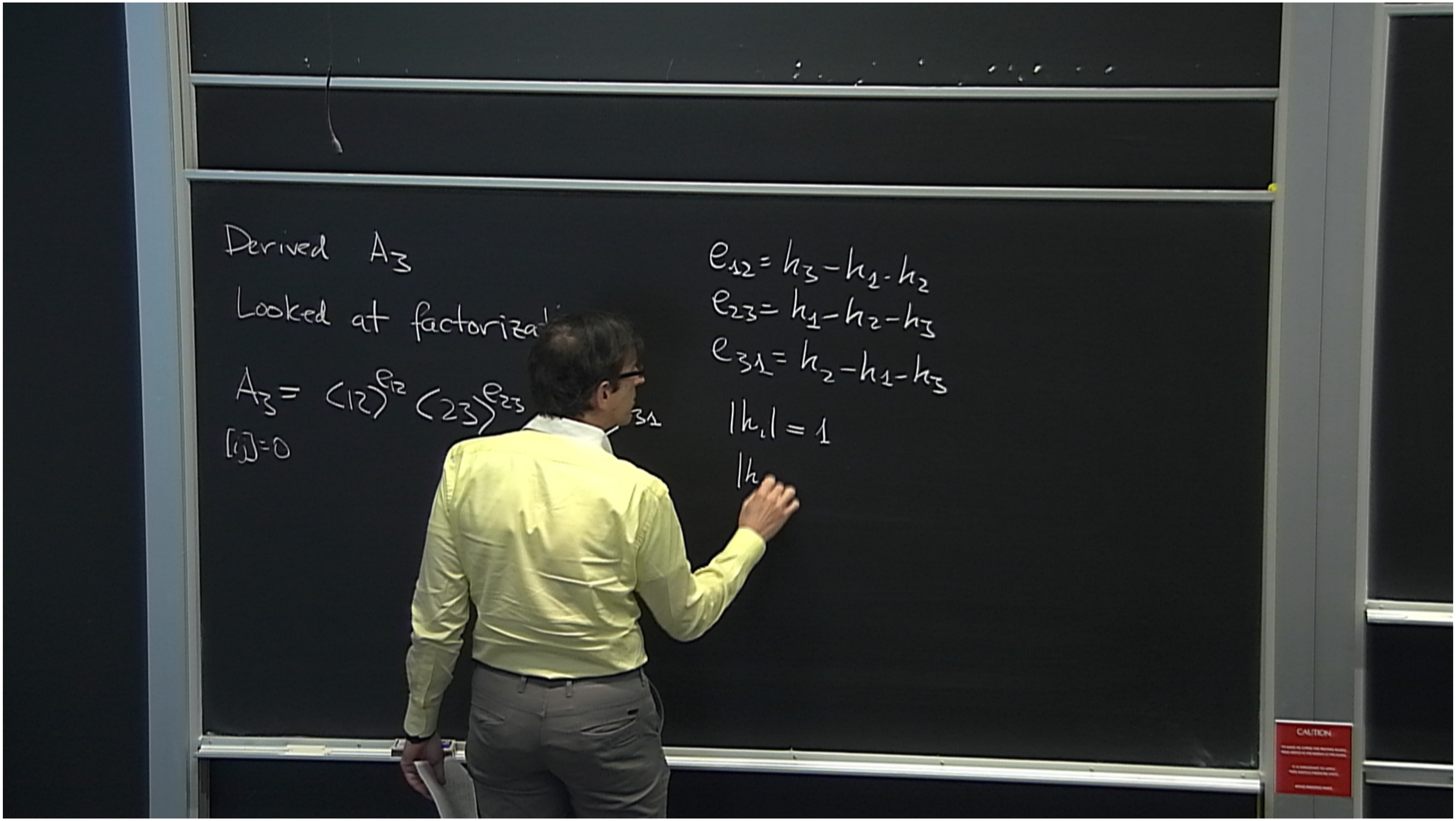
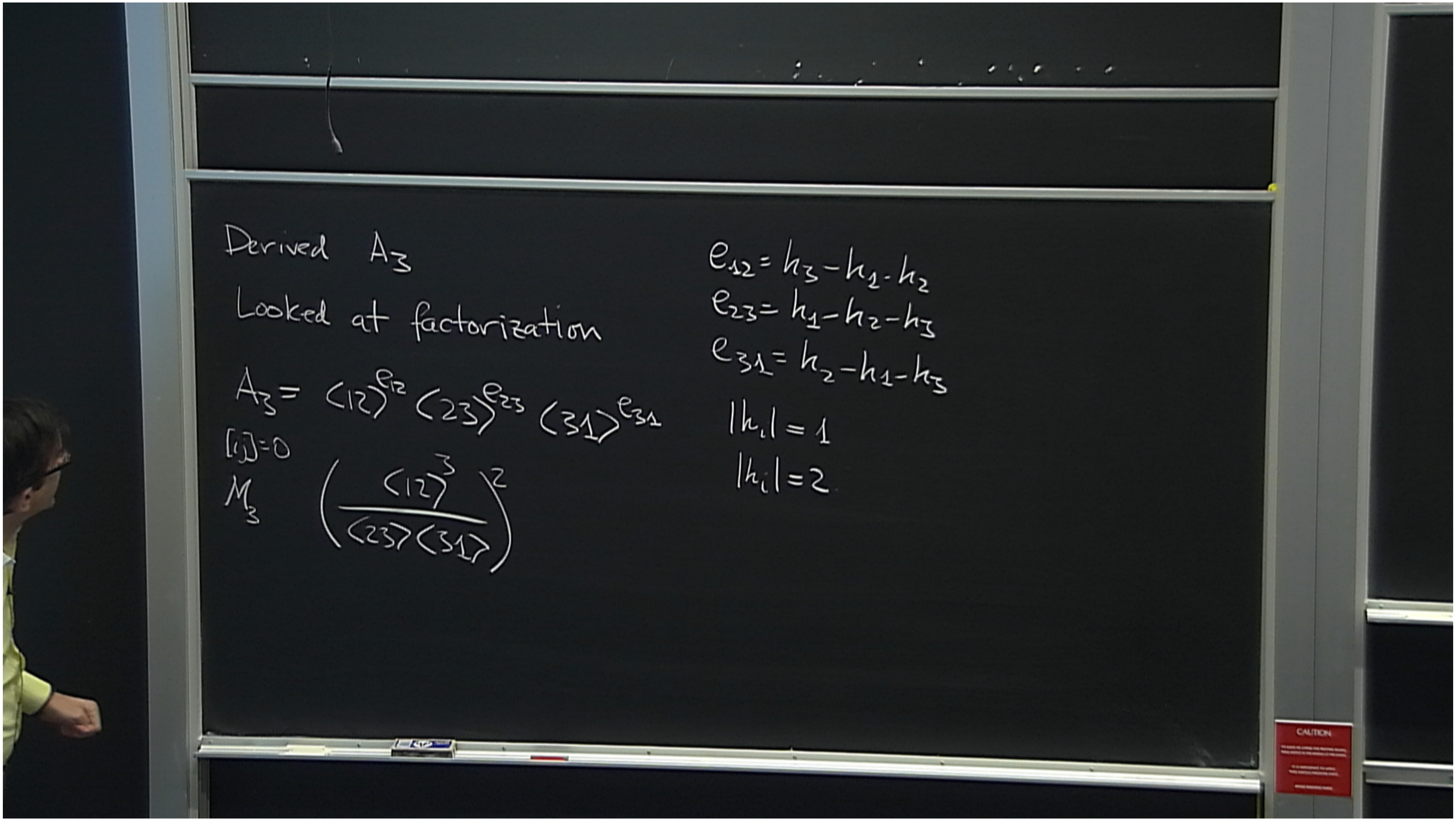
Title: Amplitudes: BCFW Recursion

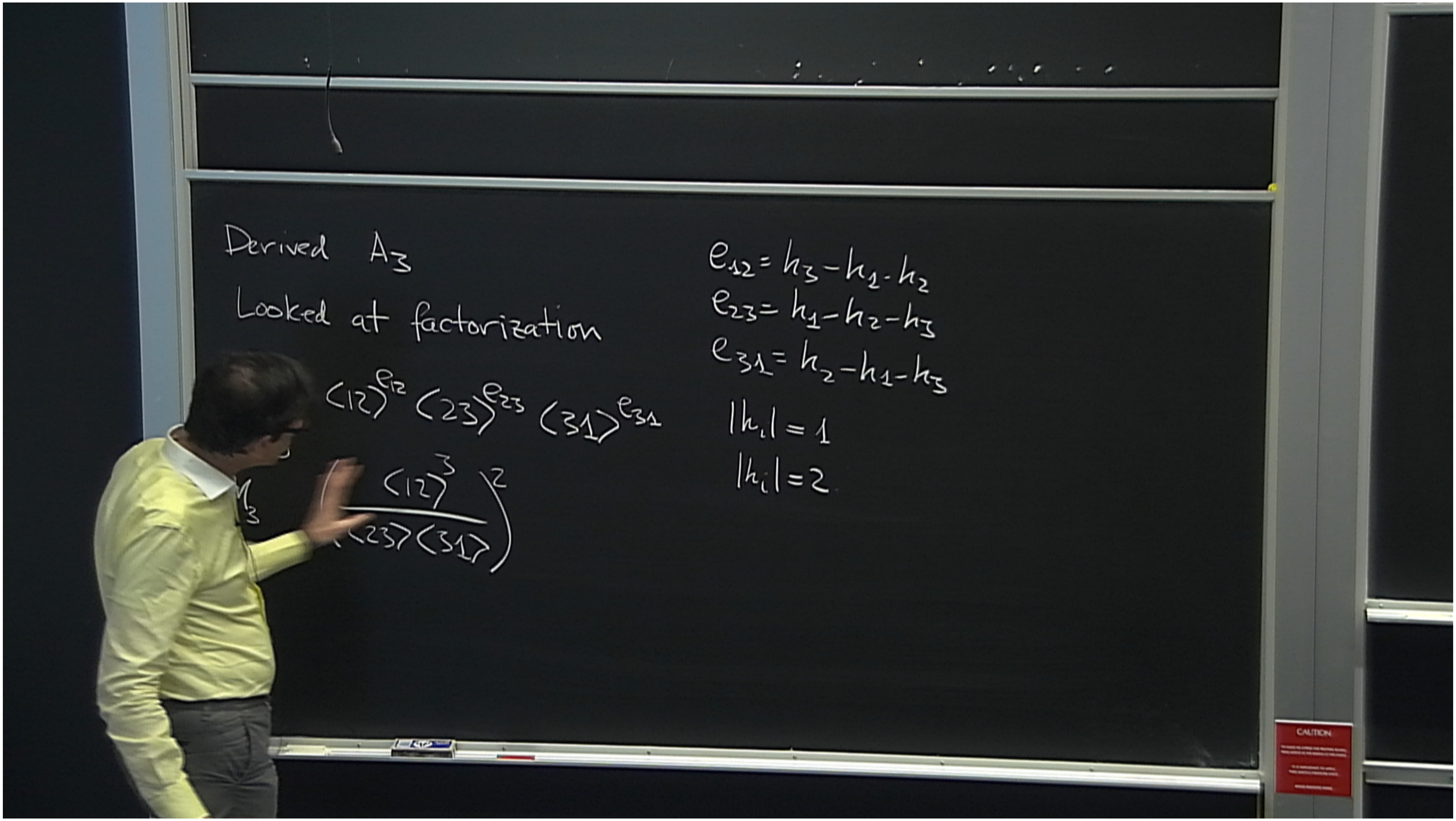
Date: Jul 15, 2015 02:30 PM

URL: http://pirsa.org/15070050

Abstract:


Derived A3
Looked at factoriza
A3 = ⟨12⟩^e12 ⟨23⟩^e23
[ij]=0
e12 = h3 - h1 - h2
e23 = h1 - h2 - h3
e31 = h2 - h1 - h3
|hi| = 1


Derived A_3
Looked at factorization
$A_3 = \langle 12\rangle^{e_{12}} \langle 23\rangle^{e_{23}} \langle 31\rangle^{e_{31}}$
$[ij]=0$
$M_3 \quad \left(\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}\right)^2$
$e_{12} = h_3 - h_1 - h_2$
$e_{23} = h_1 - h_2 - h_3$
$e_{31} = h_2 - h_1 - h_3$
$|h_i| = 1$
$|h_i| = 2$
CAUTION


Derived A3
Looked at factorization
⟨12⟩^e12 ⟨23⟩^e23 ⟨31⟩^e31
e12 = h3 - h1 - h2
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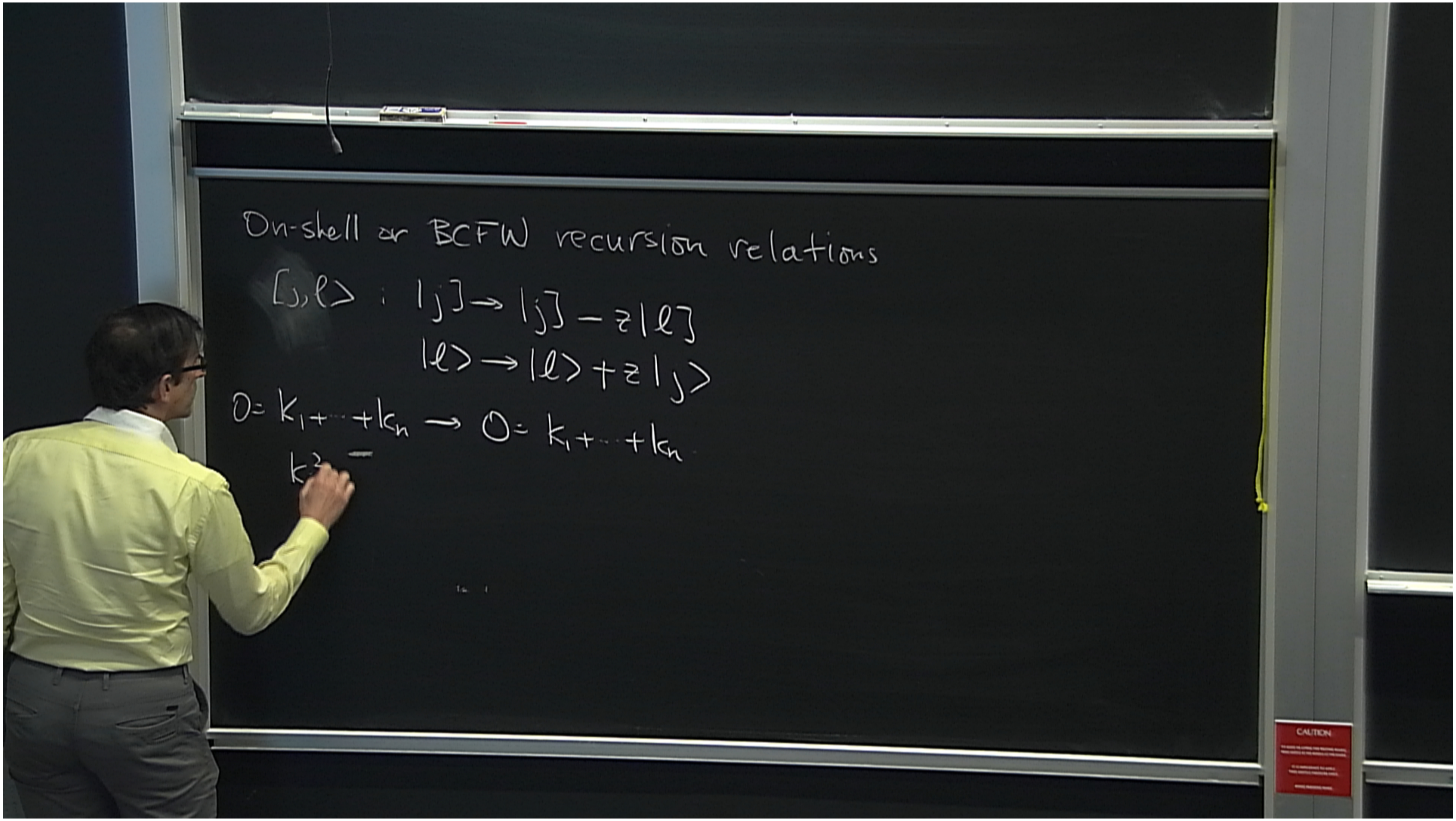
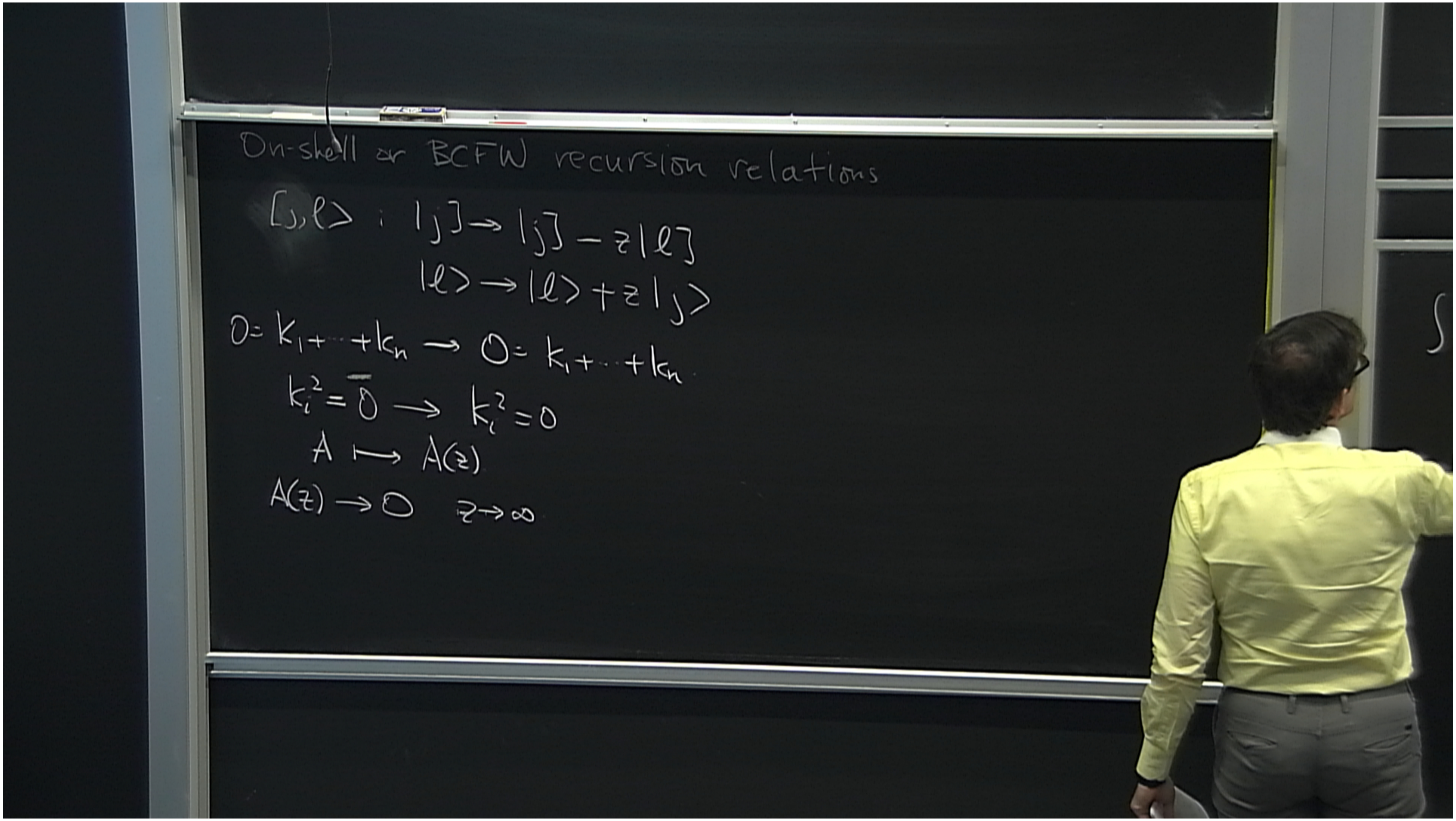
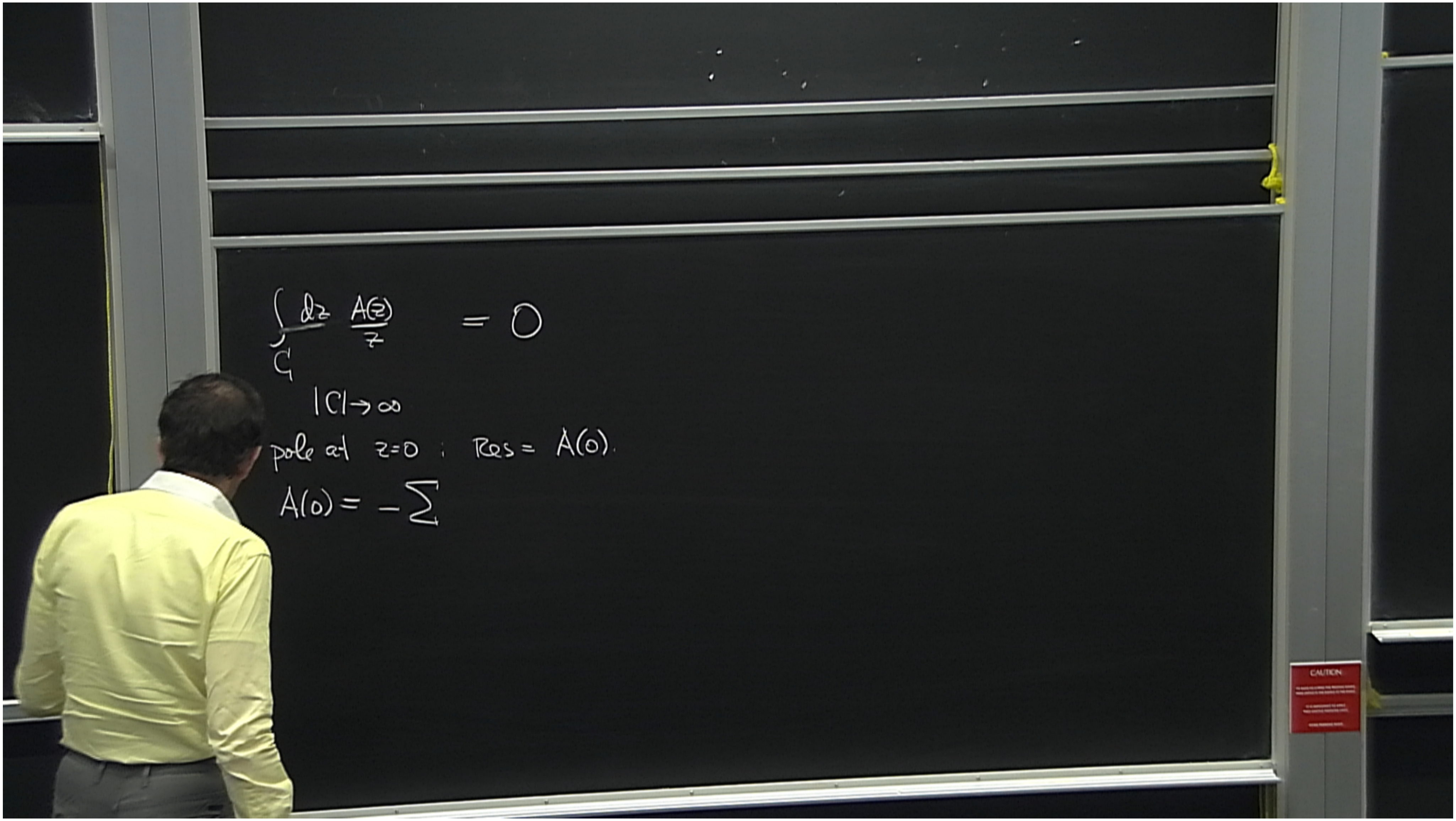

