Title: Amplitudes: Overview of Jet Physics

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Abstract:

On-Shell Methods for Scattering Amplitudes

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Trisep School
Perimeter Institute
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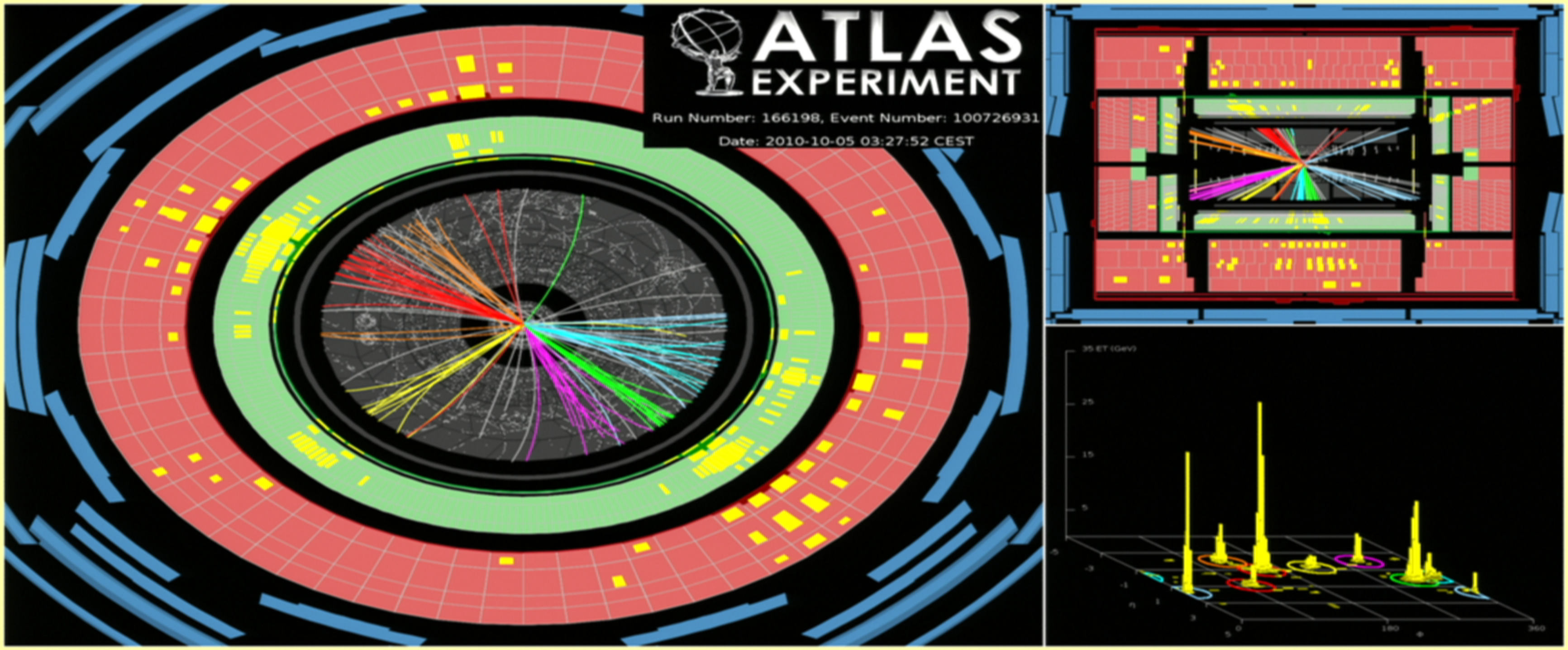
Tools for Computing Amplitudes

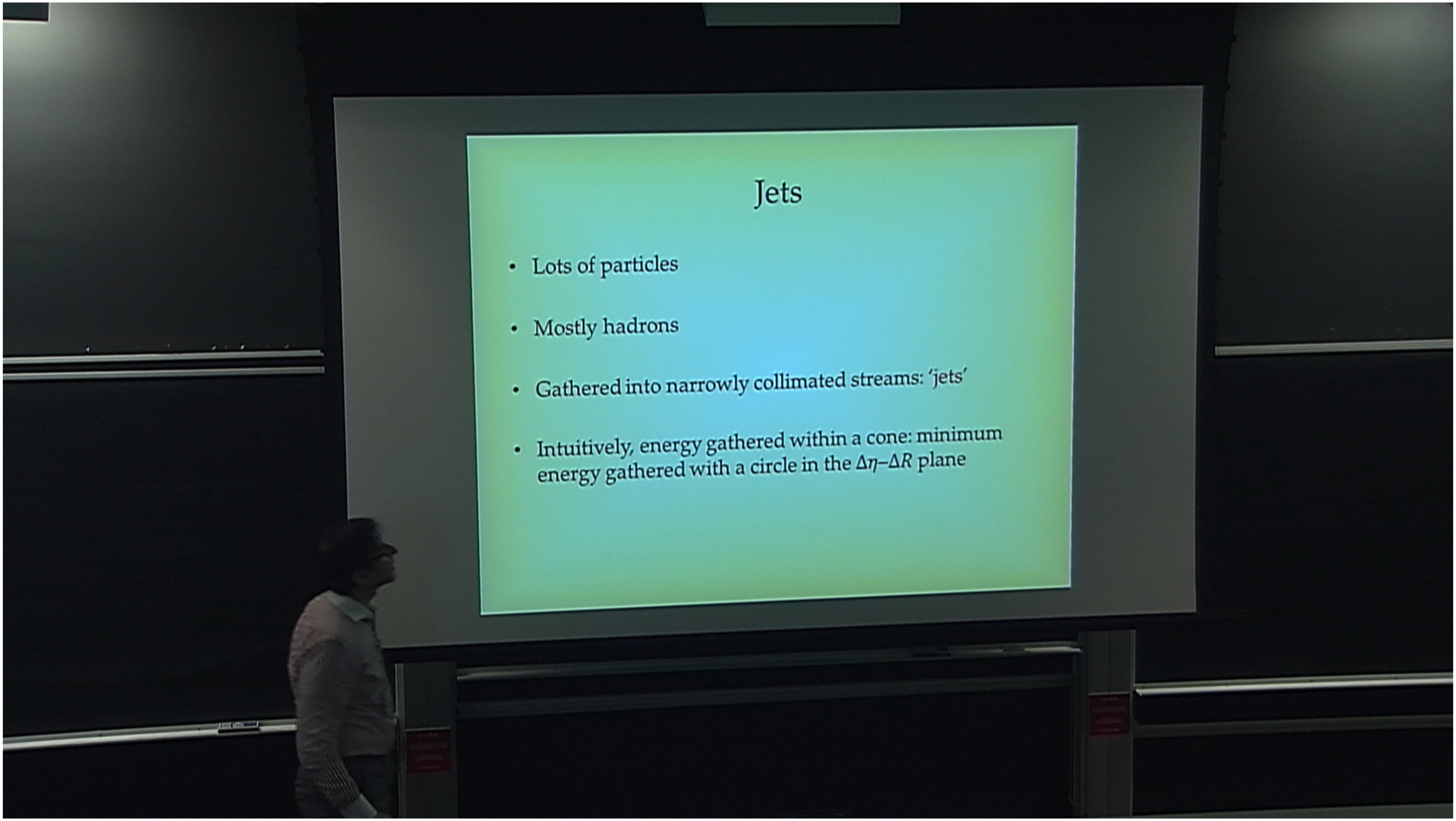
- New tools for computing in gauge theories — the core of the Standard Model

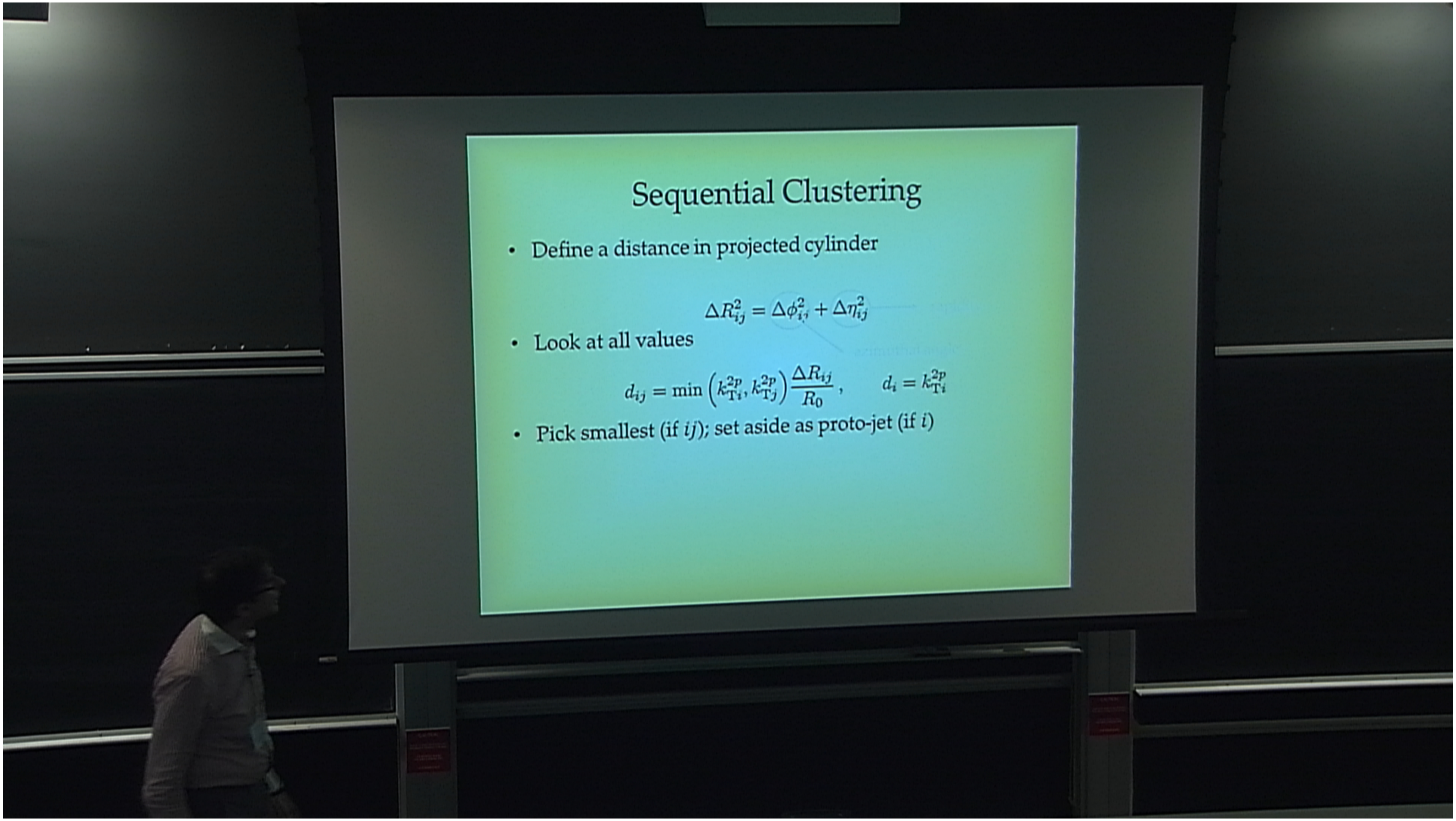
- Motivations and connections
  - Particle physics: $SU(3) \times SU(2) \times U(1)$
  - Theoretical understanding of Collider Physics
  - Precision Measurements
  - Search for physics beyond the Standard Model

- The particle content of the Standard Model is now complete, with the announcement in 2012 a Higgs-like boson by the ATLAS and CMS collaborations, looking more and more SM-like
- Every discovery opens new doors, and raises new questions
- How Standard-Model-like is the new boson?
  - We'll need precision calculations to see

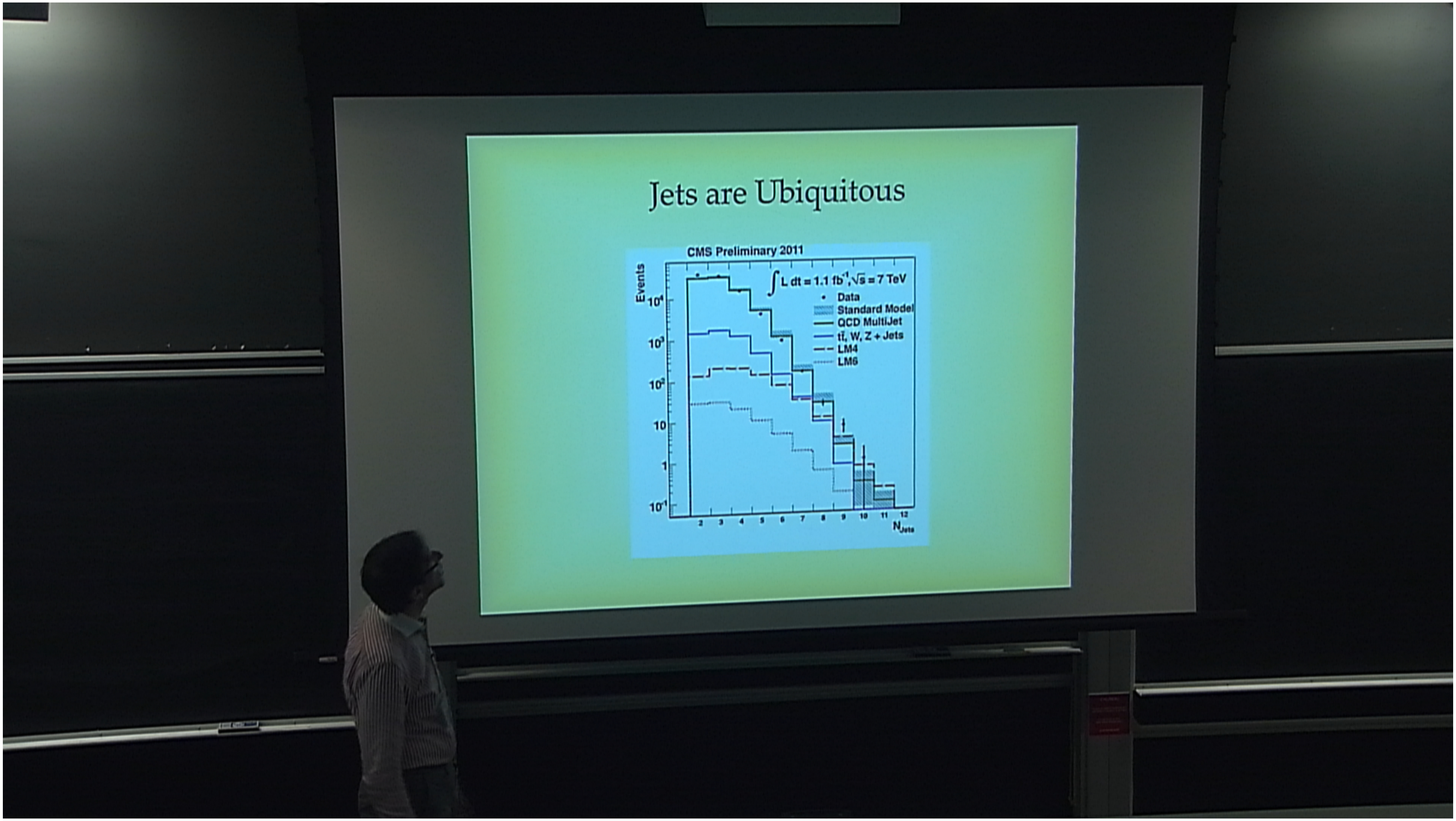
An Eight-Jet Event



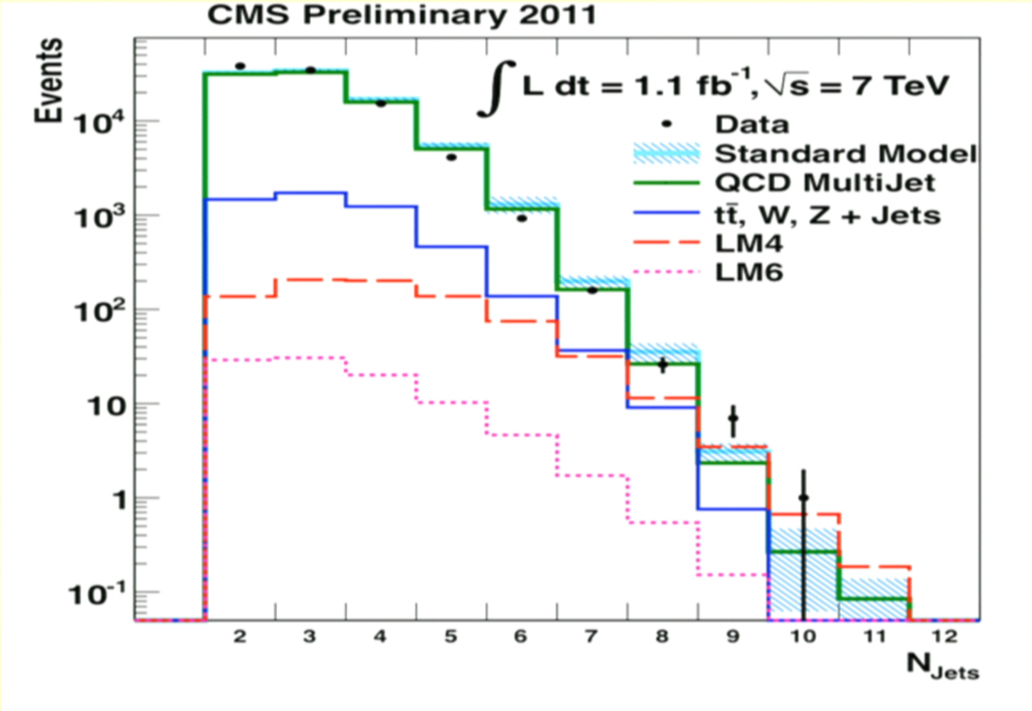

Jets
Lots of particles
Mostly hadrons
Gathered into narrowly collimated streams: 'jets'
Intuitively, energy gathered within a cone: minimum energy gathered with a circle in the Δη–ΔR plane


