

Title: Amplitudes: BCFW Recursion

Date: Jul 15, 2015 02:30 PM

URL: <http://pirsa.org/15070050>

Abstract:

Derived A_3

Looked at factorization

$$A_3 = (12)^{e_{12}} (23)^{e_{23}} (31)^{e_{31}}$$

$[i,j]=0$

$$e_{12} = h_3 - h_1 - h_2$$
$$e_{23} = h_1 - h_2 - h_3$$
$$e_{31} = h_2 - h_1 - h_3$$

$$|h_i| = 1$$

$$|h$$

Derived A_3

Looked at factorization

$$A_3 = (12)^{e_{12}} (23)^{e_{23}} (31)^{e_{31}}$$

$$[i] = 0$$

M_3

$$\left(\frac{(12)^3}{(23)(31)} \right)^2$$

$$e_{12} = h_3 - h_1 - h_2$$

$$e_{23} = h_1 - h_2 - h_3$$

$$e_{31} = h_2 - h_1 - h_3$$

$$|h_i| = 1$$

$$|h_i| = 2$$

Derived A_3

Looked at factorization

$$\langle 12 \rangle^{e_{12}} \langle 23 \rangle^{e_{23}} \langle 31 \rangle^{e_{31}}$$

$$\left(\begin{array}{c} \langle 12 \rangle^3 \\ \hline \langle 23 \rangle \langle 31 \rangle \end{array} \right)^2$$

$$e_{12} = h_3 - h_1 - h_2$$

$$e_{23} = h_1 - h_2 - h_3$$

$$e_{31} = h_2 - h_1 - h_3$$

$$|h_i| = 1$$

$$|h_i| = 2$$

On-shell or BCFW recursion relations

$$[j, l] : |j] \rightarrow |j] - z|l]$$

$$|l] \rightarrow |l] + z|j]$$

$$0 = k_1 + \dots + k_n \rightarrow 0 = k_1 + \dots + k_n$$

$$k^2 =$$

On-shell or BCFW recursion relations

$$[j, l] \rightarrow [j] - z[l]$$

$$|l\rangle \rightarrow |l\rangle + z|j\rangle$$

$$0 = k_1 + \dots + k_n \rightarrow 0 = k_1 + \dots + k_n$$

$$k_i^2 = 0 \rightarrow k_i^2 = 0$$

$$A \mapsto A(z)$$

$$A(z) \rightarrow 0 \quad z \rightarrow \infty$$

$$\int_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

$$|C| \rightarrow \infty$$

pole at $z=0$: Res = $A(0)$.

$$A(0) = -\sum$$

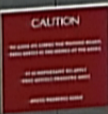
CAUTION
Do not touch the blackboard
as it is very hot.
Please do not touch the blackboard
as it is very hot.
Please do not touch the blackboard
as it is very hot.

$$\int_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

$$|C| \rightarrow \infty$$

pole at $z=0$: Res = $A(0)$.

$$A(0) = - \sum_{\substack{\text{poles } z \\ z \neq 0}} \text{Res}_{z=z_a} \frac{A(z)}{z}$$



$$\int_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

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pole at $z=0$: $\text{Res} = A(0)$.

$$A(0) = - \sum_{\substack{\text{poles } z \\ z \neq 0}} \text{Res}_{z=z_a} \frac{A(z)}{z}$$

poles in $A(z)$

$$\text{factorization : } (k_r + \dots + k_s)^2 = 0$$

$$|C| \rightarrow \infty$$

pole at $z=0$: Res = $A(0)$.

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poles in $A(z)$

$$\text{factorization: } (k_r + \dots + k_s)^2 = 0$$

$$\sum_{\substack{j, l \in \{1, \dots, n\} \\ j \neq l}} k_j k_l$$

$$\int_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

$$|C| \rightarrow \infty$$

pole at $z=0$: $\text{Res} = A(0)$.

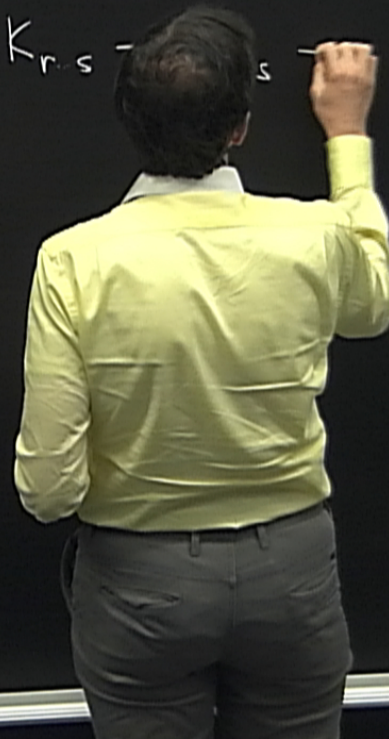
$$A(0) = - \sum_{\substack{\text{poles} \\ z \neq 0}} \text{Res}_{z=z_a} \frac{A(z)}{z}$$

poles in $A(z)$.

$$\text{factorization} : (k_r + \dots + k_s)^2 = 0$$

$$\sum_{j \in S} \sum_{l \in S} \sum_{j \in E} \{r_l, s\}$$

$$k_r s - s$$



CAUTION
Do not lean on display. The display board may become damaged if the board is not supported by the stand.

