Title: Cosmology Theory: Large Scale Structure

Date: Jul 10, 2015 04:00 PM

URL: http://pirsa.org/15070040

Abstract:

Standard set up

$$\nabla^2 \Phi_g = \frac{3}{2}\mathcal{H}^2 \Omega_m \delta$$

Poisson equation for Gravity

$$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\delta = -(1+\delta)\boldsymbol{\nabla}\cdot\boldsymbol{v}$$
$$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} = -\mathcal{H}\boldsymbol{v} - \boldsymbol{\nabla}\Phi_g$$

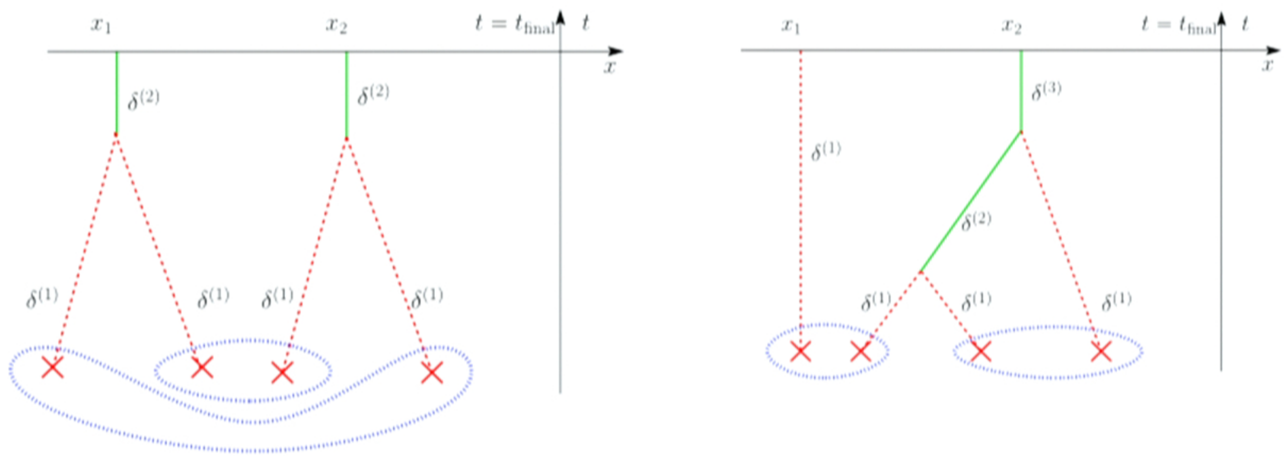
Eulerian Equations for Fluid

+ Stochastic Initial conditions

$$\boldsymbol{x}(\boldsymbol{q},\tau) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau)$$
$$\frac{d^2\boldsymbol{s}}{d\tau^2} + \mathcal{H}\frac{d\boldsymbol{s}}{d\tau} = \nabla\Phi_g(\boldsymbol{q}+\boldsymbol{s},\tau)$$
$$1+\delta(\boldsymbol{x},\tau) = \int d^3q\ \delta^D(\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{s}(\boldsymbol{q},\tau))$$

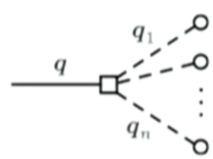
Lagrangian Equations a set of particles

Perturbative results



Compute the effect of modes on different scales and take expectation value over them.
Loops count the number of power spectra of the initial conditions in the answer.
Not necessarily a small parameter, and mixes effects that could be of different sizes. So results are misleading.

$\delta_{\boldsymbol{q}}^{(n)} =$



$= F_n(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_n)\,(2\pi)^3 \delta_D(\boldsymbol{q}_1 + \cdots + \boldsymbol{q}_n - \boldsymbol{q})$

$$P_{mm}(q) = P_{11}(q) + P_{13}(q) + P_{22}(q)$$

Eulerian Perturbation theory

$$\dot{\delta} + \theta = -\theta \star \delta \qquad [\theta \star \delta]_{\boldsymbol{q}} \equiv \int_{\boldsymbol{q}_1} \alpha(\boldsymbol{q}, \boldsymbol{q}_1)\, \theta_{\boldsymbol{q}_1} \delta_{\boldsymbol{q}-\boldsymbol{q}_1}\,, \qquad \alpha(\boldsymbol{q}, \boldsymbol{q}_1) \equiv \frac{\boldsymbol{q} \cdot \boldsymbol{q}_1}{q_1^2}\,,$$

$$\dot{\theta} + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2 \Omega_m \delta = -\theta \star \theta \qquad [\theta \star \theta]_{\boldsymbol{q}} \equiv \int_{\boldsymbol{q}_1} \beta(\boldsymbol{q}, \boldsymbol{q}_1, \boldsymbol{q} - \boldsymbol{q}_1)\, \theta_{\boldsymbol{q}_1} \delta_{\boldsymbol{q}-\boldsymbol{q}_1}\,, \quad \beta(\boldsymbol{q}, \boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{q^2 (\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)}{2 q_1^2 q_2^2}$$

$$\delta(\boldsymbol{x}, \tau) = \sum_{n=1}^{\infty} a^n(\tau)\, \delta^{(n)}(\boldsymbol{x}, \tau_{\rm in}) \qquad \text{and} \qquad \theta(\boldsymbol{x}, \tau) = -\mathcal{H}(\tau) \sum_{n=1}^{\infty} a^n(\tau)\, \theta^{(n)}(\boldsymbol{x}, \tau_{\rm in})$$

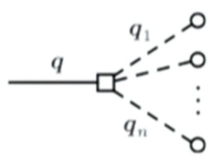
$$\delta^{(n)}_{\boldsymbol{q}}(\tau_{\rm in}) = \int_{\boldsymbol{q}_1} \cdots \int_{\boldsymbol{q}_n} (2\pi)^3 \delta_D(\boldsymbol{q}_1 + \cdots + \boldsymbol{q}_n - \boldsymbol{q})\, F_n(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_n)\, \delta^{(1)}_{\boldsymbol{q}_1}(\tau_{\rm in}) \cdots \delta^{(1)}_{\boldsymbol{q}_n}(\tau_{\rm in})$$

$$\theta^{(n)}_{\boldsymbol{q}}(\tau_{\rm in}) = \int_{\boldsymbol{q}_1} \cdots \int_{\boldsymbol{q}_n} (2\pi)^3 \delta_D(\boldsymbol{q}_1 + \cdots + \boldsymbol{q}_n - \boldsymbol{q})\, G_n(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_n)\, \delta^{(1)}_{\boldsymbol{q}_1}(\tau_{\rm in}) \cdots \delta^{(1)}_{\boldsymbol{q}_n}(\tau_{\rm in})$$

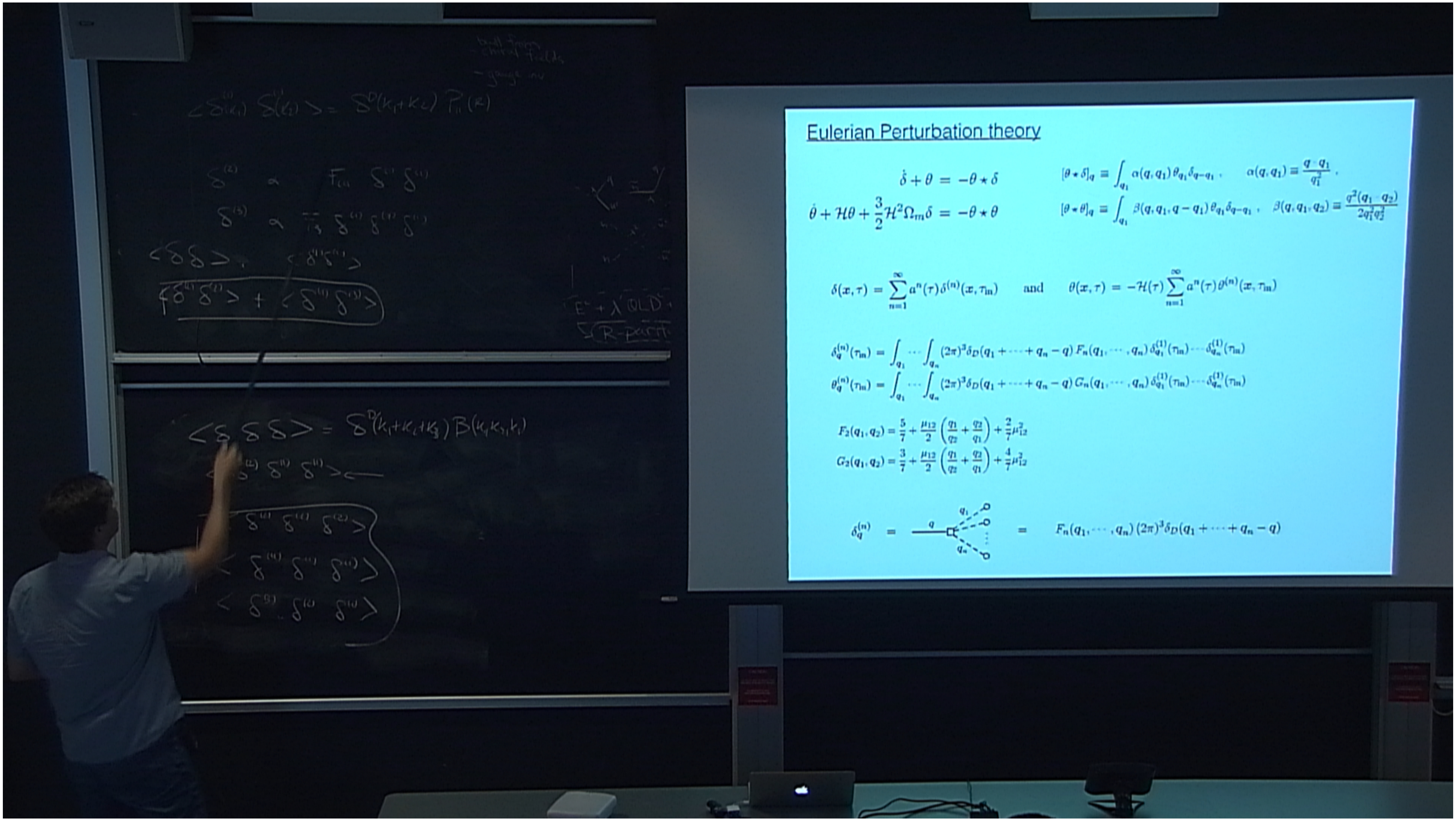
$$F_2(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{5}{7} + \frac{\mu_{12}}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\mu_{12}^2$$

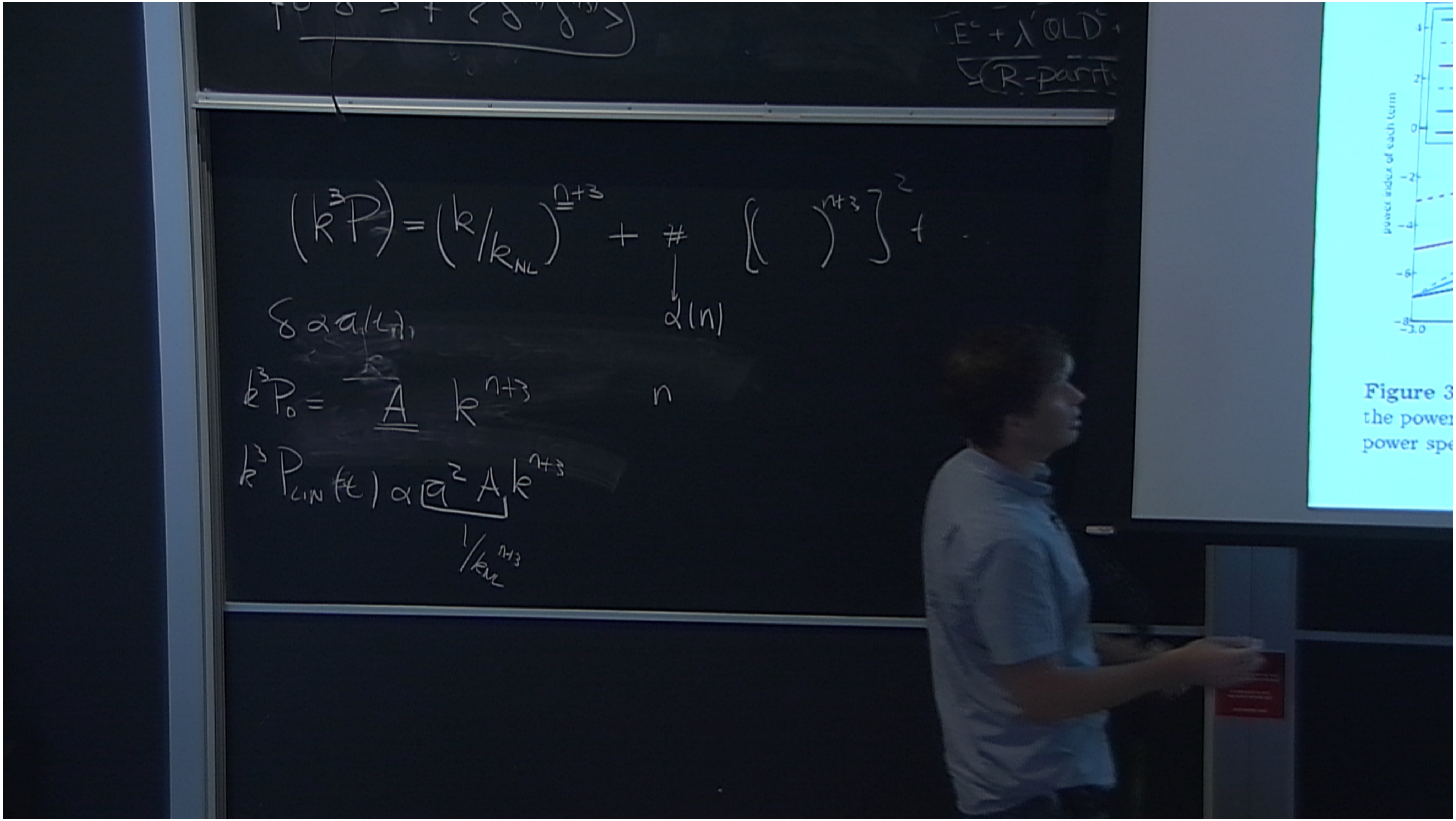
$$G_2(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{3}{7} + \frac{\mu_{12}}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{4}{7}\mu_{12}^2$$

$\delta^{(n)}_{\boldsymbol{q}}$ =



$= F_n(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_n)\, (2\pi)^3 \delta_D(\boldsymbol{q}_1 + \cdots + \boldsymbol{q}_n - \boldsymbol{q})$


Eulerian Perturbation theory
$\dot{\delta} + \theta = -\theta \star \delta$
$\dot{\theta} + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\Omega_m\delta = -\theta \star \theta$
$[\theta \star \delta]_q \equiv \int_{q_1} \alpha(q, q_1)\, \theta_{q_1} \delta_{q-q_1}$, $\alpha(q, q_1) \equiv \frac{q \cdot q_1}{q_1^2}$,
$[\theta \star \theta]_q \equiv \int_{q_1} \beta(q, q_1, q - q_1)\, \theta_{q_1} \delta_{q-q_1}$, $\beta(q, q_1, q_2) \equiv \frac{q^2 (q_1 \cdot q_2)}{2 q_1^2 q_2^2}$
$\delta(x, \tau) = \sum_{n=1}^{\infty} a^n(\tau)\, \delta^{(n)}(x, \tau_{in})$ and $\theta(x, \tau) = -\mathcal{H}(\tau) \sum_{n=1}^{\infty} a^n(\tau)\, \theta^{(n)}(x, \tau_{in})$
$\delta_q^{(n)}(\tau_{in}) = \int_{q_1} \cdots \int_{q_n} (2\pi)^3 \delta_D(q_1 + \cdots + q_n - q)\, F_n(q_1, \cdots, q_n)\, \delta_{q_1}^{(1)}(\tau_{in}) \cdots \delta_{q_n}^{(1)}(\tau_{in})$
$\theta_q^{(n)}(\tau_{in}) = \int_{q_1} \cdots \int_{q_n} (2\pi)^3 \delta_D(q_1 + \cdots + q_n - q)\, G_n(q_1, \cdots, q_n)\, \delta_{q_1}^{(1)}(\tau_{in}) \cdots \delta_{q_n}^{(1)}(\tau_{in})$
$F_2(q_1, q_2) = \frac{5}{7} + \frac{\mu_{12}}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\mu_{12}^2$
$G_2(q_1, q_2) = \frac{3}{7} + \frac{\mu_{12}}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{4}{7}\mu_{12}^2$
$\delta_q^{(n)}$ = $F_n(q_1, \cdots, q_n)\, (2\pi)^3 \delta_D(q_1 + \cdots + q_n - q)$


