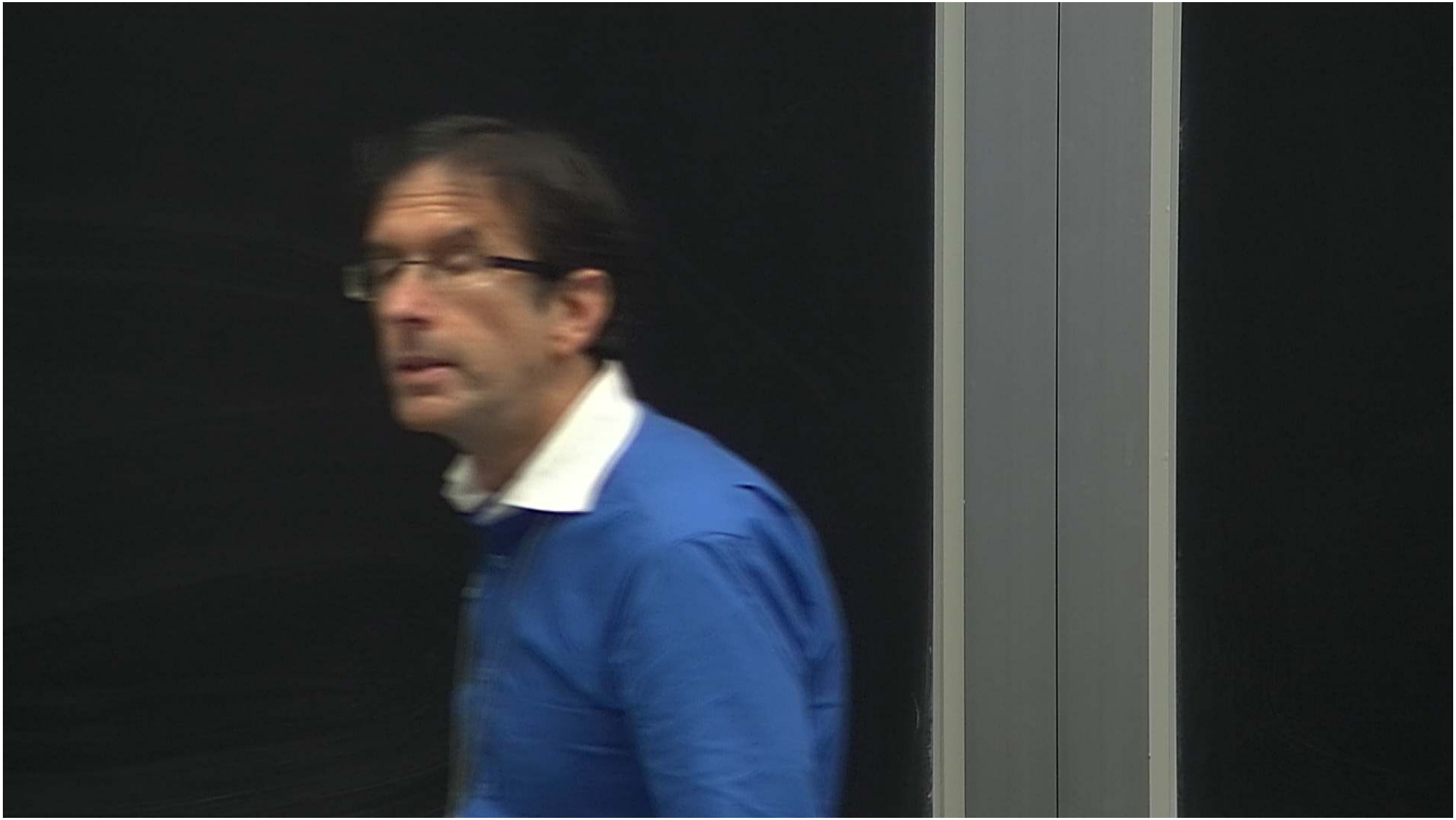
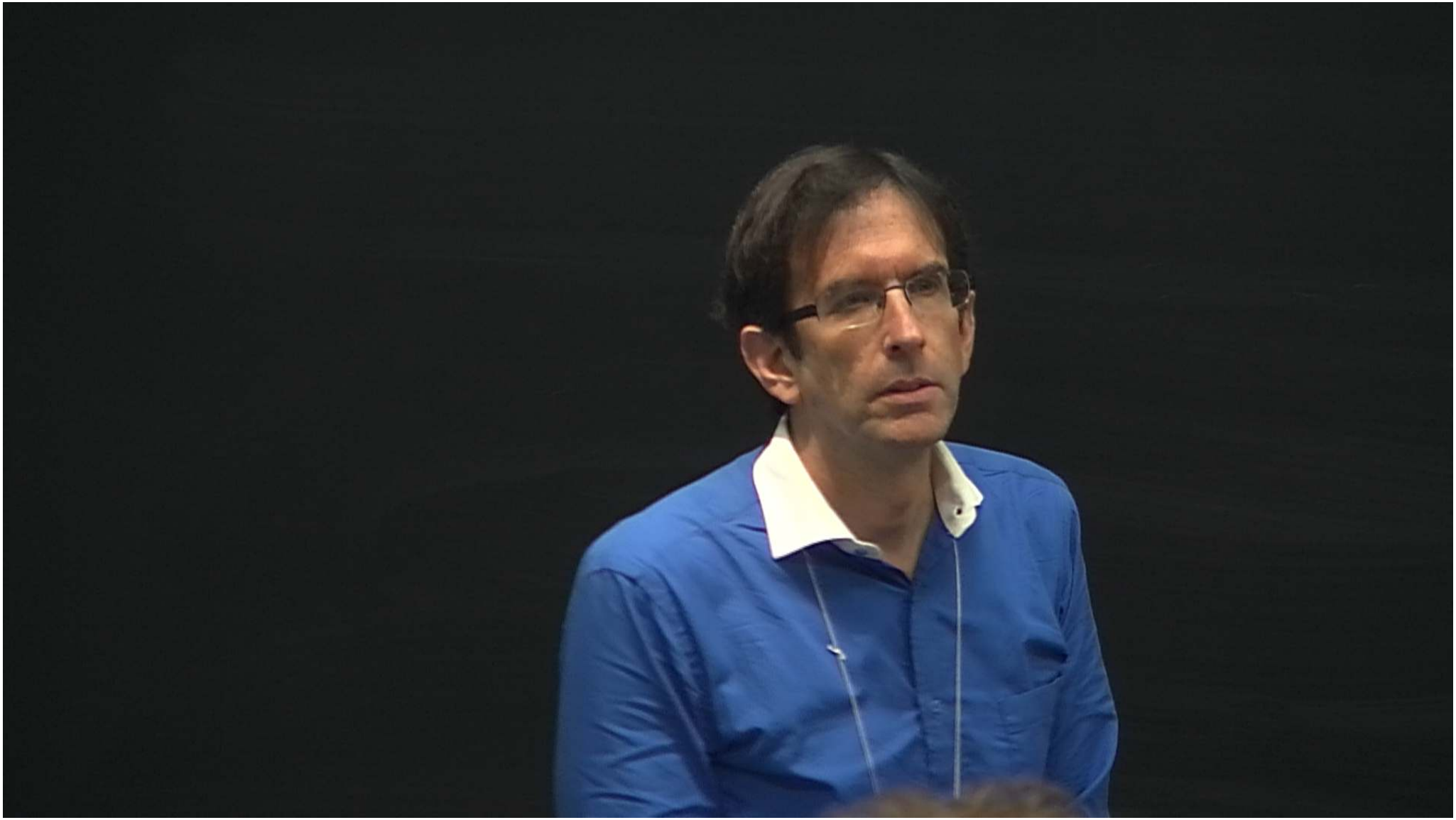
Title: Amplitudes: Spinor Helicity

Date: Jul 14, 2015 02:30 PM

URL: http://pirsa.org/15070045

Abstract:





$k \cdot \sigma = k_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ $(k^2 = 0)$

$\varepsilon^{\alpha\beta} \lambda_\alpha \lambda'_\beta \equiv \langle \lambda \lambda' \rangle$

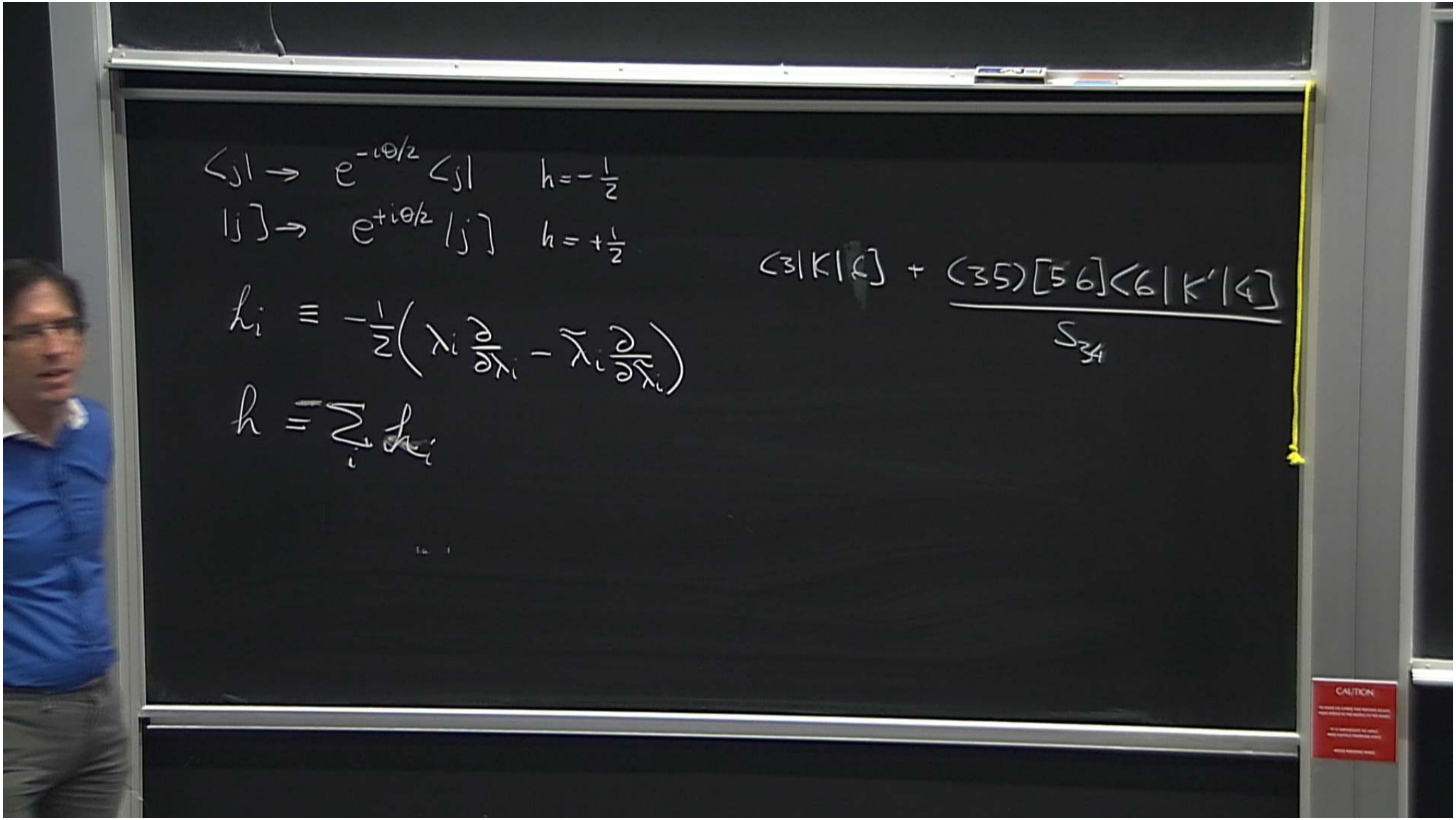
$\langle ij \rangle [ji] = 2 k_i \cdot k_j$

$\tilde{\lambda}_i = \mathrm{sign}(k_i^0)\, \lambda_i^*$

$\lambda_i \to \tau_i \lambda_i$

$k_i \to |\tau_i|^2 k_i$

Invariant if τ_i is a pure phase


$\langle j| \to e^{-i\theta/2}\langle j|$ $\quad h=-\frac{1}{2}$
$|j] \to e^{+i\theta/2}|j]$ $\quad h=+\frac{1}{2}$
$h_i \equiv -\frac{1}{2}\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right)$
$h = \sum_i h_i$
$\langle 3|K|4] + \frac{\langle 35\rangle[56]\langle 6|K'|4]}{s_{34}}$
CAUTION

