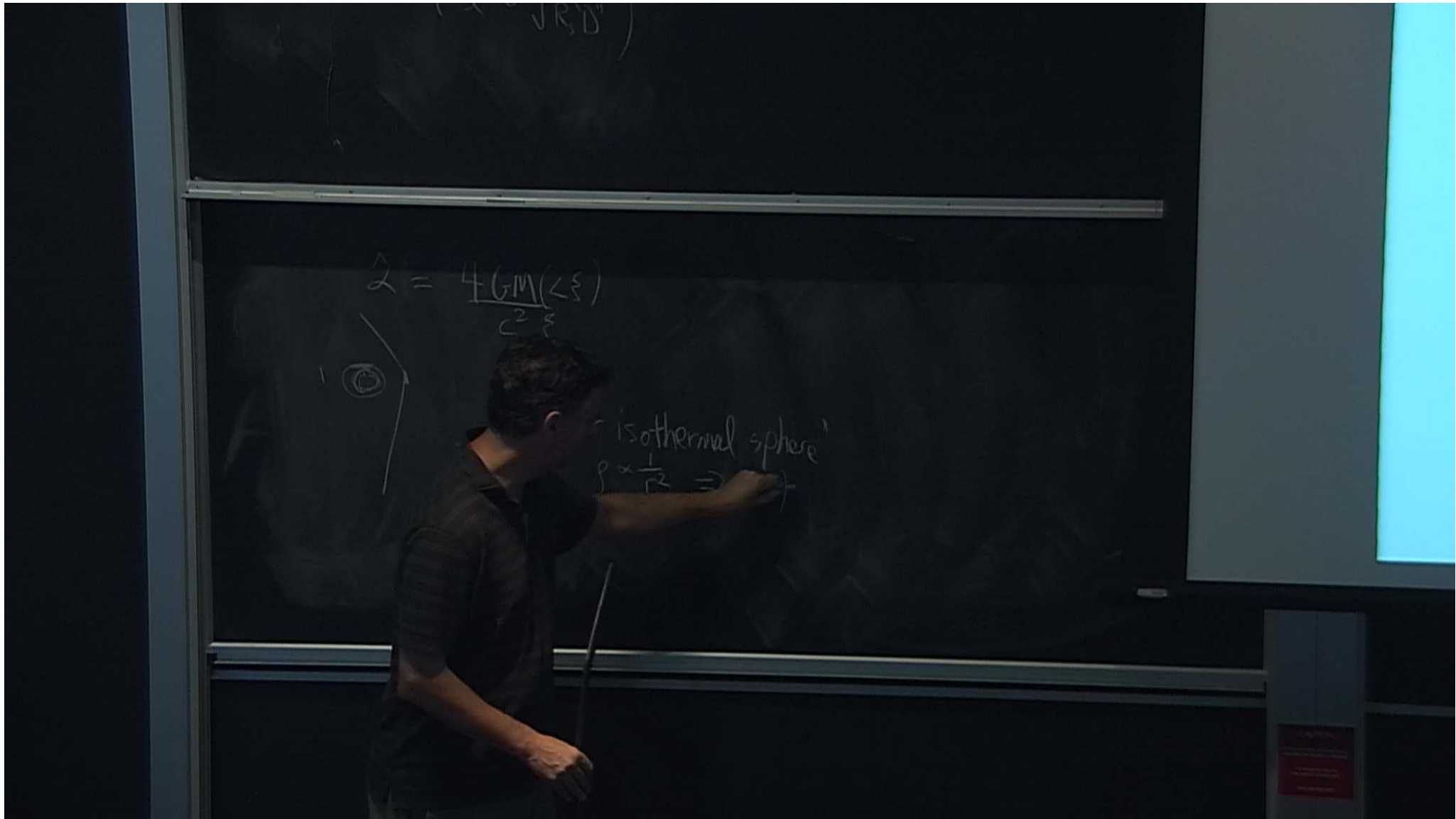
source position
image position
deflection angle