On-shell or BCFW recursion relations
[j,l> : |j] → |j] − z|l]
|l> → |l> + z|j>
0 = k1 + ··· + kn → 0 = k1 + ··· + kn


On-shell or BCFW recursion relations
[j,l> : |j] → |j] − z|l]
|l> → |l> + z|j>
0 = k1 + ··· + kn → 0 = k1 + ··· + kn
ki² = 0 → ki² = 0
A ↦ A(z)
A(z) → 0   z → ∞

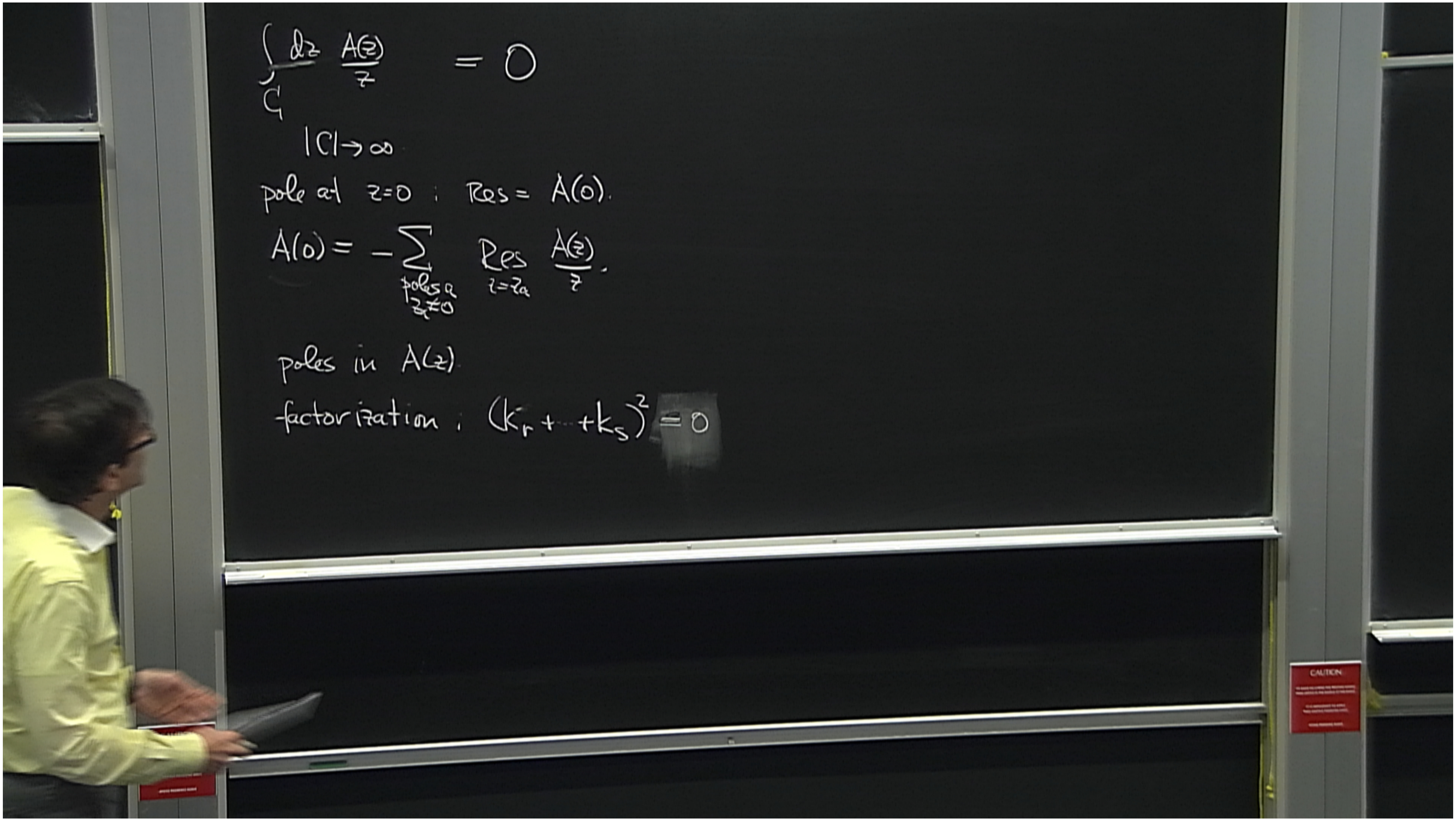

$\int_C dz \frac{A(z)}{z} = 0$
$|C| \to \infty$
pole at $z=0$; Res = $A(0)$.
$A(0) = -\sum$

$$\oint_C dz\, \frac{A(z)}{z} = 0$$

$$|C| \to \infty$$

pole at $z=0$: $\text{Res} = A(0)$.

$$A(0) = -\sum_{\substack{\text{poles } a \\ z_a \neq 0}} \operatorname*{Res}_{z=z_a} \frac{A(z)}{z}.$$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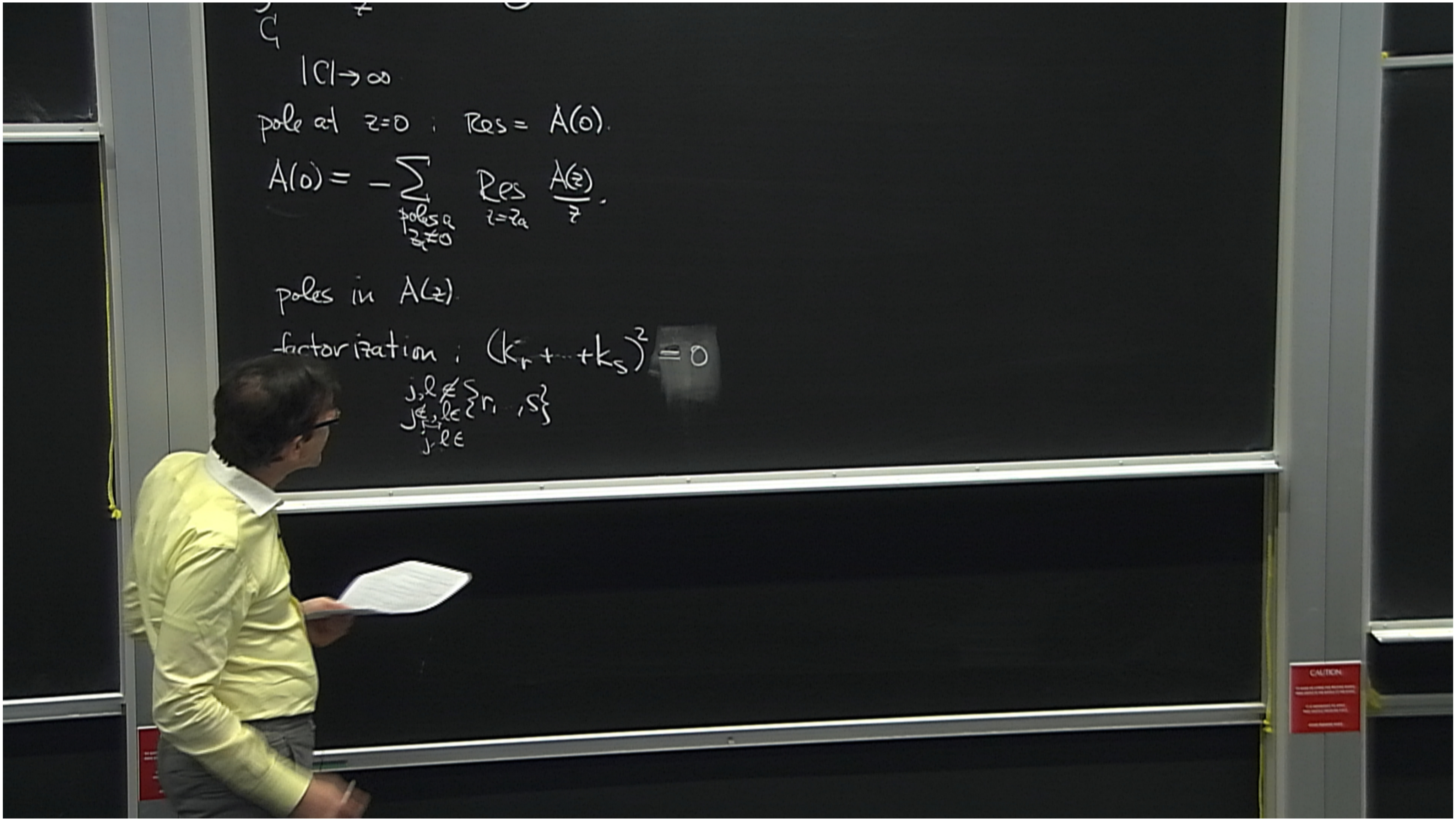
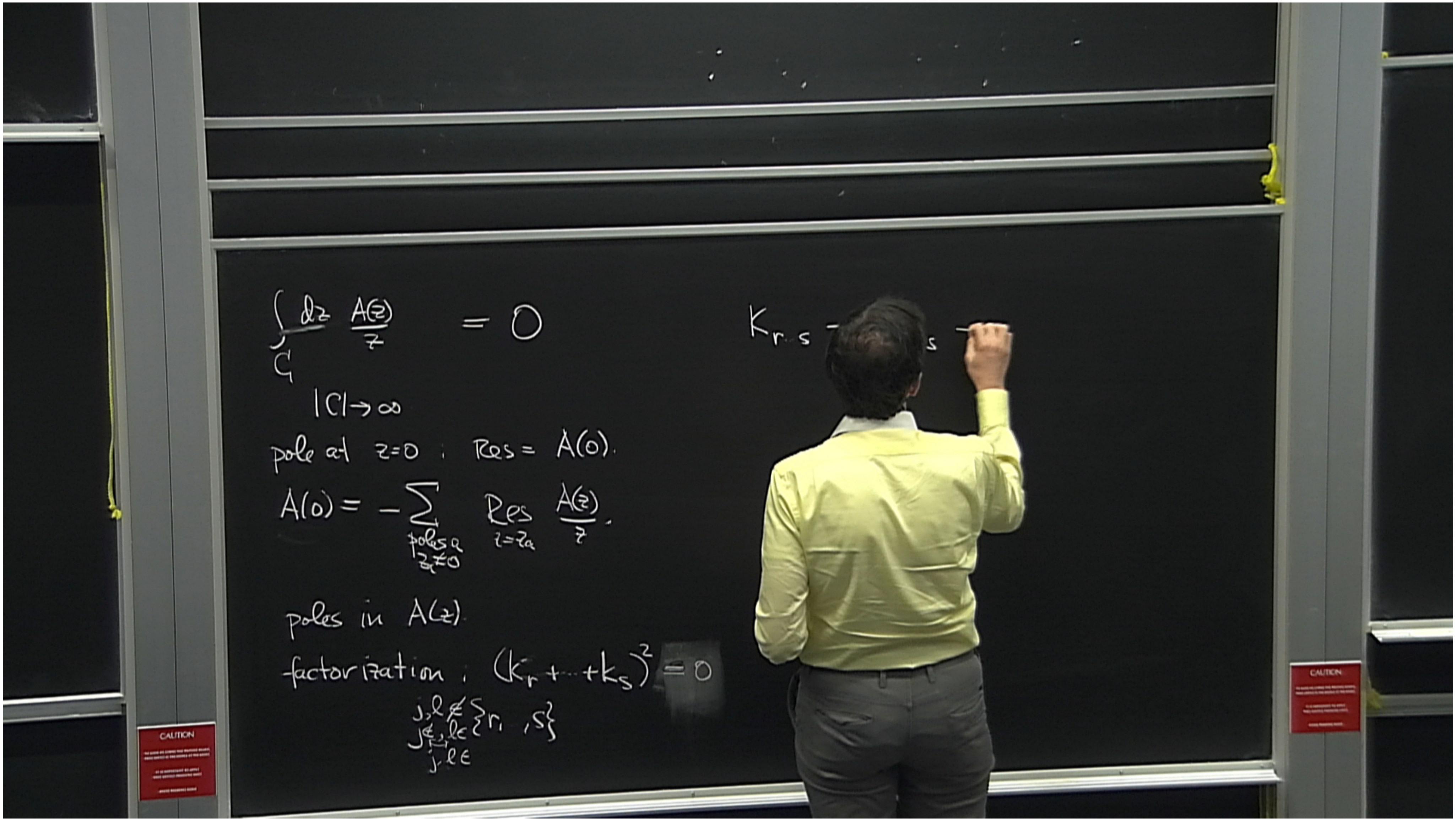
$A(0) = -\sum_{\text{poles } a,\, z_a \neq 0} \text{Res}_{z=z_a} \frac{A(z)}{z}$.
poles in $A(z)$
factorization : $(k_r + \cdots + k_s)^2 = 0$