$\langle j| \to e^{-i\theta/2}\langle j| \quad h=-\frac{1}{2}$

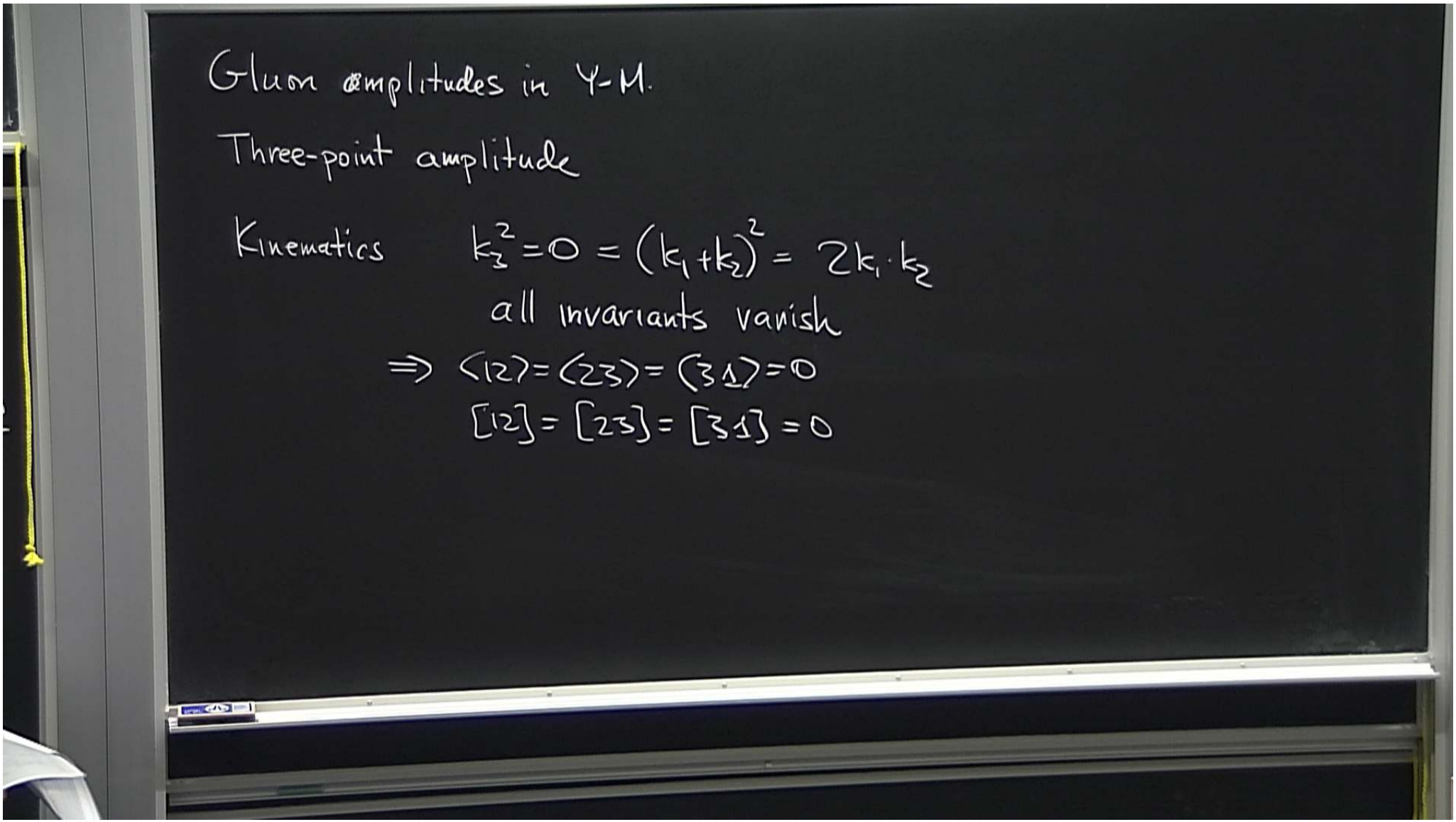
$|j] \to e^{+i\theta/2}|j] \quad h=+\frac{1}{2}$

$$h_i \equiv -\frac{1}{2}\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right)$$

$$h = \sum_i h_i$$

$$\langle 3|K|4] + \frac{\langle 35\rangle[56]\langle 6|K'|4]}{s_{34}}$$

CAUTION


Gluon amplitudes in Y-M.
Three-point amplitude
Kinematics $k_3^2 = 0 = (k_1 + k_2)^2 = 2k_1 \cdot k_2$
all invariants vanish
$\Rightarrow \langle 12\rangle = \langle 23\rangle = \langle 31\rangle = 0$
$[12] = [23] = [31] = 0$

Three-point amplitude

Kinematics $k_3^2 = 0 = (k_1 + k_2)^2 = 2k_1 \cdot k_2$

all invariants vanish

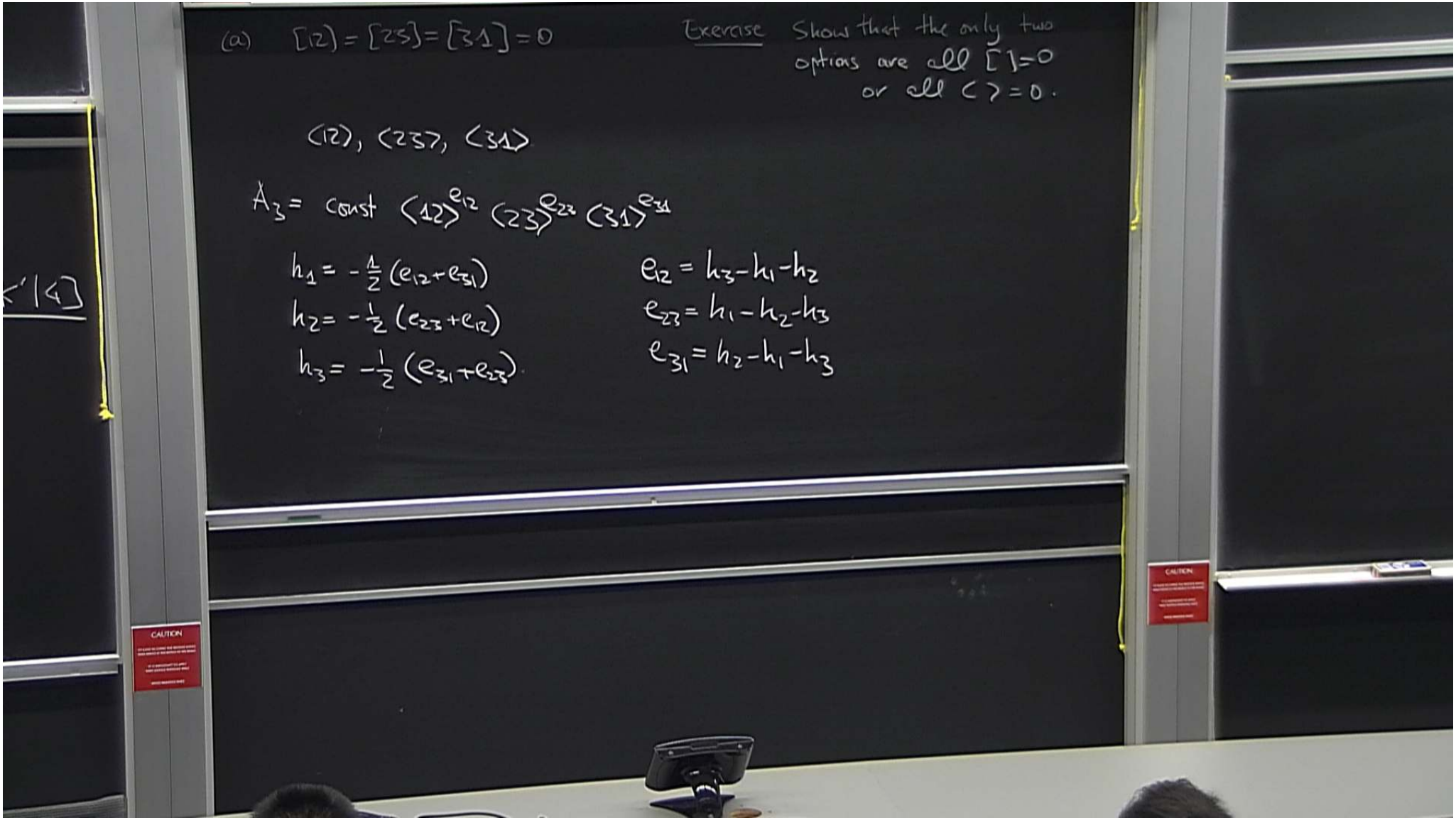
$\Rightarrow \langle 12\rangle = \langle 23\rangle = \langle 31\rangle = 0$

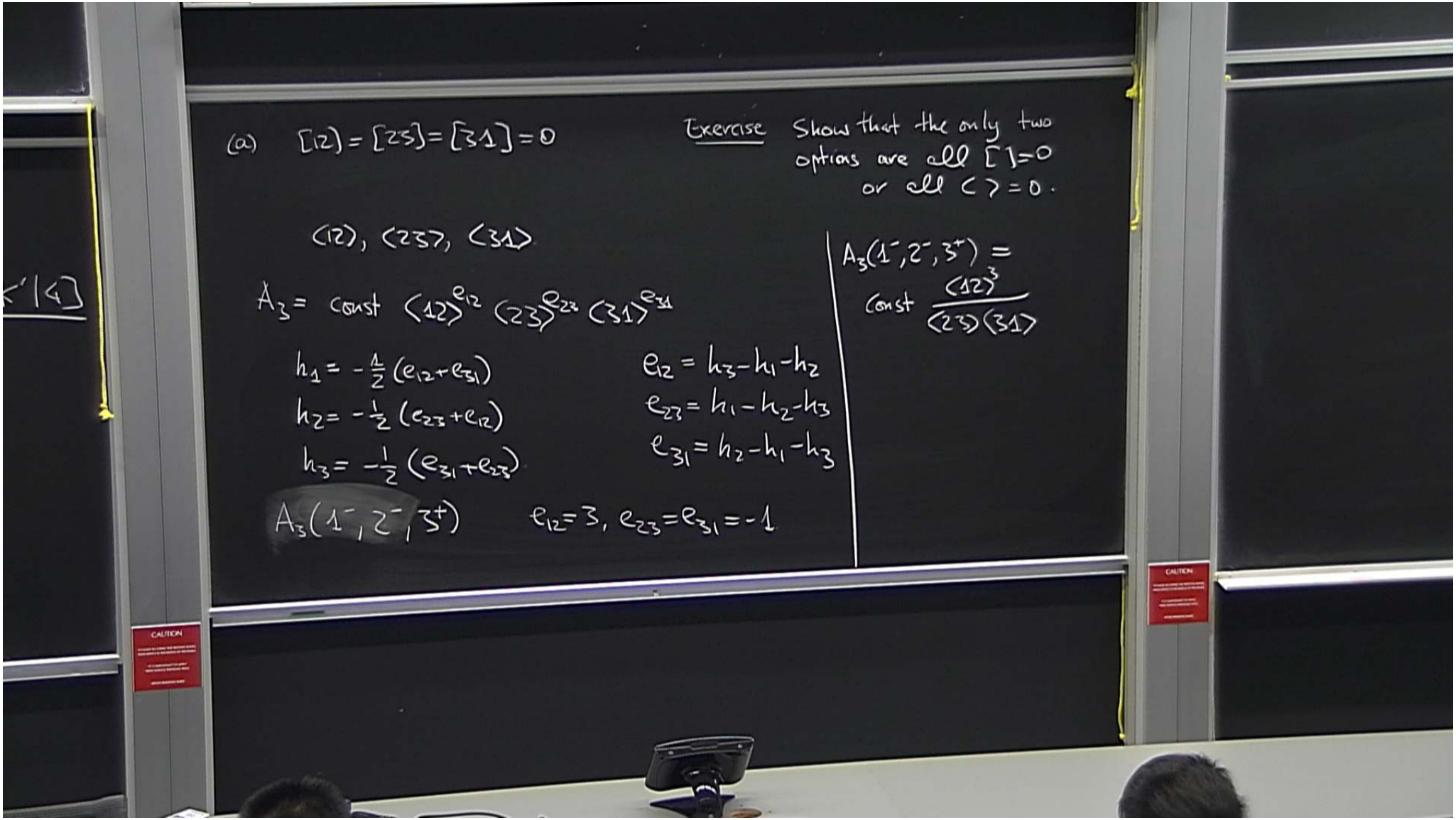
$[12] = [23] = [31] = 0$

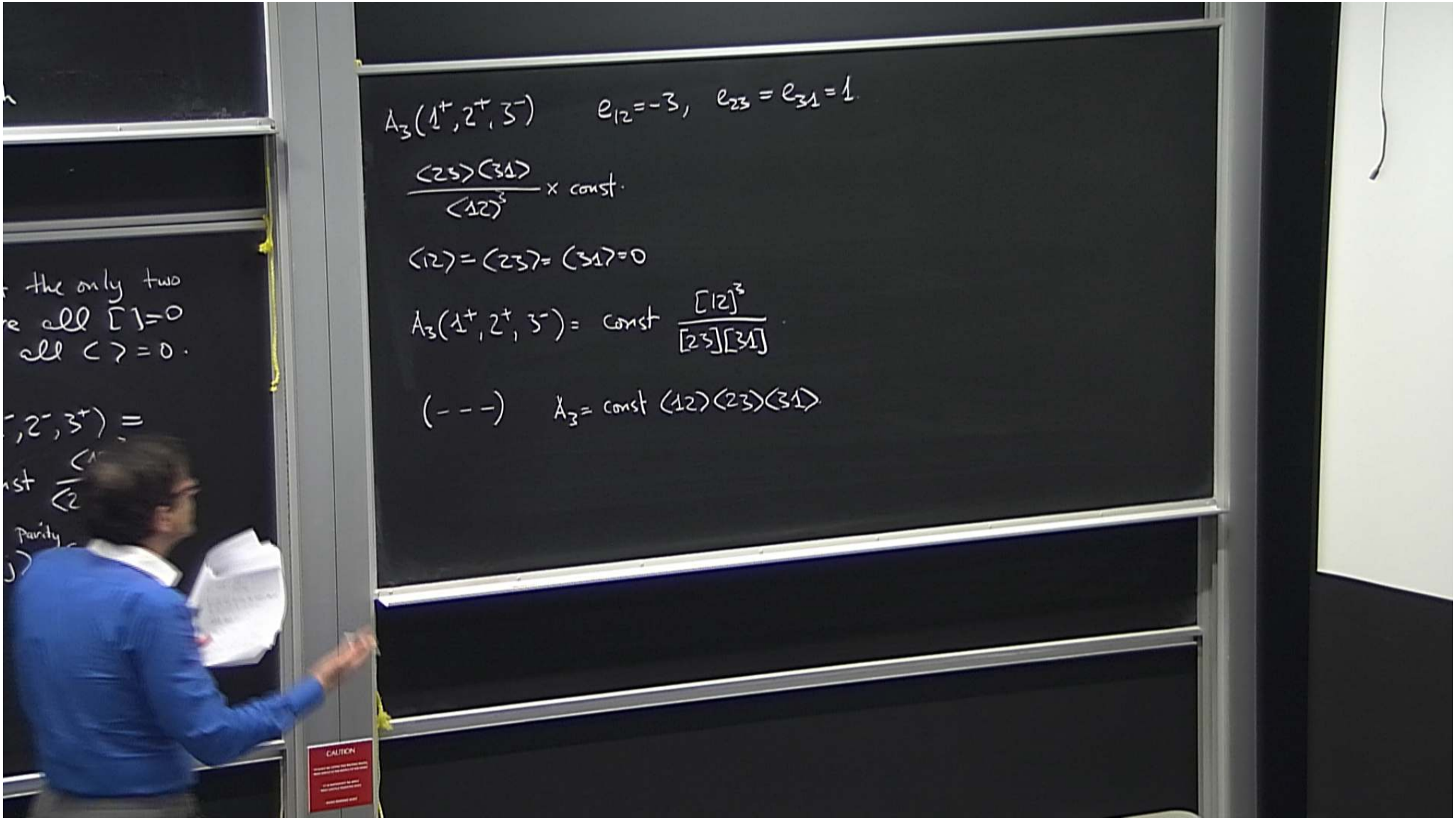
$A(1,2,3) = 0$

$\hat{\lambda} \neq \pm\lambda^*$ $k_i \cdot k_j = 0 \Rightarrow$ $\langle ij\rangle = 0$ or $[ij] = 0$

