

Title: Cosmology & Observations: Gravitational lensing

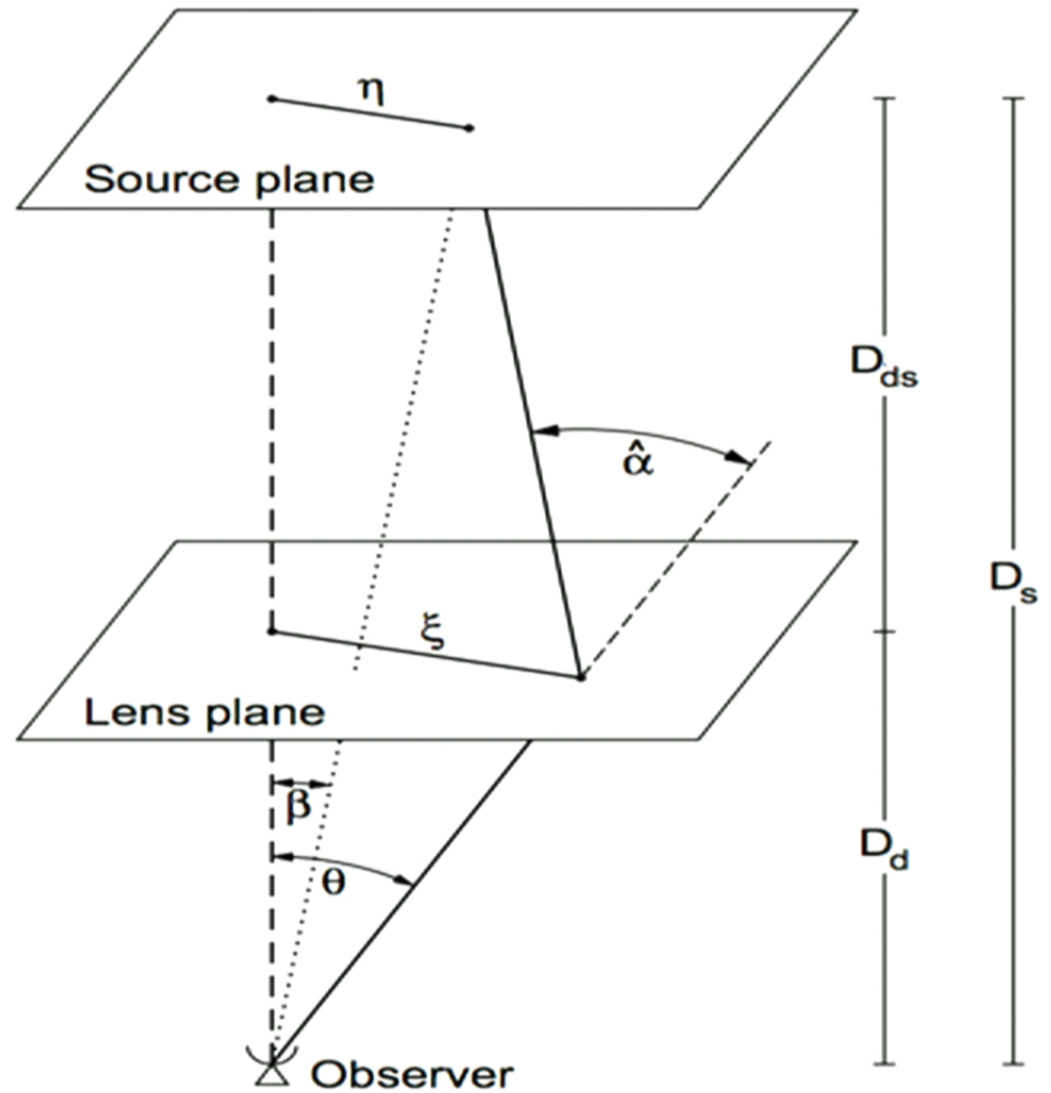
Date: Jul 15, 2015 04:00 PM

URL: <http://pirsa.org/15070051>

Abstract:

