

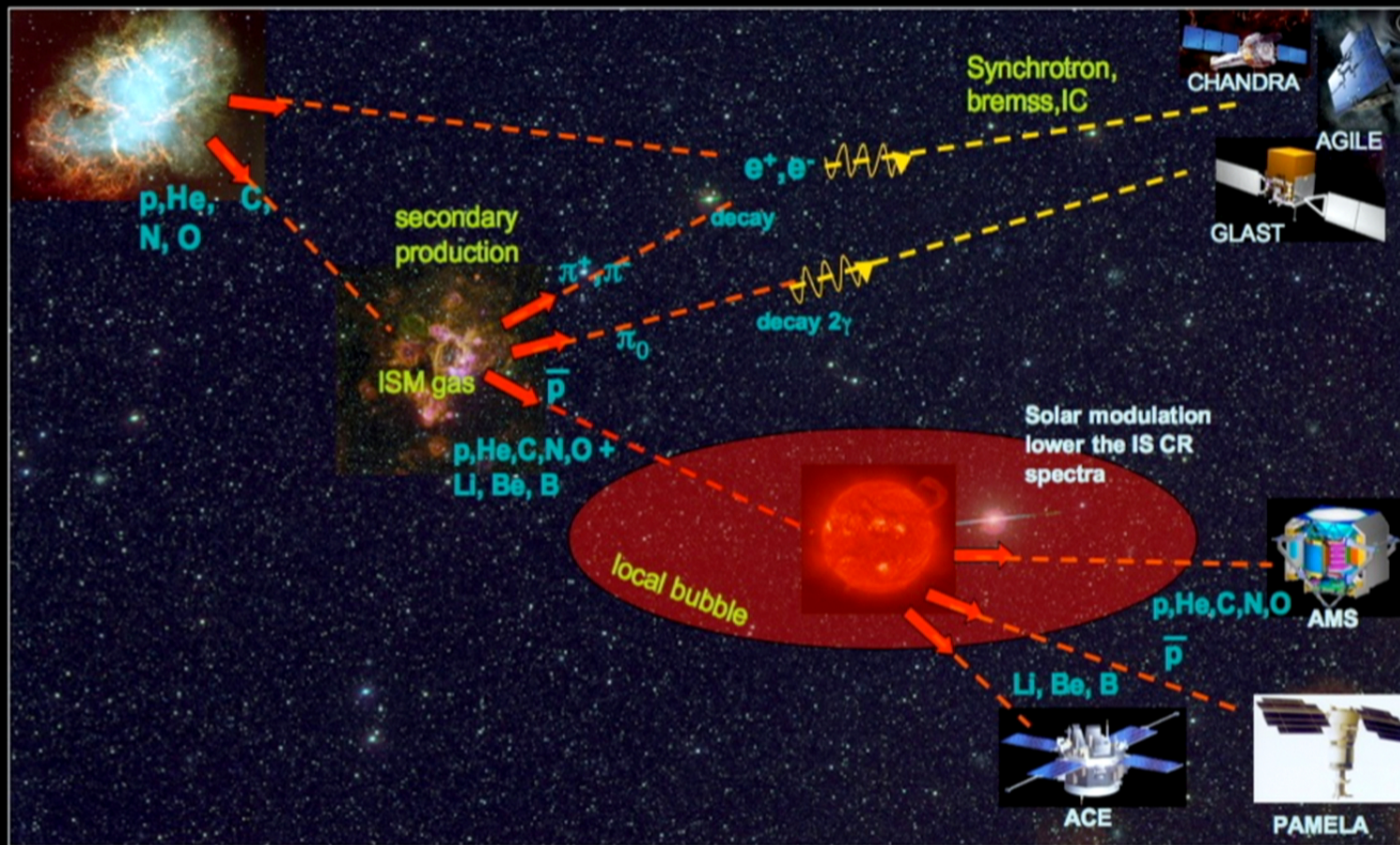
Title: Astroparticle Physics â€™ Observations: Dark matter

Date: Jul 13, 2015 04:00 PM

URL: <http://pirsa.org/15070041>

Abstract:

# COSMIC-RAY PROPAGATION



# COSMIC-RAY PROPAGATION

---

- After escaping the source, CRs travel into interstellar space
- As they propagate through the galaxy, they experience energy losses and changes in trajectory because of interactions with the interstellar medium and magnetic fields
- At the Earth, they are (mainly) isotropically distributed
- How well do we understand propagation? We infer a lot of information through the products of the interaction of the primary CRs (accelerated at the source and measured locally) as they propagate in interstellar space: secondary nuclei , anti-particles, gamma rays, synchrotron radiation

# DIFFUSION

- The isotropy of CRs and abundance of secondary CRs (i.e. nuclei accelerated at the source like p, C, Fe,... vs nuclei produced by spallation in the interstellar gas, like B, Be, ...) indicates that CRs must travel diffusively through the Galaxy
  - ➔ **Galactic magnetic fields are critical for this process**
- The measured boron to carbon ratio (B/C) determines the *grammage*  $X$ , which is related to the amount of material traversed and the escape time  $\tau_{esc}$ :

$$X(E) = \bar{n} \mu v \tau_{esc}(E) \quad \bar{n} : \text{mean gas density}$$

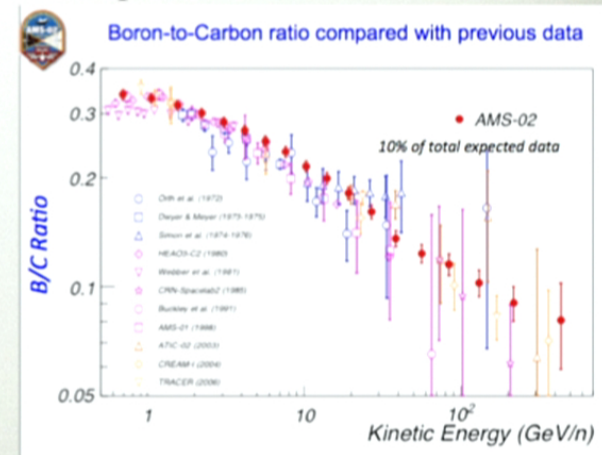
density x thickness

$$X(10 \text{ GeV}/n) \sim 10g/cm^2 \quad \mu : \text{mean gas mass}$$

v : particle speed

- This yields escape times much larger (~100 Myr) than determined by propagation in straight lines (~10-100 kyr)

➔ **CRs diffuse through the Galaxy**



# COSMIC-RAY PROPAGATION

- B/C also indicates that the grammage decreases as a function of the energy (more precisely rigidity  $R=pc/Ze$ ). At high energy:

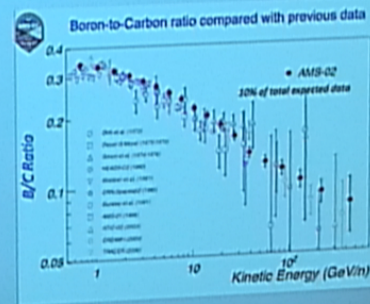
$$X(R) \sim R^{-\delta} \quad \delta = 0.3 - 0.6$$

- The spectrum of CRs measured at Earth includes propagation effects

$$N(E) \propto E^{-\alpha} \quad \alpha = \gamma + \delta$$

$\gamma$  = index of CR spectrum at the source  
 $\delta$  = slope of diffusion coefficient energy dependence

i.e. steeper than spectrum at the source!



$$\langle j_i \rangle = \text{sign}(k_i^0 k_j^0) \langle j_i \rangle^*$$

$$\langle j_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{2k} \rangle$$

symmetry  $\langle j_i \rangle = -\langle j_i \rangle$ ,  $\langle j_i \rangle = -\langle j_i \rangle$   
 $\langle i_i \rangle = 0 = \langle j_i \rangle$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu | j \rangle \equiv \langle j | \mu | j \rangle = 2 k_j^{\mu}$$

commutation  $\langle i | \mu | j \rangle = \langle j | \mu | i \rangle$

$$\langle i | \mu | j \rangle \langle j | \mu | i \rangle = 2 \langle i | j \rangle \langle j | i \rangle$$

# COSMIC-RAY PROPAGATION

- B/C also indicates that the grammage decreases as a function of the energy (more precisely *rigidity*  $R=pc/Ze$ ). At high energy:

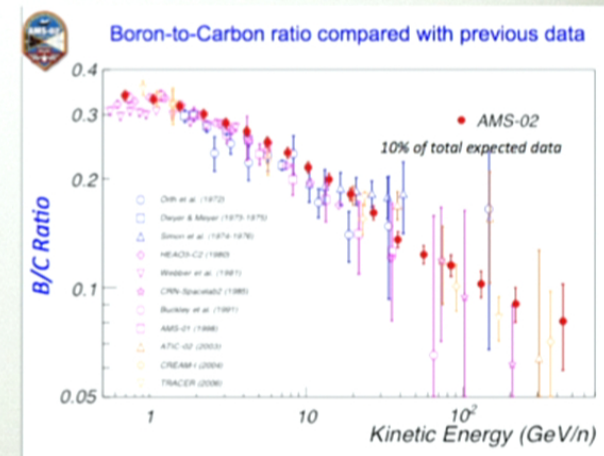
$$X(R) \sim R^{-\delta} \quad \delta = 0.3 - 0.6$$

- The spectrum of CRs measured at Earth includes propagation effects

$$N(E) \propto E^{-\alpha} \quad \alpha = \gamma + \delta$$

$\gamma$  = index of CR spectrum at the source  
 $\delta$  = slope of diffusion coefficient energy dependence

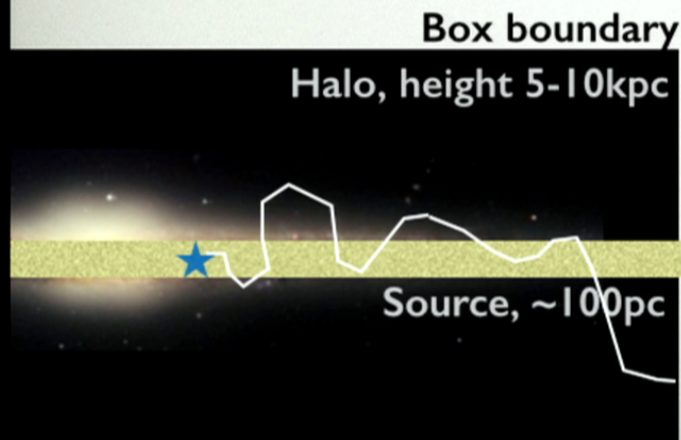
i.e. steeper than spectrum at the source!



# COSMIC-RAY PROPAGATION

- More complete formulation for CR propagation in the galaxy:

$$\begin{aligned}
 \frac{\partial \psi(\vec{r}, p, t)}{\partial t} &= \underbrace{q(\vec{r}, p, t)}_{\text{source term (includes primary source+spallation and decay)}} + \underbrace{\vec{\nabla} \cdot (D_{xx} \vec{\nabla} \psi - \vec{V} \psi)}_{\text{spatial diffusion coefficient}} \\
 &+ \underbrace{\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi}_{\text{re-acceleration}} - \underbrace{\frac{\partial}{\partial p} \left[ \dot{p} \psi - \frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \psi \right]}_{\text{convection in galactic winds}} - \underbrace{\frac{1}{\tau_f} \psi}_{\text{fragmentation}} - \underbrace{\frac{1}{\tau_r} \psi}_{\text{radioactive decay}}
 \end{aligned}$$



Leaky box: assume some escape of CRs into intergalactic space, characterized by the escape time  $\tau_{esc}$

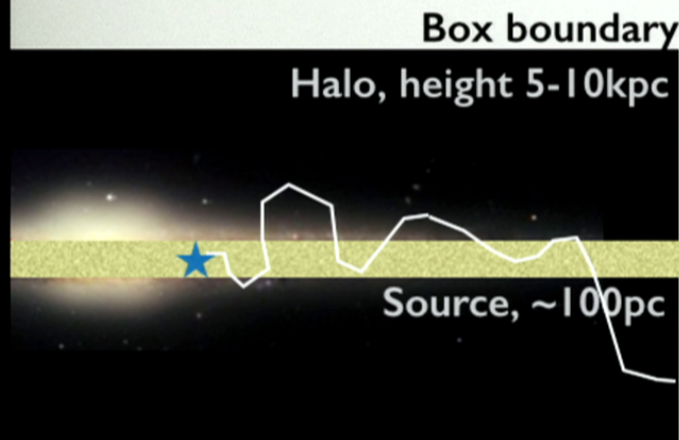
Steady-state: assume  $d\psi/dt=0$

From CR data:  $D_{xx} \sim (3 - 5) \times 10^{28} \text{cm}^2/\text{s}$

# COSMIC-RAY PROPAGATION

- More complete formulation for CR propagation in the galaxy:

$$\begin{aligned}
 \frac{\partial \psi(\vec{r}, p, t)}{\partial t} &= \underbrace{q(\vec{r}, p, t)}_{\text{source term (includes primary source+spallation and decay)}} + \underbrace{\vec{\nabla} \cdot (D_{xx} \vec{\nabla} \psi - \vec{V} \psi)}_{\text{spatial diffusion coefficient}} \\
 &+ \underbrace{\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi}_{\text{re-acceleration}} - \underbrace{\frac{\partial}{\partial p} \left[ \dot{p} \psi - \frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \psi \right]}_{\text{convection in galactic winds}} - \underbrace{\frac{1}{\tau_f} \psi}_{\text{fragmentation}} - \underbrace{\frac{1}{\tau_r} \psi}_{\text{radioactive decay}}
 \end{aligned}$$