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$$\oint_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

$|C| \rightarrow \infty$

pole at $z=0$: $\text{Res} = A(0)$

$$A(0) = - \sum_{\substack{\text{poles} \\ z \neq 0}} \text{Res}_{z=z_a} \frac{A(z)}{z}$$

poles in $A(z)$

factorisation : $(k_r + tk_s)^2 = 0$

$$\sum_{j \in \mathbb{C}} \sum_{l \in \mathbb{N}} \{r_l, s_l\}$$

$$K_{r,s} \rightarrow K_{r,s} - \frac{z}{2} \langle j | \mu | l \rangle$$

$$K_{r,s}^2 \rightarrow K_{r,s}^2 - z \langle j | K_{r,s} | l \rangle$$

CAUTION

$$\int_C \frac{dz}{z} \frac{A(z)}{z} = 0$$

$$|C| \rightarrow \infty$$

pole at $z=0$: $\text{Res} = A(0)$.

$$A(0) = - \sum_{\substack{\text{poles} \\ z \neq 0}} \text{Res}_{z=z_a} \frac{A(z)}{z}$$

poles in $A(z)$.

$$\text{factorization: } (k_r + \dots + k_s)^2 = 0$$

$$\left\{ \begin{array}{l} j, l \in \mathbb{S} \\ j \neq l \\ j, l \in \mathbb{N} \end{array} \right\}$$

$$k_{r,s} \rightarrow k_{r,s} - \frac{z}{z} \langle j | \mu | l \rangle$$

$$k_{r,s}^2 \rightarrow k_{r,s}^2 - z \langle j | k_{r,s} | l \rangle = 0$$

$$z = z_{rs} \equiv \frac{k_{r,s}^2}{\langle j | k_{r,s} | l \rangle}$$

$$\{l, m\} \rightarrow P U \bar{P}$$

$$\downarrow$$

$J \neq P$	$J \neq \bar{P}$
$L \neq P$	$L \neq \bar{P}$

CAUTION
Do not touch the surface of the board
as it may be hot or cold.
Do not touch the board
if it is glowing or hot.

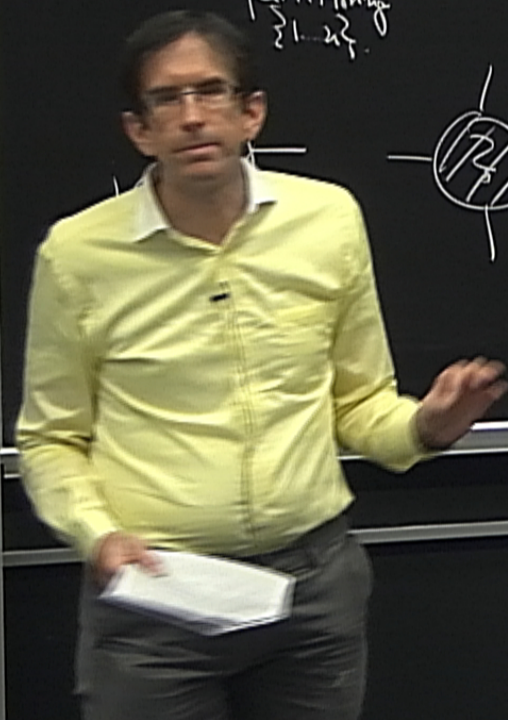
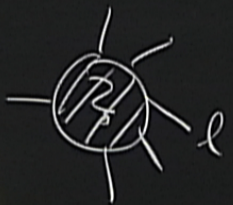
CAUTION
Do not touch the surface of the board
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Do not touch the board
if it is glowing or hot.

$$\{1, m\} \rightarrow P U \bar{P}$$

$$\downarrow$$

$J \in P$	$J \notin P$
$R \in P$	$R \in P$

$$\sum_{\text{polosa}} = \sum_{\text{partitining } \{1, 2\}}$$



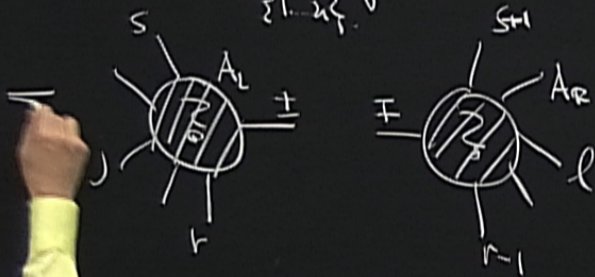
CAUTION
Do not touch the chalkboard surface
as it may be hot or contain sharp objects.
Do not use the chalkboard as a desk.
Please do not drink water.

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$$\{l, m\} \rightarrow P \cup \bar{P}$$

$$\begin{array}{l} \downarrow \\ j \in P \quad j \in \bar{P} \\ l \in P \quad l \in \bar{P} \end{array}$$

$$\sum_{\text{poles } a} = \sum_{\text{partitioning } \{l, m\}}$$



$$K_{r,s}^z = K_{(s+1) \dots (r-1)}^z$$

$$\text{Res}_{z=z_{rs}} \left(- \frac{A_L(z) z A_R(z)}{z (z-z_{rs}) \langle j | K_{r,s} | l \rangle} \right)$$

$$= - \frac{z}{z_{rs}} \frac{A_L(z_{rs}) A_R(z_{rs})}{\langle j | K_{r,s} | l \rangle}$$