(k³P) = (k/k_NL)^{n+3} + # [( )^{n+3}]² + ...
α(n)
δ ∝ a(t)
k³P₀ = A k^{n+3}
n
k³P_LIN(t) ∝ a² A k^{n+3}
1/k_NL^{n+3}
R-parit
Figure 3
the power
power spe
power index of each term

Relative size of loop corrections

Simplest estimate is in EDS with power law initial conditions.
Symmetries imply that the answer needs to be a function of k/kNL.
Power spectrum is:

$$\Delta_{\rm linear} = (\frac{k}{k_{NL}})^{n+3} \qquad \Delta_{\rm N-loop} = (\frac{k}{k_{NL}})^{(n+3)(L+1)}$$

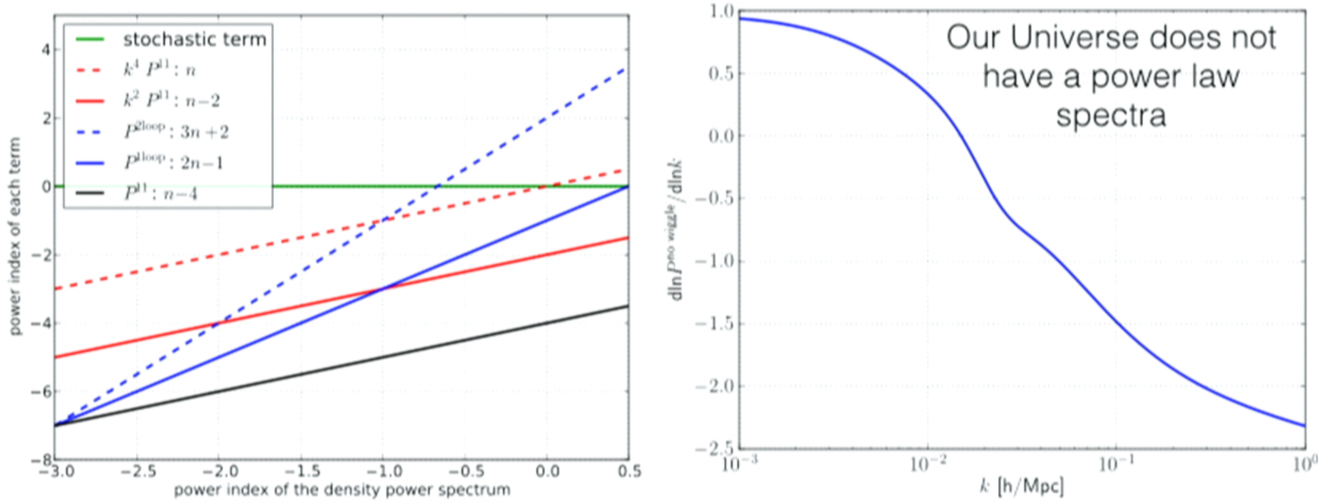
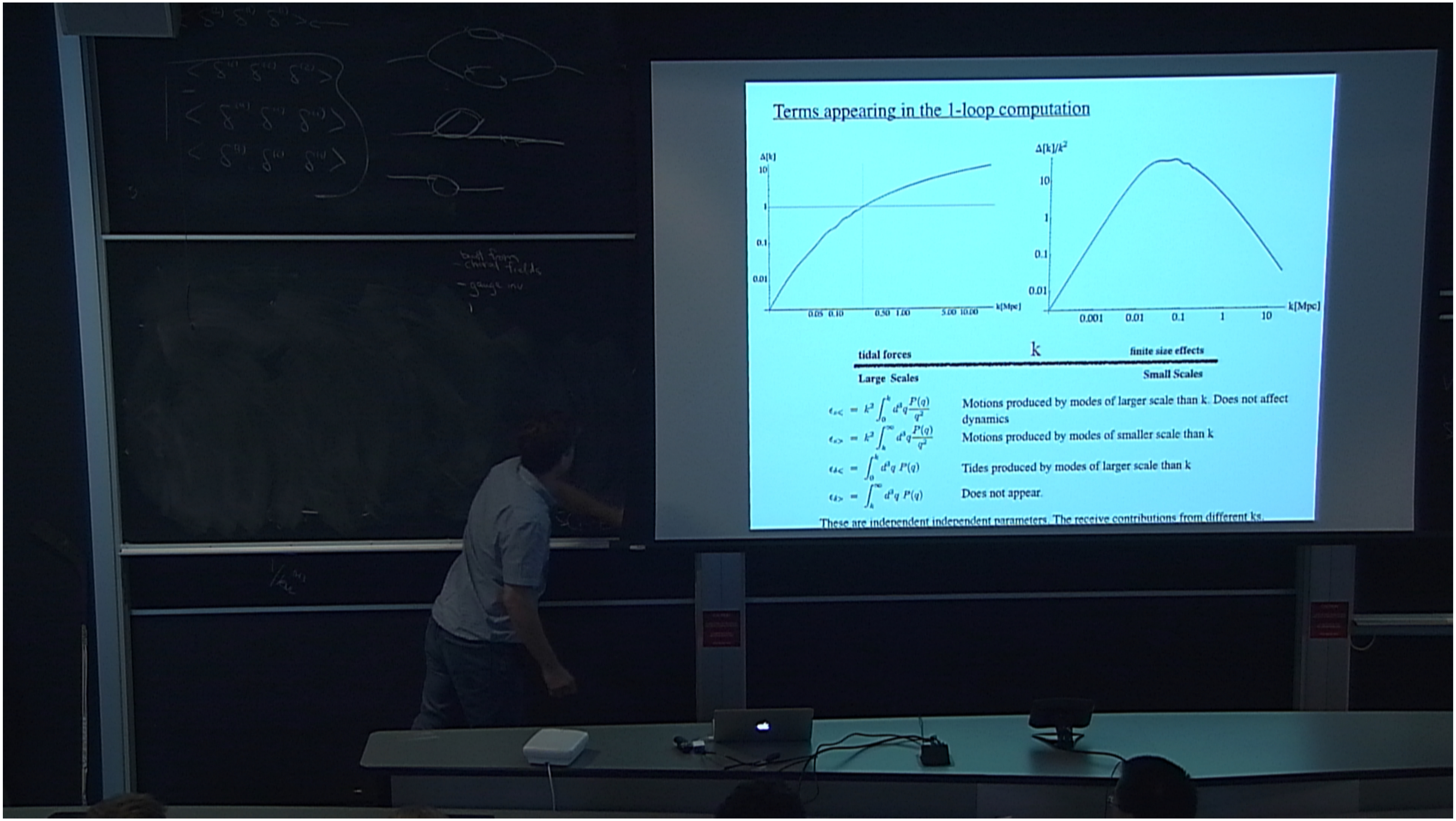
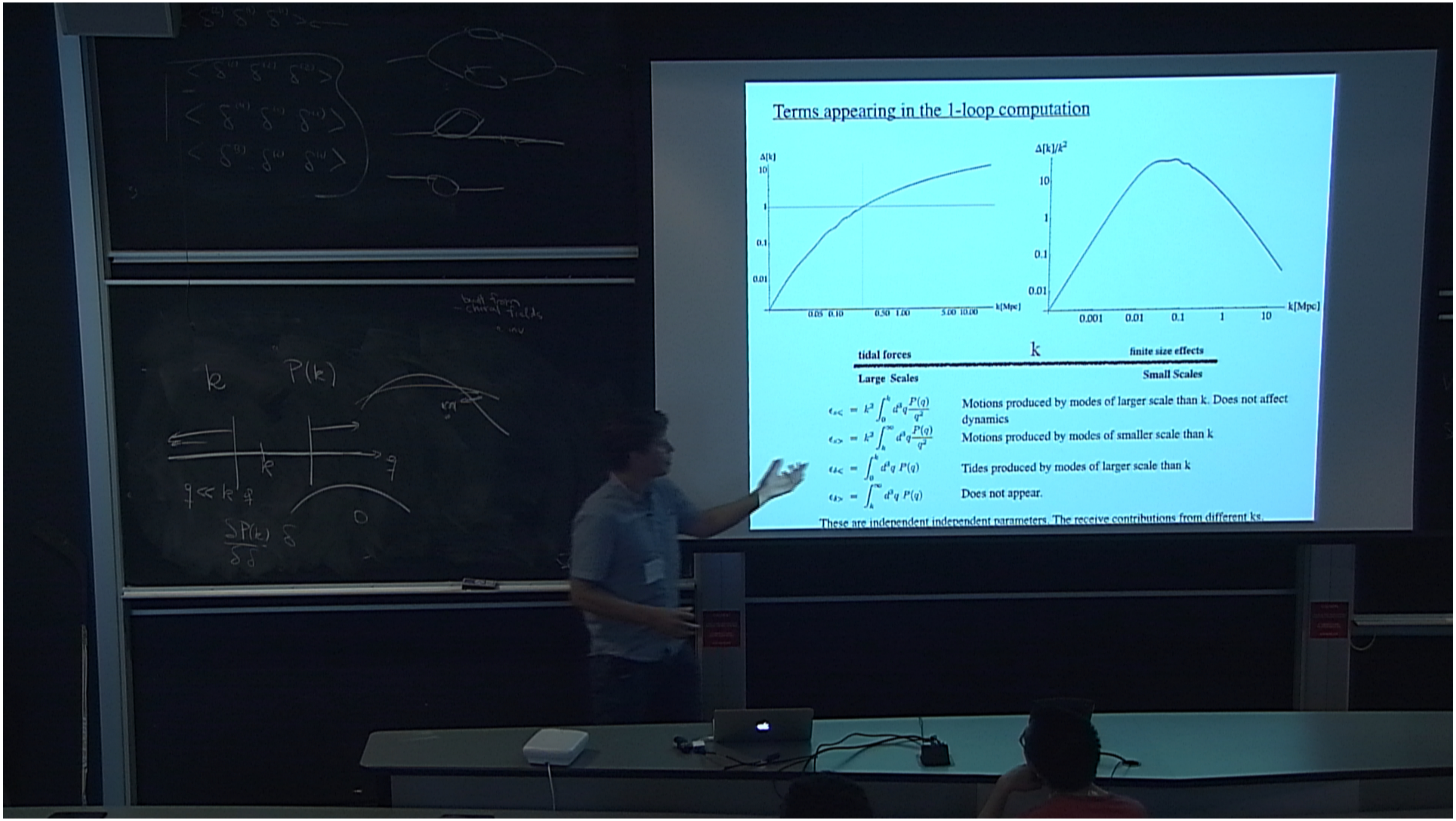


Figure 3. *Left panel:* Power index for each contribution to the power spectrum of ϕ, as a function of the power index of the linear density power spectrum $P_{\rm lin}$. *Right panel:* Power index of the no-wiggle power spectrum from Eisenstein & Hu [18].


Terms appearing in the 1-loop computation
Δ[k]
k[Mpc]
Δ[k]/k²
k[Mpc]
tidal forces
k
finite size effects
Large Scales
Small Scales
Motions produced by modes of larger scale than k. Does not affect dynamics
Motions produced by modes of smaller scale than k
Tides produced by modes of larger scale than k
Does not appear.
These are independent independent parameters. The receive contributions from different ks.


Terms appearing in the 1-loop computation
k[Mpc]
0.001
0.01
0.1
1
10
tidal forces
k
finite size effects
Large Scales
Small Scales
Motions produced by modes of larger scale than k. Does not affect dynamics
Motions produced by modes of smaller scale than k
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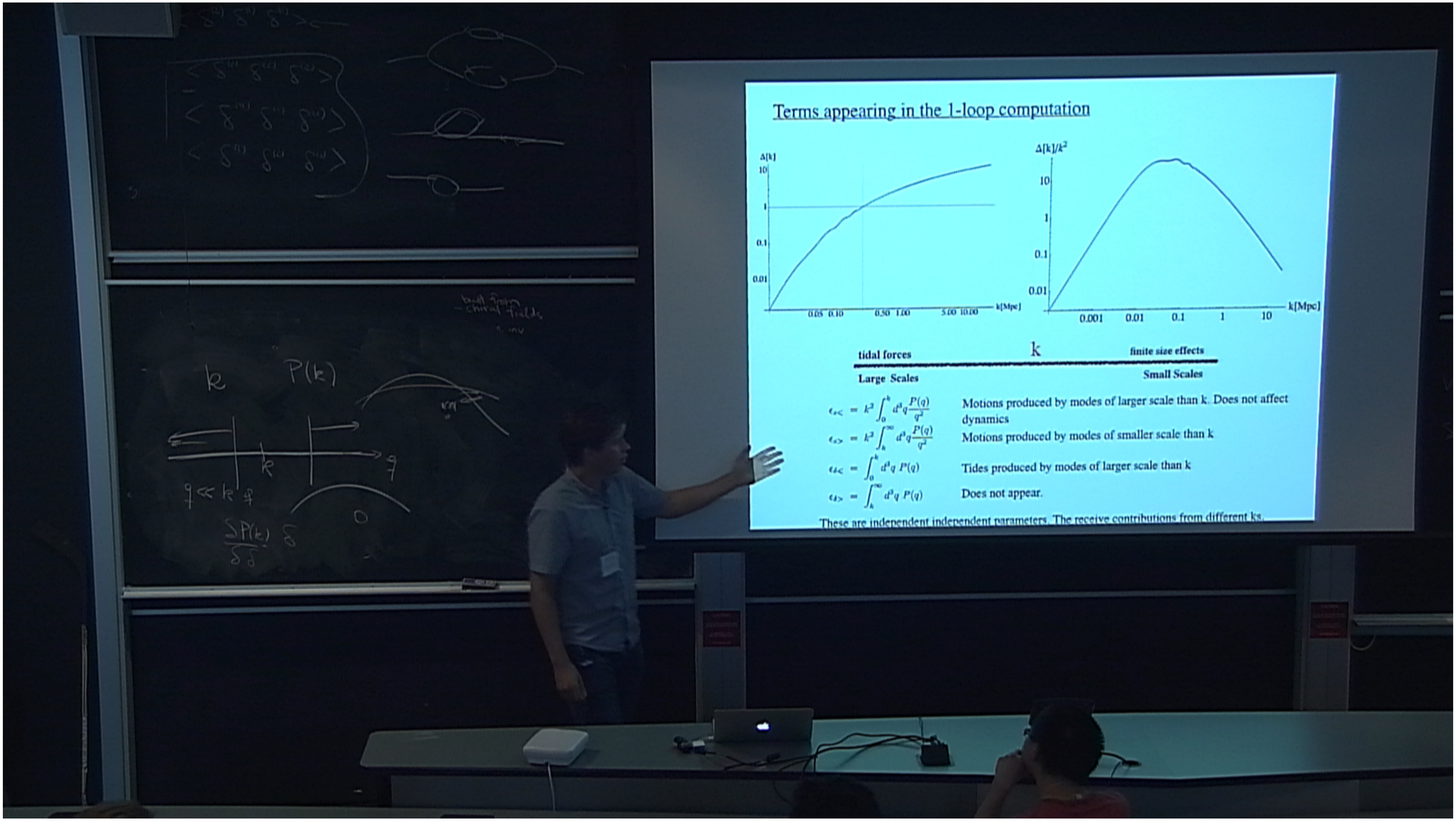