but not necessarily both


(a) $[12]=[23]=[31]=0$
Exercise Show that the only two options are all $[\,]=0$ or all $\langle\,\rangle=0$.
$\langle 12\rangle, \langle 23\rangle, \langle 31\rangle$
$A_3 = \text{const}\ \langle 12\rangle^{e_{12}} \langle 23\rangle^{e_{23}} \langle 31\rangle^{e_{31}}$
$h_1 = -\frac{1}{2}(e_{12}+e_{31})$
$h_2 = -\frac{1}{2}(e_{23}+e_{12})$
$h_3 = -\frac{1}{2}(e_{31}+e_{23})$
$e_{12} = h_3-h_1-h_2$
$e_{23} = h_1-h_2-h_3$
$e_{31} = h_2-h_1-h_3$
CAUTION
CAUTION


(a) $[12]=[23]=[31]=0$
Exercise Show that the only two options are all $[\,]=0$ or all $\langle\,\rangle=0$.
$\langle 12\rangle, \langle 23\rangle, \langle 31\rangle$
$A_3 = \text{const}\ \langle 12\rangle^{e_{12}} \langle 23\rangle^{e_{23}} \langle 31\rangle^{e_{31}}$
$h_1 = -\frac{1}{2}(e_{12}+e_{31})$
$h_2 = -\frac{1}{2}(e_{23}+e_{12})$
$h_3 = -\frac{1}{2}(e_{31}+e_{23})$
$e_{12} = h_3 - h_1 - h_2$
$e_{23} = h_1 - h_2 - h_3$
$e_{31} = h_2 - h_1 - h_3$
$A_3(1^-,2^-,3^+)$ $e_{12}=3,\ e_{23}=e_{31}=-1$
$A_3(1^-,2^-,3^+) = \text{const}\ \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$
CAUTION
CAUTION


$A_3(1^+,2^+,3^-)$ $e_{12}=-3,\ e_{23}=e_{31}=1$
$\frac{\langle 23\rangle\langle 31\rangle}{\langle 12\rangle^3}\times$ const.
$\langle 12\rangle=\langle 23\rangle=\langle 31\rangle=0$
$A_3(1^+,2^+,3^-)=$ const $\frac{[12]^3}{[23][31]}$
$(---)$ $A_3=$ const $\langle 12\rangle\langle 23\rangle\langle 31\rangle$
the only two
all $[\,]=0$
all $\langle\,\rangle=0$.
CAUTION

Exercise Compute A_3 from the Feynman 3-pt vertex.

Introduce reference vector q_μ : $q^2=0$

$$\varepsilon_\mu^{(-)}(k_j,q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}\,[jq]} \quad , \quad \varepsilon_\mu^{(+)}(k_j,q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}\,\langle qj\rangle}$$

X2C "Chinese Magic"

Exercises : Show that $\varepsilon^{(\pm)}$ have the correct form to be pol vectors for circular polarizations

: A_3(fermion, fermions, gluon).

Exercise Compute A_3 from the Feynman 3-pt vertex.

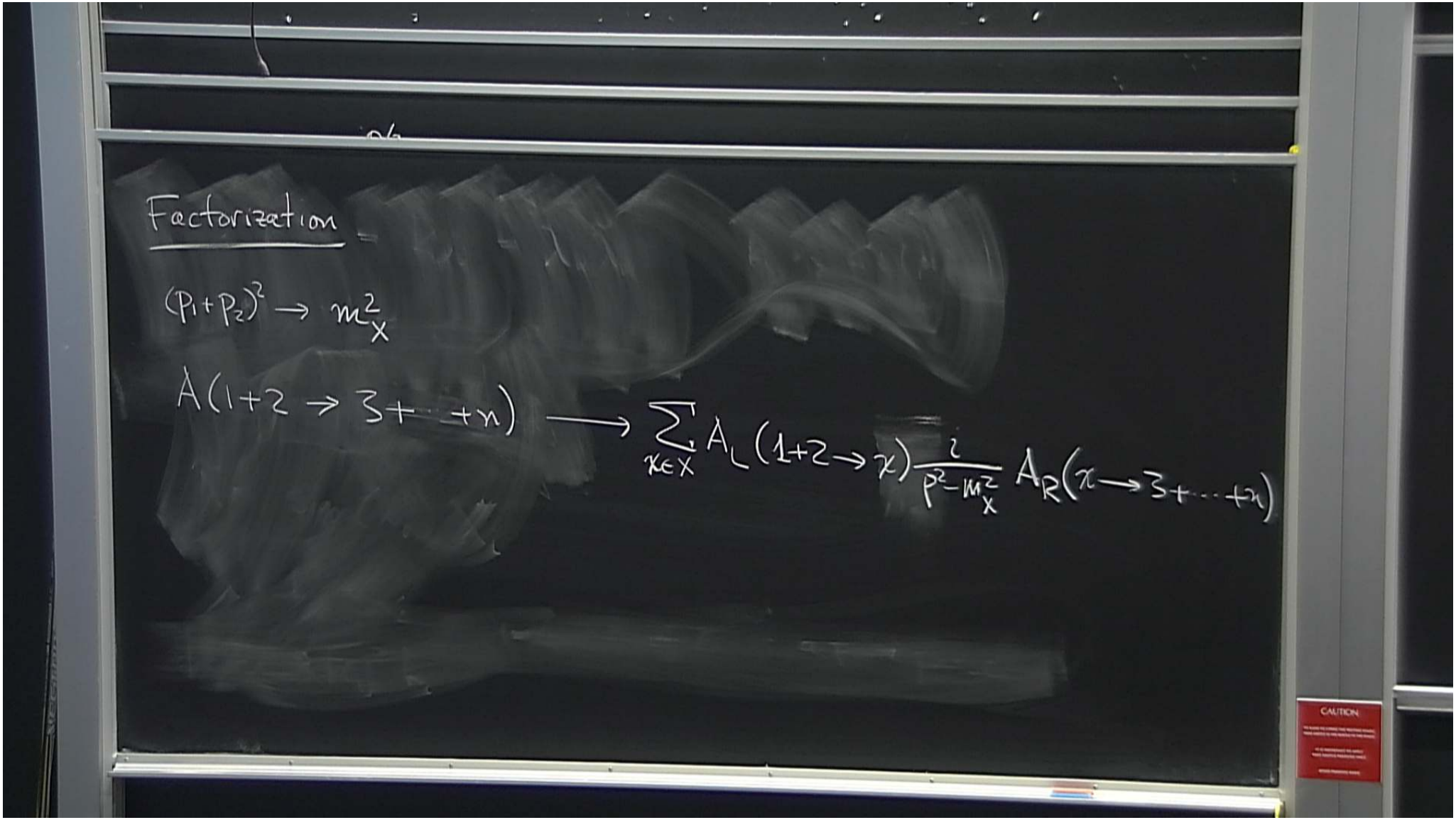
Introduce reference vector q_μ : $q^2=0$

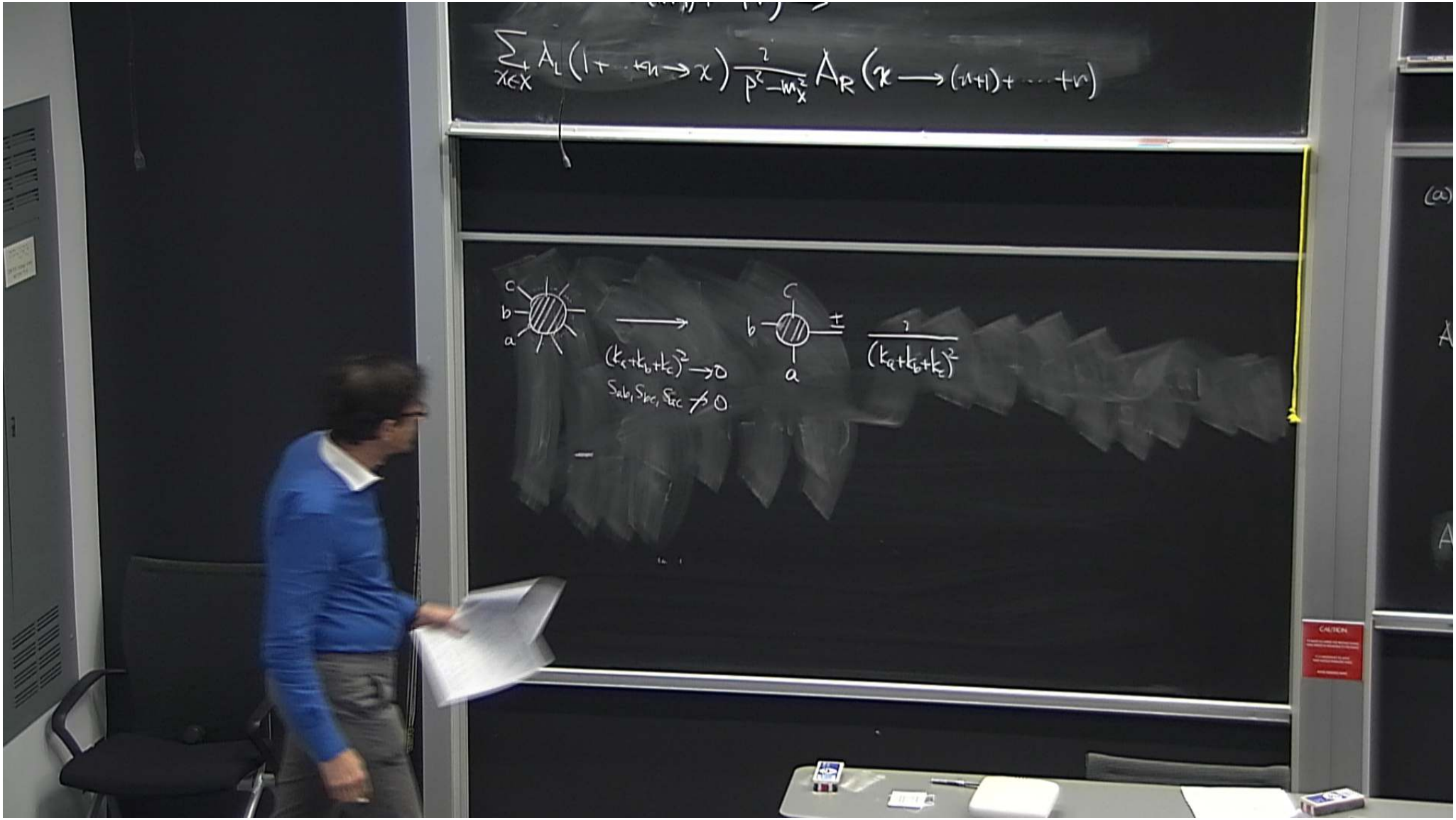
$$\varepsilon_\mu^{(-)}(k_j,q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}\,[jq]} \quad , \quad \varepsilon_\mu^{(+)}(k_j,q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}\,\langle qj\rangle}$$