Sequential Clustering
• Define a distance in projected cylinder
$\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta\eta_{ij}^2$
• Look at all values
$d_{ij} = \min\left(k_{\mathrm{T}i}^{2p}, k_{\mathrm{T}j}^{2p}\right)\frac{\Delta R_{ij}}{R_0}, \qquad d_i = k_{\mathrm{T}i}^{2p}$
• Pick smallest (if ij); set aside as proto-jet (if i)


ATLAS
EXPERIMENT

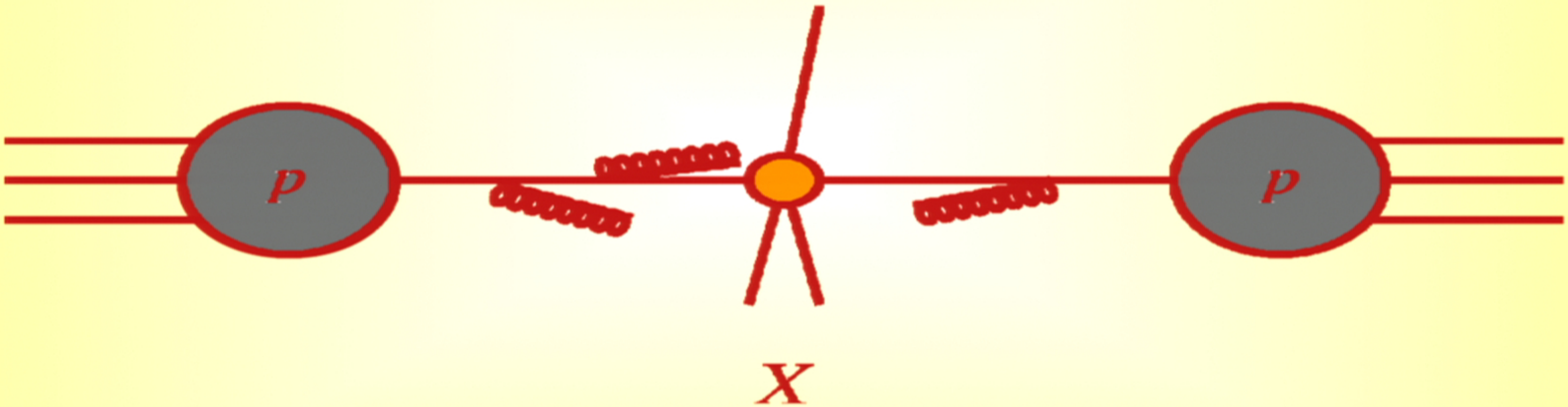

Jets are Ubiquitous
CMS Preliminary 2011
$\int L\,dt = 1.1\ fb^{-1}, \sqrt{s} = 7$ TeV
Data
Standard Model
QCD MultiJet
$t\bar{t}$, W, Z + Jets
LM4
LM6
Events
10^{4}
10^{3}
10^{2}
10
1
10^{-1}
2 3 4 5 6 7 8 9 10 11 12
N_{Jets}

Jets are Ubiquitous

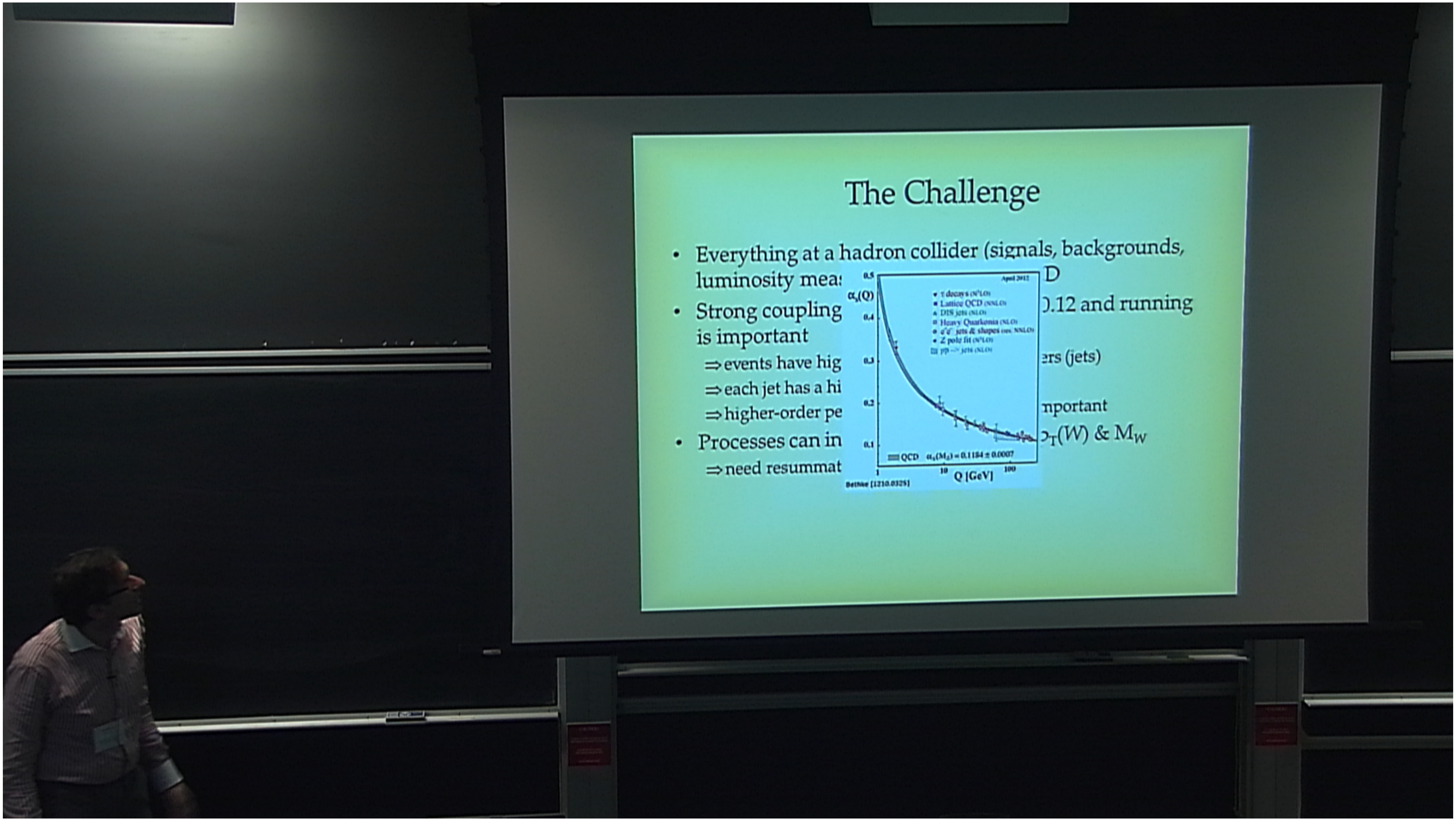


- Jets originate in the physics of QCD
- Full QCD is complicated
- Only perturbative QCD is really tractable today for these events
- Calculate in the QCD-improved parton model: for suitably averaged quantities ("infrared & collinear safe"), non-perturbative corrections are power-like, and smaller than higher-order perturbative corrections

QCD-Improved Parton Model



$$\sum_{a,b} \int dx_a dx_b \, d\text{Phase} \; f_a f_b \, \sigma_{ab} \delta(v - \text{Observable})$$


The Challenge
• Everything at a hadron collider (signals, backgrounds,
luminosity mea: D
• Strong coupling .12 and running
is important
⇒events have hig ers (jets)
⇒each jet has a hi
⇒higher-order pe mportant
• Processes can in $p_T(W)$ & M_W
⇒need resummat
$\alpha_s(Q)$
April 2012
Lattice QCD (NNLO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
Z pole fit
QCD $\alpha_s(M_Z) = 0.1184 \pm 0.0007$
Q [GeV]
Bethke [1210.0325]

The Challenge

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- Strong coupling [...]0.12 and running is important
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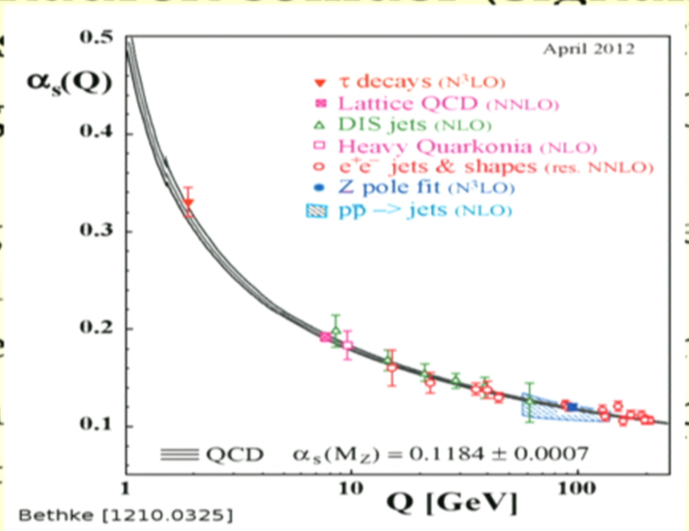

Approaches

- General parton-level fixed-order calculations
  - Numerical jet programs: general observables
  - Systematic to higher order/high multiplicity in perturbation theory
  - Parton-level, approximate jet algorithm; match detector events only statistically
- Parton showers
  - General observables
  - Leading- or next-to-leading logs only, approximate for higher order/high multiplicity
  - Can hadronize & look at detector response event-by-event
  - Understood how to match to matrix elements at leading order
- Semi-analytic calculations/resummations
  - Specific observable, for high-value targets
  - Checks on general fixed-order calculations

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Renormalization Scale

- Needed to define the coupling
- Physical quantities should be independent of it
- Truncated perturbation theory isn't
- Dependence is ~ the first missing order * logs
- Similarly for factorization scale — define parton distributions

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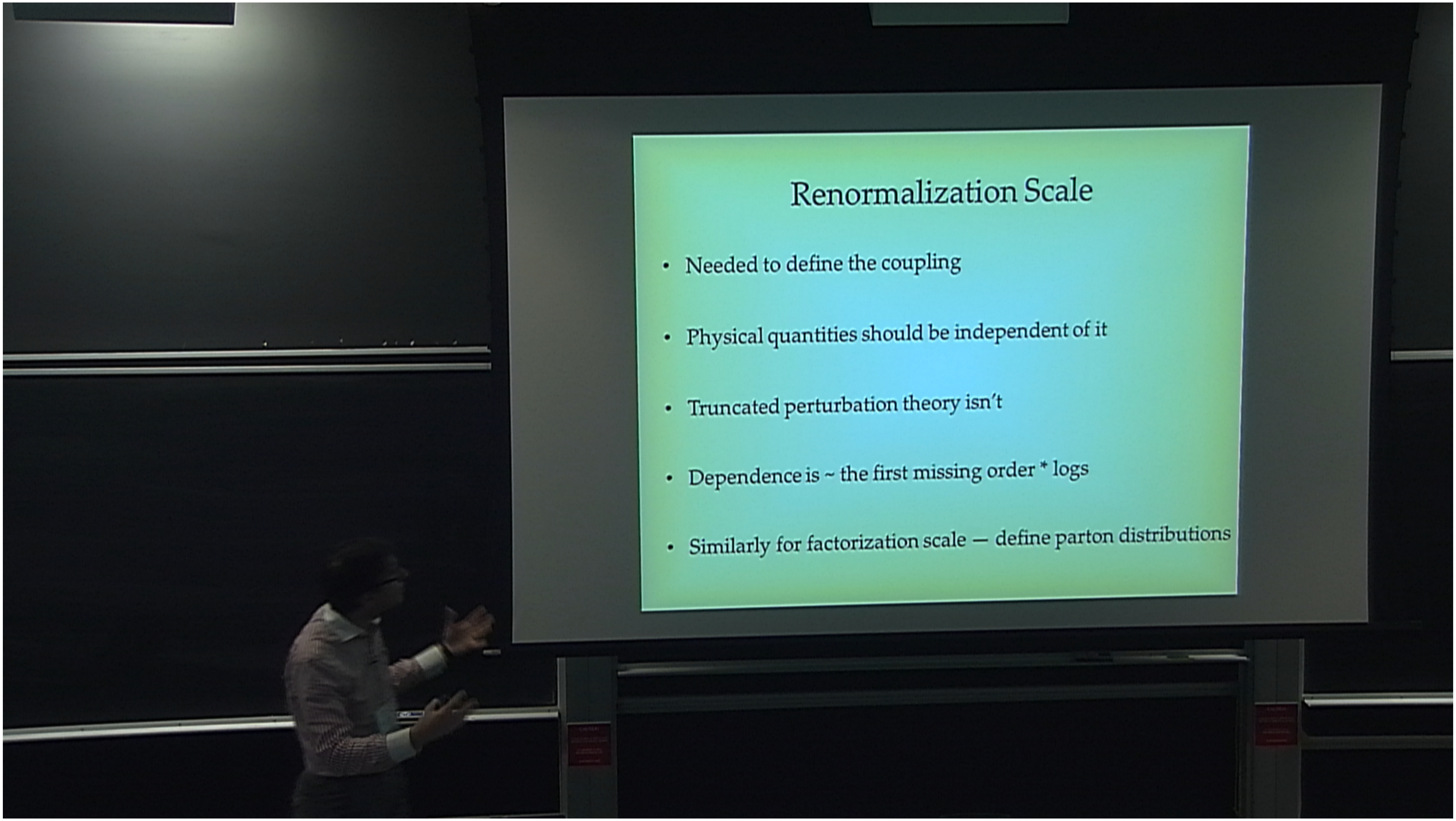
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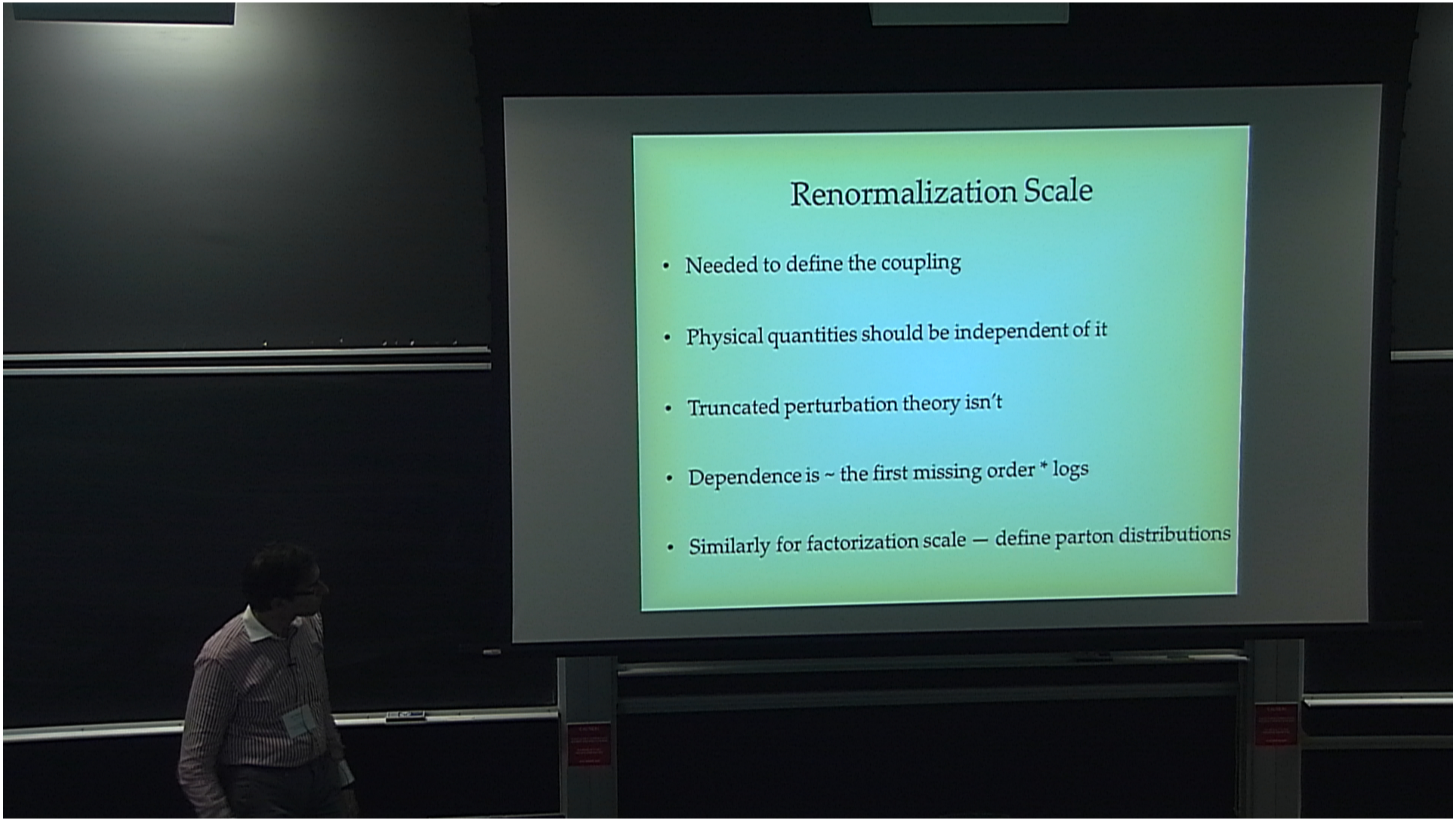
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Every sensible observable has an expansion in α_s

$$\frac{d\sigma}{d\mathcal{O}} = \alpha_s^{n_0}(\mu)\frac{d\hat{\sigma}^{\rm LO}}{d\mathcal{O}} + \alpha_s^{n_0+1}(\mu)\frac{d\hat{\sigma}^{\rm NLO}(\mu)}{d\mathcal{O}} + \alpha_s^{n_0+2}(\mu)\frac{d\hat{\sigma}^{\rm NNLO}(\mu)}{d\mathcal{O}}$$