|C| → ∞
pole at z=0 : Res = A(0).
A(0) = − Σ_{poles a, z_a ≠ 0} Res_{z=z_a} A(z)/z .
poles in A(z)
factorization : (k_r + ⋯ + k_s)^2 = 0
j, l ∉ {r, …, s}
j ∉, l ∈
j, l ∈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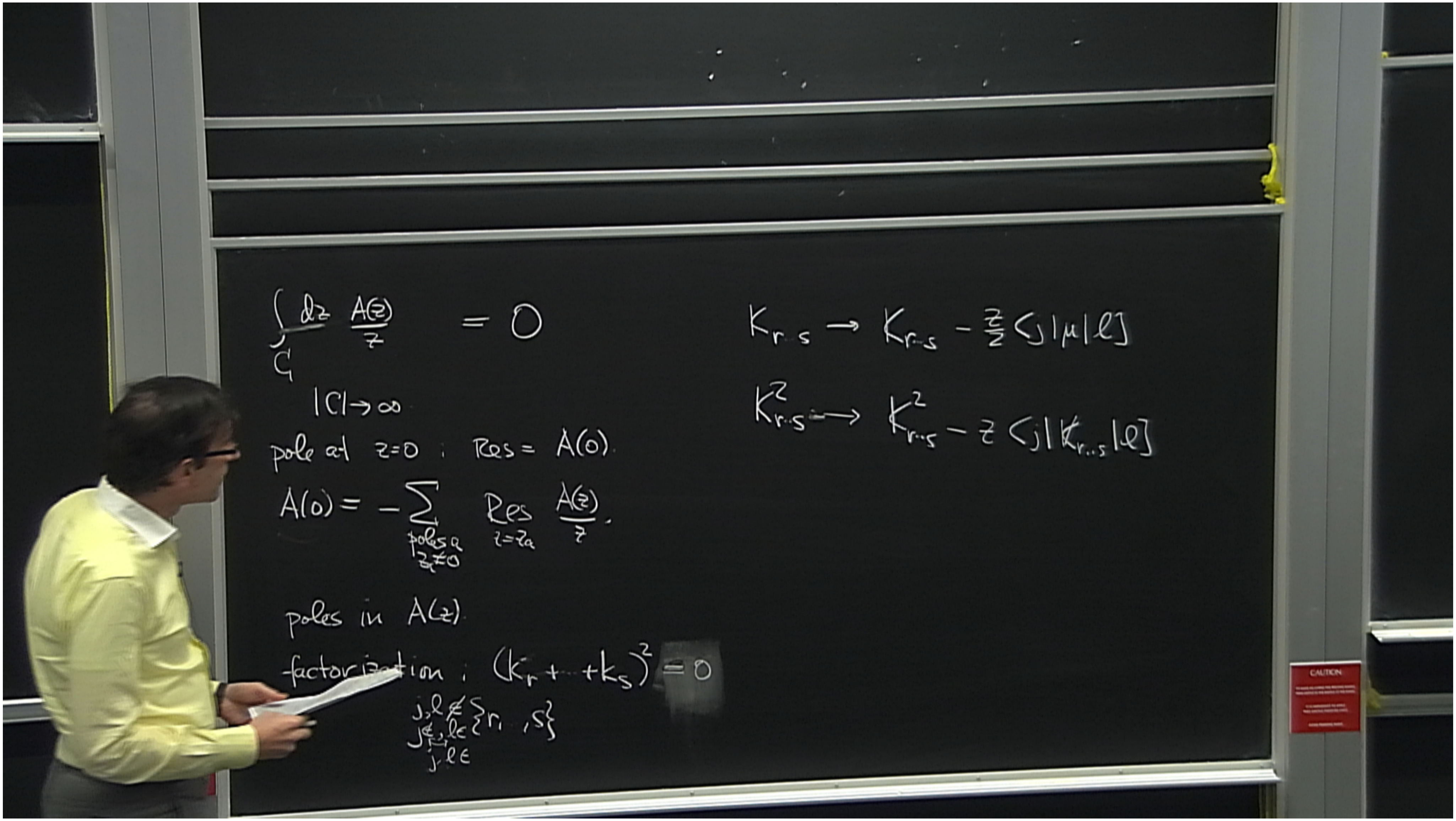
K_{r..s}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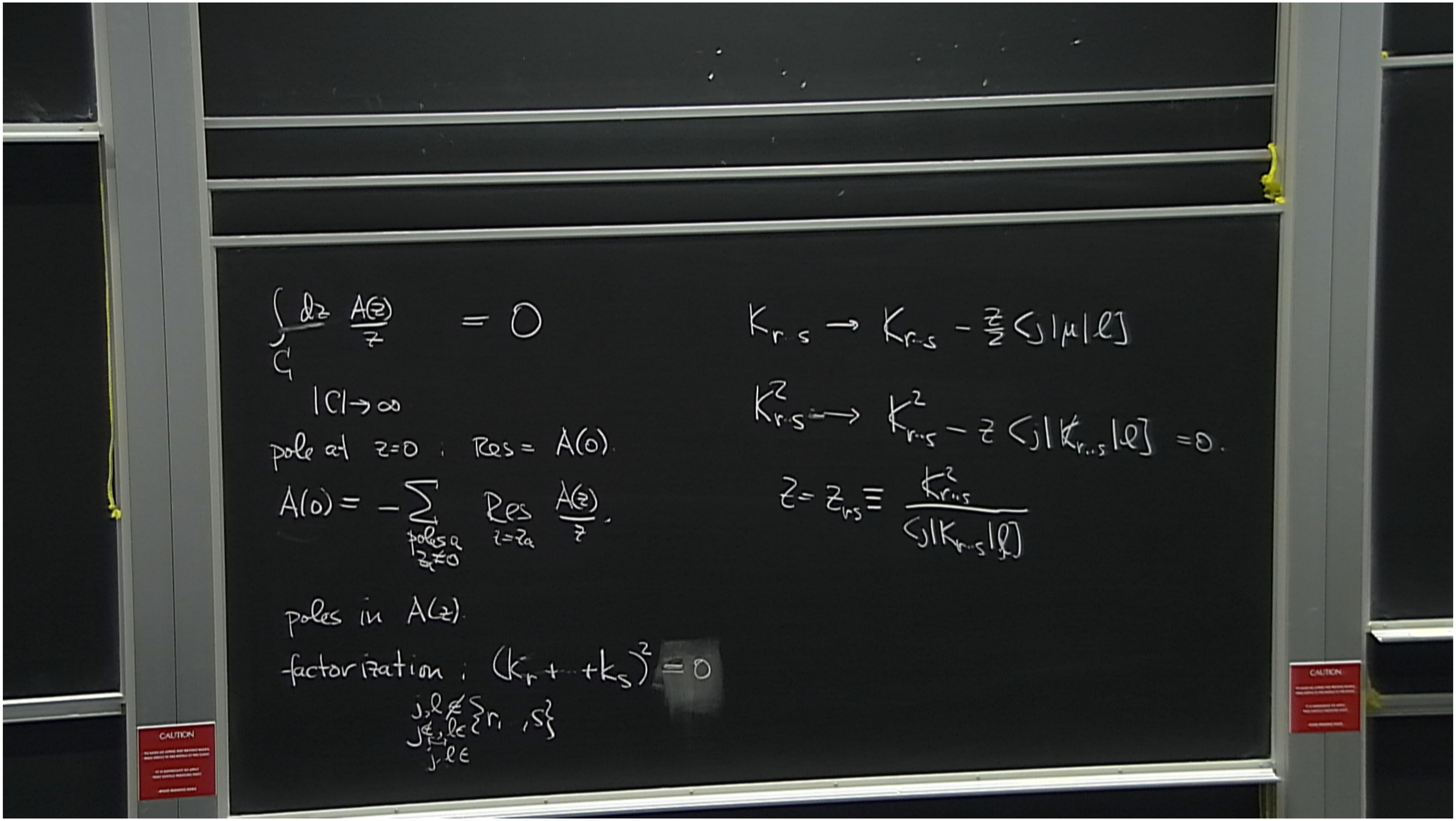
poles in A(z)
factorization : (k_r + ... + k_s)^2 = 0
j, ℓ ∉ {r, ..., s}
K_{r..s} → K_{r..s} − z/2 ⟨j|μ|ℓ]
K^2_{r..s} → K^2_{r..s} − z ⟨j|K_{r..s}|ℓ]


$\oint_C dz \frac{A(z)}{z} = 0$
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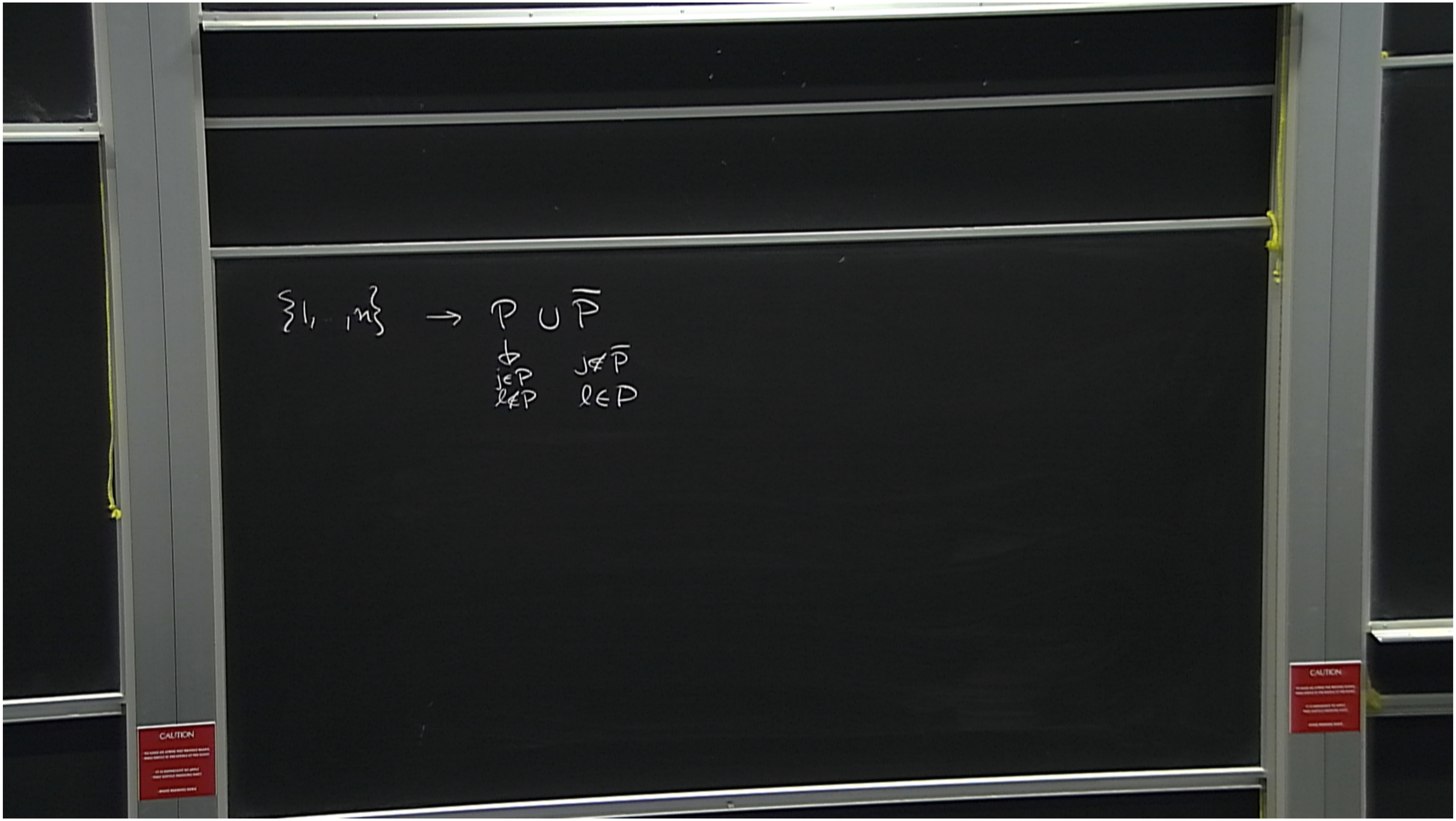
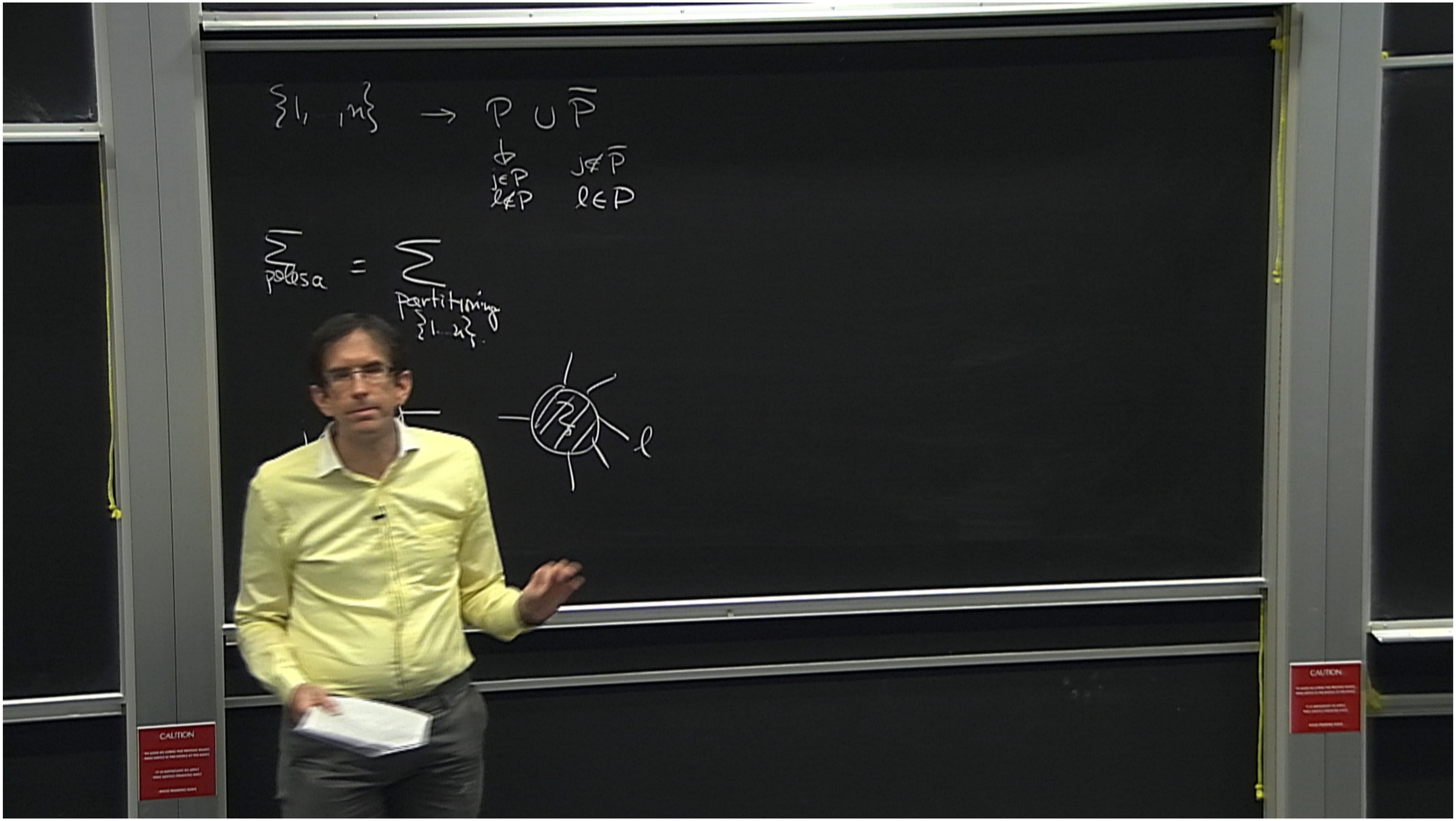
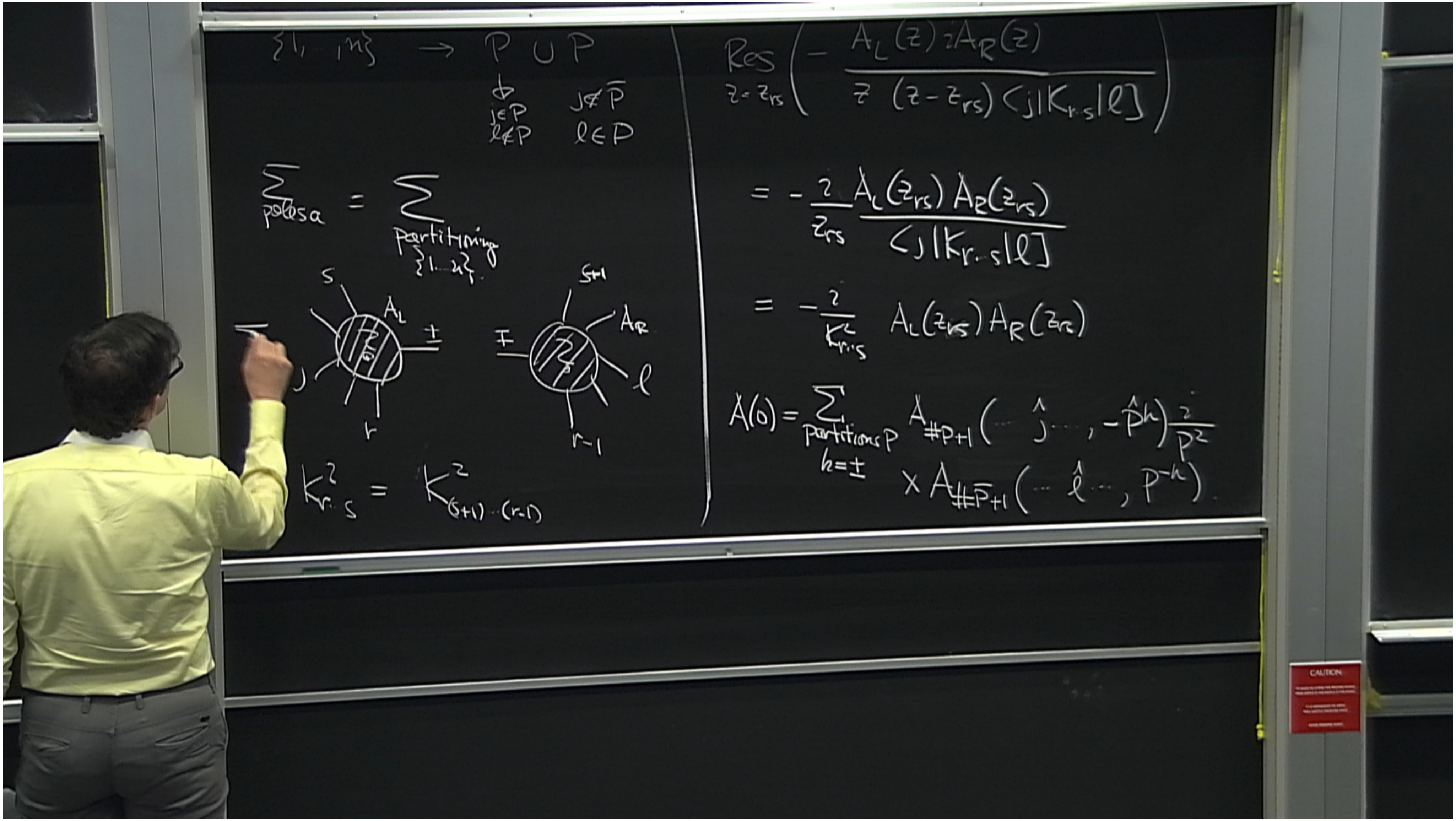
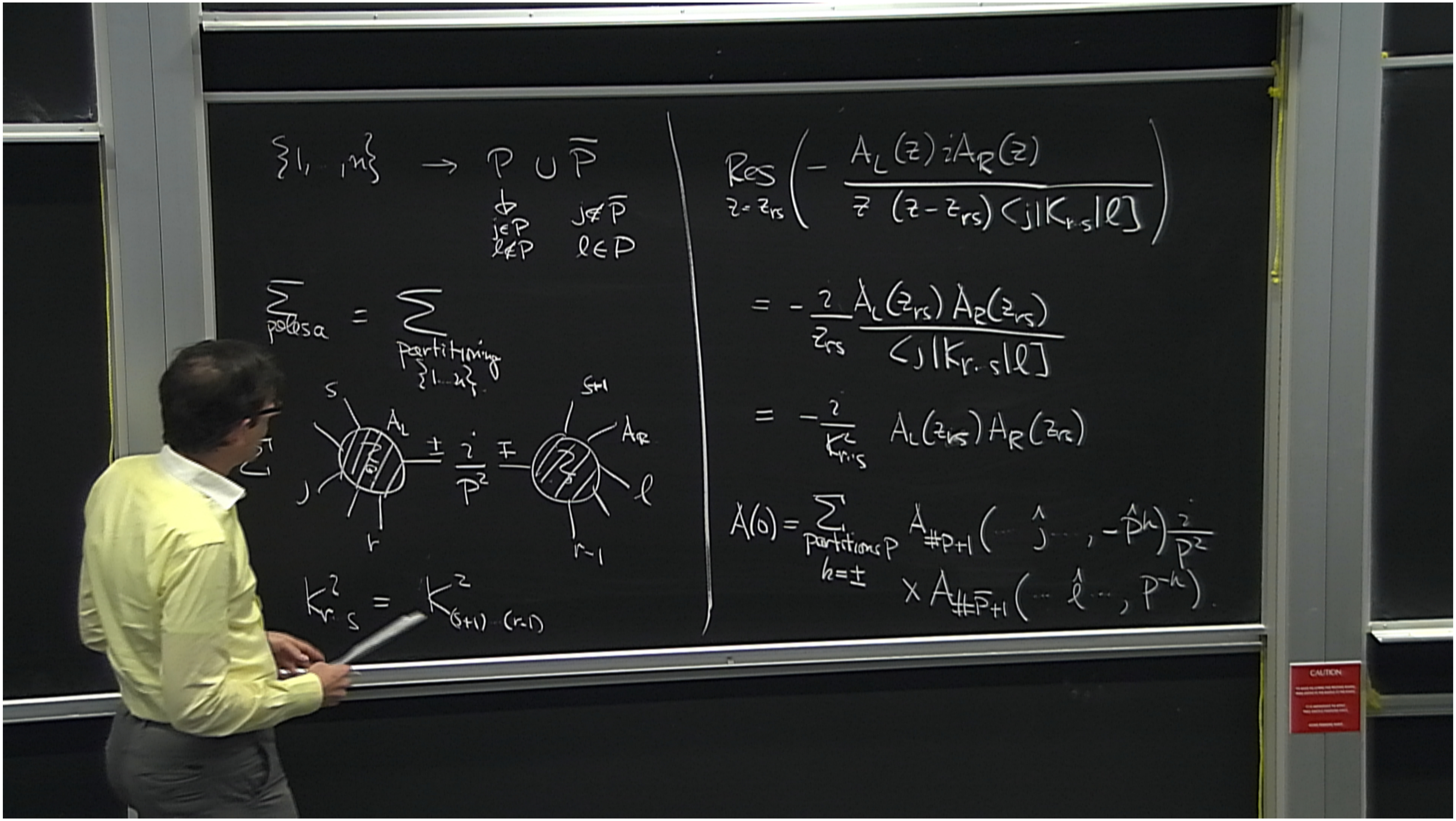
pole at $z=0$: $\mathrm{Res} = A(0)$.
$A(0) = -\sum_{\substack{\text{poles } a \\ z_a \neq 0}} \mathrm{Res}_{z=z_a} \frac{A(z)}{z}$.
poles in $A(z)$.
factorization : $(k_r + \cdots + k_s)^2 = 0$
$j, \ell \notin \{r, \ldots, s\}$
$j \notin, \ell \in$
$j, \ell \in$
$K_{r..s} \to K_{r..s} - \frac{z}{2} \langle j|\mu|\ell]$
$K^2_{r..s} \to K^2_{r..s} - z \langle j|K_{r..s}|\ell] = 0$.
$z = z_{rs} \equiv \frac{K^2_{r..s}}{\langle j|K_{r..s}|\ell]}$