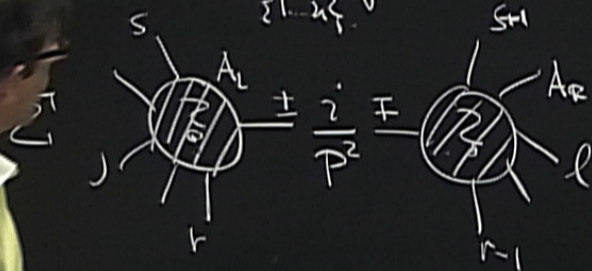
$$= - \frac{z}{K_{r,s}} A_L(z_{rs}) A_R(z_{rs})$$

$$A(0) = \sum_{\text{partitions } P} \sum_{h=\pm} A_{\#P+1}(\dots, \hat{j}, \dots, -\hat{p}^h) \frac{z}{P^z} \times A_{\#\bar{P}+1}(\dots, \hat{l}, \dots, P^{-h})$$

$$\{1, \dots, m\} \rightarrow P \cup \bar{P}$$

$$\begin{array}{l} \downarrow \\ j \in P \\ k \in P \end{array} \quad \begin{array}{l} j \notin \bar{P} \\ l \in \bar{P} \end{array}$$

$$\sum_{\text{poles } a} = \sum_{\text{partitioning } \{1, \dots, m\}}$$



$$K_{r,s}^2 = K_{(s+1)-(r-1)}^2$$

$$\text{Res}_{z=z_{rs}} \left(- \frac{A_L(z) i A_R(z)}{z (z-z_{rs}) \langle j | K_{r,s} | l \rangle} \right)$$

$$= - \frac{z}{z_{rs}} \frac{A_L(z_{rs}) A_R(z_{rs})}{\langle j | K_{r,s} | l \rangle}$$

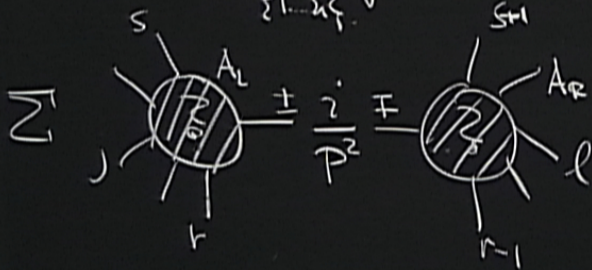
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$$\{1, \dots, m\} \rightarrow P \cup \bar{P}$$

$$\begin{array}{cc} \downarrow & \downarrow \\ j \in P & j \notin P \\ l \in P & l \in \bar{P} \end{array}$$

$$\sum_{\text{poles } a} = \sum_{\text{partitioning } \{1, \dots, m\}}$$



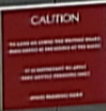
$$K_{r,s}^z = K_{(s+1), (r-1)}^z$$

$$\text{Res}_{z=z_{rs}} \left(- \frac{A_L(z) i A_R(z)}{z (z - z_{rs}) \langle j | K_{r,s} | l \rangle} \right)$$

$$= - \frac{z}{z_{rs}} \frac{A_L(z_{rs}) A_R(z_{rs})}{\langle j | K_{r,s} | l \rangle}$$

$$= - \frac{z}{K_{r,s}} A_L(z_{rs}) A_R(z_{rs}) \quad \hat{\chi} = \chi(z_{rs})$$

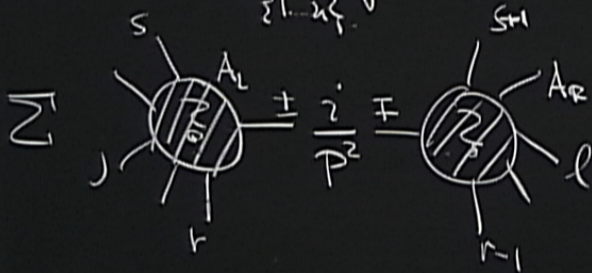
$$A(0) = \sum_{\text{partitions } P} \sum_{h=\pm} A_{\#P+1}(\hat{j}, \dots, -\hat{P}^h) \frac{z}{P^z} \times A_{\#\bar{P}+1}(\hat{l}, \dots, \hat{P}^h)$$



$$\{1, \dots, m\} \rightarrow P \cup \bar{P}$$

$$\begin{array}{cc} \downarrow & \downarrow \\ j \in P & j \notin P \\ k \in P & k \in \bar{P} \end{array}$$

$$\sum_{\text{poles } a} = \sum_{\text{partitioning } \{1, \dots, m\}}$$



$$K_{r,s}^2 = K_{(s+1), (r-1)}^2$$

$$\text{Res}_{z=z_{rs}} \left(- \frac{A_L(z) i A_R(z)}{z (z-z_{rs}) \langle j | K_{r,s} | l \rangle} \right)$$