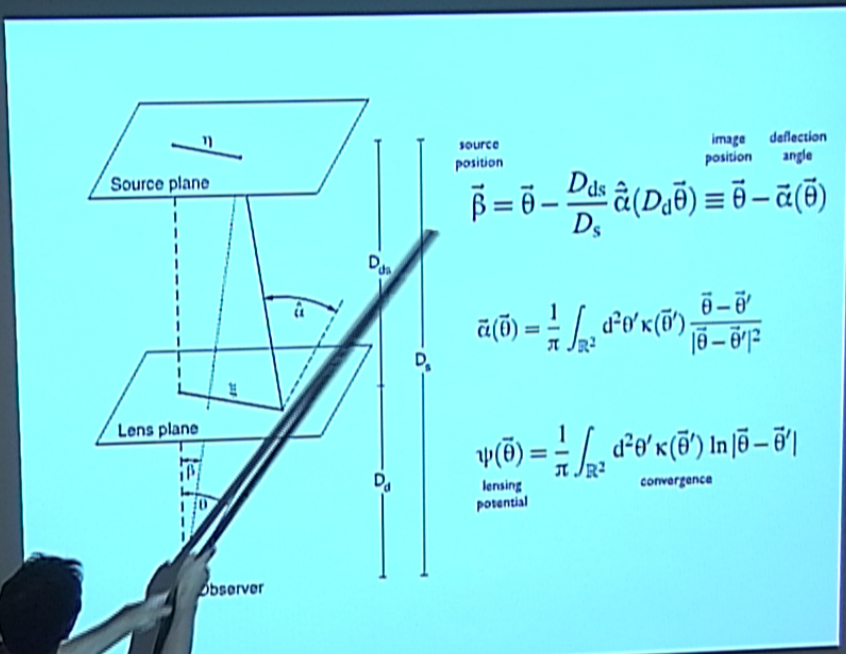
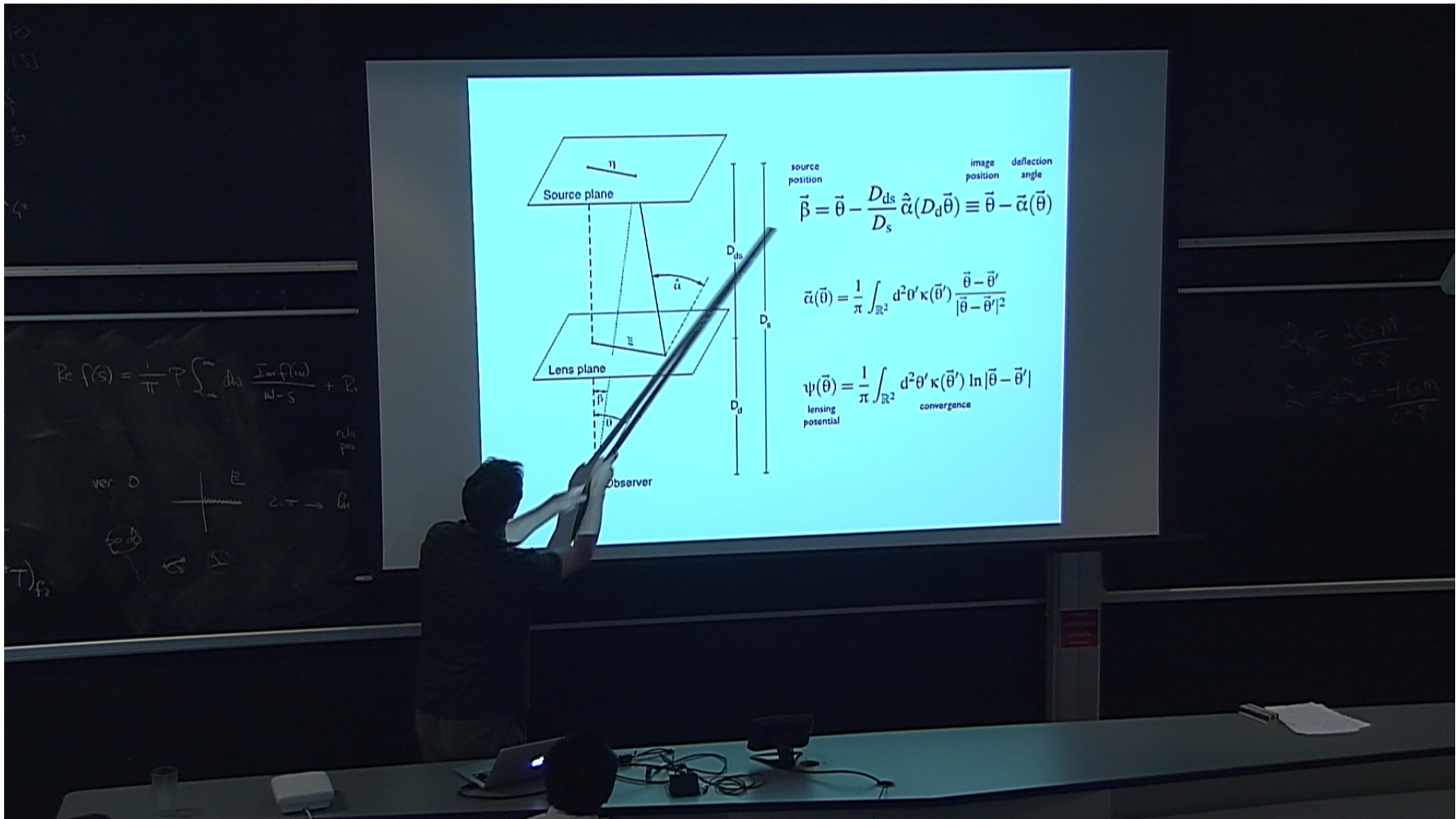
# Gravitational lensing

$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$



Bartelmann & Schneider  
<http://arxiv.org/abs/astro-ph/9912508>





source position                      image position      deflection angle

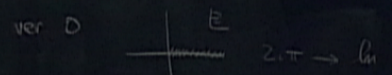
$$\vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \vec{\theta}) \equiv \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$

$$\psi(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$

lensing potential                      convergence

$$\text{Re } f(s) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dx \frac{\text{Im } f(x)}{x-s} + \mathcal{P}$$

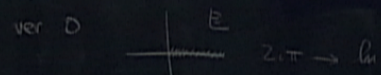


$$\Sigma = \frac{2GM}{r^2}$$

$$\Sigma = \frac{2GM}{r^2} = \frac{4GM}{c^2 r^2}$$



$$\text{Re } f(s) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dx \frac{\text{Im } f(x)}{x-s} + \mathcal{P}$$



source position  $\vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \vec{\theta}) \equiv \vec{\theta} - \vec{\alpha}(\vec{\theta})$  image position deflection angle