Terms appearing in the 1-loop computation
tidal forces
k
finite size effects
Large Scales
Small Scales
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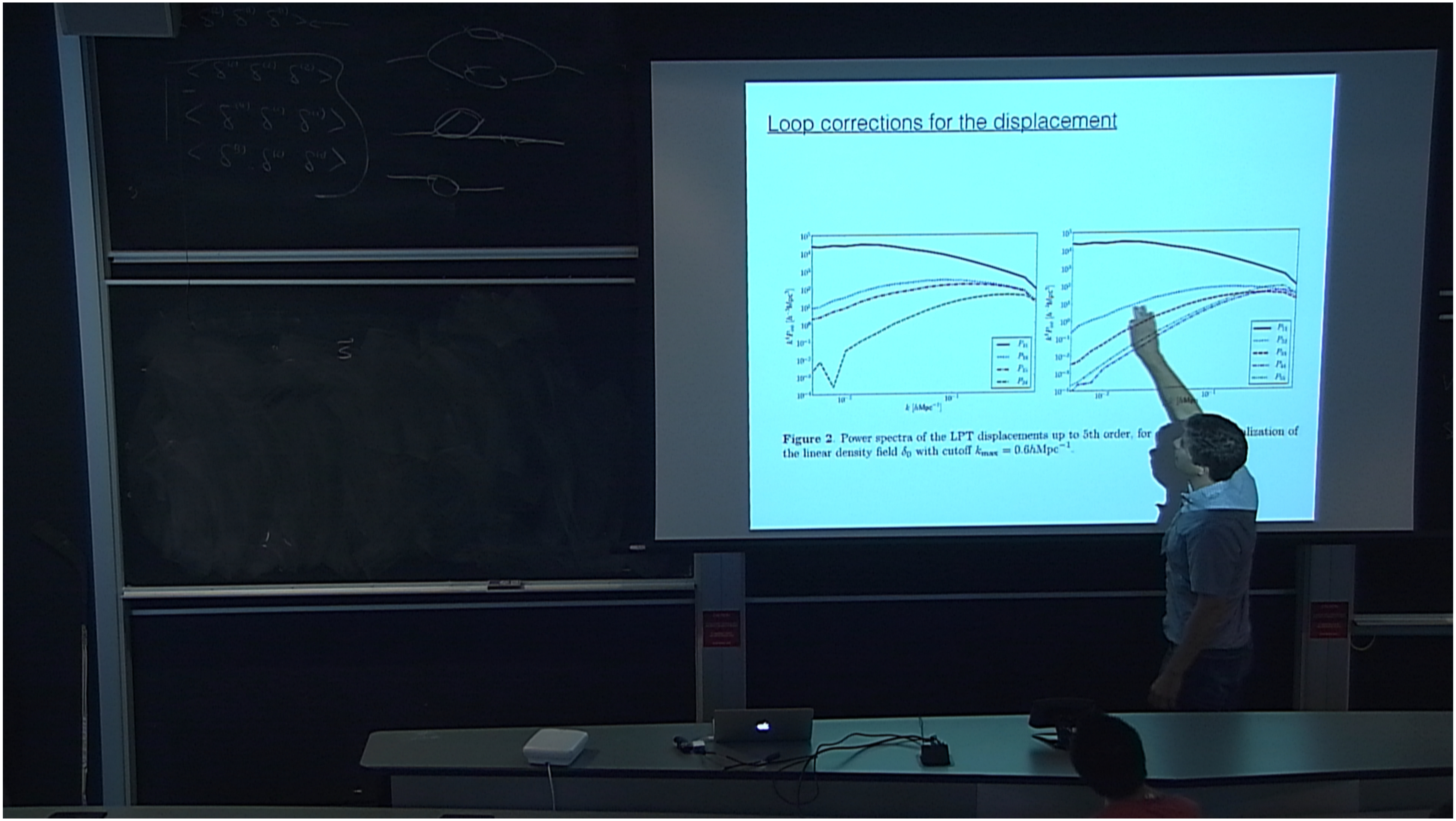
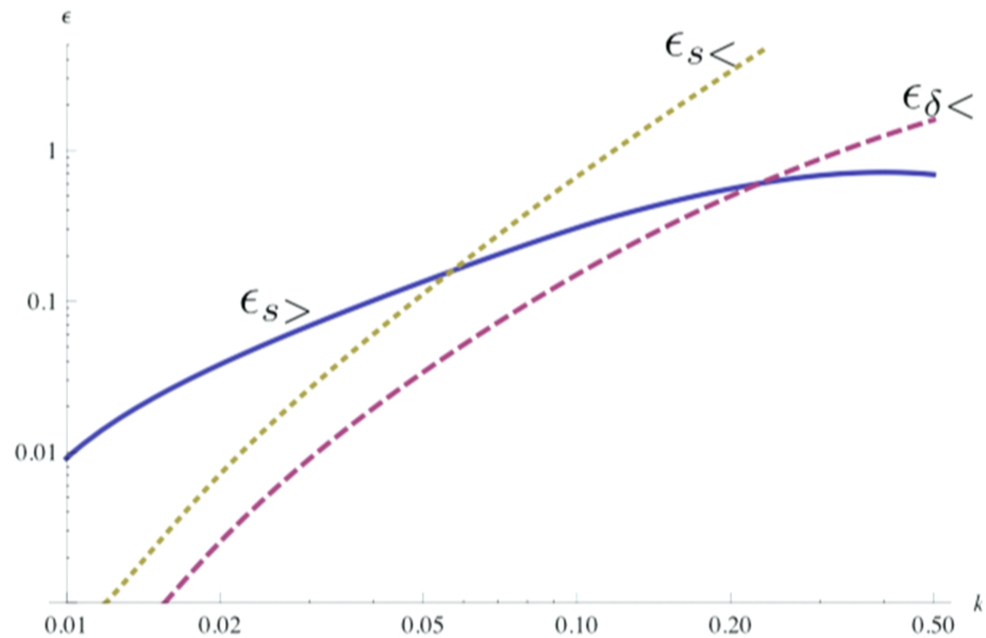

Loop corrections for the displacement
Figure 2. Power spectra of the LPT displacements up to 5th order, for
the linear density field δ_0 with cutoff $k_{max} = 0.6h\mathrm{Mpc}^{-1}$.



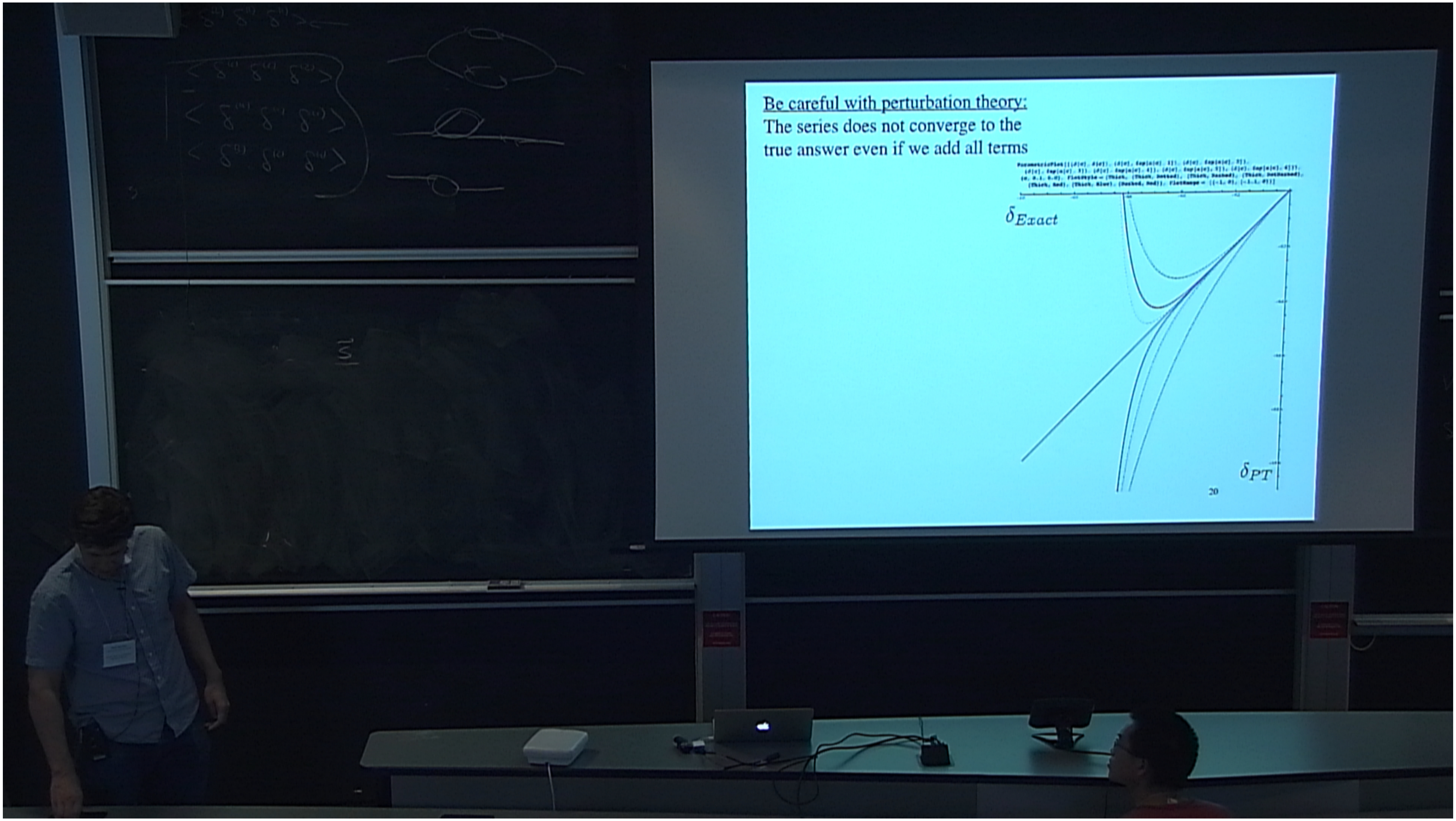
$$\epsilon_{s<} = k^2 \int_0^k d^3q \frac{P(q)}{q^2}$$ Motions produced by modes of larger scale than k

$$\epsilon_{s>} = k^2 \int_k^\infty d^3q \frac{P(q)}{q^2}$$ Motions produced by modes of smaller scale than k

$$\epsilon_{\delta<} = \int_0^k d^3q \; P(q)$$ Tides produced by modes of larger scale than k

$$\epsilon_{\delta>} = \int_k^\infty d^3q \; P(q)$$ Does not appear.

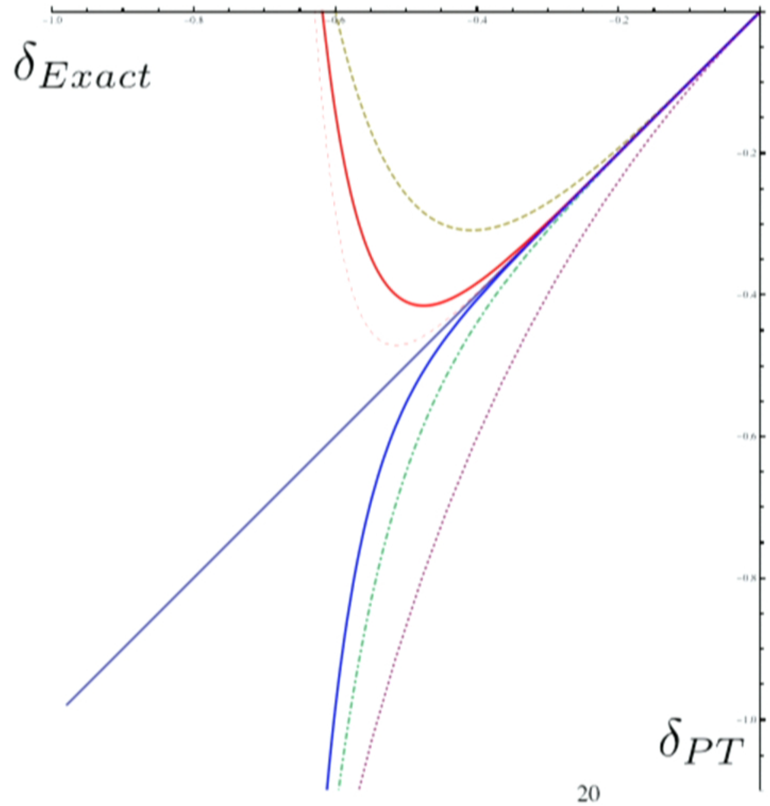
9


Be careful with perturbation theory:
The series does not converge to the
true answer even if we add all terms
δ_{Exact}
δ_{PT}
20

Be careful with perturbation theory:

The series does not converge to the true answer even if we add all terms

```
ParametricPlot[{{δ[c], δ[c]}, {δ[c], fap[a[c], 1]}, {δ[c], fap[a[c], 2]},
  {δ[c], fap[a[c], 3]}, {δ[c], fap[a[c], 4]}, {δ[c], fap[a[c], 5]}, {δ[c], fap[a[c], 6]}},
 {c, 0.1, 6.0}, PlotStyle → {Thick, {Thick, Dotted}, {Thick, Dashed}, {Thick, DotDashed},
   {Thick, Red}, {Thick, Blue}, {Dashed, Red}}, PlotRange → {{-1, 0}, {-1.1, 0}}]
```



