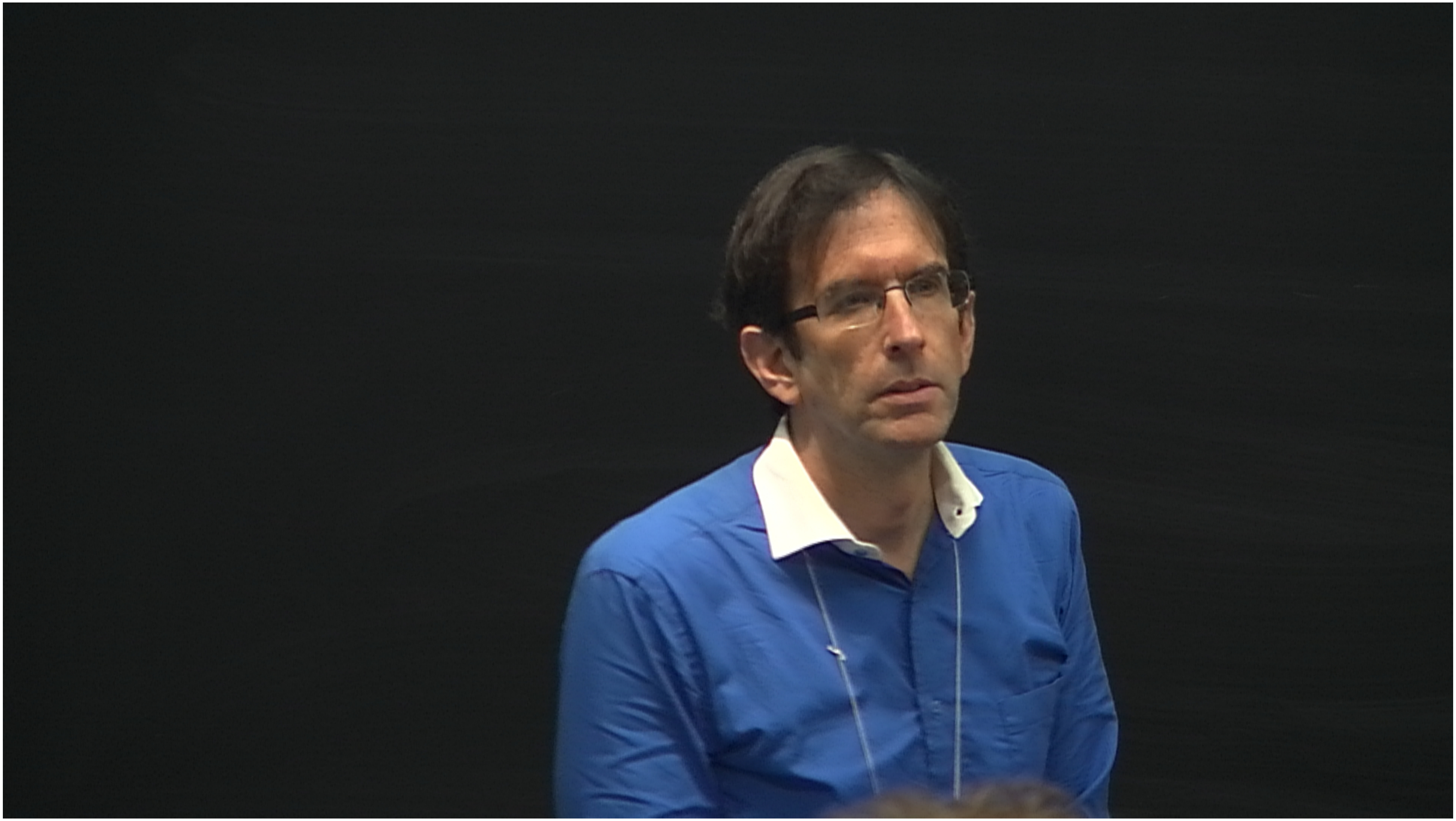
Title: Amplitudes: Spinor Helicity

Date: Jul 14, 2015 02:30 PM

URL: http://pirsa.org/15070045

Abstract:



$k^2=0$

$$\cdot\sigma = k_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}$$

$$\varepsilon^{\alpha\beta}\lambda_\alpha \lambda'_\beta \equiv \langle \lambda \lambda' \rangle$$

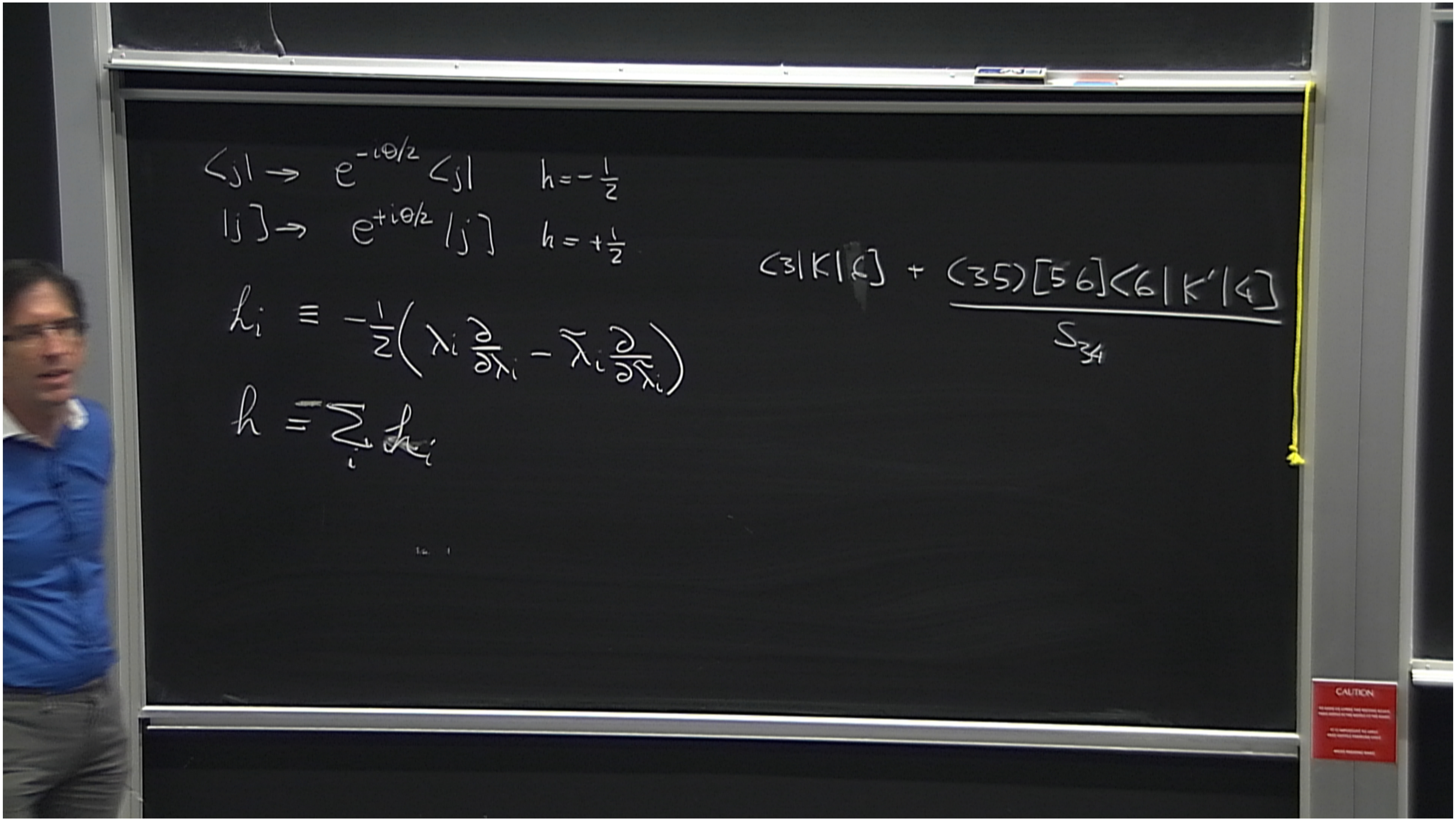
$$\langle i j \rangle \underbrace{[j i]} = 2\, k_i \cdot k_j$$

$$\tilde\lambda_i = \mathrm{sign}(k_i^0)\, \lambda_i^*$$

$$\lambda_i \to \tau_i \lambda_i$$

$$k_i \to |\tau_i|^2 k_i$$

Invariant if τ_i is a pure phase


$\langle j| \to e^{-i\theta/2} \langle j| \qquad h = -\frac{1}{2}$
$|j] \to e^{+i\theta/2} |j] \qquad h = +\frac{1}{2}$
$h_i \equiv -\frac{1}{2}\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right)$
$h = \sum_i h_i$
$\langle 3|K|4] + \frac{\langle 35\rangle [56] \langle 6|K'|4]}{s_{34}}$
CAUTION

$$\langle j| \to e^{-i\theta/2} \langle j| \qquad h = -\tfrac{1}{2}$$

$$|j] \to e^{+i\theta/2} |j] \qquad h = +\tfrac{1}{2}$$

$$h_i \equiv -\frac{1}{2}\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right)$$

$$h = \sum_i h_i$$

$$\langle 3|K|4] + \frac{\langle 35\rangle [56] \langle 6|K'|4]}{S_{34}}$$

Gluon amplitudes in Y-M.

Three-point amplitude

Kinematics $\quad k_3^2 = 0 = (k_1 + k_2)^2 = 2 k_1 \cdot k_2$

all invariants vanish

$$\Rightarrow \langle 12 \rangle = \langle 23 \rangle = \langle 31 \rangle = 0$$

$$[12] = [23] = [31] = 0$$

Three-point amplitude

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$$[12] = [23] = [31] = 0$$

$A(1,2,3) = 0$.

$$\tilde\lambda \neq \pm \lambda^* \qquad k_i\cdot k_j = 0 \Rightarrow \begin{array}{l} \langle ij\rangle = 0 \\ \underline{\text{or}} \\ [ij] = 0 \end{array}$$

but <u>not</u> necessarily both

(a) $[12] = [23] = [31] = 0$

Exercise Show that the only two options are all $[\ ]=0$ or all $\langle\ \rangle = 0$.