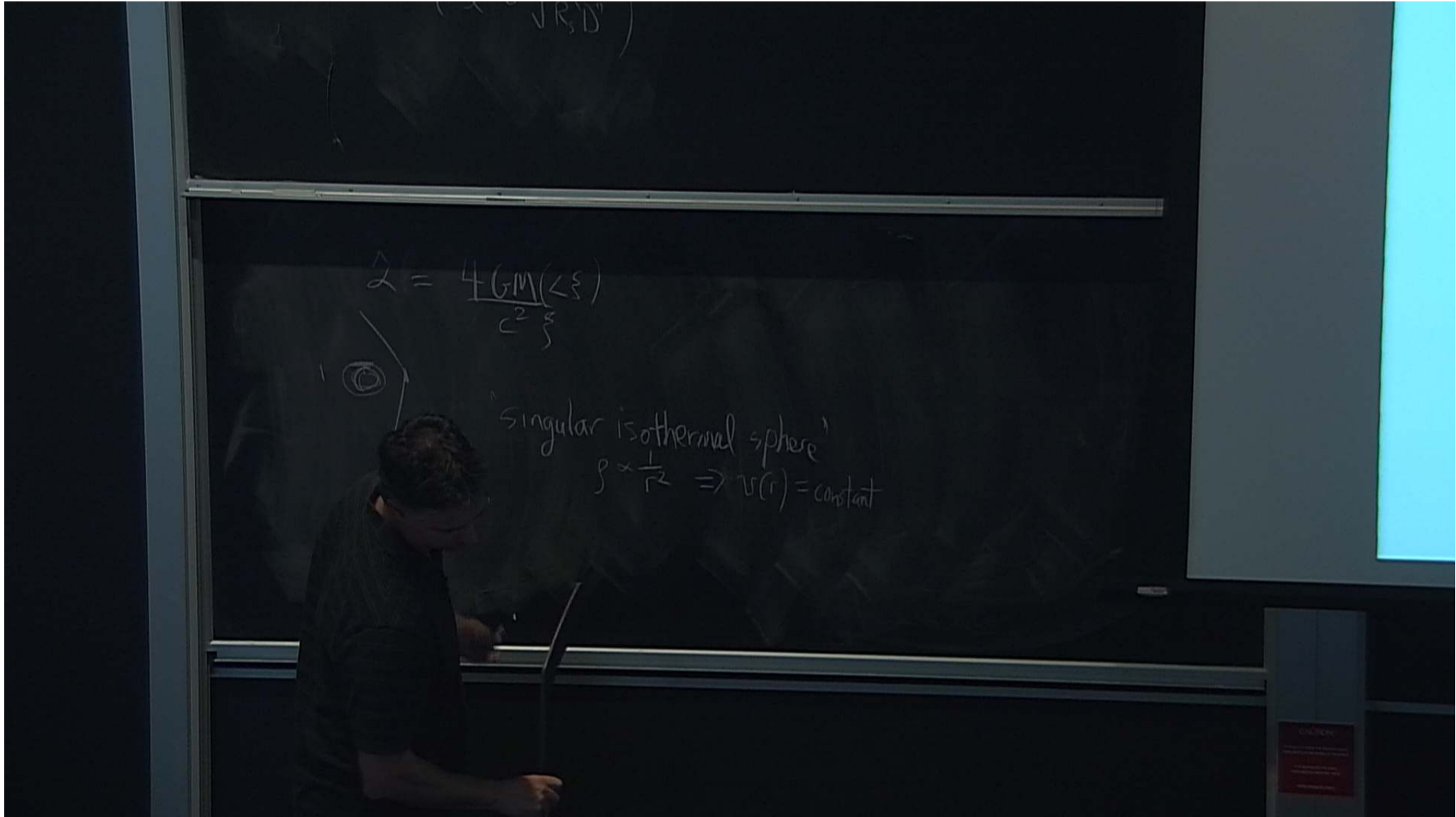
$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$

$$\psi(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$

lensing potential
convergence







$$\lambda = \frac{4GM(\rho_s)}{c^2 \xi}$$



'singular isothermal sphere'

$$\rho \propto \frac{1}{r^2} \Rightarrow v(r) = \text{constant}$$

$$\Sigma(\rho) = \frac{\sigma_v^2}{2G} \frac{1}{\xi}$$

$$\Rightarrow \lambda = \frac{4\pi \sigma_v^2}{c^2} \frac{D_{dr}}{D_s}$$

$$\lambda = \frac{4GM(\rho)}{c^2 \xi}$$



'singular isothermal sphere'