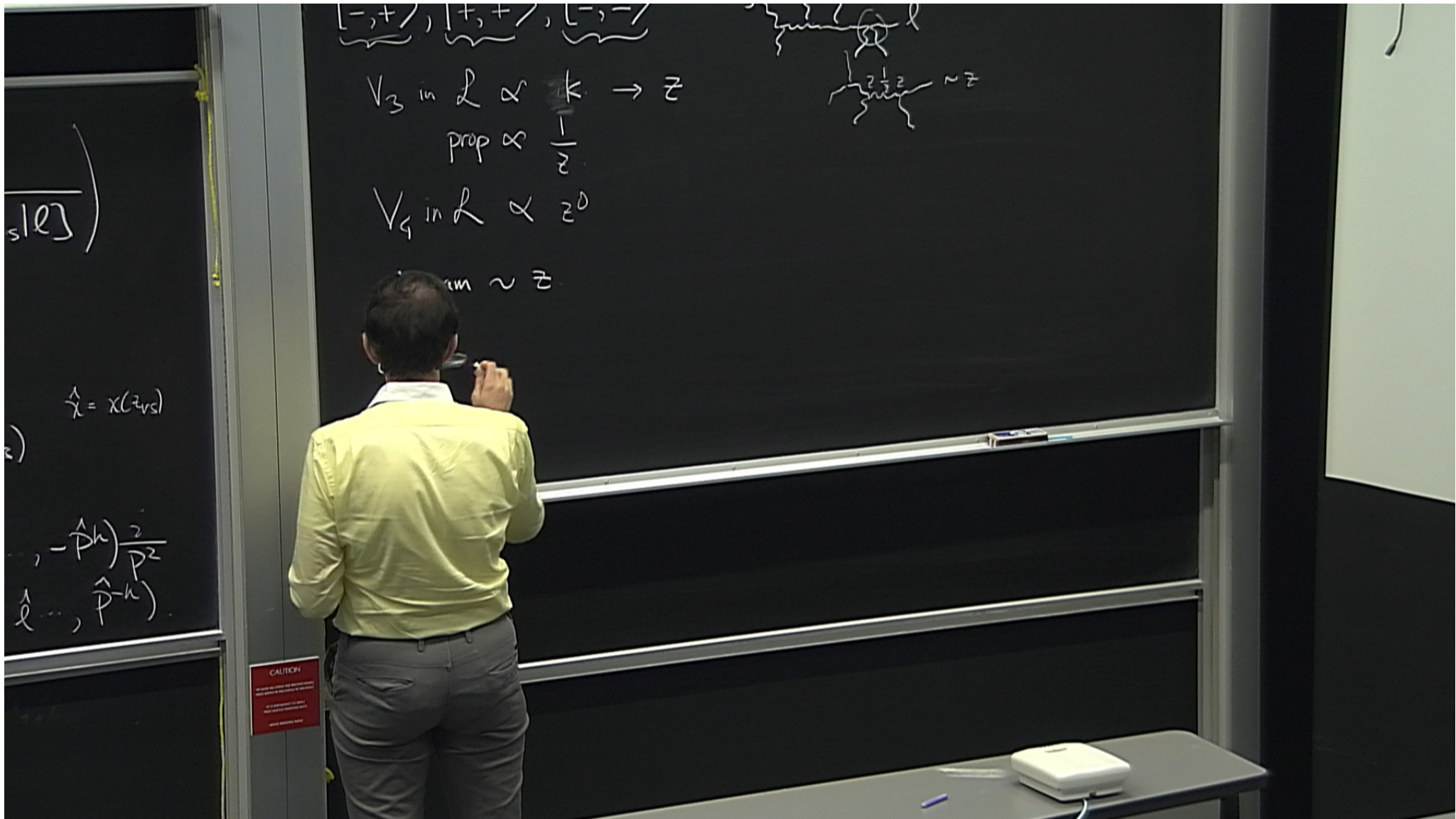
$$= - \frac{z}{z_{rs}} \frac{A_L(z_{rs}) A_R(z_{rs})}{\langle j | K_{r,s} | l \rangle}$$

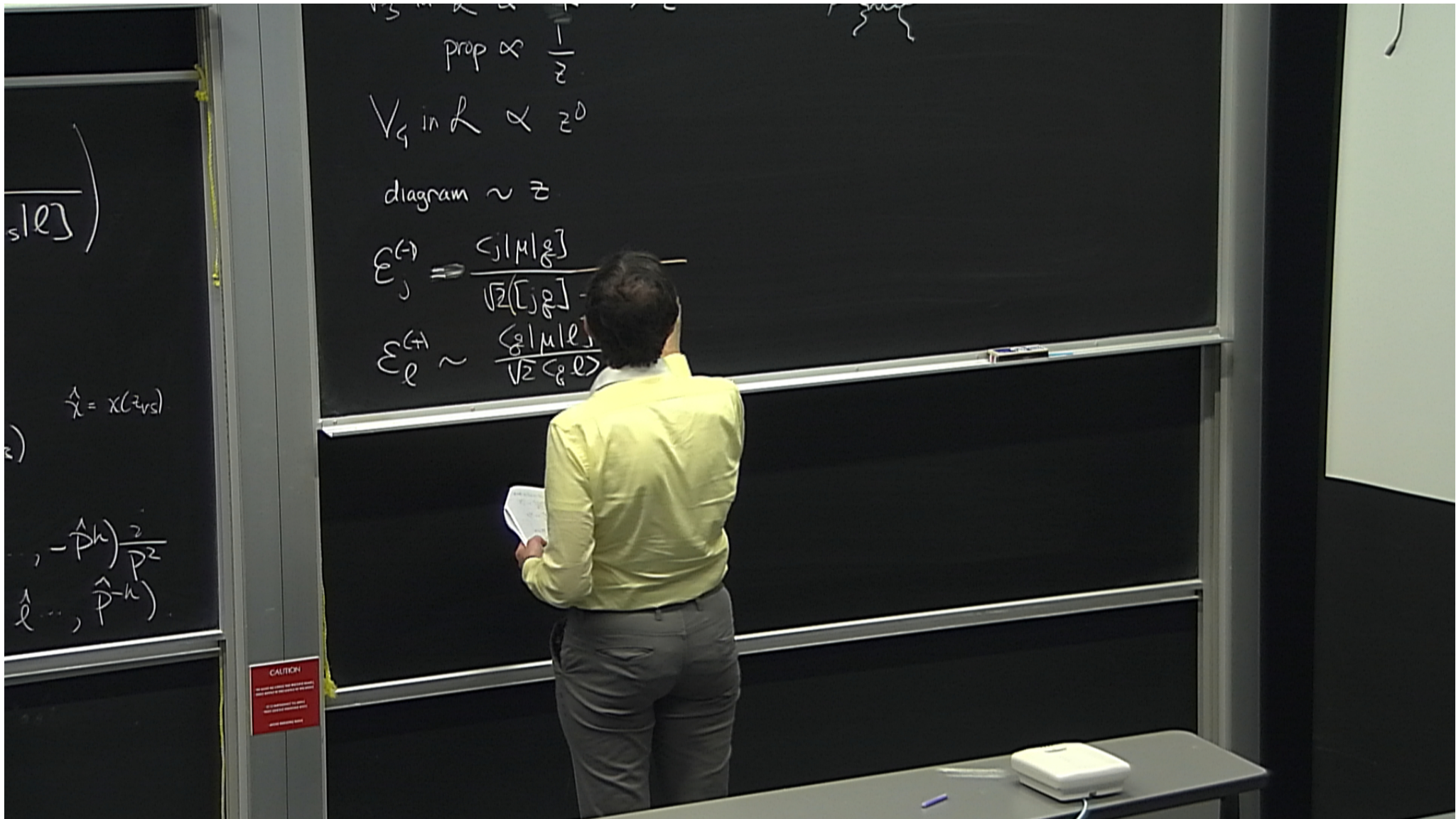
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CAUTION
Do not lean on the board when writing.
Do not use the board as a desk.
Do not use the board as a shelf.
Do not use the board as a support for equipment.