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$

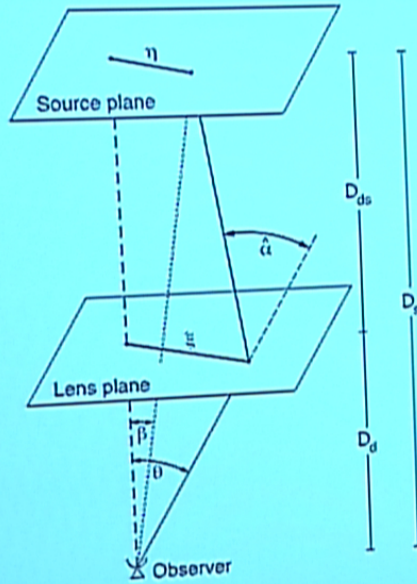
$$\psi(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$

lensing potential convergence

$$\Phi = \frac{2GM}{r}$$

$$\Phi = -\frac{GM}{r} = -\frac{GM}{\frac{c^2}{2g}}$$





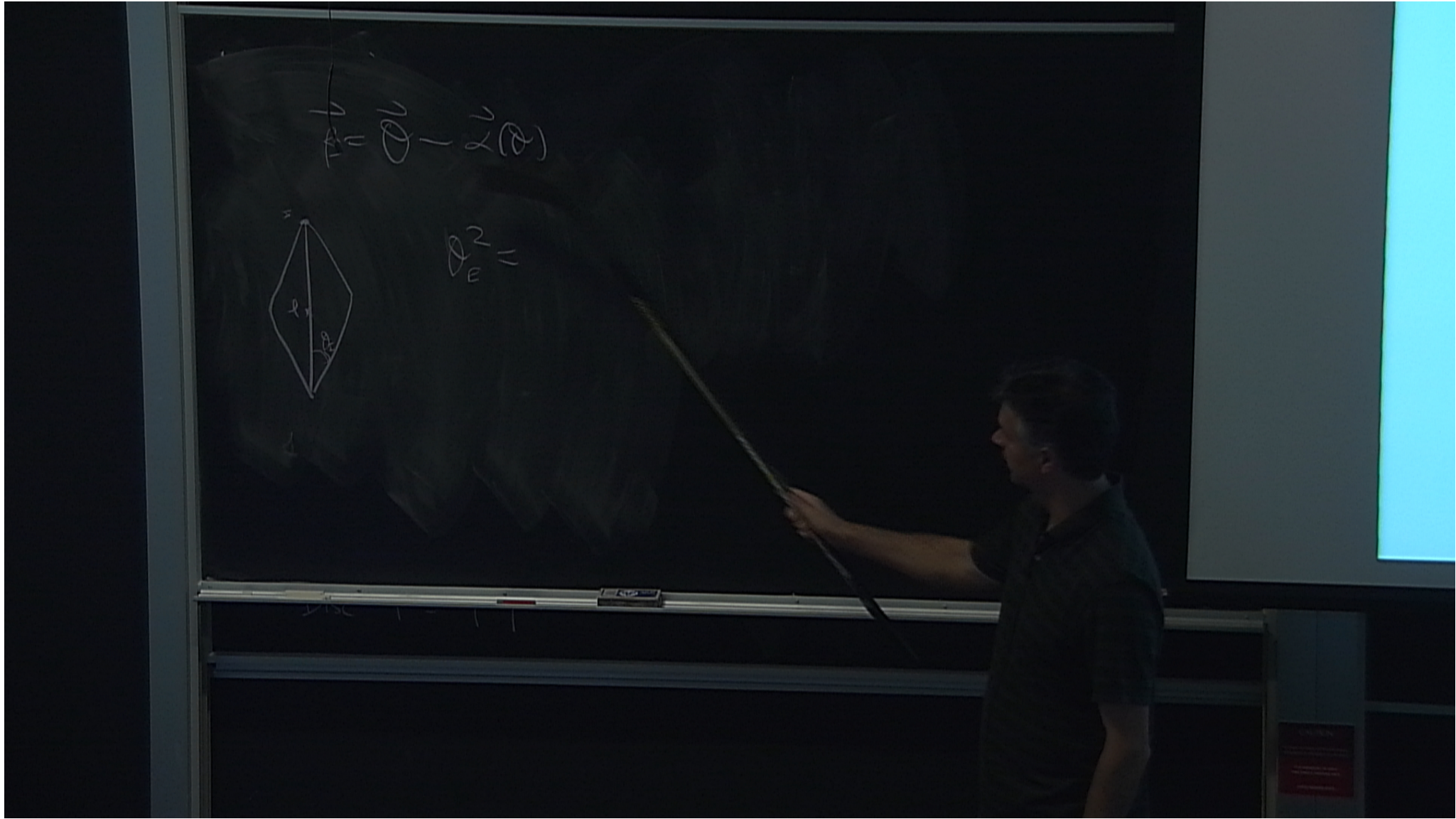
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source position
image position
deflection angle