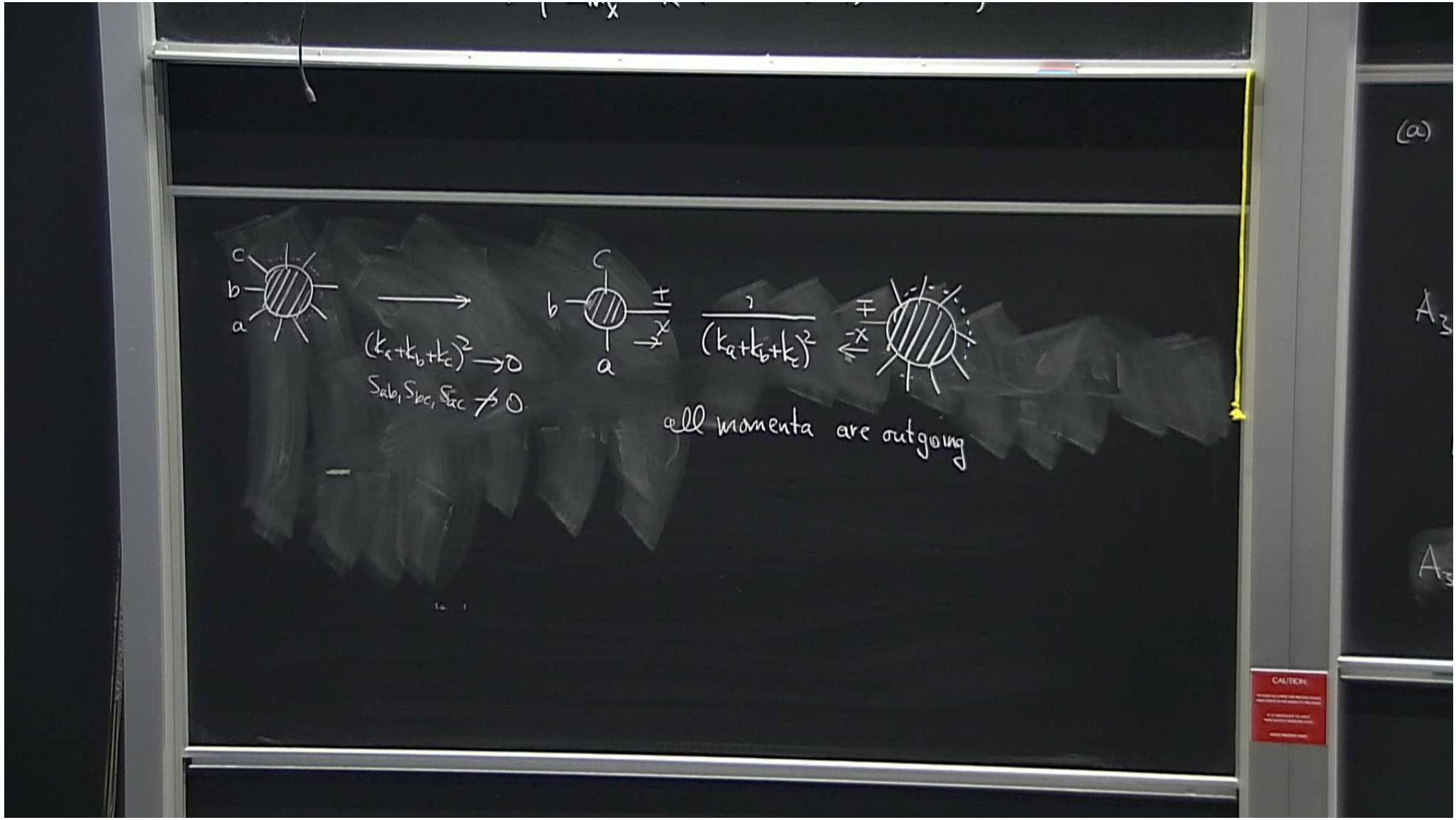
X2C "Chinese Magic"

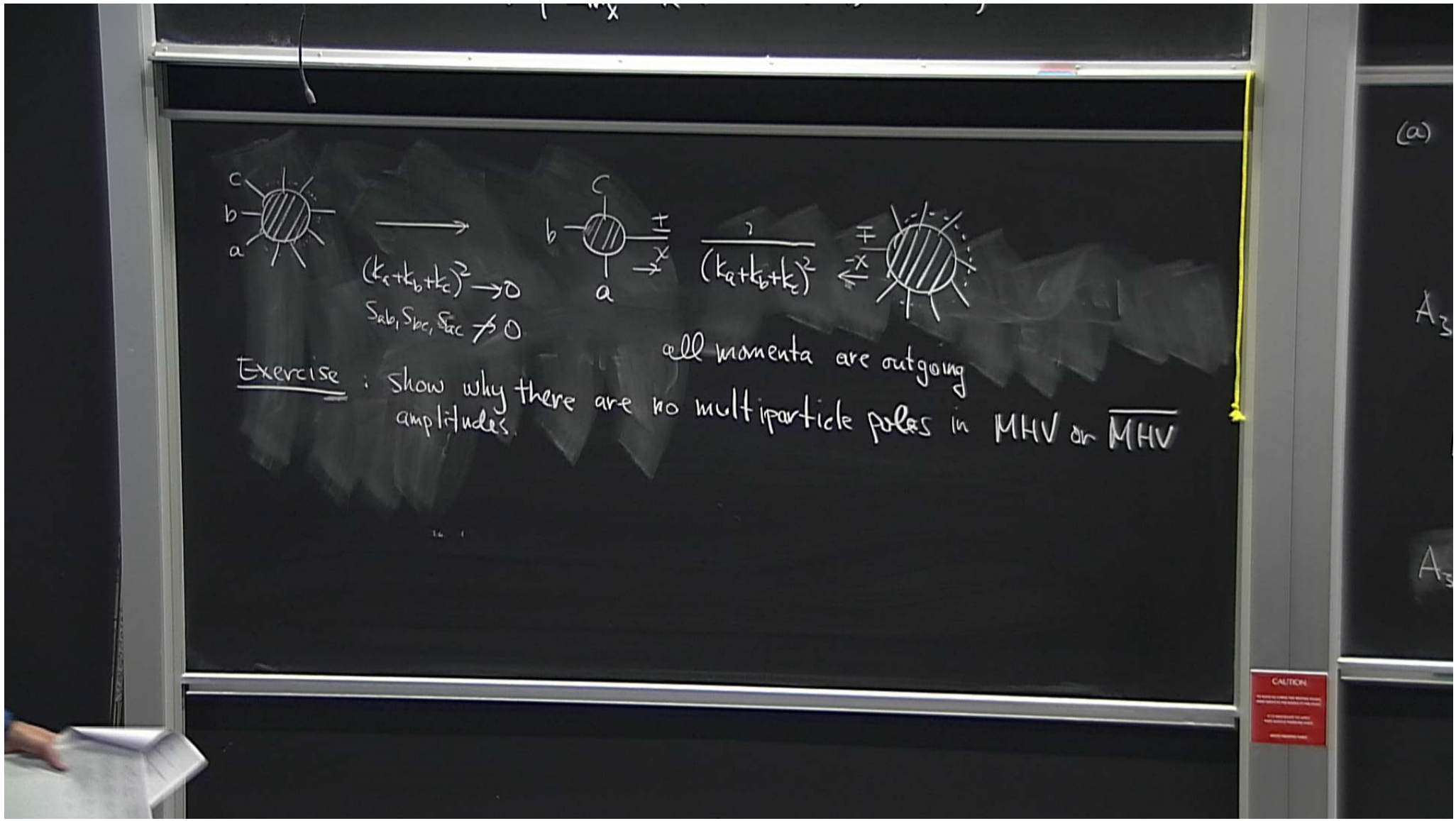
Exercises : Show that $\varepsilon^{(\pm)}$ have the correct form to be pol vectors for circular polarizations

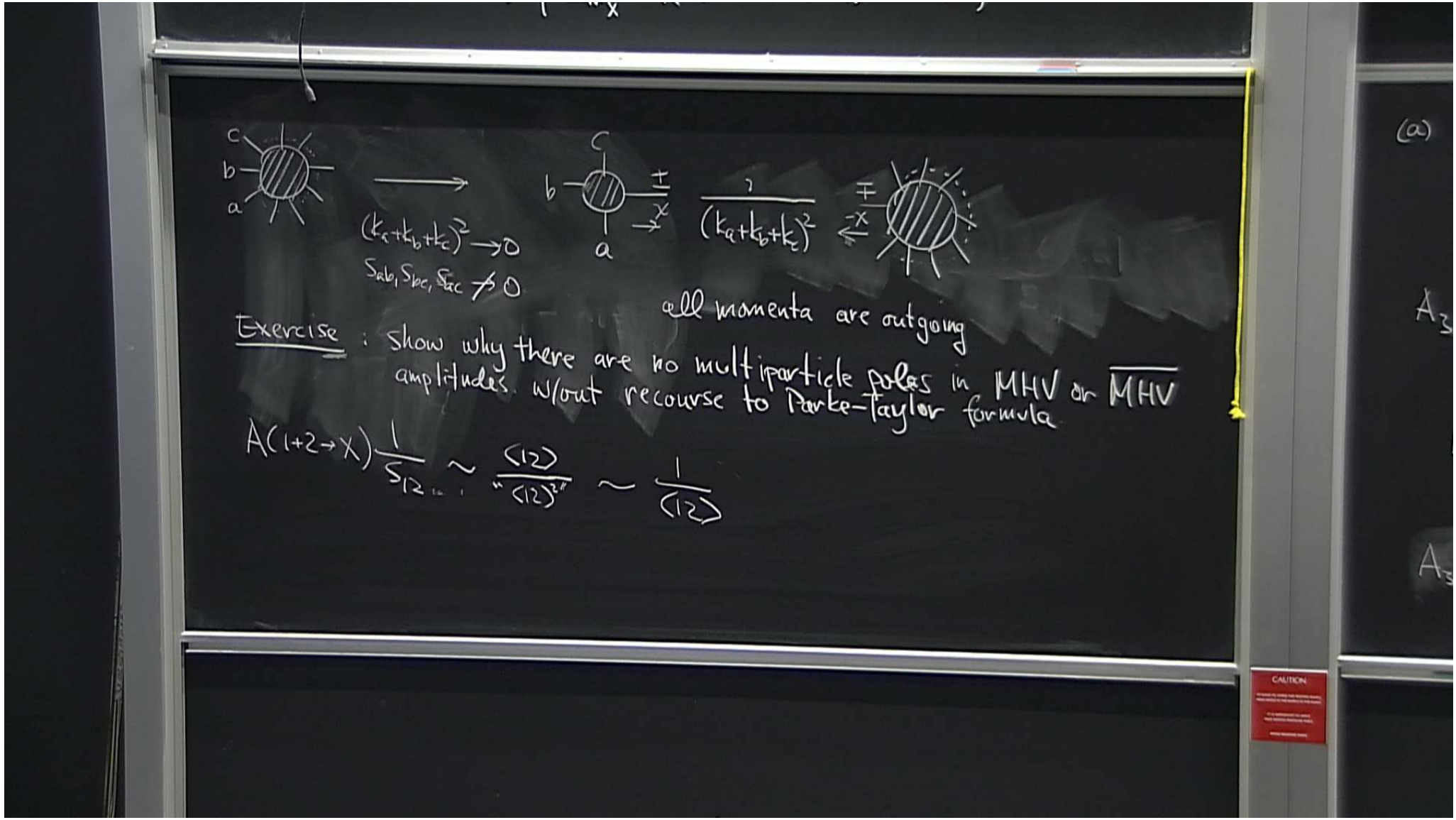
: A_3(fermion, fermions, gluon).

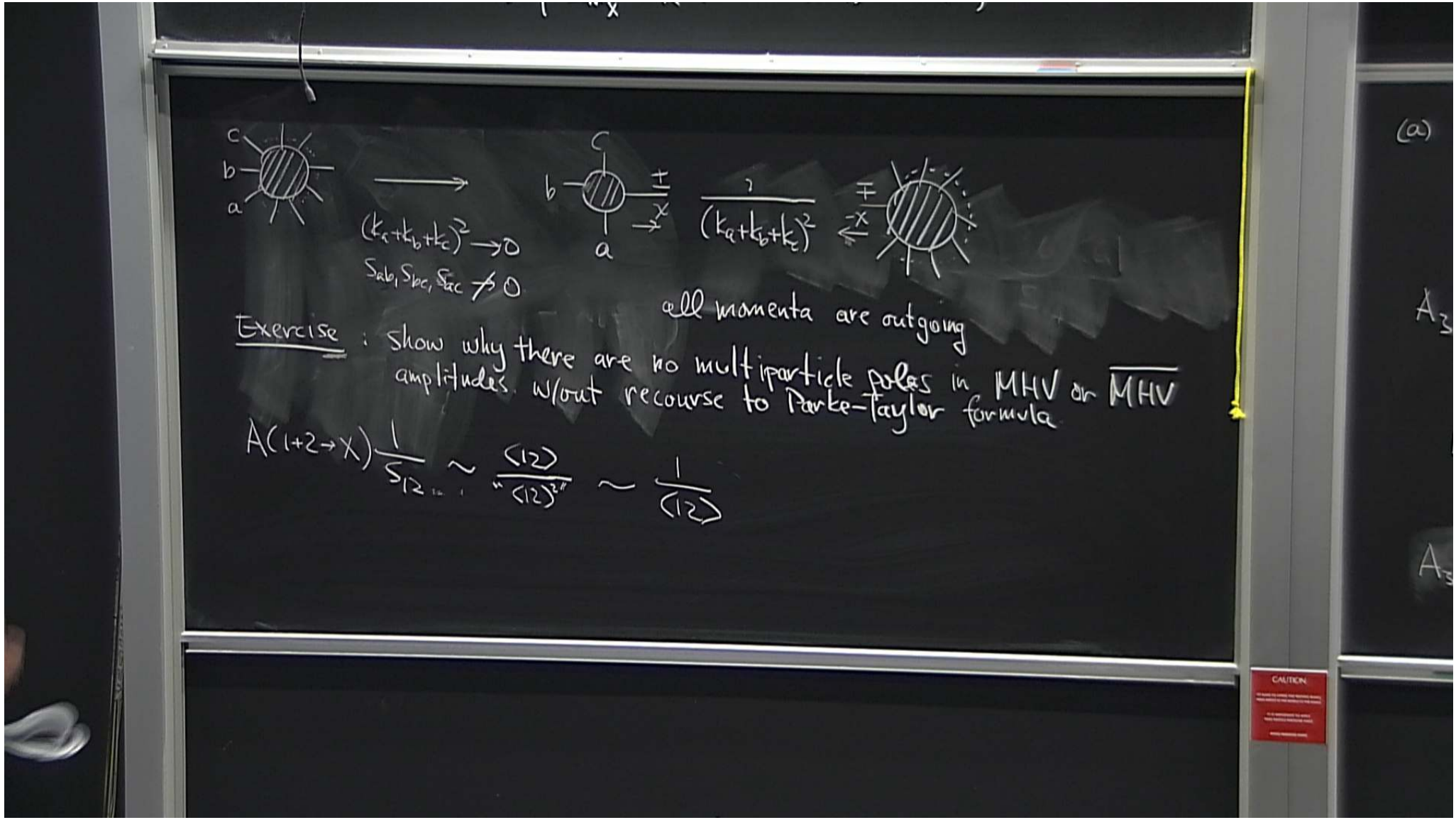

Factorization
$(p_1+p_2)^2 \to m_X^2$
$A(1+2 \to 3+\cdots+n) \longrightarrow \sum_{X \in X} A_L(1+2 \to X) \frac{i}{p^2-m_X^2} A_R(X \to 3+\cdots+n)$
CAUTION