Examples

$$\frac{d\sigma^{W+1\,\rm jet}}{dp_{\rm T}^{\rm jet}} = \alpha_s(\mu)\frac{d\hat{\sigma}^{\rm LO}}{dp_{\rm T}^{\rm jet}} + \alpha_s^2(\mu)\frac{d\hat{\sigma}^{\rm NLO}(\mu)}{dp_{\rm T}^{\rm jet}} + \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\rm NNLO}(\mu)}{dp_{\rm T}^{\rm jet}}$$

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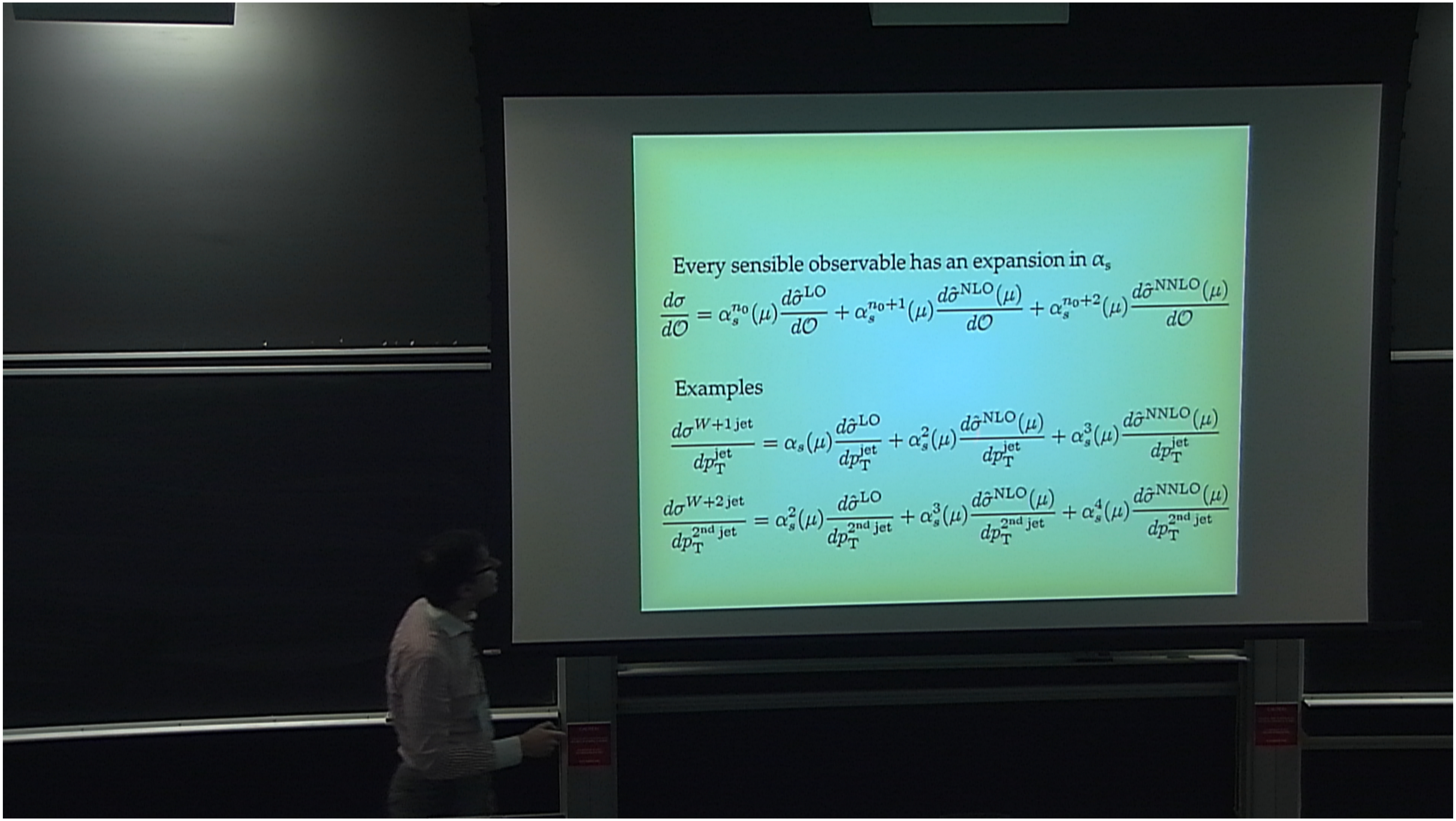
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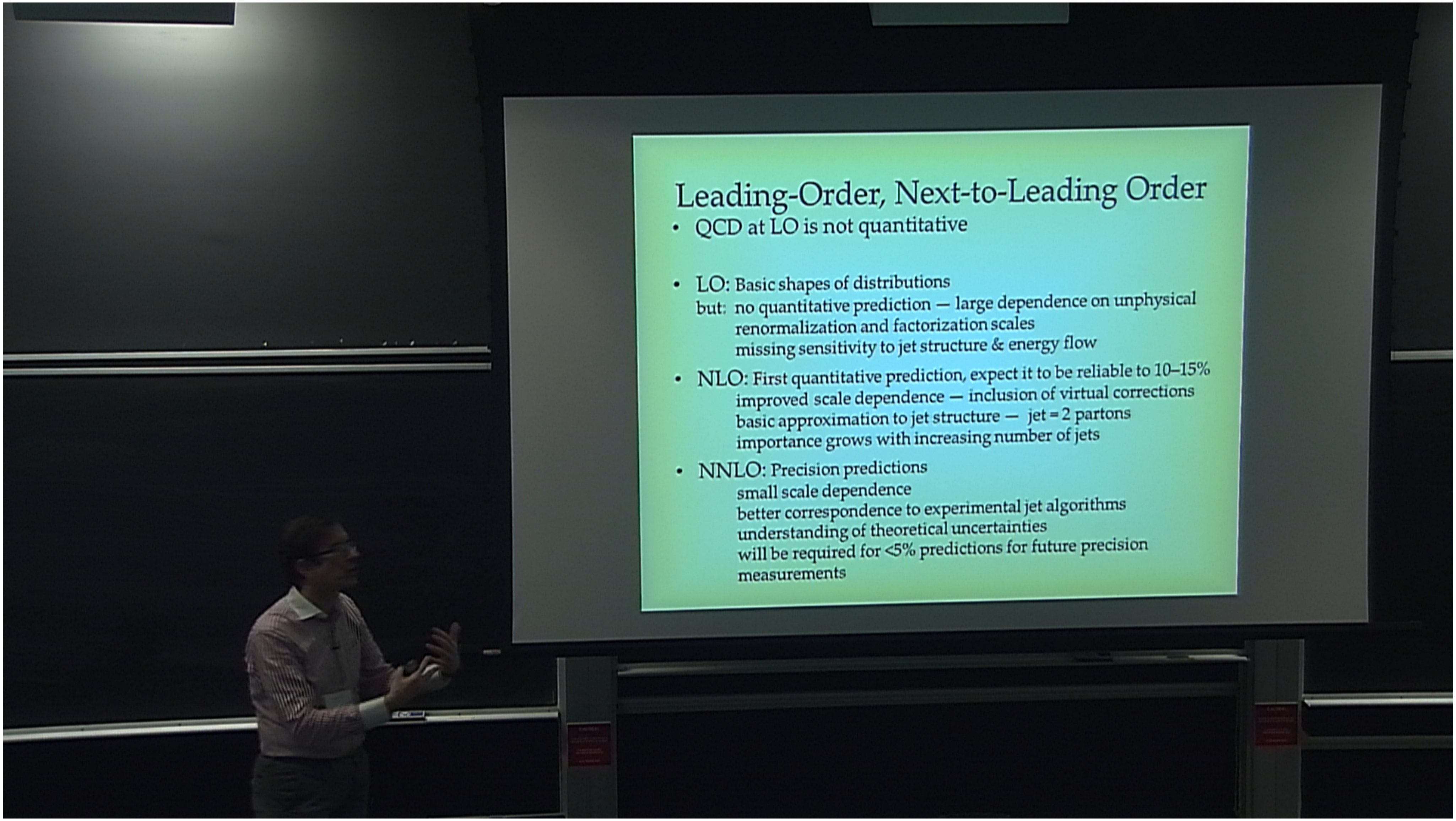
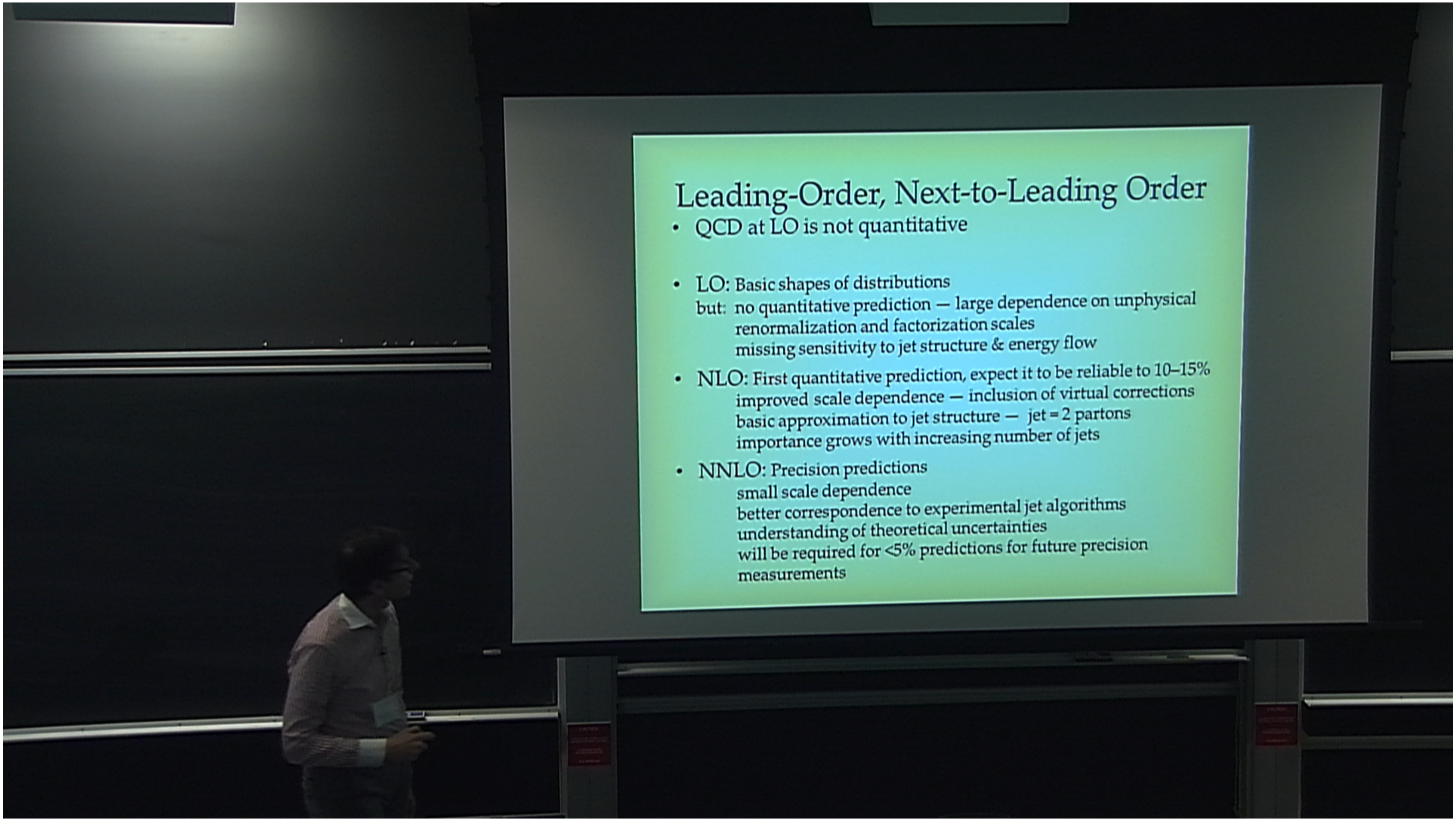