$\langle 12\rangle,\ \langle 23\rangle,\ \langle 31\rangle$

$$A_3 = \text{const}\ \langle 12\rangle^{e_{12}} \langle 23\rangle^{e_{23}} \langle 31\rangle^{e_{31}}$$

$$h_1 = -\tfrac{1}{2}(e_{12}+e_{31}) \qquad e_{12} = h_3 - h_1 - h_2$$

$$h_2 = -\tfrac{1}{2}(e_{23}+e_{12}) \qquad e_{23} = h_1 - h_2 - h_3$$

$$h_3 = -\tfrac{1}{2}(e_{31}+e_{23}). \qquad e_{31} = h_2 - h_1 - h_3$$

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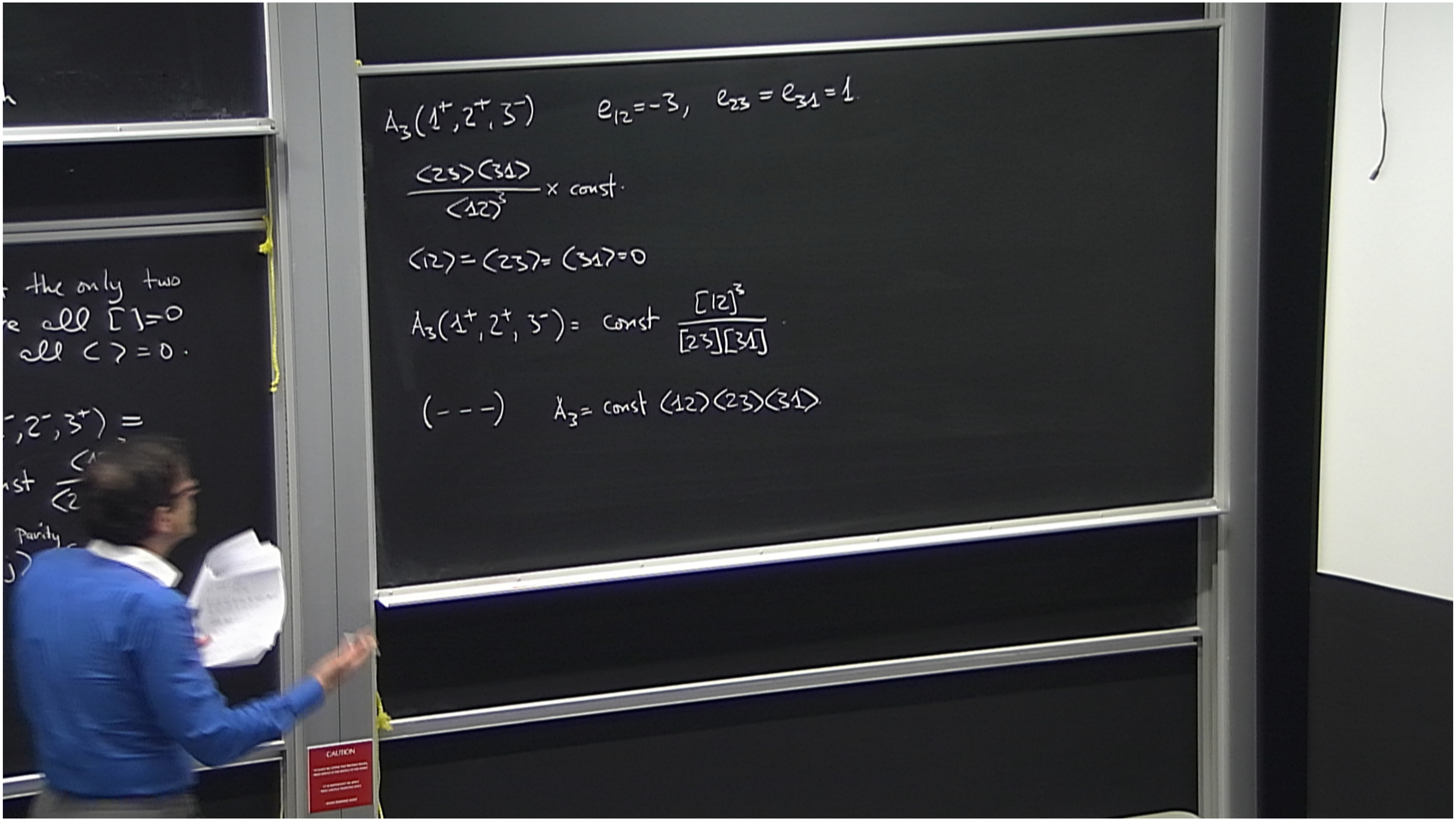
$$h_2 = -\tfrac{1}{2}(e_{23}+e_{12}) \qquad e_{23} = h_1 - h_2 - h_3$$

$$h_3 = -\tfrac{1}{2}(e_{31}+e_{23}) \qquad e_{31} = h_2 - h_1 - h_3$$

$A_3(1^-, 2^-, 3^+)$ $\quad e_{12} = 3,\ e_{23} = e_{31} = -1$

$$A_3(1^-, 2^-, 3^+) = \text{const}\ \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$


$A_3(1^+,2^+,3^-)$ $e_{12}=-3,\ e_{23}=e_{31}=1.$
$\frac{\langle 23\rangle\langle 31\rangle}{\langle 12\rangle^3}\times \text{const}.$
$\langle 12\rangle=\langle 23\rangle=\langle 31\rangle=0$
$A_3(1^+,2^+,3^-)=\text{const}\ \frac{[12]^3}{[23][31]}.$
$(---)$ $A_3=\text{const}\ \langle 12\rangle\langle 23\rangle\langle 31\rangle.$
the only two
e all [ ]=0
all < > = 0.
parity

Exercise Compute A_3 from the Feynman 3-pt vertex.

Introduce reference vector q_μ, $q^2=0$

$$\varepsilon^{(-)}_\mu(k_j, q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}\,[j q]} , \quad \varepsilon^{(+)}_\mu(k_j, q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}\,\langle q j\rangle}$$

XZC "Chinese Magic"

Exercises : Show that $\varepsilon^{(\pm)}$ have the correct form to be pol vectors for circular polarizations

: A_3(fermion, fermion, gluon).

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Introduce reference vector q_μ : $q^2=0$

$$\varepsilon^{(-)}_\mu(k_j,q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}\,[j\,q]} \quad , \quad \varepsilon^{(+)}_\mu(k_j,q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}\,\langle q\,j\rangle}$$