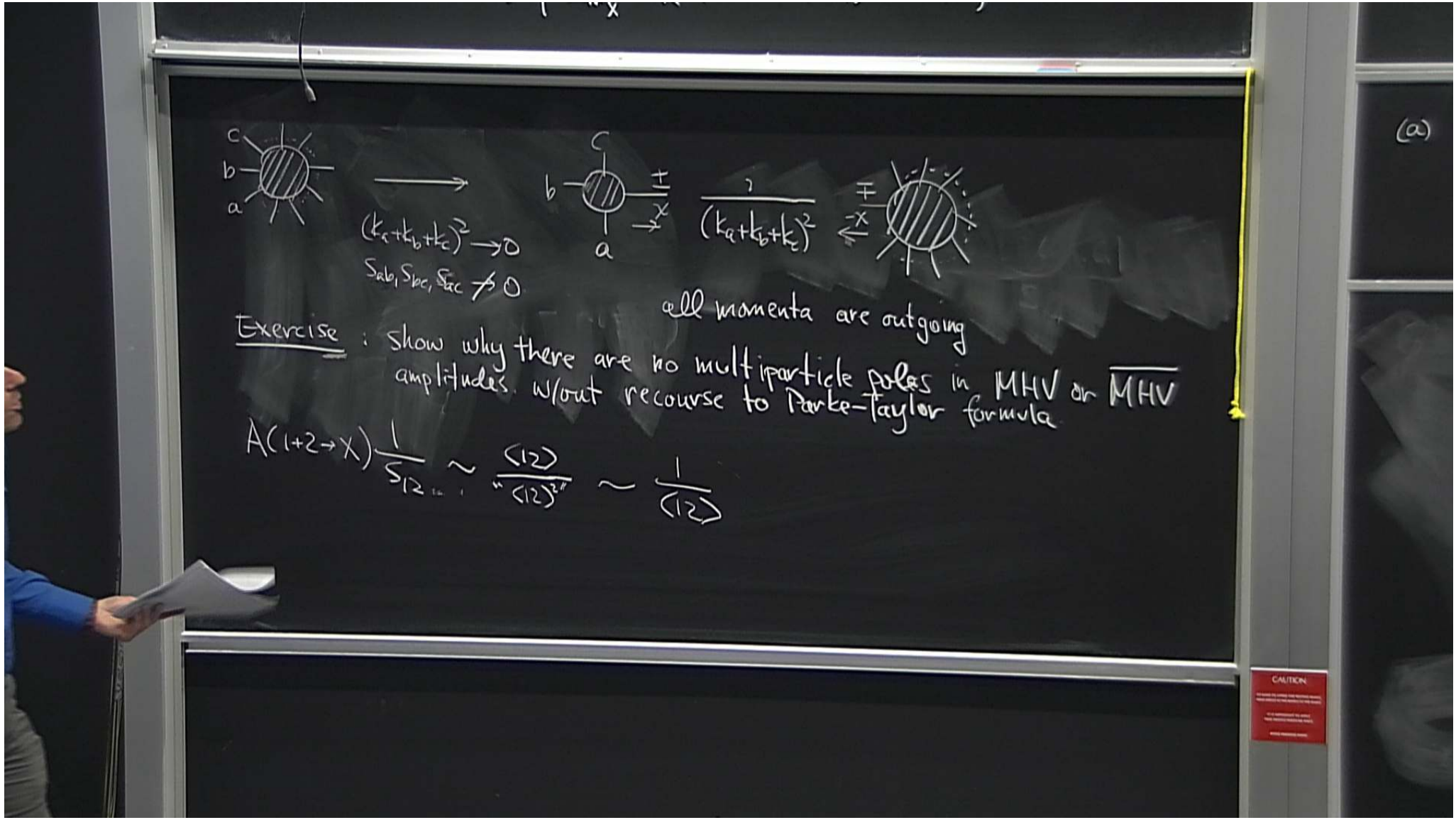

$\sum_{X \in \mathcal{X}} A_L(1+\cdots+n \to X) \frac{i}{p^2 - m_X^2} A_R(X \to (n+1)+\cdots+r)$
c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$S_{ab}, S_{bc}, S_{ac} \neq 0$
c
b
a
$\pm$
$\frac{i}{(k_a+k_b+k_c)^2}$
(a)
CAUTION

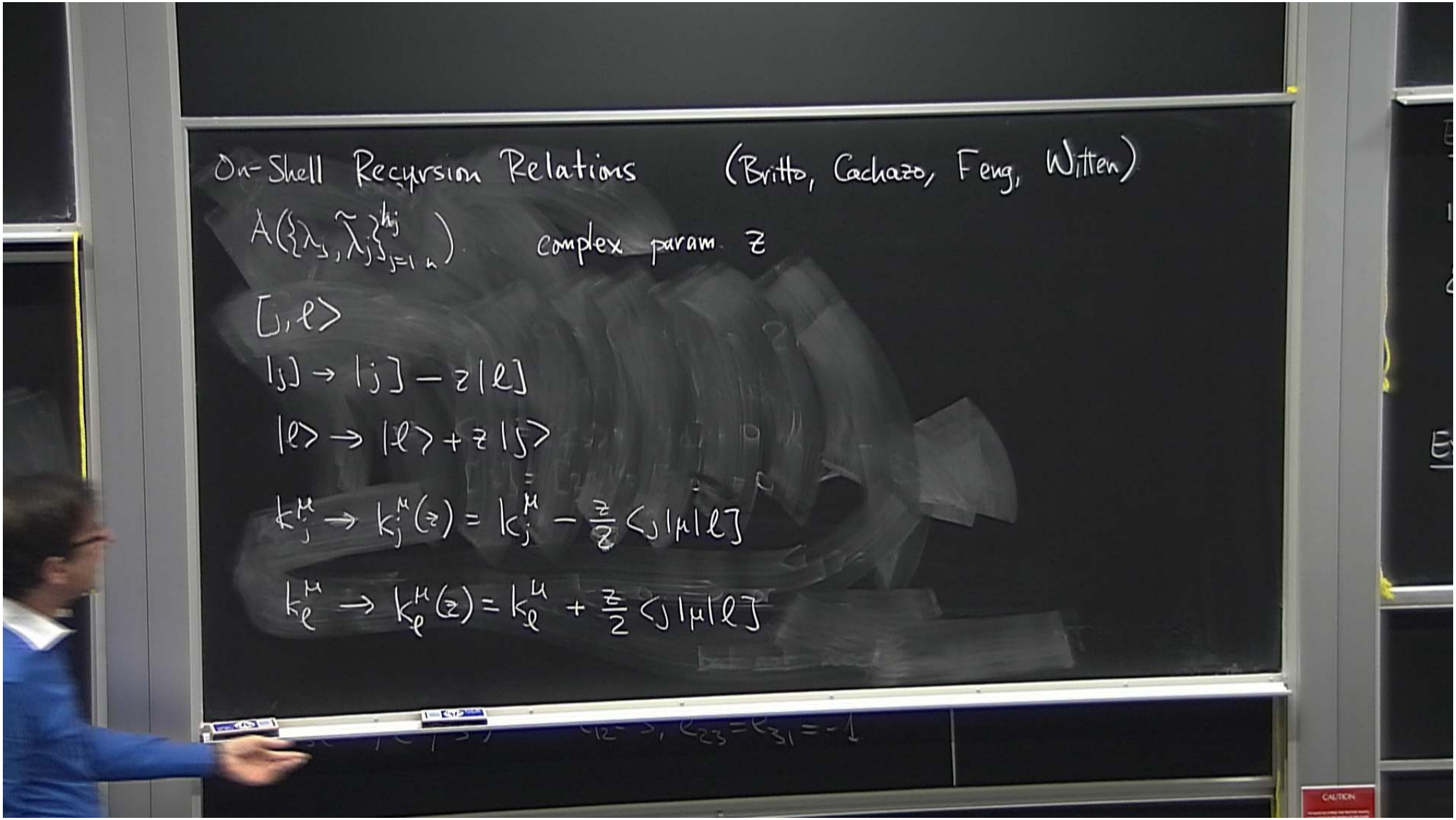

c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$S_{ab}, S_{bc}, S_{ac} \not\to 0$
c
b
a
$\frac{1}{(k_a+k_b+k_c)^2}$
all momenta are outgoing
(a)

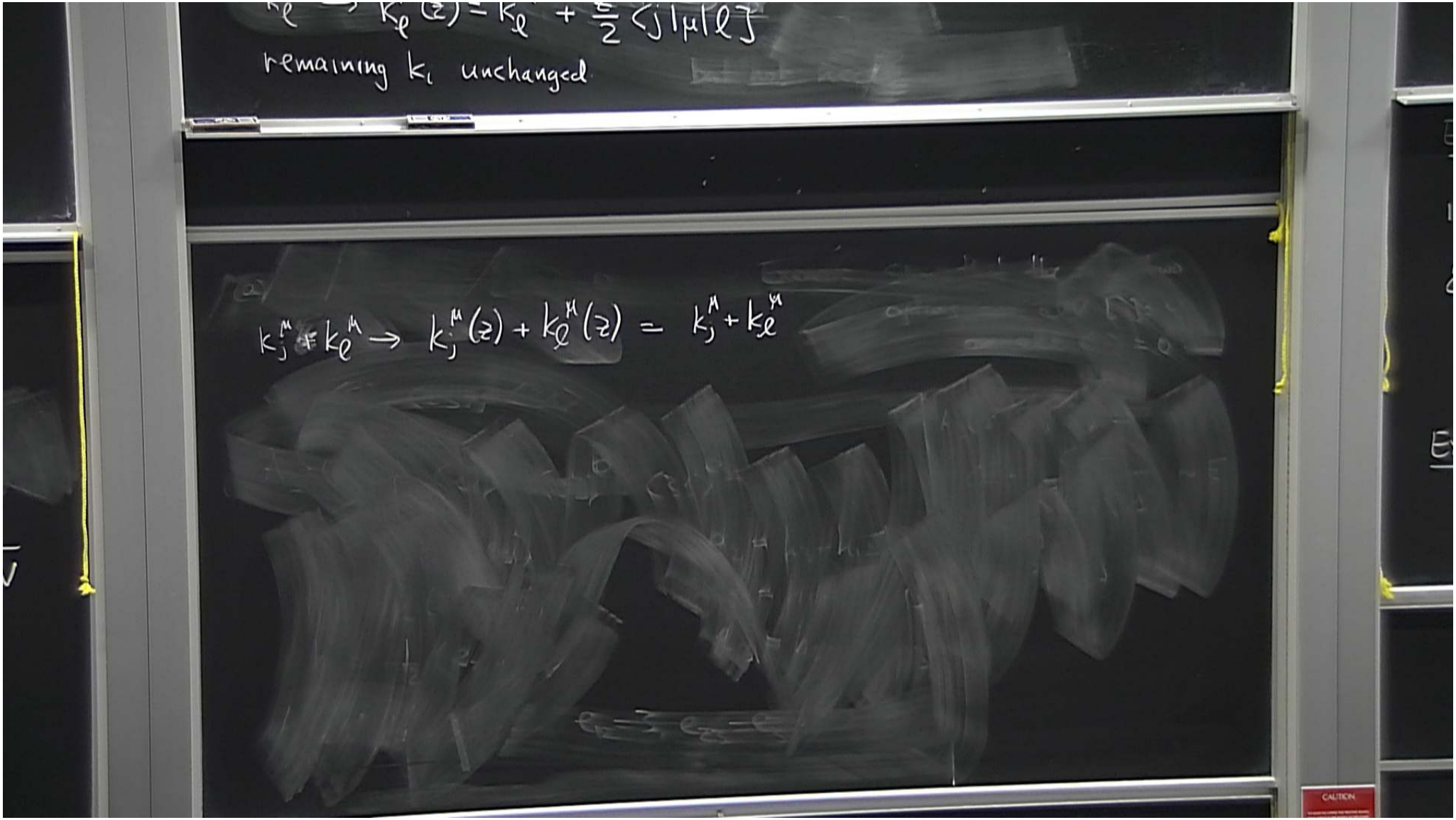

c
b
a
$(k_c+k_b+k_c)^2 \to 0$
$S_{ab}, S_{bc}, S_{ac} \not\to 0$
c
b
a
$\frac{1}{(k_a+k_b+k_c)^2}$
all momenta are outgoing
Exercise : Show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$