Leading-Order, Next-to-Leading Order
• QCD at LO is not quantitative
• LO: Basic shapes of distributions
but: no quantitative prediction — large dependence on unphysical
renormalization and factorization scales
missing sensitivity to jet structure & energy flow
• NLO: First quantitative prediction, expect it to be reliable to 10–15%
improved scale dependence — inclusion of virtual corrections
basic approximation to jet structure — jet = 2 partons
importance grows with increasing number of jets
• NNLO: Precision predictions
small scale dependence
better correspondence to experimental jet algorithms
understanding of theoretical uncertainties
will be required for <5% predictions for future precision
measurements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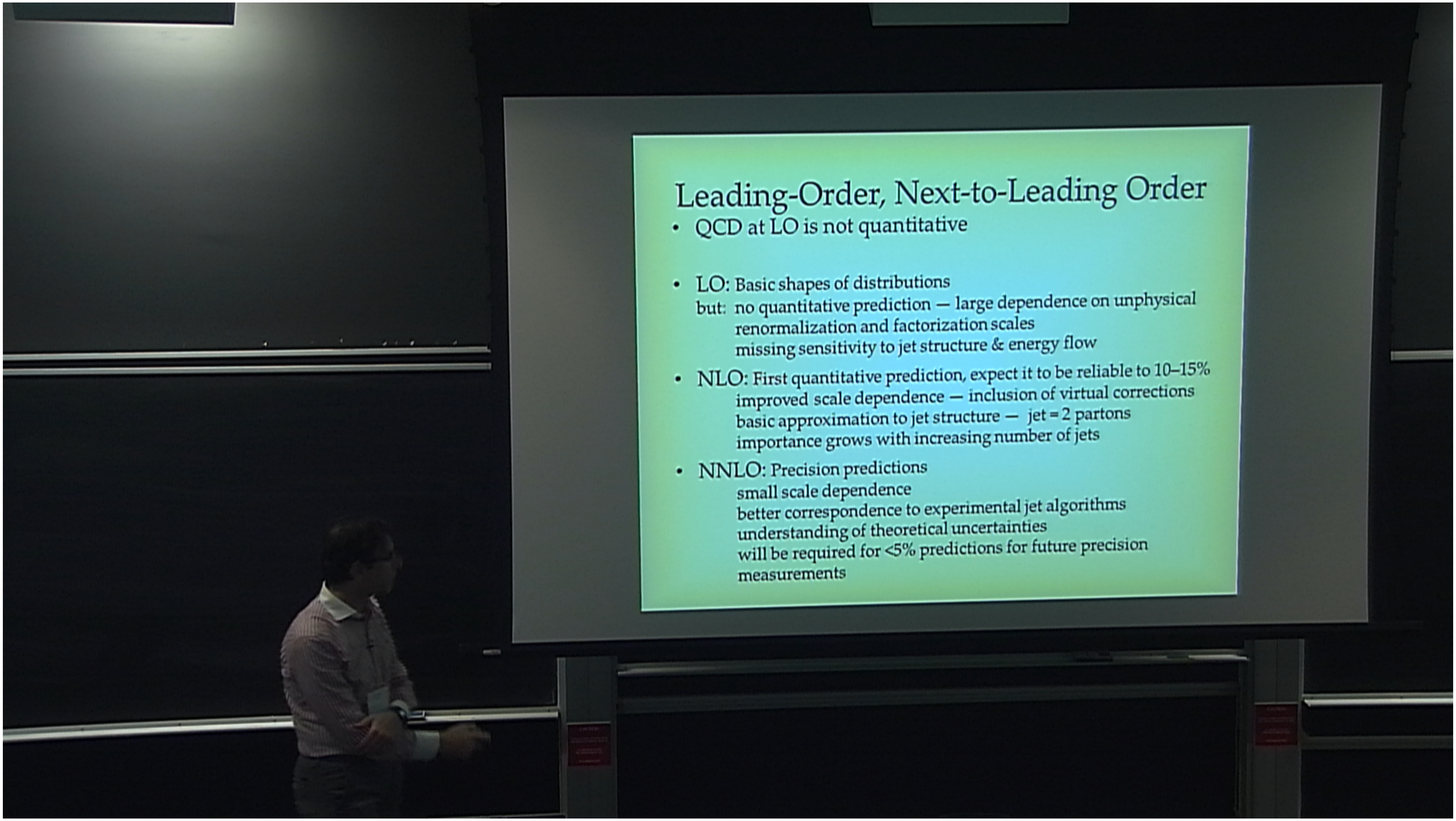
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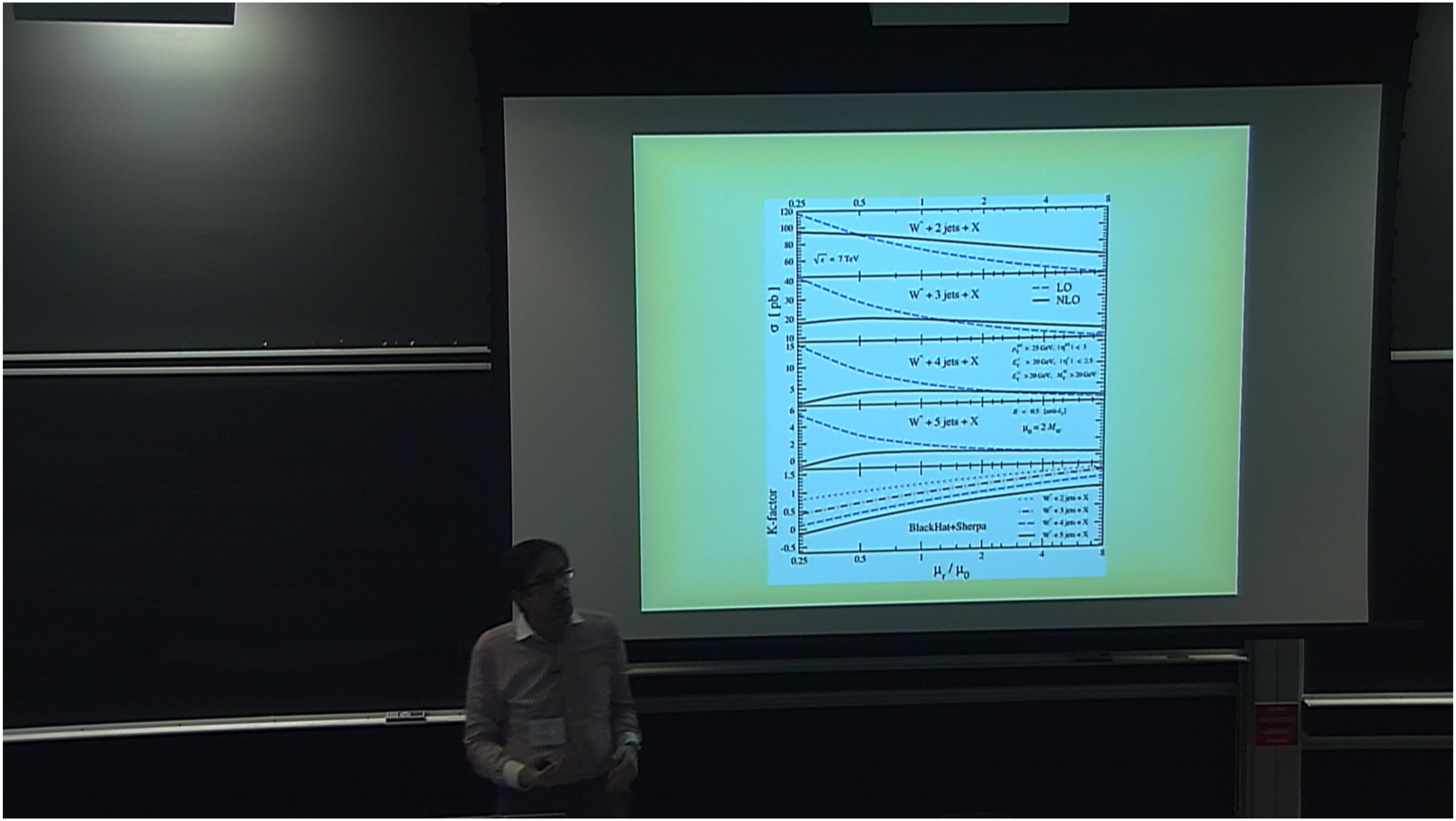
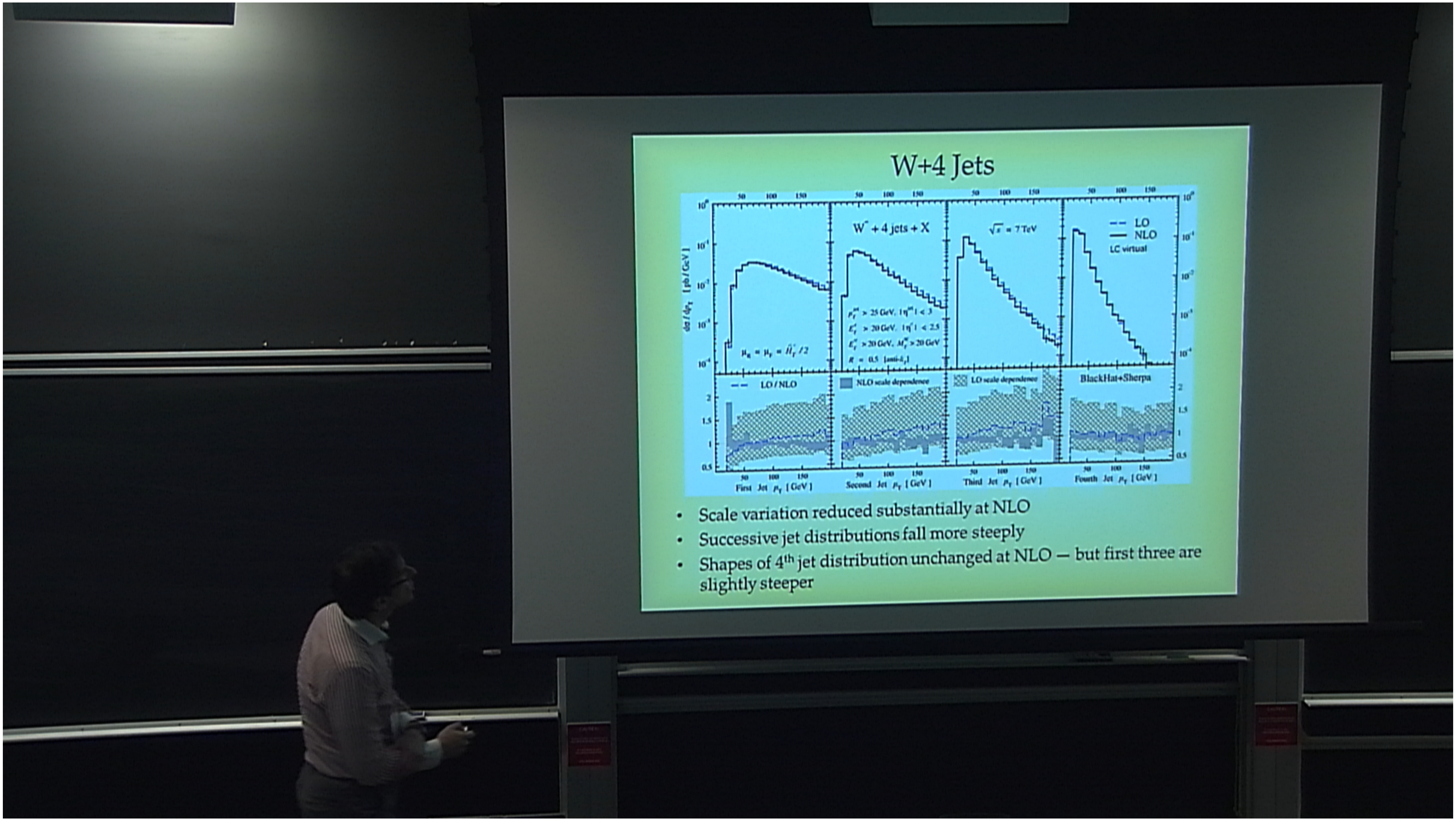
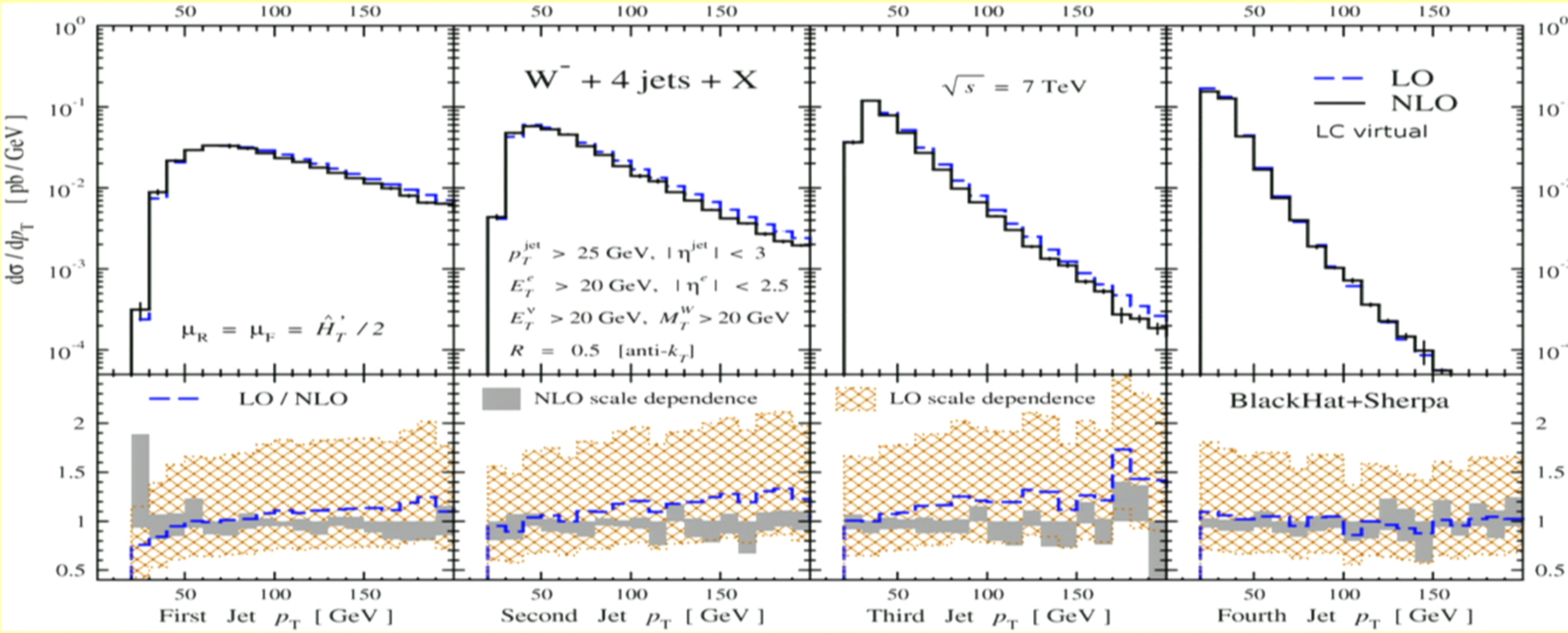


W + 2 jets + X
W + 3 jets + X
W + 4 jets + X
W + 5 jets + X
LO
NLO
σ [pb]
K-factor
BlackHat+Sherpa
μr / μ0


W+4 Jets
W⁻ + 4 jets + X
√s = 7 TeV
LO
NLO
LC virtual
LO / NLO
NLO scale dependence
LO scale dependence
BlackHat+Sherpa
First Jet p_T [GeV]
Second Jet p_T [GeV]
Third Jet p_T [GeV]
Fourth Jet p_T [GeV]
• Scale variation reduced substantially at NLO
• Successive jet distributions fall more steeply
• Shapes of 4th jet distribution unchanged at NLO — but first three are slightly steeper

W+4 Jets



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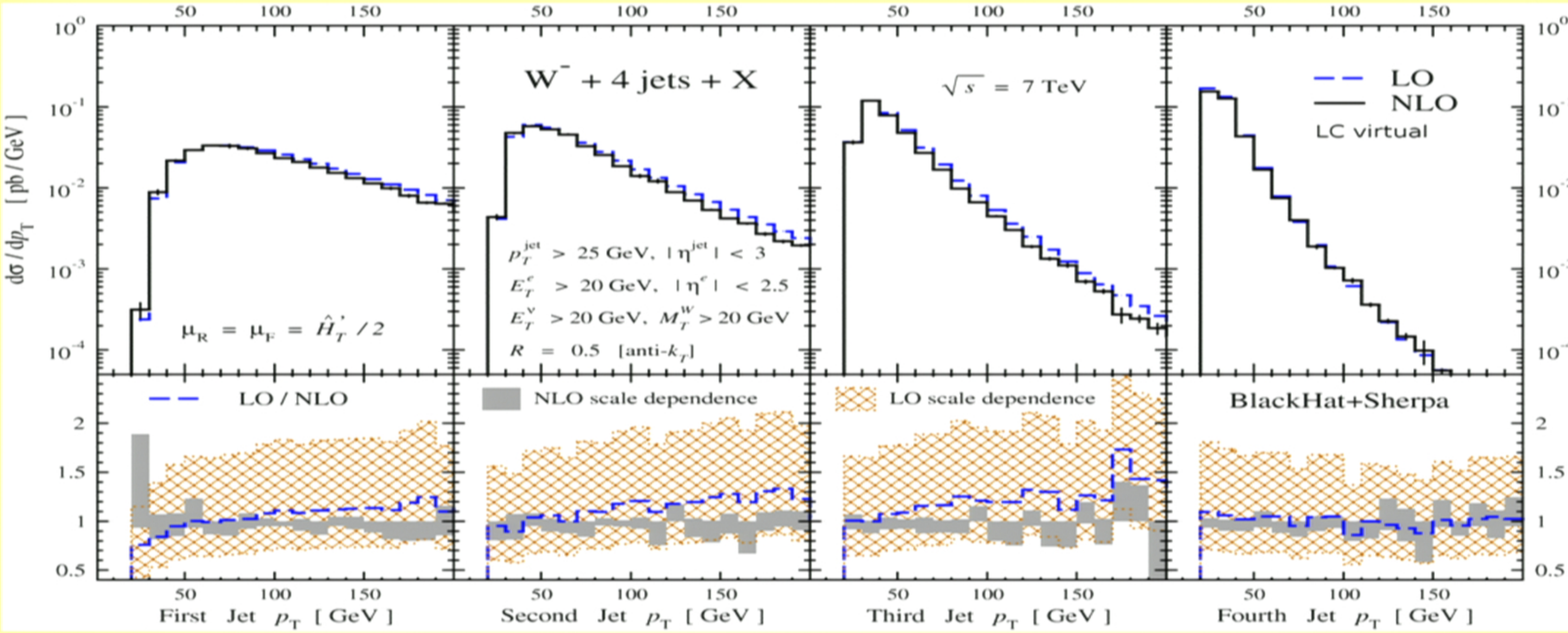

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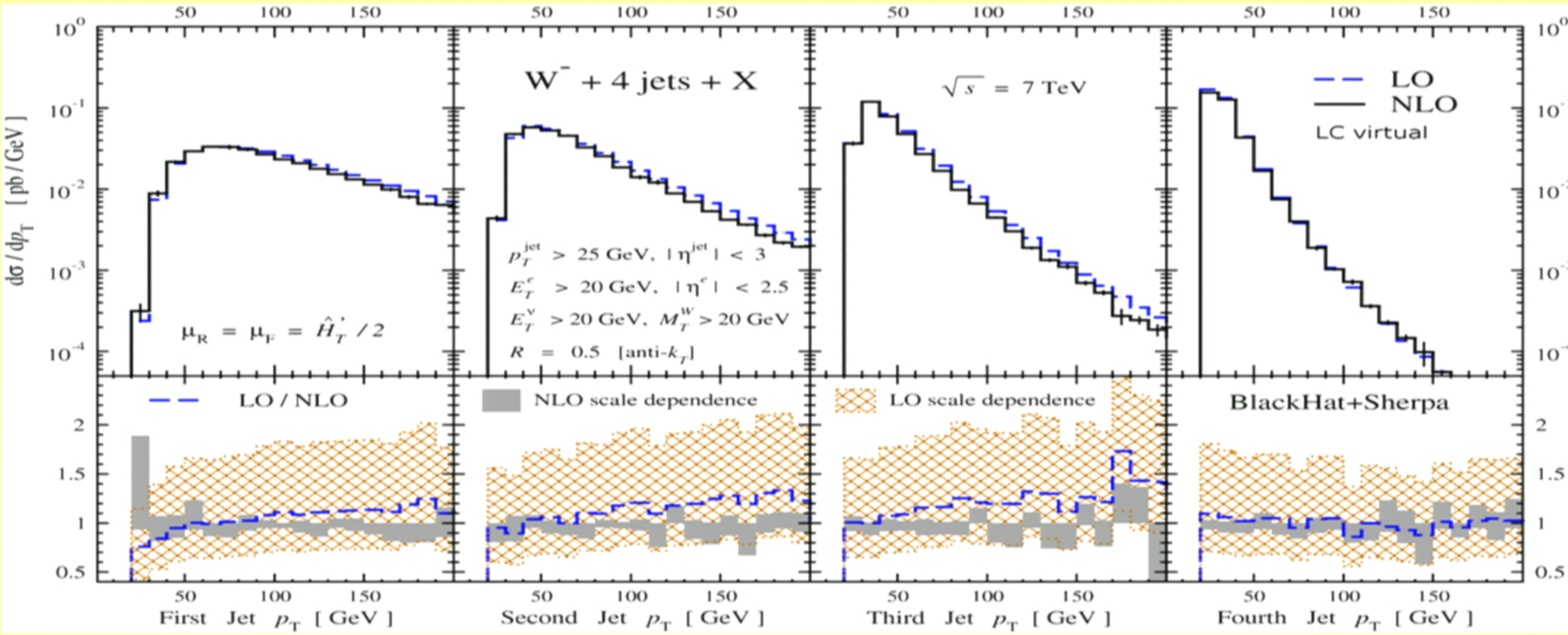