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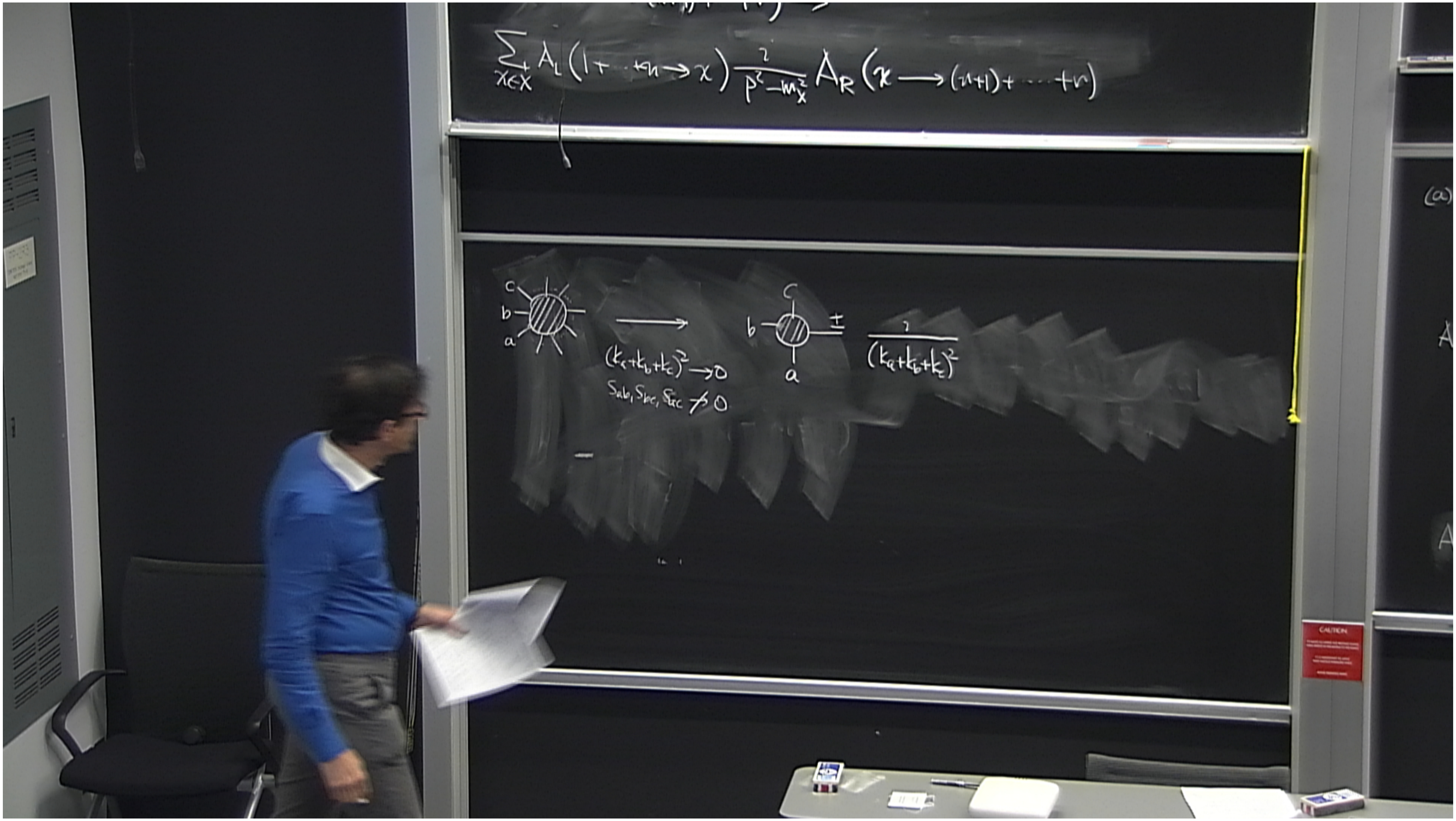
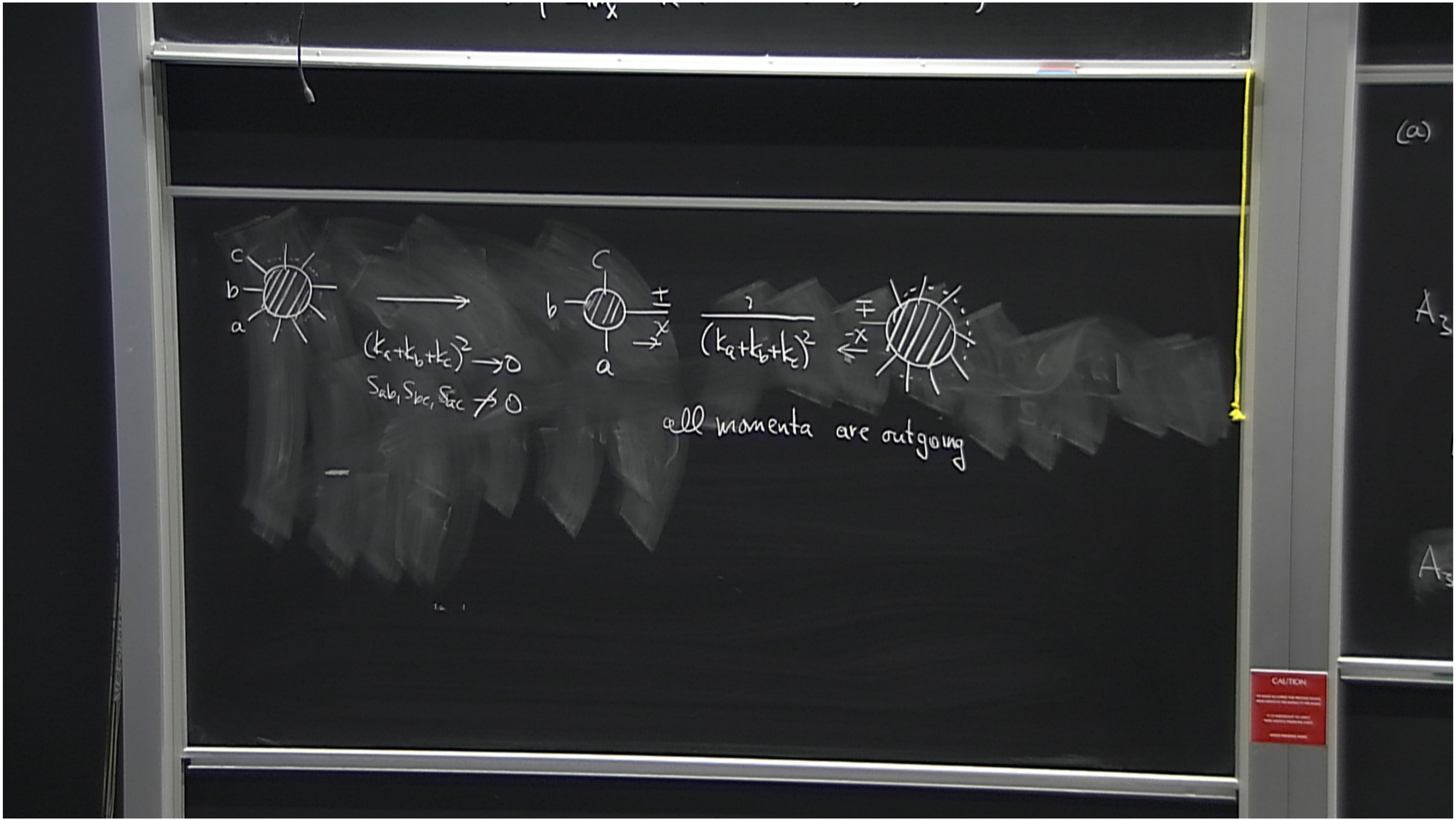
Factorization

$$(p_1+p_2)^2 \to m_X^2$$

$$A(1+2 \to 3+\cdots+n) \longrightarrow \sum_{x\in X} A_L(1+2 \to x)\frac{i}{P^2-m_X^2}A_R(x \to 3+\cdots+n)$$


$\sum_{X \in \mathcal{X}} A_L(1+\cdots+n \to X) \frac{i}{p^2 - m_X^2} A_R(X \to (n+1)+\cdots+r)$
c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$s_{ab}, s_{bc}, s_{ac} \not\to 0$
c
b
a
±
$\frac{i}{(k_a+k_b+k_c)^2}$


c
b
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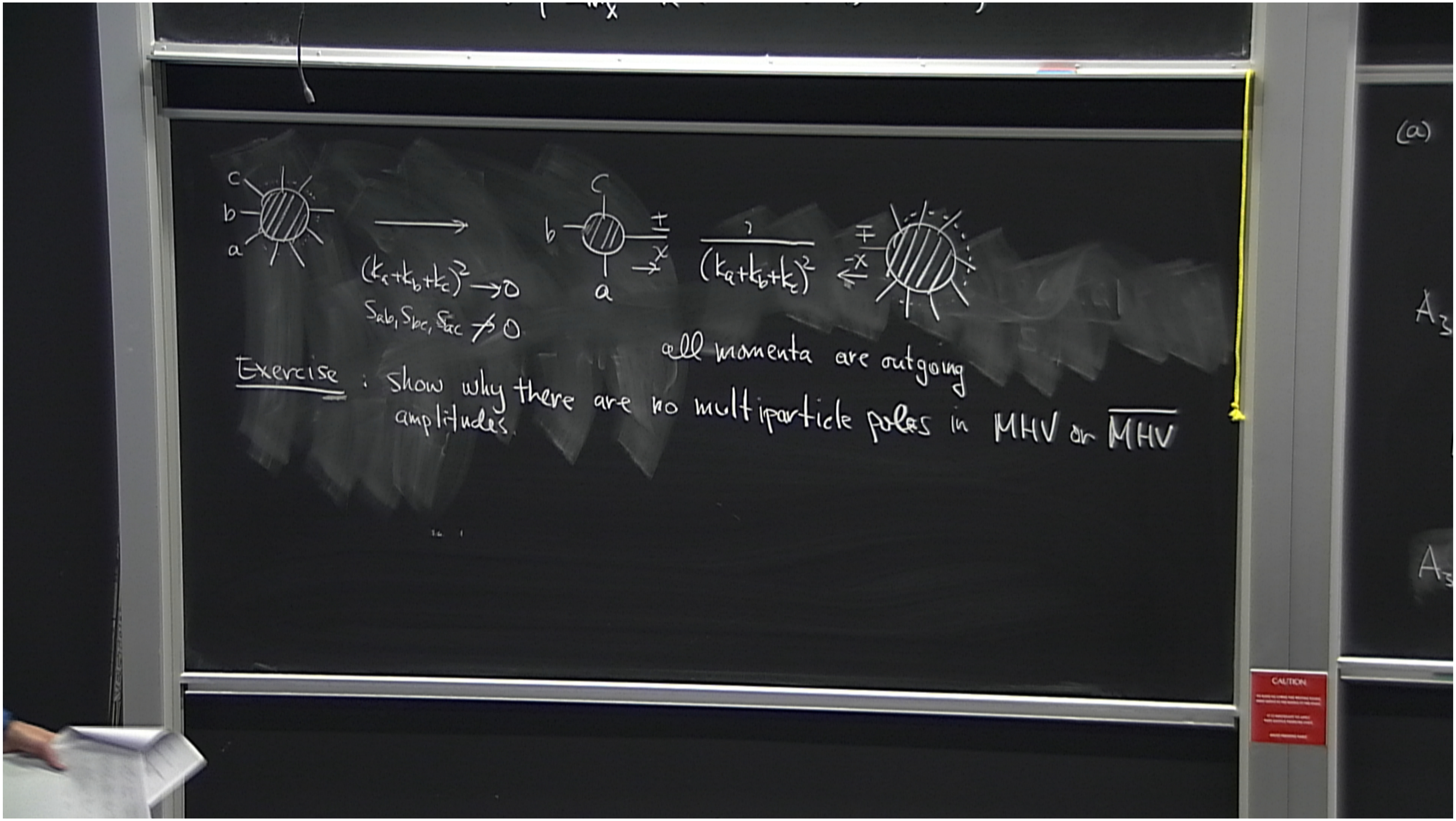
c
b
a
$\pm$
x
$\frac{i}{(k_a+k_b+k_c)^2}$
$\mp$
$-x$
all momenta are outgoing
(a)
A_3
A_3