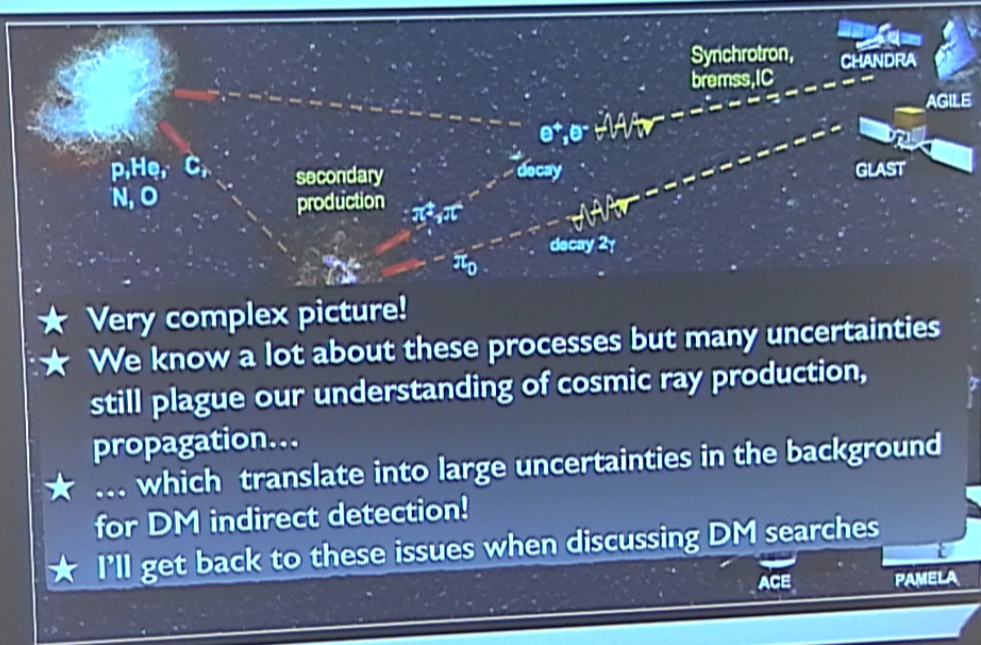


**Leaky box:** assume some escape of CRs into intergalactic space, characterized by the escape time  $\tau_{esc}$

**Steady-state:** assume  $d\psi/dt=0$

**From CR data:**  $D_{xx} \sim (3 - 5) \times 10^{28} \text{cm}^2/\text{s}$





- ★ Very complex picture!
- ★ We know a lot about these processes but many uncertainties still plague our understanding of cosmic ray production, propagation...
- ★ ... which translate into large uncertainties in the background for DM indirect detection!
- ★ I'll get back to these issues when discussing DM searches

$$\langle j_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle j_i \rangle^*$$

$$\langle j_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{2k} \rangle$$

symmetry  $\langle j_i \rangle = - \langle j_i \rangle$ ,  $\langle j_i \rangle = - \langle j_i \rangle$

$$\langle j_i \rangle = 0 = \langle j_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle j_i \rangle = \langle j_i | \mu | j_i \rangle = 2 k_j^i$$

conjugation  $\langle j_i | \mu | j_i \rangle = \langle j_i | \mu | j_i \rangle$

identity  $\langle j_i | \mu | j_i \rangle \langle j_i | \mu | j_i \rangle = 2 \langle j_i \rangle \langle j_i \rangle$

$$\sigma_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\lambda = k \pm k^*, \quad e^{-i\phi k} = \frac{k^0 \pm i k^3}{\sqrt{k^0 k^0}}$$

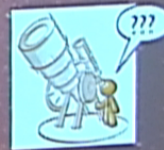
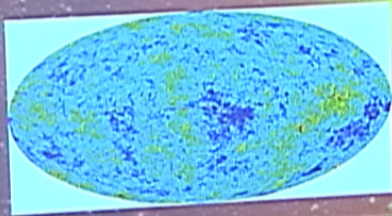
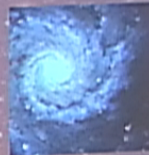
$$\lambda_{\pm} = \begin{pmatrix} -e^{\pm i\phi} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda}_{\pm} = \begin{pmatrix} -e^{\mp i\phi} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise: show that  $\phi \in \mathbb{R}$

# OUTLINE

- Evidence for dark matter at very different scales

- ▶ Galaxies
- ▶ Clusters of galaxies
- ▶ Universe



$$\langle \psi_j | \psi_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle \psi_i | \psi_j \rangle$$

$$\langle \psi_j | \psi_i \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{k^0}$$

symmetry  $\langle \psi_j | \psi_i \rangle = - \langle \psi_i | \psi_j \rangle$ ,  $[ \psi_j ] = - [ \psi_i ]$

$$\langle \psi_i | \psi_i \rangle = 0 = [ \psi_i ]$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$[ \psi_i | \psi_j ] \equiv \langle \psi_i | \mu | \psi_j \rangle = 2 k_j^i$$

conjugation  $\langle \mu | \psi_j \rangle = [ \psi_i | \mu ]$

$$\langle \psi_i | \mu | \psi_j \rangle \langle \psi_l | \mu | \psi_k \rangle = 2 \langle \psi_i | \psi_l \rangle \langle \psi_j | \psi_k \rangle$$

$$\psi_j = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

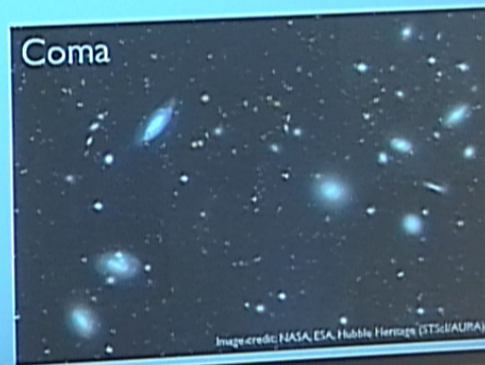
$$\psi_{\pm} = k \pm k^0, \quad e^{\pm i \psi k} = \frac{k^0 \pm k^z}{\sqrt{k_x^2 + k_y^2}}$$

Exercise show that  $\psi \in \mathbb{R}$

$$\lambda_{\pm} = \begin{pmatrix} -e^{\pm i \psi k} \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}, \quad \tilde{\lambda}_{\pm} = \begin{pmatrix} -e^{\mp i \psi k} \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}$$

# ZWICKY AND THE COMA CLUSTER

- The existence of dark matter was postulated by Zwicky in the 1930's to explain the dynamics of galaxies in the Coma galaxy cluster.
- (Clusters of galaxies are the largest gravitationally bound system known in the Universe. They contain ~10s to 1000s of galaxies.)
- Zwicky first inferred the total mass of the cluster by measuring the velocities of its galaxies



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2k_i \cdot k_j$$

$$-\frac{k}{2k}$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu_i \mu_j \rangle = \langle v_j \mu_i \mu_j \rangle = 2k_j^i$$

conjugation  $\langle \mu_i \mu_j \rangle = \langle \mu_j \mu_i \rangle$

$$\langle \mu_i \mu_j \rangle \langle \mu_j \mu_i \rangle = 2 \langle v_i \rangle \langle v_j \rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$e^{-i\mathbf{v} \cdot \mathbf{k}} = \frac{k^0 \pm k^z}{\sqrt{k_x^2 + k_y^2}}$$

$$\lambda_k = \begin{pmatrix} -\sigma^3 \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}, \quad \tilde{\lambda}_k = \begin{pmatrix} -\sigma^3 \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}$$

# ZWICKY AND THE COMA CLUSTER

- For systems in dynamical equilibrium and held together by gravity, the virial theorem becomes:

$$\frac{1}{2}m(3\sigma^2) \quad \leftarrow \quad G \frac{M_{tot}(r)m}{r}$$

$$2\langle T \rangle = -\langle V \rangle$$

Velocities ~ 1000 km/s  
 R ~ Mpc  
 Distance ~ 100 Mpc  
 (1 pc = 3.26 light yrs)

- By measuring the velocity (dispersion) of the galaxies in the Coma cluster, Zwicky could infer its total mass.
- However, the luminous mass (the galaxies in the cluster) was far smaller!

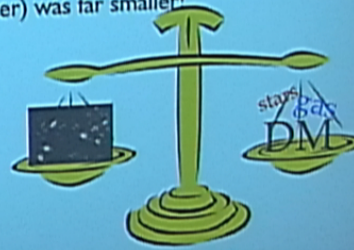
F. Zwicky, *Astrophysical Journal*, vol. 86, p.217 (1937)

$$M > 9 \times 10^{44} \text{ gr.} \quad (35)$$

The Coma cluster contains about one thousand nebulae. The average mass of one of these nebulae is therefore

$$\bar{M} > 9 \times 10^{44} \text{ gr} = 4.5 \times 10^{14} M_{\odot} \quad (36)$$

the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about  $8.5 \times 10^7$  suns. According



$$\langle v_j \rangle = \text{sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k_n} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$\langle v_i \rangle = 0 = \langle v_i \rangle$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu_i \mu_j \rangle = \langle v_i v_j \rangle = 2 k_i \cdot k_j$$

conjugation  $\langle \mu_i \mu_j \rangle = \langle v_i v_j \rangle$

$$\langle \mu_i \mu_j \rangle = 2 \langle v_i \rangle \langle v_j \rangle$$

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$k^0 \pm k^1 = \frac{k^0 \pm k^1}{\sqrt{E}}$$

$$\lambda = \begin{pmatrix} -\sigma^{12} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} \sigma^{12} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

# ZWICKY AND THE COMA CLUSTER

- For systems in dynamical equilibrium and held together by gravity, the virial theorem becomes:

$$\frac{1}{2}m(3\sigma^2) \quad \leftarrow \quad G \frac{M_{tot}(r)m}{r}$$

$$2\langle T \rangle = -\langle V \rangle$$

Velocities ~ 1000 km/s  
 R ~ Mpc  
 Distance ~ 100 Mpc  
 (1 pc = 3.26 light yrs)

- By measuring the velocity (dispersion) of the galaxies in the Coma cluster, Zwicky could infer its total mass.
- However, the luminous mass (the galaxies in the cluster) was far smaller!

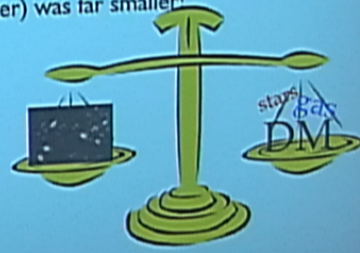
F. Zwicky, *Astrophysical Journal*, vol. 86, p.217 (1937)

$M > 9 \times 10^{16} \text{ gr.}$  (35)

The Coma cluster contains about one thousand nebulae. The average mass of one of these nebulae is therefore

$\bar{M} > 9 \times 10^{16} \text{ gr} = 4.5 \times 10^{11} M_{\odot}$  (36)

the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about  $8.5 \times 10^7$  suns. According



$$\langle v_i \rangle = \text{sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k_n} \rangle$$

symmetry  $\langle v_i \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_i \rangle$

$\langle v_i \rangle = 0 = \langle v_i \rangle$

Exercise show that  $\lambda, \bar{\lambda}$  are solutions for

bracket momenta "Gordon identity"

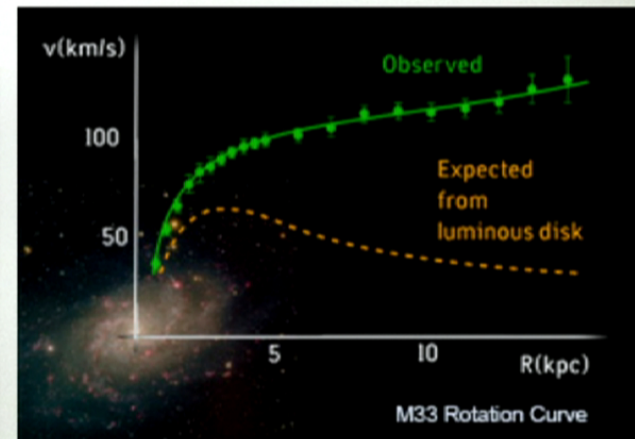
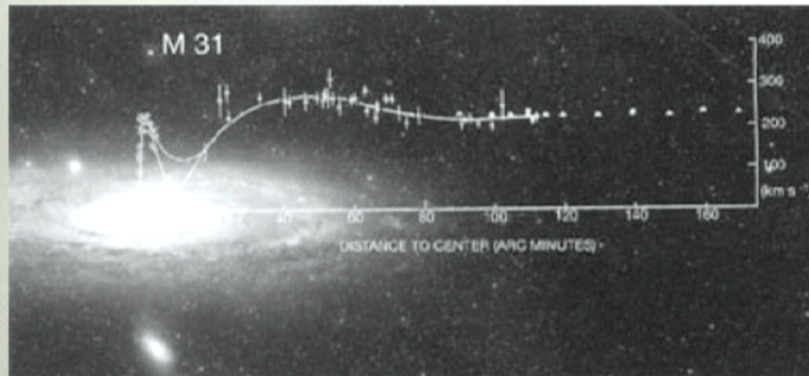
$\langle v_i \rangle = \langle v_i \rangle = 2 k_i^*$

conjugation  $\langle v_i \rangle = \langle v_i \rangle$

$\langle v_i \rangle \langle v_i \rangle = 2 \langle v_i \rangle \langle v_i \rangle$