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```
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 {c, 0.1, 6.0}, PlotStyle → {Thick, {Thick, Dotted}, {Thick, Dashed}, {Thick, DotDashed},
   {Thick, Red}, {Thick, Blue}, {Dashed, Red}}, PlotRange → {{-1, 0}, {-1.1, 0}}]
```
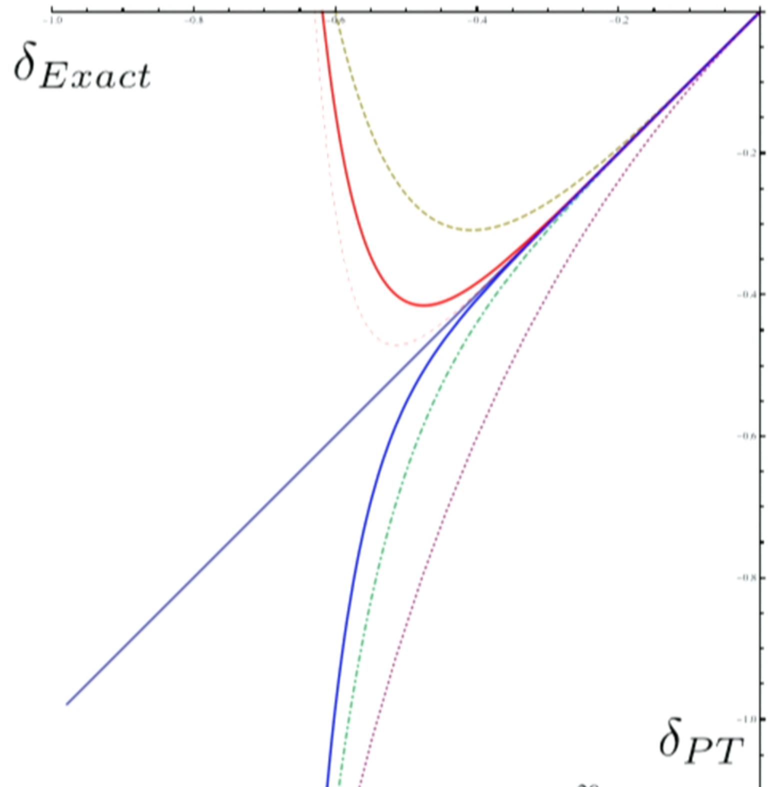
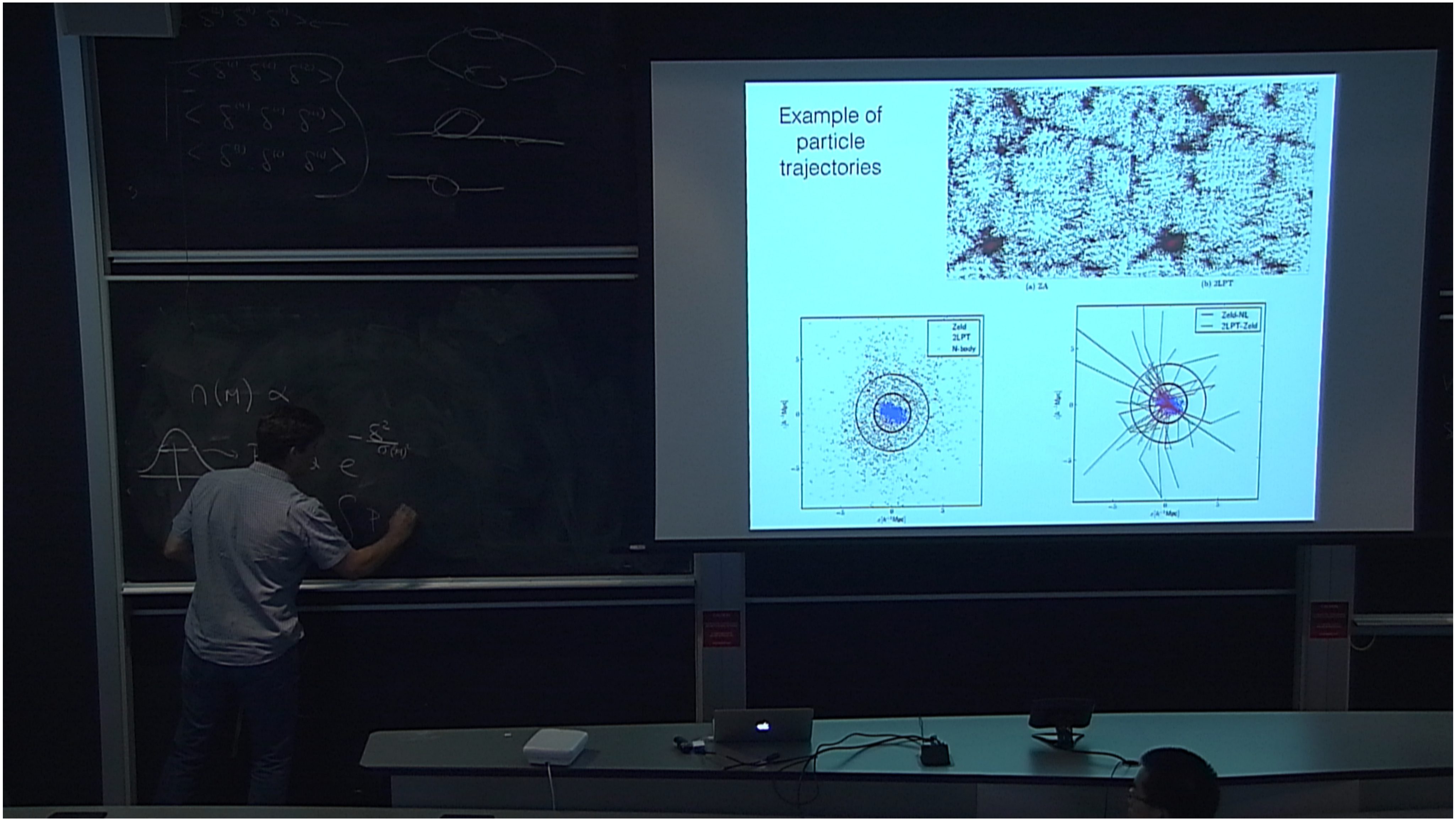
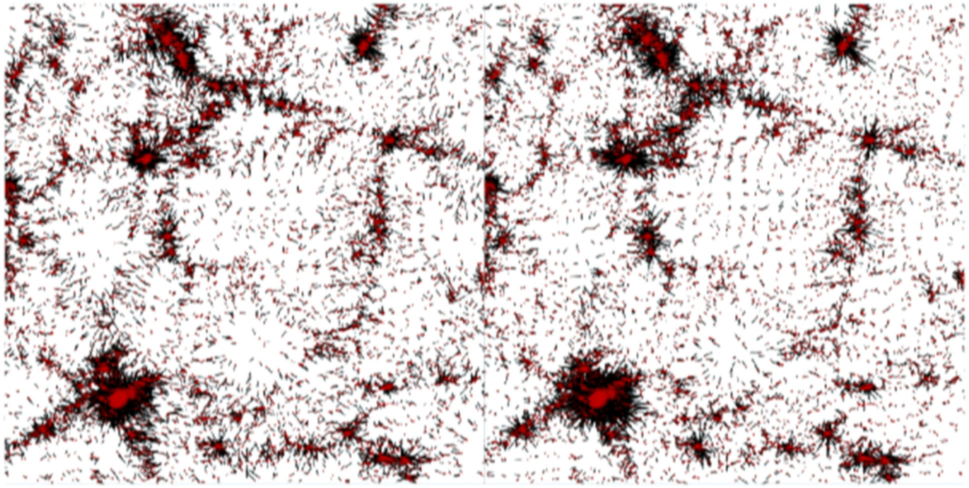
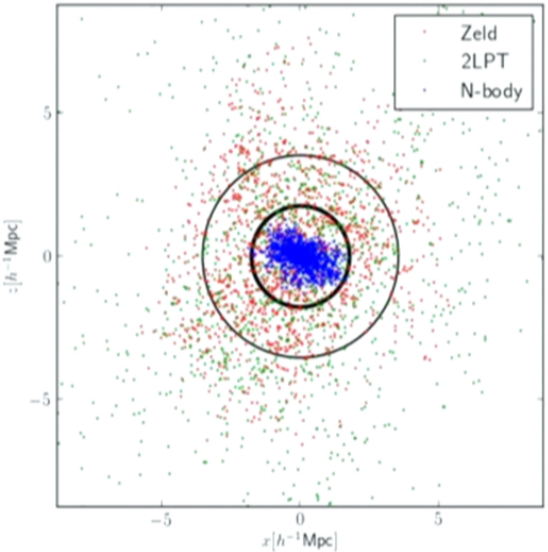



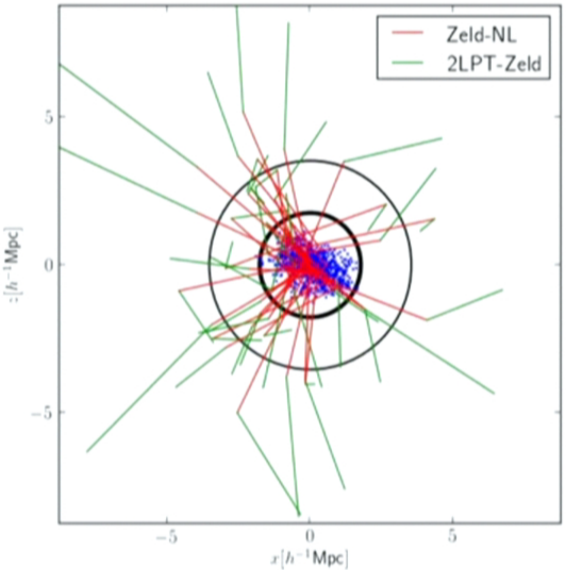

Example of particle trajectories
(a) ZA
(b) 2LPT
Zeld
2LPT
N-body
Zeld-NL
2LPT-Zeld

Example of particle trajectories

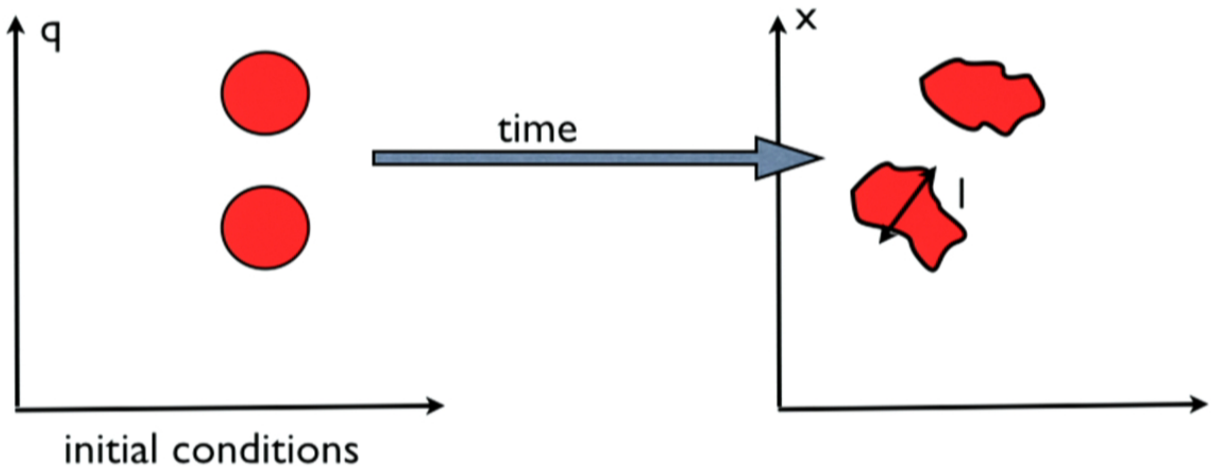


(a) ZA (b) 2LPT





The EFT in Lagrangian space

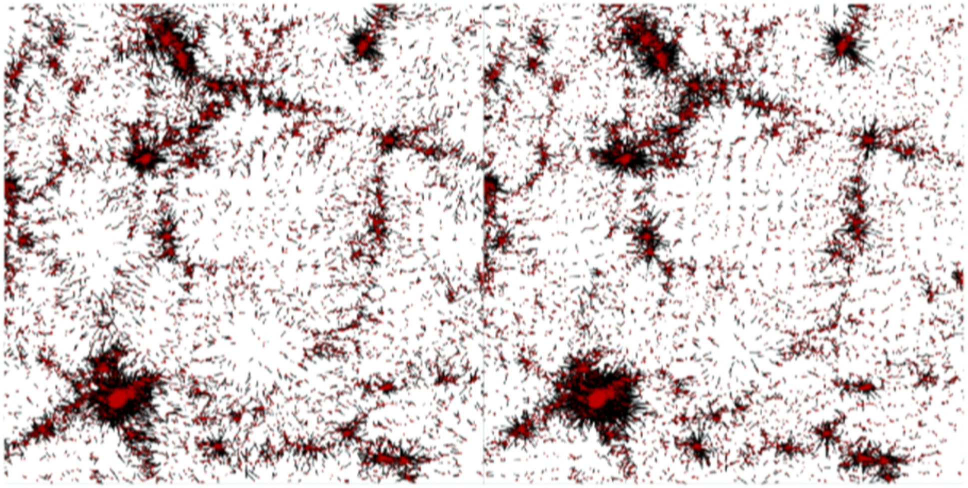


Finite size effects in the interaction. At the lowest order basically just the quadrupole moment.

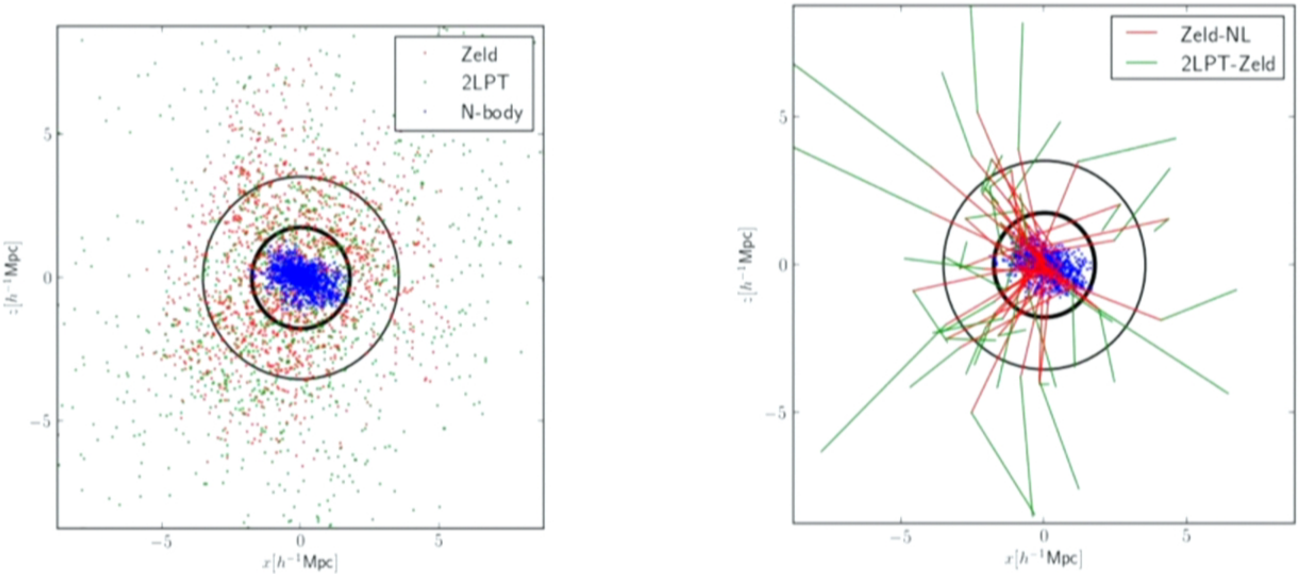
Derivative expansion, powers of k l. This expansion is different that the expansion in delta. Both are necessary in perturbation theory.

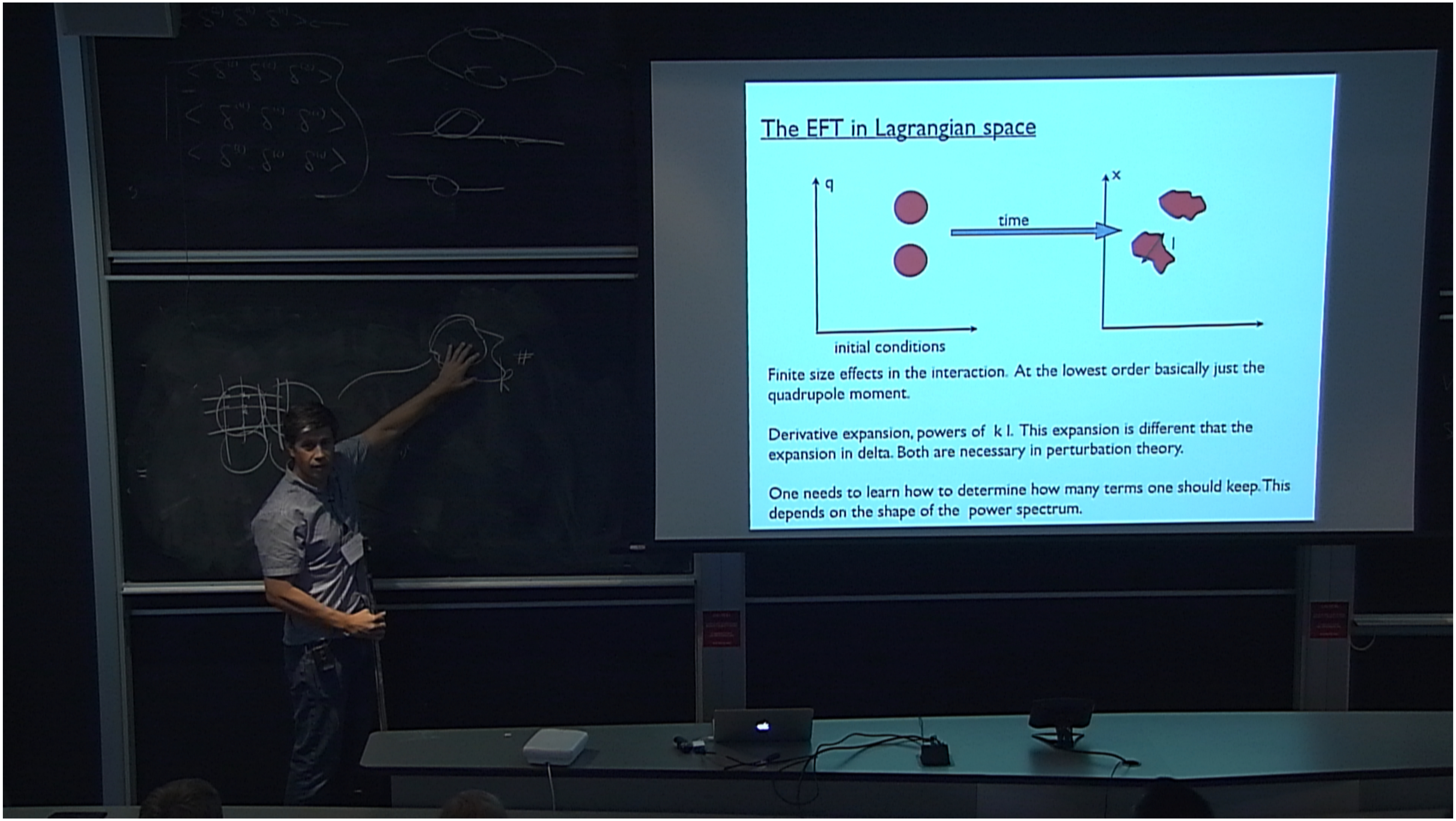
One needs to learn how to determine how many terms one should keep. This depends on the shape of the power spectrum.

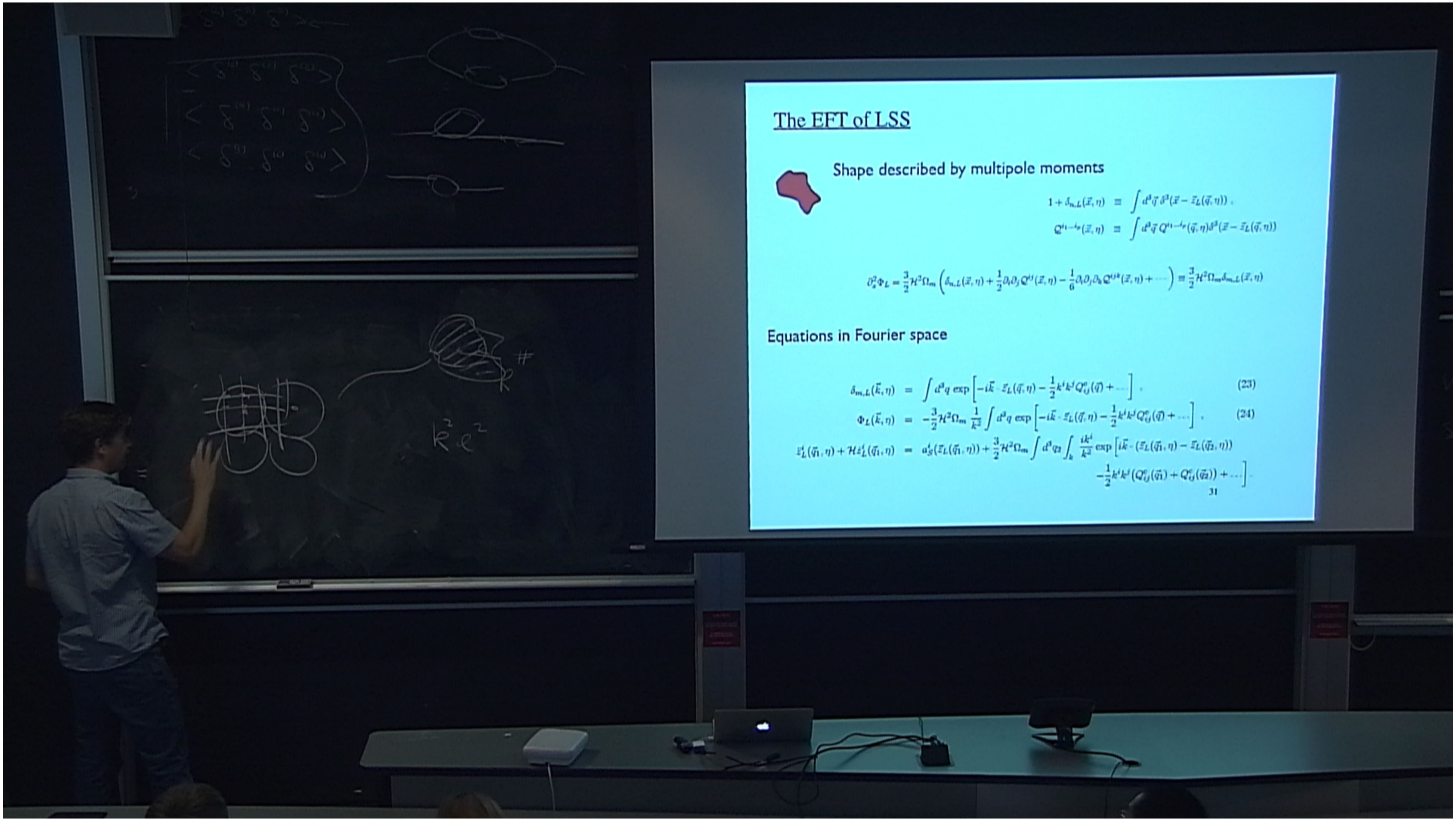
Example of particle trajectories

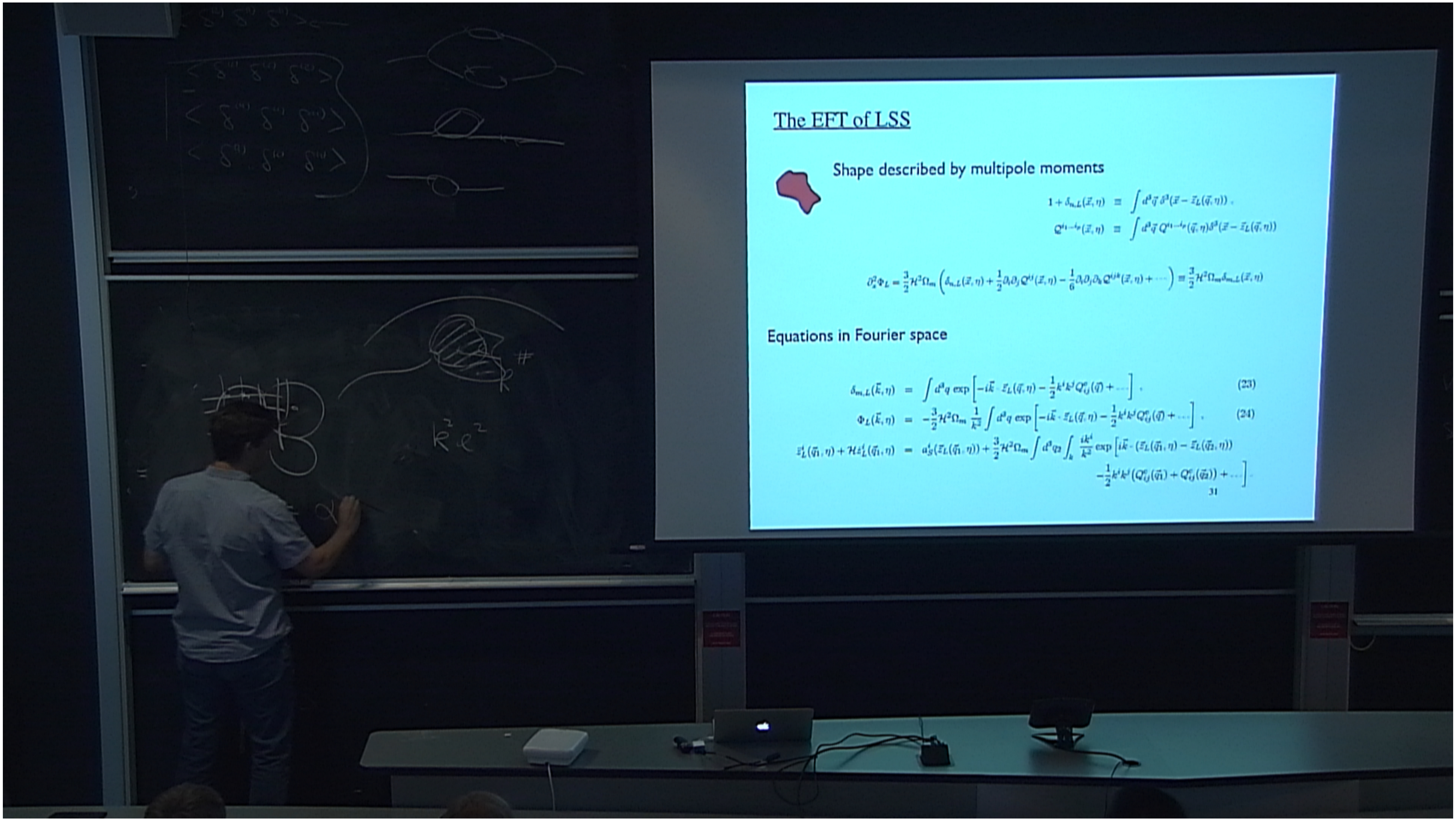


(a) ZA (b) 2LPT

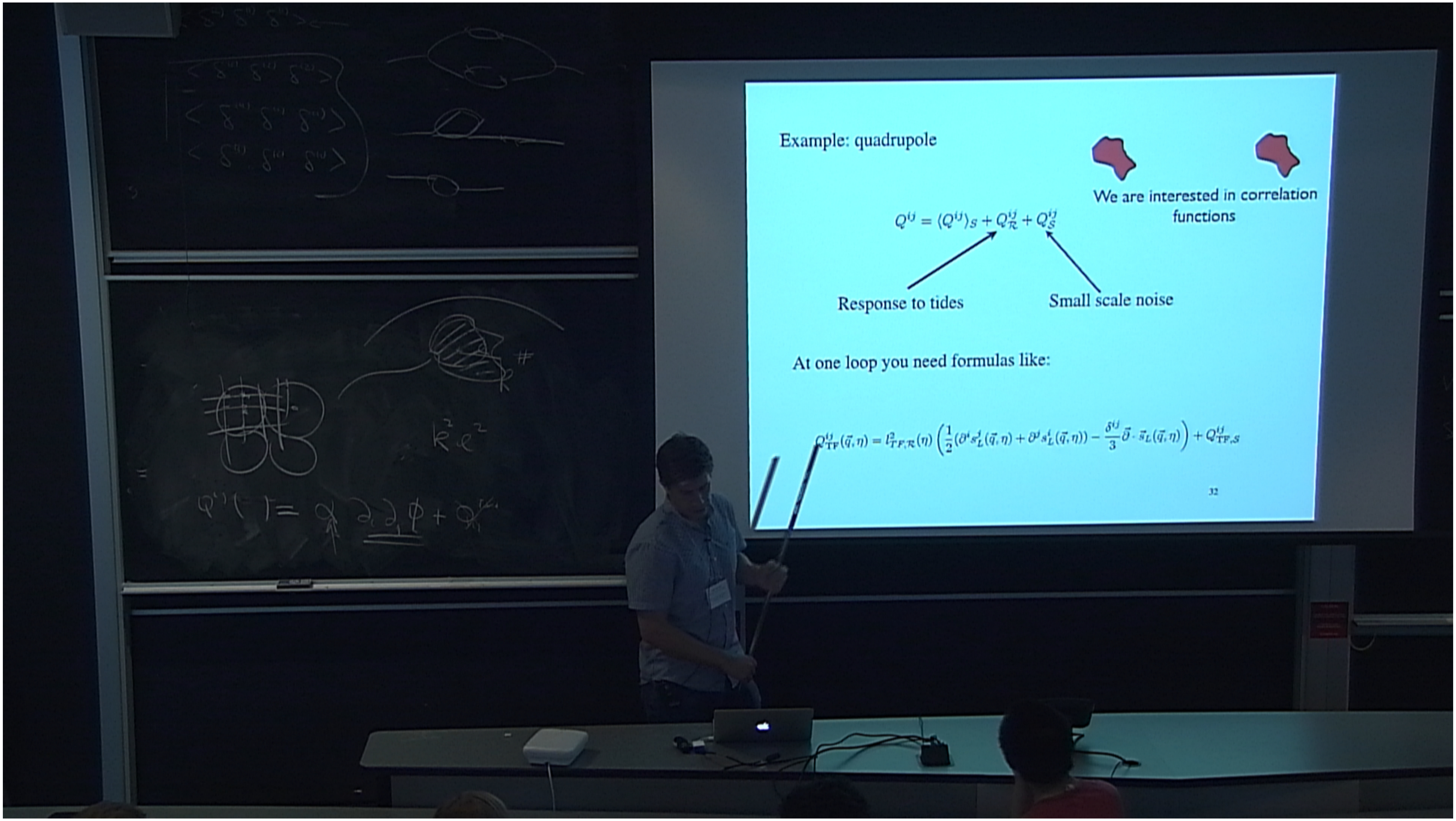



The EFT in Lagrangian space
q
time
x
l
initial conditions
Finite size effects in the interaction. At the lowest order basically just the quadrupole moment.
Derivative expansion, powers of k l. This expansion is different that the expansion in delta. Both are necessary in perturbation theory.
One needs to learn how to determine how many terms one should keep. This depends on the shape of the power spectrum.


The EFT of LSS
Shape described by multipole moments
Equations in Fourier space
(23)
(24)
31



The EFT of LSS
Shape described by multipole moments
$1 + \delta_{m,L}(\vec{x},\eta) \equiv \int d^3\vec{q}\, \delta^3(\vec{x} - \vec{z}_L(\vec{q},\eta))$,