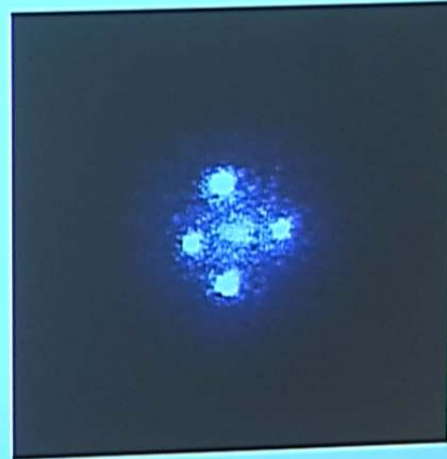
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$$\lambda = \frac{4\pi \sigma_v^2}{c^2} \frac{D_{dr}}{D_s} = \frac{\rho}{\xi}$$

Strong lensing

- multiple paths from source to observer due to gravitational lens along line of sight
- lenses have been discovered in many wavelengths: radio, mm, submm, optical



isothermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{v_{\text{sc}}^2}{2G} \frac{1}{r} \Rightarrow \theta = \sqrt{\frac{4\pi G}{c^2} \frac{D_{\text{sc}}}{D_s}} = \theta_E$

Strong lensing

- multiple paths from source to observer due to gravitational lens along line of sight
- sensitive to the **total mass** inside the Einstein radius, but that is typically $\sim 1/2$ dark matter, $1/2$ normal baryons that we can see already



Strong lensing



multiple paths from source to observer due to gravitational lens along line of sight

Point MASS

other 'mass sphere'
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$

$$\frac{G_M^2}{2G} \frac{1}{3} \Rightarrow \alpha = \frac{4\pi G_M^2}{c^2} \frac{D_d}{D_s} = \theta$$

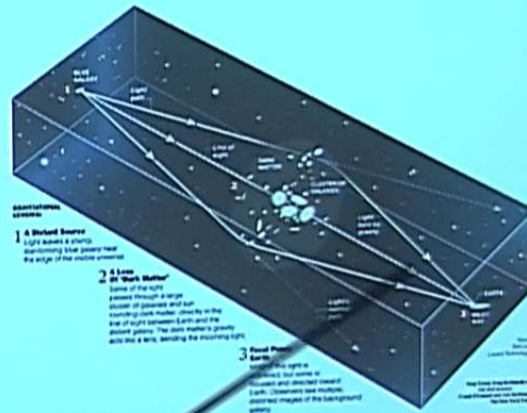






Time Delays

- two sources of time delays:
- geometry (longer and shorter paths)
- Shapiro delay (propagating through a gravitational potential)



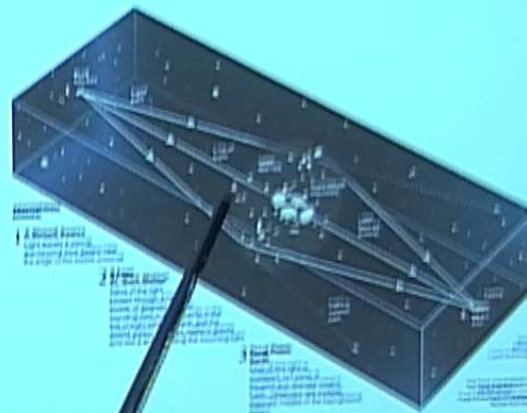
$$r - \theta_s)^2 - \psi(\theta_1)$$

http://www.lsst.org/lsst/science/scientist_dark_matter

isothermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{G M}{2G} \frac{1}{r^3} \Rightarrow \rho = \frac{4\pi G}{c^2} \frac{D_s}{D_s} = \rho$

Time Delays

- two sources of time delays:
- geometry (longer and shorter paths)
- Shapiro delay (propagating through a