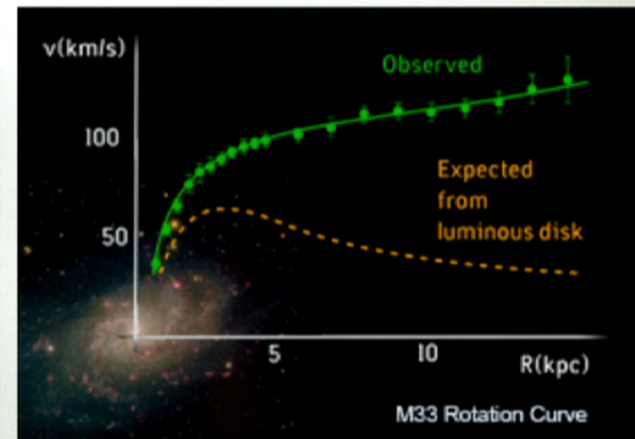
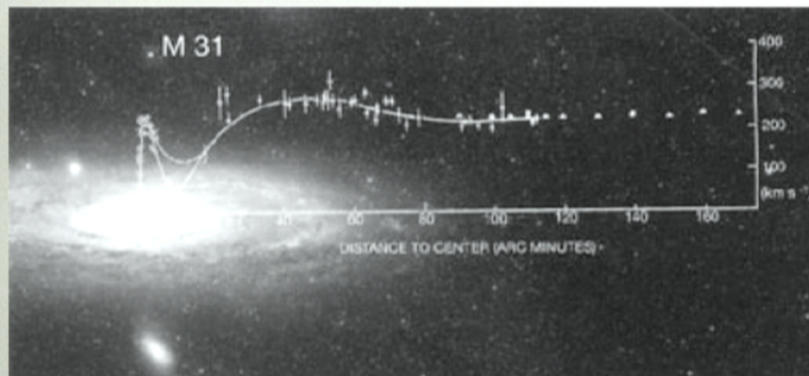
# ROTATION CURVES OF GALAXIES

- Departures from the predictions of newtonian gravity became apparent also at galactic scales with the measurement of rotation curves of galaxies (Rubin et al, 1970)



# ROTATION CURVES OF GALAXIES

- Departures from the predictions of newtonian gravity became apparent also at galactic scales with the measurement of rotation curves of galaxies (Rubin et al, 1970)

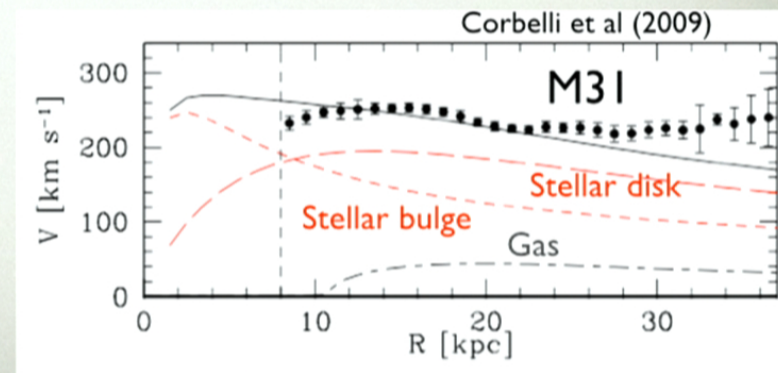
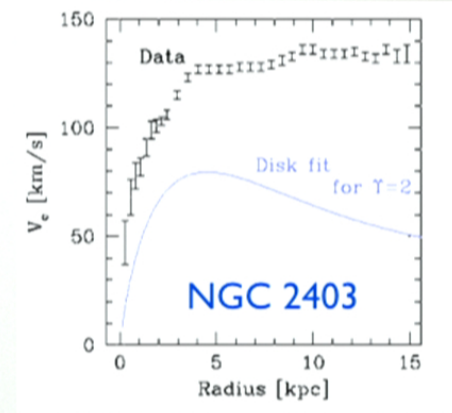


# ROTATION CURVES OF GALAXIES

From newtonian dynamics:

$$F = \frac{mv^2}{r} = G \frac{mM}{r^2}$$

$$v(r) \propto r^{-1/2}$$





# ROTATION CURVES OF GALAXIES

- From newtonian dynamics:

$$F = \frac{mv^2}{r} = G \frac{mM}{r^2}$$

$$v(r) \propto r^{-1/2}$$

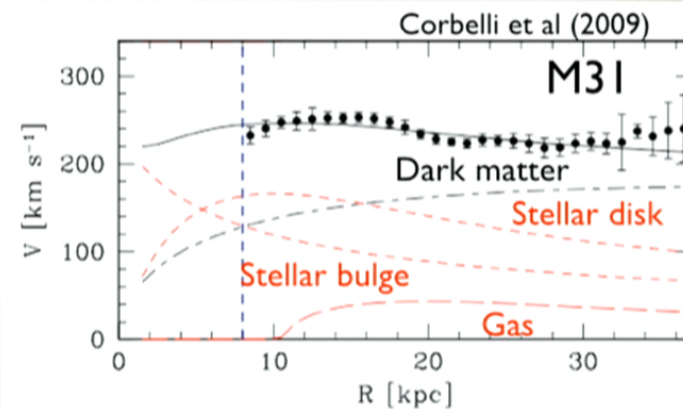
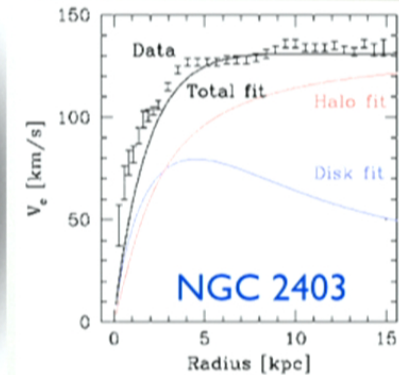
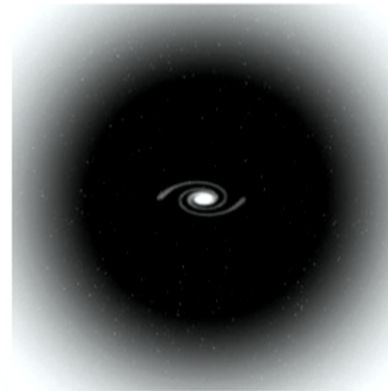
- For constant v:

$$M(r) \propto r \quad \rho(r) \propto r^{-2}$$

Mass density not as steeply falling as star density (exponential)!

➔ By adding extended dark matter halo get good fit to the data.

Similar exercise for the Milky Way yields local DM density:  
 $\rho(8.5 \text{ kpc}) \sim 0.2\text{-}0.5 \text{ GeV/cm}^3$  (direct detection)



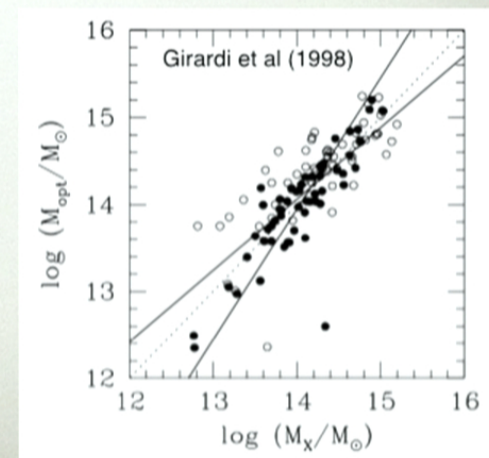
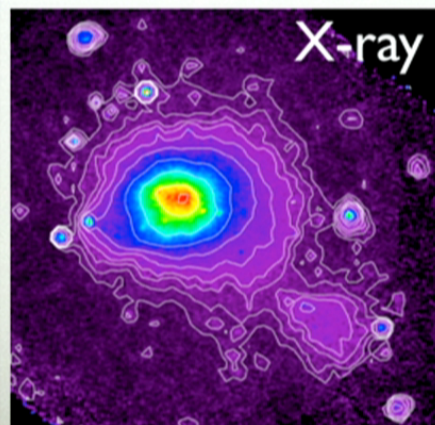
# GALAXY CLUSTERS

- X-ray emitted by very hot intracluster gas ( $10^7$ - $10^8$ K) through bremsstrahlung.
- Gas mass and total mass in galaxy clusters measured by X-ray (assuming thermal equilibrium), lensing
- Mass determination consistent with clusters being dark matter dominated

Galaxy cluster:

~1-2% stars, ~5-15% gas, remaining is dark matter

Coma galaxy cluster



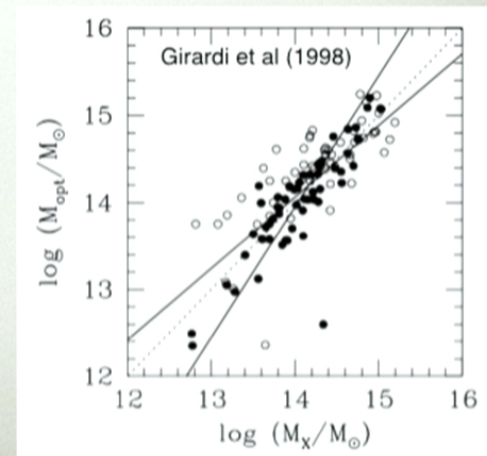
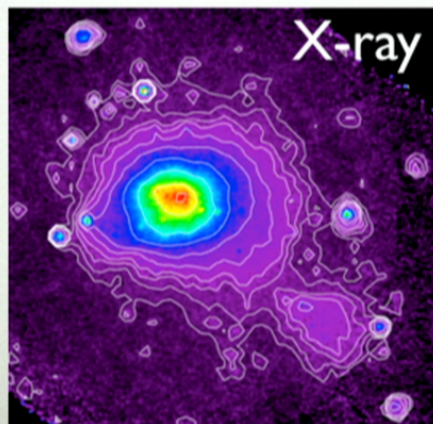
# GALAXY CLUSTERS

- X-ray emitted by very hot intracluster gas ( $10^7$ - $10^8$ K) through bremsstrahlung.
- Gas mass and total mass in galaxy clusters measured by X-ray (assuming thermal equilibrium), lensing
- Mass determination consistent with clusters being dark matter dominated

Galaxy cluster:

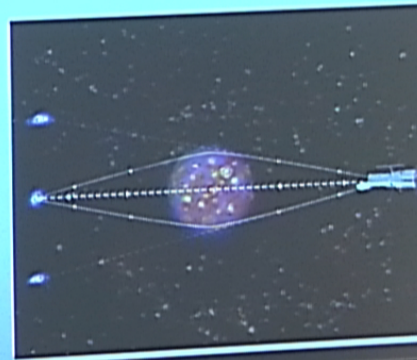
~1-2% stars, ~5-15% gas, remaining is dark matter

## Coma galaxy cluster



# GRAVITATIONAL LENSING

- Image distortion caused by intervening gravitational potential
- Sensitive to total mass



$$\langle j_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle j_i \rangle^*$$

$$\langle j_i \rangle = 2k_i \cdot k_j$$

$$-\frac{k}{k^0}$$

symmetry  $\langle j_i \rangle = -\langle j_i \rangle$ ,  $[j_i] = -[j_i]$

$$\langle i_i \rangle = 0 = [i_i]$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle j_i | j_j \rangle = \langle j_i | \mu | j_j \rangle = 2k_j^i$$

conjugation  $\langle i | \mu | j \rangle = [i | \mu | j]$

identity  $\langle i | \mu | j \rangle \langle j | \mu | i \rangle = 2 \langle i | g \rangle [j | j]$

$$\sigma_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Exercise show that  $A \in \mathbb{R}$

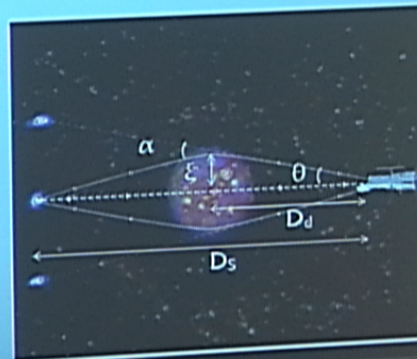
$$e^{-i\phi k} = \frac{k^0 \pm k^c}{\sqrt{k \cdot k}}$$

$$\lambda_{\pm} = \begin{pmatrix} -e^{\pm i\phi} \sqrt{k \cdot k} \\ \sqrt{k \cdot k} \end{pmatrix}, \quad \tilde{\lambda}_{\pm} = \begin{pmatrix} -e^{\mp i\phi} \sqrt{k \cdot k} \\ \sqrt{k \cdot k} \end{pmatrix}$$

# GRAVITATIONAL LENSING

- Image distortion caused by intervening gravitational potential
- Sensitive to total mass
- From general relativity:

$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$



$$\langle i, j \rangle = \text{sign}(k_i^0 k_j^0) \langle j, i \rangle^*$$

$$\langle j, i \rangle = 2k_i \cdot k_j$$

$$-\frac{k}{k^0}$$

symmetry  $\langle i, j \rangle = -\langle j, i \rangle$ ,  $[i, j] = -[j, i]$   
 $\langle i, i \rangle = 0 = [i, i]$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu | j \rangle \equiv \langle j | \mu | j \rangle = 2k_j^{\mu}$$

conjugation  $\langle i | \mu | j \rangle = [i | \mu | j]$

$$\langle i | \mu | j \rangle \langle j | \mu | i \rangle = 2 \langle i | j \rangle [i | j]$$