$Q^{i_1 \dots i_p}(\vec{x},\eta) \equiv \int d^3\vec{q}\, Q^{i_1 \dots i_p}(\vec{q},\eta)\delta^3(\vec{x} - \vec{z}_L(\vec{q},\eta))$
$\partial_x^2 \Phi_L = \frac{3}{2}\mathcal{H}^2\Omega_m \left(\delta_{m,L}(\vec{x},\eta) + \frac{1}{2}\partial_i\partial_j Q^{ij}(\vec{x},\eta) - \frac{1}{6}\partial_i\partial_j\partial_k Q^{ijk}(\vec{x},\eta) + \cdots\right) \equiv \frac{3}{2}\mathcal{H}^2\Omega_m \delta_{m,L}(\vec{x},\eta)$
Equations in Fourier space
$\delta_{m,L}(\vec{k},\eta) = \int d^3q \exp\left[-i\vec{k}\cdot\vec{z}_L(\vec{q},\eta) - \frac{1}{2}k^i k^j Q^e_{ij}(\vec{q}) + \cdots\right]$, (23)
$\Phi_L(\vec{k},\eta) = -\frac{3}{2}\mathcal{H}^2\Omega_m \frac{1}{k^2}\int d^3q \exp\left[-i\vec{k}\cdot\vec{z}_L(\vec{q},\eta) - \frac{1}{2}k^i k^j Q^e_{ij}(\vec{q}) + \cdots\right]$, (24)
$\ddot{z}^i_L(\vec{q}_1,\eta) + \mathcal{H}\dot{z}^i_L(\vec{q}_1,\eta) = a^i_S(\vec{z}_L(\vec{q}_1,\eta)) + \frac{3}{2}\mathcal{H}^2\Omega_m \int d^3q_2 \int_k \frac{ik^i}{k^2} \exp\left[i\vec{k}\cdot(\vec{z}_L(\vec{q}_1,\eta) - \vec{z}_L(\vec{q}_2,\eta)) - \frac{1}{2}k^i k^j (Q^e_{ij}(\vec{q}_1) + Q^e_{ij}(\vec{q}_2)) + \cdots\right]$
31


Example: quadrupole
We are interested in correlation functions
$Q^{ij} = \langle Q^{ij} \rangle_S + Q^{ij}_{\mathcal{R}} + Q^{ij}_S$
Response to tides
Small scale noise
At one loop you need formulas like:
32

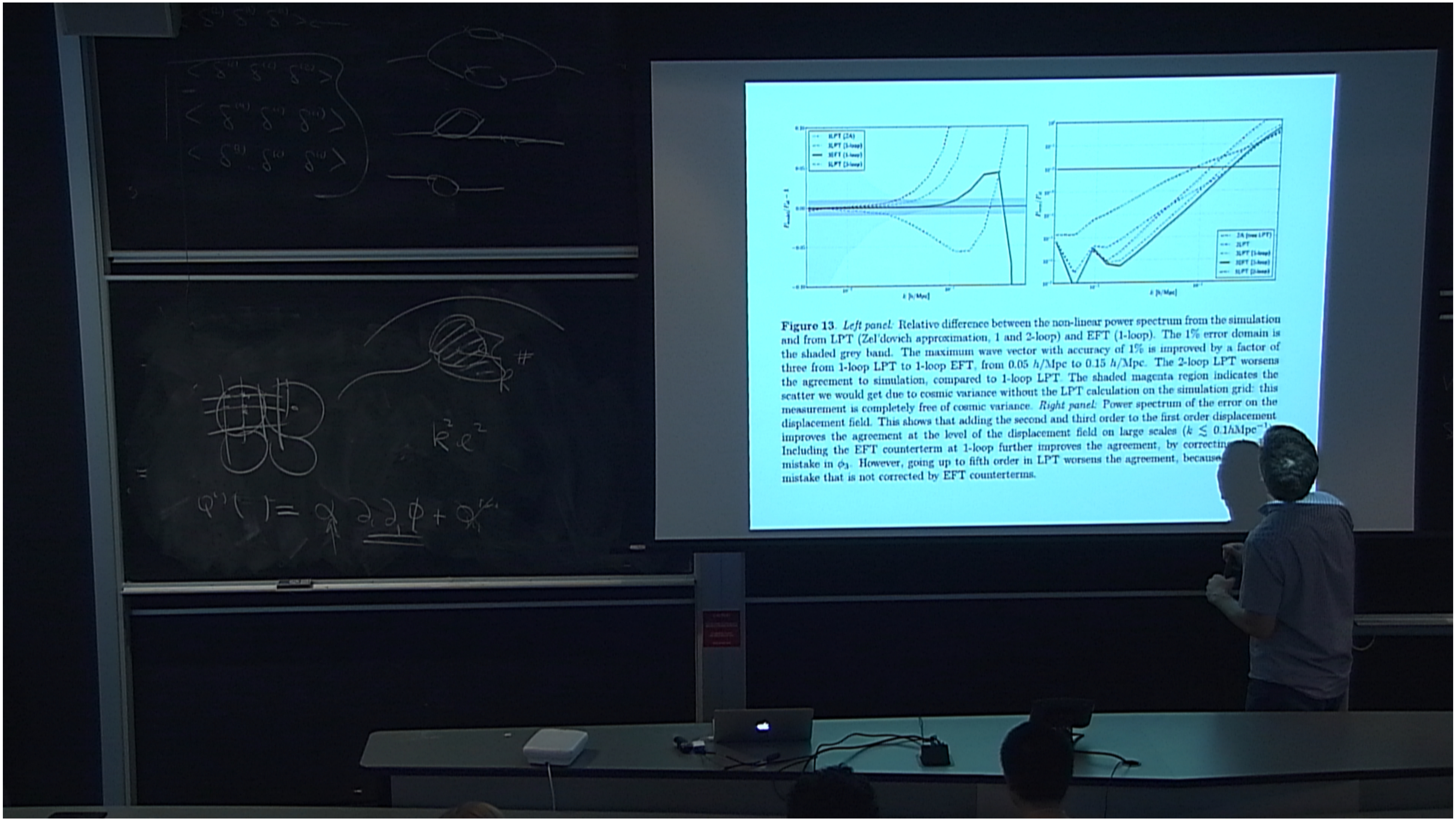


Figure 13. Left panel: Relative difference between the non-linear power spectrum from the simulation and from LPT (Zel'dovich approximation, 1 and 2-loop) and EFT (1-loop). The 1% error domain is the shaded grey band. The maximum wave vector with accuracy of 1% is improved by a factor of three from 1-loop LPT to 1-loop EFT, from 0.05 h/Mpc to 0.15 h/Mpc. The 2-loop LPT worsens the agreement to simulation, compared to 1-loop LPT. The shaded magenta region indicates the scatter we would get due to cosmic variance without the LPT calculation on the simulation grid: this measurement is completely free of cosmic variance. Right panel: Power spectrum of the error on the displacement field. This shows that adding the second and third order to the first order displacement improves the agreement at the level of the displacement field on large scales (k ≲ 0.1hMpc