{1, ..., n} → P ∪ P̄
j ∈ P   j ∉ P̄
l ∉ P   l ∈ P̄
CAUTION
CAUTION


{1, ,n} → P ∪ P̄
j∈P
l∉P
j∉P̄
l∈P
Σ poles a = Σ partitioning {1...n}
l


{1, ..., n} → P ∪ P̄
j ∈ P, l ∉ P
j ∉ P̄, l ∈ P̄
Σ poles a = Σ partitioning {1...n}
s, A_L, ±, j, r
s+1, A_R, ∓, l, r−1
K²_{r..s} = K²_{(s+1)..(r−1)}
Res_{z=z_rs} (− A_L(z) A_R(z) / z (z − z_rs) ⟨j|K_{r..s}|l])
= − (1/z_rs) A_L(z_rs) A_R(z_rs) / ⟨j|K_{r..s}|l]
= − (1/K²_{r..s}) A_L(z_rs) A_R(z_rs)
A(0) = Σ_{partitions P, h=±} A_{#P+1}(... ĵ ..., −P̂^h) (1/P²) × A_{#P̄+1}(... l̂ ..., P^{−h})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Σ poles a = Σ partitions {1...n}
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A_L
± i/P² ∓
s+1
A_R
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j
r
r-1
K²_{r..s} = K²_{(s+1)..(r-1)}
Res_{z=z_rs} (− A_L(z) i A_R(z) / z (z − z_rs) ⟨j|K_{r..s}|l])
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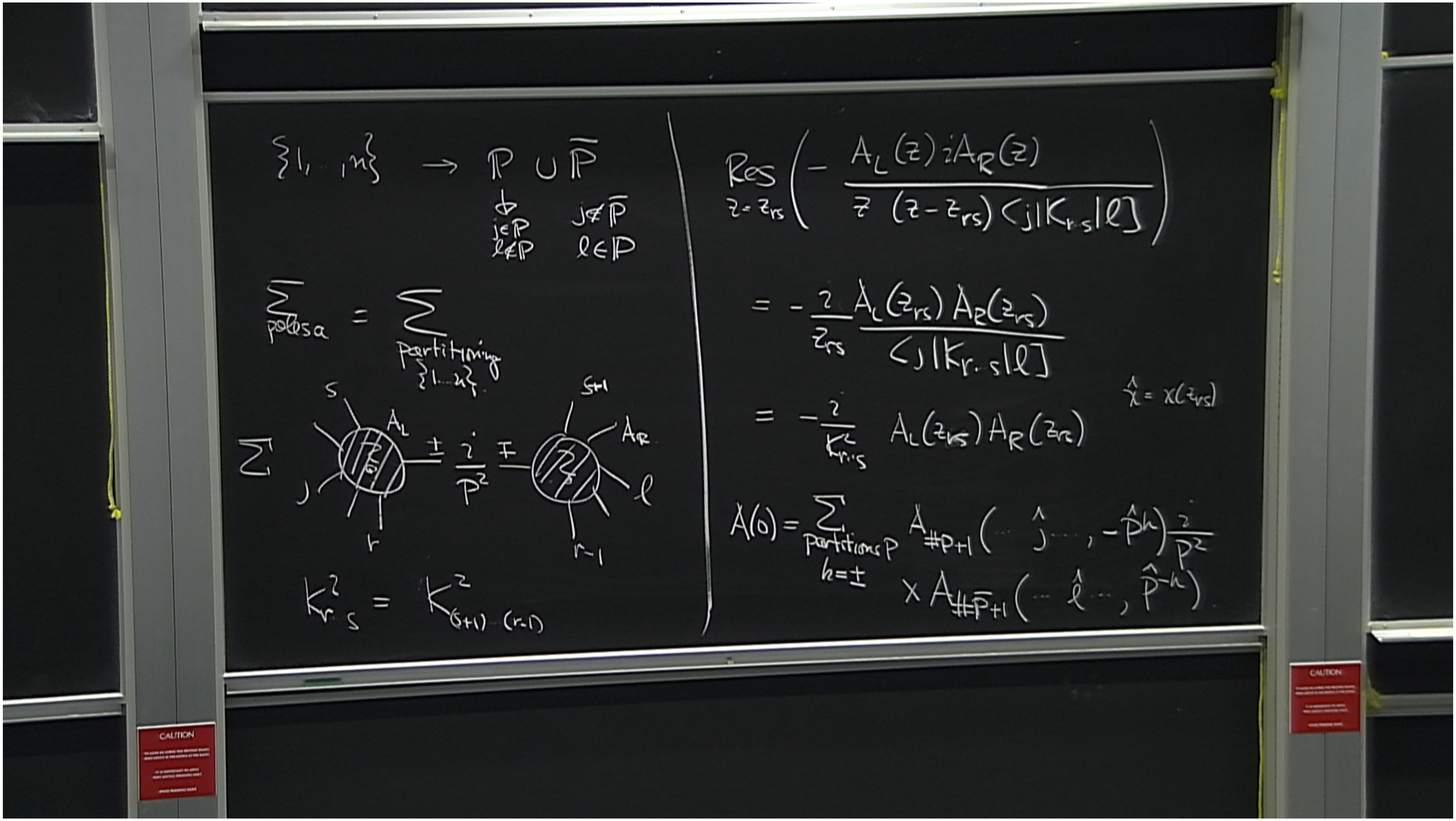
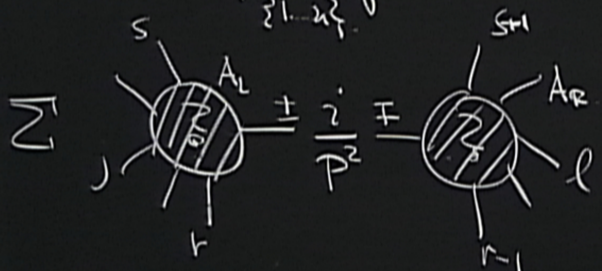
× A_{#P̄+1}(... l̂ ..., P^{−h})


{1, ..., n} → P ∪ P̄
j∈P l∉P
j∉P̄ l∈P
Σ poles a = Σ partitioning {1...n}
s
A_L
± i/P² ∓
s+1
A_R
j
r
l
r-1
K²_{r...s} = K²_{(s+1)...(r-1)}
Res z=z_rs (− A_L(z) i A_R(z) / z (z − z_rs) ⟨j|K_{r..s}|l])
= − i/z_rs · A_L(z_rs) A_R(z_rs) / ⟨j|K_{r..s}|l]
= − i/K²_{r..s} A_L(z_rs) A_R(z_rs)
x̂ = x(z_rs)
A(0) = Σ partitions P, h=± A_{#P+1}(... ĵ ..., −P̂^h) i/P²
× A_{#P̄+1}(... l̂ ..., P̂^{−h})

$$\{1, \ldots, n\} \to \mathbb{P} \cup \bar{\mathbb{P}}$$

$j \in \mathbb{P}$, $\ell \notin \mathbb{P}$ $\qquad$ $j \notin \bar{\mathbb{P}}$, $\ell \in \bar{\mathbb{P}}$

$$\sum_{\text{poles } a} = \sum_{\substack{\text{partitions} \\ \{1 \ldots n\}}}$$



$$K_{r \ldots s}^2 = K^2_{(s+1) \ldots (r-1)}$$

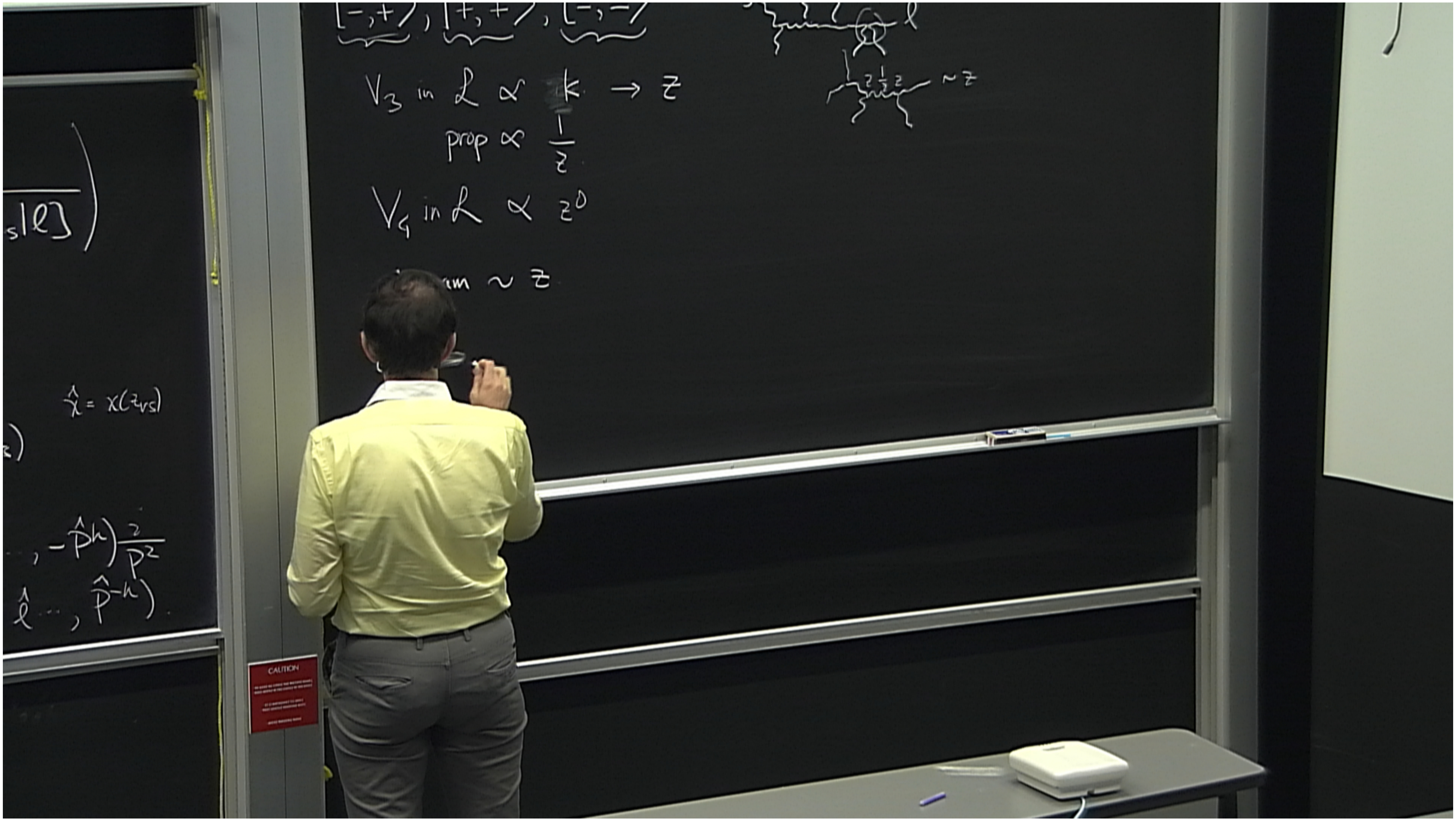
$$\operatorname*{Res}_{z = z_{rs}} \left(- \frac{A_L(z)\, i A_R(z)}{z\,(z - z_{rs})\, \langle j | K_{r \ldots s} | \ell]} \right)$$

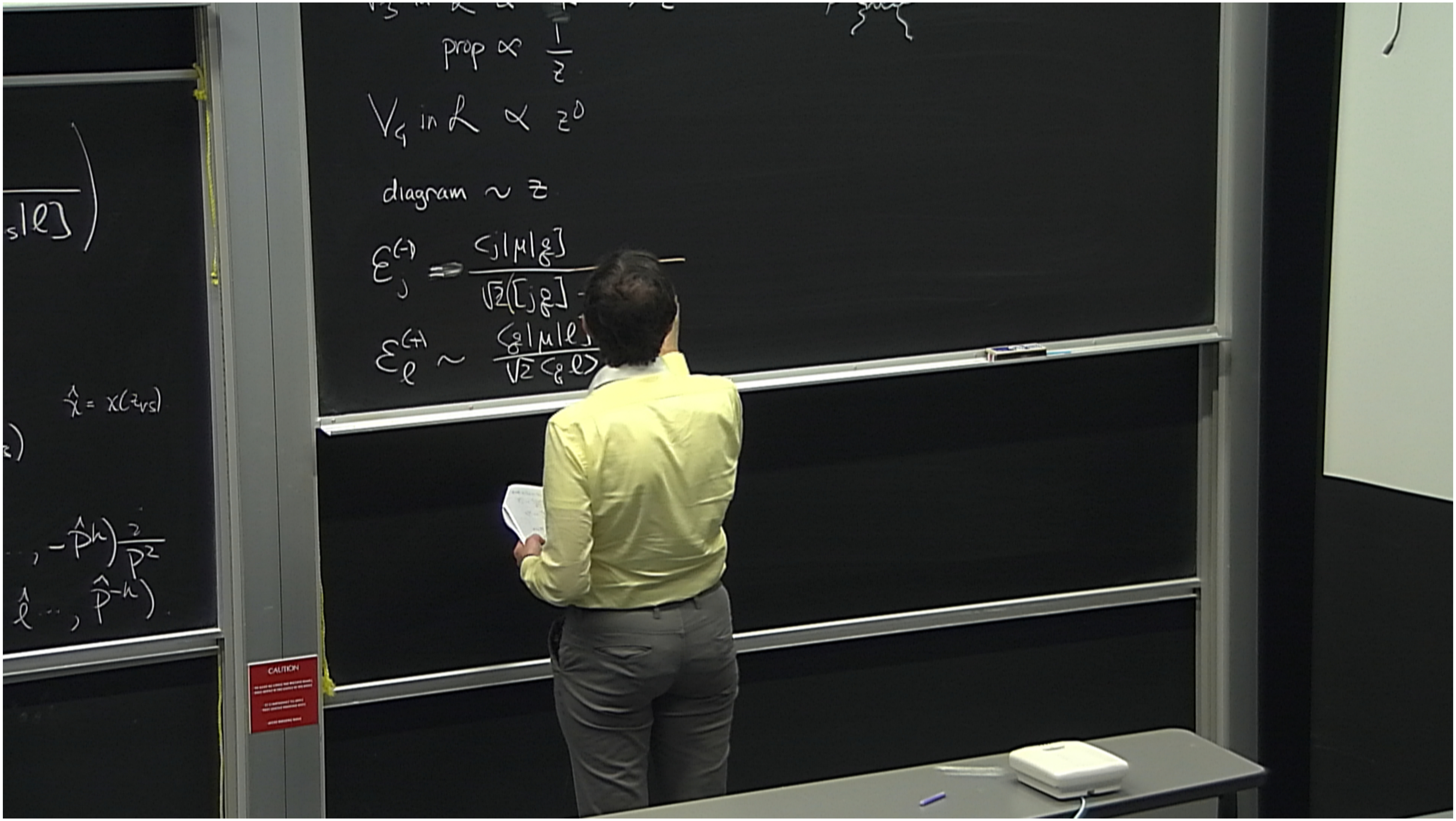
$$= - \frac{i}{z_{rs}} \frac{\hat{A}_L(z_{rs})\, \hat{A}_R(z_{rs})}{\langle j | K_{r \ldots s} | \ell]}$$

$$\hat{x} = x(z_{rs})$$

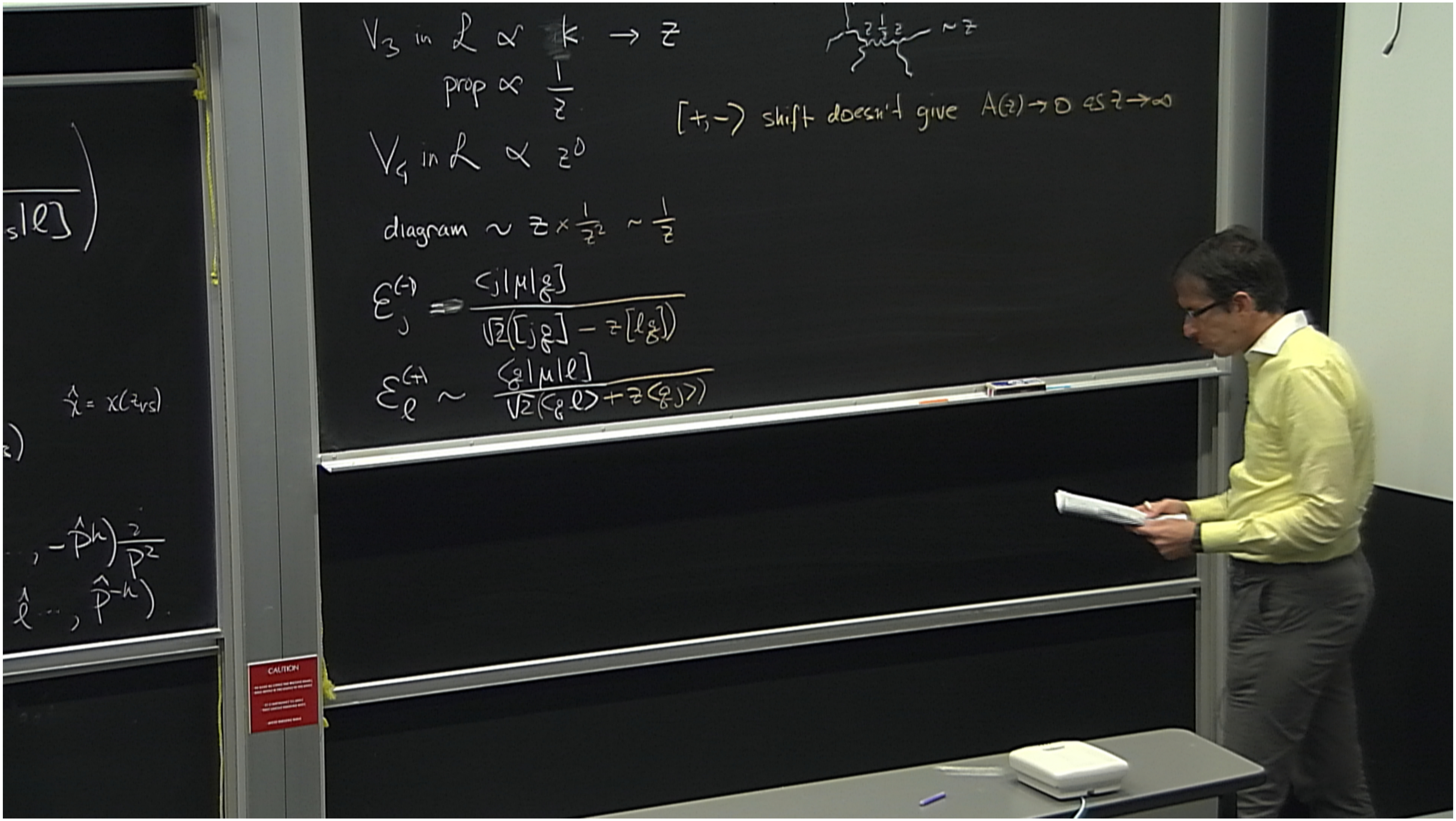
$$= - \frac{i}{K^2_{r \ldots s}} A_L(z_{rs})\, A_R(z_{rs})$$