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$$\text{prop} \propto \frac{1}{z}$$

$$V_g \text{ in } R \propto z^0$$

diagram $\sim z$

$$E_j^{(-)} = \frac{\langle j | M | g \rangle}{\sqrt{2} \langle j | g \rangle}$$

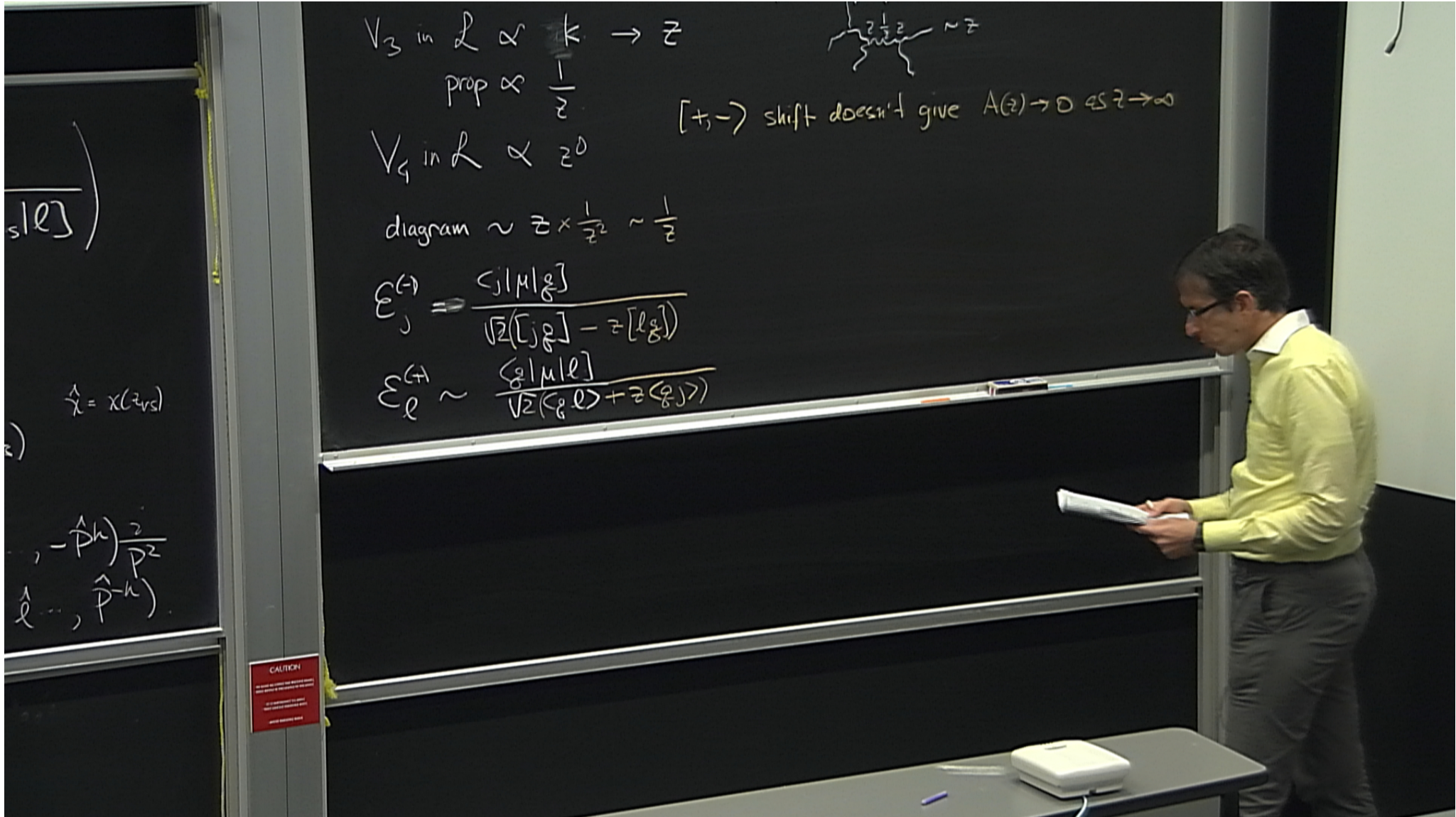
$$E_e^{(+)} \sim \frac{\langle g | M | e \rangle}{\sqrt{2} \langle g | e \rangle}$$

$$\overline{s | e \rangle}$$

$$\hat{\chi} = \chi(z, v, s)$$

$$-\hat{p}^h \frac{z}{p^z}$$

CAUTION



$$V_3 \text{ in } \mathcal{L} \propto k \rightarrow z$$
$$\text{prop} \propto \frac{1}{z}$$