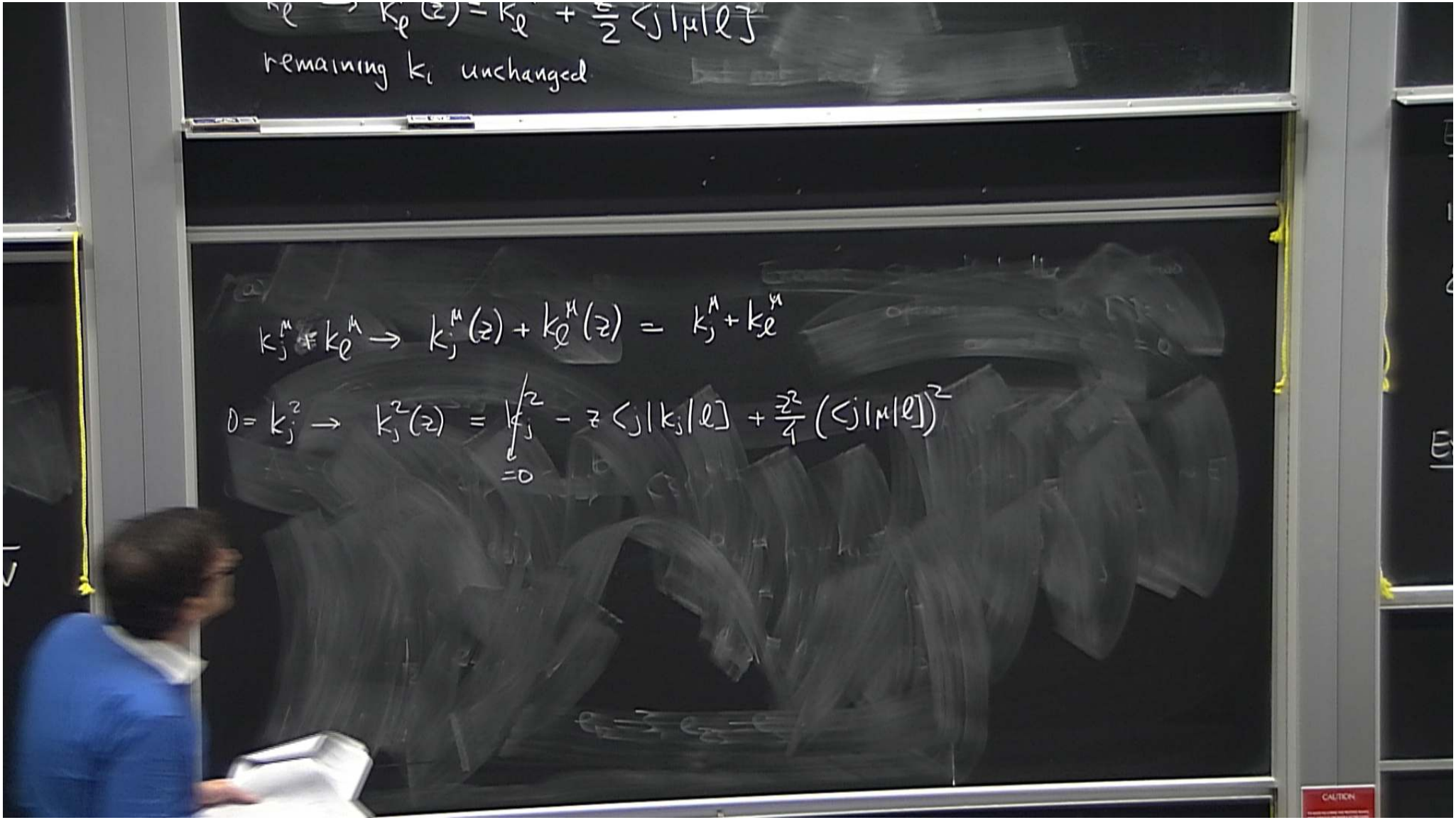

c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$s_{ab}, s_{bc}, s_{ac} \not\to 0$
c
b
a
+
-x
$\frac{1}{(k_a+k_b+k_c)^2}$
+
-x
all momenta are outgoing
Exercise : Show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$ amplitudes w/out recourse to Parke-Taylor formula
$A(1+2\to X)\frac{1}{s_{12\ldots}} \sim \frac{\langle 12\rangle}{\text{"}\langle 12\rangle^{2}\text{"}} \sim \frac{1}{\langle 12\rangle}$
(a)
A_3
A_3
CAUTION

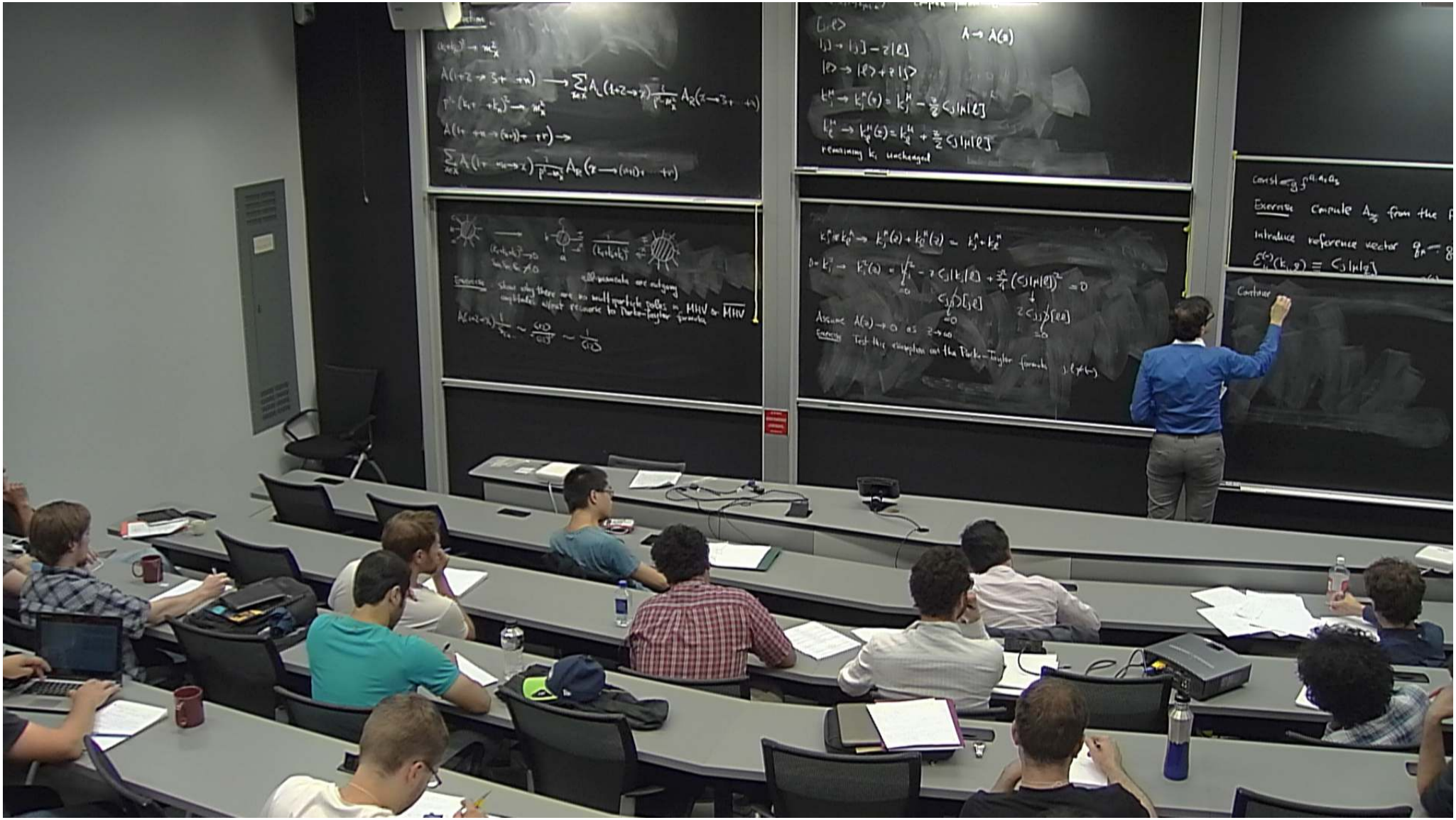

c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$s_{ab}, s_{bc}, s_{ac} \not\to 0$
$\pm$
x
$\frac{1}{(k_a+k_b+k_c)^2}$
$\mp$
$-x$
all momenta are outgoing
Exercise : show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$ amplitudes w/out recourse to Parke-Taylor formula
$A(1+2\to X)\frac{1}{s_{12}} \sim \frac{\langle 12\rangle}{\text{"}\langle 12\rangle^2\text{"}} \sim \frac{1}{\langle 12\rangle}$
(a)
A_3
A_3


c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$s_{ab}, s_{bc}, s_{ac} \not\to 0$
c
b
a
$\frac{1}{(k_a+k_b+k_c)^2}$
all momenta are outgoing
Exercise : Show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$ amplitudes w/out recourse to Parke-Taylor formula
$A(1+2\to X)\frac{1}{s_{12\ldots}} \sim \frac{\langle 12\rangle}{\text{“}\langle 12\rangle^{2}\text{”}} \sim \frac{1}{\langle 12\rangle}$
(a)
CAUTION


On-Shell Recursion Relations (Britto, Cachazo, Feng, Witten)
$A(\{\lambda_j, \tilde{\lambda}_j, h_j\}_{j=1\ldots n})$ complex param z
$[\,,\ell\rangle$
$|j] \to |j] - z|\ell]$
$|\ell\rangle \to |\ell\rangle + z|j\rangle$
$k_j^\mu \to k_j^\mu(z) = k_j^\mu - \frac{z}{2}\langle j|\mu|\ell]$
$k_\ell^\mu \to k_\ell^\mu(z) = k_\ell^\mu + \frac{z}{2}\langle j|\mu|\ell]$


$k_\ell \to k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$
remaining k_i unchanged
$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$


$k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$
remaining k_i unchanged
$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$
$0 = k_j^2 \to k_j^2(z) = \not{k}_j^2 - z\langle j|k_j|\ell] + \frac{z^2}{4}(\langle j|\mu|\ell])^2$
$=0$



$k_\ell \to k_\ell(z) = k_\ell + \frac{z}{2}\langle j|\mu|\ell]$

remaining k_i unchanged

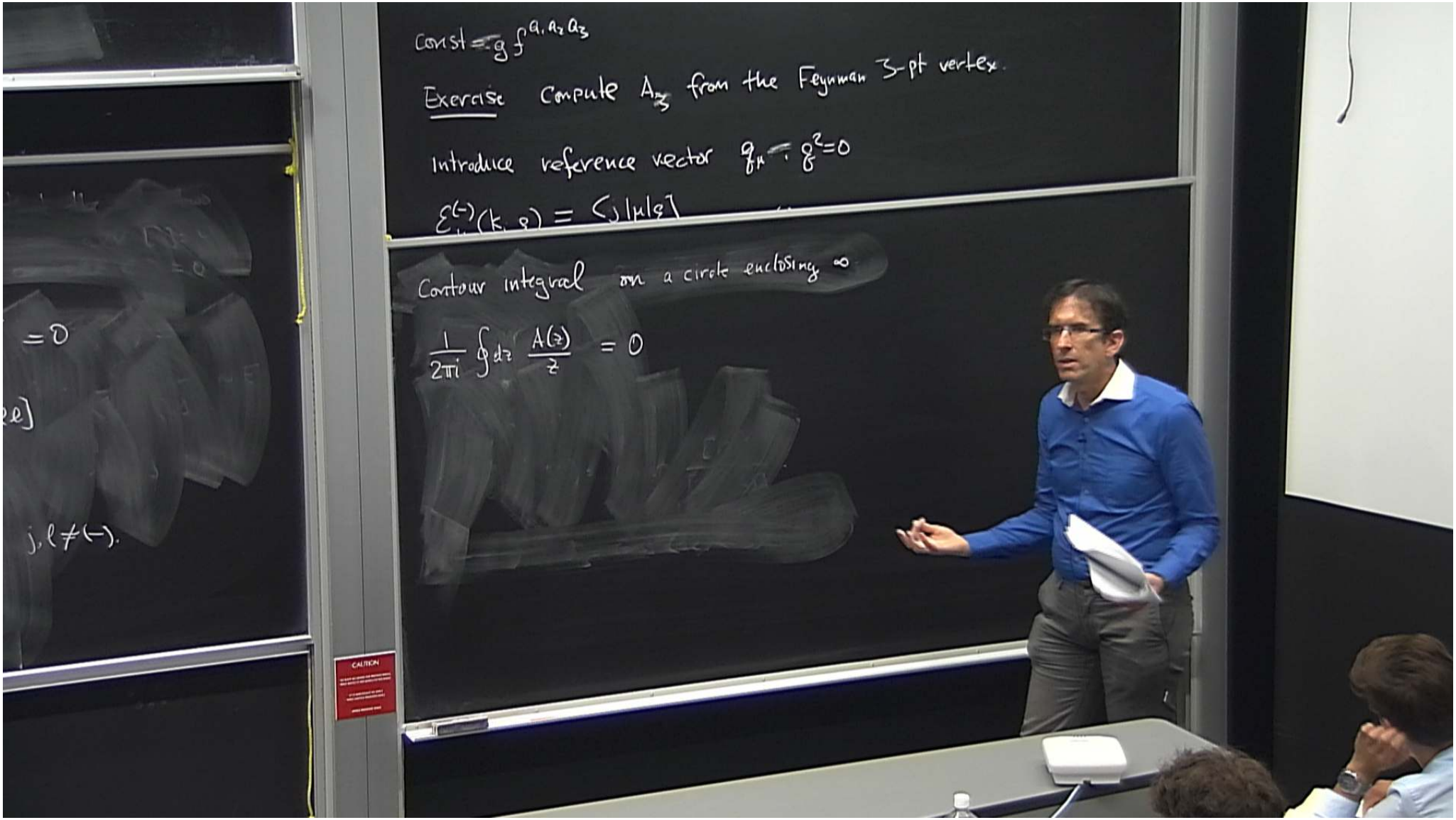
$k_j^\mu + k_\ell^\mu \to k_j^\mu(z) + k_\ell^\mu(z) = k_j^\mu + k_\ell^\mu$