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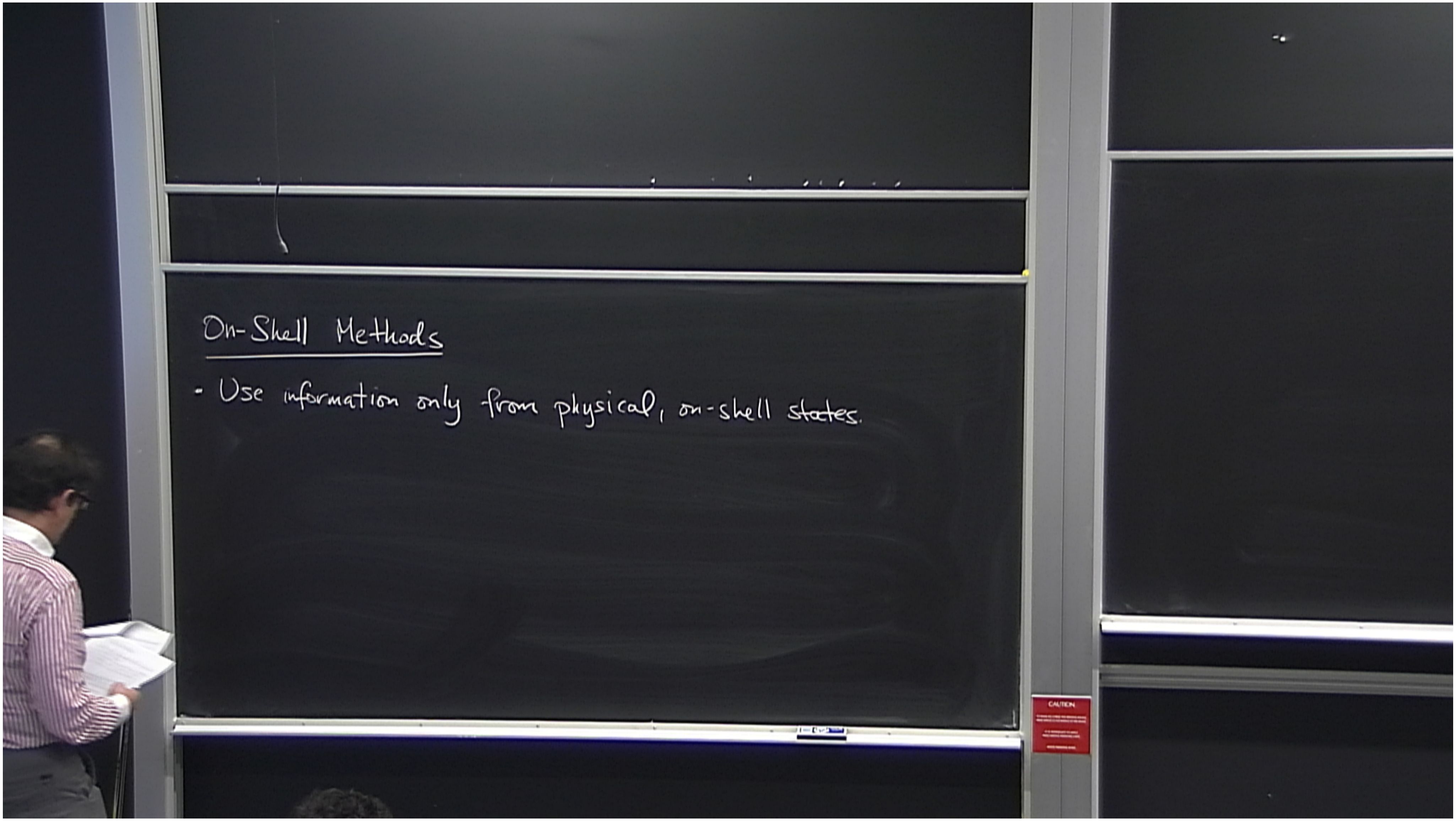
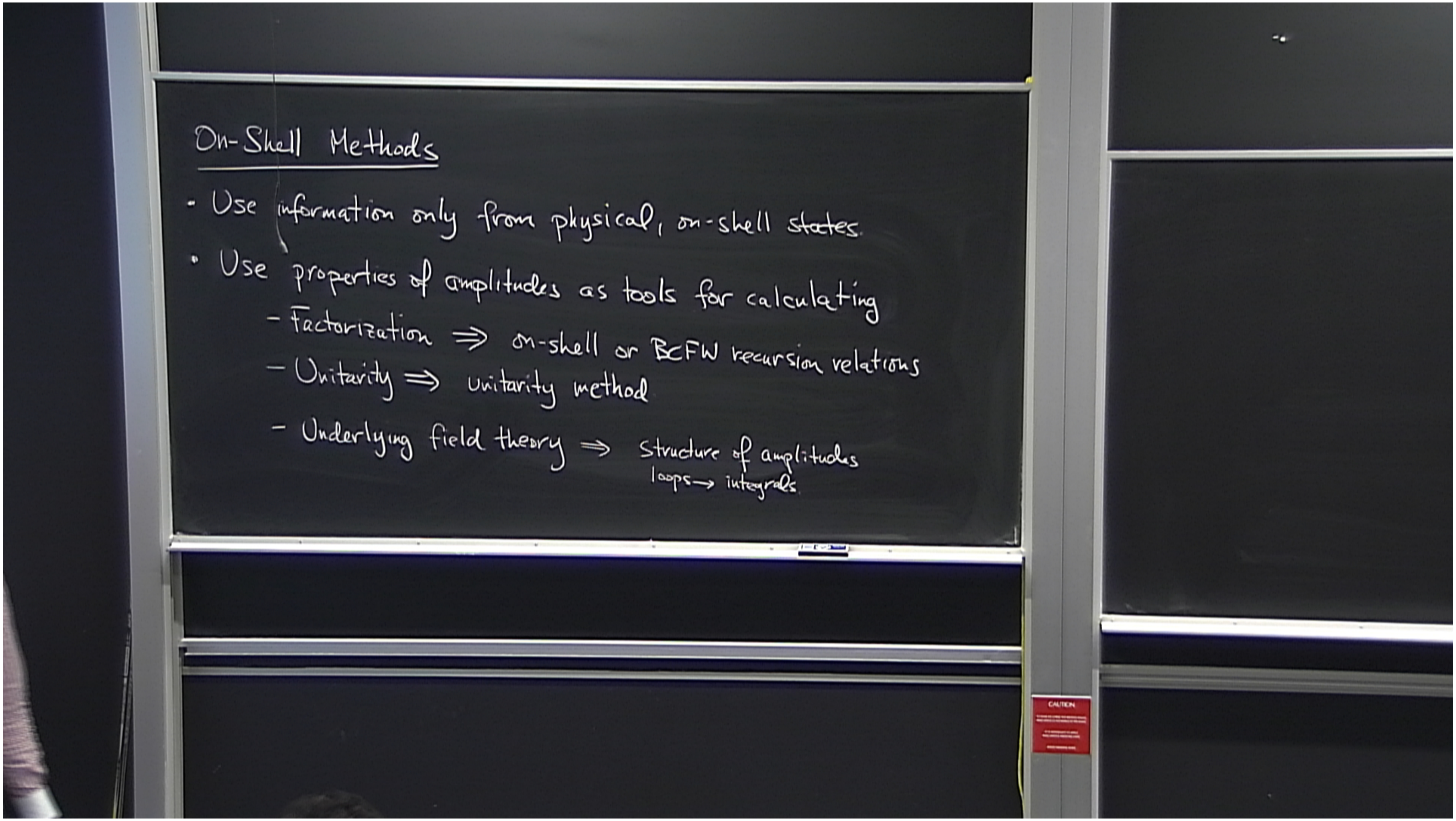
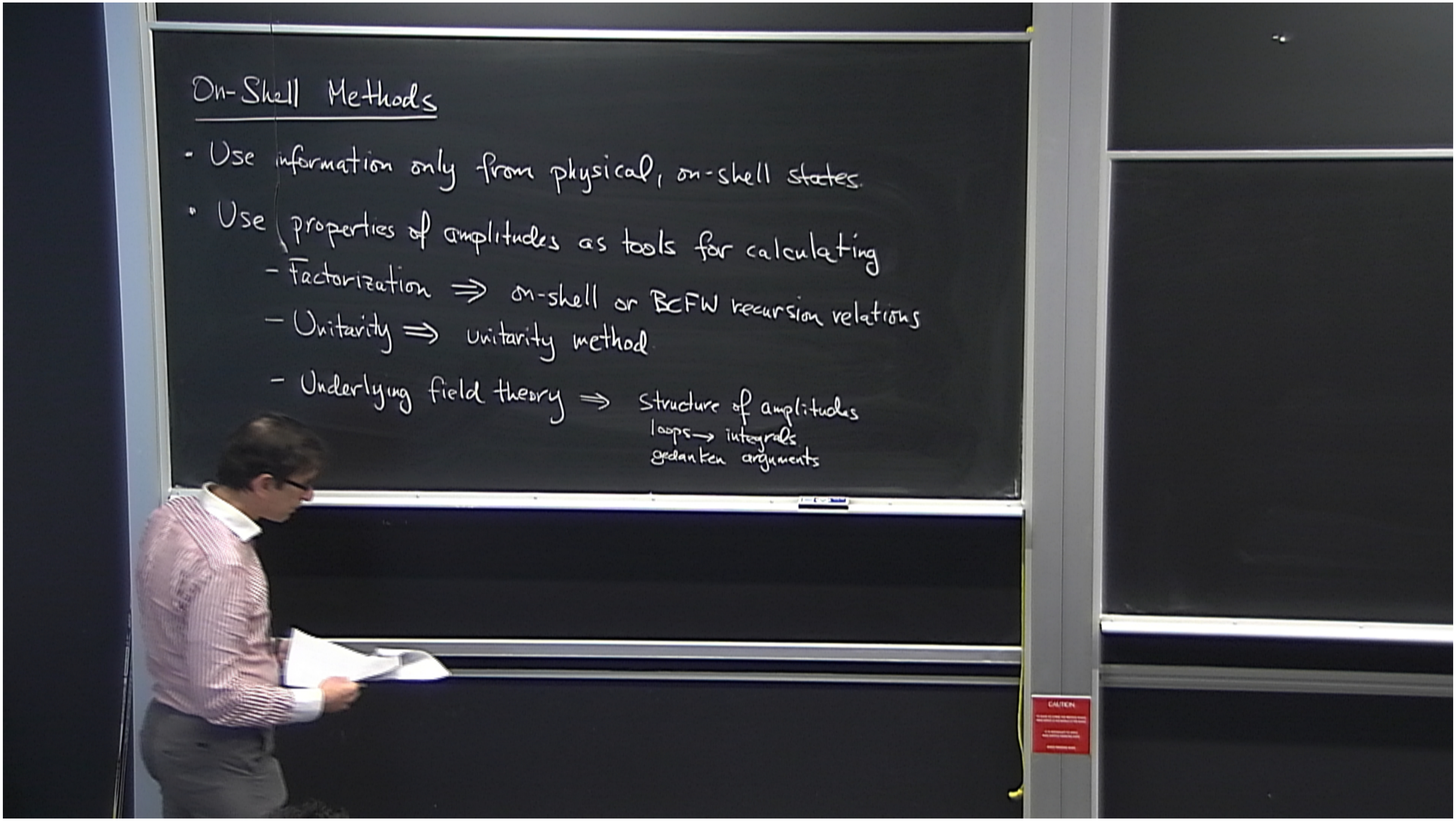

On-Shell Methods
- Use information only from physical, on-shell states.