c
b
a
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$\pm$
x
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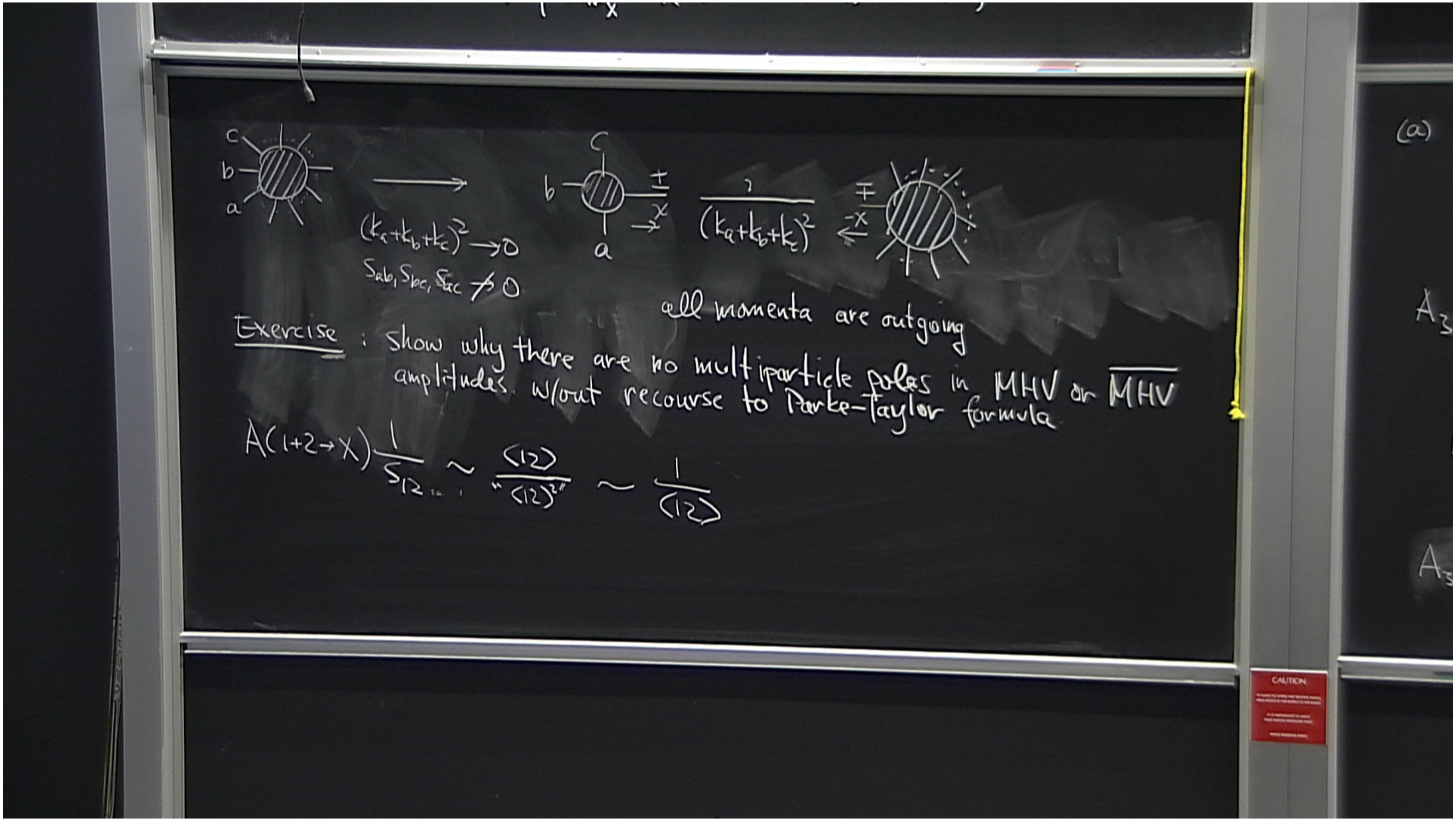
$-x$
all momenta are outgoing
Exercise: show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$ amplitudes.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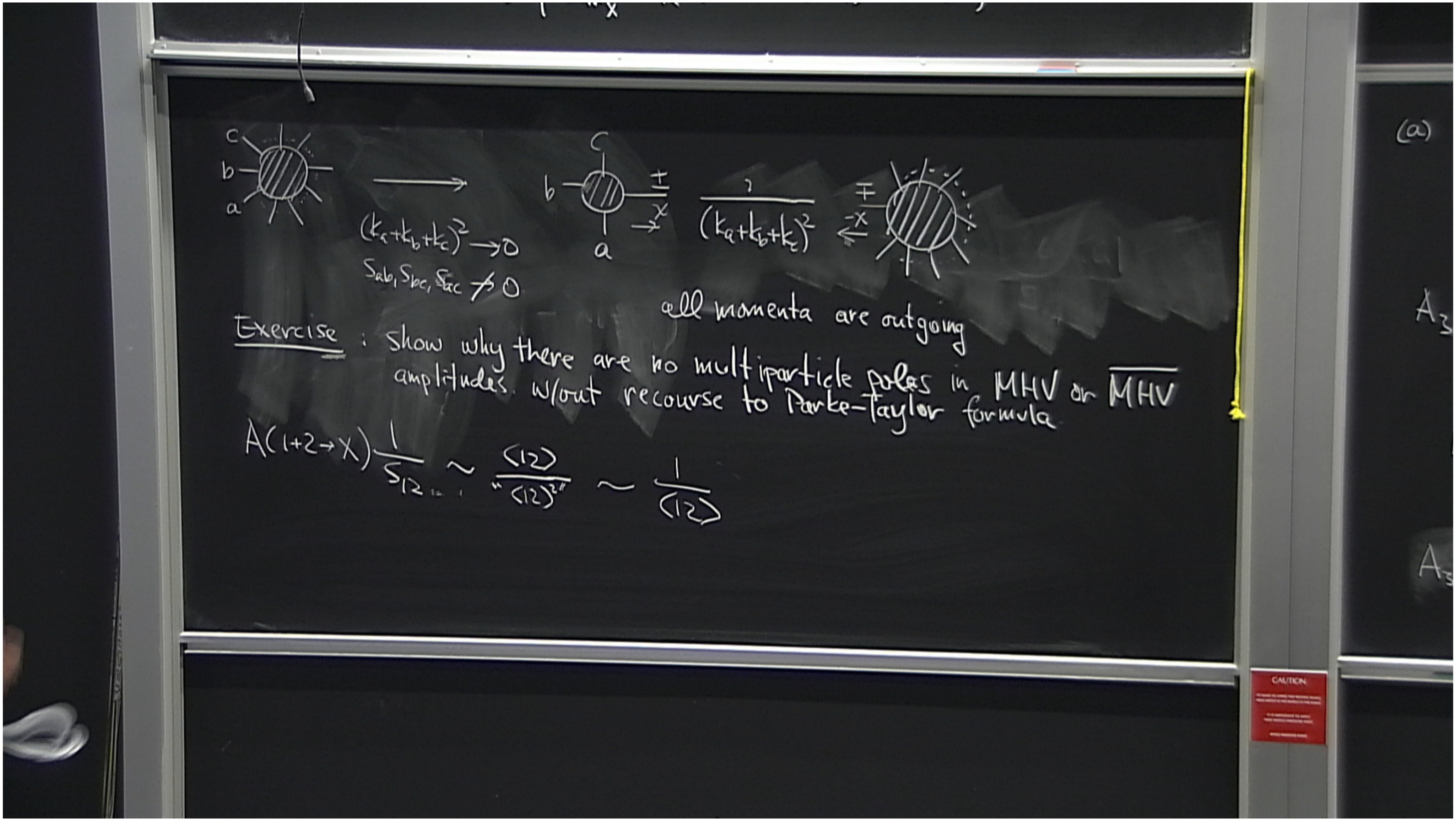
$A(1+2\to X)\frac{1}{s_{12}} \sim \frac{\langle 12\rangle}{\text{"}\langle 12\rangle^2\text{"}} \sim \frac{1}{\langle 12\rangle}$
(a)