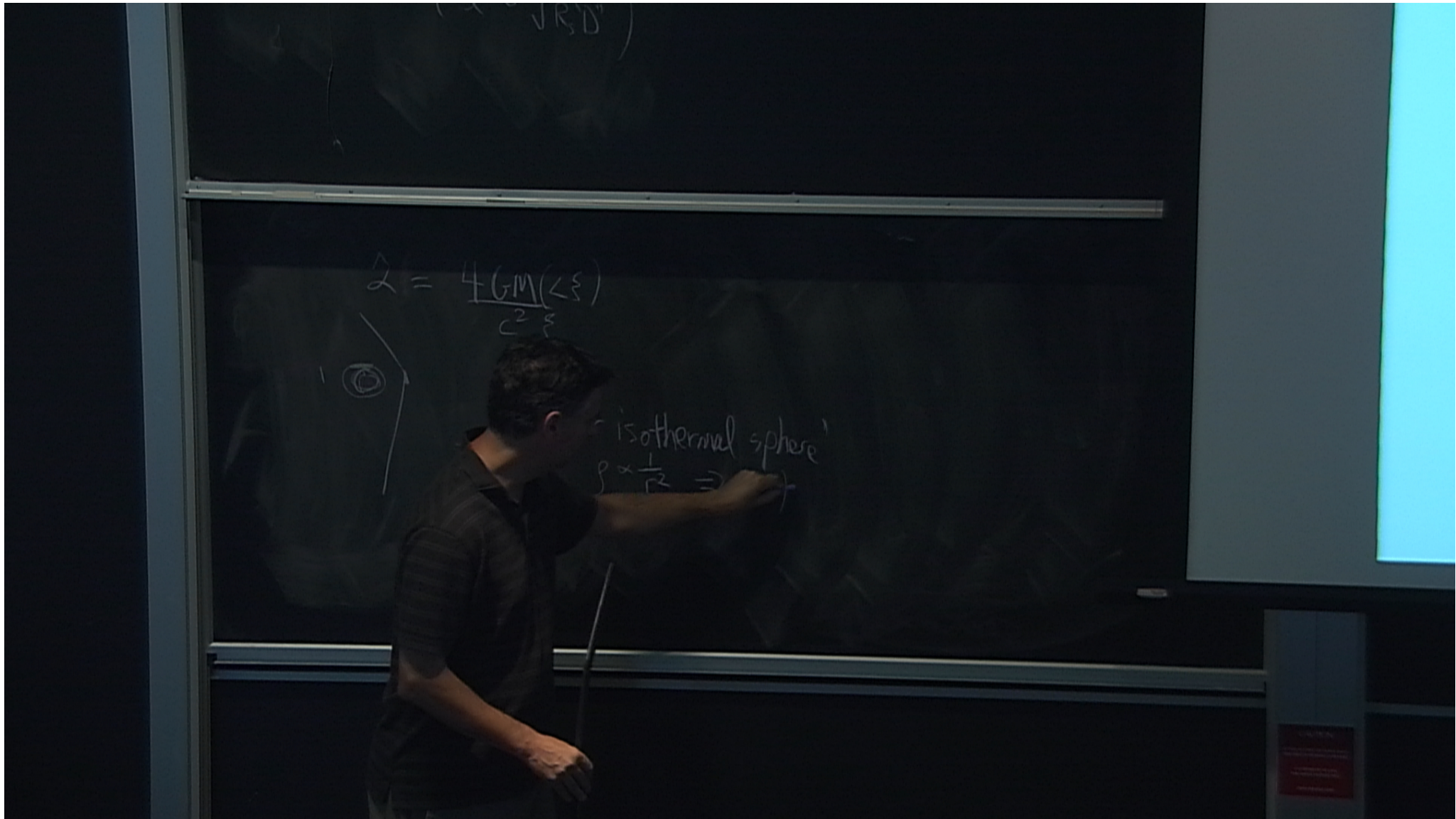
$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$

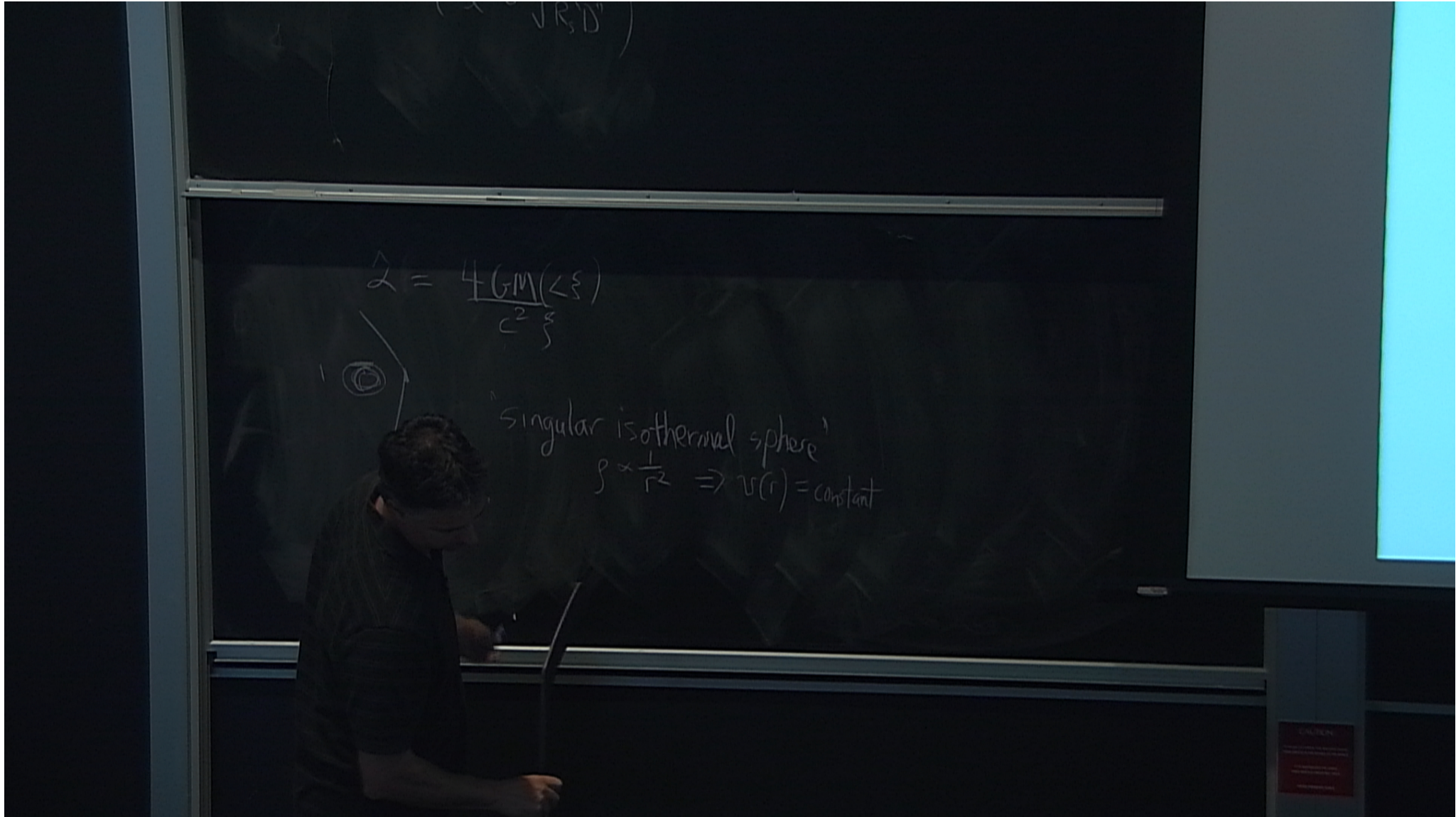
$$\psi(\vec{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$