On-Shell Methods
• Use information only from physical, on-shell states.
• Use properties of amplitudes as tools for calculating
– Factorization ⇒ on-shell or BCFW recursion relations
– Unitarity ⇒ unitarity method
– Underlying field theory ⇒ structure of amplitudes
loops → integrals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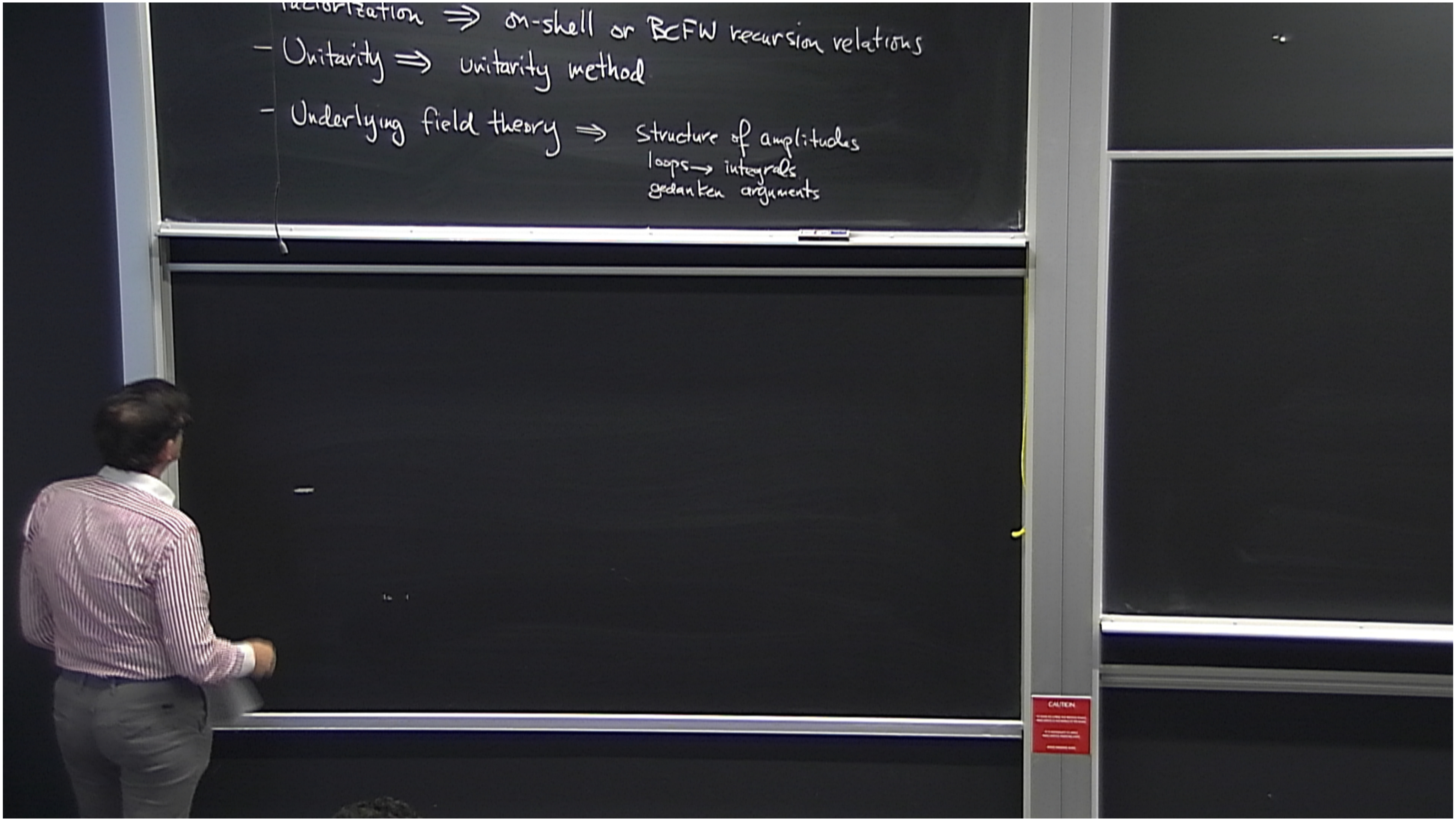
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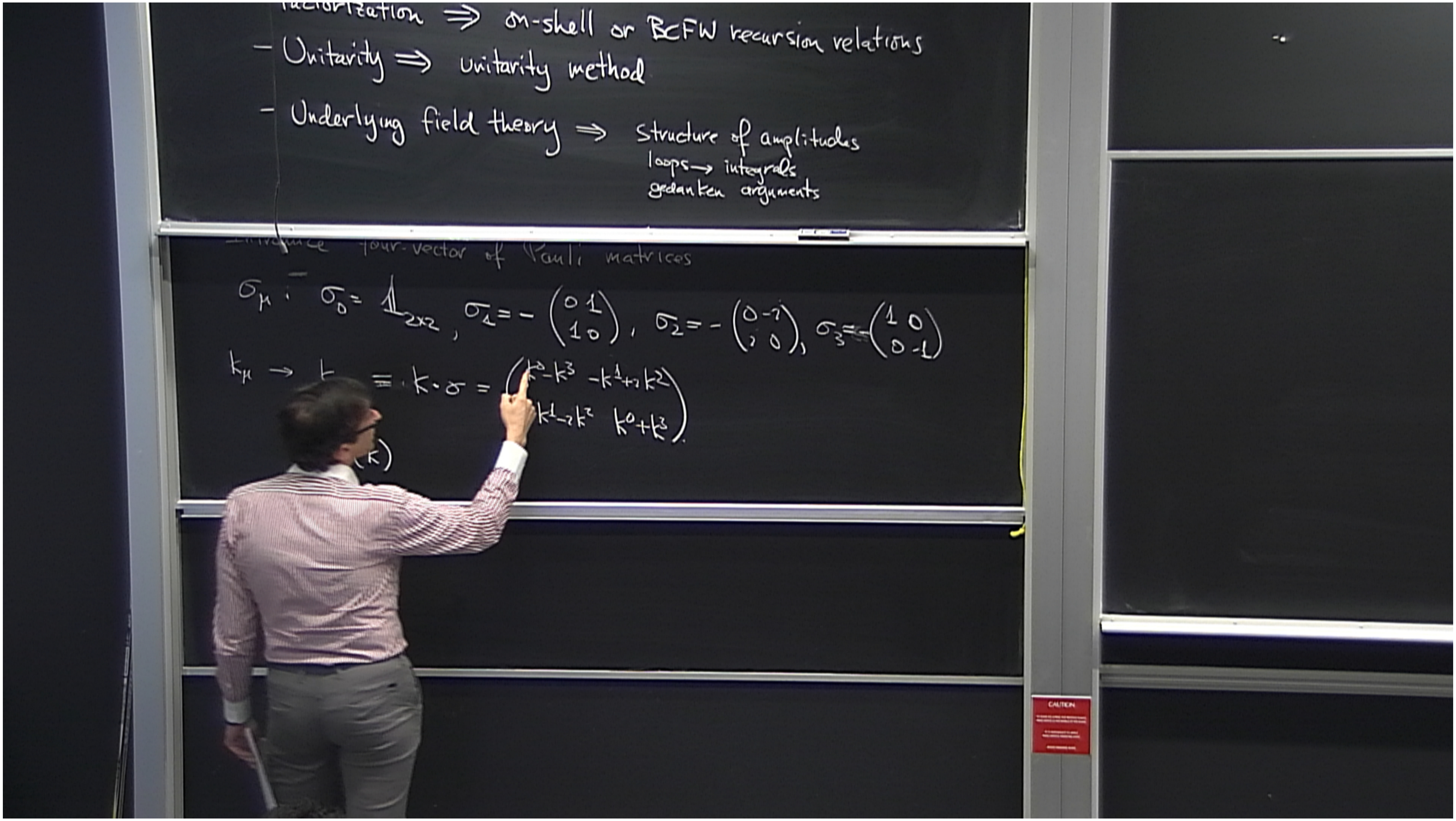
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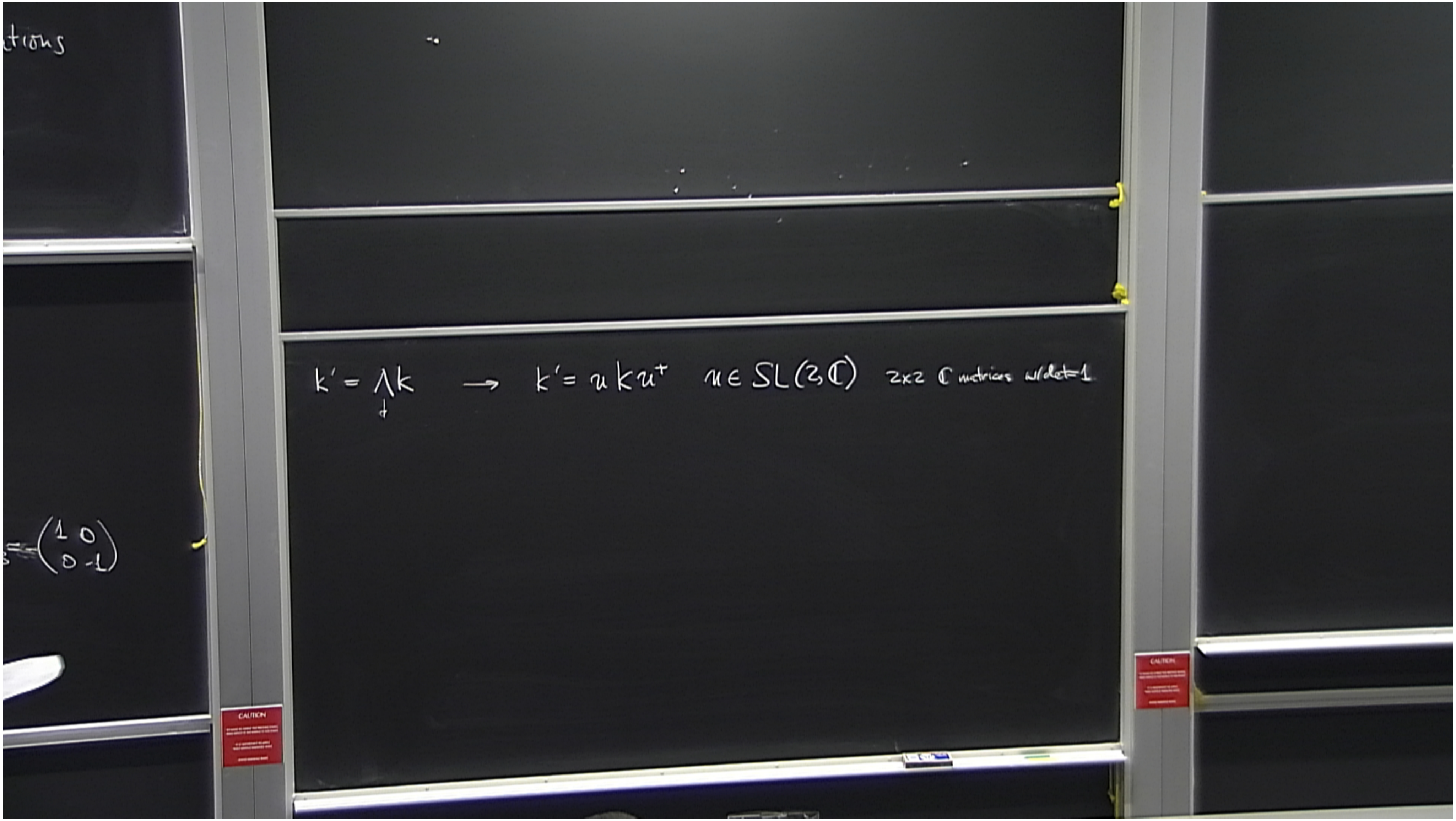
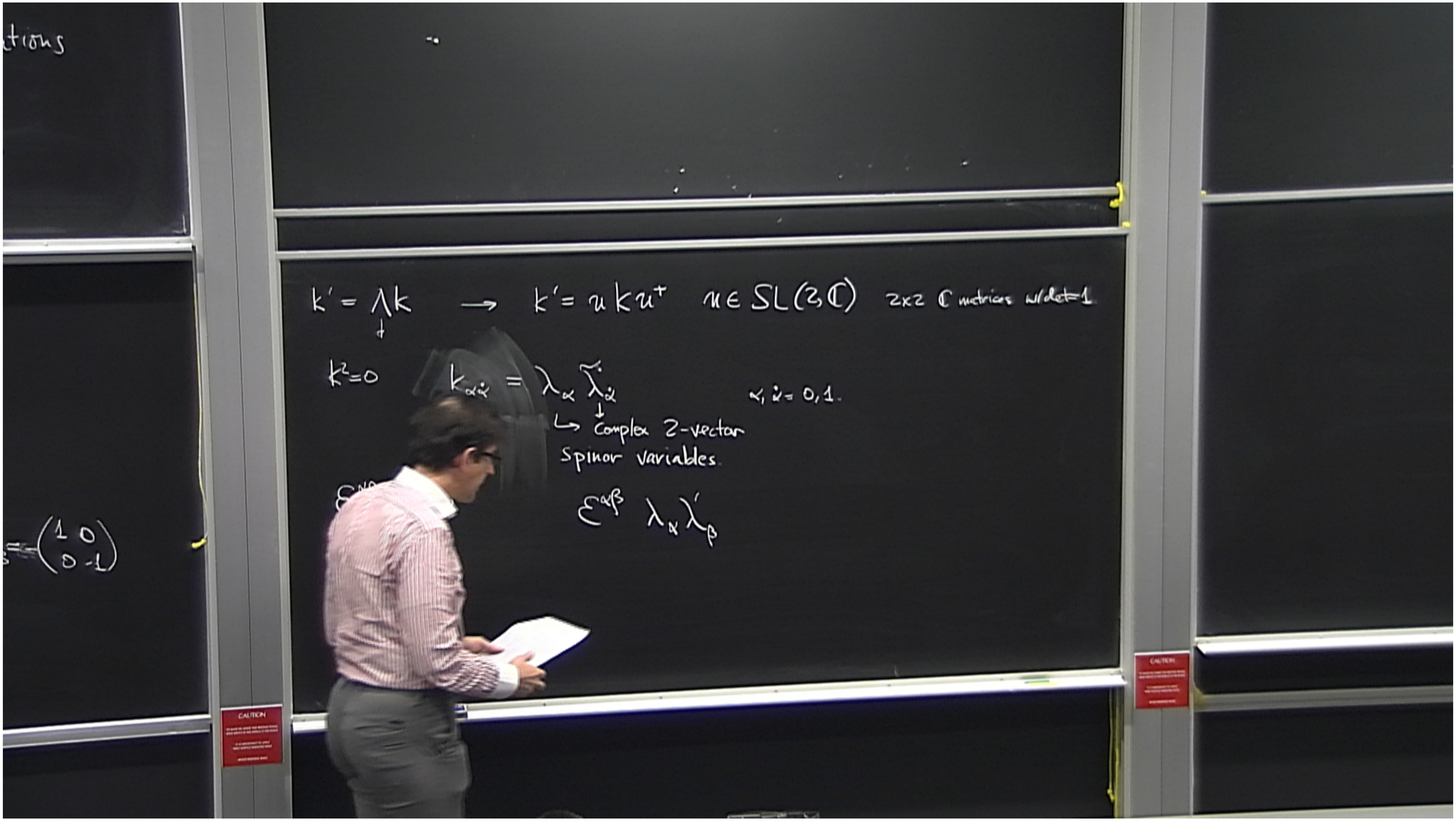
four-vector of Pauli matrices
$\sigma_\mu : \sigma_0 = \mathbb{1}_{2\times2}, \sigma_1 = -\begin{pmatrix}0&1\\1&0\end{pmatrix}, \sigma_2 = -\begin{pmatrix}0&-i\\i&0\end{pmatrix}, \sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$
$k_\mu \rightarrow$
$= k\cdot\sigma = \begin{pmatrix}k^0-k^3 & -k^1+ik^2\\ -k^1-ik^2 & k^0+k^3\end{pmatrix}$


k' = Λk → k' = u k u⁺ u ∈ SL(2,ℂ) 2x2 ℂ matrices w/det=1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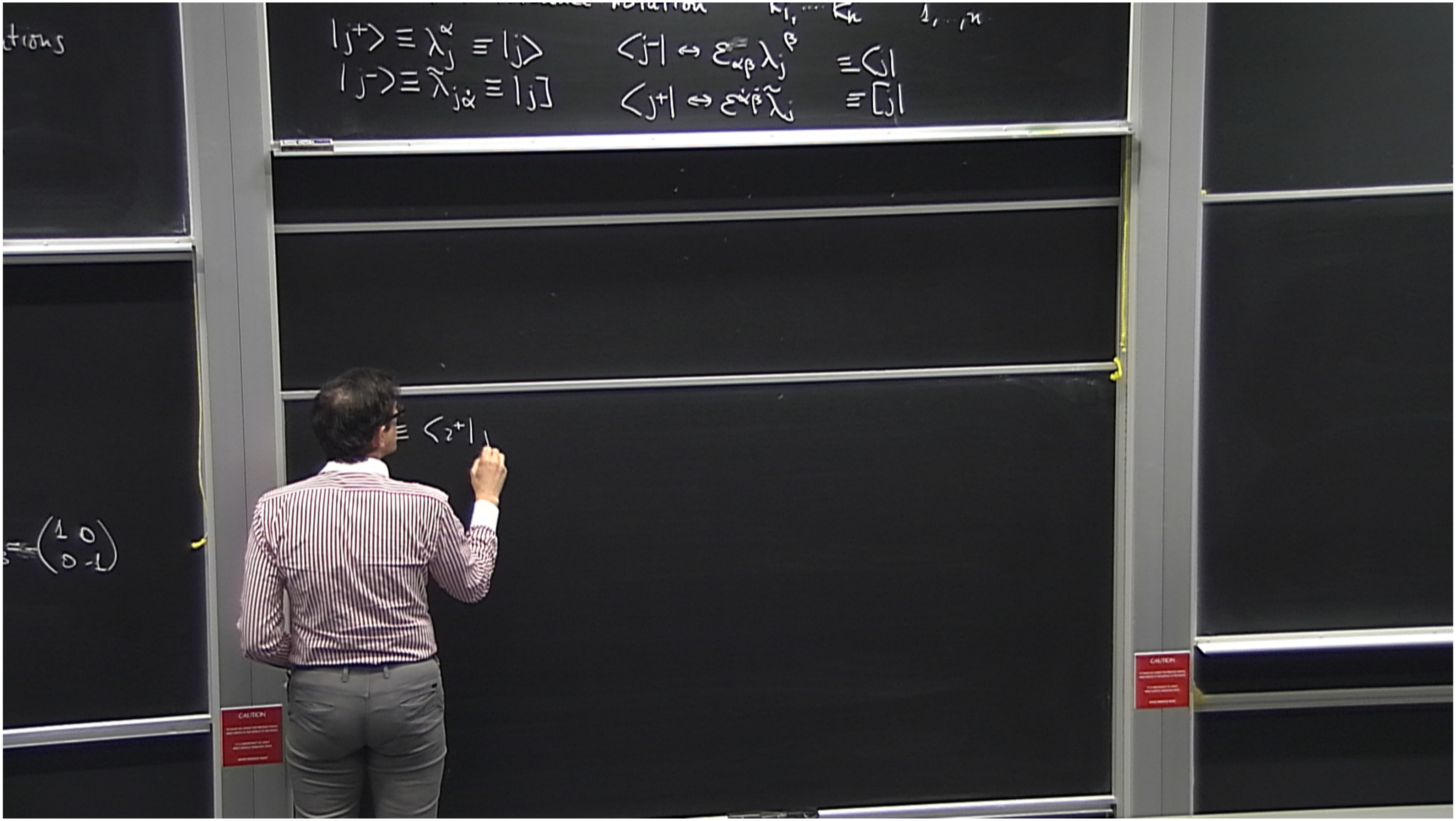
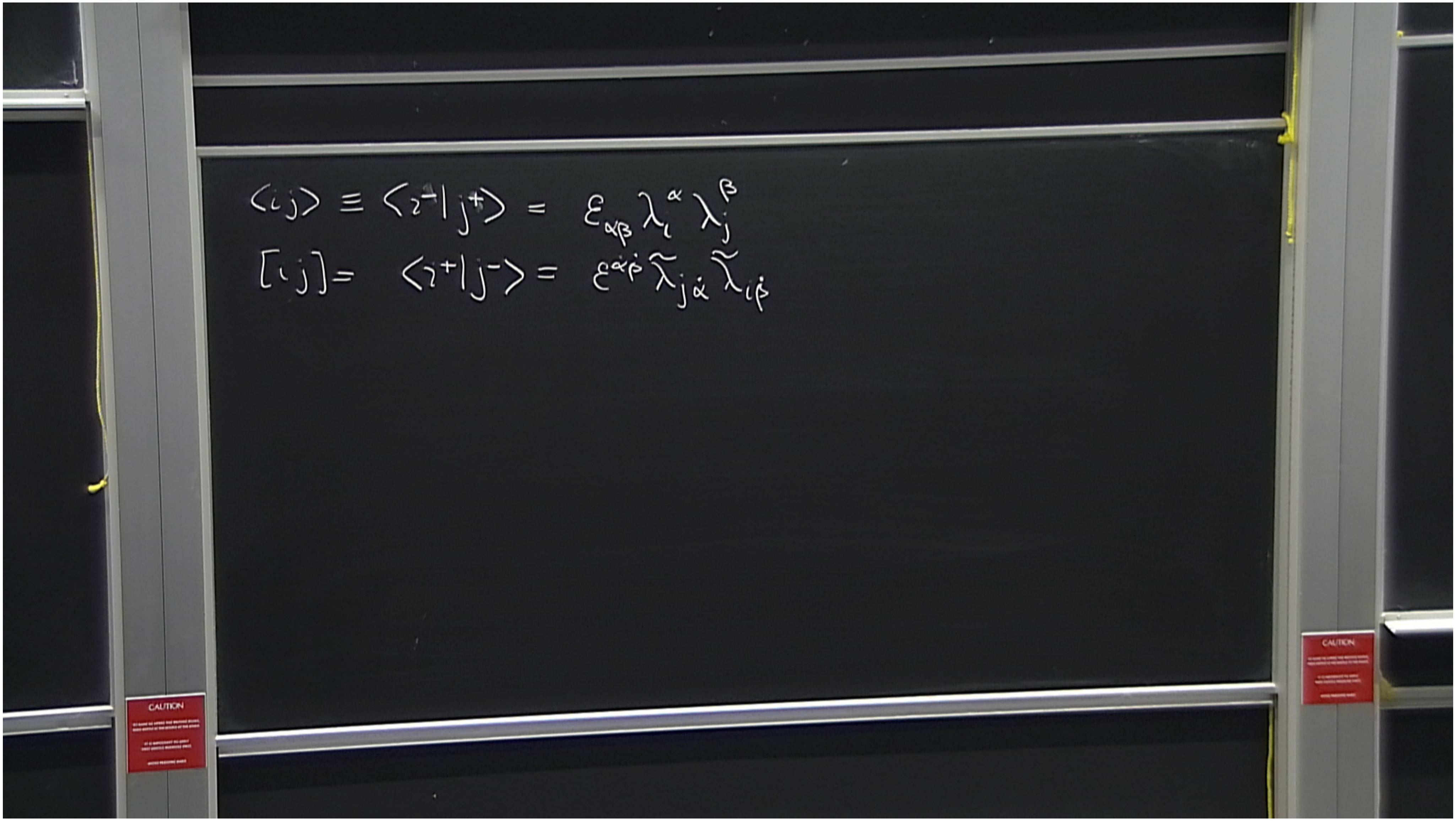
k²=0 k_{αα̇} = λ_α λ̃_α̇ α, α̇ = 0,1.
↳ complex 2-vector
spinor variables.
ε^{αβ} λ_α λ'_β