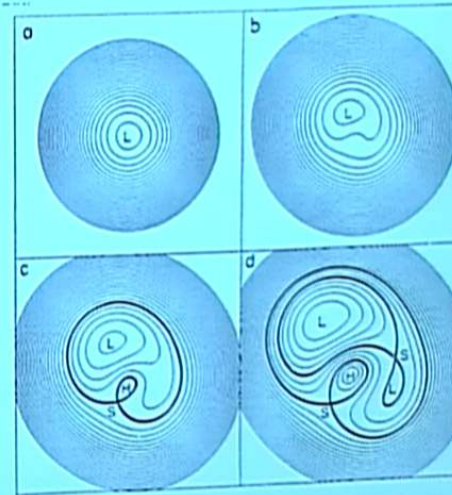


$$\frac{1}{2}(\theta_1 - \theta_s)^2 = \psi(\theta_1)$$

http://www.ligo.caltech.edu/~science/scientist_dark_matter

Fermat's Principle

- images form at stationary points of the arrival time surface
- maxima, minima, saddle points all cause images
- lumps of mass perturb arrival time surface, moving images and distorting their shapes

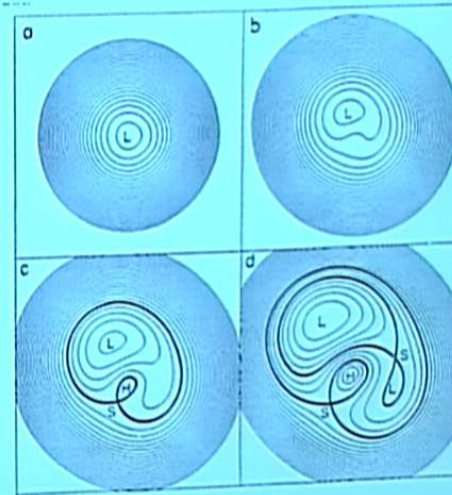


$$\tau(\theta_I; \theta_S) = \frac{1}{2}(\theta_I - \theta_S)^2 - \psi(\theta_I)$$

Blandford & Narayan 1986

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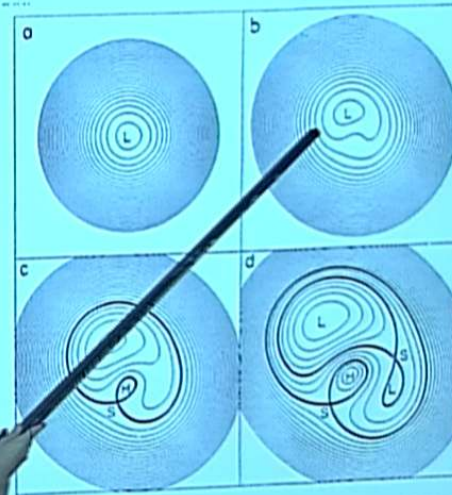


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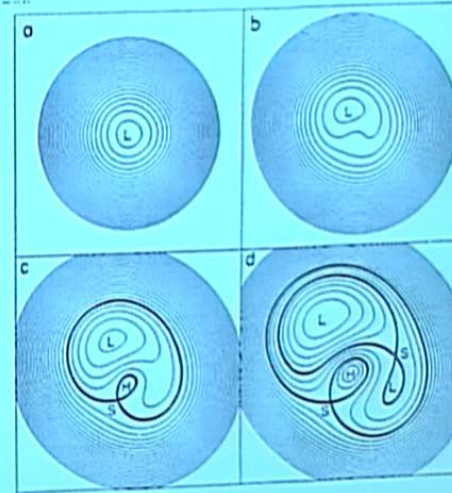
Blandford & Narayan 1986

$$\tau(\theta_1; \theta_2, s) = \int_{\theta_1}^{\theta_2} \sqrt{1 - \frac{D_s^2}{D_l^2} \sin^2 \theta} d\theta$$

other 'sphere'
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{G_M^2}{2G} \frac{1}{3} \Rightarrow \chi = \frac{4\pi G_M^2 D_s}{c^2 D_l} = \theta$

