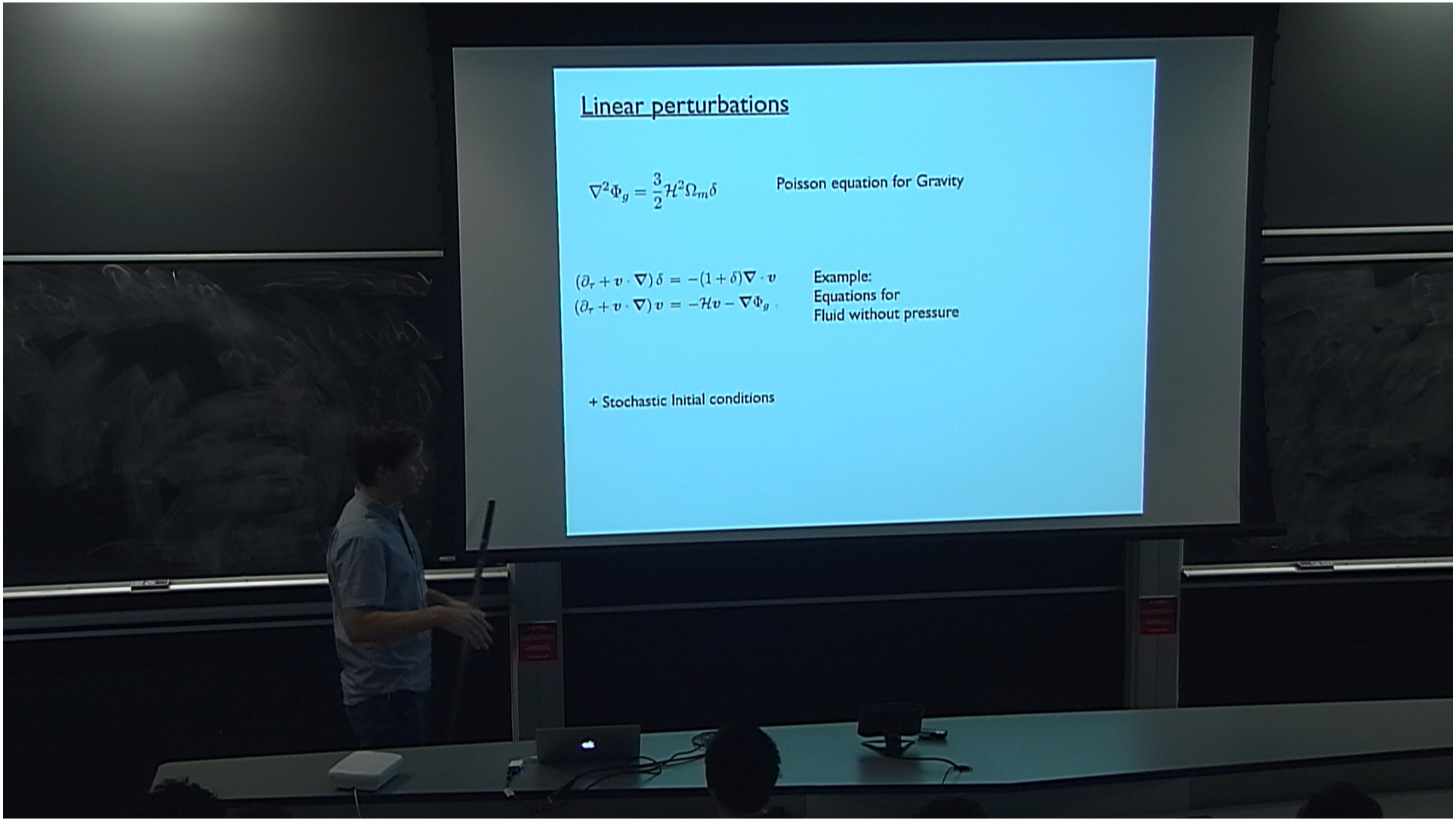
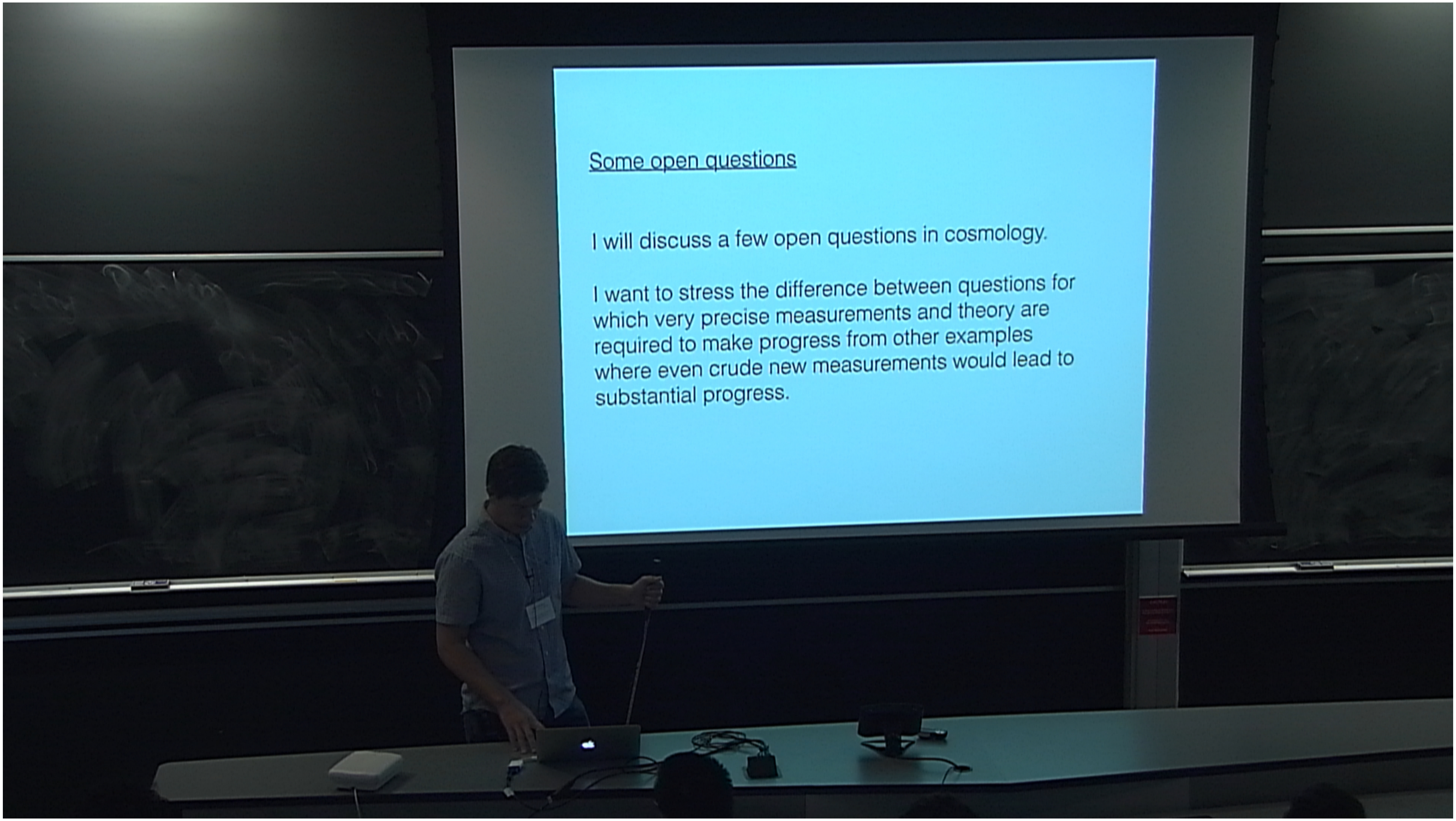
Title: Cosmology Theory: Large Scale Structure