$|j^+\rangle \equiv \lambda_j^\alpha \equiv |j\rangle$
$\langle j^-| \leftrightarrow \varepsilon_{\alpha\beta}\lambda_j^\beta \equiv \langle j|$
$|j^-\rangle \equiv \tilde\lambda_{j\dot\alpha} \equiv |j]$
$\langle j^+| \leftrightarrow \varepsilon^{\dot\alpha\dot\beta}\tilde\lambda_j \equiv [j|$
$\equiv \langle 2^+|$


$\langle ij \rangle \equiv \langle i^- | j^+ \rangle = \varepsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$
$[ij] = \langle i^+ | j^- \rangle = \varepsilon^{\dot\alpha\dot\beta} \tilde\lambda_{j\dot\alpha} \tilde\lambda_{i\dot\beta}$

$|j\rangle \equiv \lambda_{j\dot\alpha} \equiv |j]$ $\quad \langle j^+| \leftrightarrow \varepsilon^{\dot\alpha\dot\beta}\tilde\lambda_j \equiv [j|$

$$\langle ij\rangle \equiv \langle i^-|j^+\rangle = \varepsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta$$

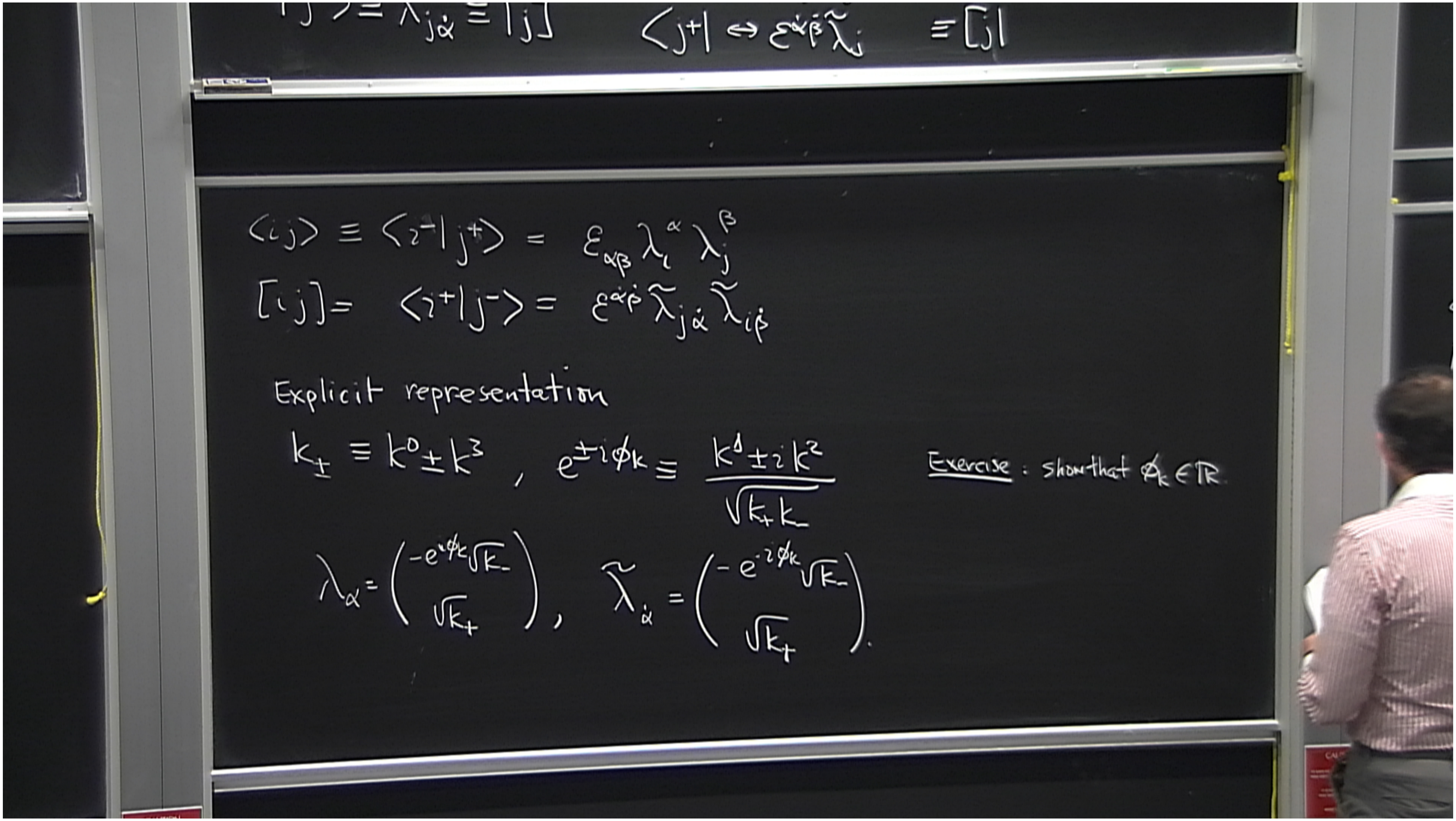
$$[ij] = \langle i^+|j^-\rangle = \varepsilon^{\dot\alpha\dot\beta}\tilde\lambda_{j\dot\alpha}\tilde\lambda_{i\dot\beta}$$

Explicit representation

$$k_\pm \equiv k^0 \pm k^3, \quad e^{\pm i\phi_k} \equiv \frac{k^1 \pm i k^2}{\sqrt{k_+ k_-}}$$

Exercise: show that $\phi_k \in \mathbb{R}$.

$$\lambda_\alpha = \begin{pmatrix} -e^{i\phi_k}\sqrt{k_-} \\ \sqrt{k_+} \end{pmatrix}, \quad \tilde\lambda_{\dot\alpha} = \begin{pmatrix} -e^{-i\phi_k}\sqrt{k_-} \\ \sqrt{k_+} \end{pmatrix}.$$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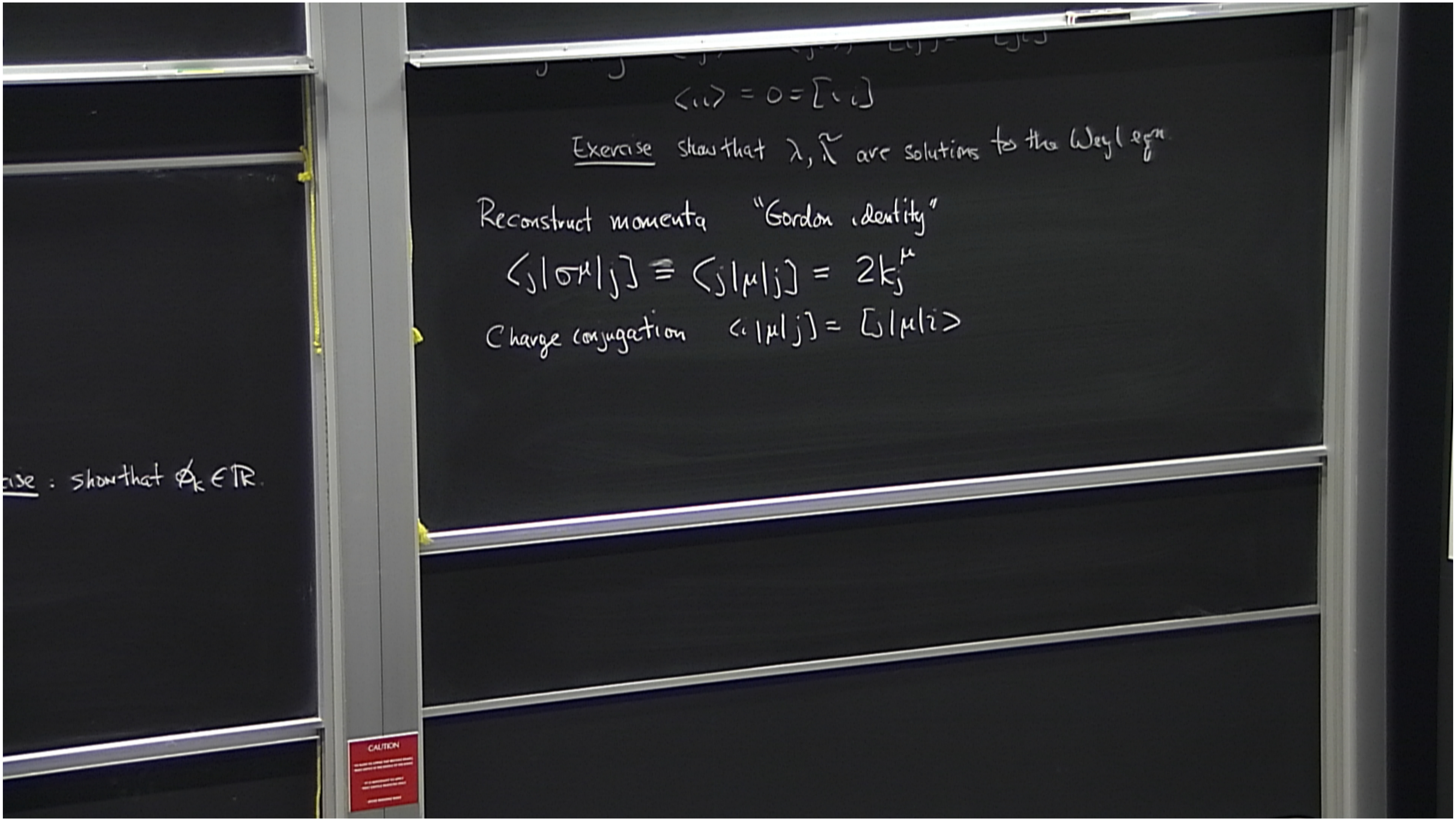
...ise : Show that $\phi_k \in \mathbb{R}$.

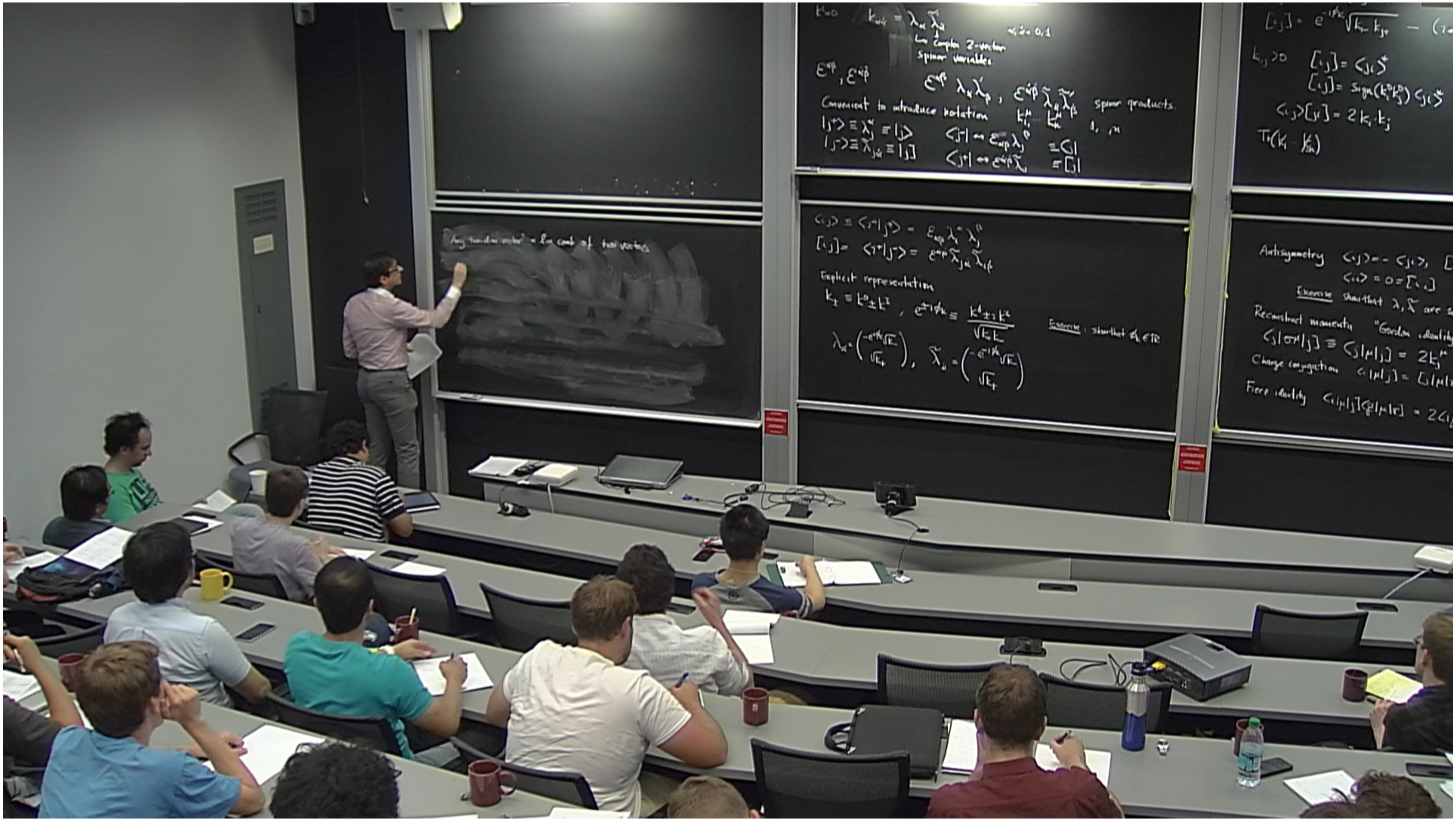
Antisymmetry $\langle ij \rangle = -\langle ji \rangle$, $[ij] = -[ji]$