Fermat's Principle

- images form at stationary points of the arrival time surface
- maxima, minima, saddle points all cause images
- lumps of mass perturb arrival time surface, moving images and changing their shapes



Blandford & Narayan 1986

$$(\theta_s) = \frac{1}{2}(\theta_s^2 - \psi(\theta_s))$$

other 'thermal sphere'
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{G_M^2}{2G} \frac{1}{3} \Rightarrow \chi = \frac{4\pi G_M^2 D_s}{c^2 D_s} = \theta$

SN Refsdal: Lensing of a Supernova!

caught at
different
times in
different
images

likely missed
one a few
decades
ago, will see
another
image in a
few years

et al 2014



Point mass

isothermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$

$$\frac{G M^2}{2G} \frac{1}{r^2} \Rightarrow \frac{4\pi G^2}{r^2} \frac{D_s}{D_s} = \frac{1}{r^2} \frac{D_s}{D_s}$$

$$\mathcal{A}(\vec{\theta}) = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix},$$

where we have introduced the components of the shear $\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma|e^{2i\varphi}$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}), \quad \gamma_2 = \psi_{,12},$$

$$\mathcal{A} = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix} \quad \begin{array}{l} \text{distortion has overall} \\ \text{magnification} \\ \text{(hard to observe)} \end{array}$$

$$g(\vec{\theta}) \equiv \frac{\gamma(\vec{\theta})}{1 - \kappa(\vec{\theta})} \quad \text{reduced shear}$$

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distortion has overall magnification
(hard to observe)

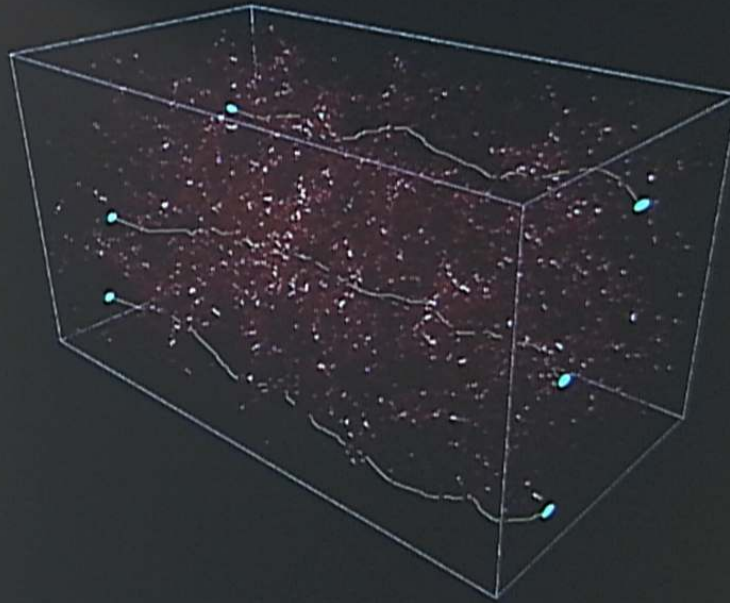
$$g(\vec{\theta}) \equiv \frac{\gamma(\vec{\theta})}{1 - \kappa(\vec{\theta})}$$

reduced shear



isothermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{G_0^2}{2G} \frac{1}{3} \Rightarrow \kappa = \frac{4\pi G_0^2 D_s}{2} \frac{D_s}{D_s} = 1$

DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES



SIMULATION COURTESY MCG GROUP @ COLEMAN IAP

$\frac{1}{2} \frac{d^2x}{dt^2}$
Point
mass

athermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{G M^2}{2G} \frac{1}{3} \Rightarrow \angle = \frac{4\pi G^2 D_0^2}{r^2 D_0} = \theta$

Galaxies are not round

- individual galaxies have complex morphologies
- solution: average over many galaxies