Date: Jul 10, 2015 10:15 AM

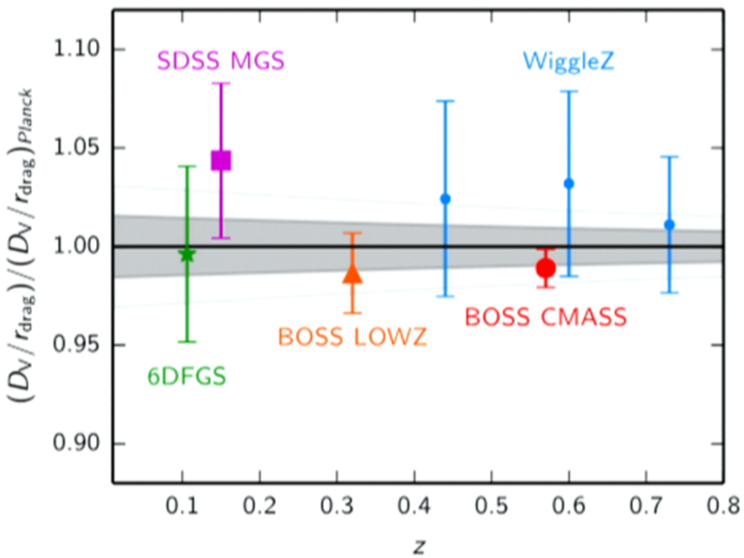
URL: http://pirsa.org/15070032

Abstract:


Linear perturbations
$\nabla^2\Phi_g = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$
Poisson equation for Gravity
$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\delta = -(1+\delta)\boldsymbol{\nabla}\cdot\boldsymbol{v}$
$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} = -\mathcal{H}\boldsymbol{v} - \boldsymbol{\nabla}\Phi_g$
Example:
Equations for
Fluid without pressure
+ Stochastic Initial conditions


Some open questions
I will discuss a few open questions in cosmology.
I want to stress the difference between questions for which very precise measurements and theory are required to make progress from other examples where even crude new measurements would lead to substantial progress.

# Dark Energy: The  BAO Scale



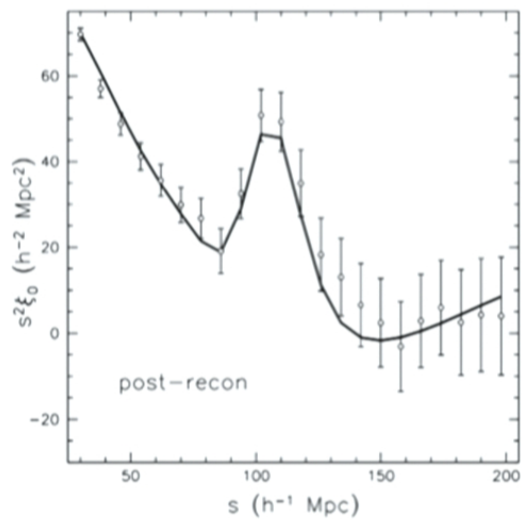
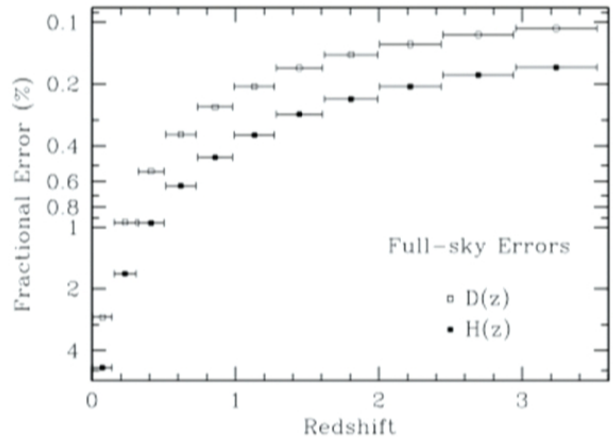
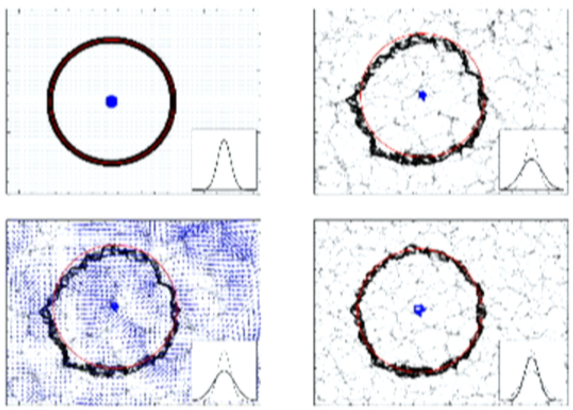
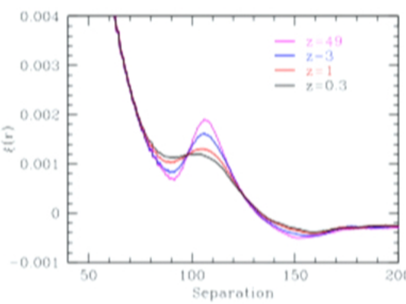