Including the EFT counterterm at 1-loop further improves the agreement, by correctin
mistake in φ3. However, going up to fifth order in LPT worsens the agreement, because
mistake that is not corrected by EFT counterterms.

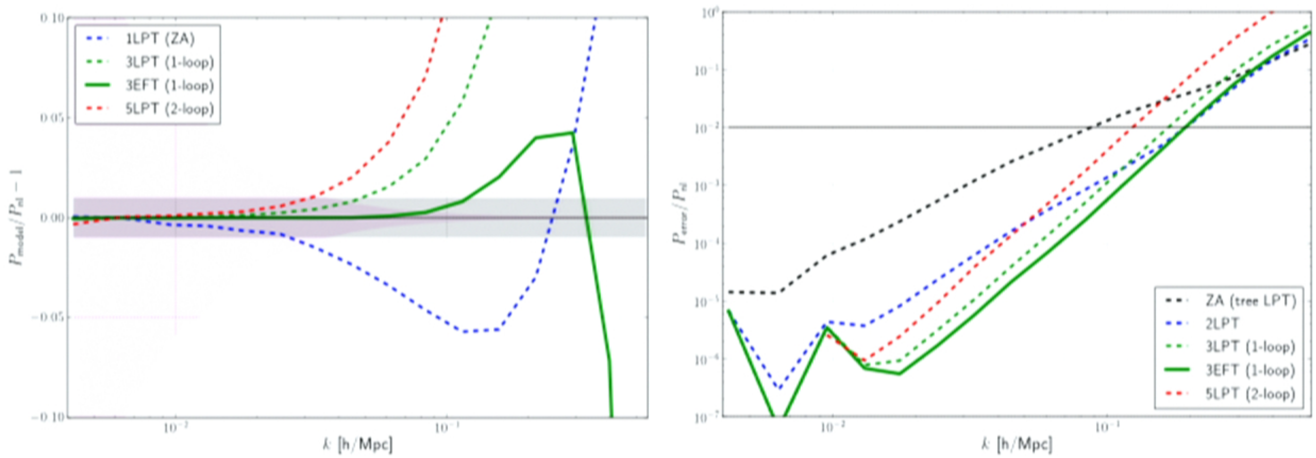


Figure 13. *Left panel:* Relative difference between the non-linear power spectrum from the simulation and from LPT (Zel'dovich approximation, 1 and 2-loop) and EFT (1-loop). The 1% error domain is the shaded grey band. The maximum wave vector with accuracy of 1% is improved by a factor of three from 1-loop LPT to 1-loop EFT, from 0.05 h/Mpc to 0.15 h/Mpc. The 2-loop LPT worsens the agreement to simulation, compared to 1-loop LPT. The shaded magenta region indicates the scatter we would get due to cosmic variance without the LPT calculation on the simulation grid: this measurement is completely free of cosmic variance. *Right panel:* Power spectrum of the error on the displacement field. This shows that adding the second and third order to the first order displacement improves the agreement at the level of the displacement field on large scales ($k \lesssim 0.1h\text{Mpc}^{-1}$). Including the EFT counterterm at 1-loop further improves the agreement, by correcting the UV mistake in ϕ_3. However, going up to fifth order in LPT worsens the agreement, because of the UV mistake that is not corrected by EFT counterterms.

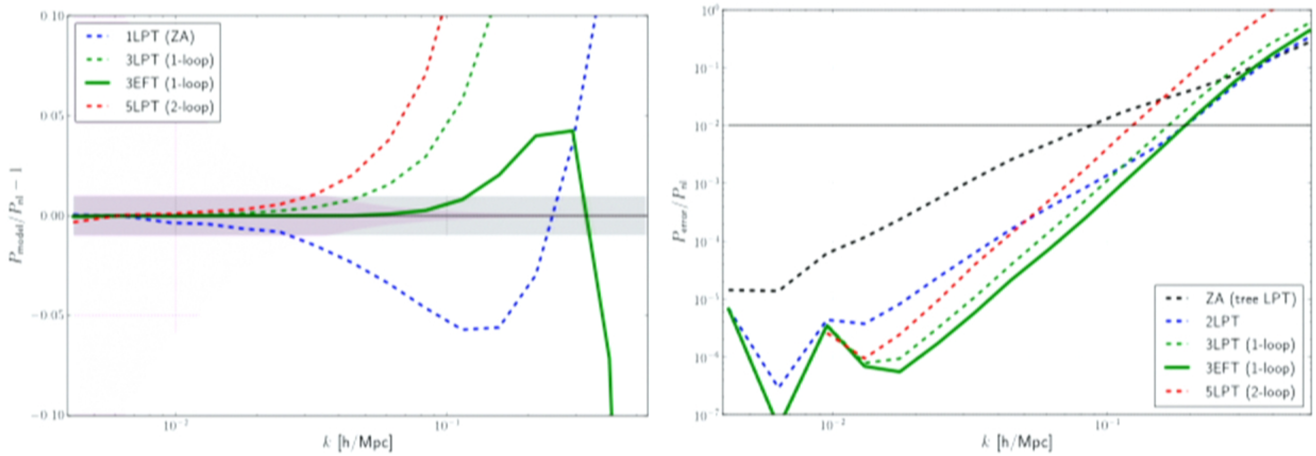
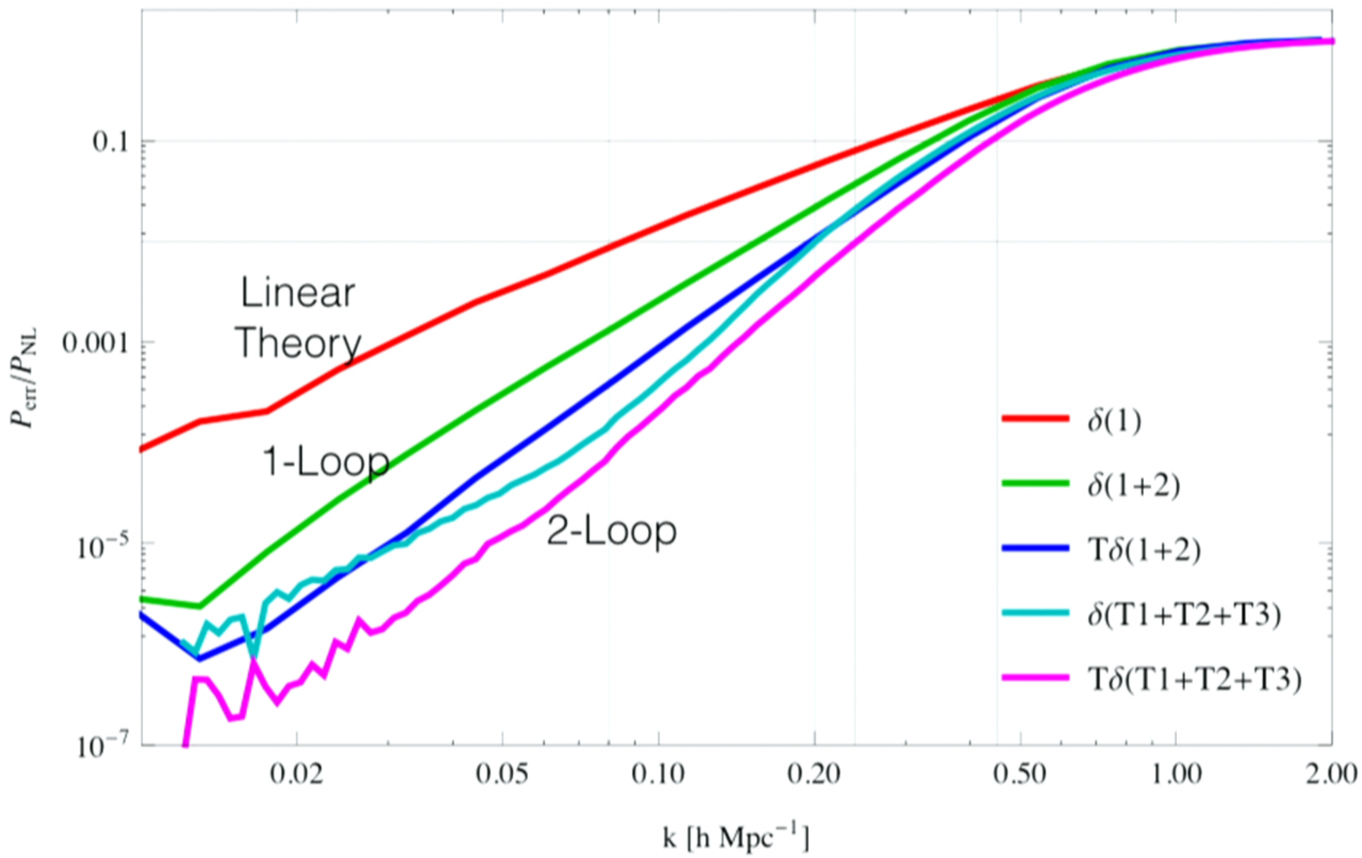
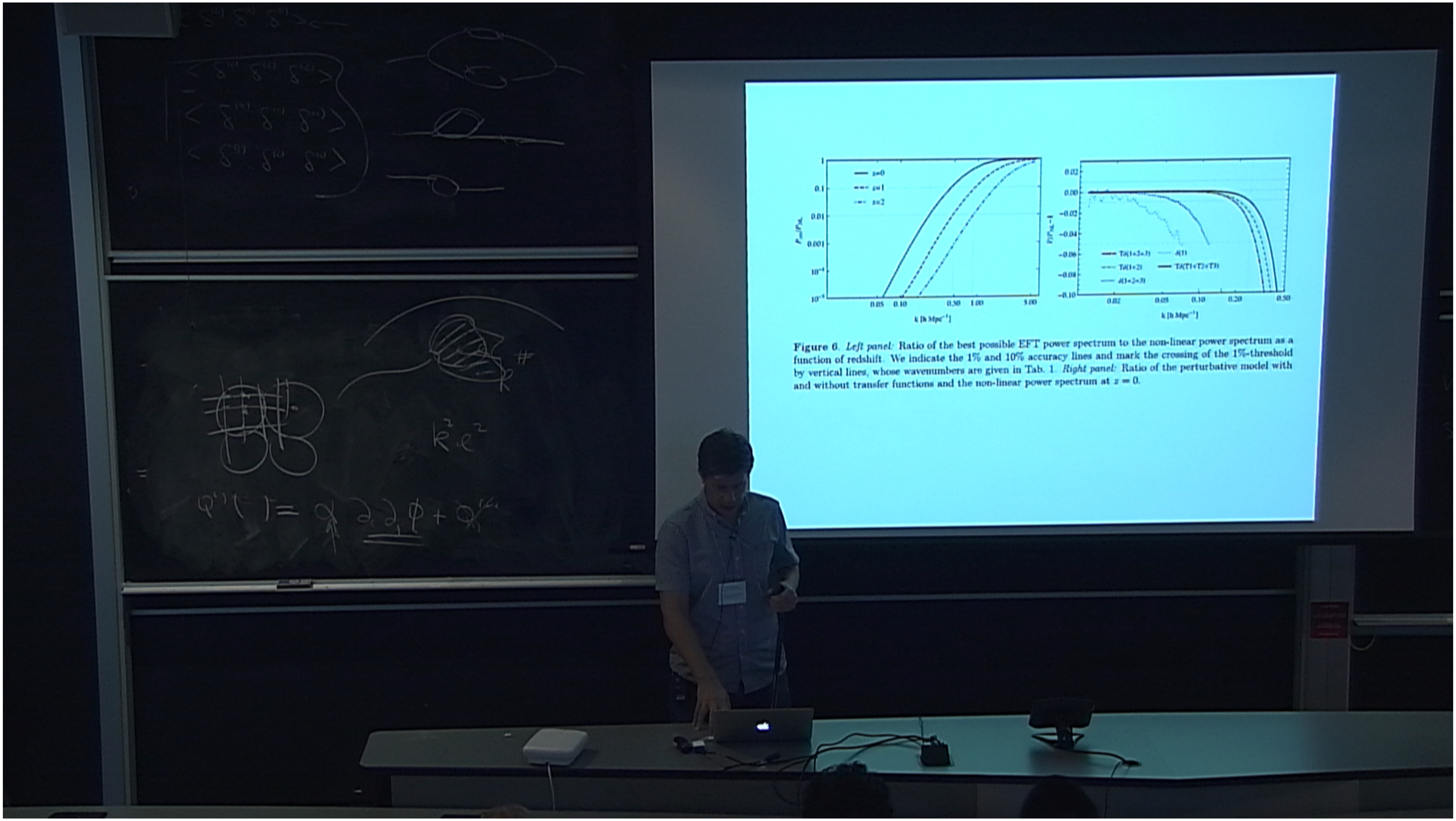
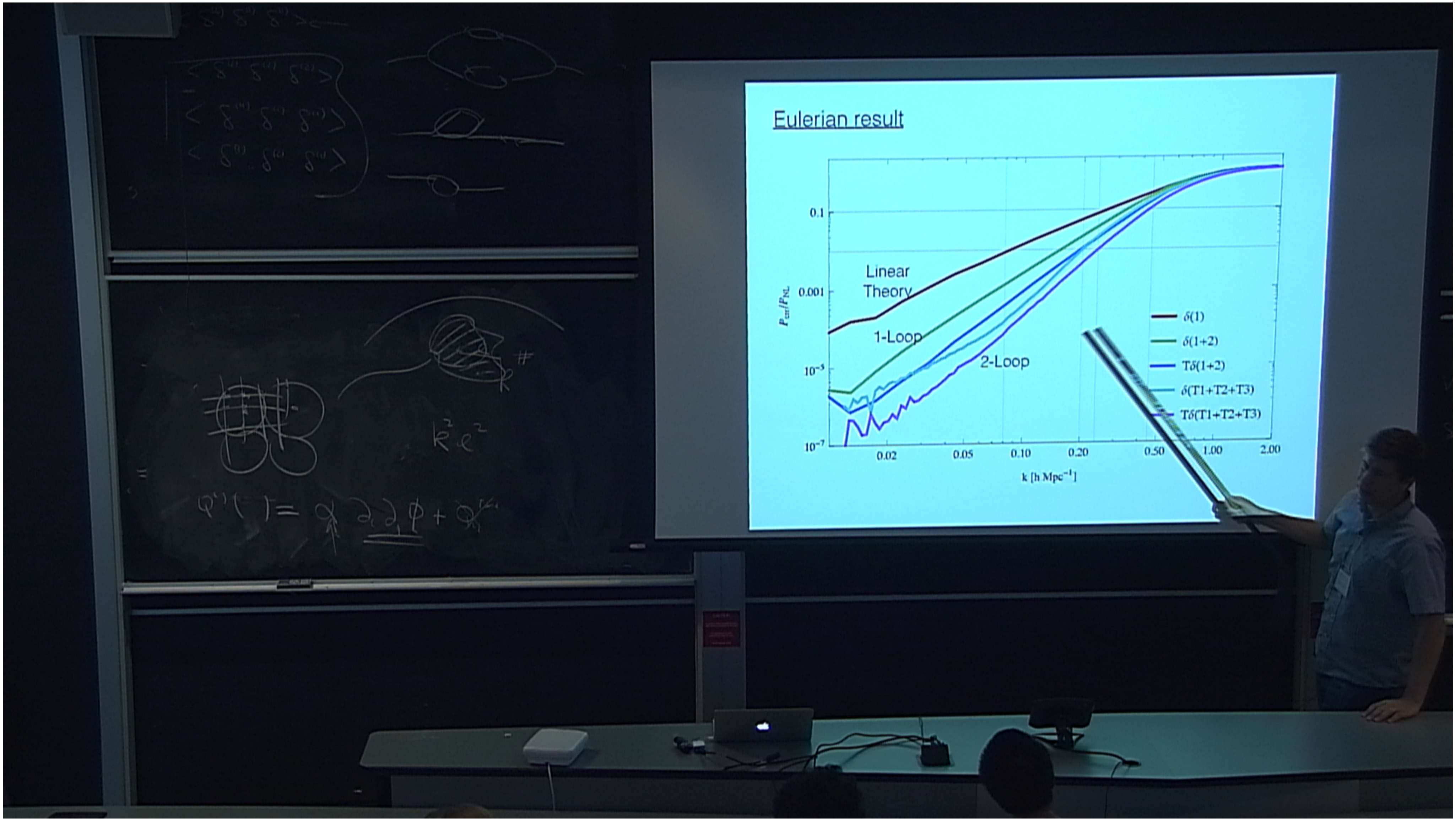