[+, -] shift doesn't give $A(z) \rightarrow 0$ as $z \rightarrow \infty$

$$V_4 \text{ in } \mathcal{L} \propto z^0$$

$$\text{diagram} \sim z \times \frac{1}{z^2} \sim \frac{1}{z}$$

$$E_j^{(-)} = \frac{\langle j | M | g \rangle}{\sqrt{2}(\langle j | g \rangle - z \langle l | g \rangle)}$$

$$E_l^{(+)} \sim \frac{\langle g | M | l \rangle}{\sqrt{2}(\langle g | l \rangle + z \langle g | j \rangle)}$$

$$\overline{s | l \rangle}$$

$$\hat{\chi} = \chi(z, v, s)$$

z)

$$, -\hat{p}^h) \frac{z}{p^z}$$
$$\hat{p}^h, \hat{p}^h)$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER
OR THE BOARDER

$$\overline{s|l\rangle}$$

$$\hat{\chi} = \chi(z, vs)$$

$$-\hat{p}^h \frac{z}{p^z}$$

$$\hat{p}^h$$

$[-,+>, [+,>], [-,->$

$$V_3 \text{ in } \mathcal{L} \propto k \rightarrow z$$

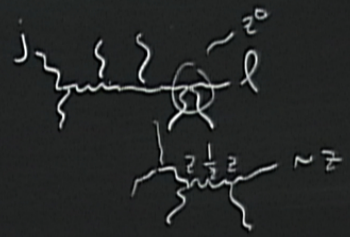
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$[+,->$ shift doesn't give $A(z) \rightarrow 0$ as $z \rightarrow \infty$

Exercise show the large- z behavior for the different shifts in the MHV & $\overline{\text{MHV}}$ amplitudes

CAUTION

$$A_4(1^- 2^- 3^+ 4^+)$$

Pick $[2, 3]$ shift

$$|2\rangle \rightarrow |2\rangle - 2|3\rangle$$

$$|3\rangle \rightarrow |3\rangle + 2|2\rangle$$

$$\{1, 2, 3, 4\}$$

$$|2\rangle \rightarrow |2\rangle$$

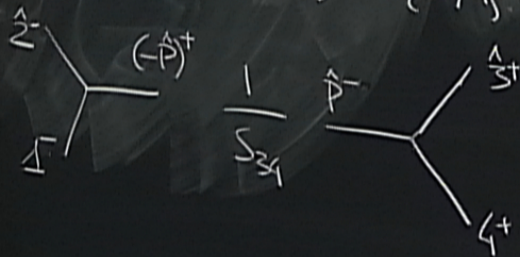
$$|3\rangle \rightarrow |3\rangle$$

$$A_4(1^- 2^- 3^+ 4^+)$$

Pick $[2,3]$ shift

$$\begin{aligned} |2\rangle &\rightarrow |2\rangle - 2|3\rangle & |2\rangle &\rightarrow |2\rangle \\ |3\rangle &\rightarrow |3\rangle + 2|2\rangle & |3\rangle &\rightarrow |3\rangle \end{aligned}$$

$$\{1, 2, 3, 4\} \quad \{1, 2\} \cup \{3, 4\}$$



$$\frac{(12)^3}{(2(-1))(-1)1}$$

$$\frac{7}{S_{34}}$$

$$(-2) \frac{[34]^3}{[13][14]}$$

CAUTION
 PLEASE DO NOT TOUCH THE BOARD SURFACE
 UNLESS YOU ARE INSTRUCTED TO DO SO
 ALL INFORMATION ON THIS BOARD IS UNCLASSIFIED
 UNLESS INDICATED OTHERWISE

$$|(-\hat{P})\rangle \rightarrow ? |\hat{P}\rangle$$

$$\rightarrow \frac{\langle 12 \rangle^3 \langle 34 \rangle^3}{\langle 43 \rangle \langle 34 \rangle} \langle 1|\hat{P}|3 \rangle \langle 2|\hat{P}|4 \rangle$$

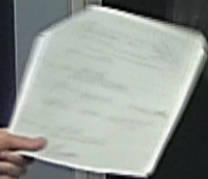
$$\hat{P} = P - \frac{z_0}{2} \langle 2|P|3 \rangle$$

$$|(-\hat{P})\rangle \rightarrow ? |\hat{P}\rangle$$

$$\rightarrow \langle 12 \rangle^3 \langle 34 \rangle^3$$

$$\frac{\langle 43 \rangle \langle 34 \rangle}{\langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle}$$

$$\hat{P} = P - \frac{z_0}{k_{12}} \langle 21 | \mu | 3 \rangle$$



CAUTION
 TO AVOID ACCIDENTS AND INJURIES, PLEASE
 FOLLOW THE SAFETY INSTRUCTIONS ON THE
 EQUIPMENT AND THE SAFETY DATA SHEET.
 ALWAYS WEAR YOUR SAFETY GOGGLES.