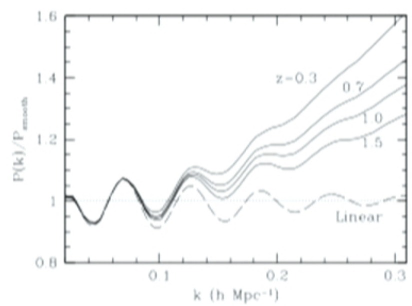
Table 2. BAO Forecasts for a Full-Sky BAO Survey

| $z_{min}$ | $z_{max}$ | Volume | % Err $D_A(z)$ | % Err $H(z)$ | $\Omega_\Lambda(z)$ | S/N | $\sigma_w$ |
|---|---|---|---|---|---|---|---|
| 0.00 | 0.15 | 0.33 | 2.8 | 4.9 | 0.708 | 7.3 | 0.64 |
| 0.15 | 0.32 | 2.62 | 0.95 | 1.7 | 0.616 | 18.2 | 0.088 |
| 0.32 | 0.51 | 7.89 | 0.53 | 0.96 | 0.515 | 27.0 | 0.036 |
| 0.51 | 0.73 | 16.5 | 0.35 | 0.63 | 0.413 | 32.9 | 0.021 |
| 0.73 | 0.99 | 28.4 | 0.26 | 0.46 | 0.318 | 34.9 | 0.015 |
| 0.99 | 1.28 | 42.9 | 0.21 | 0.36 | 0.236 | 33.3 | 0.013 |
| 1.28 | 1.62 | 59.0 | 0.17 | 0.28 | 0.170 | 30.2 | 0.012 |
| 1.62 | 2.00 | 75.8 | 0.14 | 0.24 | 0.119 | 25.2 | 0.013 |
| 2.00 | 2.44 | 92.3 | 0.13 | 0.21 | 0.082 | 20.0 | 0.014 |
| 2.44 | 2.95 | 108 | 0.12 | 0.18 | 0.056 | 15.5 | 0.016 |
| 2.95 | 3.53 | 121 | 0.11 | 0.17 | 0.038 | 11.4 | 0.020 |
| 3.53 | 4.20 | 133 | 0.10 | 0.15 | 0.025 | 8.3 | 0.025 |
| 4.20 | 4.96 | 142 | 0.10 | 0.15 | 0.017 | 5.8 | 0.033 |



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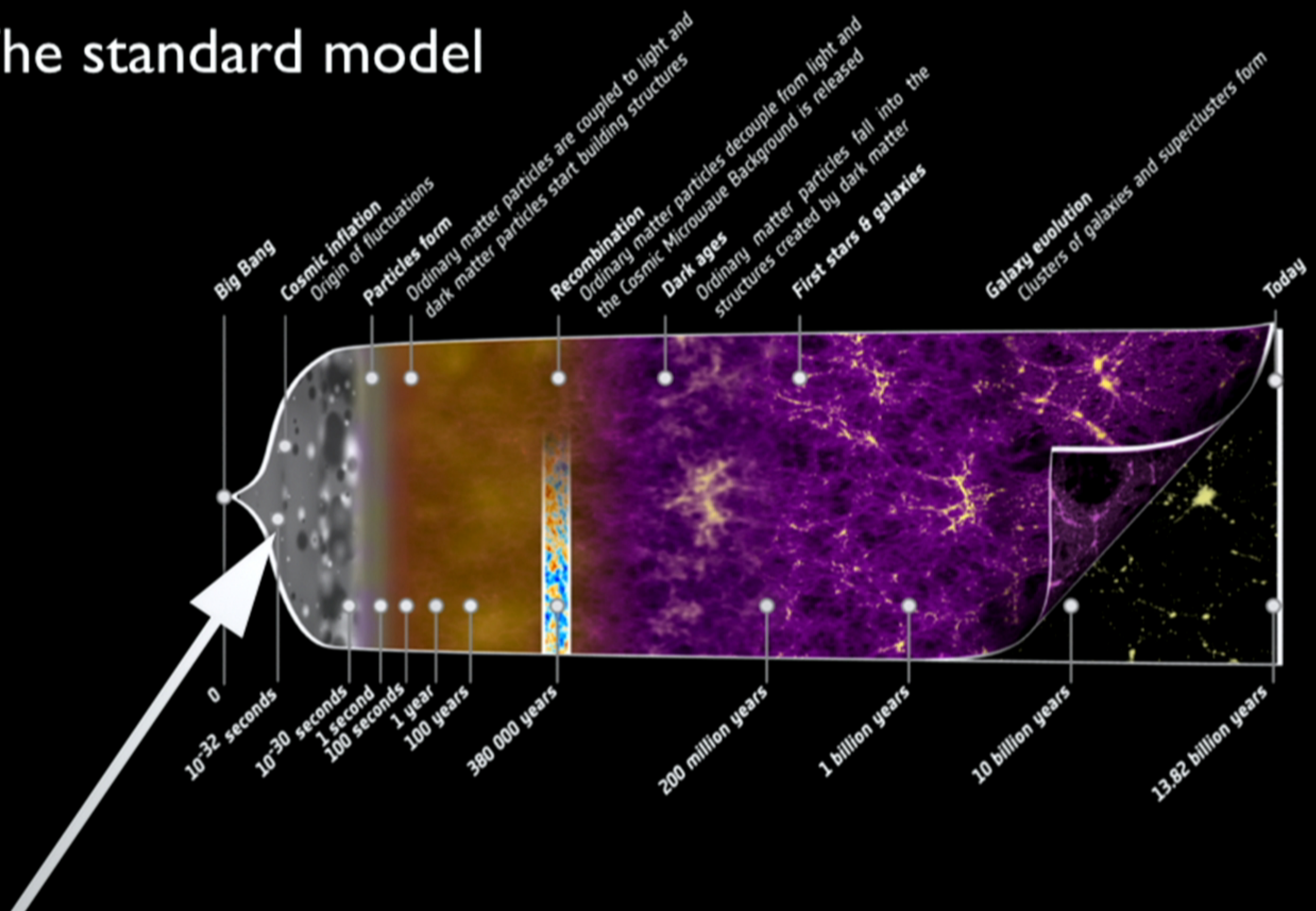
# The standard model