Figure 13. *Left panel:* Relative difference between the non-linear power spectrum from the simulation and from LPT (Zel'dovich approximation, 1 and 2-loop) and EFT (1-loop). The 1% error domain is the shaded grey band. The maximum wave vector with accuracy of 1% is improved by a factor of three from 1-loop LPT to 1-loop EFT, from 0.05 h/Mpc to 0.15 h/Mpc. The 2-loop LPT worsens the agreement to simulation, compared to 1-loop LPT. The shaded magenta region indicates the scatter we would get due to cosmic variance without the LPT calculation on the simulation grid: this measurement is completely free of cosmic variance. *Right panel:* Power spectrum of the error on the displacement field. This shows that adding the second and third order to the first order displacement improves the agreement at the level of the displacement field on large scales ($k \lesssim 0.1 h\mathrm{Mpc}^{-1}$). Including the EFT counterterm at 1-loop further improves the agreement, by correcting the UV mistake in ϕ_3. However, going up to fifth order in LPT worsens the agreement, because of the UV mistake that is not corrected by EFT counterterms.

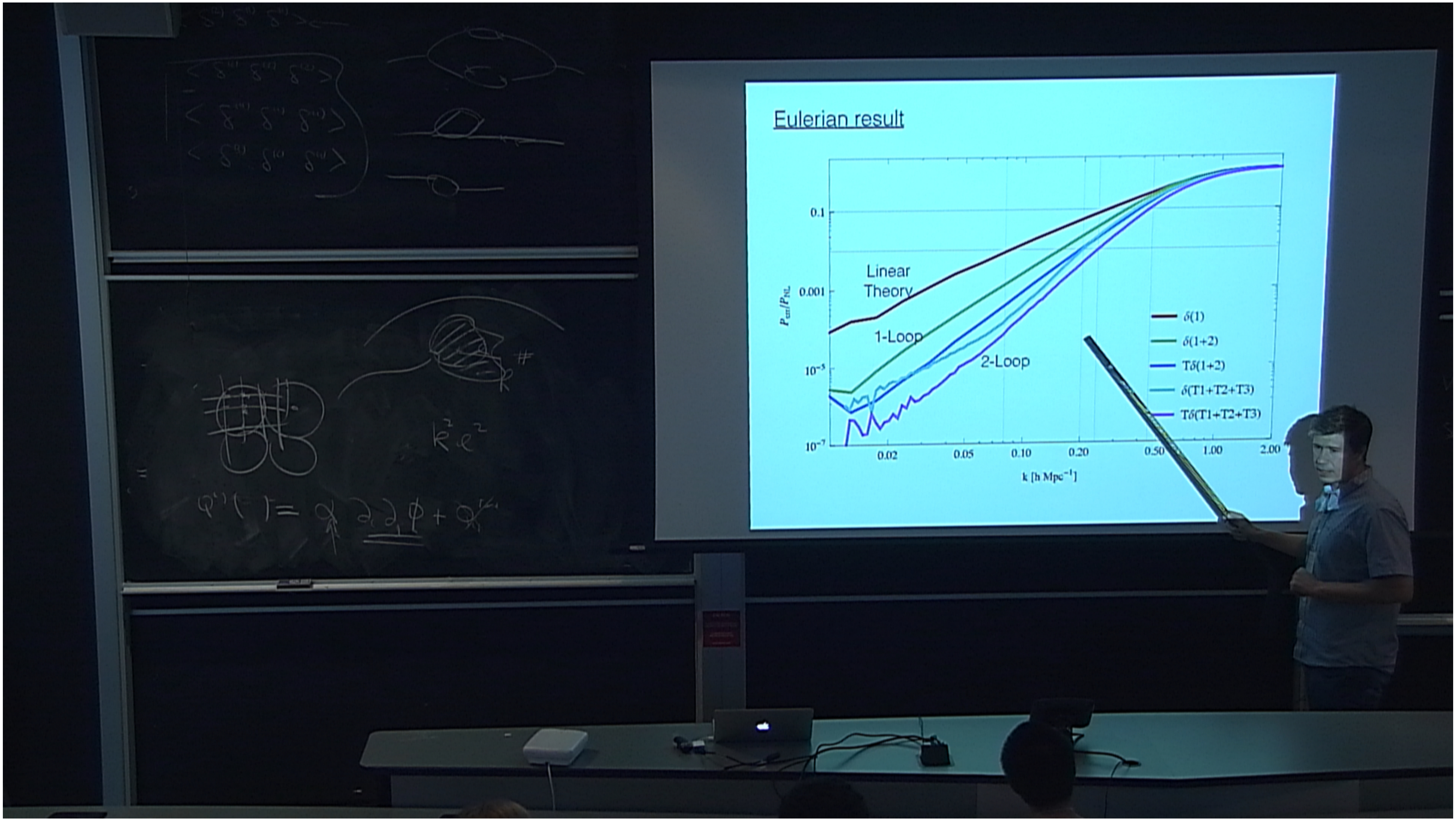
Eulerian result




z=0
z=1
z=2
k [h Mpc⁻¹]
k [h Mpc⁻¹]
Figure 6. Left panel: Ratio of the best possible EFT power spectrum to the non-linear power spectrum as a function of redshift. We indicate the 1% and 10% accuracy lines and mark the crossing of the 1%-threshold by vertical lines, whose wavenumbers are given in Tab. 1. Right panel: Ratio of the perturbative model with and without transfer functions and the non-linear power spectrum at z = 0.



Eulerian result
Linear Theory
1-Loop
2-Loop
0.1
0.001
10⁻⁵
10⁻⁷
0.02
0.05
0.10
0.20
0.50
1.00
2.00
k [h Mpc⁻¹]
δ(1)
δ(1+2)
Tδ(1+2)
δ(T1+T2+T3)
Tδ(T1+T2+T3)



Eulerian result
Linear Theory
1-Loop
2-Loop
0.1
0.001
10^{-5}
10^{-7}
P_{err}/P_{NL}
0.02
0.05
0.10
0.20
0.50
1.00
2.00
k [h Mpc^{-1}]
δ(1)
δ(1+2)
Tδ(1+2)
δ(T1+T2+T3)
Tδ(T1+T2+T3)

General lessons from EFT

- The small scale dynamics that is not captured by perturbation theory introduces a small number of free parameters that need to be fitted from simulation or data
- We understand the structure of these new terms, their dependence with scale is fixed.