(a)
c
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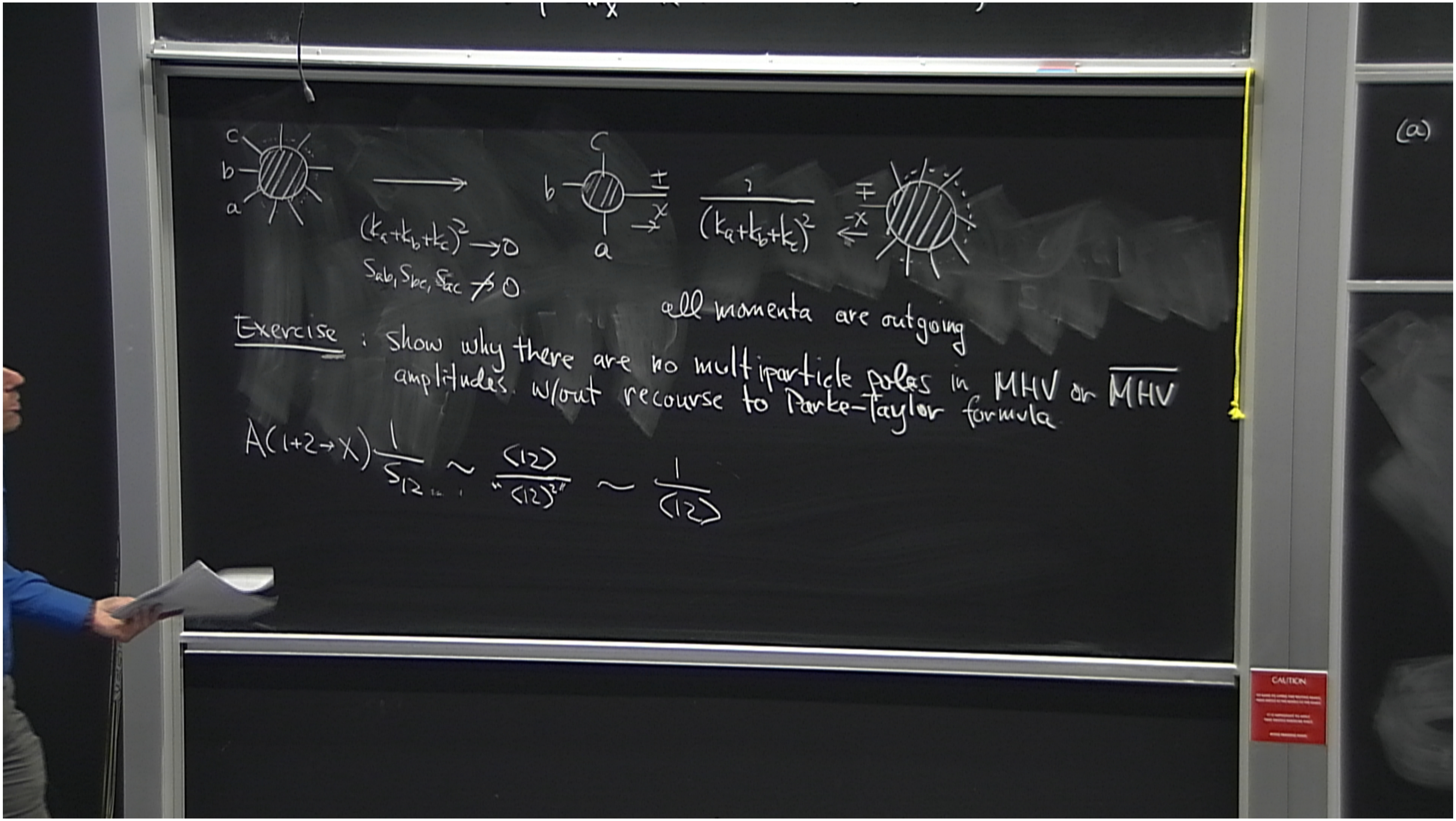
$A(1+2\to X)\frac{1}{s_{12}} \sim \frac{\langle 12\rangle}{``\langle 12\rangle^2"} \sim \frac{1}{\langle 12\rangle}$
A_3
A_3
CAUTION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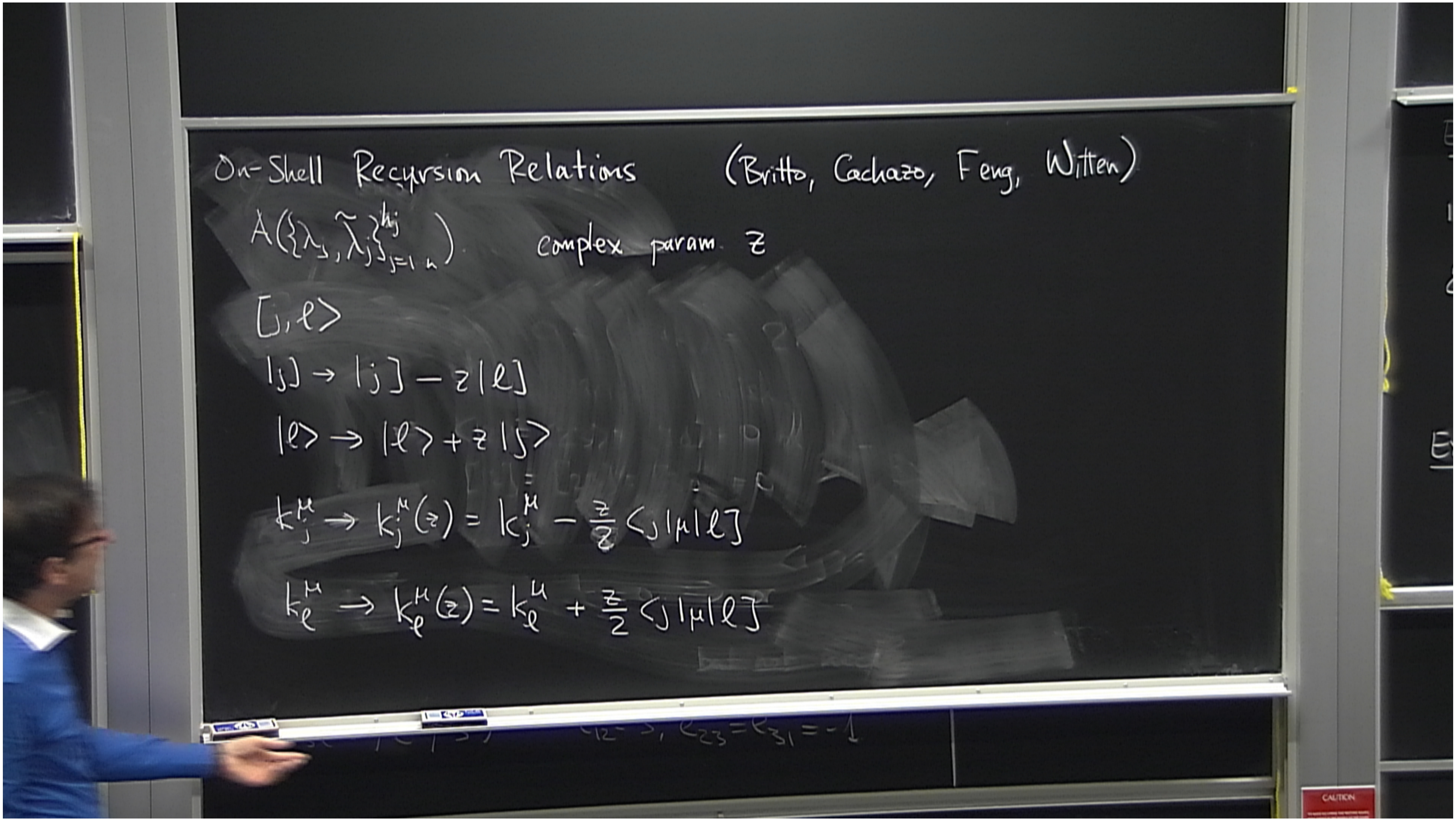
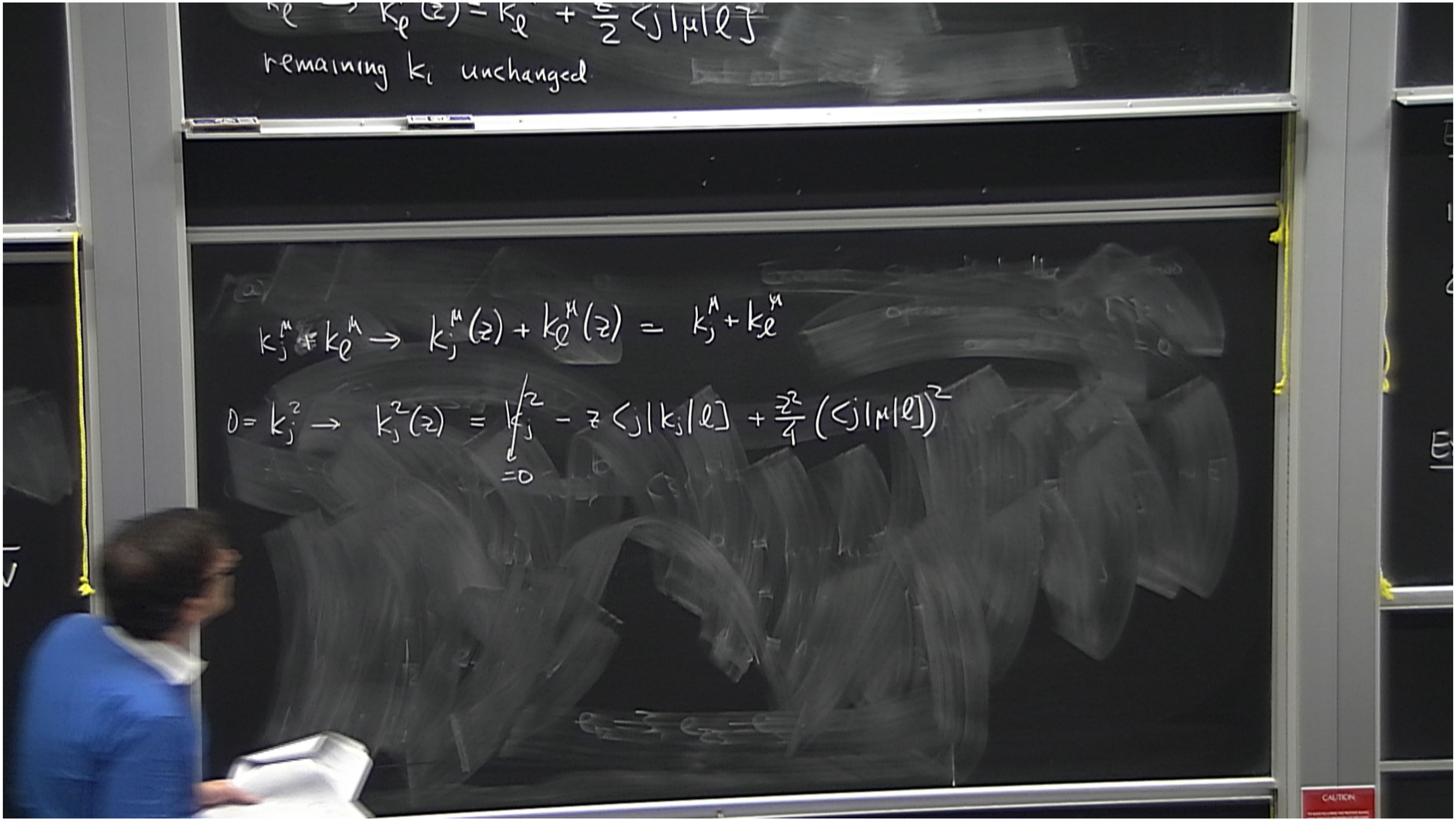

On-Shell Recursion Relations (Britto, Cachazo, Feng, Witten)
$A(\{\lambda_j, \tilde{\lambda}_j\}^{h_j}_{j=1..n})$ complex param z
$[j, \ell\rangle$
$|j] \to |j] - z|\ell]$
$|\ell\rangle \to |\ell\rangle + z|j\rangle$
$k_j^\mu \to k_j^\mu(z) = k_j^\mu - \frac{z}{2}\langle j|\mu|\ell]$
$k_\ell^\mu \to k_\ell^\mu(z) = k_\ell^\mu + \frac{z}{2}\langle j|\mu|\ell]$

$k_\ell \to k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$

remaining k_i unchanged

$$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$$


$k_\ell \to k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$
remaining k_i unchanged
$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$
$0 = k_j^2 \to k_j^2(z) = \underbrace{k_j^2}_{=0} - z\langle j|k_j|\ell] + \frac{z^2}{4}(\langle j|\mu|\ell])^2$