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Exercise show that  $\lambda \in \mathbb{R}$

$$e^{-i\lambda k} = \frac{k^0 \pm k^z}{\sqrt{E}}$$

$$\lambda_{\pm} = \begin{pmatrix} -c^{\pm 1/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda}_{\pm} = \begin{pmatrix} -c^{\pm 1/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

# GRAVITATIONAL LENSING

- Image distortion caused by intervening gravitational potential
- Sensitive to total mass
- From general relativity:

Lens mass

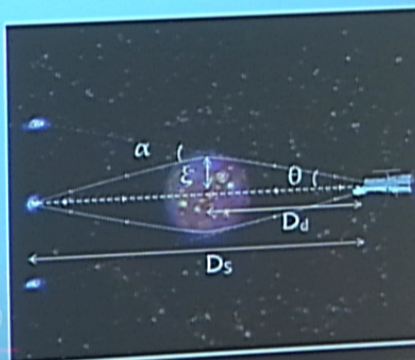
$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$

Impact parameter

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

$\sin(\hat{\alpha}) \approx \tan(\hat{\alpha}) \approx \hat{\alpha}$

- Image separation proportional to sqrt(M)



$$\langle i, j \rangle = \text{Sign}(k_i^0 k_j^0) \langle j, i \rangle^*$$

$$\langle j, i \rangle = 2k_i \cdot k_j$$

$$\langle \frac{k}{k^0} \rangle$$

symmetry  $\langle i, j \rangle = -\langle j, i \rangle$ ,  $[i, j] = -[j, i]$

$$\langle i, i \rangle = 0 = [i, i]$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu | j \rangle \equiv \langle j | \mu | j \rangle = 2k_j^{\mu}$$

conjugation  $\langle i | \mu | j \rangle = [i | \mu | j]$

identity  $\langle i | \mu | j \rangle \langle j | \mu | i \rangle = 2 \langle i, j \rangle [i, j]$

# GRAVITATIONAL LENSING

- Image distortion caused by intervening gravitational potential
- Sensitive to total mass
- From general relativity:

Lens mass

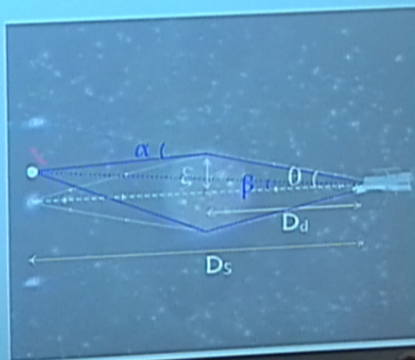
$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$

Impact parameter

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

$\sin(\hat{\alpha}) \approx \tan(\hat{\alpha}) \approx \hat{\alpha}$

$$\theta - \beta = \frac{\theta_E^2}{\theta}$$



$$\langle \epsilon_j \rangle = \text{sign}(k_i^0 k_j^0) \langle \epsilon_i \rangle^*$$

$$\langle \epsilon_i \rangle = 2k_i \cdot k_j$$

$$\langle \frac{k}{k_n} \rangle$$

symmetry  $\langle \epsilon_j \rangle = -\langle \epsilon_i \rangle$ ,  $\langle \epsilon_i \rangle = -\langle \epsilon_j \rangle$

$\langle \epsilon_i \rangle = 0 = \langle \epsilon_i \rangle$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \epsilon_i | j \rangle \equiv \langle \epsilon_j | \mu | j \rangle = 2k_j^{\mu}$$

conjugation  $\langle \epsilon_i | \mu | j \rangle = \langle \epsilon_j | \mu | i \rangle$

$$\langle \epsilon_i | \mu | j \rangle \langle \epsilon_j | \mu | i \rangle = 2 \langle \epsilon_i | \epsilon_j \rangle \langle \epsilon_i | \epsilon_j \rangle$$

$$\epsilon_i = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Exercise show that  $A \in \mathbb{R}$

$$e^{-i\phi k} = \frac{k^0 \pm k^c}{\sqrt{k \cdot k}}$$

$$\lambda_{\pm} = \begin{pmatrix} -c^{1/2} \sqrt{k \cdot k} \\ \sqrt{k \cdot k} \end{pmatrix}, \tilde{\lambda}_{\pm} = \begin{pmatrix} -c^{1/2} \sqrt{k \cdot k} \\ \sqrt{k \cdot k} \end{pmatrix}$$

# GRAVITATIONAL LENSING

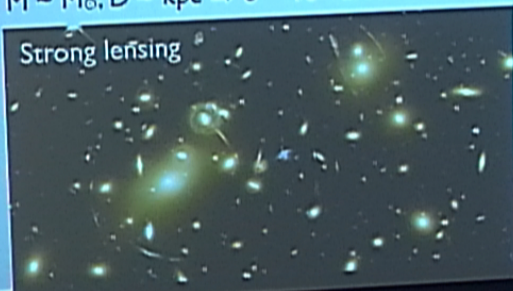
- Strong (large mass densities; multiple images, rings, ...), weak (smaller mass densities; distortion observed with statistical approach), microlensing

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

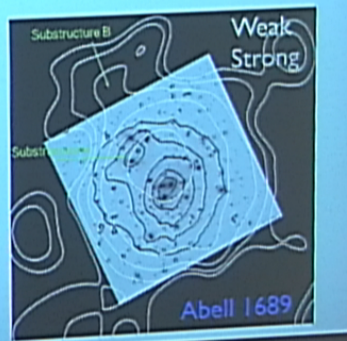
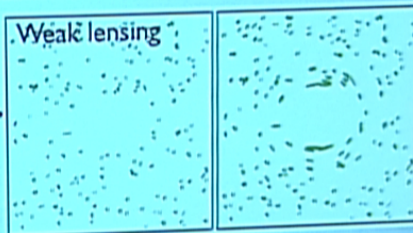
$$M \sim 10^{15} M_\odot, D \sim \text{Gpc} \Rightarrow \theta \sim 100 \text{ arcsec}$$

$$M \sim M_\odot, D \sim \text{kpc} \Rightarrow \theta \sim 10^{-3} \text{ arcsec}$$

Strong lensing



Weak lensing



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k^2} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle, \langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu_j \rangle = \langle v_j | \mu_j \rangle = 2 k_j^2$$

$$\langle \mu_j \rangle = \langle v_j | \mu_j \rangle$$

$$\langle \mu_j \rangle = 2 \langle v_j \rangle \langle v_j \rangle$$

$$\langle v_j \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$e^{-i\mu_j} = \frac{k^0 \pm k^z}{\sqrt{k^2}}$$

$$\lambda = \begin{pmatrix} -e^{-i\mu_j} \sqrt{k^2} \\ \sqrt{k^2} \end{pmatrix}, \tilde{\lambda} = \begin{pmatrix} -e^{i\mu_j} \sqrt{k^2} \\ \sqrt{k^2} \end{pmatrix}$$

Exercise show that  $\lambda \in \mathbb{R}$



# GRAVITATIONAL LENSING

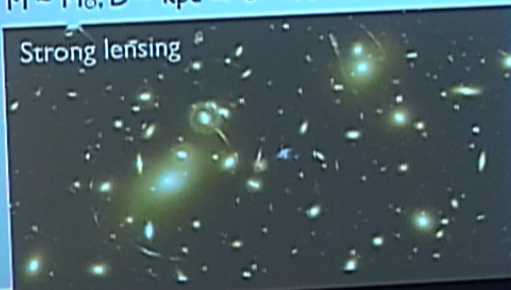
- Strong (large mass densities; multiple images, rings, ...), weak (smaller mass densities; distortion observed with statistical approach), microlensing

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

$$M \sim 10^{15} M_\odot, D \sim \text{Gpc} \Rightarrow \theta \sim 100 \text{ arcsec}$$

$$M \sim M_\odot, D \sim \text{kpc} \Rightarrow \theta \sim 10^{-3} \text{ arcsec}$$

Strong lensing



Weak lensing



Abell 1689

$$\langle \epsilon_{ij} \rangle = \text{sign}(k_i^0 k_j^0) \langle \epsilon_{ij} \rangle^*$$

$$\langle \epsilon_{ij} \rangle = 2 k_i \cdot k_j$$

$$\frac{k}{|k|}$$

symmetry  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle, \langle \epsilon_{ii} \rangle = -\langle \epsilon_{jj} \rangle$

$$\langle \epsilon_{ii} \rangle = 0 = \langle \epsilon_{jj} \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions to

trace momenta "Gordon identity"

$$\langle \epsilon_{ij} | \mu | \epsilon_{ij} \rangle = 2 k_j^i k_i^j$$

conjugation  $\langle \epsilon_{ij} | \mu | \epsilon_{ij} \rangle = \langle \epsilon_{ij} | \mu | \epsilon_{ij} \rangle^*$

trity  $\langle \epsilon_{ij} | \mu | \epsilon_{ij} \rangle = 2 \langle \epsilon_{ij} | \mu | \epsilon_{ij} \rangle$

$$e^{-i\mu \cdot x} = \frac{k^0 \pm k^c}{\sqrt{k_L}}$$

$$\lambda_\pm = \begin{pmatrix} -e^{-i\mu \cdot x} \sqrt{k_L} \\ \sqrt{k_L} \end{pmatrix}, \tilde{\lambda}_\pm = \begin{pmatrix} -e^{i\mu \cdot x} \sqrt{k_L} \\ \sqrt{k_L} \end{pmatrix}$$

Exercise show that  $\lambda \in \mathbb{R}$

# GRAVITATIONAL LENSING

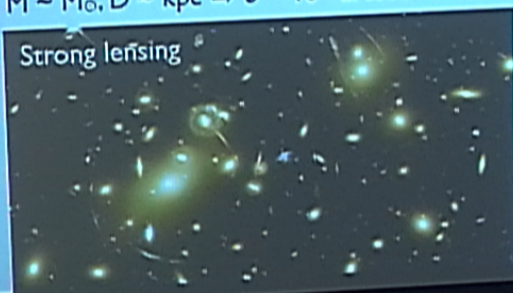
- Strong (large mass densities; multiple images, rings, ...), weak (smaller mass densities; distortion observed with statistical approach), microlensing

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

$$M \sim 10^{15} M_\odot, D \sim \text{Gpc} \Rightarrow \theta \sim 100 \text{ arcsec}$$

$$M \sim M_\odot, D \sim \text{kpc} \Rightarrow \theta \sim 10^{-3} \text{ arcsec}$$

Strong lensing



Weak lensing



$$\langle \epsilon_{ij} \rangle = \text{Sign}(k_i^0 k_j^0) \langle \epsilon_{ij} \rangle^*$$

$$\langle \epsilon_{ij} \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{2k}$$

symmetry  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$ ,  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$

$$\langle \epsilon_{ii} \rangle = 0 = \langle \epsilon_{jj} \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

inert moments "Gordon identity"

$$\langle \epsilon_{ij} \rangle = \langle \epsilon_{ij} | \mu_{ij} \rangle = 2 k_j^i$$

conjugation  $\langle \epsilon_{ij} | \mu_{ij} \rangle = \langle \epsilon_{ij} | \mu_{ji} \rangle$

identity  $\langle \epsilon_{ij} | \mu_{ij} \rangle \langle \epsilon_{kl} | \mu_{kl} \rangle = 2 \langle \epsilon_{ij} \rangle \langle \epsilon_{kl} \rangle$

Exercise show that  $A \in \mathbb{R}$

$$\lambda = \frac{k^0 \pm k^z}{\sqrt{k_x^2 + k_y^2}}$$

$$\tilde{\lambda} = \frac{-i k_x \pm k_y}{\sqrt{k_x^2 + k_y^2}}$$

# GRAVITATIONAL LENSING

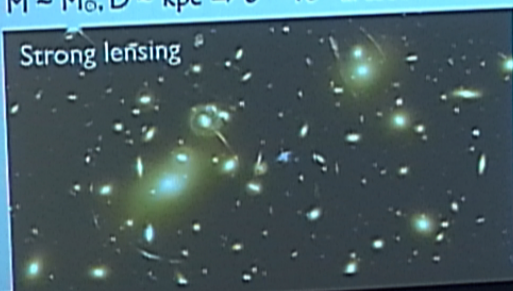
- Strong (large mass densities; multiple images, rings, ...), weak (smaller mass densities; distortion observed with statistical approach), microlensing

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

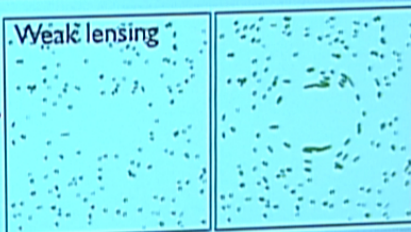
$$M \sim 10^{15} M_\odot, D \sim \text{Gpc} \Rightarrow \theta \sim 100 \text{ arcsec}$$

$$M \sim M_\odot, D \sim \text{kpc} \Rightarrow \theta \sim 10^{-3} \text{ arcsec}$$