Why is the Universe so old/big? Attractor solution.
Seeds for structure formation are quantum fluctuations of the clock.

What we have learned from the CMB:

0. No curvature (1/2 percent level)

1. The seeds are primordial

2. Amplitude: $\ln A_s = -19.932 \pm 0.034$

3. Slope: $1 - n_s = 0.0355 \pm 0.0049$

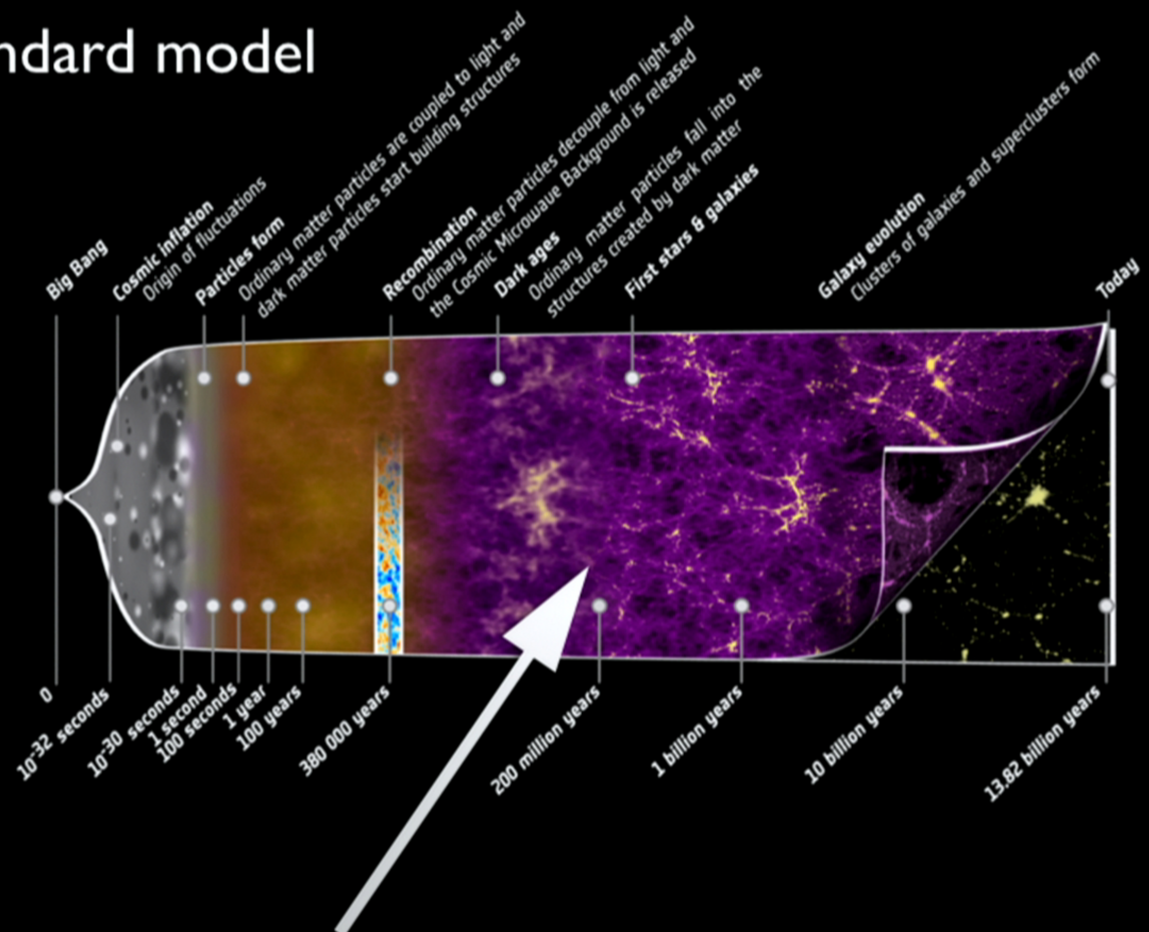
4. No gravitational waves (10 percent level)