- Calculations come with theoretical error bars.
- We are not strangers to these type of things, bias, higher dimension operators in particle physics.

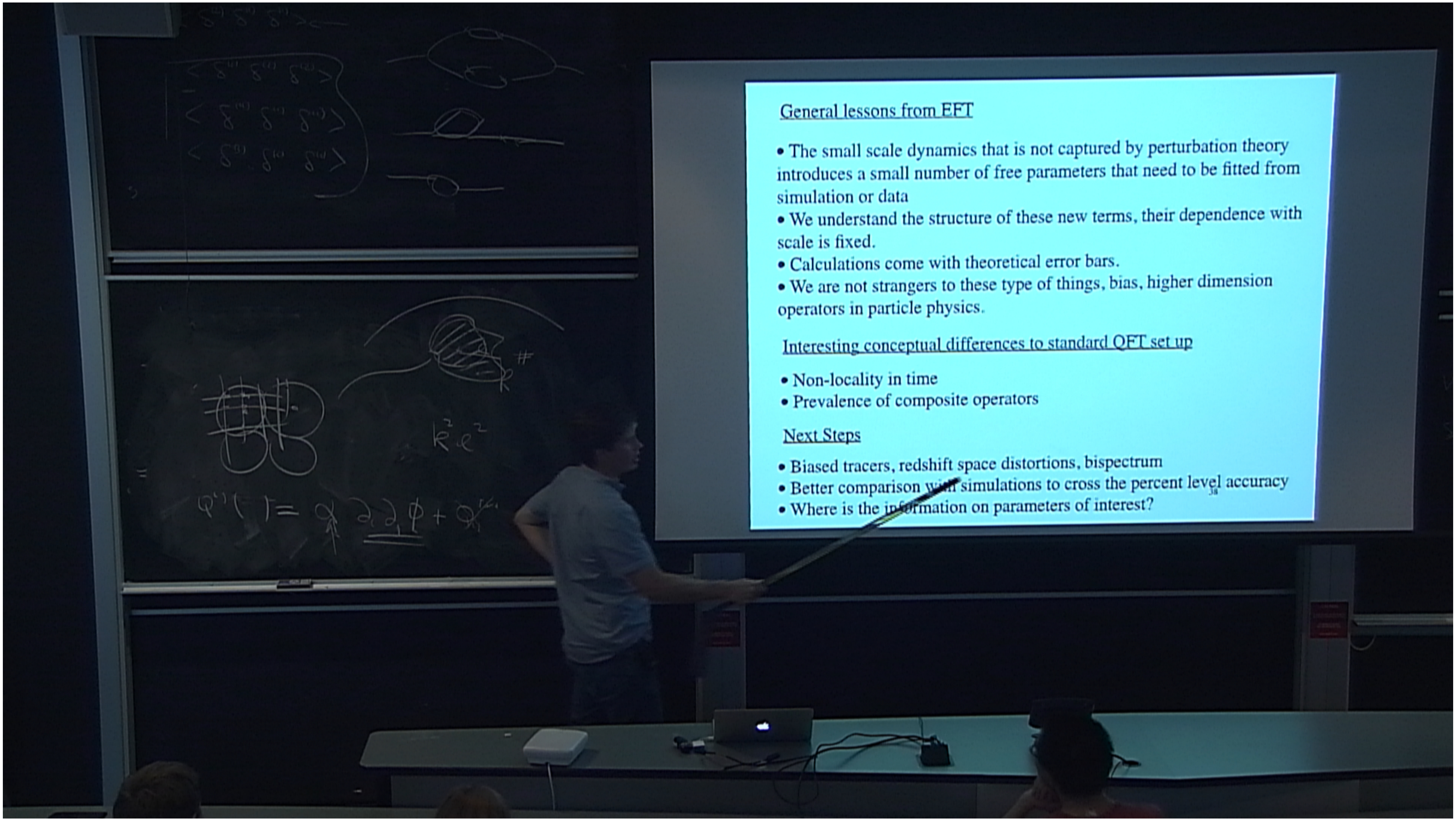
Interesting conceptual differences to standard QFT set up

- Non-locality in time
- Prevalence of composite operators

Next Steps

- Biased tracers, redshift space distortions, bispectrum
- Better comparison with simulations to cross the percent level accuracy
- Where is the information on parameters of interest?

38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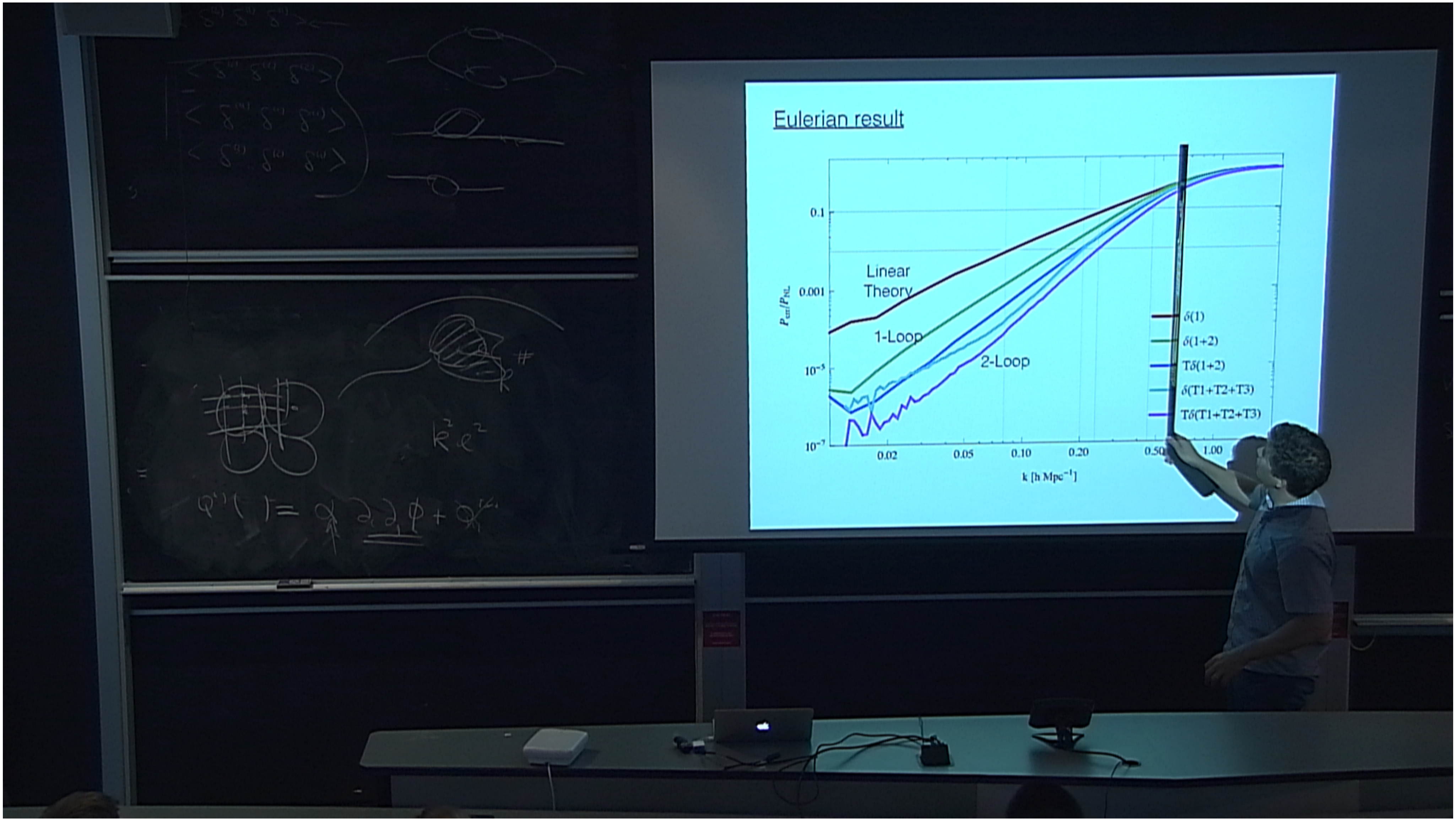
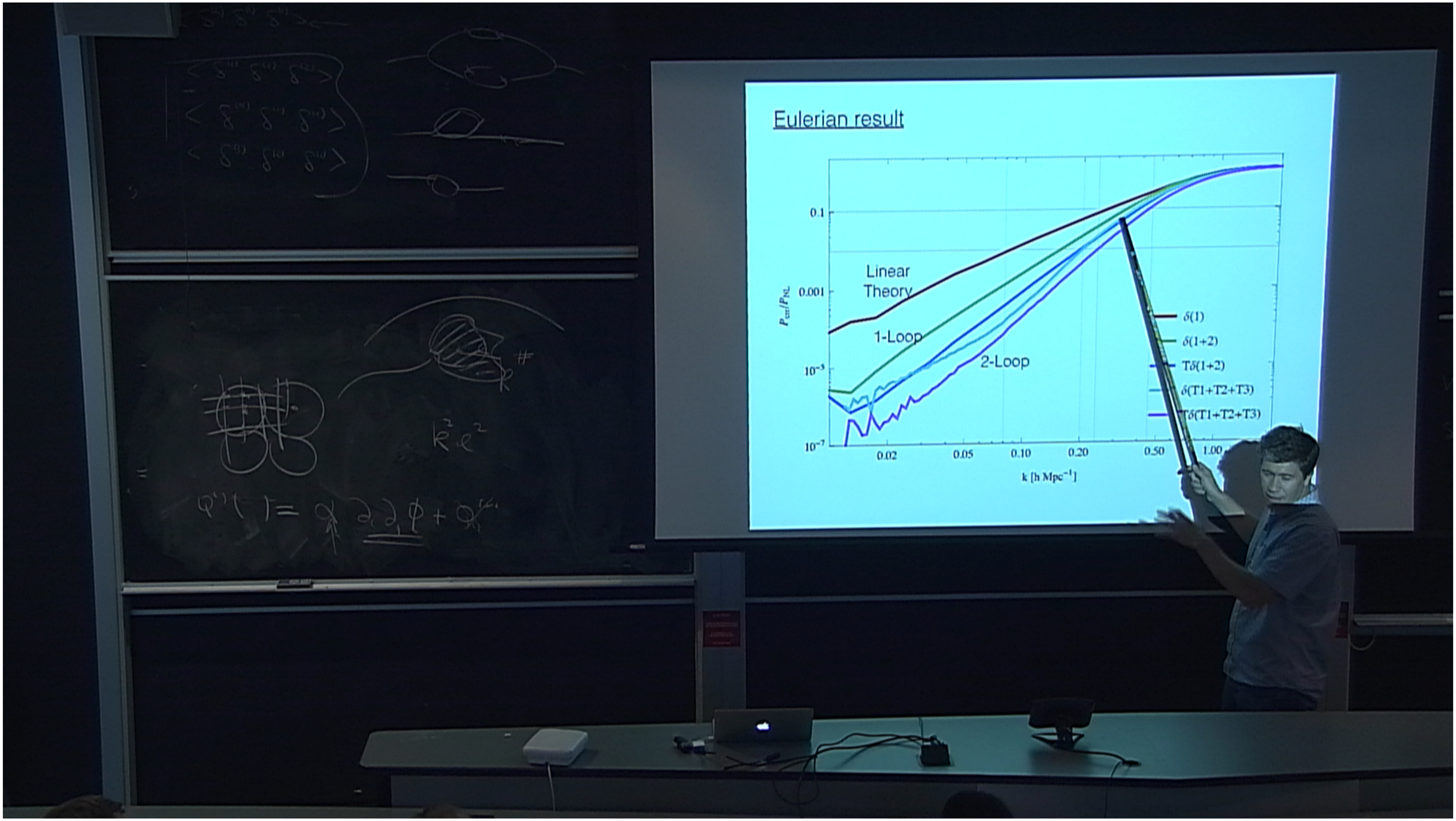


Eulerian result
Linear Theory
1-Loop
2-Loop
P_{err}/P_{NL}
0.1
0.001
10^{-5}
10^{-7}
0.02
0.05
0.10
0.20
0.50
1.00
k [h Mpc^{-1}]
δ(1)
δ(1+2)
Tδ(1+2)
δ(T1+T2+T3)
Tδ(T1+T2+T3)



Eulerian result
0.1
0.001
10^{-5}
10^{-7}
P_{ctr}/P_{NL}
Linear Theory
1-Loop
2-Loop
0.02
0.05
0.10
0.20
0.50
1.00
k [h Mpc^{-1}]
δ(1)
δ(1+2)
Tδ(1+2)
δ(T1+T2+T3)
Tδ(T1+T2+T3)

IR Resummation

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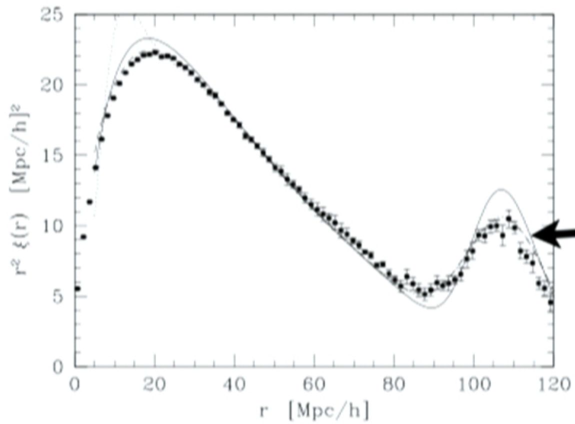
Existing perturbation theory does not organize terms according to their size.

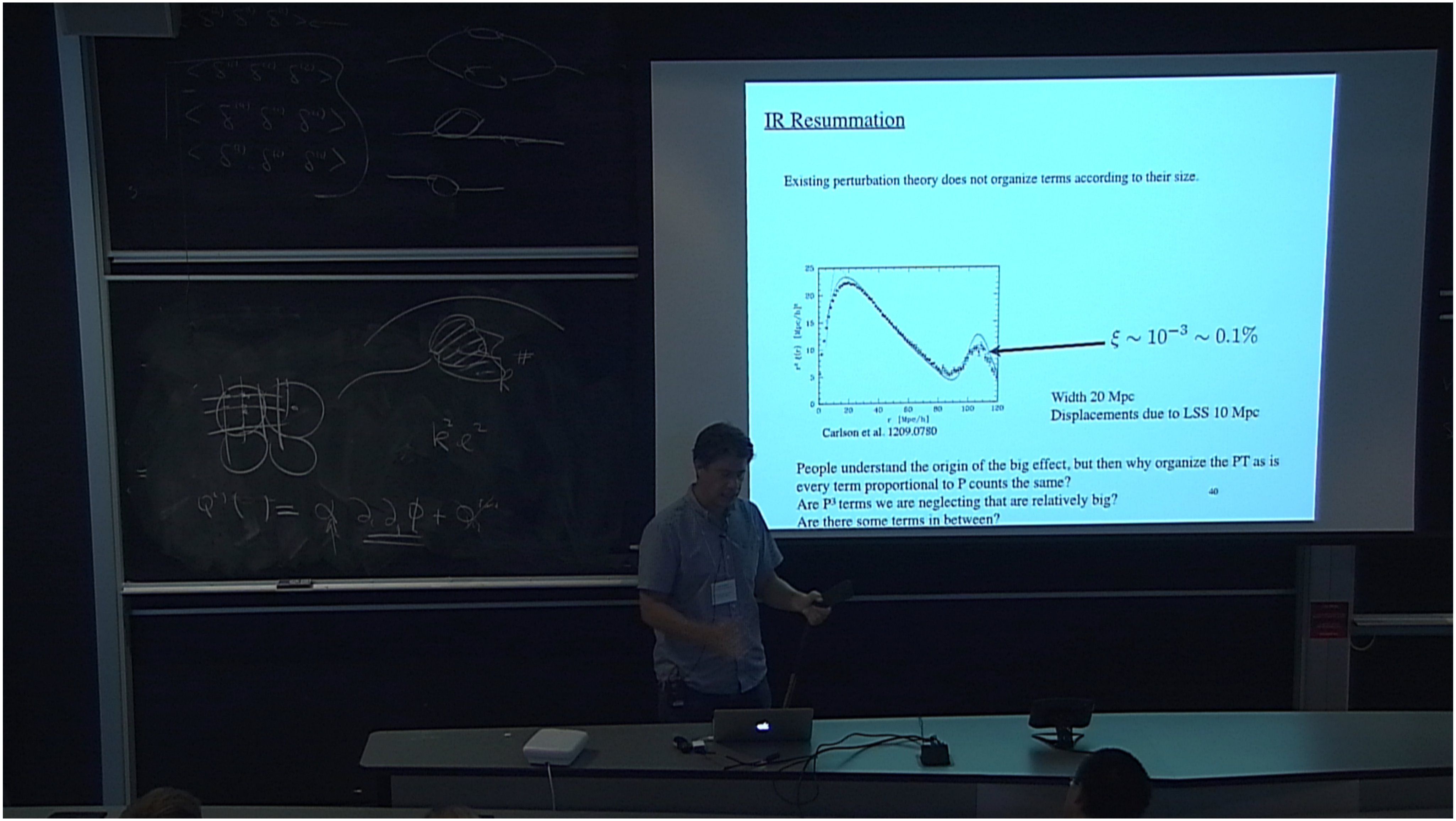


Carlson et al. 1209.0780

$\xi \sim 10^{-3} \sim 0.1\%$

Width 20 Mpc
Displacements due to LSS 10 Mpc

People understand the origin of the big effect, but then why organize the PT as is every term proportional to P counts the same?
Are P^3 terms we are neglecting that are relatively big?
Are there some terms in between?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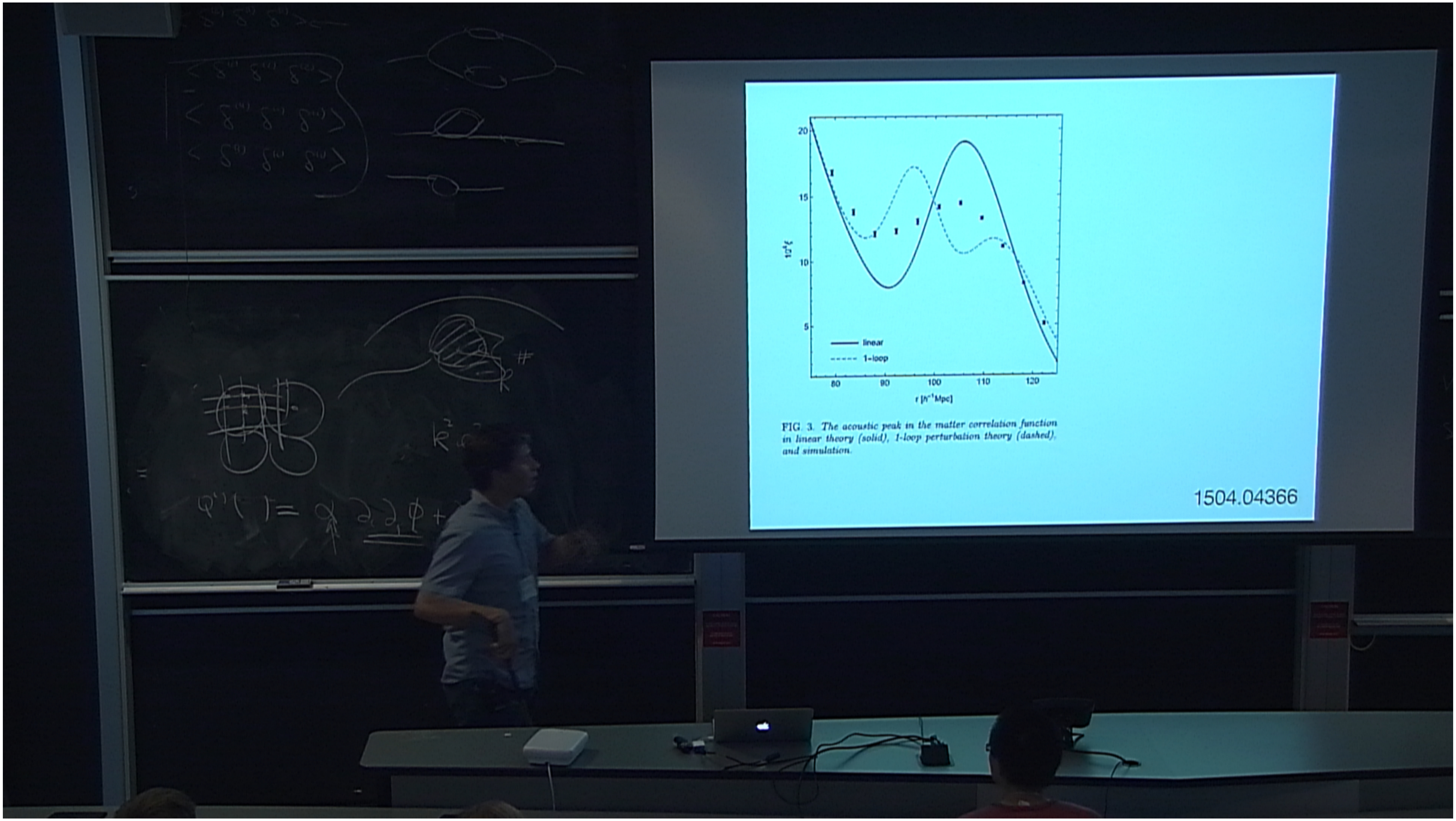
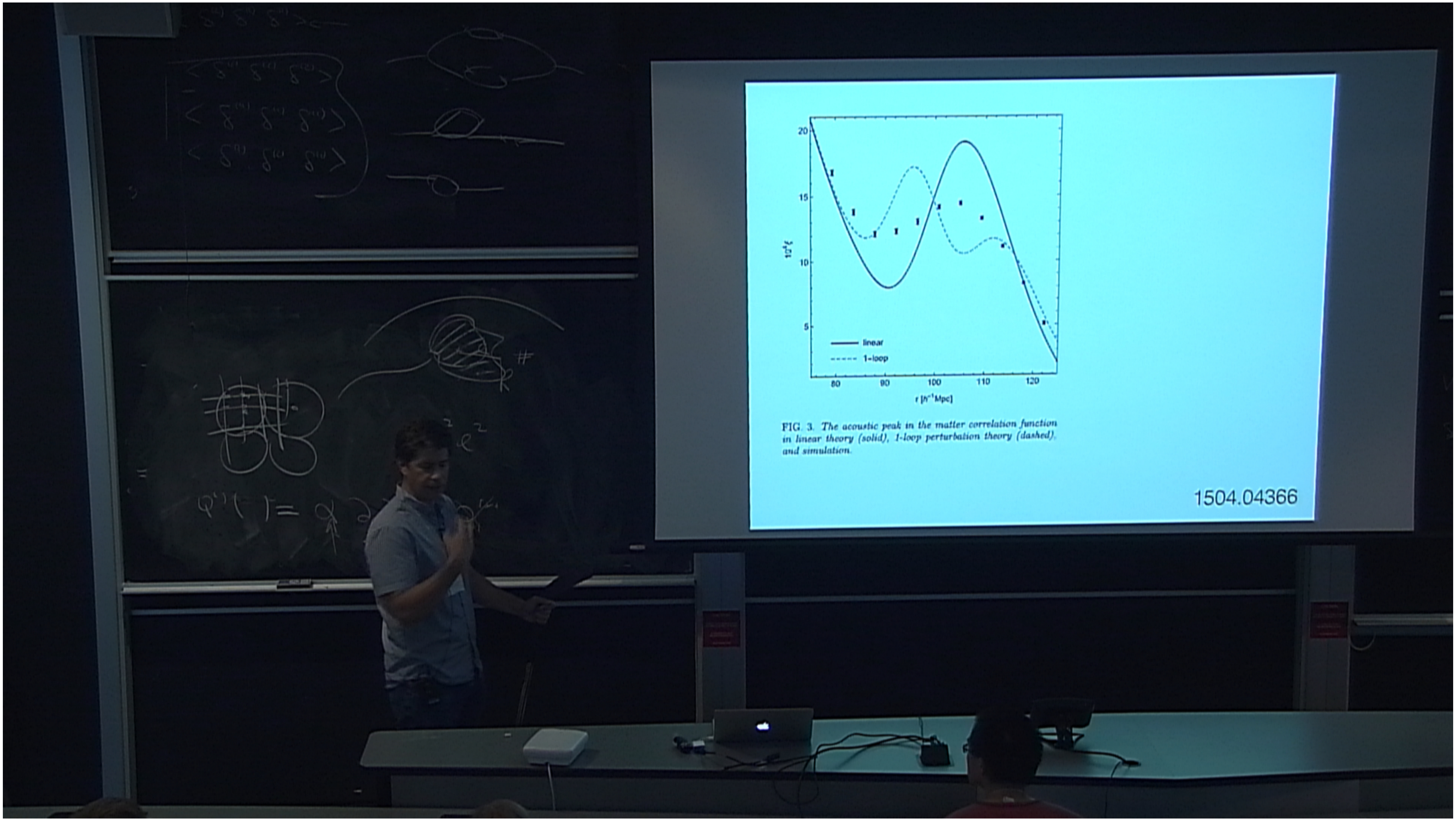
40


20
15
10
5
linear
1-loop
80
90
100
110
120
r [h^-1 Mpc]
FIG. 3. The acoustic peak in the matter correlation function in linear theory (solid), 1-loop perturbation theory (dashed), and simulation.
1504.04366



linear
1-loop
r [h^{-1} Mpc]
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