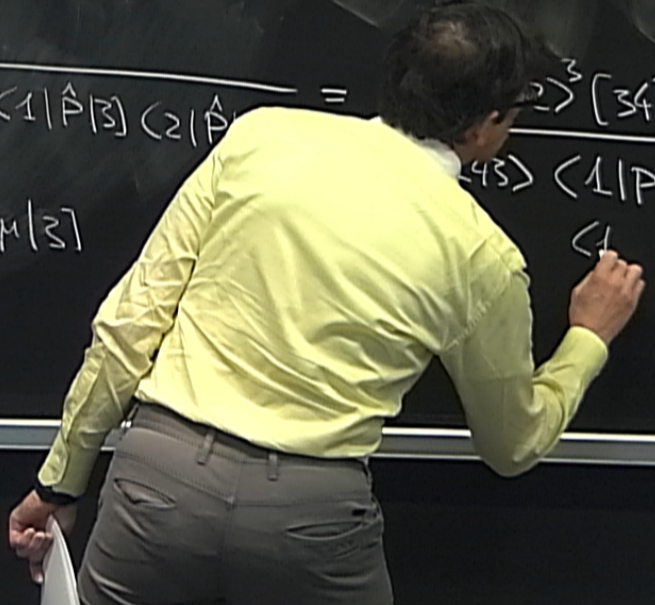
$$i \frac{\langle 12 \rangle^3}{\langle 2(-\hat{P}) \rangle \langle (-\hat{P})1 \rangle} \quad \frac{?}{s_{34}} \quad (-?) \quad \frac{[34]^3}{[\hat{P}3][4\hat{P}]}$$

$$|(-\hat{P})\rangle \rightarrow i |\hat{P}\rangle$$

$$\rightarrow i \frac{\langle 12 \rangle^3 [34]^3}{\langle 43 \rangle [34] \langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle} = \frac{2^3 [34]^2}{\langle 43 \rangle \langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle}$$

$$\hat{P} = P - \frac{z_0}{k_{12}} \langle 2|P|3 \rangle$$

$$\frac{2^3 [34]^2}{\langle 43 \rangle \langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle}$$



$$i \frac{\langle 12 \rangle^3}{\langle 2(-\hat{P}) \rangle \langle (-\hat{P})1 \rangle} \xrightarrow{S_{34}} (-i) \frac{[34]^3}{[\hat{P}3][4\hat{P}]}$$

$$|(-\hat{P})\rangle \rightarrow i |\hat{P}\rangle$$

$$\rightarrow i \frac{\langle 12 \rangle^3 [34]^3}{\langle 43 \rangle [34] \langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle} = +i \frac{\langle 12 \rangle^3 [34]^3}{\langle 43 \rangle \langle 1\hat{P}|3 \rangle \langle 2\hat{P}|4 \rangle}$$

$$\hat{P} = P - \frac{z_0}{k_{12}} \langle 2|p|3 \rangle$$

$$= +i \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

This formula generalizes to the so-called Parke-Taylor formula

$$A_{n=2}^{m_1, m_2} = \frac{\langle m_1, m_2 \rangle^2}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle \langle n1 \rangle}$$

Exercise Derive this using BCFW (induction)

This formula generalizes to the so-called Parke-Taylor formula

$$A_{n+1} = i \frac{\langle m_1, m_2 \rangle^2}{\langle 12 \rangle \langle 23 \rangle \dots \langle n-1, n \rangle \langle n1 \rangle}$$

Exercise Derive this using BCFW (induction)

This formula generalizes to the so-called Parke-Taylor formula

$$A_{n=2}^{MHV} = \frac{\langle m_1, m_2 \rangle^2}{\langle 12 \rangle \langle 23 \rangle \dots \langle n-1, n \rangle \langle n1 \rangle} (1^+ \dots m_1^- \dots m_2^- \dots n^+)$$

Exercise Derive this using BCFW (induction)

Unitarity

One-Loop Computations

$$S^\dagger S = 1$$

$$T = S - 1$$

$$A_{n, \text{MHV}} =$$

Ex

CAUTION
DO NOT TOUCH THE BOARD
IF YOU ARE NOT A TEACHER
OR A STUDENT OF THE COURSE

Unitarity

One-Loop Computations

$$S^\dagger S = \mathbb{1}$$

$$\Rightarrow T = S^{-1}$$

$$\Rightarrow (T - T^\dagger) = T^\dagger T$$

$$A_{\mu\nu} =$$

Ex

CAUTION
THE BOARD IS HOT AND MAY BE DAMAGED BY EXCESSIVE HEAT.
DO NOT TOUCH THE BOARD.
DO NOT TOUCH THE BOARD.
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One-loop Computations