5. No fluctuation in composition (percent level)

6. No departures from Gaussianity $\frac{\mathrm{Non-Gaussian}}{\mathrm{Gaussian}} < 10^{-3} - 10^{-4}$

*Anything that the CMB is sensitive to requires a lot of precision to improve*

# The standard model




The end of the dark ages

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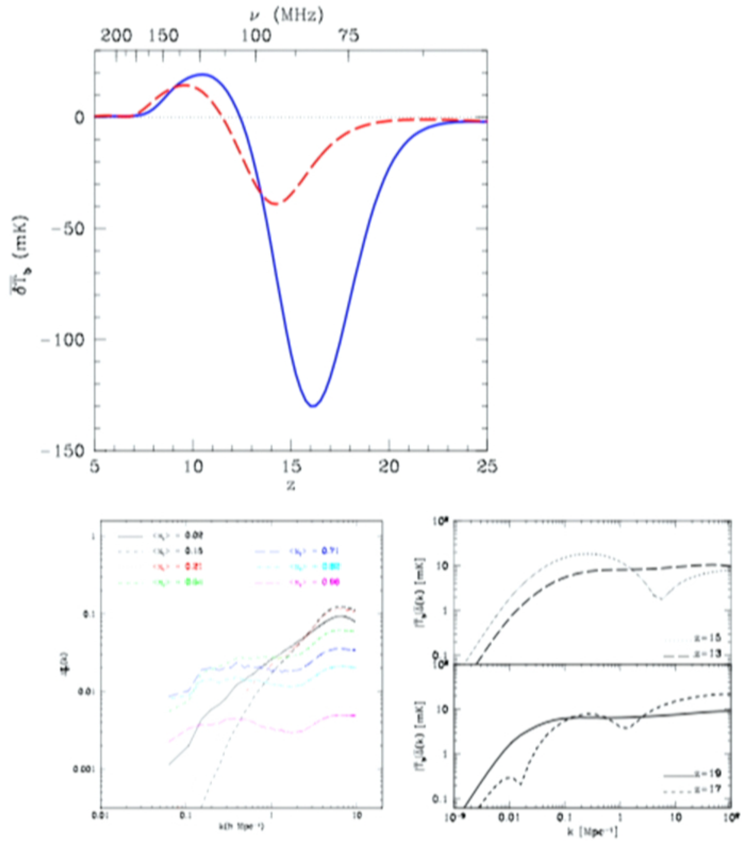
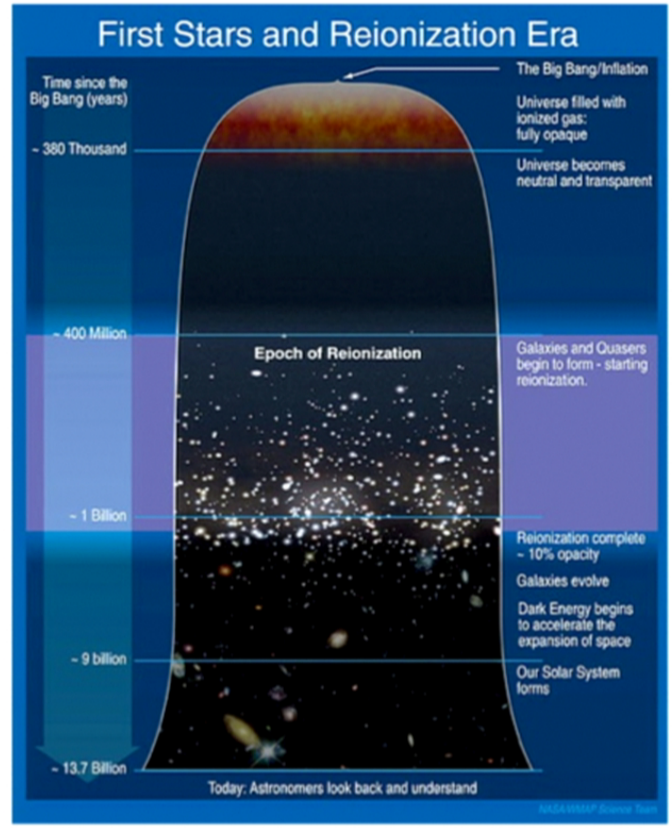
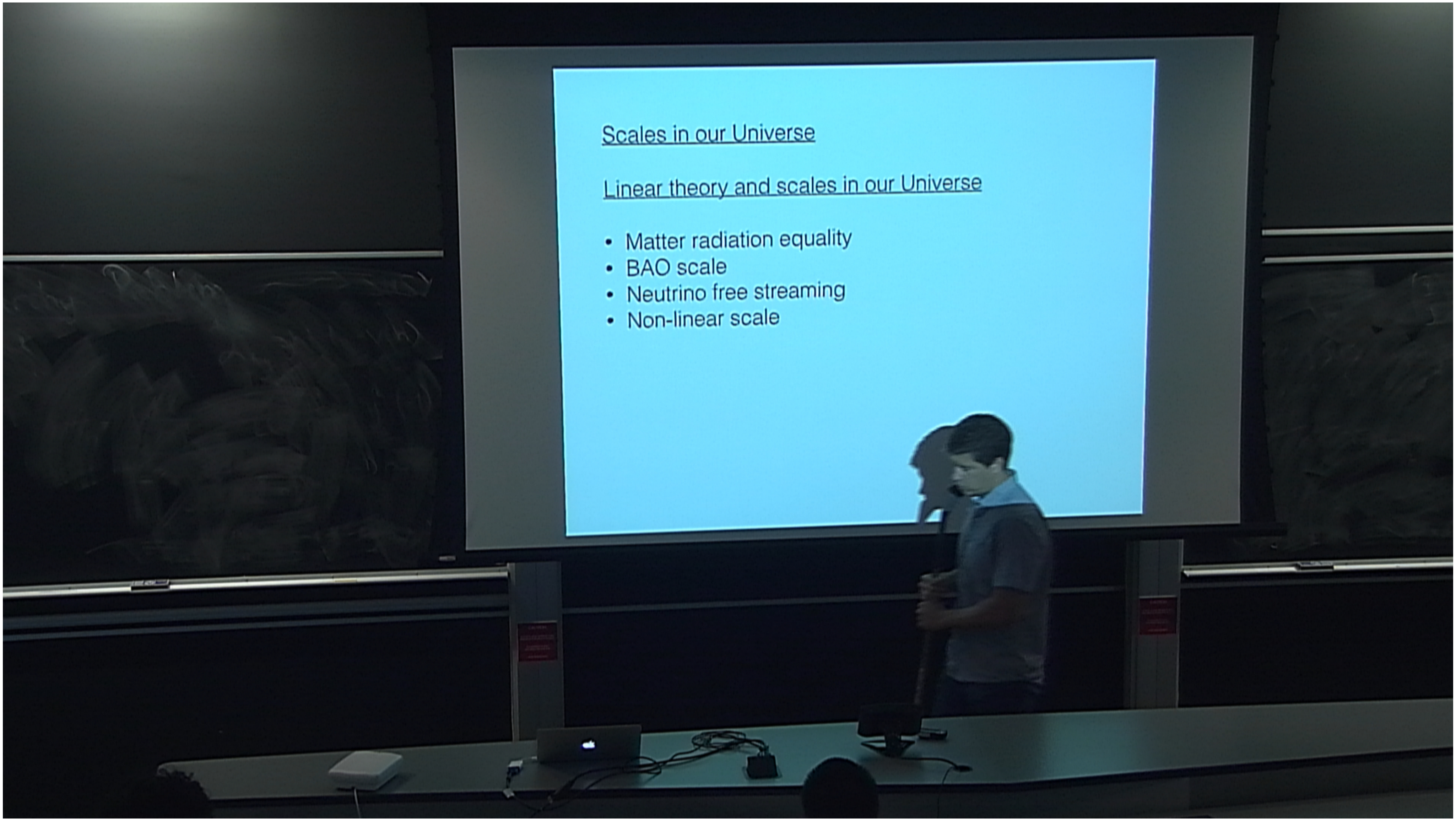
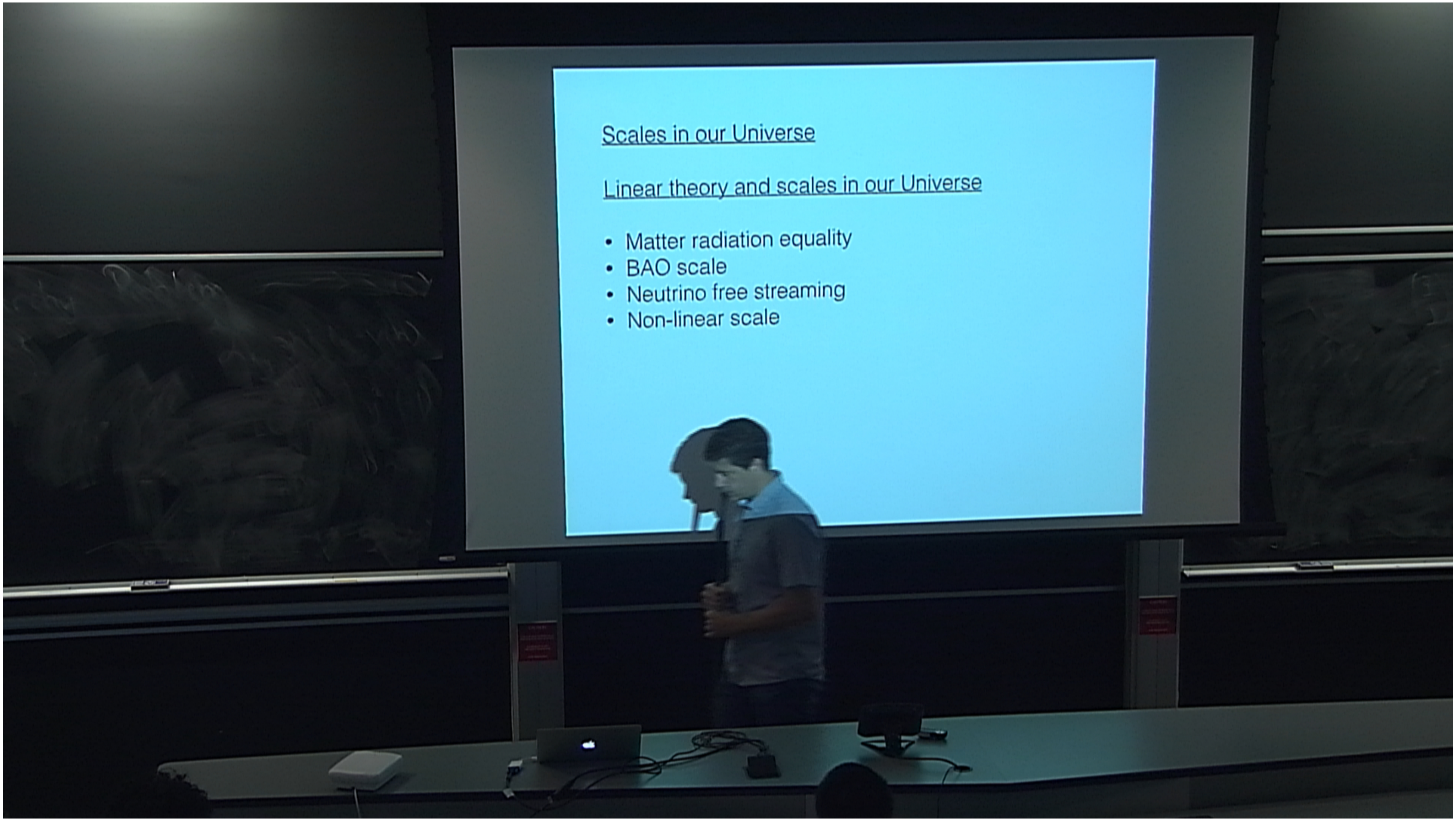