$k_\ell \to k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$

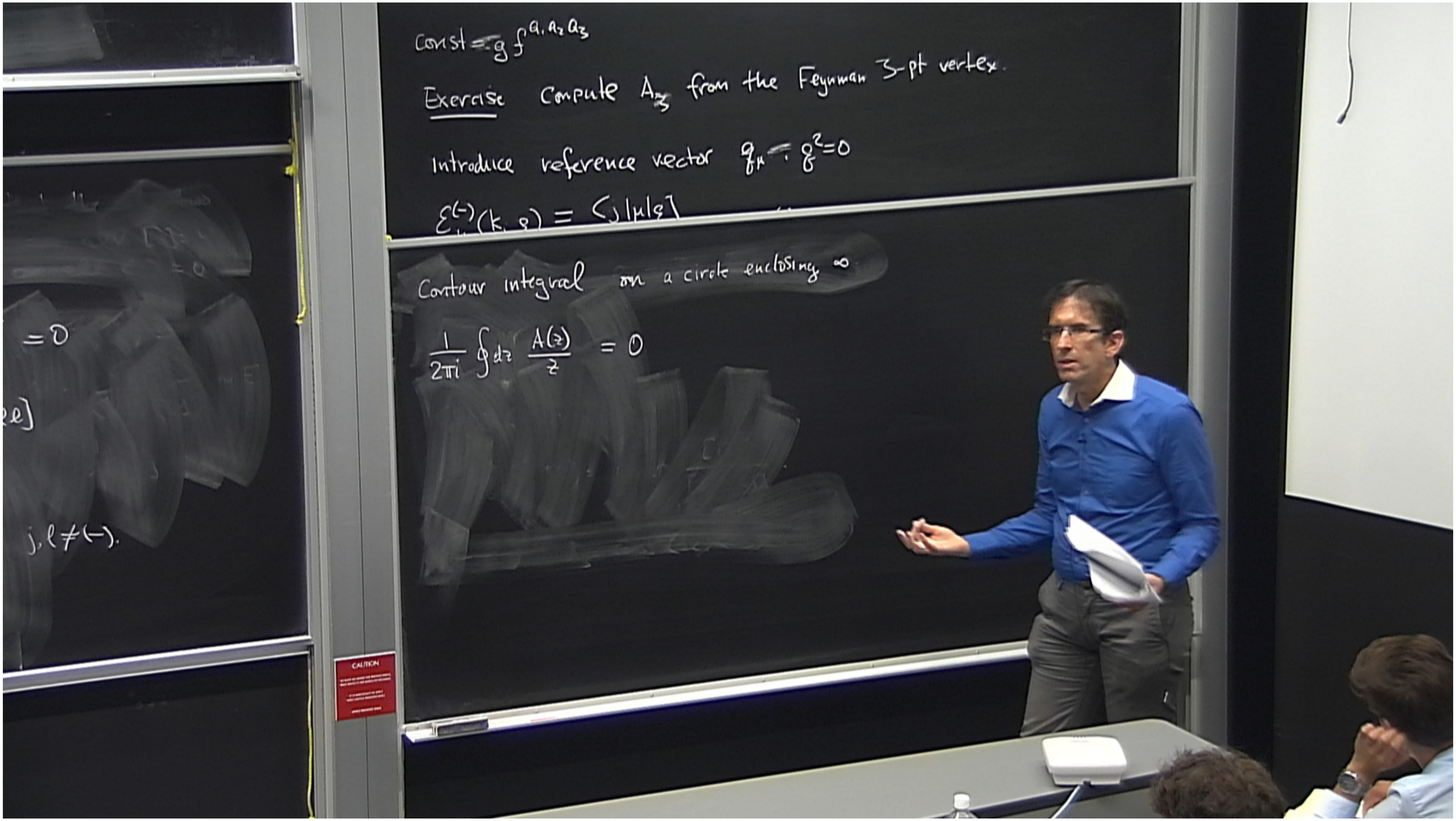
remaining k_i unchanged

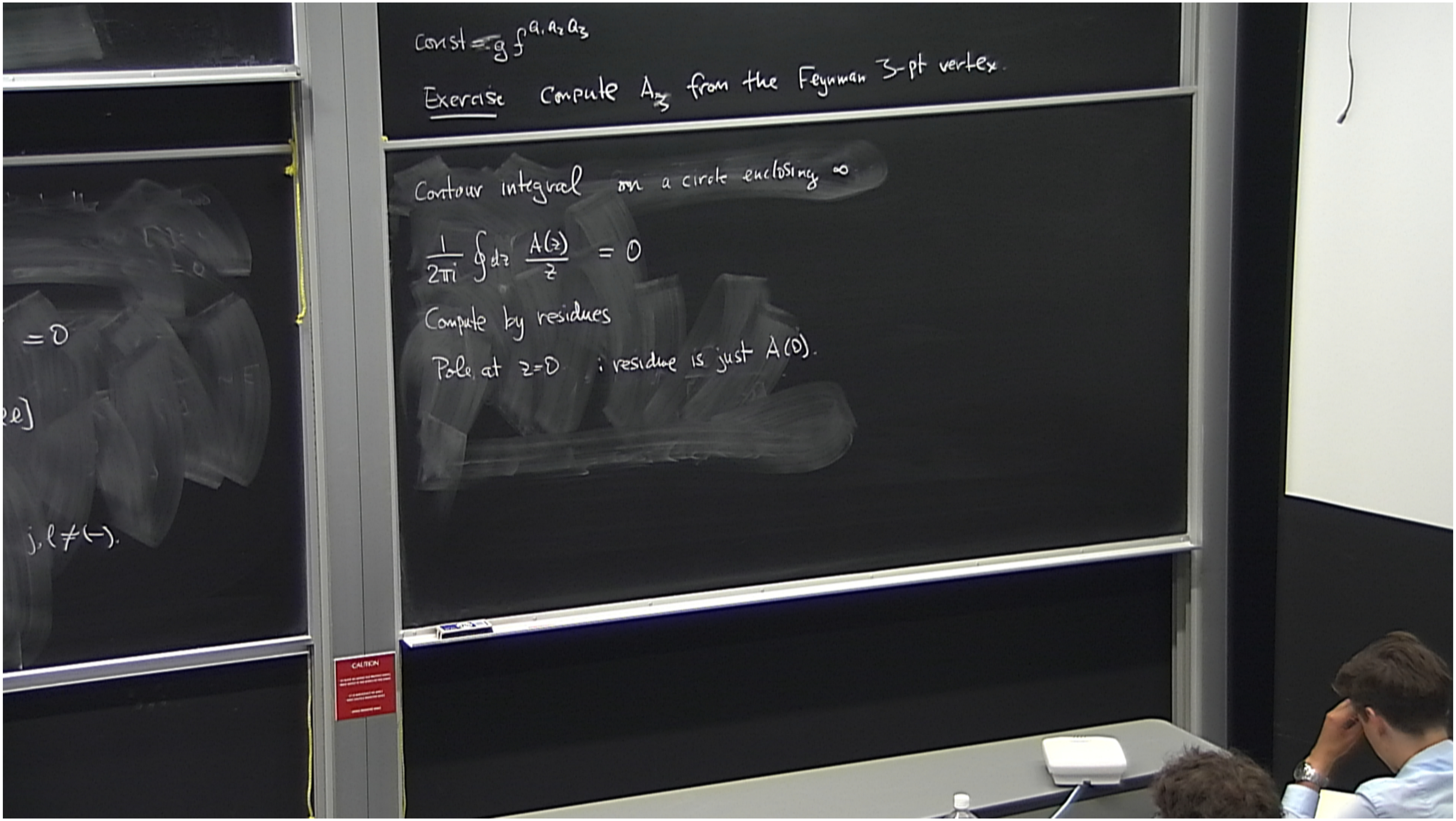
$$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$$

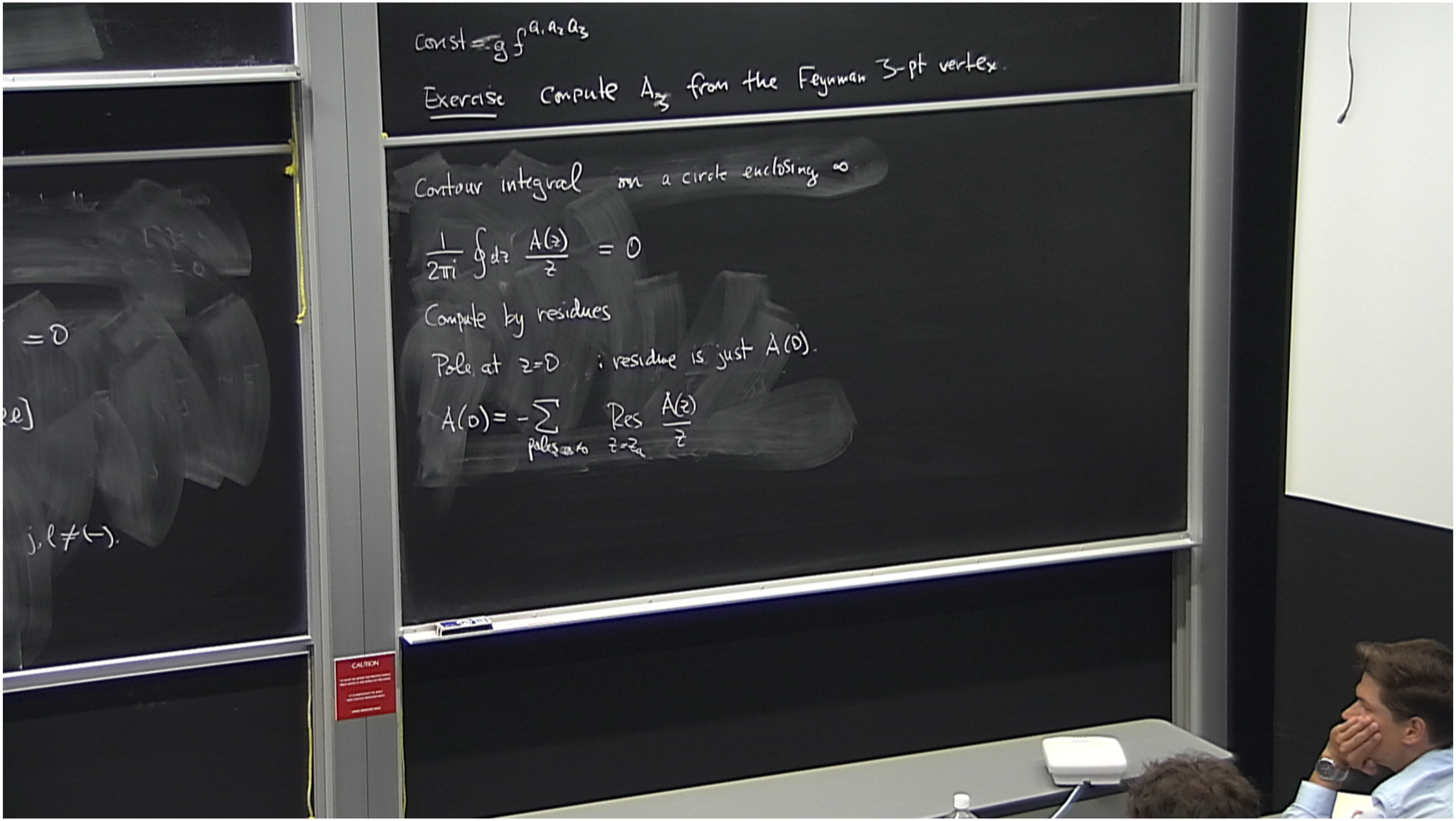
$$0 = k_j^2 \to k_j^2(z) = \underbrace{\cancel{k_j^2}}_{=0} - z\underbrace{\langle j|k_j|\ell]}_{\langle j\cancel{j}\rangle[j\ell]\,=\,0} + \frac{z^2}{4}\underbrace{(\langle j|\mu|\ell])^2}_{z\langle jj\cancel{\rangle}[\ell\ell]\,=\,0} = 0$$