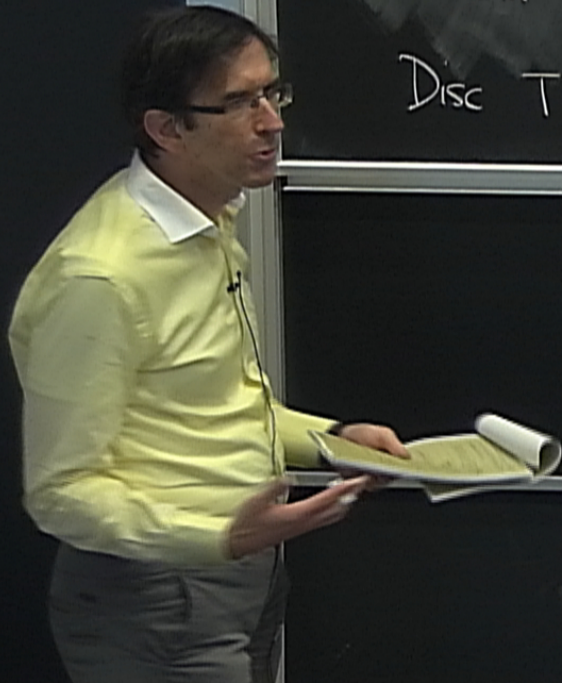
$$S^T S = \mathbb{1}$$

$$T = S^{-1}$$

$$\rightarrow (T - T^T) = T^T T$$

$$2 \text{ "Im" } T_{fi} = (T^T T)_{fi}$$

$$\text{Disc } T = T^T T$$



CAUTION
DO NOT TOUCH THE BOARD SURFACE
IT IS PROTECTED BY A
THIN LAYER OF POLYMER
WHICH COULD BE DAMAGED

Unitarity

One-Loop Computations

$$S^\dagger S = 1$$

$$2T = S - 1$$

$$\rightarrow (T - T^\dagger) = T^\dagger T$$

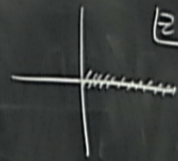
$$2 \text{ "Im" } T_{fi} = (T^\dagger T)_{fi}$$

$$\text{Disc } T = T^\dagger T$$

$$\text{Re } f(s) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega \frac{\text{Im } f(\omega)}{\omega - s} + \text{Re } C_\infty$$

related to UV properties

ver. 0



$$2i\pi \rightarrow \ln\left(\frac{-s}{-t}\right)_{t \rightarrow 0}$$

This formula generalizes to the so-called Parke-Taylor formula

$$\text{Disc} \frac{1}{X+i\varepsilon} = \lim_{\varepsilon \rightarrow 0^+} \frac{1}{2i} \left[\frac{1}{X+i\varepsilon} - \frac{1}{X-i\varepsilon} \right] = -\frac{\varepsilon}{X^2 + \varepsilon^2}$$

$$\text{Disc } \frac{1}{X + i\varepsilon} = \frac{1}{2i} \left[\frac{1}{X + i\varepsilon} - \frac{1}{X - i\varepsilon} \right] = -\frac{\varepsilon}{X^2 + \varepsilon^2} = -\pi \delta(X)$$

$\lim_{\varepsilon \rightarrow 0^+}$

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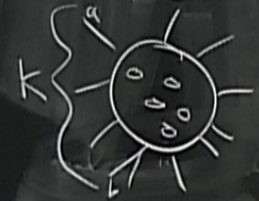
Cutkosky ('60s)



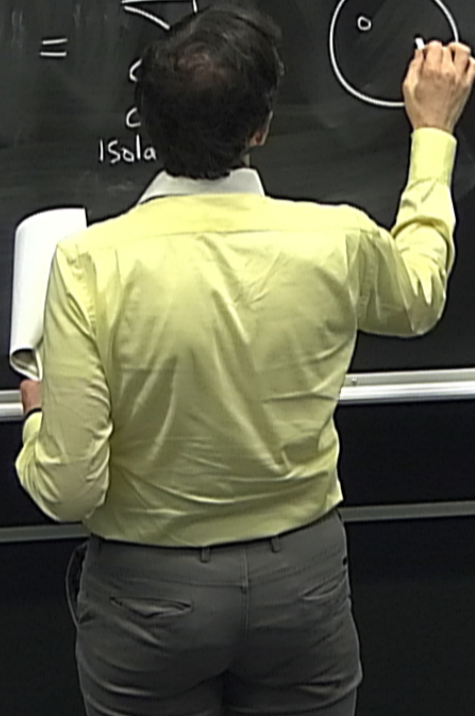
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Cutkosky ('60s)



Disc
 K^2



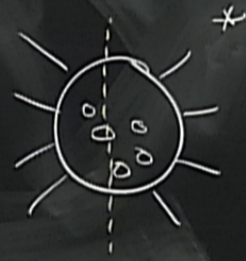
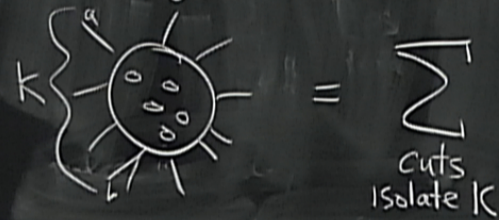
CAUTION
Do not touch the surface when the surface is hot.
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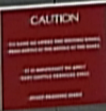
$\lim_{\epsilon \rightarrow 0^+}$

Cutkosky ('60s)



Disc
 k^2

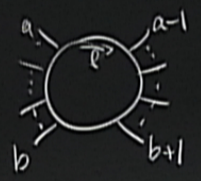
$$\frac{1}{q^2 - m^2 + i\epsilon} = -2\pi i \int_0^{(+)} \delta(q^2 - m^2) = -2\pi i \Theta(q^0) \delta(q^2 - m^2)$$



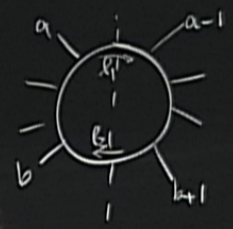
$$-\pi \delta(x)$$

$$\xi_{\ell}^{(A)} \sim \frac{\langle g | M | \ell \rangle}{\sqrt{2}(\langle g | \ell \rangle + z \langle g | J \rangle)}$$

Disc
 K^2



=



$$K = K_{a \cdot b} = K_{a+}$$

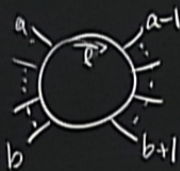
$$\tau_i \Theta(\ell) \delta(\ell^2 - m^2)$$

CAUTION

$$-\pi \delta(x)$$

$$\tau_i \Theta(p) \delta(p^2 - m^2)$$

Disc
 K^2



=



$$K = K_{a,b} = k_a + \dots + k_b$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER
IF YOU HAVE ANY
QUESTIONS
PLEASE ASK THE
LECTURER