FIG. 3: *Left panel:* Evolution of the spherically-averaged 21 cm power spectrum during reionization, from a numerical simulation of that process. Here the fluctuations are presented in units of $\delta T_b^2$ for a fully neutral universe (see eq. 1): $\langle x_i \rangle$ is the ionized fraction during reionization. From [22]. *Right panels:* Evolution of the spherically-averaged power spectrum *before* reionization begins in earnest, including X-ray heating and Wouthuysen-Field coupling fluctuations. From [28]. In all panels, we use models with parameters similar to the solid curve in Fig. 2.




Scales in our Universe
Linear theory and scales in our Universe
• Matter radiation equality
• BAO scale
• Neutrino free streaming
• Non-linear scale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## Scales in our Universe

## Linear theory and scales in our Universe

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Scales in our Universe

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# Standard set up

$$\nabla^2\Phi_g = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$$

Poisson equation for Gravity

$$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\delta \;=\; -(1+\delta)\boldsymbol{\nabla}\cdot\boldsymbol{v}$$
$$(\partial_\tau + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} \;=\; -\mathcal{H}\boldsymbol{v} - \boldsymbol{\nabla}\Phi_g\;.$$

Eulerian Equations for Fluid

+ Stochastic Initial conditions

$$\boldsymbol{x}(\boldsymbol{q},\tau) \;=\; \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau)$$
$$\frac{d^2\boldsymbol{s}}{d\tau^2} + \mathcal{H}\frac{d\boldsymbol{s}}{d\tau} \;=\; \nabla\Phi_g(\boldsymbol{q}+\boldsymbol{s},\tau)$$
$$1+\delta(\boldsymbol{x},\tau) \;=\; \int d^3q\;\delta^D(\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{s}(\boldsymbol{q},\tau))$$

Lagrangian Equations a set of particles

# Standard set up

$$\nabla^2 \Phi_g = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta$$

Poisson equation for Gravity

$$(\partial_\tau + \boldsymbol{v} \cdot \boldsymbol{\nabla})\, \delta \;=\; -(1+\delta) \boldsymbol{\nabla} \cdot \boldsymbol{v}$$
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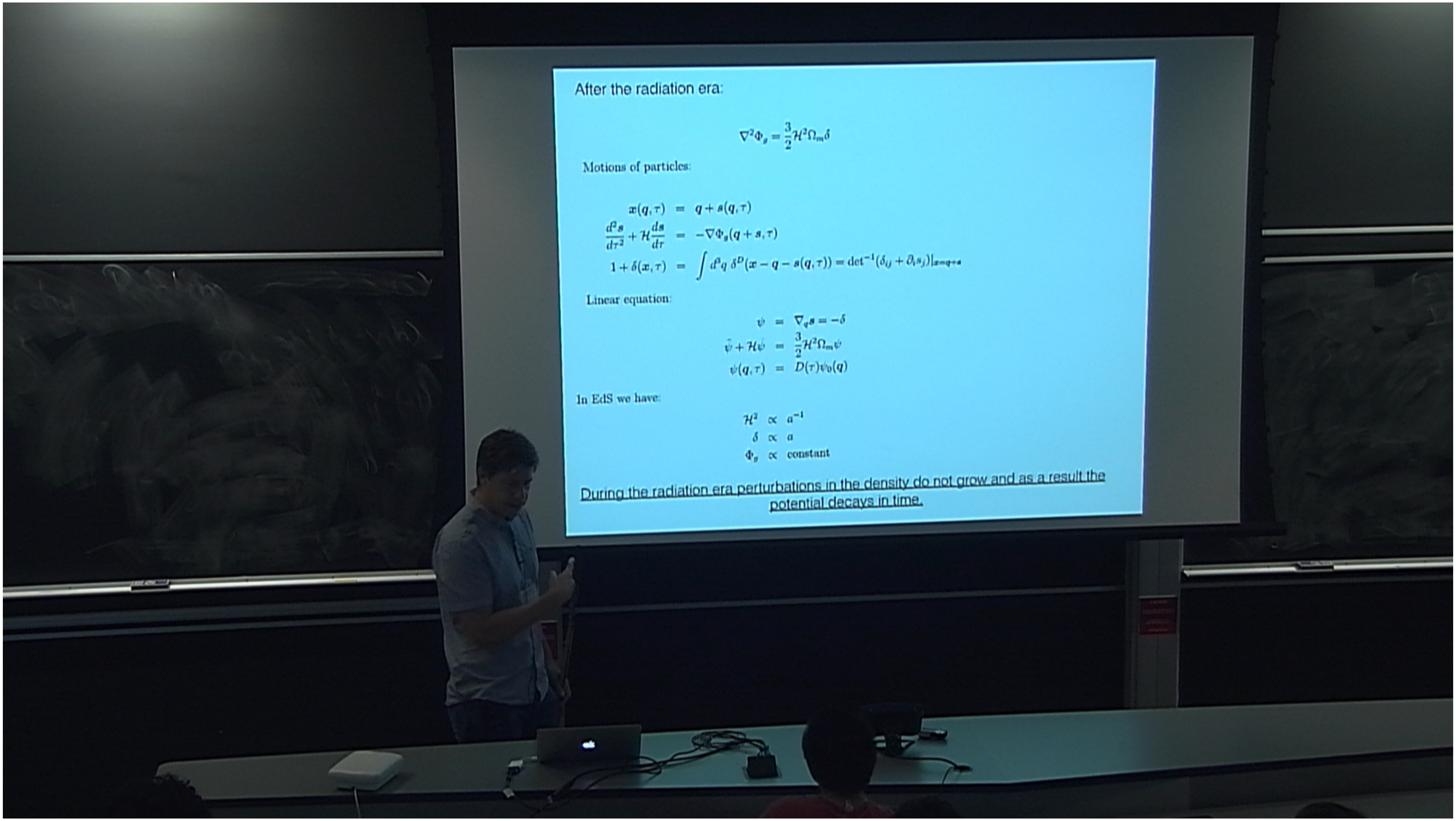
Eulerian Equations for Fluid

+ Stochastic Initial conditions

$$\boldsymbol{x}(\boldsymbol{q}, \tau) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q}, \tau)$$
$$\frac{d^2 \boldsymbol{s}}{d\tau^2} + \mathcal{H} \frac{d\boldsymbol{s}}{d\tau} = \nabla \Phi_g(\boldsymbol{q} + \boldsymbol{s}, \tau)$$
$$1 + \delta(\boldsymbol{x}, \tau) = \int d^3 q\; \delta^D(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{s}(\boldsymbol{q}, \tau))$$