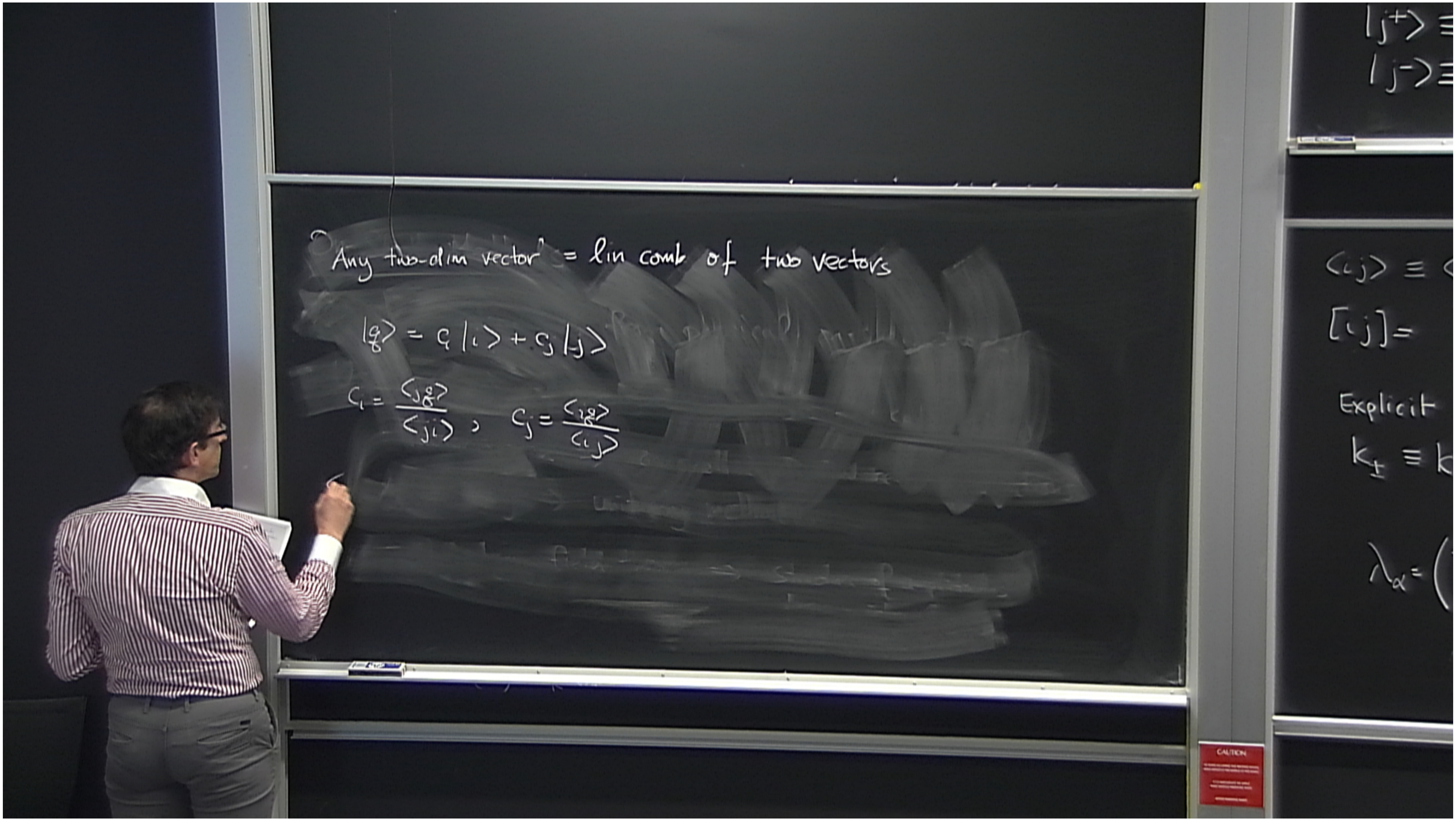
$\langle ii \rangle = 0 = [ii]$

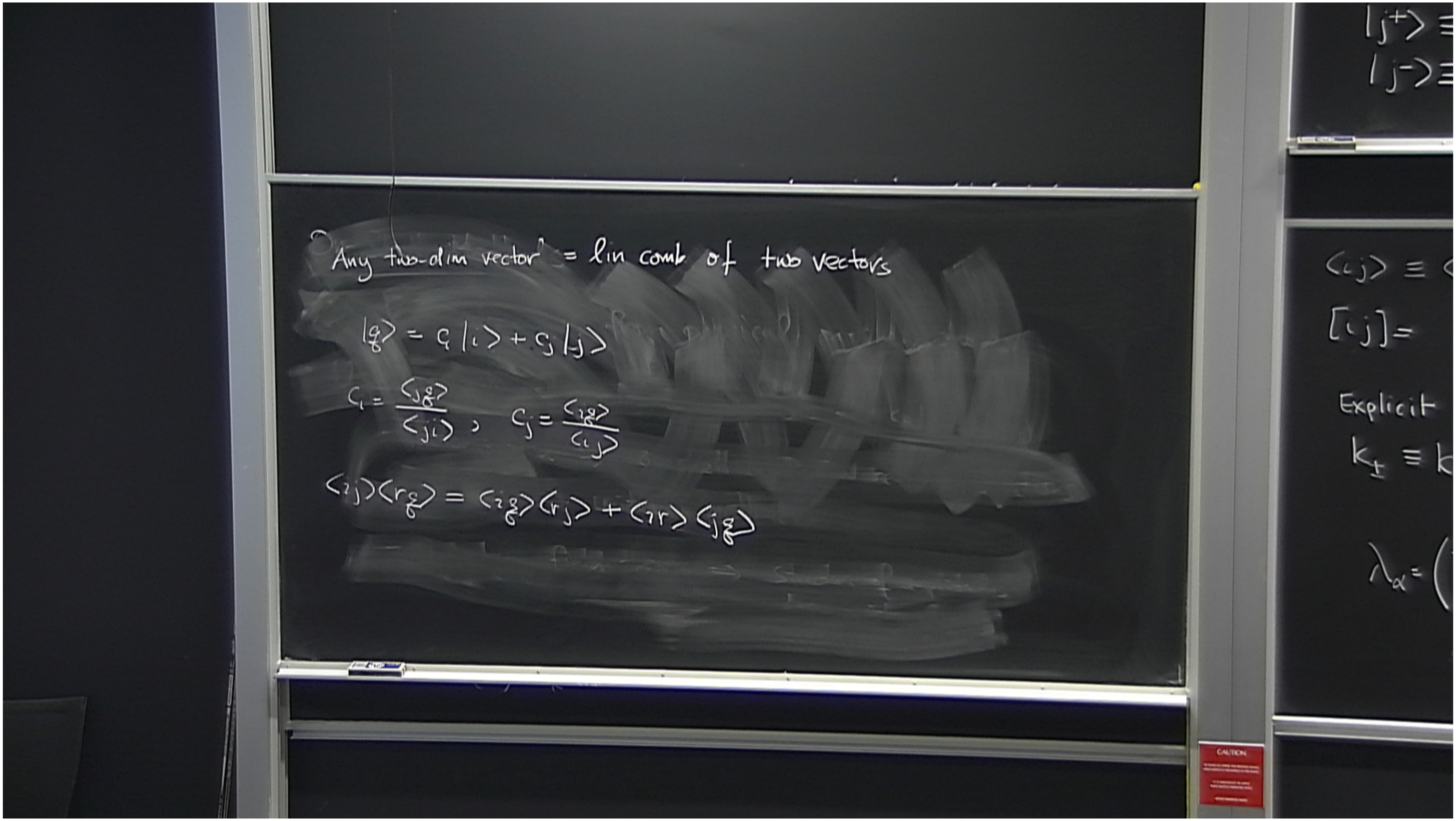
<u>Exercise</u> Show that $\lambda, \tilde{\lambda}$ are solutions to the Weyl eqn.

Reconstruct momenta


⟨ii⟩ = 0 = [ii]
Exercise show that λ, λ̃ are solutions to the Weyl eqn
Reconstruct momenta "Gordon identity"
⟨j|σ^μ|j] ≡ ⟨j|μ|j] = 2k_j^μ
Charge conjugation ⟨i|μ|j] = [j|μ|i⟩
ise : show that φ_k ∈ ℝ.
CAUTION




Any two-dim vector = lin comb of two vectors
|g> = c_i |i> + c_j |j>
c_i = <jg>/<ji> , c_j = <ig>/<ij>
|j+> ≡
|j-> ≡
<ij> ≡
[ij] =
Explicit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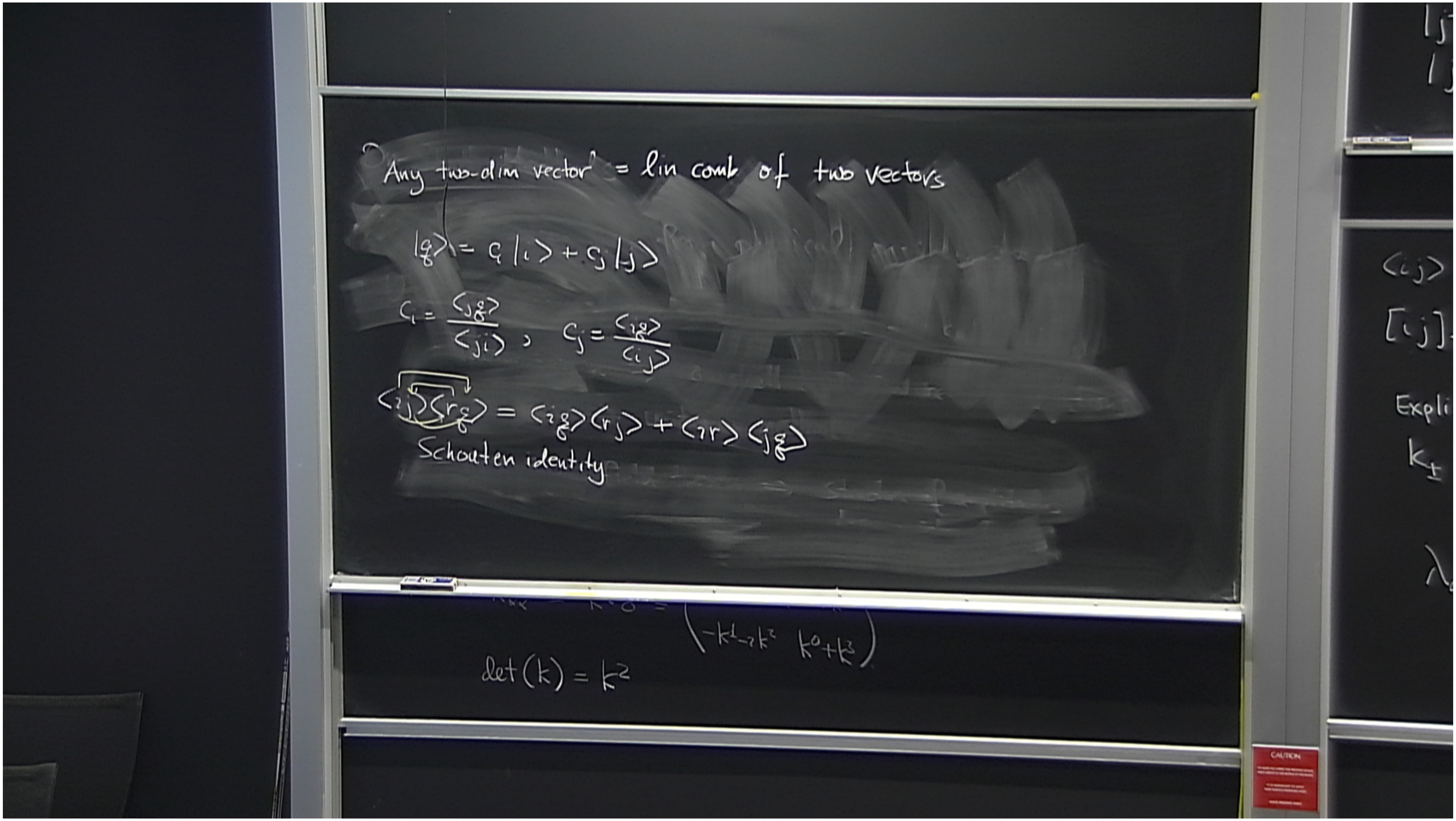
CAUTION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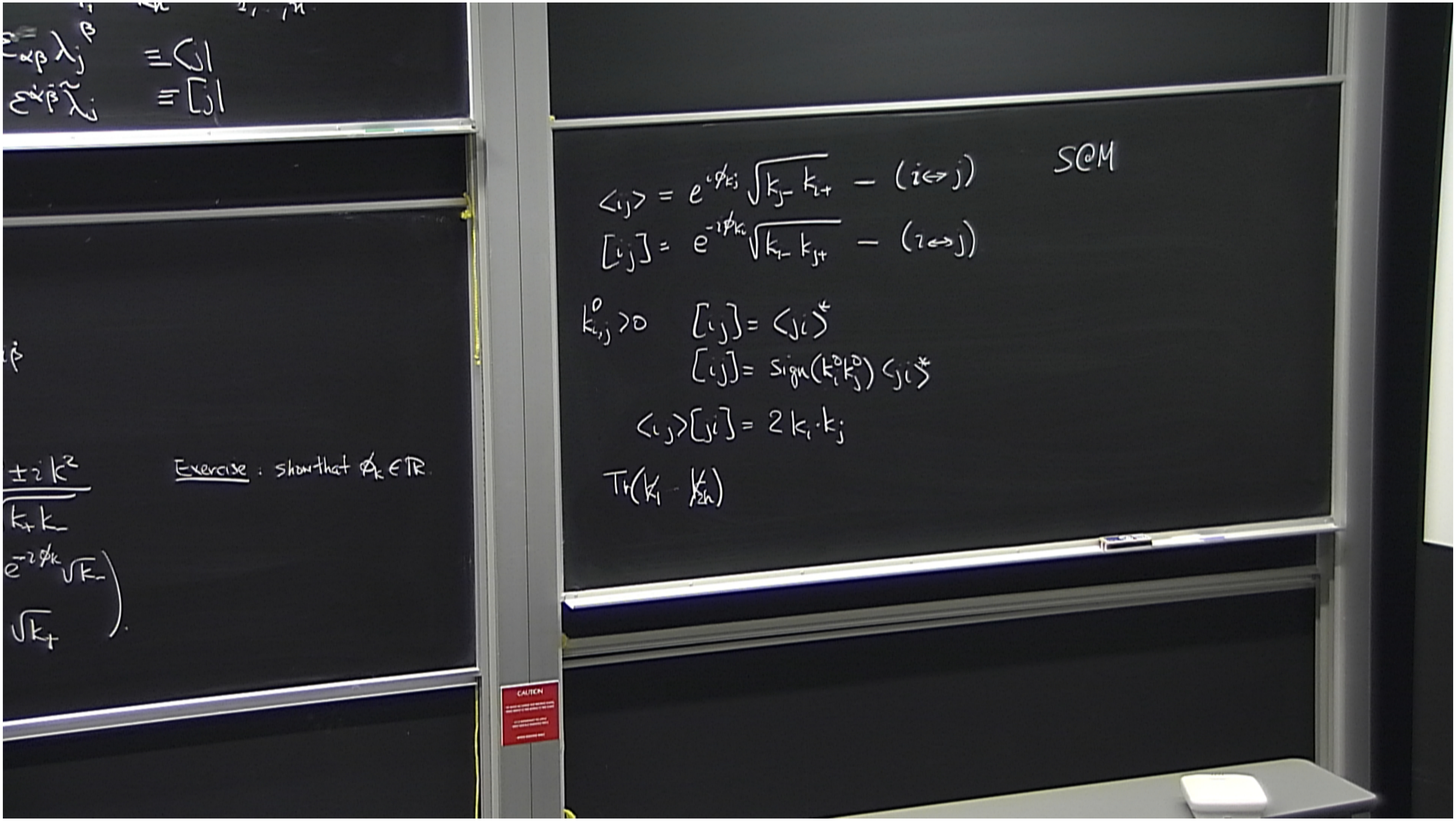
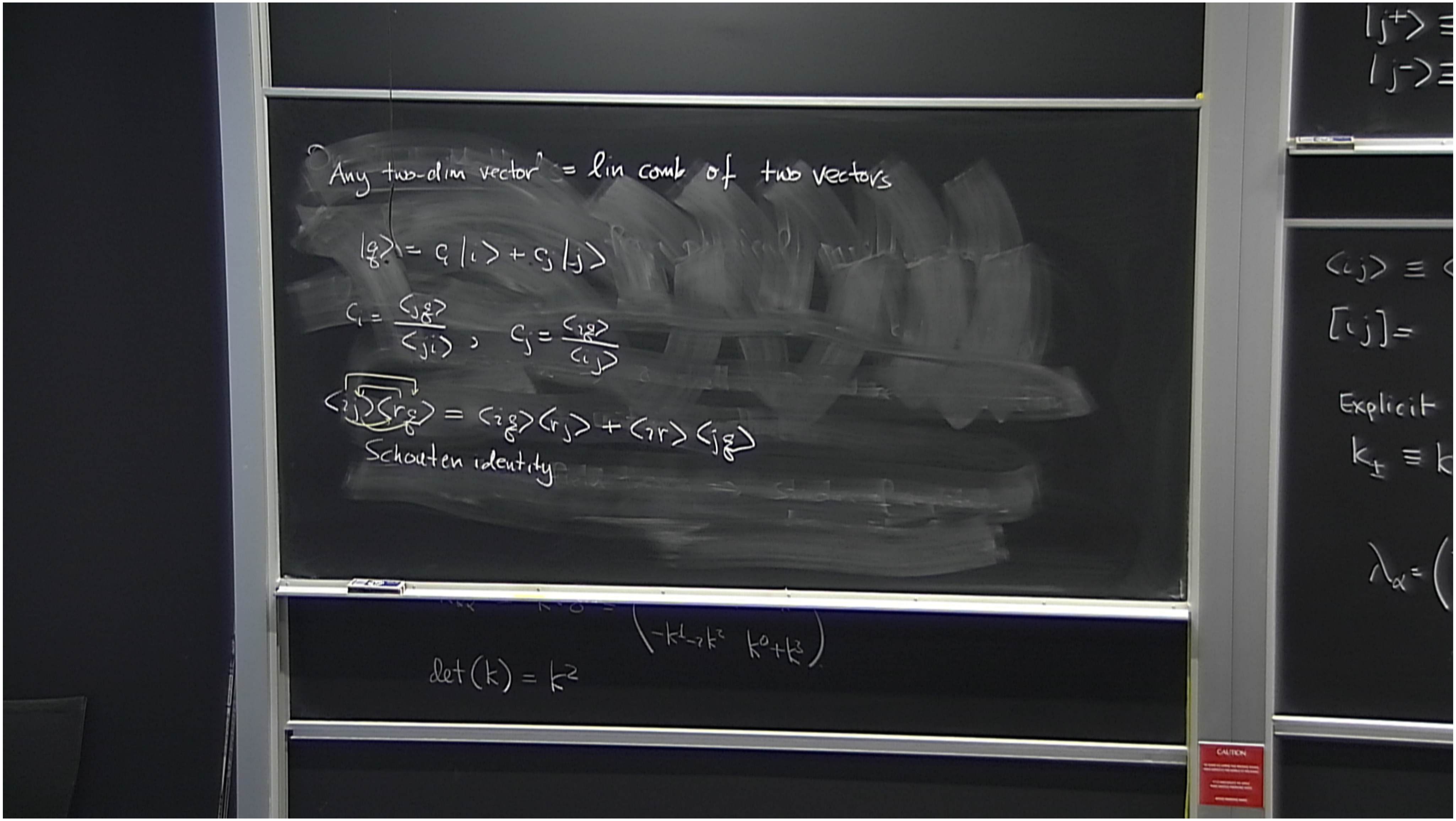
<ij><rq> = <iq><rj> + <ir><jq>
Schouten identity
det(k) = k^2


Exercise : show that $\phi_k \in \mathbb{R}$
$\langle ij \rangle = e^{i\phi_{k_j}} \sqrt{k_{j-} k_{i+}} - (i \leftrightarrow j)$
S@M
$[ij] = e^{-i\phi_{k_i}} \sqrt{k_{i-} k_{j+}} - (i \leftrightarrow j)$
$k^0_{i,j} > 0 \quad [ij] = \langle ji \rangle^*$
$[ij] = \text{sign}(k^0_i k^0_j) \langle ji \rangle^*$
$\langle ij \rangle [ji] = 2 k_i \cdot k_j$
$\text{Tr}(\not{k}_1 \cdots \not{k}_{2n})$


Spinor products
Convenient to introduce notation
Explicit representation
Schouten identity
Antisymmetry
Charge conjugation
Fierz identity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Explicit