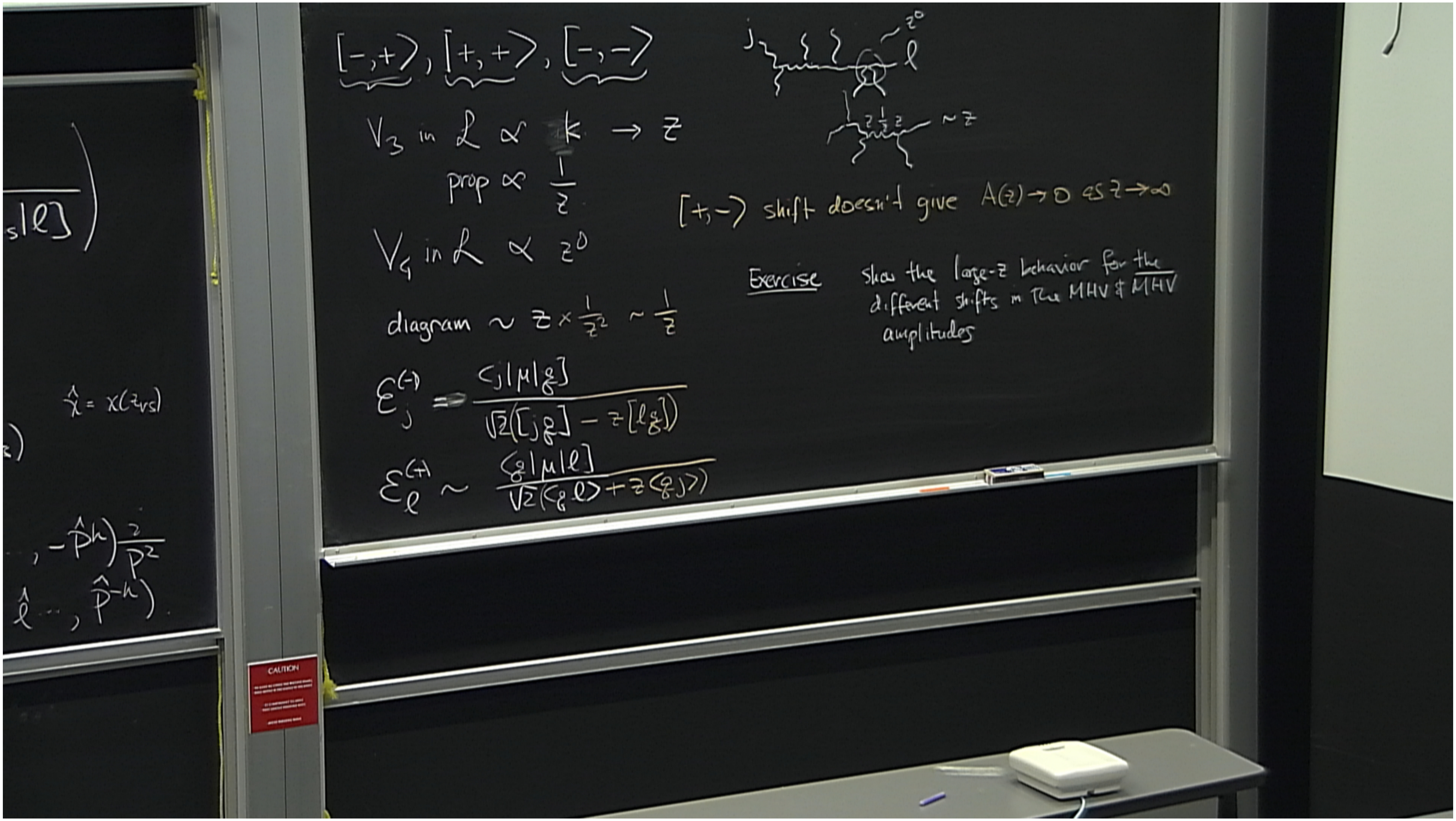
$$A(0) = \sum_{\substack{\text{partitions } \mathbb{P} \\ h = \pm}} \hat{A}_{\# \mathbb{P}+1}(\ldots \hat{j} \ldots, -\hat{P}^h) \frac{i}{P^2} \times \hat{A}_{\# \bar{\mathbb{P}}+1}(\ldots \hat{\ell} \ldots, \hat{P}^{-h})$$


V3 in L ∝ k → z
prop ∝ 1/z
V4 in L ∝ z0
CAUTION



prop ∝ 1/z
V4 in L ∝ z0
diagram ~ z
CAUTION


V3 in L ∝ k → z
prop ∝ 1/z
V4 in L ∝ z^0
diagram ~ z × 1/z^2 ~ 1/z
ε_j^(−) = ⟨j|μ|g] / √2([jg] − z[ℓg])
ε_ℓ^(+) ~ ⟨g|μ|ℓ] / √2(⟨gℓ⟩ + z⟨gj⟩)
[+,−⟩ shift doesn't give A(z) → 0 as z → ∞
~ z
CAUTION


[−,+⟩, [+,+⟩, [−,−⟩
V3 in L ∝ k → z
prop ∝ 1/z
V4 in L ∝ z^0
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ε_j^(−) = ⟨j|μ|g] / √2([jg] − z[ℓg])
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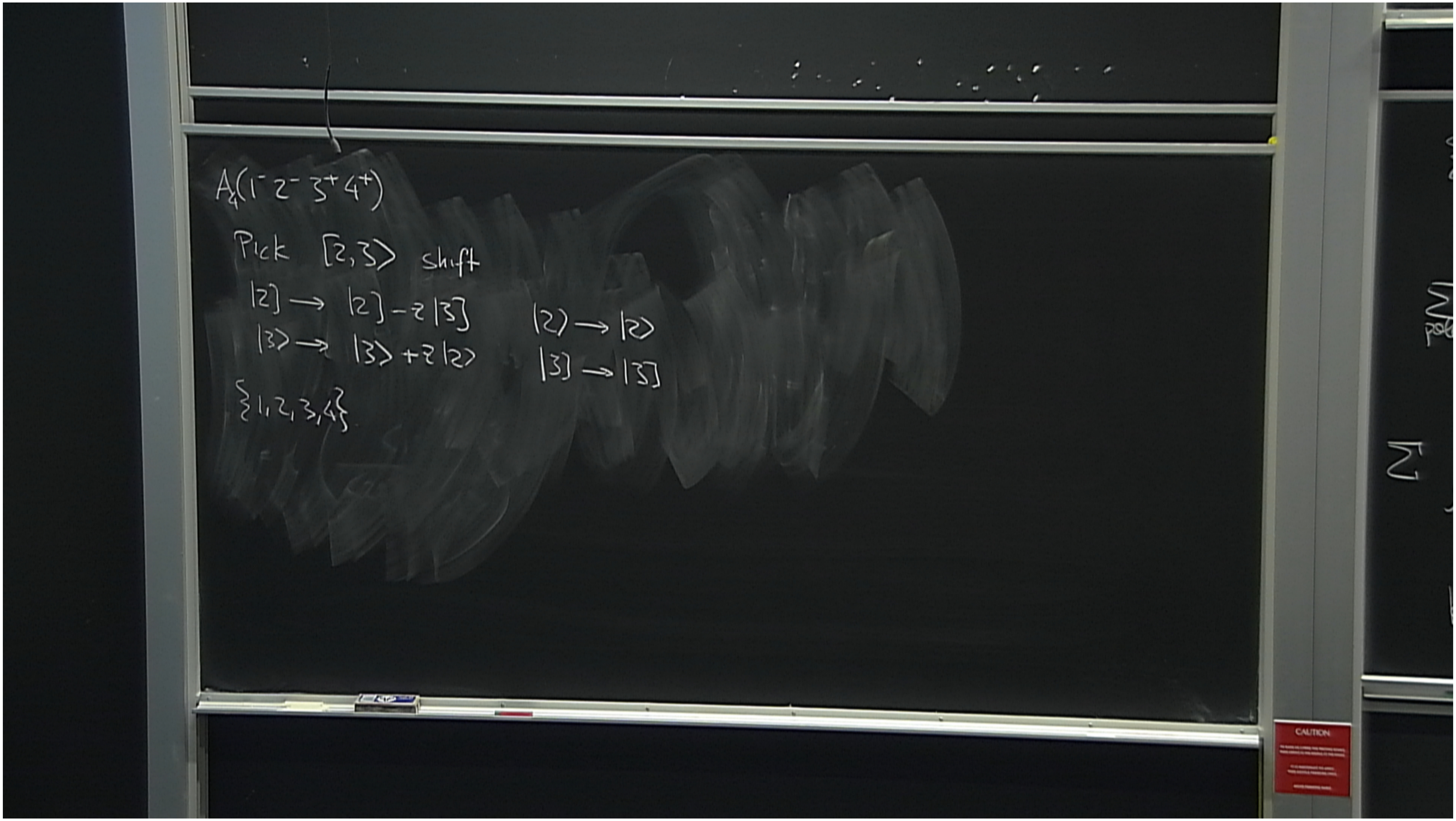
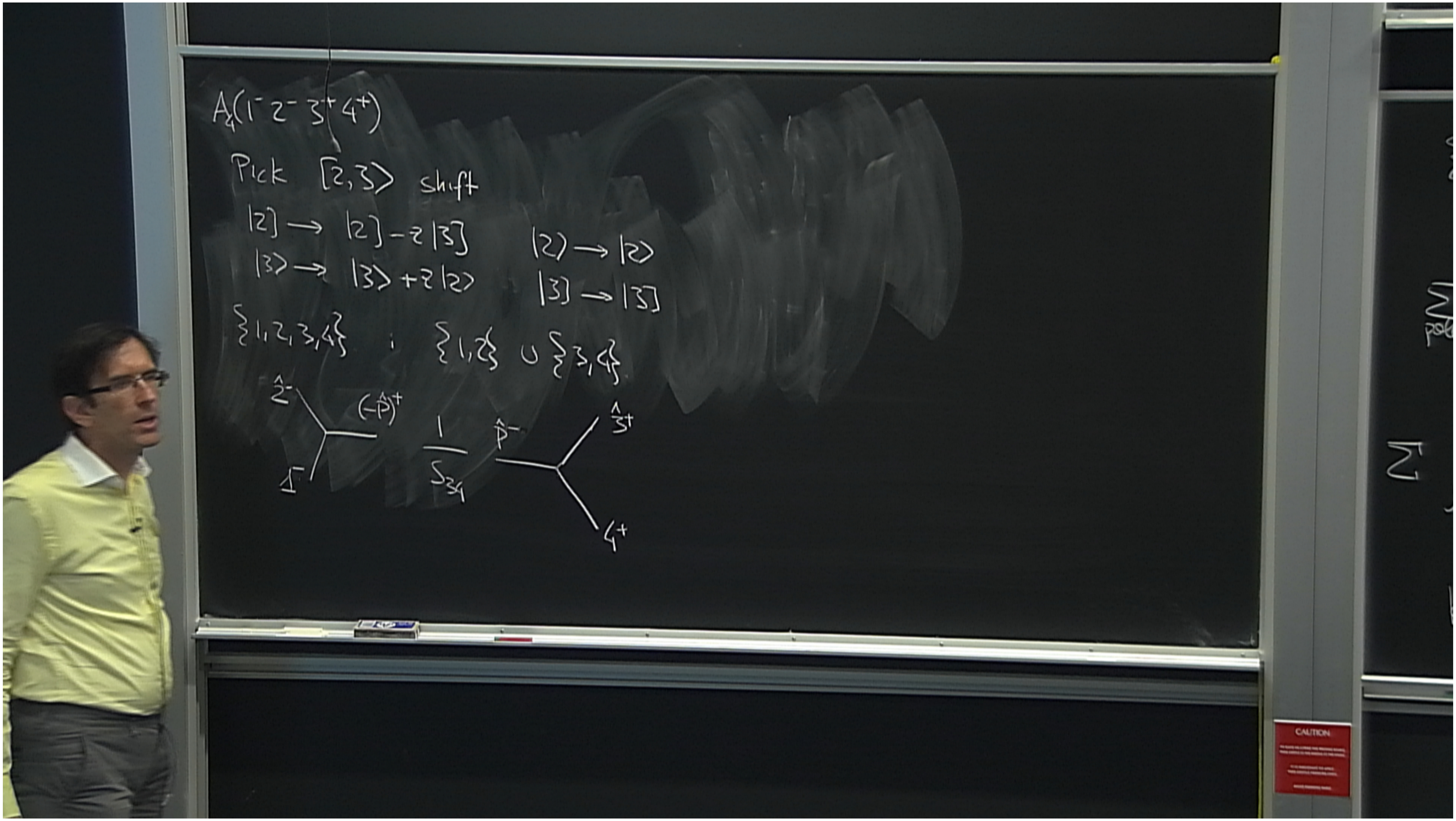
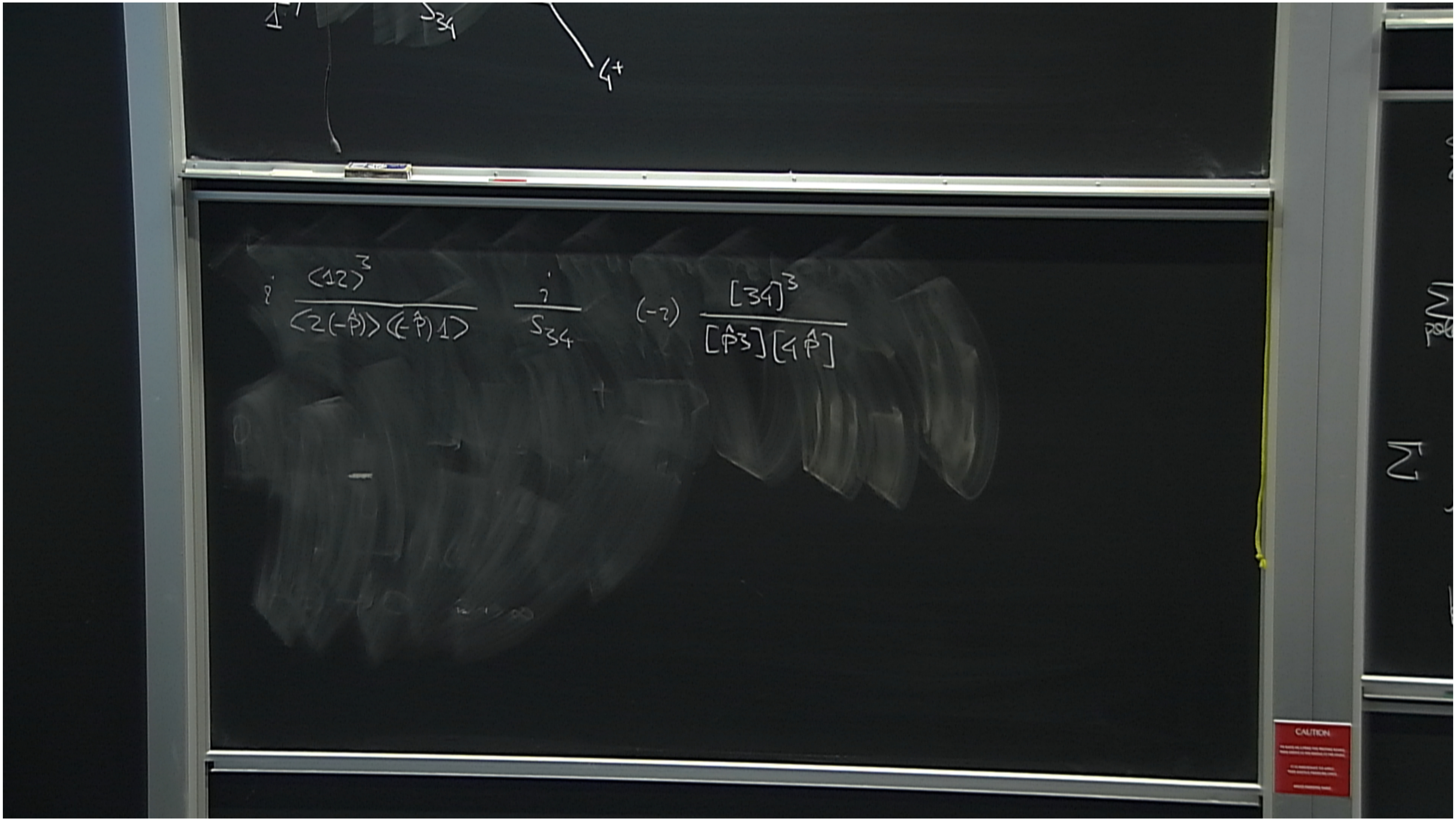
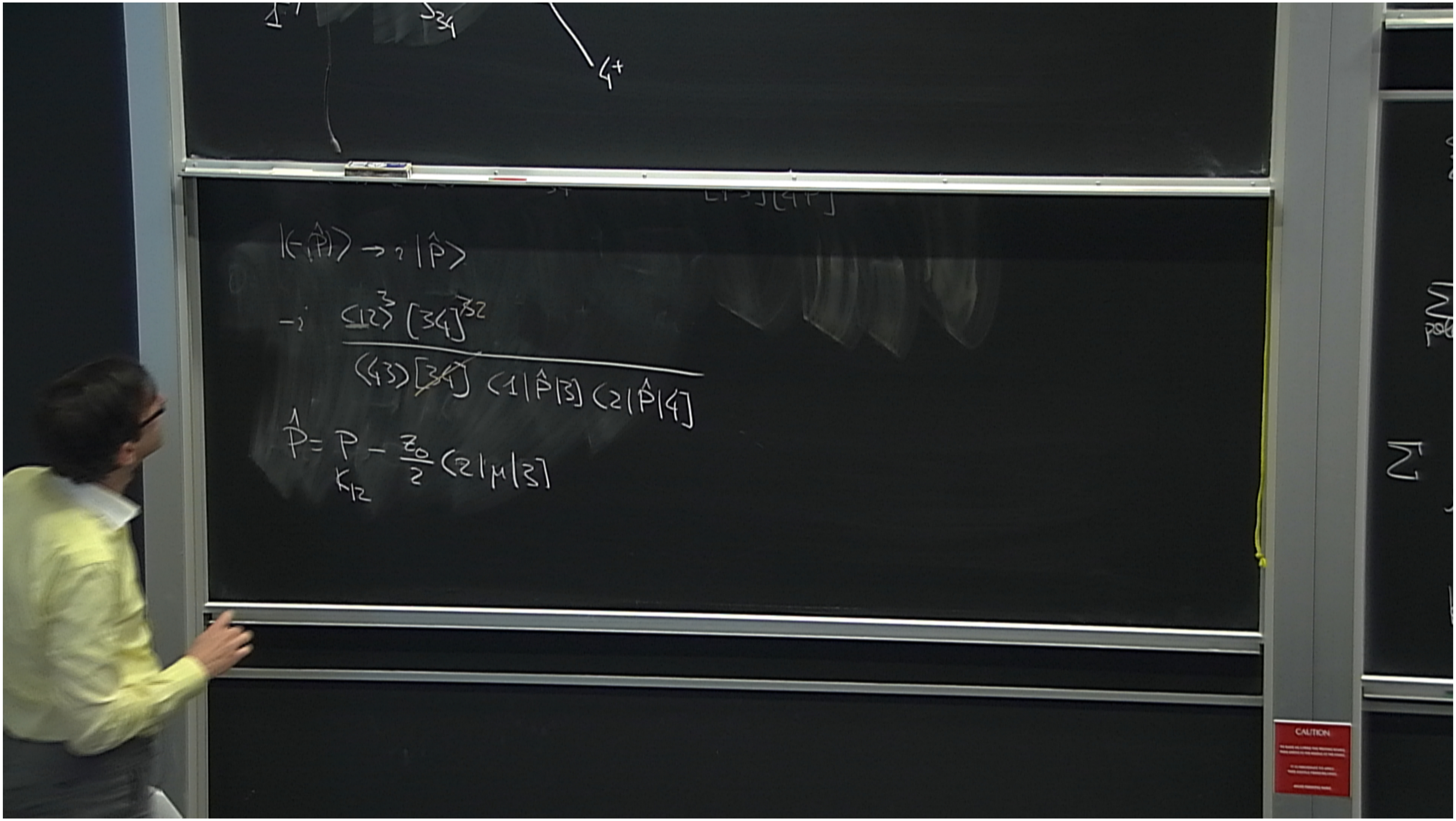
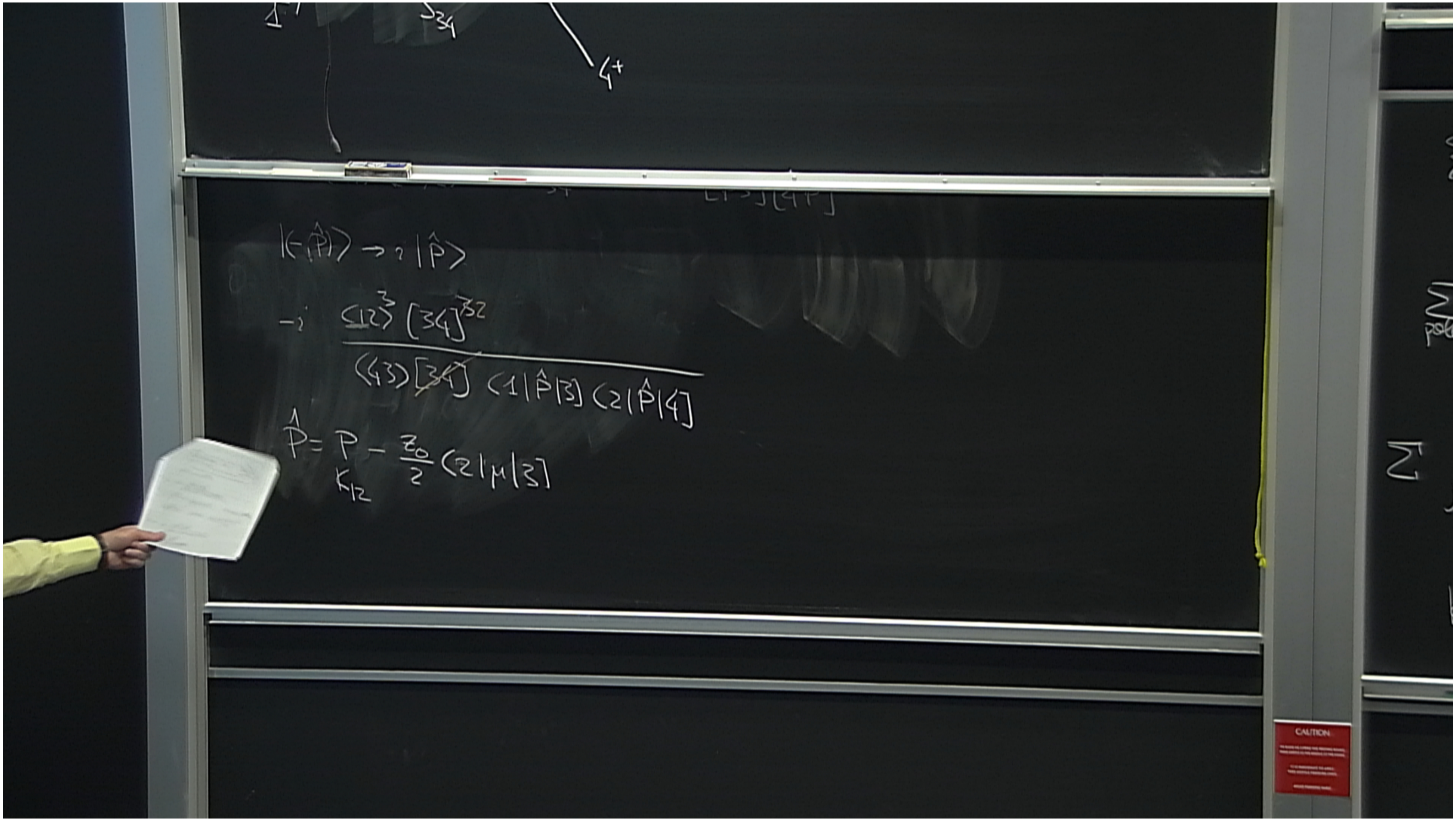
[+,−⟩ shift doesn't give A(z) → 0 as z → ∞
Exercise
Show the large-z behavior for the different shifts in the MHV & MHV amplitudes
CAUTION