lensing potential
convergence











$$\lambda = \frac{4GM(\rho_s)}{c^2 \xi}$$



'singular isothermal sphere'

$$\rho \propto \frac{1}{r^2} \Rightarrow v(r) = \text{constant}$$

$$\Sigma(\rho) = \frac{\sigma_v^2}{2G} \frac{1}{\xi}$$

$$\lambda = \frac{4\pi \sigma_v^2}{c^2} \frac{D_{ds}}{D_s}$$



$$\lambda = \frac{4GM(\rho)}{c^2 \xi}$$



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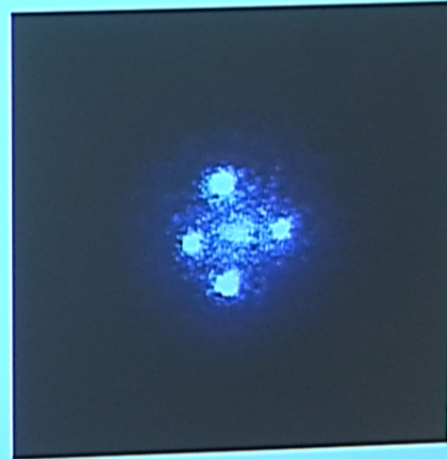
$$\Sigma(\rho) = \frac{\sigma_v^2}{2G} \frac{1}{\xi}$$

$$\Rightarrow \lambda = \frac{4\pi \sigma_v^2}{c^2} \frac{D_{dr}}{D_s} = \frac{\rho}{E}$$



# Strong lensing

- multiple paths from source to observer due to gravitational lens along line of sight
- lenses have been discovered in many wavelengths: radio, mm, submm, optical

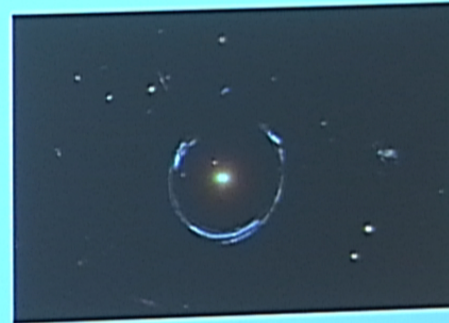


othermel sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$   
 $\frac{G_M^2}{2G} \frac{1}{r^3} \Rightarrow \left[ \frac{4\pi G_M^2}{c^2} \frac{D_d}{D_s} \right] = \theta^2$



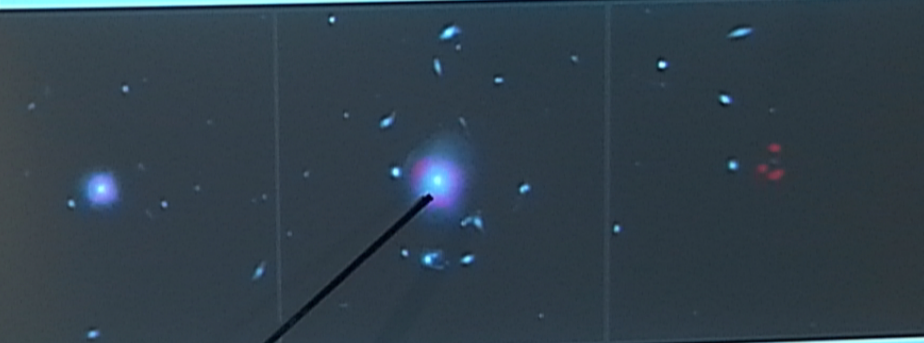
# Strong lensing

- multiple paths from source to observer due to gravitational lens along line of sight
- sensitive to the **total mass** inside the Einstein radius, but that is typically  $\sim 1/2$  dark matter,  $1/2$  normal baryons that we can see already





# Strong lensing



multiple paths from source to observer due to gravitational lens along line of sight

Point MASS

isothermal sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$

$$\frac{v_{\text{sc}}^2}{2G} \frac{1}{s} \Rightarrow \alpha = \frac{4\pi G^2 D_d}{c^2 D_s} = \theta$$









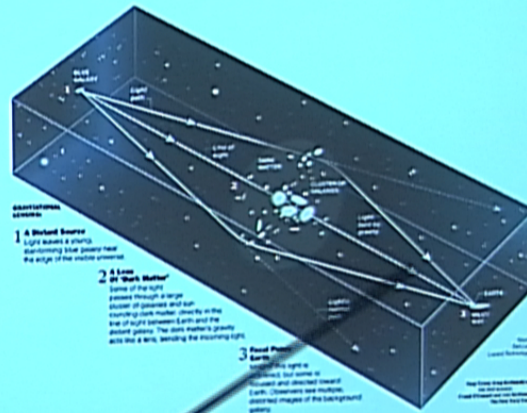






# Time Delays

- two sources of time delays:
- geometry (longer and shorter paths)
- Shapiro delay (propagating through a gravitational potential)



$$r - \theta_s)^2 - \psi(\theta_s)$$

[http://www.lsst.org/lsst/science/scientist\\_dark\\_matter](http://www.lsst.org/lsst/science/scientist_dark_matter)