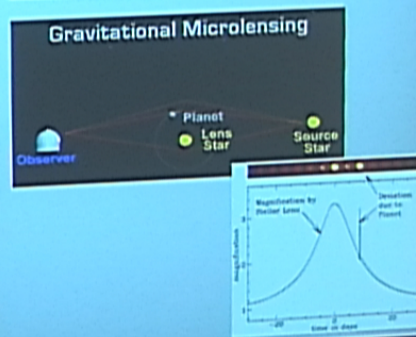
Strong lensing



Weak lensing



Gravitational Microlensing



$$\langle \epsilon_{ij} \rangle = \text{Sign}(k_i^0 k_j^0) \langle \epsilon_{ij} \rangle^*$$

$$\langle \epsilon_{ij} \rangle = 2 k_i \cdot k_j$$

$$k_i$$

symmetry  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$ ,  $\langle \epsilon_{ii} \rangle = -\langle \epsilon_{jj} \rangle$

$$\langle \epsilon_{ii} \rangle = 0 = \langle \epsilon_{jj} \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

inert momenta "Gordon identity"

$$\langle \epsilon_{ij} \rangle = \langle \epsilon_{ij} | \mu_{ij} \rangle = 2 k_i^0 k_j^0$$

$$\langle \epsilon_{ij} \rangle = \langle \epsilon_{ij} | \mu_{ij} \rangle$$

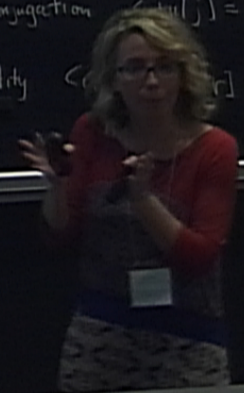
$$\langle \epsilon_{ij} \rangle = 2 \langle \epsilon_{ij} \rangle \langle \epsilon_{ij} \rangle$$

$$\epsilon_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\epsilon_{ij} = k_i \pm k_j, \quad e^{i\mu_{ij} x} = \frac{k_i \pm k_j}{\sqrt{k_i k_j}}$$

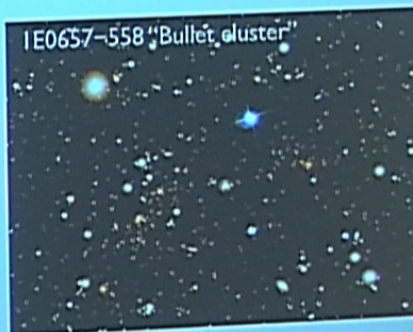
$$\lambda = \begin{pmatrix} -c^2 \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} -c^2 \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise show that  $A \in \mathbb{R}$



# COSMIC SUPERCOLLIDERS

- Systems where the presence of dark matter can be inferred and it is not positionally coincident with ordinary matter strongly endorse the dark matter hypothesis
- Galaxy cluster mergers



$$\langle j \rangle = \text{Sign}(k_i^0 k_j^0) \langle j_i \rangle^*$$

$$\langle j_i \rangle = 2k_i \cdot k_j$$

$$-\frac{k}{k^0}$$

symmetry  $\langle j \rangle = -\langle j_i \rangle$ ,  $[j] = -[j_i]$

$$\langle j_i \rangle = 0 = [j_i]$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$[j_i] \equiv \langle j_i | \mu | j \rangle = 2k_j^i k_j^0$$

conjugation  $\langle j_i | \mu | j \rangle = [j_i | \mu | j]$

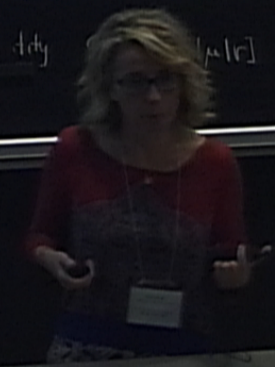
identity  $\langle j_i | \mu | j \rangle = 2 \langle j_i \rangle [j_j]$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$e^{-i\phi k} = \frac{k^0 \pm k^z}{\sqrt{k^0 k^0 - k^z k^z}}$$

$$\lambda = \begin{pmatrix} -e^{i\phi/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} -e^{-i\phi/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise: show that  $\lambda \in \mathbb{R}$



# COSMIC SUPERCOLLIDERS

---

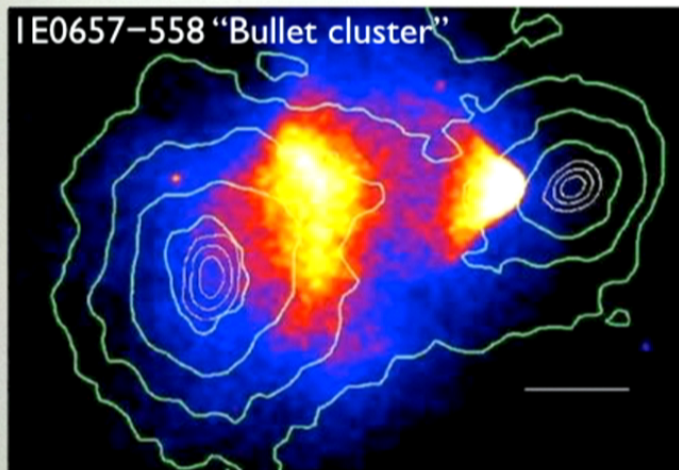
- Systems where the presence of dark matter can be inferred and it is not positionally coincident with ordinary matter strongly endorse the dark matter hypothesis
- Galaxy cluster mergers



# COSMIC SUPERCOLLIDERS

Weak lensing

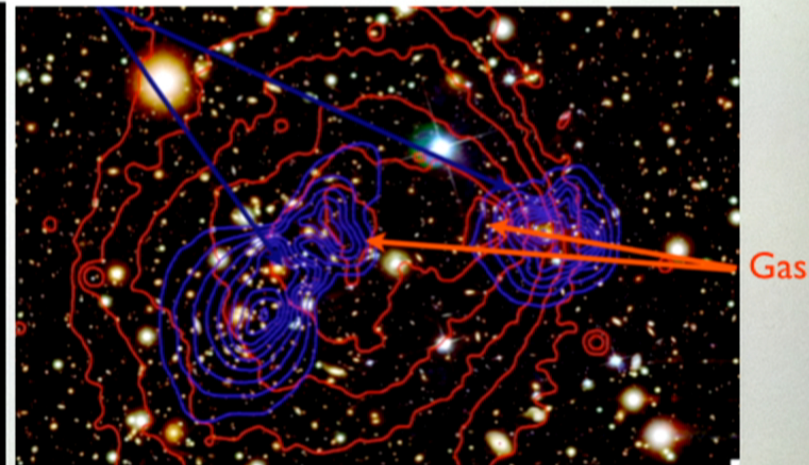
Clowe et al 2006



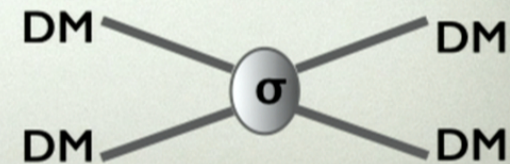
Weak and strong lensing

Bradač et al 2006

Total mass

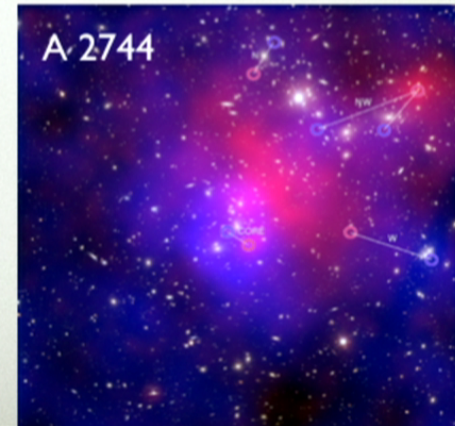


(\*) Constraints on the self-interaction cross section:  
 $\sigma/m < 1.3 \text{ barn/GeV}$  (Randall et al 2008)



# MORE COSMIC SUPERCOLLIDERS

Mahdavi et al 2007



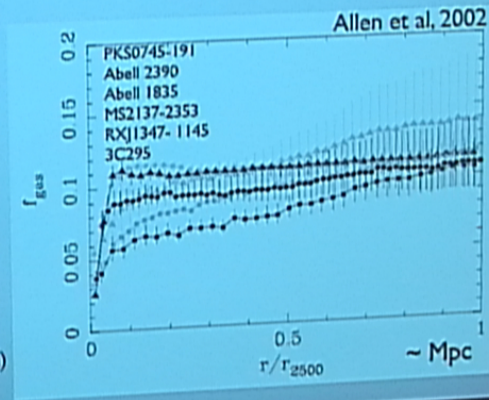
# GALAXY CLUSTERS

- Gas mass and total mass in galaxy clusters measured by X-ray, lensing
- Assume the matter content in galaxy clusters is representative of the Universe  
 $\Rightarrow$  constrain the Universe total matter density!

Constrain matter density:  
 $\Omega_M (\Omega_B \rho_M / \rho_B - \Omega_B / f_{gas}) \sim 0.3$

$$\Omega = \frac{\rho}{\rho_c}$$

$\rho_c$ : Critical energy density of the Universe (flat)



$$\langle \epsilon_{ij} \rangle = \text{Sign}(k_i^0 k_j^0) \langle \epsilon_{ij} \rangle^*$$

$$\langle \epsilon_{ij} \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{k_0}$$

symmetry  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$ ,  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$   
 $\langle \epsilon_{ii} \rangle = 0 = \langle \epsilon_{ii} \rangle$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \epsilon_{ij} | \mu | j \rangle = \langle \epsilon_{ij} | \mu | j \rangle = 2 k_j^i$$

conjugation  $\langle \epsilon | \mu | j \rangle = \langle \epsilon | \mu | i \rangle$

identity  $\langle \epsilon | \mu | i \rangle = 2 \langle \epsilon | \mu | j \rangle$



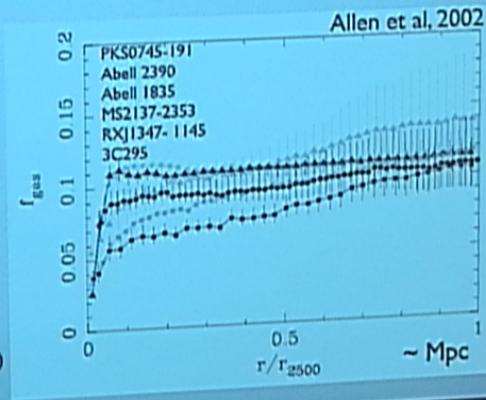
# GALAXY CLUSTERS

- Gas mass and total mass in galaxy clusters measured by X-ray, lensing
- Assume the matter content in galaxy clusters is representative of the Universe  
 $\Rightarrow$  constrain the Universe total matter density!

Constrain matter density:  
 $\Omega_M (\Omega_B \rho_M / \rho_B - \Omega_B / f_{gas}) \sim 0.3$

$$\Omega = \frac{\rho}{\rho_c}$$

$\rho_c$ : Critical energy density of the Universe (flat)



$$\langle \epsilon_{ij} \rangle = \text{sign}(k_i^0 k_j^0) \langle \epsilon_{ij} \rangle^*$$

$$\langle \epsilon_{ij} \rangle = 2 k_i \cdot k_j$$

$$-\frac{k_i}{k_i}$$

symmetry  $\langle \epsilon_{ij} \rangle = -\langle \epsilon_{ji} \rangle$ ,  $\langle \epsilon_{ii} \rangle = -\langle \epsilon_{ii} \rangle$   
 $\langle \epsilon_{ii} \rangle = 0 = \langle \epsilon_{ii} \rangle$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \epsilon_{ij} | \mu_j \rangle \equiv \langle \epsilon_{ij} | \mu_j \rangle = 2 k_j^i$$

conjugation  $\langle \epsilon_{ij} | \mu_j \rangle = \langle \mu_i | \epsilon_{ij} \rangle$

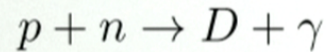
identity  $\langle \epsilon_{ij} | \mu_j \rangle \langle \mu_i | \epsilon_{ij} \rangle = 2 \langle \epsilon_{ij} \rangle \langle \mu_i | \mu_j \rangle$

$\epsilon_{ij} = k_i \otimes k_j$ ,  $e^{-i\mu \cdot x} = \frac{k^0 \pm k^3}{\sqrt{E}} e^{i\mu \cdot x}$  Exercise: show that  $A \in \mathbb{R}$

$$\lambda = \begin{pmatrix} -c^{1/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} c^{1/2} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

# BIG BANG NUCLEOSYNTHESIS

- As the Universe cools down ( $\sim 100$ s sec after Big Bang,  $\sim$  MeV), light elements form (deuterium, helium, lithium). E.g.:



- (Much longer timescales for heavier elements to form, e.g. C, N, O)

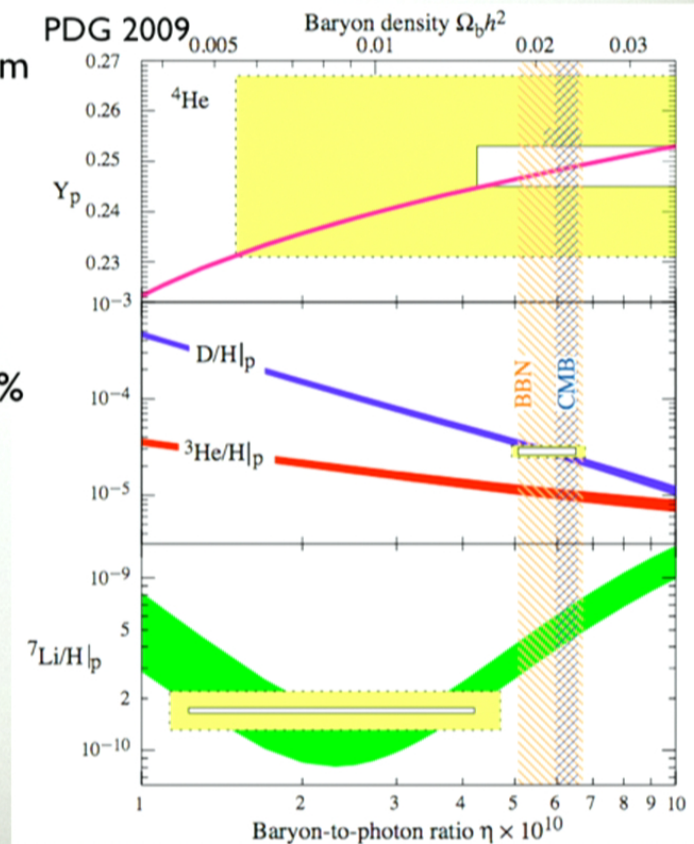
Constrains baryon density:  $\Omega_B \sim$  few %

$$\Omega = \frac{\rho}{\rho_c}$$

$\rho_c$ : Critical energy density of the Universe (flat)