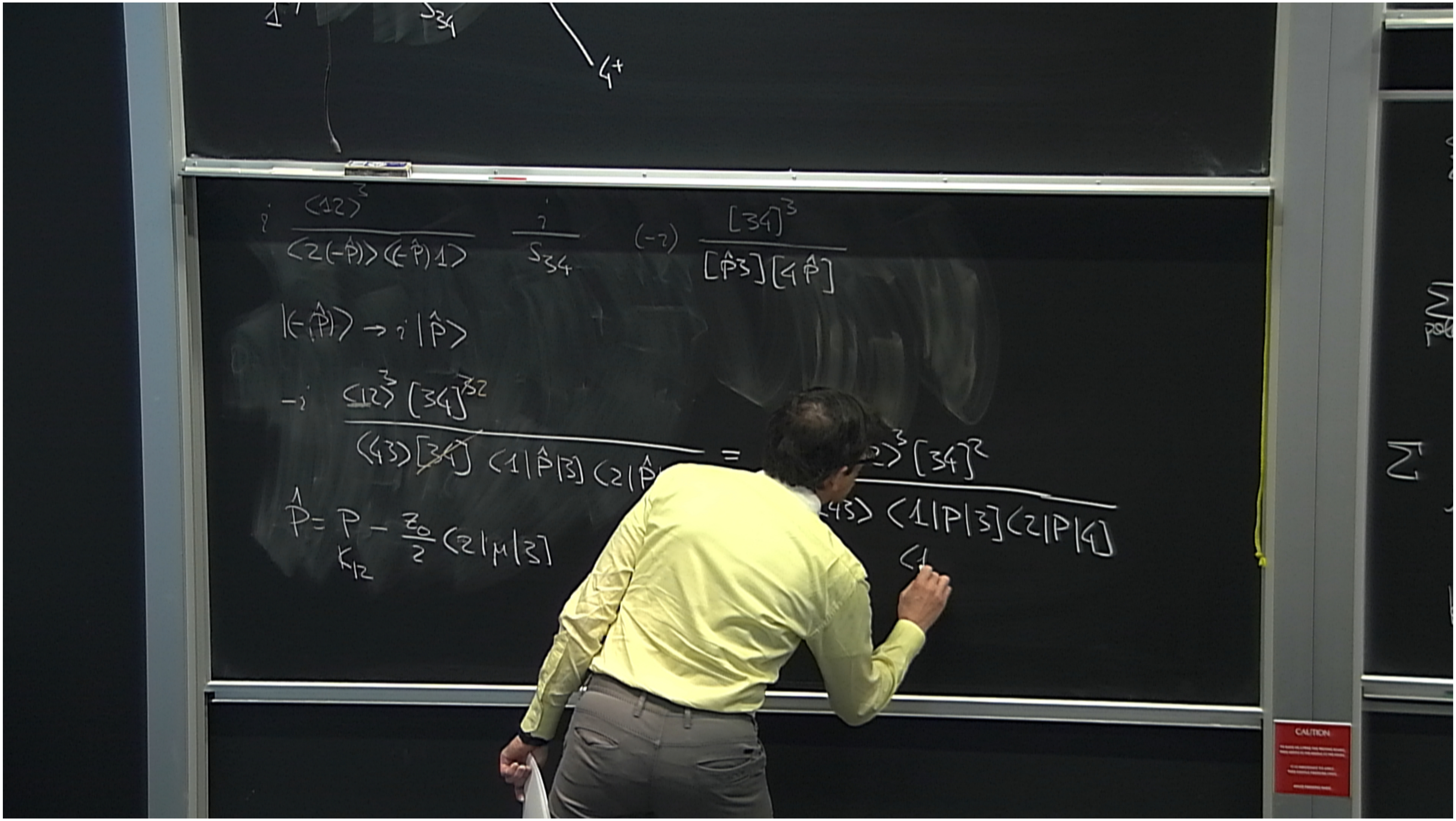

A4(1⁻ 2⁻ 3⁺ 4⁺)
Pick [2,3> shift
|2] → |2] − z|3]
|3> → |3> + z|2>
|2> → |2>
|3] → |3]
{1,2,3,4}
CAUTION

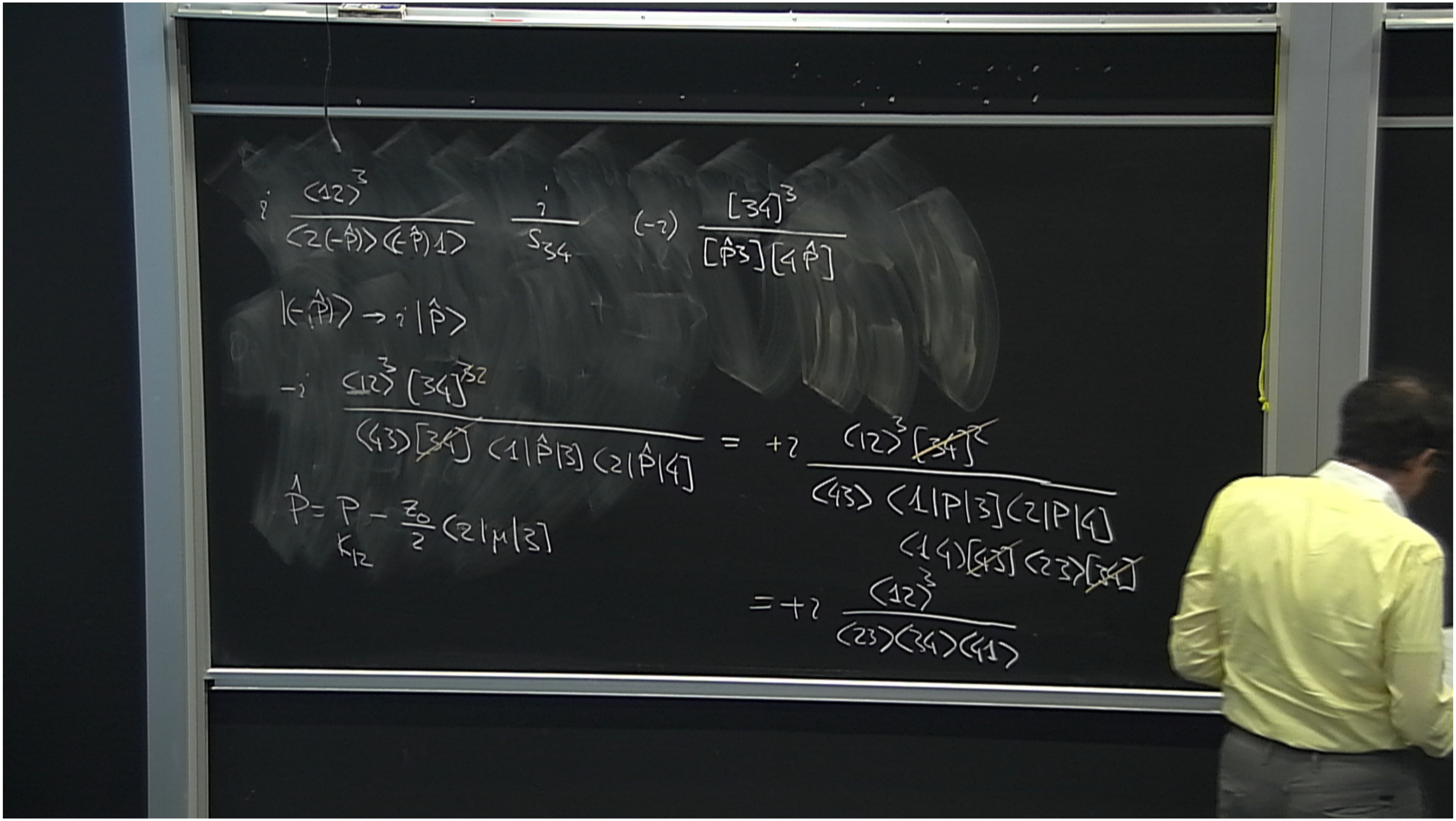

A_4(1^- 2^- 3^+ 4^+)
Pick [2,3> shift
|2] → |2] − z|3]
|3> → |3> + z|2>
|2> → |2>
|3] → |3]
{1,2,3,4} : {1,2} ∪ {3,4}
2̂^-
(−p̂)^+
1^-
1/s_34
p̂^-
3̂^+
4^+

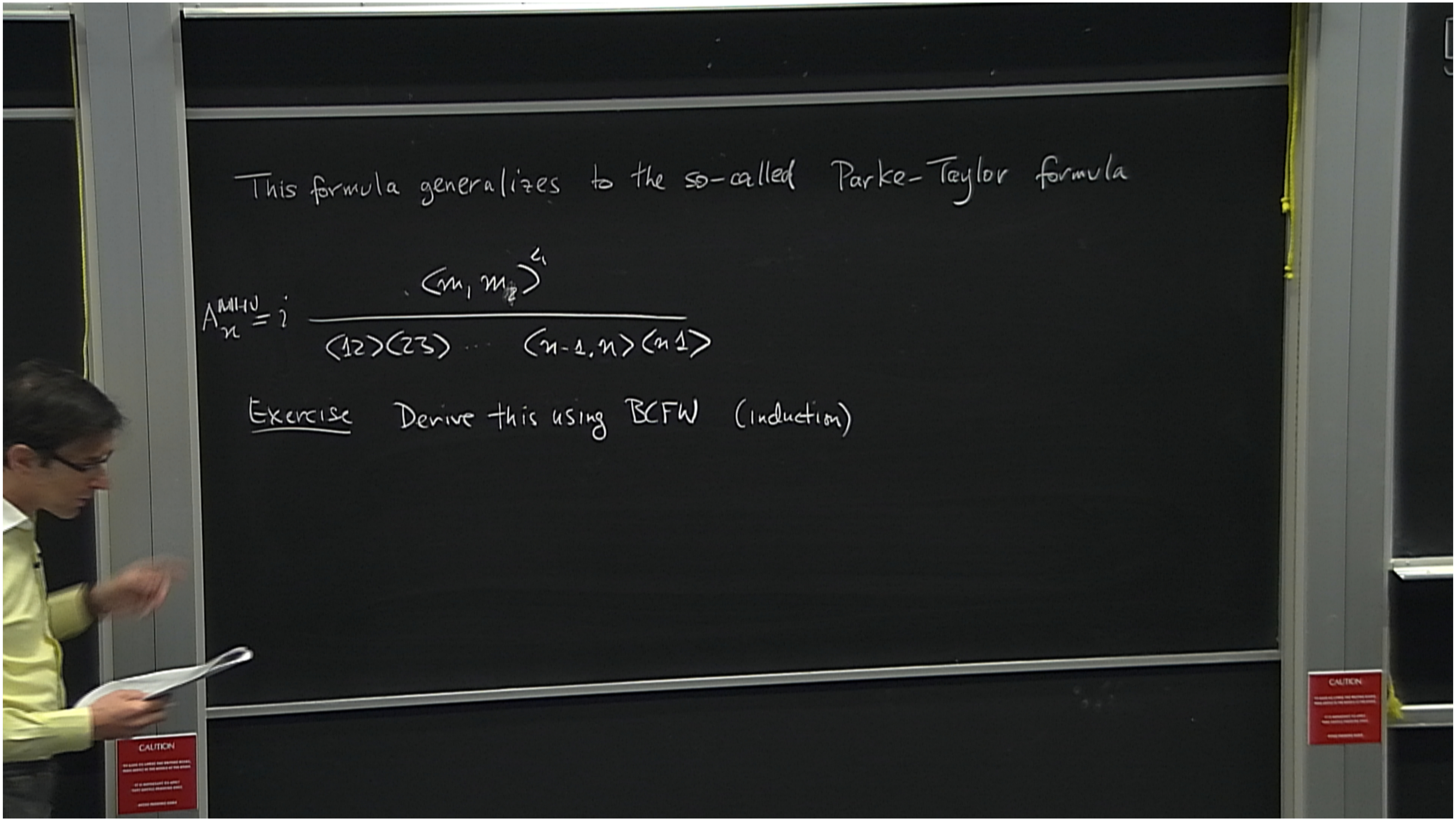

1⁻
S34
4⁺
i ⟨12⟩^3 / ⟨2(−P̂)⟩⟨(−P̂)1⟩
i / S34
(−i) [34]^3 / [P̂3][4P̂]








i ⟨12⟩³ / ⟨2(−P̂)⟩⟨(−P̂)1⟩   i / s₃₄   (−i) [34]³ / [P̂3][4P̂]
|(−P̂)⟩ → i |P̂⟩
−i ⟨12⟩³ [34]³ / ⟨43⟩[34] ⟨1|P̂|3] ⟨2|P̂|4] = +i ⟨12⟩³ [34]² / ⟨43⟩ ⟨1|P|3]⟨2|P|4]
P̂ = P_{K12} − z₀/2 ⟨2|μ|3]
⟨14⟩[43] ⟨23⟩[34]
= +i ⟨12⟩³ / ⟨23⟩⟨34⟩⟨41⟩


This formula generalizes to the so-called Parke-Taylor formula
$A_n^{MHV} = i \frac{\langle m_1 m_2 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle \langle n 1 \rangle}$
Exercise Derive this using BCFW (induction)
CAUTION