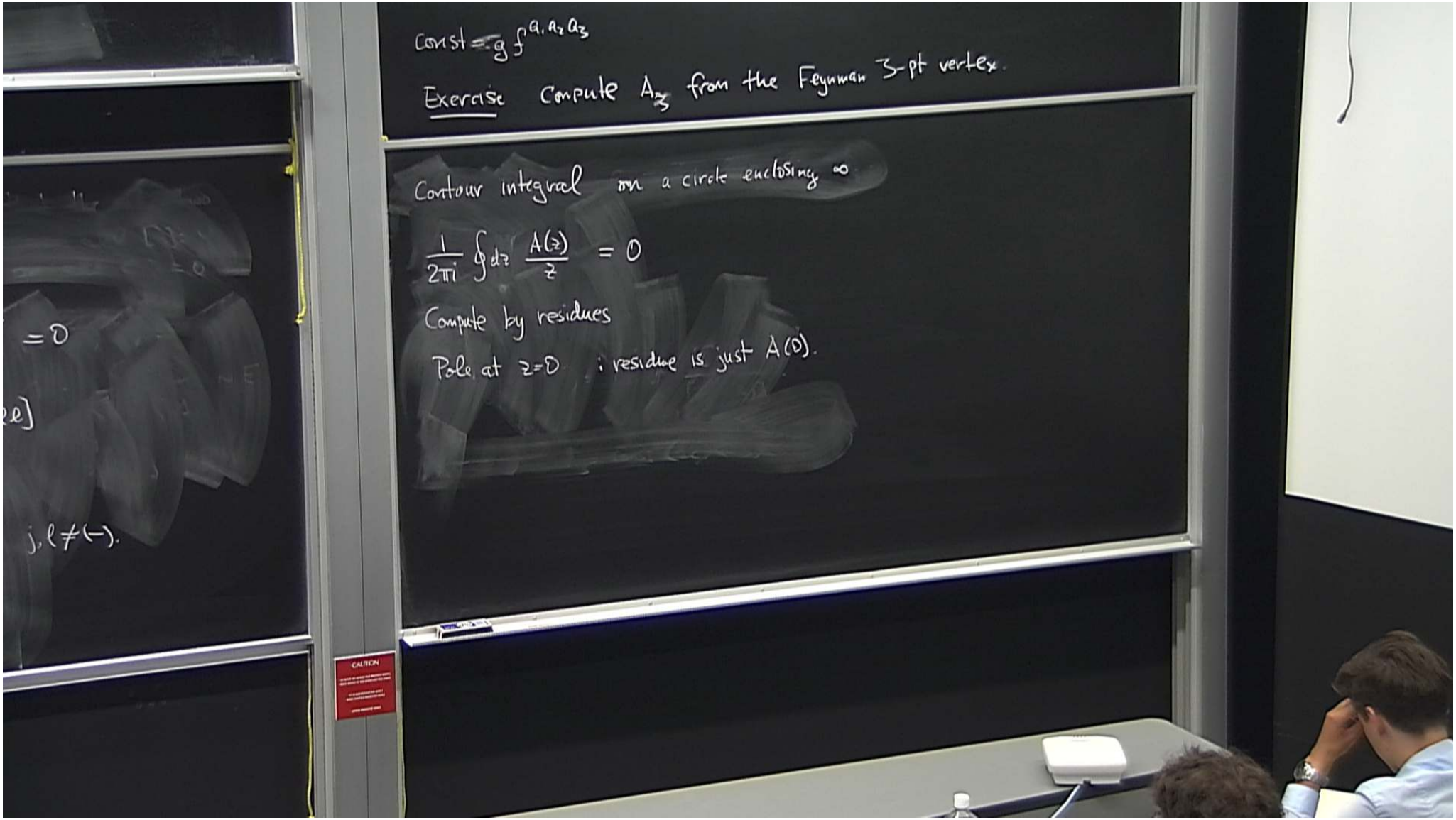
$0 = k_j^2 \to k_j^2(z) = \cancel{k_j^2}^{=0} - z\langle j|k_j|\ell] + \frac{z^2}{4}(\langle j|\mu|\ell])^2 = 0$

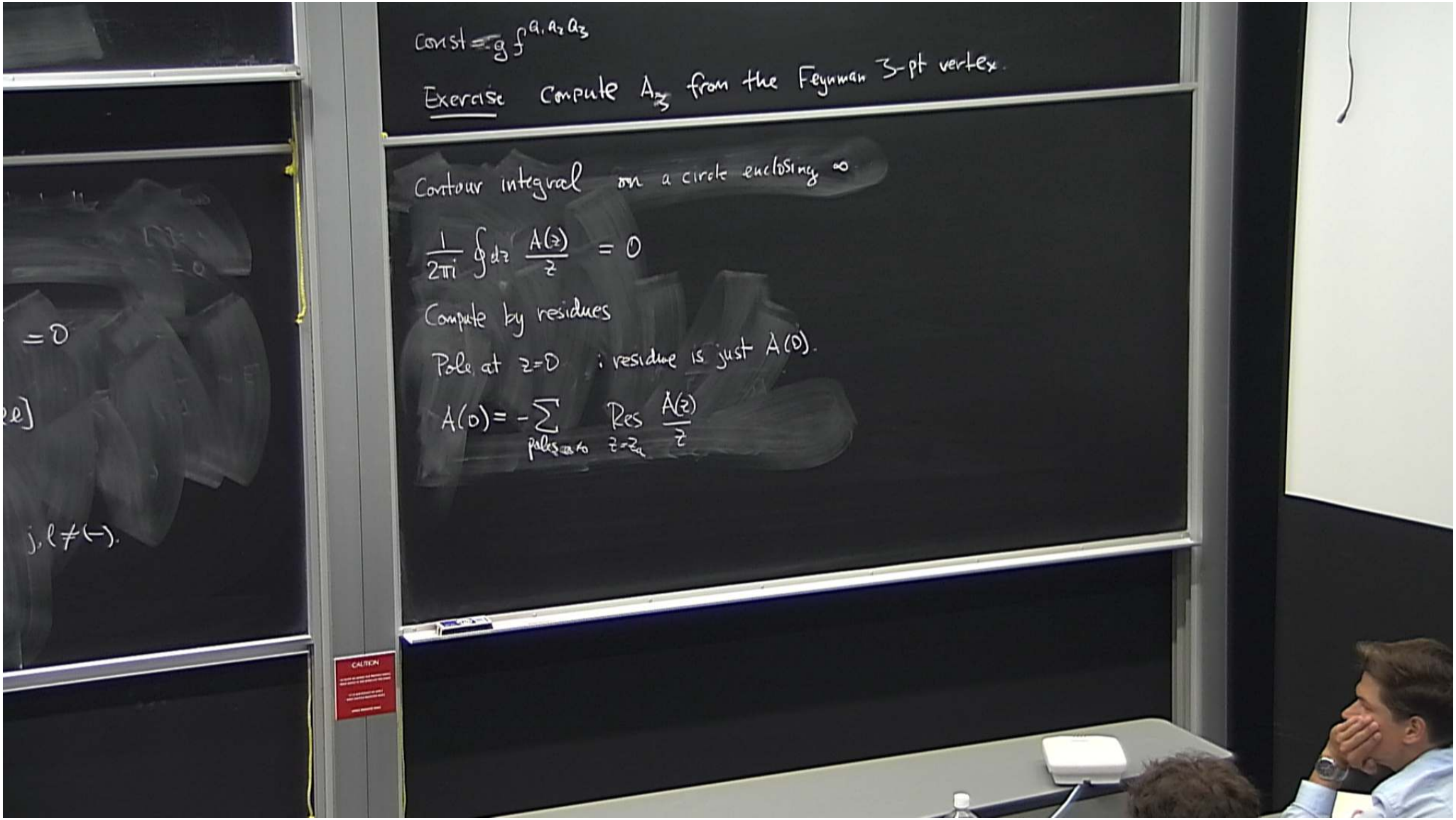
$\langle j\cancel{j}\rangle[j\ell]$ ($=0$) $\qquad z\langle jj\rangle\cancel{[\ell\ell]}$ ($=0$)

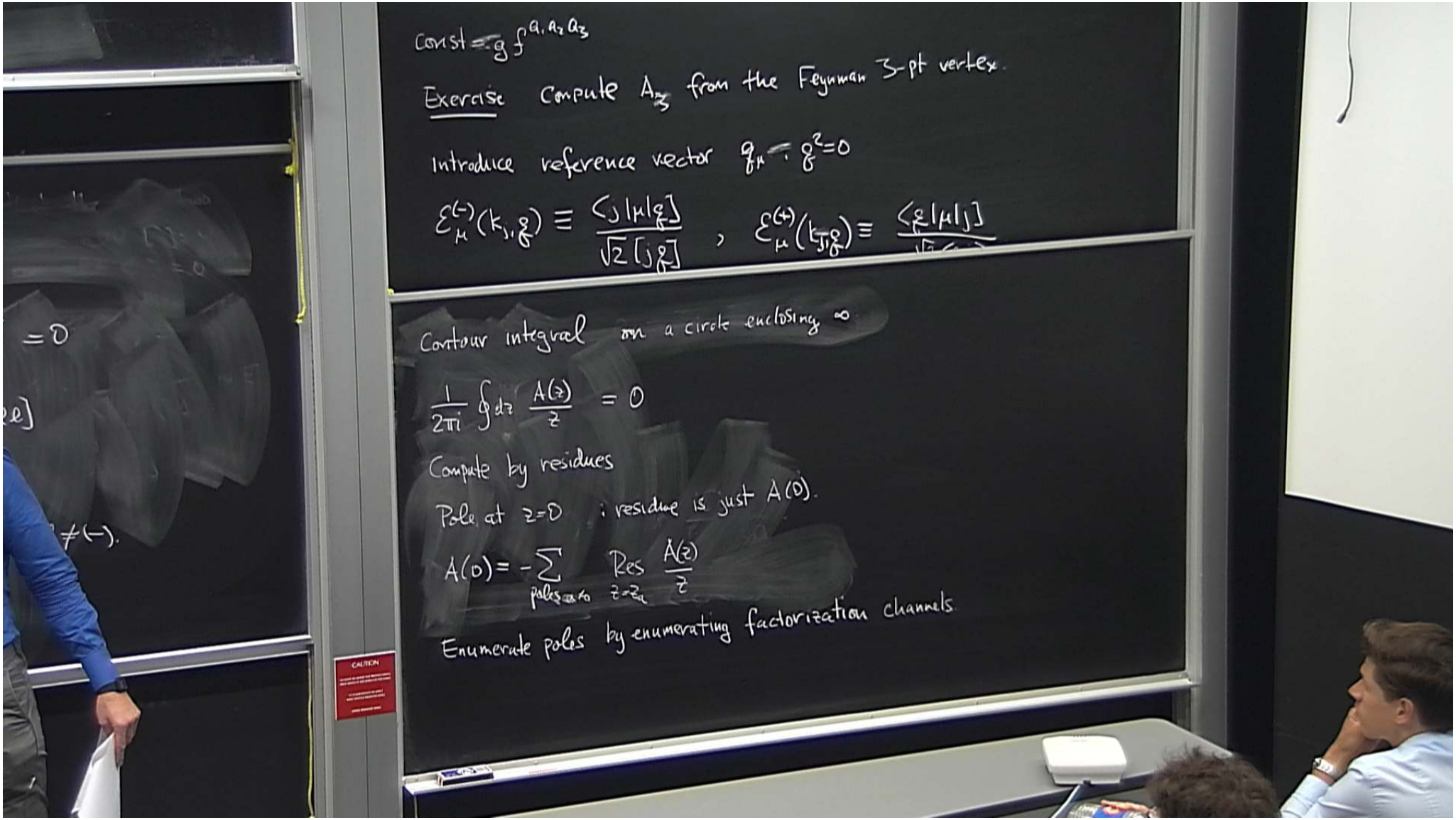
Assume $A(z) \to 0$ as $z \to \infty$

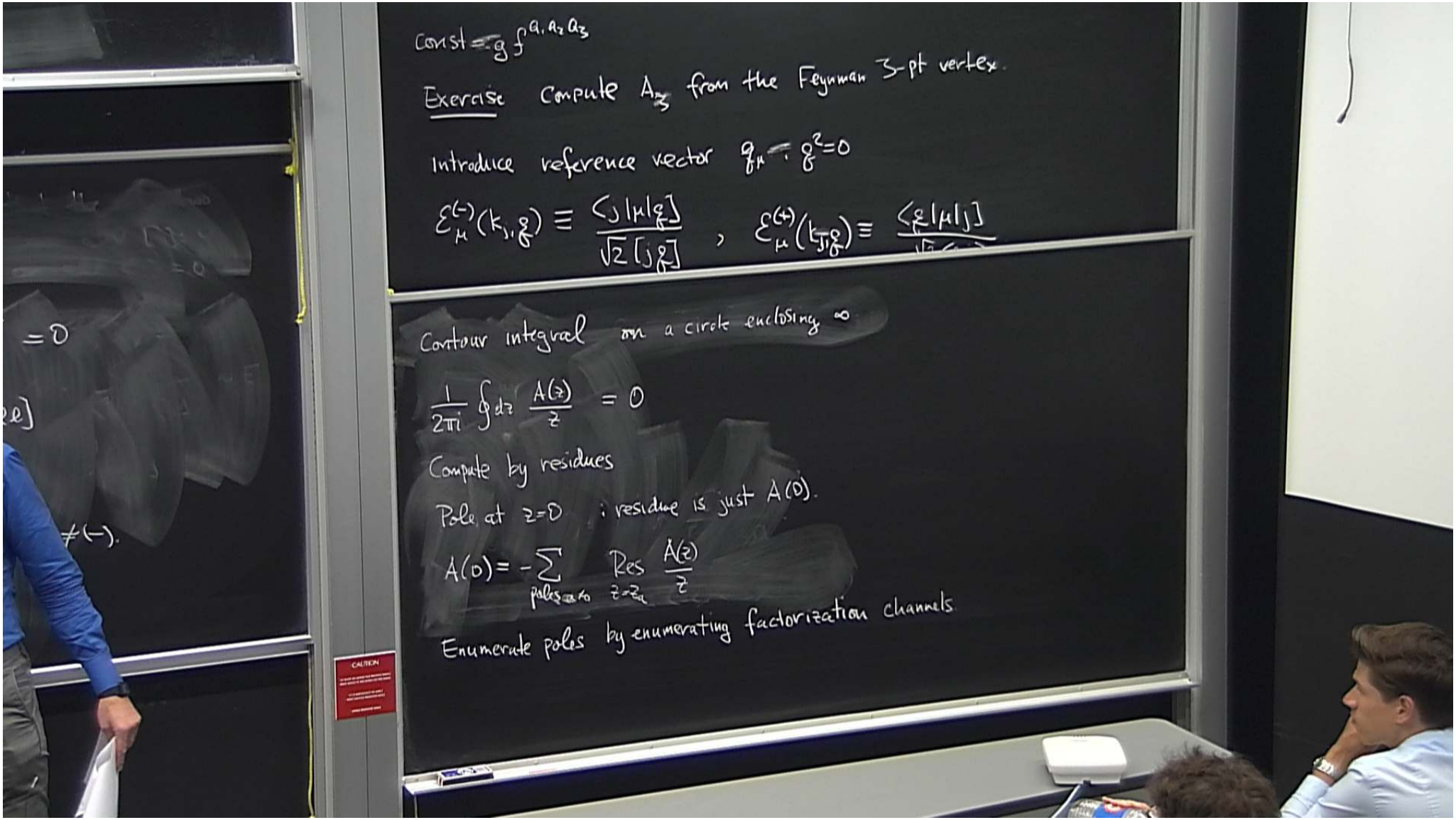
Exercise Test this assumption on the Parke–Taylor formula $j, \ell \neq (-)$.