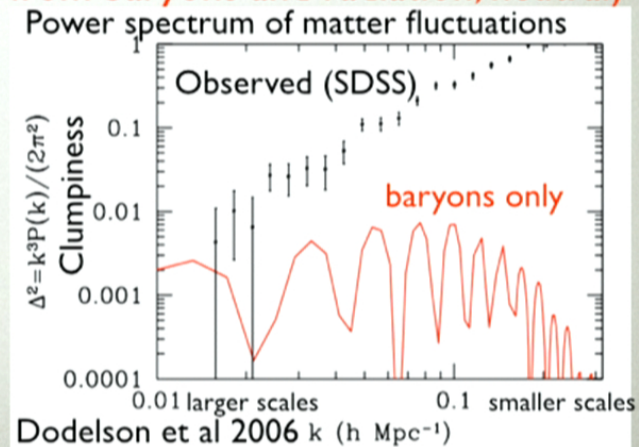
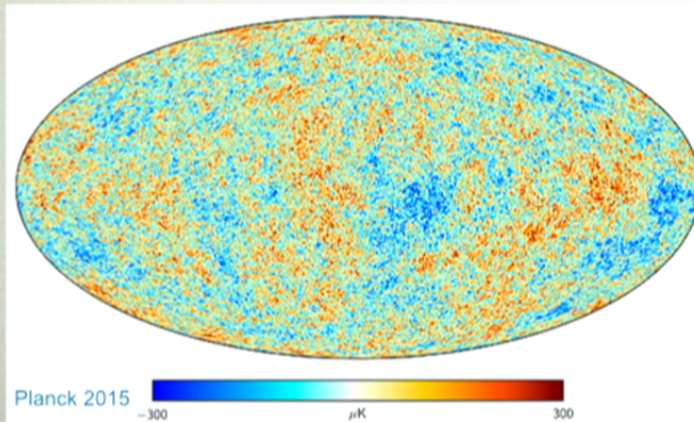
- ➔ Most matter in the Universe is non-baryonic

- Remarkable agreement with CMB estimate of baryon density (more next)



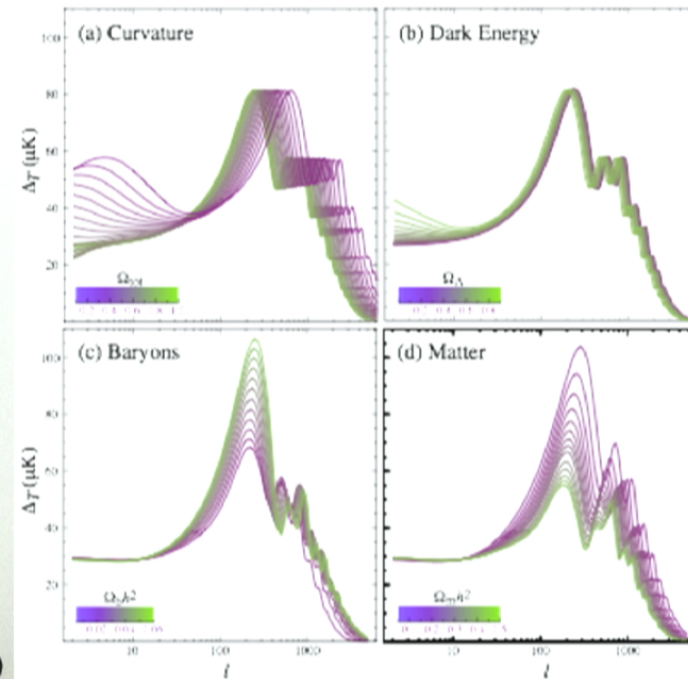
# COSMIC MICROWAVE BACKGROUND

- Relic of a time in the early Universe when matter and radiation decoupled (protons and electron form neutral hydrogen and become transparent to photons,  $\sim 100,000$ s years after Big Bang,  $\sim eV$ )
- Universe was isotropic and homogeneous at large scales
- Very small temperature fluctuations, too small to evolve into structure observed today  $T = 2.725 \text{ K}$   $\rightarrow$  Require additional matter to start forming structure earlier (decoupled from baryons and radiation, neutral)  
 $\Delta T \sim 200 \mu\text{K}$



# COSMIC MICROWAVE BACKGROUND

- The CMB angular power spectrum depends on several parameters, including  $\Omega_B, \Omega_M, \Omega_\Lambda$  ( $\Omega_\Lambda$  is the vacuum density)
- Matching location and heights of the peaks constrains these parameters and geometry of the Universe (flat,  $\Omega_{\text{total}}=1$ )



Hu et al (2002)

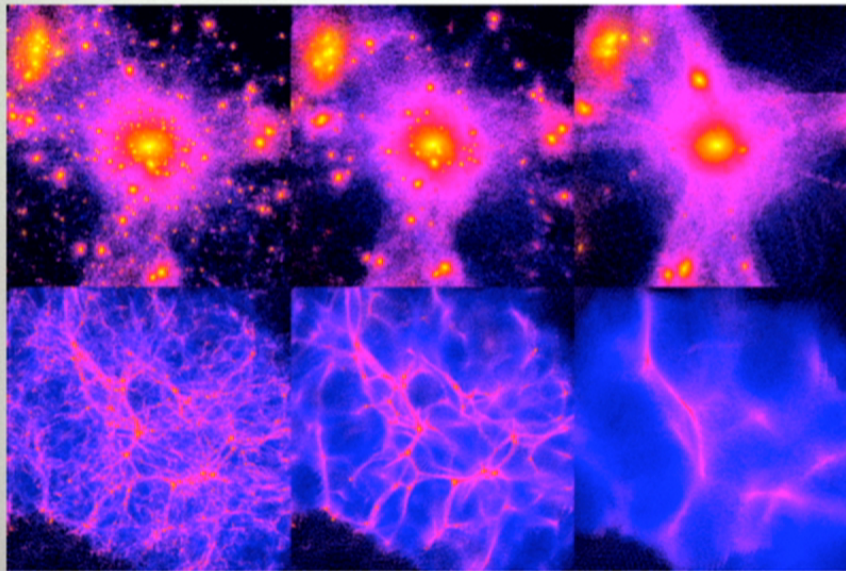
# CDM

- CDM (Cold Dark Matter), i.e. non relativistic, consistent with observations
- Hot dark matter excluded (smooths out structure)

COLD

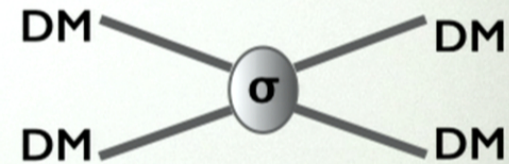
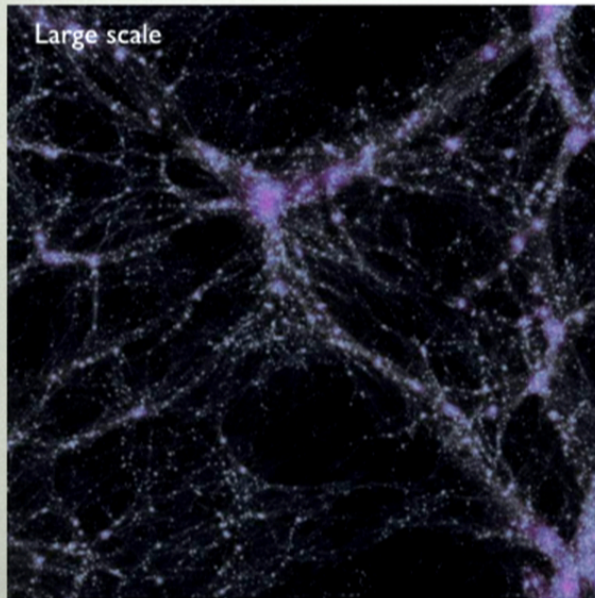
WARM

HOT

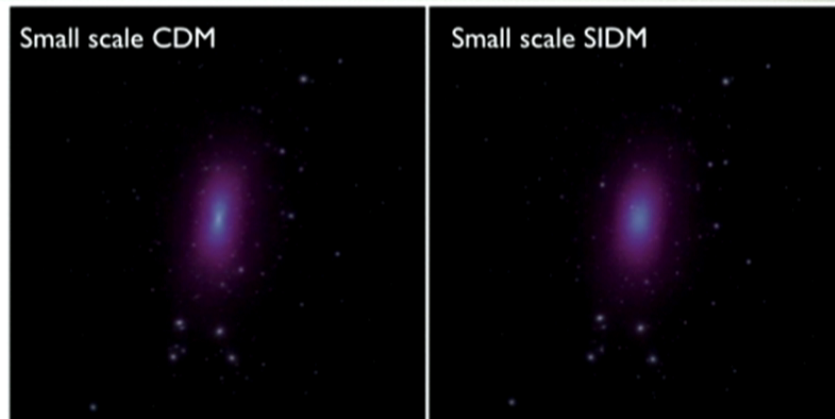


# CDM

- CDM (Cold Dark Matter), i.e. non relativistic, consistent with observations
- Hot dark matter excluded (smooths out structure)
- Self-interactions would also smooth out dense DM regions, though wouldn't significantly affect large scale structure; consistent with observation

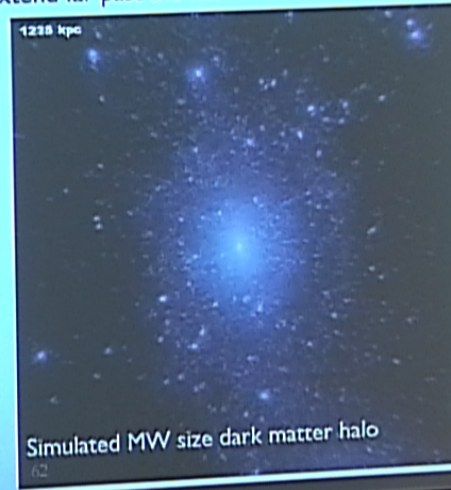
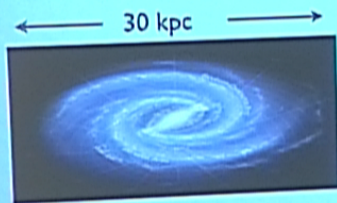


Rocha et al. 2012



# DARK MATTER DISTRIBUTION

- Milky Way galaxy stellar disk: approx. 30 kpc diameter and 300 pc thick
- The dark matter halo is predicted to extend far past the luminous matter



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2k_i \cdot k_j$$

$$\frac{k}{2k}$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i | v_j \rangle = \langle v_j | v_i \rangle = 2k_j^i$$

conjugation  $\langle v_i | v_j \rangle = \langle v_j | v_i \rangle^*$

identity  $\langle v_i | v_j \rangle \langle v_j | v_i \rangle = 2 \langle v_i | v_j \rangle \langle v_j | v_i \rangle$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

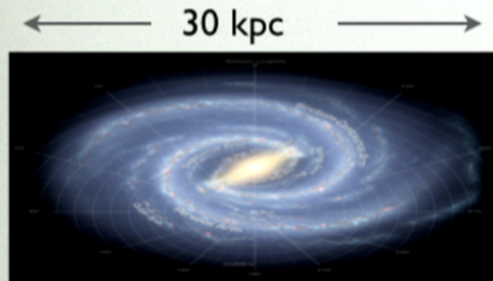
$$e^{-i\phi k} = \frac{k^0 \pm i k^c}{\sqrt{E_k}}$$

$$\lambda_k = \begin{pmatrix} -e^{-i\phi k} \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}, \tilde{\lambda}_k = \begin{pmatrix} -e^{-i\phi k} \sqrt{E_k} \\ \sqrt{E_k} \end{pmatrix}$$

Exercise: show that  $A \in \mathbb{R}$

# DARK MATTER DISTRIBUTION

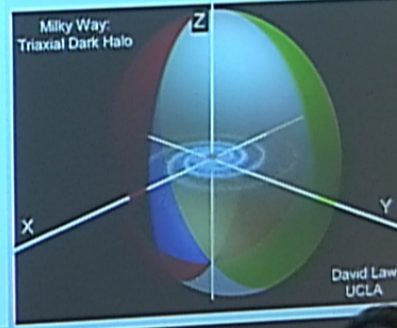
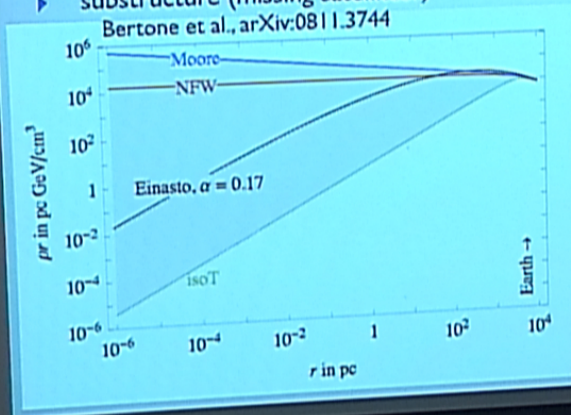
- Milky Way galaxy stellar disk: approx. 30 kpc diameter and 300 pc thick
- The dark matter halo is predicted to extend far past the luminous matter