This formula generalizes to the so-called Parke-Taylor formula

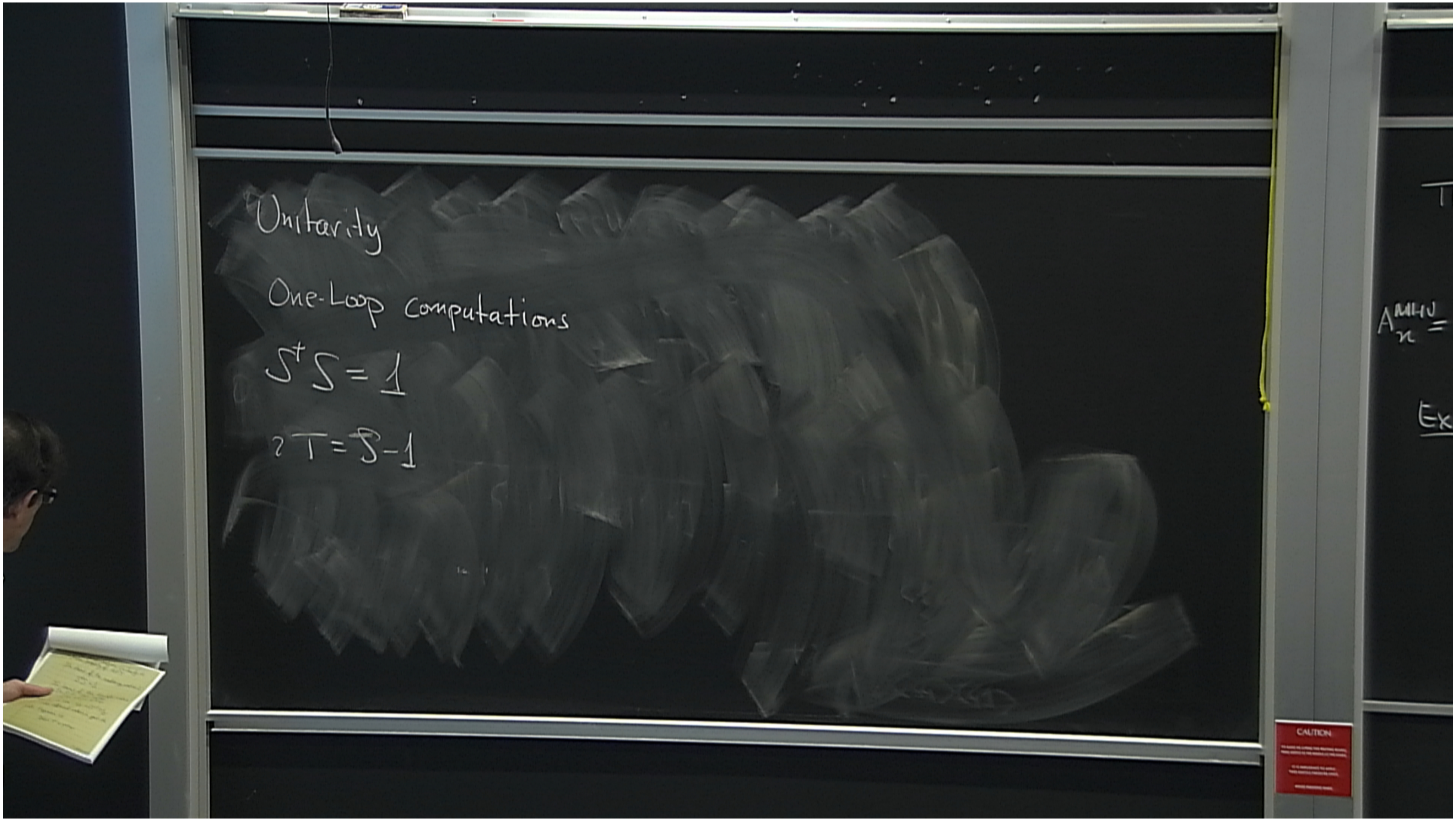
$$A_n^{MHV} = i \frac{\langle m_1 m_2 \rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n-1, n\rangle\langle n1\rangle}$$

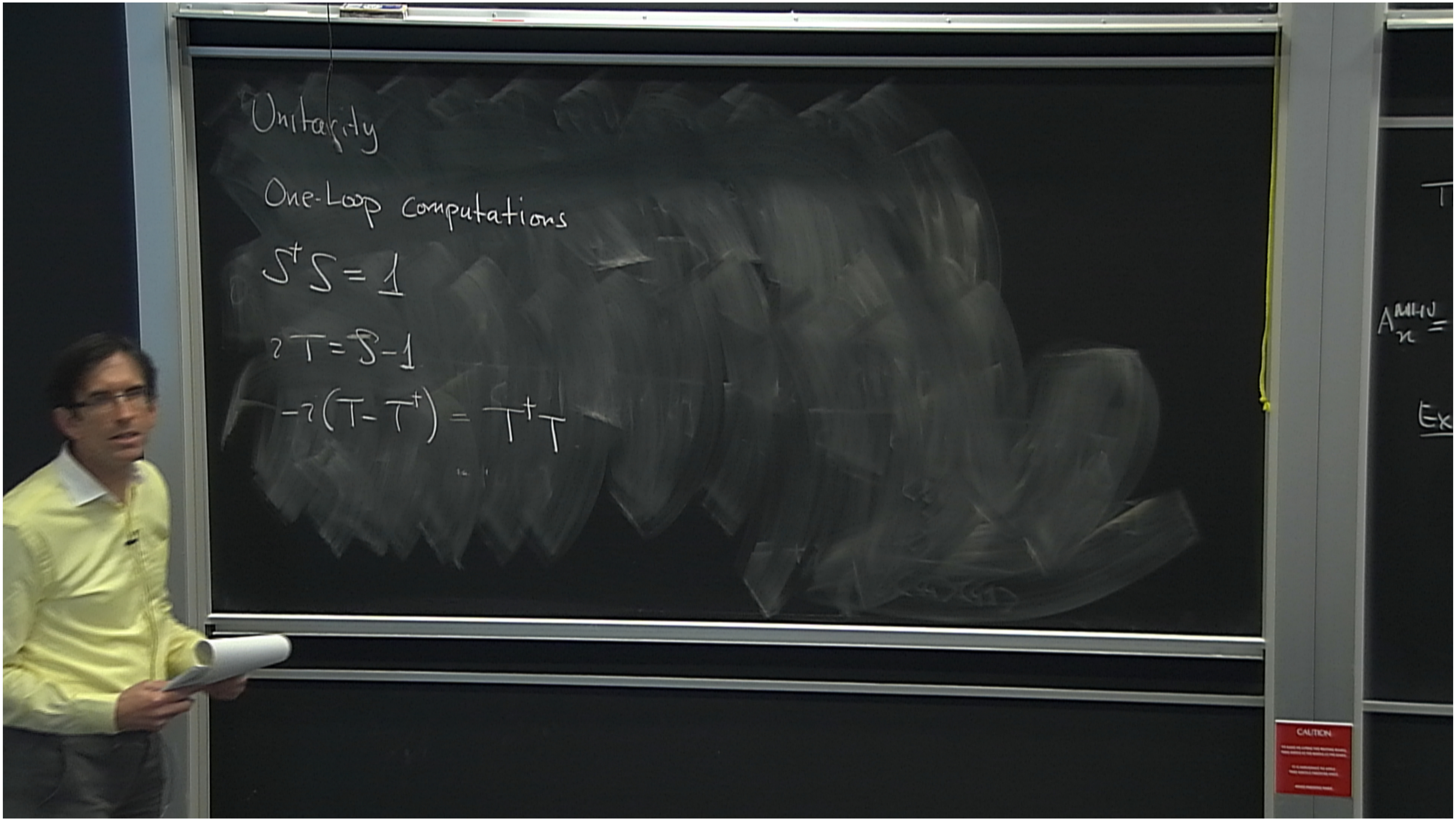
<u>Exercise</u> Derive this using BCFW (induction)

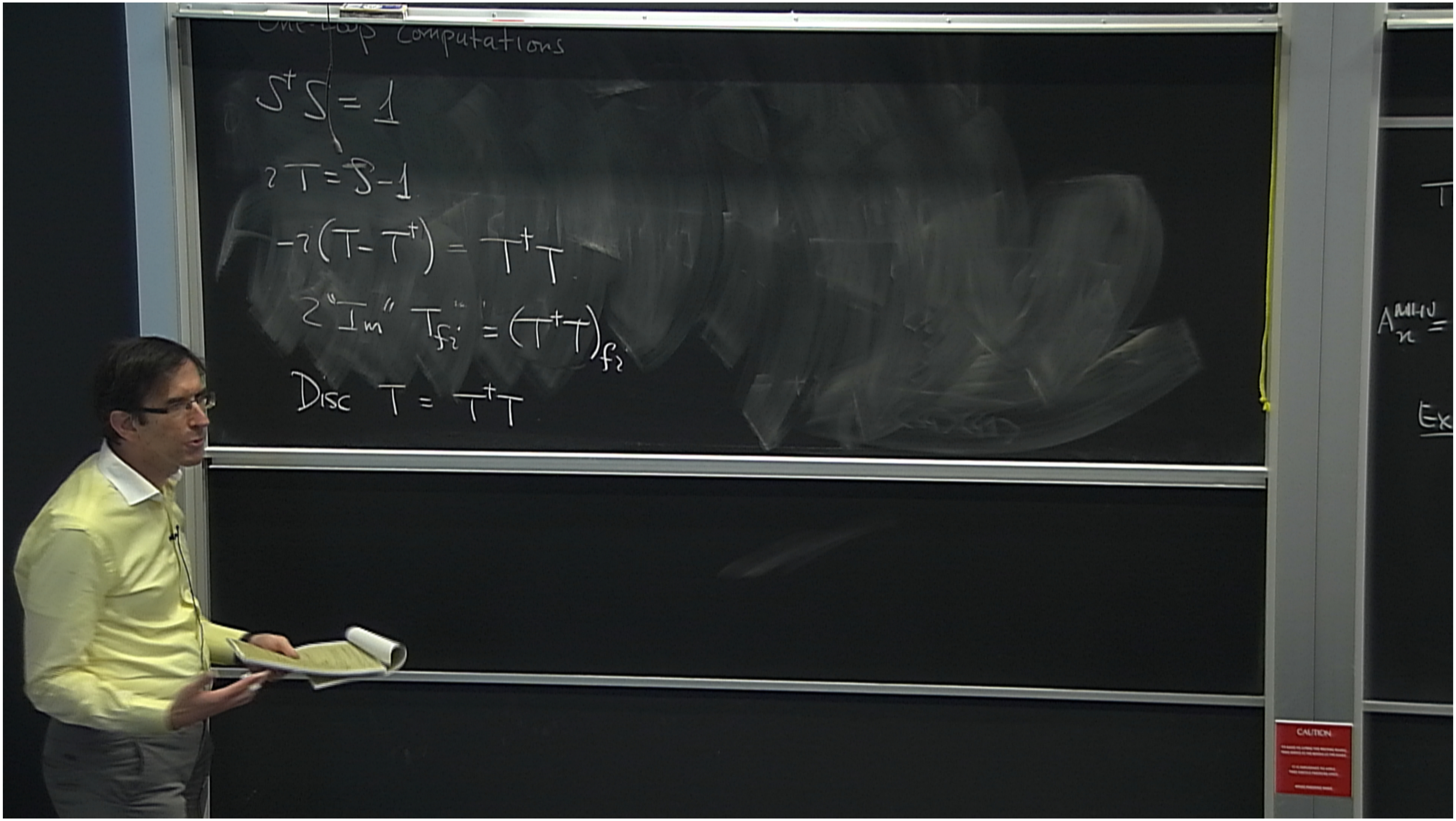
This formula generalizes to the so-called Parke-Taylor formula

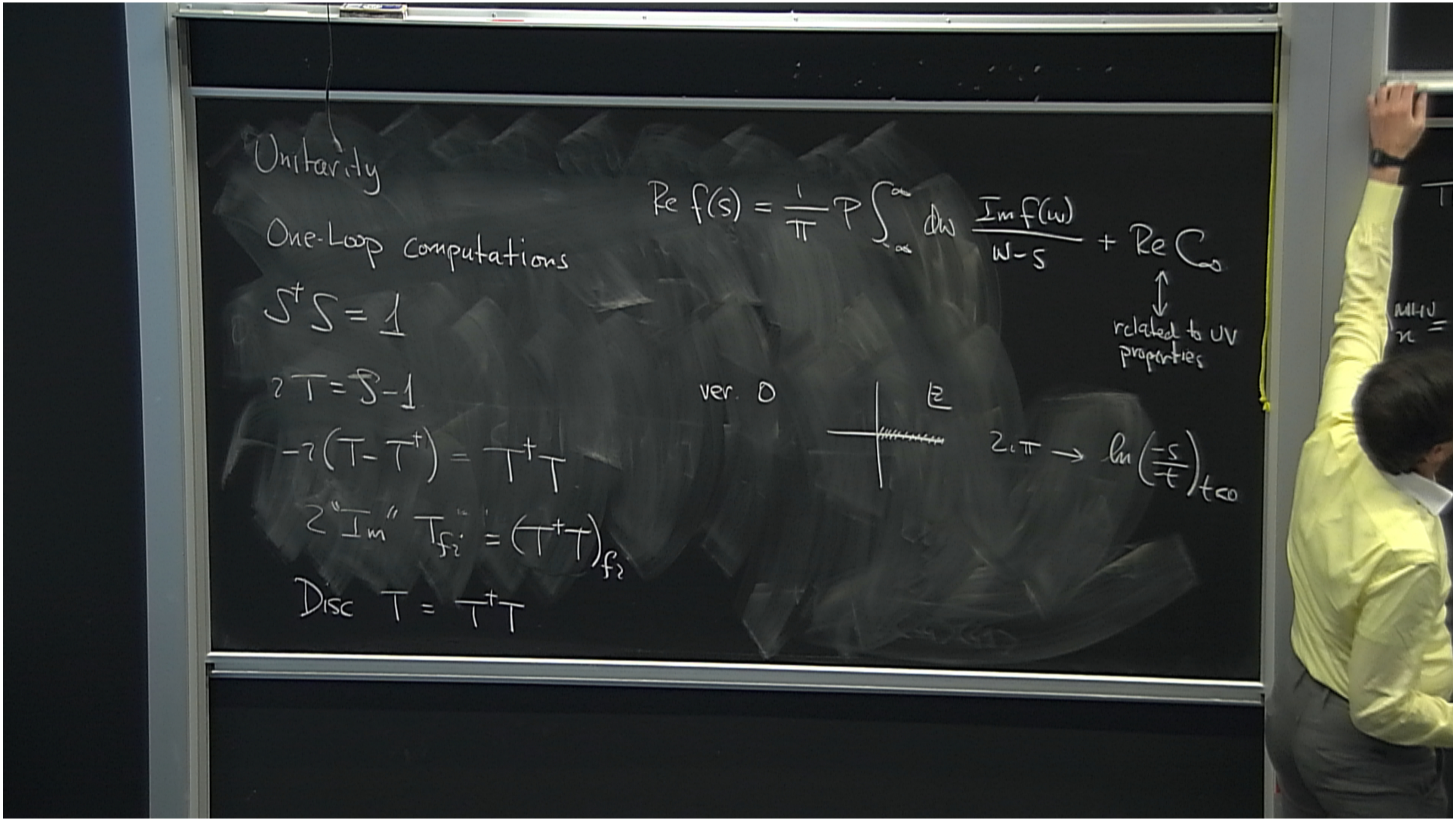
$$A_n^{MHV} = i \frac{\langle m_1 m_2 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle \langle n 1 \rangle} \qquad (1^+ \cdots m_1^- \cdots m_2^- \cdots n^+)$$

<u>Exercise</u> Derive this using BCFW (induction)


Unitarity
One-Loop computations
S†S = 1
? T = S − 1
A^MHV_n =
Ex


Unitarity
One-Loop computations
S† S = 1
i T = S − 1
−i (T − T†) = T† T

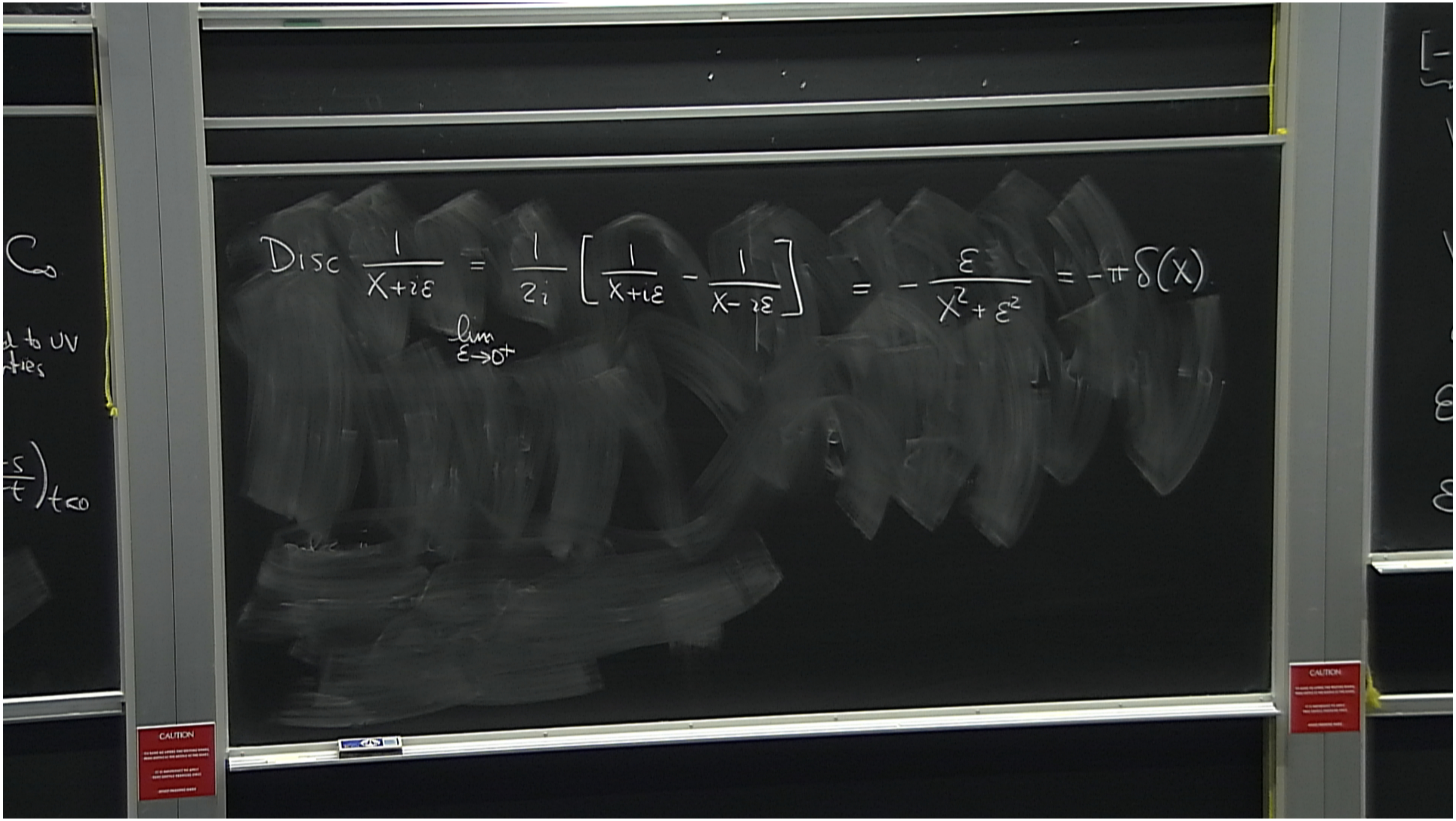

Computations
S†S = 1
iT = S − 1
−i(T − T†) = T†T
2 "Im" T_fi = (T†T)_fi
Disc T = T†T