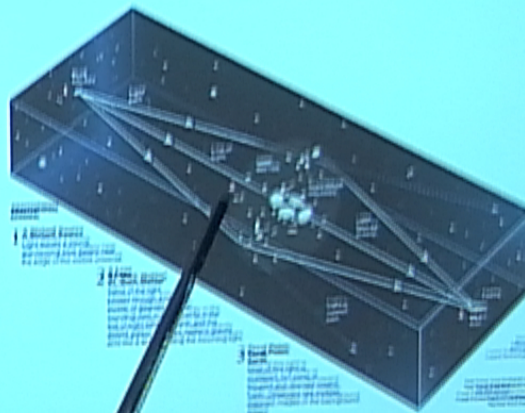
Point MASS

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# Time Delays

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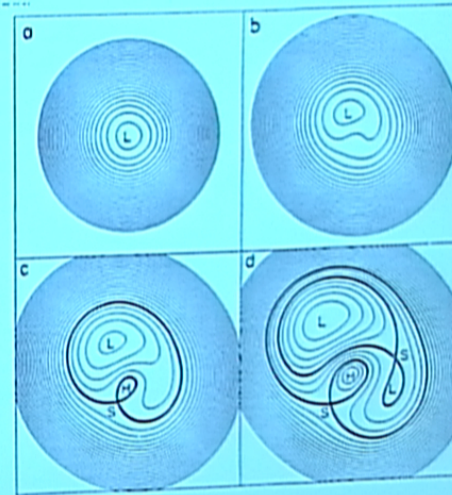
$$\psi(\theta_1 = \theta_s) = \psi(\theta_1)$$

[http://www.kit.edu/science/scientist\\_dark\\_matter](http://www.kit.edu/science/scientist_dark_matter)



# Fermat's Principle

- images form at stationary points of the arrival time surface
- maxima, minima, saddle points all cause images
- lumps of mass perturb arrival time surface, moving images and distorting their shapes



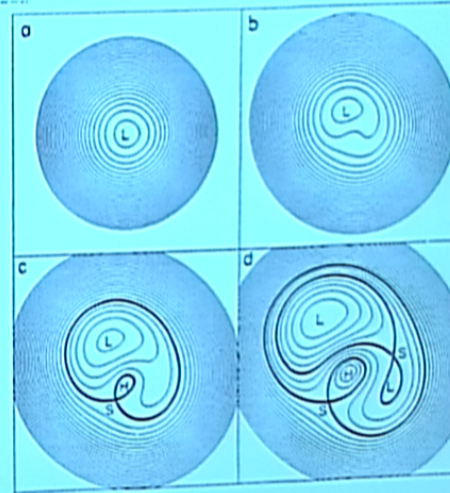
$$\tau(\theta_I; \theta_S) = \frac{1}{2}(\theta_I - \theta_S)^2 - \psi(\theta_I)$$

Blandford & Narayan 1986



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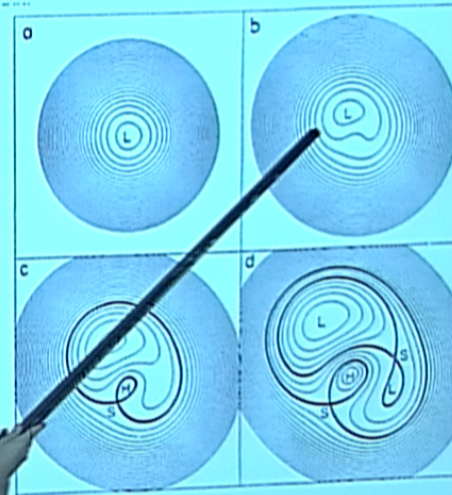
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Blandford & Narayan 1986

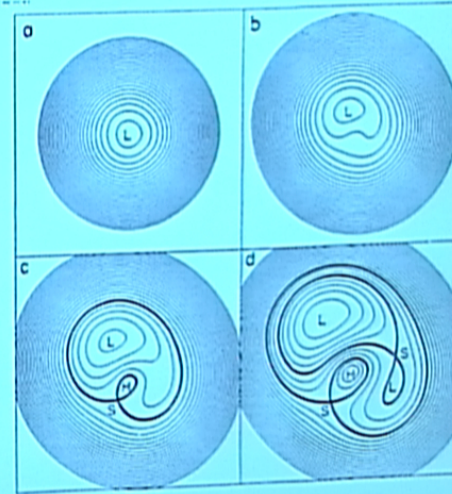
$$\tau(\theta_1; \theta_2, s) = \int_{\theta_1}^{\theta_2} \sqrt{1 - \frac{2GM}{rc^2}} \frac{D_\theta}{c} d\theta$$

other 'sphere'  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$   
 $\frac{G M^2}{2G} \frac{1}{3} \Rightarrow \langle \dots \rangle = \frac{4\pi G^2 D_\theta}{c^2 D_\theta} = \dots$



# Fermat's Principle

- images form at stationary points of the arrival time surface
- maxima, minima, saddle points all cause images
- lumps of mass perturb arrival time surface, moving images and changing their shapes



Blandford & Narayan 1986

$$t(\theta_s) = \frac{1}{c}(\theta_s^2 - \psi(\theta_s))$$

other 'mass sphere'  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$   
 $\frac{G M^2}{2G} \frac{1}{3} \Rightarrow \chi = \frac{4\pi G^2 D_s^2}{c^2 D_s} = \theta_s^2$