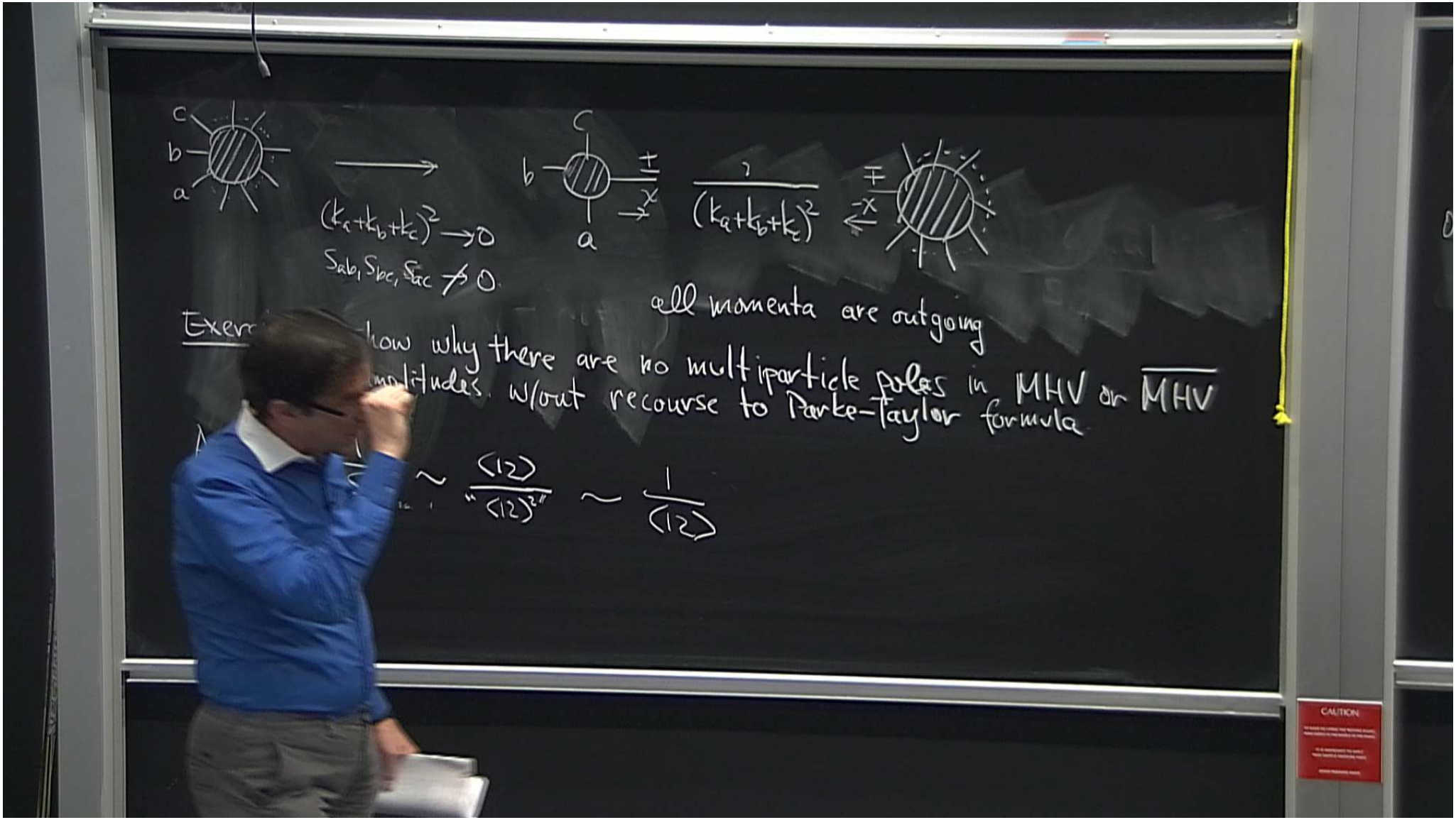

Const $= -g f^{a_1 a_2 a_3}$
Exercise Compute $A_{\pm}$ from the Feynman 3-pt vertex.
Introduce reference vector q_μ : $q^2 = 0$
$\varepsilon^{(-)}(k, q) = C_j |\mu| q]$
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = 0$
CAUTION

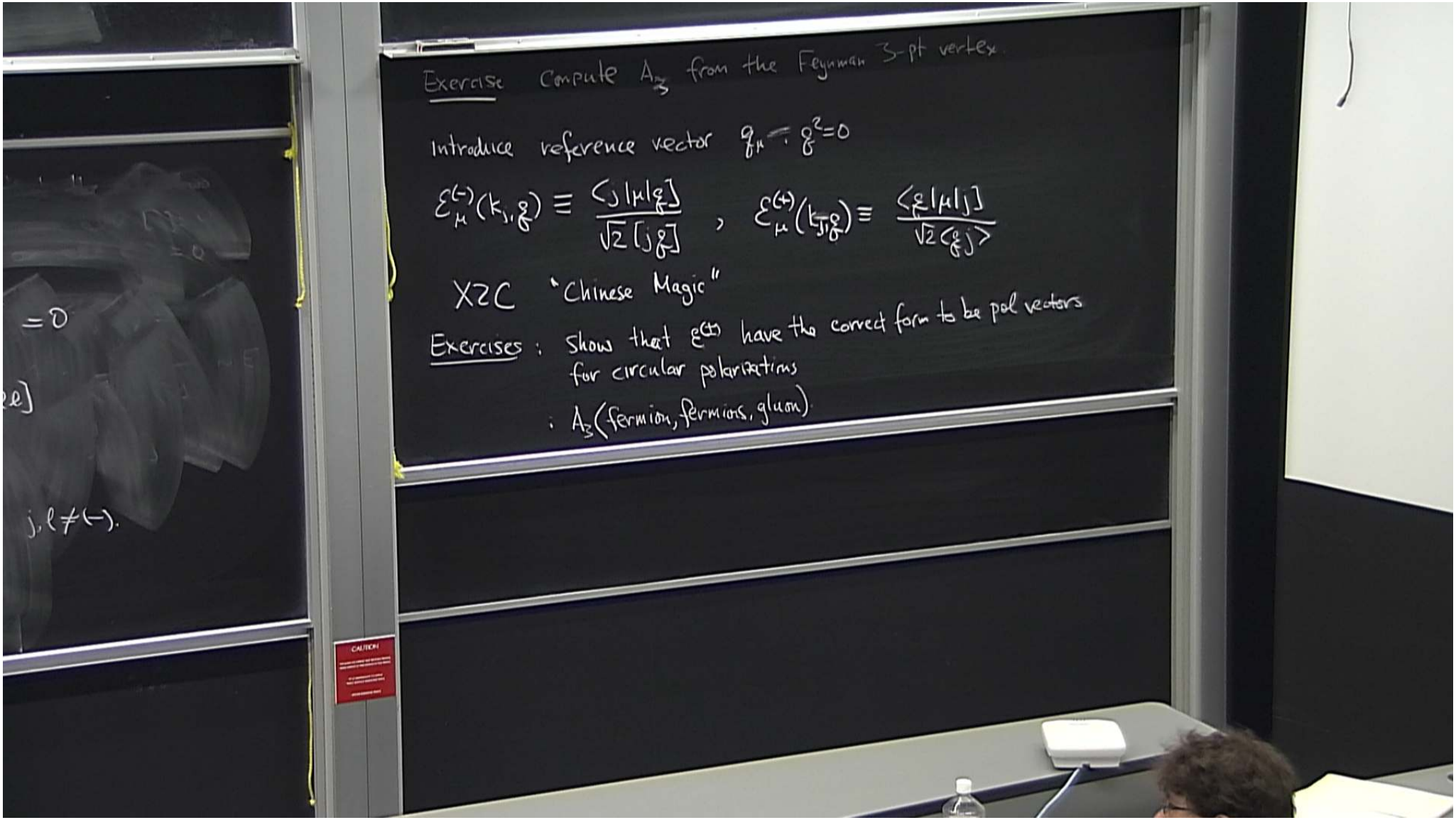

Const $= -g f^{a_1 a_2 a_3}$
Exercise Compute A_z from the Feynman 3-pt vertex.
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = 0$
Compute by residues
Pole at $z=0$; residue is just $A(0)$.
CAUTION

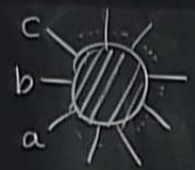

Const = -g f^{a1 a2 a3}
Exercise Compute A_3 from the Feynman 3-pt vertex.
Contour integral on a circle enclosing ∞
1/2πi ∮ dz A(z)/z = 0
Compute by residues
Pole at z=0 ; residue is just A(0).
A(0) = - Σ_{poles ≠ 0} Res_{z=z_a} A(z)/z
=0
CAUTION


Const $= g f^{a_1 a_2 a_3}$
Exercise Compute A_3 from the Feynman 3-pt vertex.
Introduce reference vector q_μ : $q^2 = 0$
$\varepsilon_\mu^{(-)}(k_j, q) \equiv \frac{\langle j | \mu | q]}{\sqrt{2}[jq]}$, $\varepsilon_\mu^{(+)}(k_j, q) \equiv \frac{\langle q | \mu | j]}{\sqrt{2}}$
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = 0$
Compute by residues
Pole at $z=0$; residue is just $A(0)$.
$A(0) = -\sum_{\text{poles} \neq 0} \text{Res}_{z=z_a} \frac{A(z)}{z}$
Enumerate poles by enumerating factorization channels.
CAUTION


Const $= -g f^{a_1 a_2 a_3}$
Exercise Compute A_3 from the Feynman 3-pt vertex.
Introduce reference vector q_μ : $q^2 = 0$
$\varepsilon^{(-)}_\mu(k_j, q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}[jq]}$, $\varepsilon^{(+)}_\mu(k_j, q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}}$
Contour integral on a circle enclosing ∞
$\frac{1}{2\pi i}\oint dz \frac{A(z)}{z} = 0$
Compute by residues
Pole at $z=0$; residue is just $A(0)$.
$A(0) = -\sum_{\text{poles} \neq 0} \text{Res}_{z=z_\alpha} \frac{A(z)}{z}$
Enumerate poles by enumerating factorization channels.
CAUTION

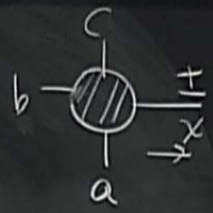

c
b
a
$(k_a+k_b+k_c)^2 \to 0$
$s_{ab}, s_{bc}, s_{ac} \not\to 0$
c
b
a
$\frac{1}{(k_a+k_b+k_c)^2}$
all momenta are outgoing
Exer
how why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$
amplitudes w/out recourse to Parke-Taylor formula
$\sim \frac{\langle 12\rangle}{\text{“}\langle 12\rangle^{2}\text{”}} \sim \frac{1}{\langle 12\rangle}$
CAUTION


Exercise Compute A_3 from the Feynman 3-pt vertex.
Introduce reference vector q_μ : $q^2=0$
$\varepsilon^{(-)}_\mu(k_j,q) \equiv \frac{\langle j|\mu|q]}{\sqrt{2}[jq]}$, $\varepsilon^{(+)}_\mu(k_j,q) \equiv \frac{\langle q|\mu|j]}{\sqrt{2}\langle qj\rangle}$
XZC "Chinese Magic"
Exercises : Show that $\varepsilon^{(\pm)}$ have the correct form to be pol vectors for circular polarizations
: A_3(fermion, fermions, gluon).
CAUTION



$(k_a+k_b+k_c)^2 \to 0$

$s_{ab}, s_{bc}, s_{ac} \not\to 0$



$\frac{1}{(k_a+k_b+k_c)^2}$

all momenta are outgoing

Exercise: Show why there are no multiparticle poles in MHV or $\overline{\text{MHV}}$ amplitudes w/out recourse to Parke-Taylor formula.

$$A(1+2\to X)\frac{1}{s_{12\ldots}} \sim \frac{\langle 12\rangle}{\text{“}\langle 12\rangle^{2}\text{”}} \sim \frac{1}{\langle 12\rangle}$$