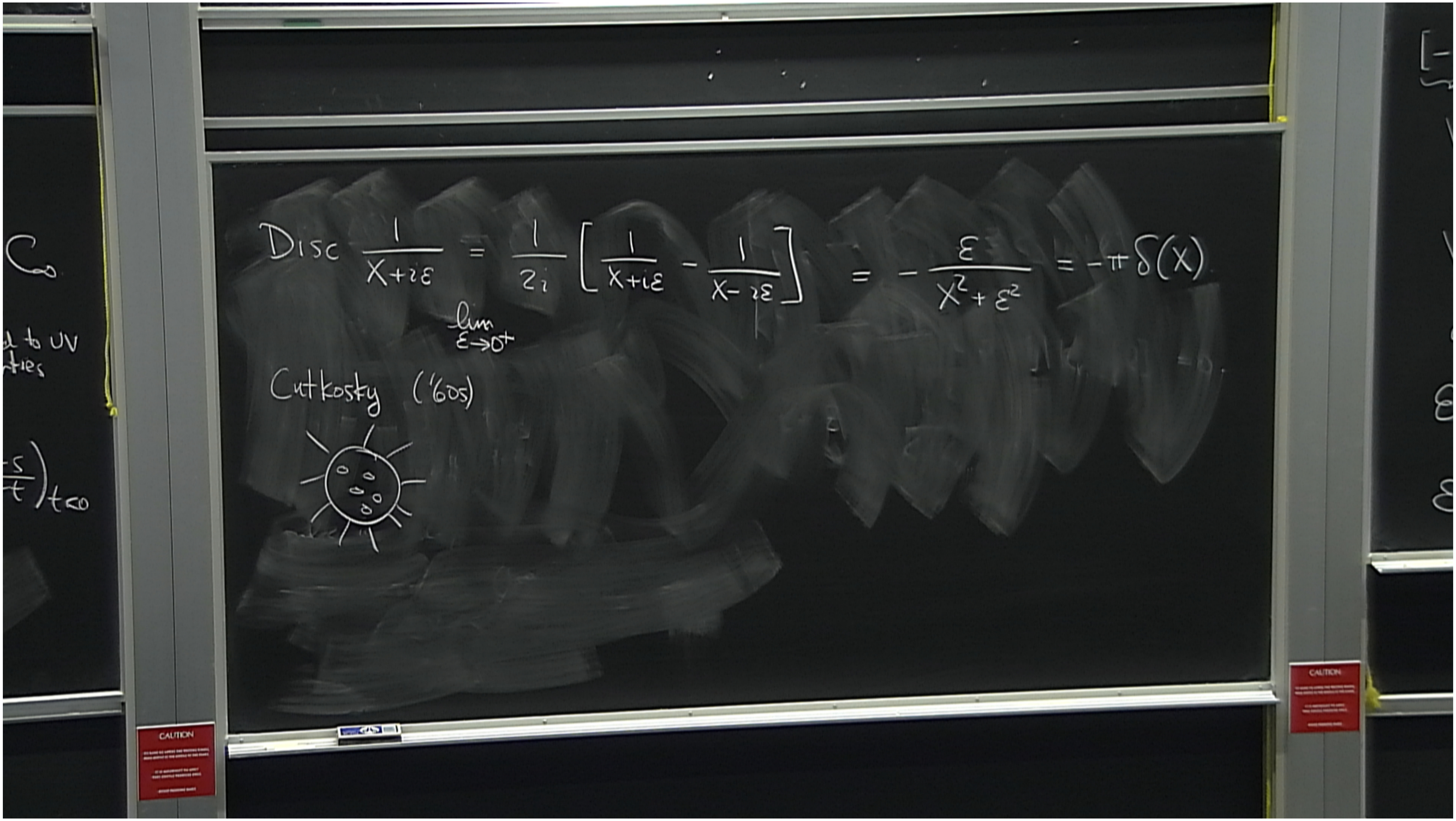

Unitarity
One-Loop computations
$S^\dagger S = 1$
$i T = S - 1$
$-i(T - T^\dagger) = T^\dagger T$
$2\,\text{"Im"}\,T_{fi} = (T^\dagger T)_{fi}$
Disc $T = T^\dagger T$
$\text{Re}\, f(s) = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega \frac{\text{Im}\, f(\omega)}{\omega - s} + \text{Re}\, C_\infty$
related to UV properties
ver. 0
$2i\pi \rightarrow \ln\left(\frac{-s}{-t}\right)_{t<0}$

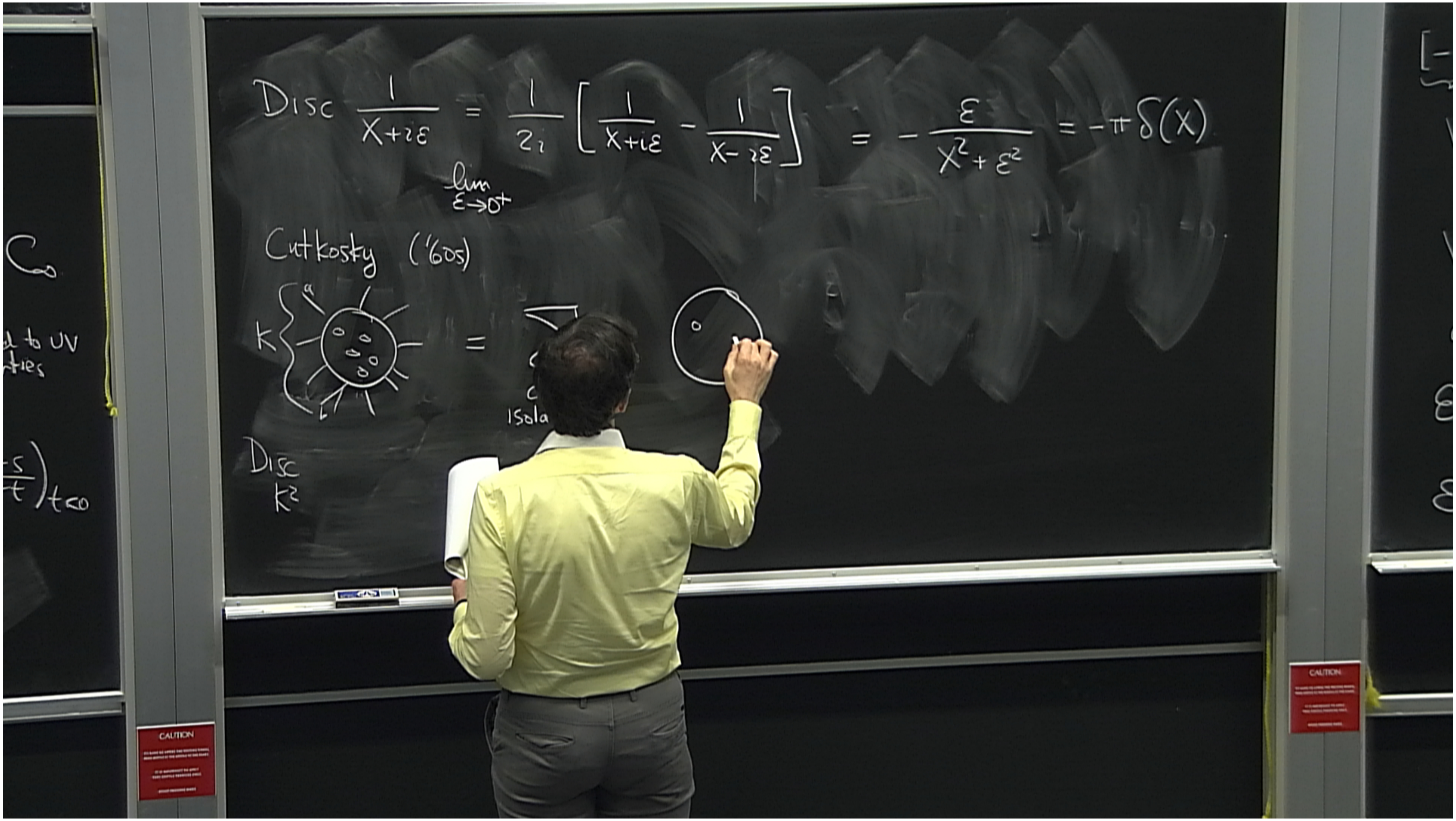
This formula generalizes to the so-called Parke-Taylor formula

$$\text{Disc}\,\frac{1}{X+i\varepsilon} = \frac{1}{2i}\left[\frac{1}{X+i\varepsilon} - \frac{1}{X-i\varepsilon}\right] = -\frac{\varepsilon}{X^2+\varepsilon^2}$$

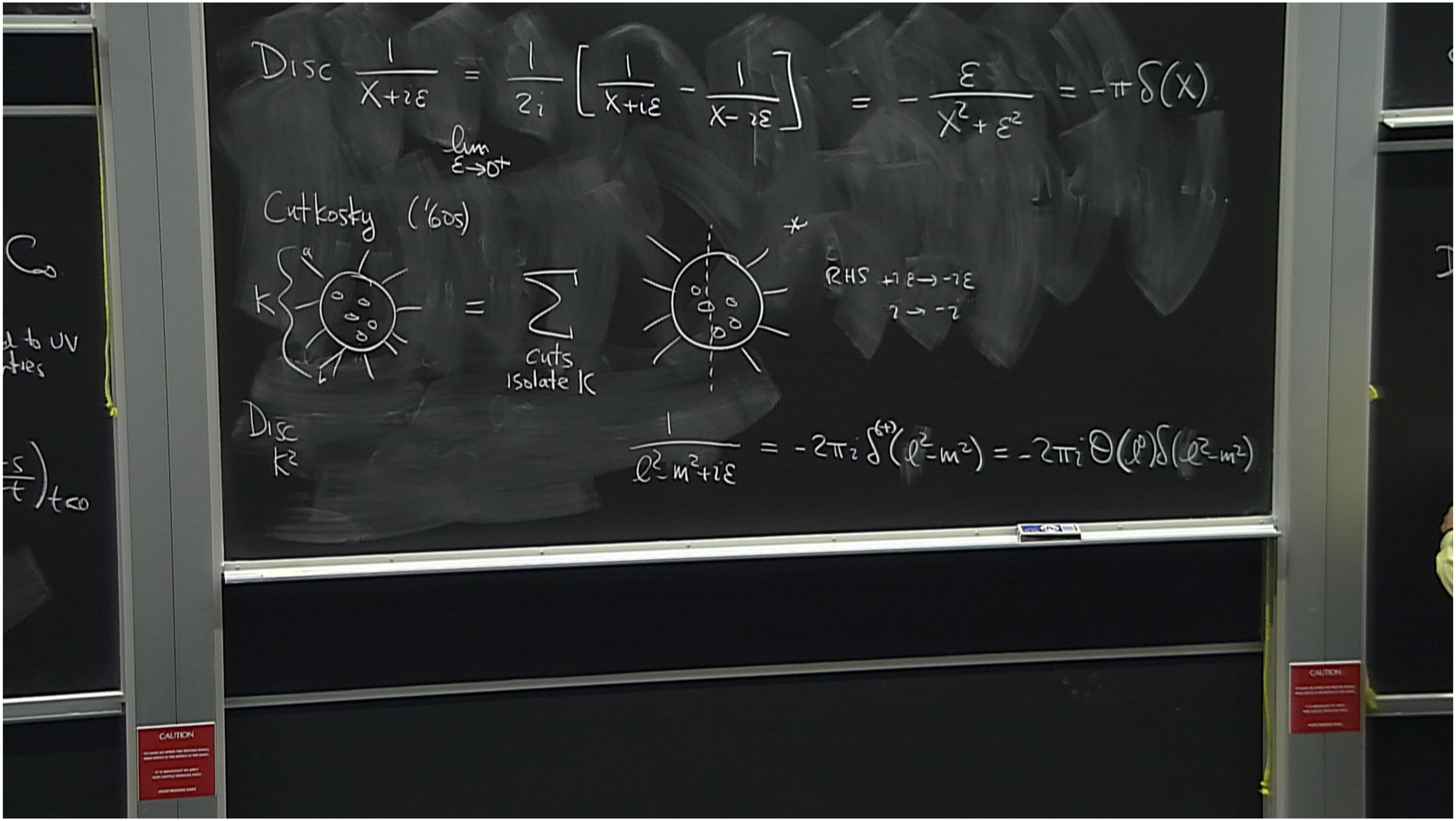
$\lim_{\varepsilon \to 0^+}$

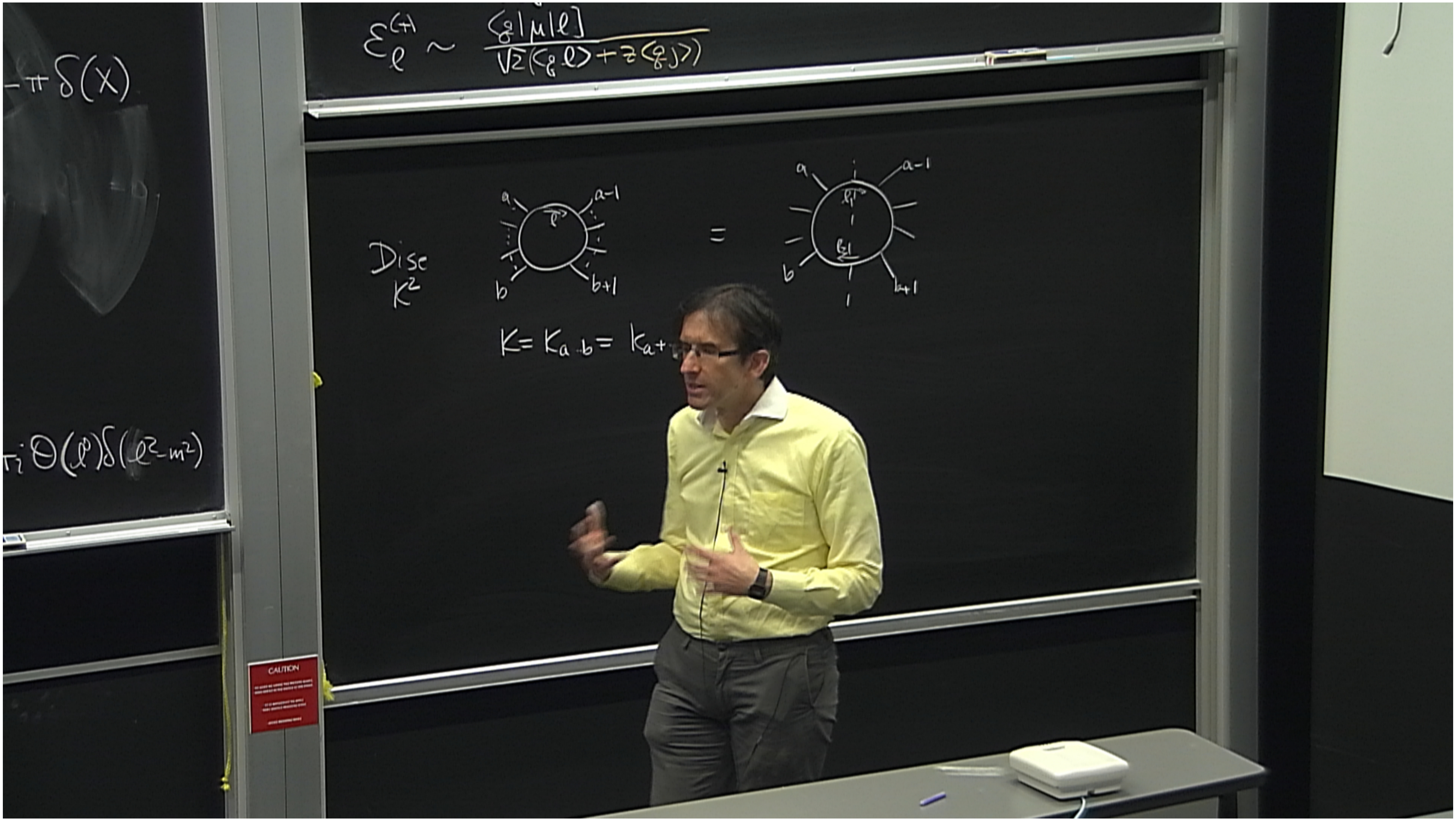

$\text{Disc}\,\frac{1}{x+i\varepsilon} = \frac{1}{2i}\left[\frac{1}{x+i\varepsilon} - \frac{1}{x-i\varepsilon}\right] = -\frac{\varepsilon}{x^2+\varepsilon^2} = -\pi\delta(x)$
$\lim_{\varepsilon\to 0^+}$
C_0
d to UV
ties
$t<0$
CAUTION


$\text{Disc}\,\frac{1}{x+i\varepsilon} = \frac{1}{2i}\left[\frac{1}{x+i\varepsilon} - \frac{1}{x-i\varepsilon}\right] = -\frac{\varepsilon}{x^2+\varepsilon^2} = -\pi\,\delta(x)$
$\lim_{\varepsilon\to 0^+}$
Cutkosky ('60s)

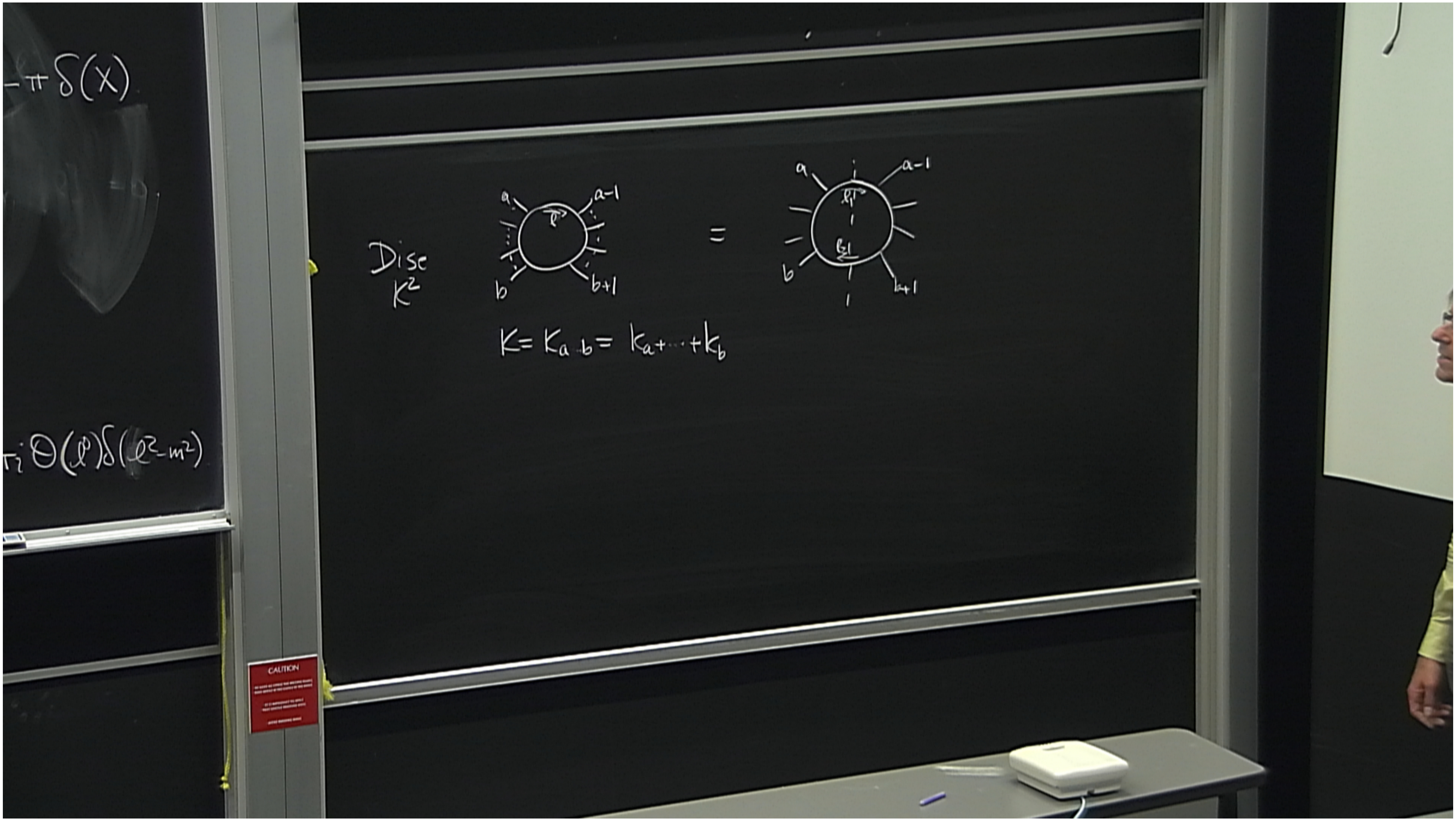


$\text{Disc} \frac{1}{x+i\varepsilon} = \frac{1}{2i}\left[\frac{1}{x+i\varepsilon} - \frac{1}{x-i\varepsilon}\right] = -\frac{\varepsilon}{x^2+\varepsilon^2} = -\pi\delta(x)$
$\lim_{\varepsilon\to 0^+}$
Cutkosky ('60s)
K
=
Isola
Disc
K^2


Disc 1/(x+iε) = 1/2i [1/(x+iε) - 1/(x-iε)] = - ε/(x²+ε²) = -π δ(x)
lim ε→0+
Cutkosky ('60s)
RHS +iε → -iε
i → -i
cuts
isolate K
Disc K²
1/(ℓ²-m²+iε) = -2πi δ(+)(ℓ²-m²) = -2πi Θ(ℓ⁰) δ(ℓ²-m²)
CAUTION



Disc
K²
K= Ka..b= Ka+
CAUTION


−π δ(X)
Disc
K²
a
a−1
b
b+1
=
K= Ka..b = ka+ ··· +kb
Θ(ℓ⁰)δ(ℓ²−m²)
CAUTION