# SN Refsdal: Lensing of a Supernova!

caught at  
different  
times in  
different  
images

likely missed  
one a few  
decades  
ago, will see  
another  
image in a  
few years

et al 2014



Point mass

isothermal sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$

$$\frac{G M^2}{2G} \frac{1}{r^3} \Rightarrow \frac{4\pi G^2}{r^2} \frac{D_s}{D_s} = \theta$$



$$\mathcal{A}(\vec{\theta}) = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left( \delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix},$$

where we have introduced the components of the shear  $\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma|e^{2i\varphi}$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}), \quad \gamma_2 = \psi_{,12},$$

$$\mathcal{A} = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix} \quad \begin{array}{l} \text{distortion has overall} \\ \text{magnification} \\ \text{(hard to observe)} \end{array}$$

$$g(\vec{\theta}) \equiv \frac{\gamma(\vec{\theta})}{1 - \kappa(\vec{\theta})} \quad \text{reduced shear}$$

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distortion has overall  
magnification  
*(hard to observe)*

$$g(\vec{\theta}) \equiv \frac{\gamma(\vec{\theta})}{1 - \kappa(\vec{\theta})}$$

reduced shear

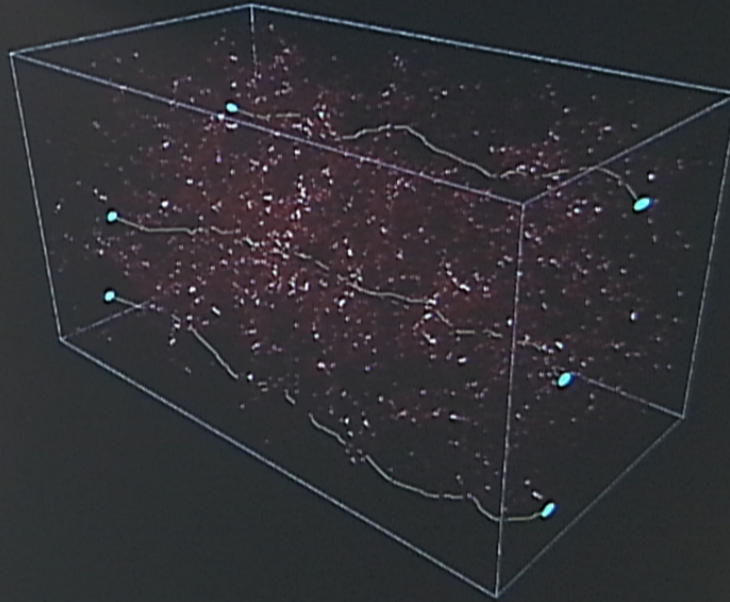


isothermal sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$

$$\frac{G^2}{2G} \frac{1}{3} \Rightarrow \kappa = \frac{4\pi G^2 D_s^2}{c^2} \frac{D_d}{D_s} = \frac{4}{3}$$



DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES



SIMULATION COURTESY MCG GROUP, © COLEMAN, IAP

Point MASS

isothermal sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$   
 $\frac{v_{\text{rot}}^2}{2G} \frac{1}{r} \Rightarrow \rho = \frac{4\pi G v_{\text{rot}}^2}{r^2} \frac{D_1}{D_2} = \frac{1}{r^2}$



# Galaxies are not round

- individual galaxies have complex morphologies
- solution: average over many galaxies



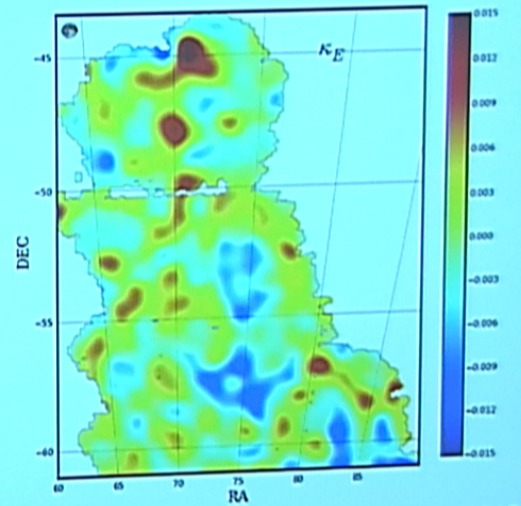
Point MASS

isothermal sphere  
 $\frac{1}{r^2} \Rightarrow v(r) = \text{constant}$   
 $\frac{G M^2}{2G} \frac{1}{r} \Rightarrow \frac{1}{2} = \frac{4\pi G^2 D_0^2}{r^2 D_0} = \frac{1}{2}$



# Cosmic mass maps

- averaging over the shapes of many galaxies provides local estimate of the cosmic shear
- shear map allows reconstruction of gravitational potential, and thus the mass

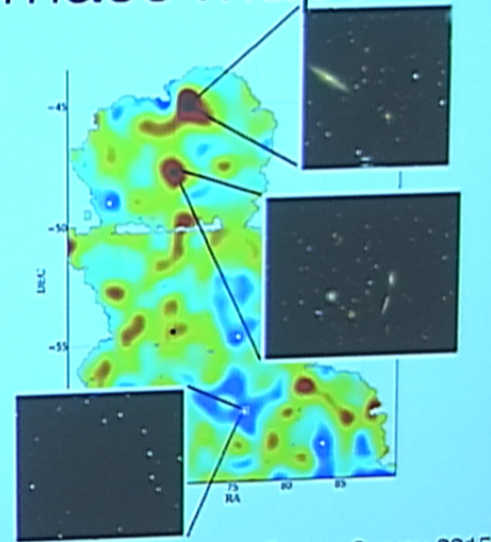


Dark Energy Survey 2015



# Cosmic mass maps

- averaging over the shapes of many galaxies provides local estimate of the cosmic shear
- shear map allows reconstruction of gravitational potential, and thus the mass
- strong correlations between where the mass is and where the galaxies are