# DARK MATTER DISTRIBUTION

- Strong predictions from  $\Lambda$ CDM on how DM is distributed
- ...but much is still unknown (affects DM indirect searches!), e.g.:
  - core-cusp profile
  - halo shape (spherical, prolate, oblate, triaxial, dark disk, ...)
  - substructure (missing satellites?)



$$\langle v_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{2k}$$

symmetry  $\langle v_i \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_i \rangle$

$$\langle v_i \rangle = 0 = \langle v_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_i^i k_j^j$$

conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

identity  $\langle v_i | \mu | v_j \rangle \langle v_j | \mu | v_i \rangle = 2 \langle v_i v_j \rangle \langle v_i v_j \rangle$

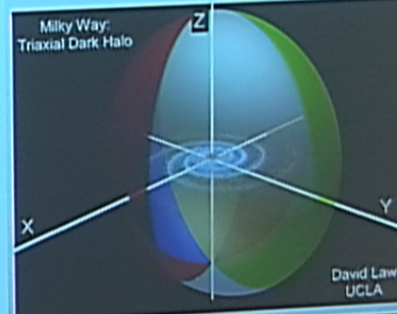
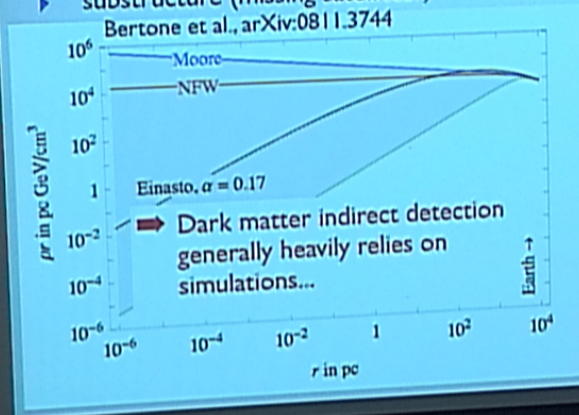
$$E = k \pm k^0, \quad e^{-i\mu k} = \frac{k^0 \pm k^0}{\sqrt{E} E}$$

$$\lambda_k = \begin{pmatrix} -0^0 \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda}_k = \begin{pmatrix} -0^0 \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise - show that  $A \in \mathbb{R}$

# DARK MATTER DISTRIBUTION

- Strong predictions from  $\Lambda$ CDM on how DM is distributed
- ...but much is still unknown (affects DM indirect searches!), e.g.:
  - core-cusp profile
  - halo shape (spherical, prolate, oblate, triaxial, dark disk, ...)
  - substructure (missing satellites?)



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k^0} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_j \rangle$ ,  $\langle v_i \rangle = -\langle v_i \rangle$

$$\langle v_i \rangle = 0 = \langle v_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions to

but momenta "Gordon identity"

$$\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_i^0 k_j^0$$

conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

diry  $\langle v_i | \mu | v_j \rangle \langle v_j | \mu | v_i \rangle = 2 \langle v_i^0 v_j^0 \rangle [v_i v_j]$

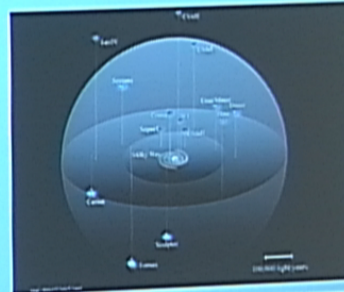
$$E^2 = k^2 + k^0^2, \quad e^{-i\mu L} = \frac{k^0 + i k^3}{\sqrt{E}}$$

$$\lambda = \begin{pmatrix} -i\sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} i\sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise show that  $A \in \mathbb{R}$

# DM SUBSTRUCTURES

- Optically observed dwarf spheroidal galaxies (dSph): largest clumps predicted by N-body simulation.
- Very large M/L ratio: 10 to  $\sim 1000$  (M/L  $\sim 10$  for Milky Way)
- DM density inferred from the stellar data!
- Excellent targets for indirect DM searches!
- Also, never before observed DM substructures:
  - Would significantly shine only in radiation produced by DM annihilation/decay
  - But we don't know where they are!



64

$$\langle v_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k} \rangle$$

symmetry  $\langle v_i \rangle = - \langle v_i \rangle$ ,  $\langle v_i \rangle = - \langle v_i \rangle$

$$\langle v_i \rangle = 0 = \langle v_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i | v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_j^i$$

conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

identity  $\langle v_i | \mu | v_i \rangle = 2 \langle v_i \rangle \langle v_i \rangle$

$$\langle v_i \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e^{-i v \cdot k} = \frac{k^0 \pm k^z}{\sqrt{E_k}}$$

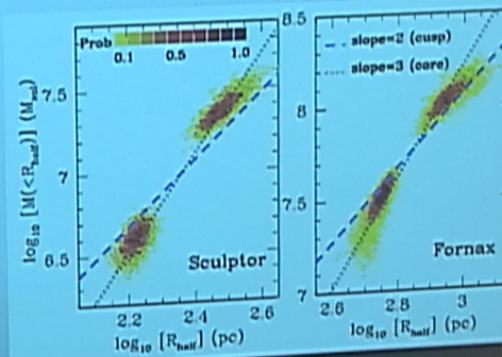
$$\lambda_\pm = \begin{pmatrix} -e^{-i v \cdot k} \\ \sqrt{E_k} \end{pmatrix}, \tilde{\lambda}_\pm = \begin{pmatrix} e^{-i v \cdot k} \\ \sqrt{E_k} \end{pmatrix}$$

Exercise show that  $\lambda \in \mathbb{R}$

# DM SUBSTRUCTURES

- Probing stellar populations with different metallicity in dwarf spheroidal galaxies allows measurements of mass enclosed within two different radii
  - ⇒ Can measure slope of mass profile!
- For Sculptor and Fornax, consistent with cored profile for inner ~100pc. Rule out NFW at CL >95%
- Baryonic feedback?

Walker & Penarrubia 2011



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k_n} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$   
 $\langle v_i \rangle = 0 = \langle v_j \rangle$   
 Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for  
 bruct momenta "Gordon identity"  
 $\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_i^* k_j^*$   
 conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$   
 dirac  $\langle v_i | \mu | v_i \rangle = 2 \langle v_i \rangle \langle v_j \rangle$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

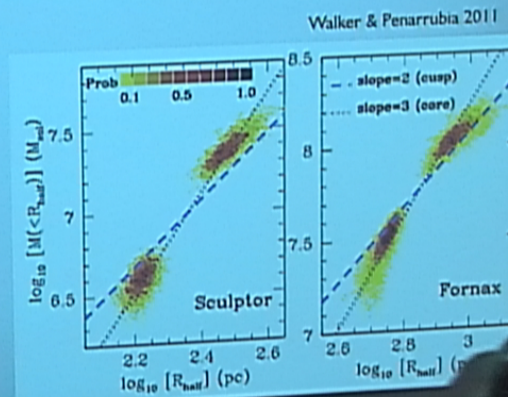
$$e^{-i\vec{k} \cdot \vec{r}} = \frac{k^0 + i k^3}{\sqrt{k^0^2 + k^3^2}}$$

$$\lambda = \begin{pmatrix} -i k^0 \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} i k^0 \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$



# DM SUBSTRUCTURES

- Probing stellar populations with different metallicity in dwarf spheroidal galaxies allows measurements of mass enclosed within two different radii
  - ⇒ Can measure slope of mass profile!
- For Sculptor and Fornax, consistent with cored profile for inner ~100pc. Rule out NFW at CL >95%
- Baryonic feedback?



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{2k}$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions to

inert momenta "Gordon identity"

$$\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_i^* k_j^*$$

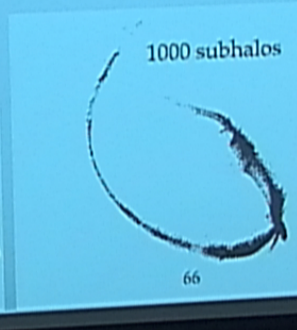
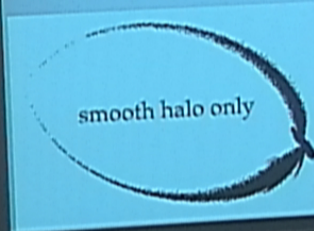
conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

diry  $\langle v_i | \mu | v_j \rangle \langle v_j | \mu | v_i \rangle = 2 \langle v_i \rangle \langle v_j \rangle$

# TESTING DM SUBSTRUCTURES

- Tidal streams cannot remain smooth in CDM
- Are observed streams smooth or have structure? Carlberg et al, arXiv:1102.3501

Simulated star stream



$$\langle v_i \rangle = \text{sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i \rangle \langle v_j \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{k_n}$$

symmetry  $\langle v_i \rangle = -\langle v_j \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i \rangle \langle v_j \rangle = \langle v_i \mu_j \rangle = 2 k_j^i$$

conjugation  $\langle v_i \mu_j \rangle = \langle v_j \mu_i \rangle$

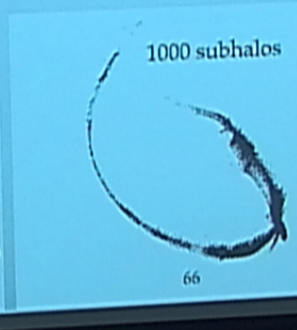
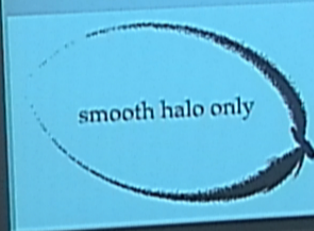
$$\langle v_i \mu_j \rangle \langle v_k \mu_l \rangle = 2 \langle v_i v_k \rangle \langle v_j v_l \rangle$$

$$\lambda = \begin{pmatrix} -v^i v^j / \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} v^i v^j / \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

# TESTING DM SUBSTRUCTURES

- Tidal streams cannot remain smooth in CDM
- Are observed streams smooth or have structure? Carlberg et al, arXiv:1102.3501

Simulated star stream



$$\langle \psi_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle \psi_i \rangle^*$$

$$\langle \psi_i \psi_j \rangle = 2 k_i \cdot k_j$$

$$-\frac{k_i}{k_i}$$

symmetry  $\langle \psi_j \rangle = -\langle \psi_i \rangle$ ,  $\langle \psi_i \rangle = -\langle \psi_j \rangle$

$$\langle \psi_i \rangle = 0 = \langle \psi_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \psi_i \psi_j \rangle \equiv \langle \psi_i | \mu | \psi_j \rangle = 2 k_i \cdot k_j$$

conjugation  $\langle \psi_i | \mu | \psi_j \rangle = \langle \psi_j | \mu | \psi_i \rangle$

identity  $\langle \psi_i | \mu | \psi_j \rangle \langle \psi_j | \mu | \psi_i \rangle = 2 \langle \psi_i \rangle \langle \psi_j \rangle$

$$k = k \pm k^*$$

$$e^{-i\psi} = \frac{k^0 \pm i k^1}{\sqrt{E}}$$

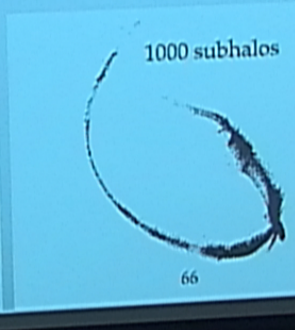
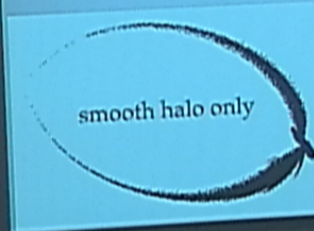
Exercise show that  $A \in \mathbb{R}$

$$\lambda = \begin{pmatrix} -\sigma^{12} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} \sigma^{12} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

# TESTING DM SUBSTRUCTURES

- Tidal streams cannot remain smooth in CDM
- Are observed streams smooth or have structure? Carlberg et al, arXiv:1102.3501

Simulated star stream



$$\langle \rho \rangle = \text{Sign}(k_i k_j) \langle \rho \rangle^*$$

$$\langle \rho \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{k}$$

symmetry  $\langle \rho \rangle = -\langle \rho \rangle$ ,  $\langle \rho \rangle = -\langle \rho \rangle$

$$\langle \rho \rangle = 0 = \langle \rho \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \rho \rangle = \langle \rho \rangle = 2 k_j^*$$

conjugation  $\langle \rho \rangle = \langle \rho \rangle$

$$\langle \rho \rangle \langle \rho \rangle = 2 \langle \rho \rangle \langle \rho \rangle$$



# TESTING DM SUBSTRUCTURES

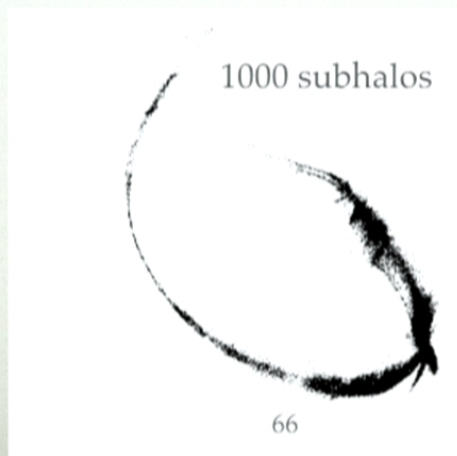
- Tidal streams cannot remain smooth in CDM
- Are observed streams smooth or have structure?