Assume $A(z) \to 0$ as $z \to \infty$

<u>Exercise</u> Test this assumption on the Parke–Taylor formula $j, \ell \neq (-)$.


Const = g f a1 a2 a3
Exercise Compute A3 from the Feynman 3-pt vertex.
Introduce reference vector qμ : q2=0
Contour integral on a circle enclosing ∞
1/2πi ∮dz A(z)/z = 0


const = $g f^{a_1 a_2 a_3}$
Exercise Compute A_3 from the Feynman 3-pt vertex.
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i}\oint dz \frac{A(z)}{z} = 0$
Compute by residues
Pole at $z=0$: residue is just $A(0)$.
= 0
$j, l \neq (-)$.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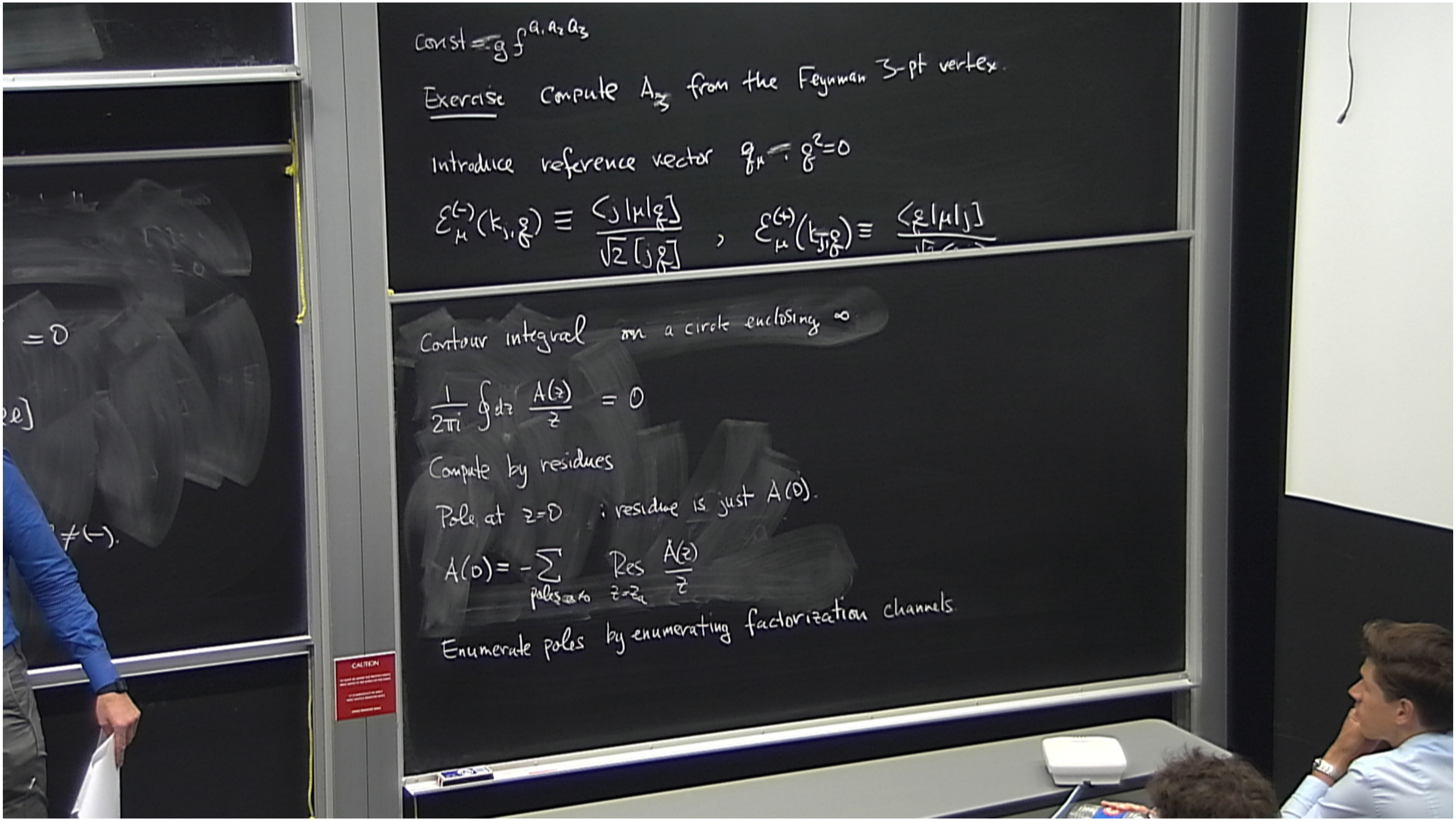
$\frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = 0$
Compute by residues
Pole at $z=0$: residue is just $A(0)$.
$A(0) = -\sum_{\text{poles} \neq 0} \operatorname{Res}_{z=z_a} \frac{A(z)}{z}$
$= 0$
$j, \ell \neq (-)$.