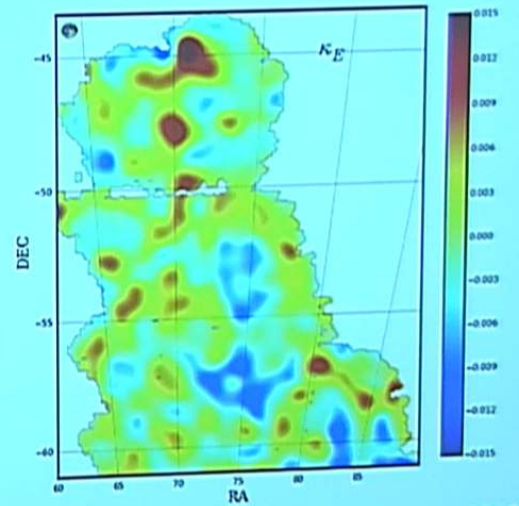


Point Mass

isothermal sphere
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$
 $\frac{v_c^2}{2G} \frac{1}{r} \Rightarrow \rho = \frac{4\pi v_c^2}{r^2} \frac{D_0}{D_0} = \frac{1}{r^2}$

Cosmic mass maps

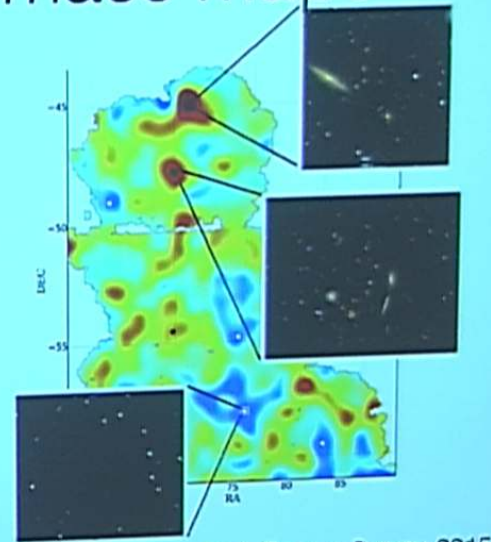
- averaging over the shapes of many galaxies provides local estimate of the cosmic shear
- shear map allows reconstruction of gravitational potential, and thus the mass



Dark Energy Survey 2015

Cosmic mass maps

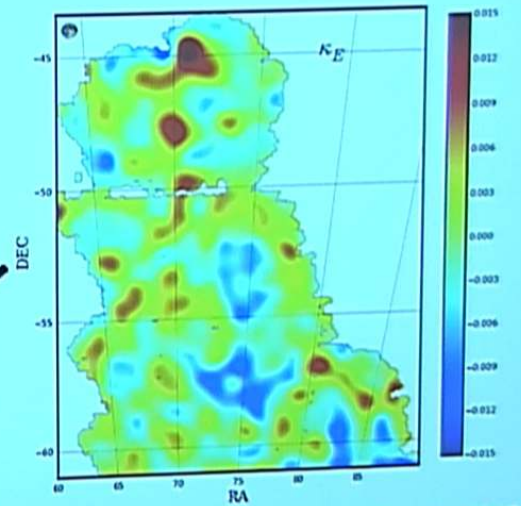
- averaging over the shapes of many galaxies provides local estimate of the cosmic shear
- shear map allows reconstruction of gravitational potential, and thus the mass
- strong correlations between where the mass is and where the galaxies are



Dark Energy Survey 2015

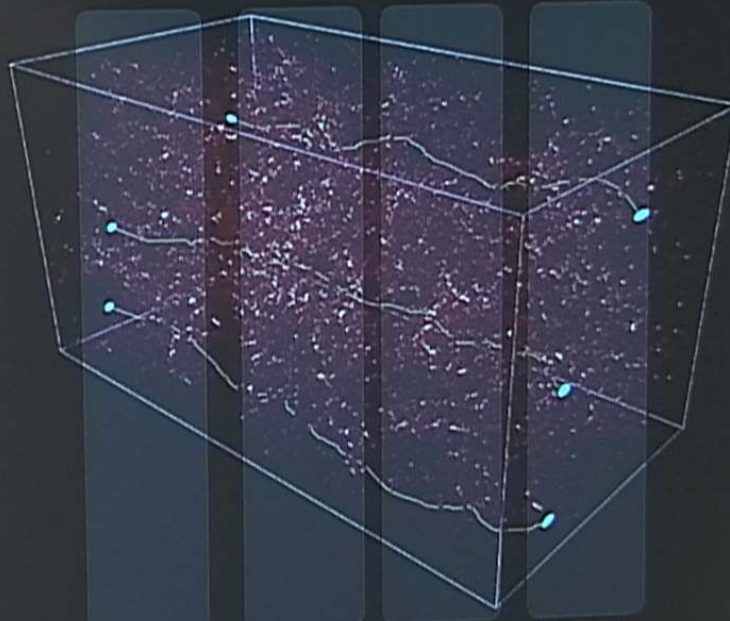
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Dark Energy Survey 2015

DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES



SIMULATION COURTESY MCG GROUP, © COLUMBIA UAP

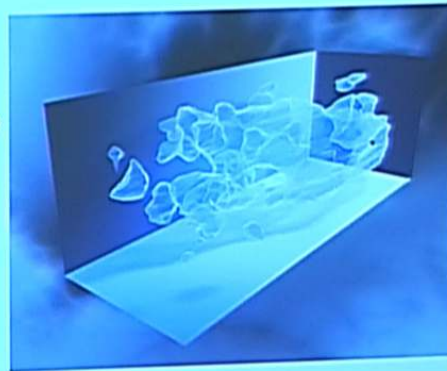
D_{th}
 R, D_d

POINT MASS

\star $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ $\frac{1}{128}$ $\frac{1}{256}$ $\frac{1}{512}$ $\frac{1}{1024}$ $\frac{1}{2048}$ $\frac{1}{4096}$ $\frac{1}{8192}$ $\frac{1}{16384}$ $\frac{1}{32768}$ $\frac{1}{65536}$ $\frac{1}{131072}$ $\frac{1}{262144}$ $\frac{1}{524288}$ $\frac{1}{1048576}$ $\frac{1}{2097152}$ $\frac{1}{4194304}$ $\frac{1}{8388608}$ $\frac{1}{16777216}$ $\frac{1}{33554432}$ $\frac{1}{67108864}$ $\frac{1}{134217728}$ $\frac{1}{268435456}$ $\frac{1}{536870912}$ $\frac{1}{1073741824}$ $\frac{1}{2147483648}$ $\frac{1}{4294967296}$ $\frac{1}{8589934592}$ $\frac{1}{17179869184}$ $\frac{1}{34359738368}$ $\frac{1}{68719476736}$ $\frac{1}{137438953472}$ $\frac{1}{274877906944}$ $\frac{1}{549755813888}$ $\frac{1}{1099511627776}$ $\frac{1}{2199023255552}$ $\frac{1}{4398046511104}$ $\frac{1}{8796093022208}$ $\frac{1}{17592186044416}$ $\frac{1}{35184372088832}$ $\frac{1}{70368744177664}$ $\frac{1}{140737488355328}$ 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Weak lensing tomography

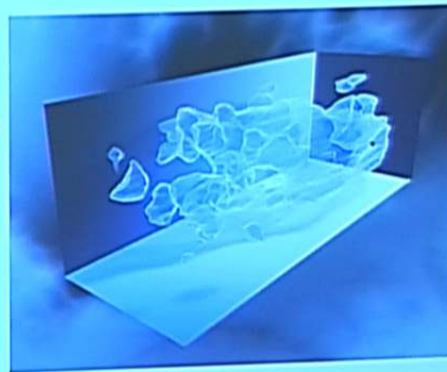
- using source galaxies at different redshifts allows one to reconstruct the 3D mass distribution
 - mass, not galaxy, density means you can measure the time evolution of the density fluctuations
- results using Hubble over
~1 sq deg



Massey et al

Weak lensing tomography

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E-modes/B-modes

- E-modes vary spatially parallel or perpendicular to polarization direction
- B-modes vary spatially at 45 degrees
- CMB
 - scalar perturbations only generate *only* E



Summary

- gravitational lensing is everywhere (galaxies, quasars, CMB, [also stars, to find planets])
- sensitive to mass, not light
- probes both amplitude of matter clustering and geometry