Measurements seem to be consistent with structure/gaps!

Simulated star stream



1000 subhalos



Star stream north-west of M31 (Andromeda)

Carlberg et al, arXiv:1102.3501

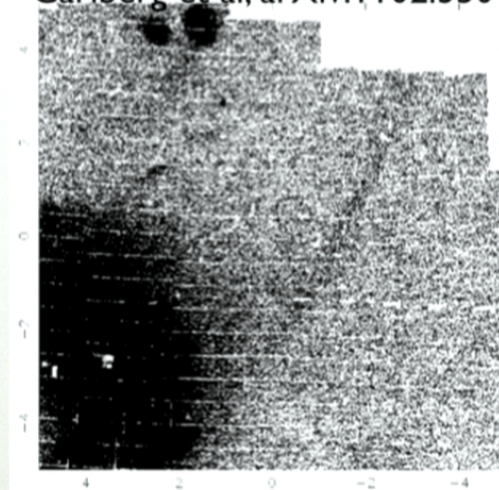


FIG. 1.— The spatial distribution of the  $[\text{Fe}/\text{H}] = [-0.6, -2.4]$  red giant stars in the NW region of M31. A full field version is presented in Richardson et al. (2011). The image is  $10^\circ$  across in the tangent projection co-ordinates, which are centered in the exact middle of this map.

# TESTING DM SUBSTRUCTURES

- Tidal streams cannot remain smooth in CDM
- Are observed streams smooth or have structure?

Pal 5 stream

Carlberg, 2012

Measurements seem to be consistent with structure/gaps!

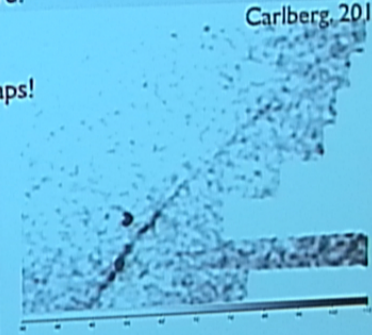


FIG. 1.— The match filtered star densities in the region of the Pal 5 stream in the SDSS  $\lambda$  and  $\eta$  co-ordinate system. The raw image has been smoothed with a 3 pixel Gaussian. The object above the stream is the foreground cluster M5.

67

$$\langle v_i \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k^0} \rangle$$

symmetry  $\langle v_i \rangle = -\langle v_j \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_i^0 k_j^0$$

conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

$$\langle v_i | \mu | v_j \rangle \langle v_j | \mu | v_k \rangle = 2 \langle v_i \rangle \langle v_k \rangle \langle v_j \rangle$$

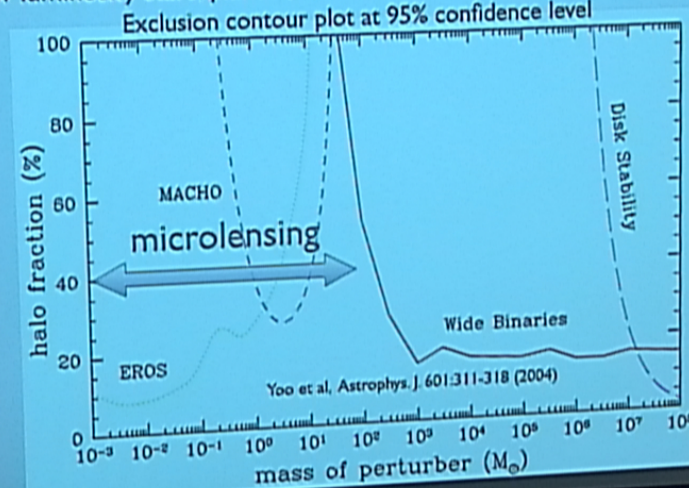
$$\langle v_i \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$v = k \pm k^0, \quad e^{-i v \cdot x} = \frac{k^0 \pm k^0}{\sqrt{E_L}}$$

$$\lambda = \begin{pmatrix} -e^{i v \cdot x} \sqrt{E_L} \\ \sqrt{E_L} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} e^{i v \cdot x} \sqrt{E_L} \\ \sqrt{E_L} \end{pmatrix}$$

# MACHOS

- MACHOs (MASSive Compact Halo Objects) are strongly disfavored as an explanation for dark matter
- E.g. low luminosity stars, planets, black holes



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{|k|} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions to

but momenta "Gordon identity"

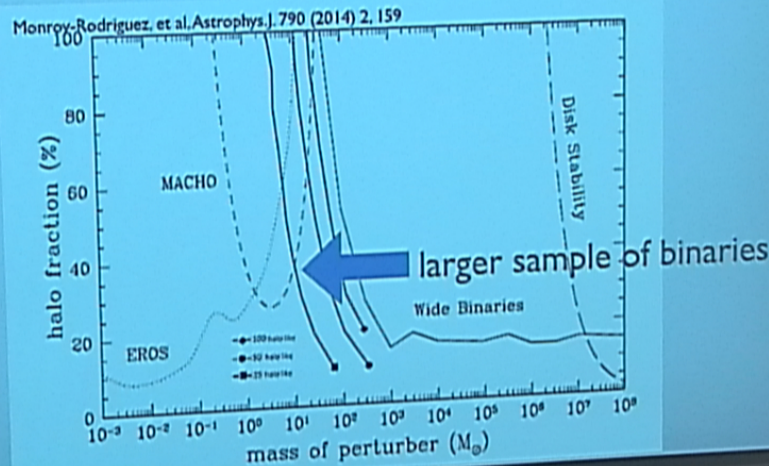
$$\langle \mu_i \mu_j \rangle = \langle v_i v_j \rangle = 2 k_i^i k_j^j$$

conjugation  $\langle \mu_i \mu_j \rangle = \langle v_i v_j \rangle$

trity  $\langle \mu_i \mu_j \rangle = 2 \langle v_i v_j \rangle$

# MACHOS

- MACHOs (MASSive Compact Halo Objects) are strongly disfavored as an explanation for dark matter
- E.g. low luminosity stars, planets, black holes



$$\langle v_j \rangle = \text{Sign}(k_i^0 k_j^0) \langle v_i \rangle^*$$

$$\langle v_i v_j \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k_x} \rangle$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_j \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions to

inert momenta "Gordon identity"

$$\langle v_i v_j \rangle \equiv \langle v_i | \mu | v_j \rangle = 2 k_j^i k_i^j$$

conjugation  $\langle v_i | \mu | v_j \rangle = \langle v_j | \mu | v_i \rangle$

diry  $\langle v_i | \mu | v_j \rangle \langle v_j | \mu | v_i \rangle = 2 \langle v_i v_j \rangle \langle v_j v_i \rangle$

# MOND

- Modified Newtonian Dynamics. Newton's law breaks down for very small accelerations
- Proposed to explain rotation curves of galaxies (Milgrom, 1983). Does a very good job! No dark matter necessary.
- Parameter  $a_0$  ( $1.2 \times 10^{-10} \text{ms}^{-2}$ , determined by observations):

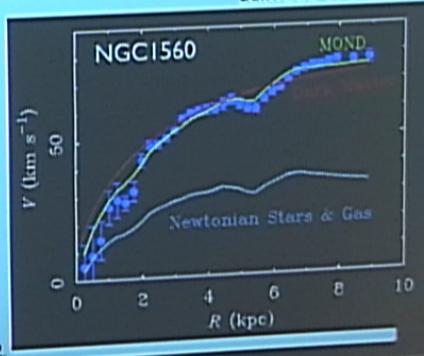
$a \gg a_0$  conventional dynamics  $a = \frac{MG}{r^2}$   
 $a \ll a_0$  modified dynamics  $\frac{a^2}{a_0} = \frac{MG}{r^2}$

Begeman et al 1991  
Sellwood et al 2005

total mass  $\rightarrow$  flat rotation velocity

$$a_0 GM_b = V_f^4$$

- MOND fails at larger scales, galaxy clusters



For a review:  
Sanders and McGaugh, Ann.Rev.Astron.Astrophys.40:263-317,2002

$$\langle v_j \rangle = \text{Sign}(k_i k_j) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2 k_i \cdot k_j$$

$$-\frac{k}{2k}$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle \mu_i \mu_j \rangle = \langle v_j | \mu_i \rangle = 2 k_j^i$$

conjugation  $\langle v_i | \mu_j \rangle = \langle v_j | \mu_i \rangle$

identity  $\langle v_i | \mu_j \rangle \langle v_j | \mu_i \rangle = 2 \langle v_i \rangle \langle v_j \rangle$

$$e^{-i\phi L} = \frac{k^+ \pm k^-}{\sqrt{E_L}}$$

$$\lambda = \begin{pmatrix} -e^{i\phi} \sqrt{E_L} \\ \sqrt{E_L} \end{pmatrix}, \quad \tilde{\lambda} = \begin{pmatrix} -e^{-i\phi} \sqrt{E_L} \\ \sqrt{E_L} \end{pmatrix}$$

Exercise show that  $A \in \mathbb{R}$

# MOND

- Modified Newtonian Dynamics. Newton's law breaks down for very small accelerations
- Proposed to explain rotation curves of galaxies (Milgrom, 1983). Does a very good job! No dark matter necessary.
- Parameter  $a_0$  ( $1.2 \times 10^{-10} \text{ms}^{-2}$ , determined by observations):

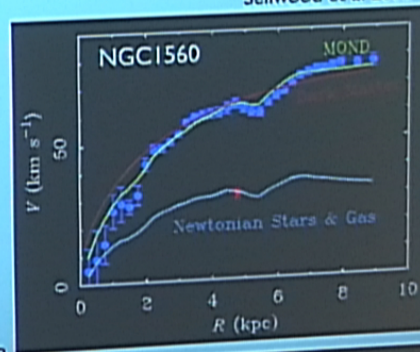
$a \gg a_0$  conventional dynamics  $a = \frac{MG}{r^2}$   
 $a \ll a_0$  modified dynamics  $\frac{a^2}{a_0} = \frac{MG}{r^2}$

Begeman et al 1991  
Sellwood et al 2005

total mass  $\rightarrow$  flat rotation velocity

$$a_0 GM_b = V_f^4$$

- MOND fails at larger scales, galaxy clusters



For a review:  
Sanders and McGaugh, Ann.Rev.Astron.Astrophys.40:263-317,2002

$$\langle v_j \rangle = \text{sign}(k_i k_j) \langle v_i \rangle^*$$

$$\langle v_i \rangle = 2 k_i \cdot k_j$$

$$\frac{k}{k_0}$$

symmetry  $\langle v_j \rangle = -\langle v_i \rangle$ ,  $\langle v_i \rangle = -\langle v_j \rangle$

$$\langle v_i \rangle = 0 = \langle v_i \rangle$$

Exercise show that  $\lambda, \tilde{\lambda}$  are solutions for

bracket momenta "Gordon identity"

$$\langle v_i | j \rangle \equiv \langle v_i | \mu | j \rangle = 2 k_j^i$$

conjugation  $\langle v_i | \mu | j \rangle = \langle v_j | \mu | i \rangle$

identity  $\langle v_i | \mu | i \rangle = 2 \langle v_i | \mu | v_i \rangle$

# SUMMARY

- Evidence for dark matter is overwhelming, e.g.:
  - ▶ Rotation curves
  - ▶ Gravitational lensing
  - ▶ Structure formation
  
- What data tells us about dark matter:
  - ▶ it makes up almost all of the matter in the Universe
  - ▶ it interacts very weakly, and at least gravitationally, with ordinary matter
  - ▶ it is cold, i.e. non-relativistic
  - ▶ it is neutral
  - ▶ it is stable (or it is very long-lived)

➔ But doesn't tell us what it is...

$$\langle j_i \rangle = \text{Sign}(k_i k_j^0) \langle j_i \rangle^*$$

$$\langle j_i \rangle = 2 k_i \cdot k_j$$

$$\langle \frac{k}{k^0} \rangle$$

symmetry  $\langle j_i \rangle = - \langle j_i \rangle$ ,  $\langle j_i \rangle = - \langle j_i \rangle$

$$\langle j_i \rangle = 0 = \langle j_i \rangle$$

Exercise show that  $\lambda, \tilde{\chi}$  are solutions for

bracket momenta "Gordon identity"

$$\langle j_i \rangle \equiv \langle j_i | \mu | j_j \rangle = 2 k_j^i$$

conjugation  $\langle j_i | \mu | j_j \rangle = \langle j_i | \mu | j_j \rangle$

$$\langle j_i | \mu | j_j \rangle \langle j_i | \mu | j_k \rangle = 2 \langle j_i \rangle \langle j_j \rangle$$

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$E = k \pm k^0, \quad e^{\pm i k \cdot x} = \frac{k^0 \pm k^0}{\sqrt{E E}}$$

$$\lambda = \begin{pmatrix} -e^{i k \cdot x} \sqrt{E} \\ \sqrt{E} \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} e^{i k \cdot x} \sqrt{E} \\ \sqrt{E} \end{pmatrix}$$

Exercise show that  $\lambda \in \mathbb{R}$