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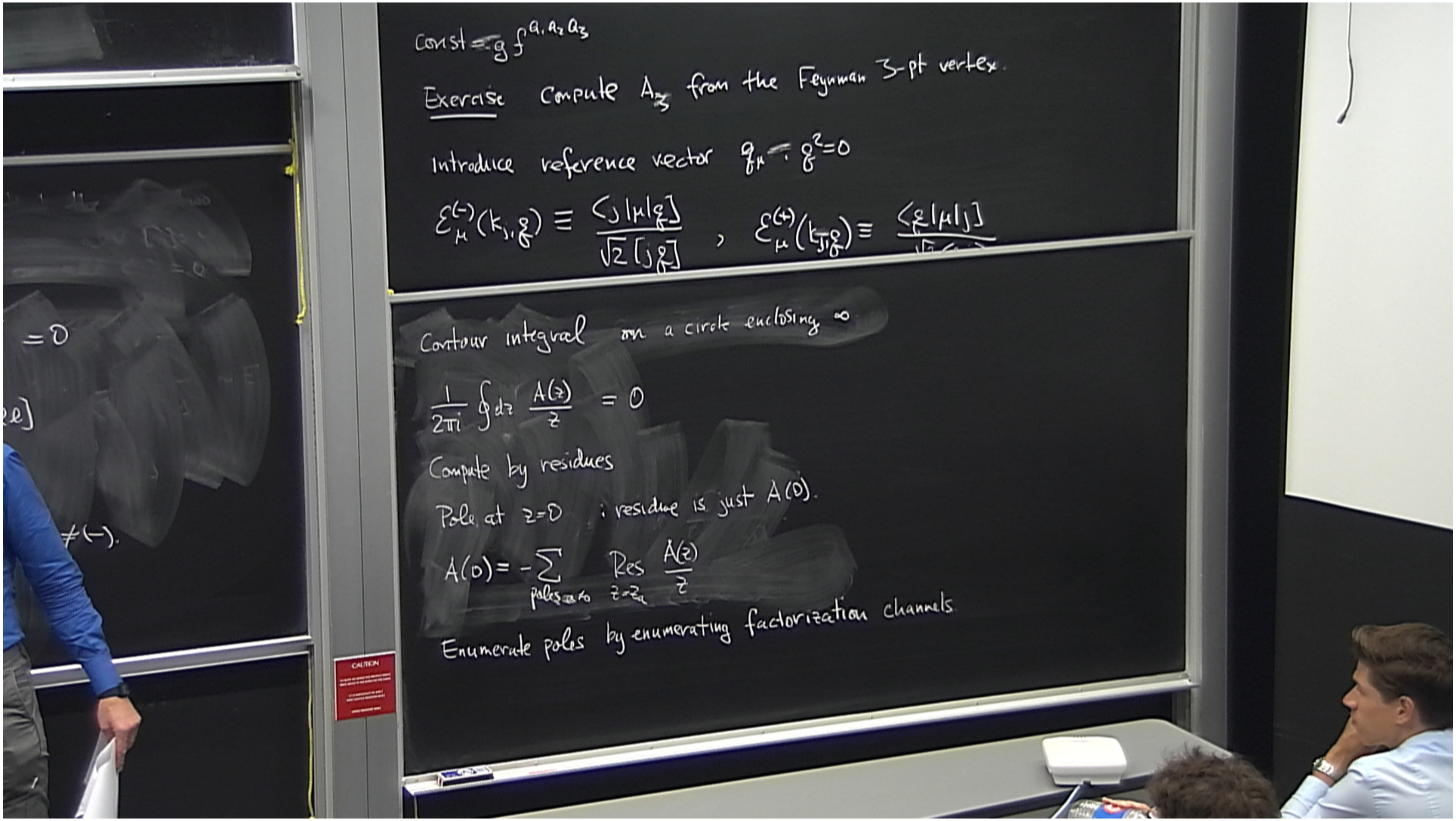
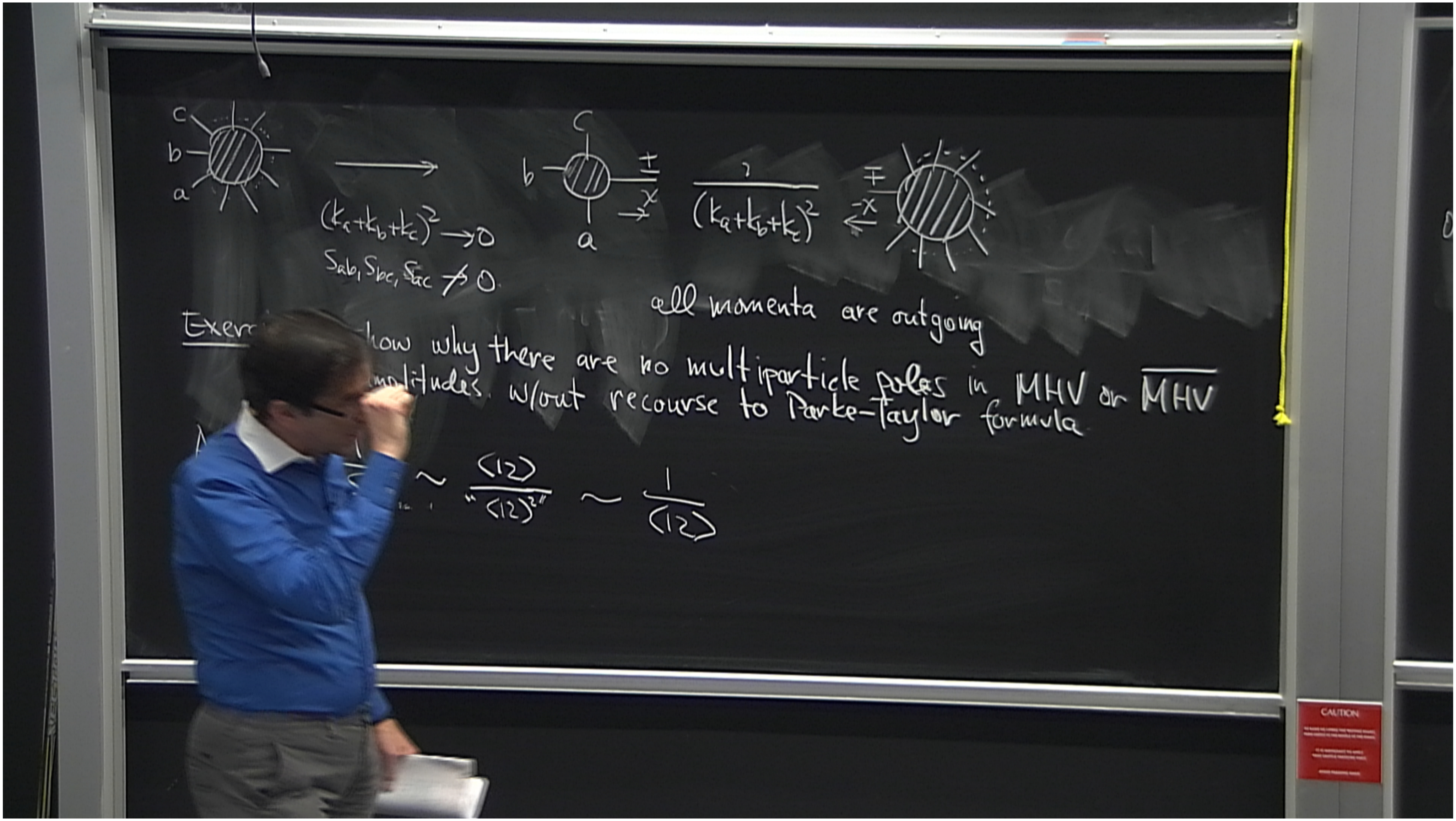
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i}\oint dz\,\frac{A(z)}{z} = 0$
Compute by residues
Pole at $z = 0$: residue is just $A(0)$.
$A(0) = -\sum_{\text{poles } z \neq 0} \operatorname{Res}_{z = z_i} \frac{A(z)}{z}$
Enumerate poles by enumerating factorization channels
$= 0$
$\neq (-).$


const = g f^{a1 a2 a3}
Exercise Compute A_3 from the Feynman 3-pt vertex.
Introduce reference vector q_μ : q^2 = 0
ε^(-)_μ(k_j, q) ≡ <j|μ|q] / √2 [j q] , ε^(+)_μ(k_j, q) ≡ <q|μ|j] / √2
Contour integral on a circle enclosing ∞
1/2πi ∮ dz A(z)/z = 0
Compute by residues
Pole at z=0 : residue is just A(0).
A(0) = − Σ_{poles a≠0} Res_{z=z_a} A(z)/z
Enumerate poles by enumerating factorization channels



c
b
a
(ka+kb+kc)² → 0
Sab, Sbc, Sac ↛ 0
±
x
∓
-x
(ka+kb+kc)²
all momenta are outgoing
how why there are no multiparticle poles in MHV or MHV
w/out recourse to Parke-Taylor formula
⟨12⟩
"⟨12⟩²"
1
⟨12⟩

Exercise: Compute A_3 from the Feynman 3-pt vertex.

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Exercises: Show that $\varepsilon^{(\pm)}$ have the correct form to be pol vectors for circular polarizations

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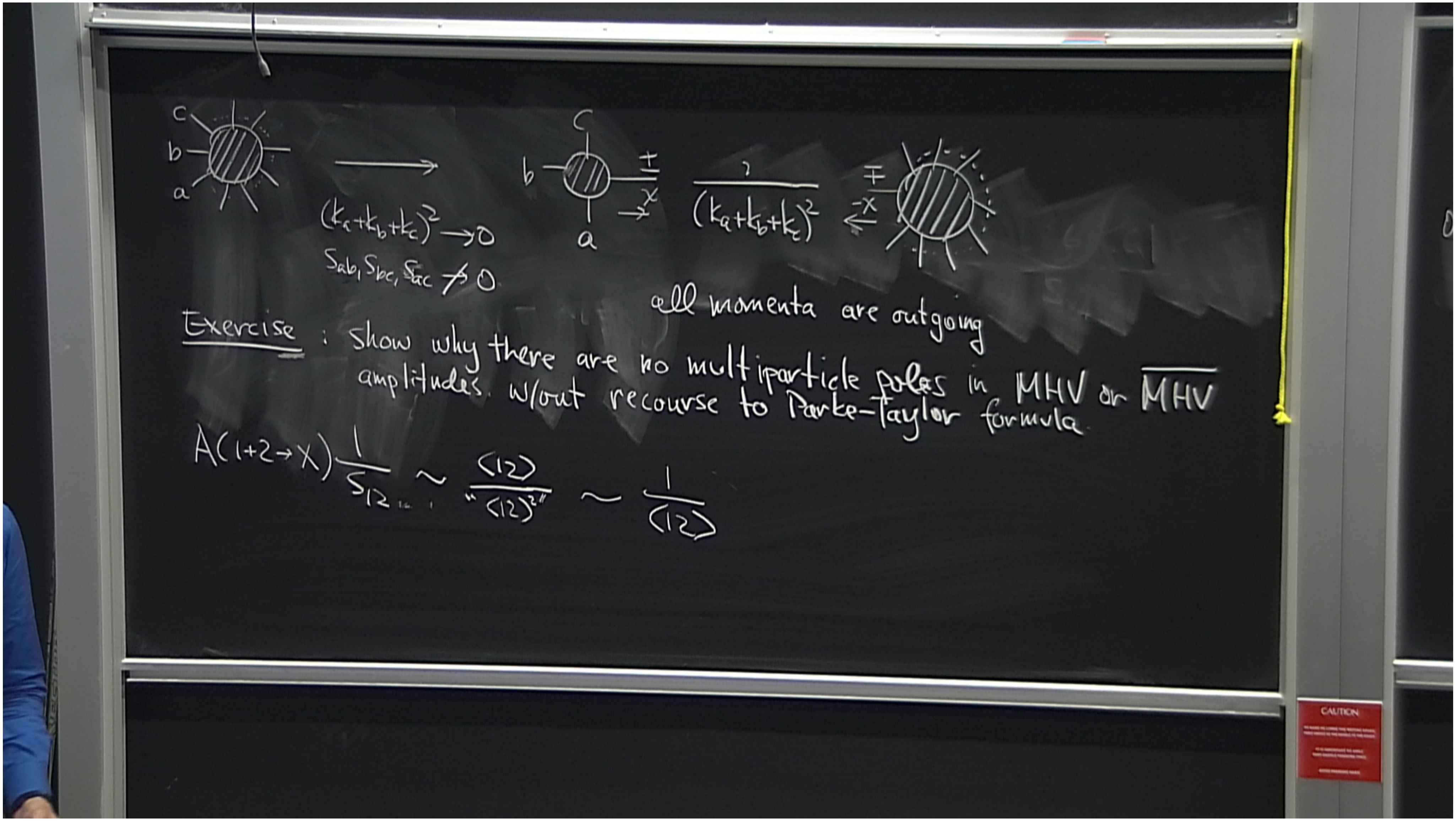
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$s_{ab}, s_{bc}, s_{ac} \not\to 0$
c
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$\pm$
x
$\frac{?}{(k_a+k_b+k_c)^2}$
$\mp$
$-x$
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$A(1+2\to X)\frac{1}{s_{12}} \sim \frac{\langle 12\rangle}{``\langle 12\rangle^2"} \sim \frac{1}{\langle 12\rangle}$