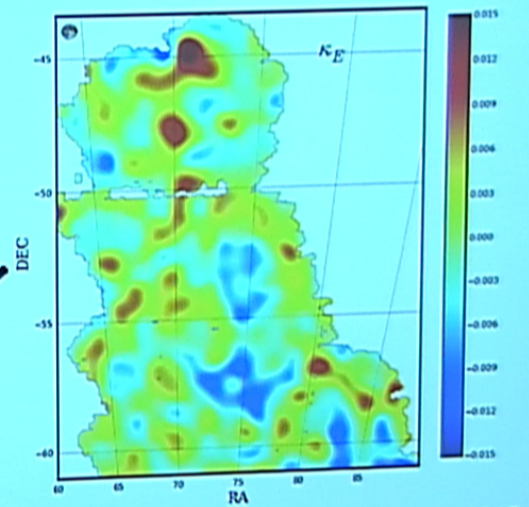


Dark Energy Survey 2015



# Cosmic mass maps

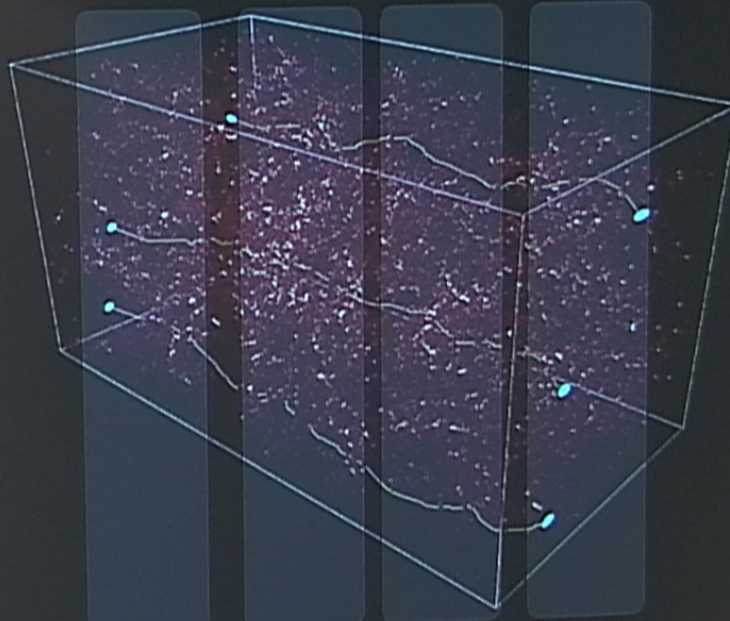
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Dark Energy Survey 2015



DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES



SIMULATION COURTESY MCG GROUP, © COLEMAN MAP

$D_{\text{th}}$   
 $D_{\text{td}}$

POINT MASS

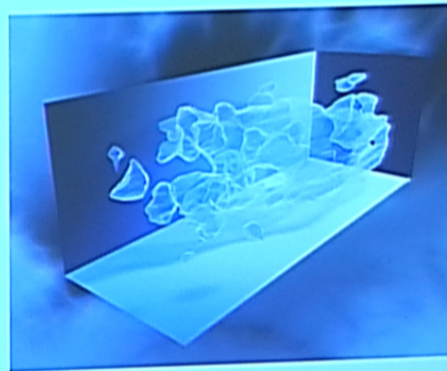
$\star$  15 15 0  
|||||  
|||||

$\kappa = 4\pi G^2 \frac{D_{\text{th}}}{c^2} \frac{D_{\text{td}}}{D_s} = 1$



# Weak lensing tomography

- using source galaxies at different redshifts allows one to reconstruct the 3D mass distribution
  - mass, not galaxy, density means you can measure the time evolution of the density fluctuations
- results using Hubble over  
~1 sq deg

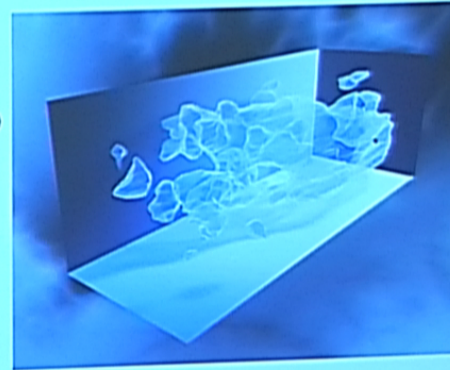


Massey et al



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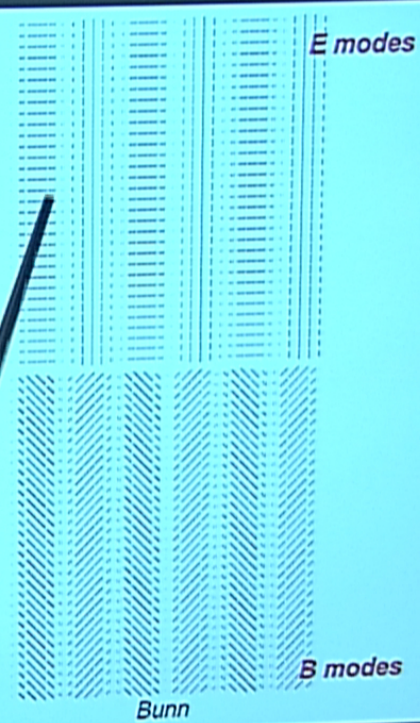


Massey et al



## E-modes/B-modes

- E-modes vary spatially parallel or perpendicular to polarization direction
- B-modes vary spatially at 45 degrees
- CMB
  - scalar perturbations only generate \*only\* E





# Summary

- gravitational lensing is everywhere (galaxies, quasars, CMB, [also stars, to find planets])
- sensitive to mass, not light
- probes both amplitude of matter clustering and geometry