Lagrangian Equations a set of particles


After the radiation era:
$\nabla^2\Phi_g = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$
Motions of particles:
$x(q,\tau) = q + s(q,\tau)$
$\frac{d^2 s}{d\tau^2} + \mathcal{H}\frac{ds}{d\tau} = -\nabla\Phi_g(q+s,\tau)$
$1+\delta(x,\tau) = \int d^3q\, \delta^D(x - q - s(q,\tau)) = \det{}^{-1}(\delta_{ij} + \partial_i s_j)|_{x=q+s}$
Linear equation:
$\psi = \nabla_q s = -\delta$
$\ddot{\psi} + \mathcal{H}\dot{\psi} = \frac{3}{2}\mathcal{H}^2\Omega_m\psi$
$\psi(q,\tau) = D(\tau)\psi_0(q)$
In EdS we have:
$\mathcal{H}^2 \propto a^{-1}$
$\delta \propto a$
$\Phi_g \propto$ constant
During the radiation era perturbations in the density do not grow and as a result the potential decays in time.

After the radiation era:

$$\nabla^2 \Phi_g = \frac{3}{2}\mathcal{H}^2 \Omega_m \delta$$

Motions of particles:

$$\begin{aligned}
\boldsymbol{x}(\boldsymbol{q},\tau) &= \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q},\tau) \\
\frac{d^2\boldsymbol{s}}{d\tau^2} + \mathcal{H}\frac{d\boldsymbol{s}}{d\tau} &= -\nabla\Phi_g(\boldsymbol{q}+\boldsymbol{s},\tau) \\
1+\delta(\boldsymbol{x},\tau) &= \int d^3q\ \delta^D(\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{s}(\boldsymbol{q},\tau)) = \det{}^{-1}(\delta_{ij}+\partial_i s_j)|_{\boldsymbol{x}=\boldsymbol{q}+\boldsymbol{s}}
\end{aligned}$$

Linear equation:

$$\begin{aligned}
\psi &= \nabla_q \boldsymbol{s} = -\delta \\
\ddot{\psi} + \mathcal{H}\dot{\psi} &= \frac{3}{2}\mathcal{H}^2\Omega_m \psi \\
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In EdS we have:

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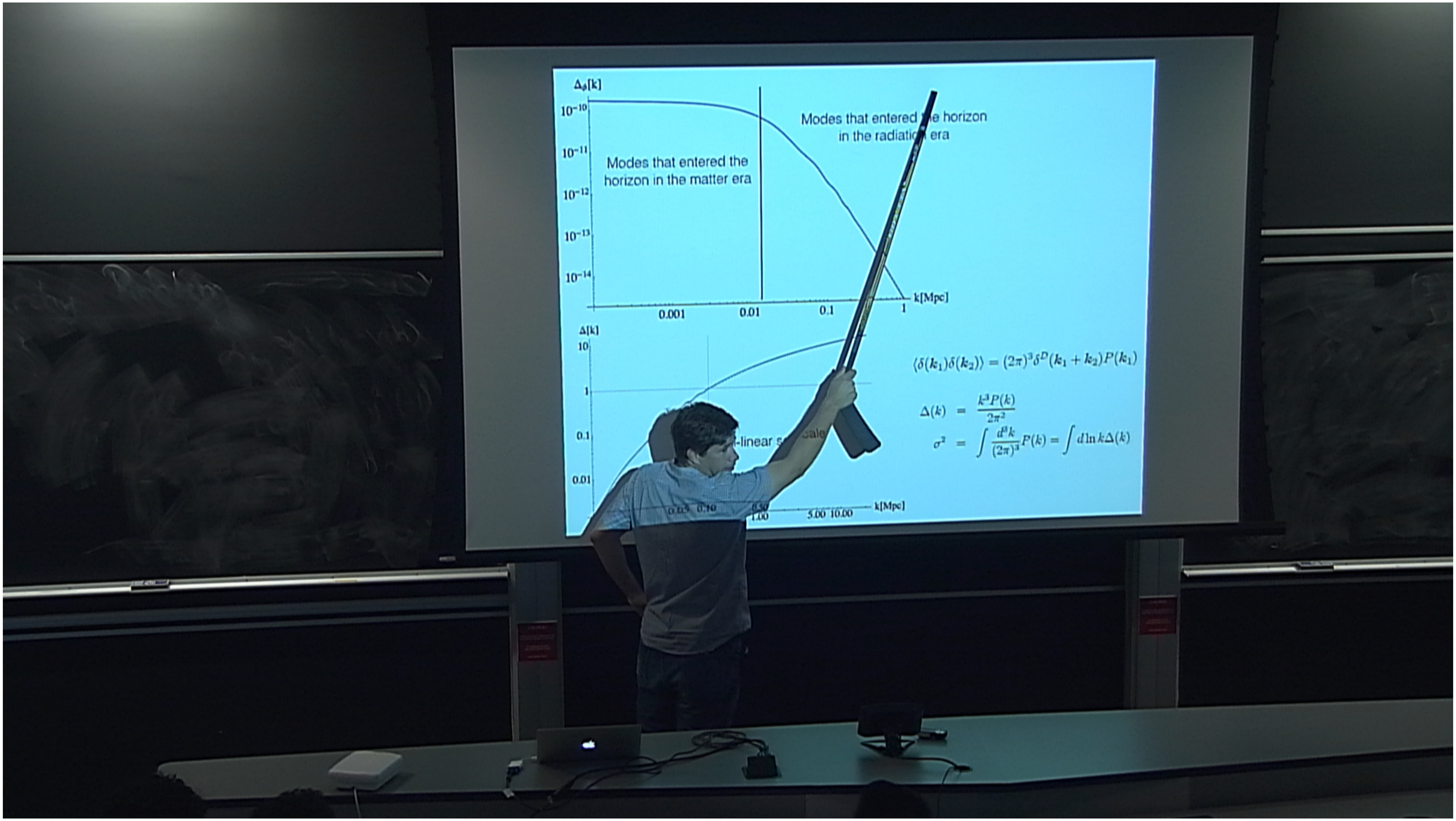
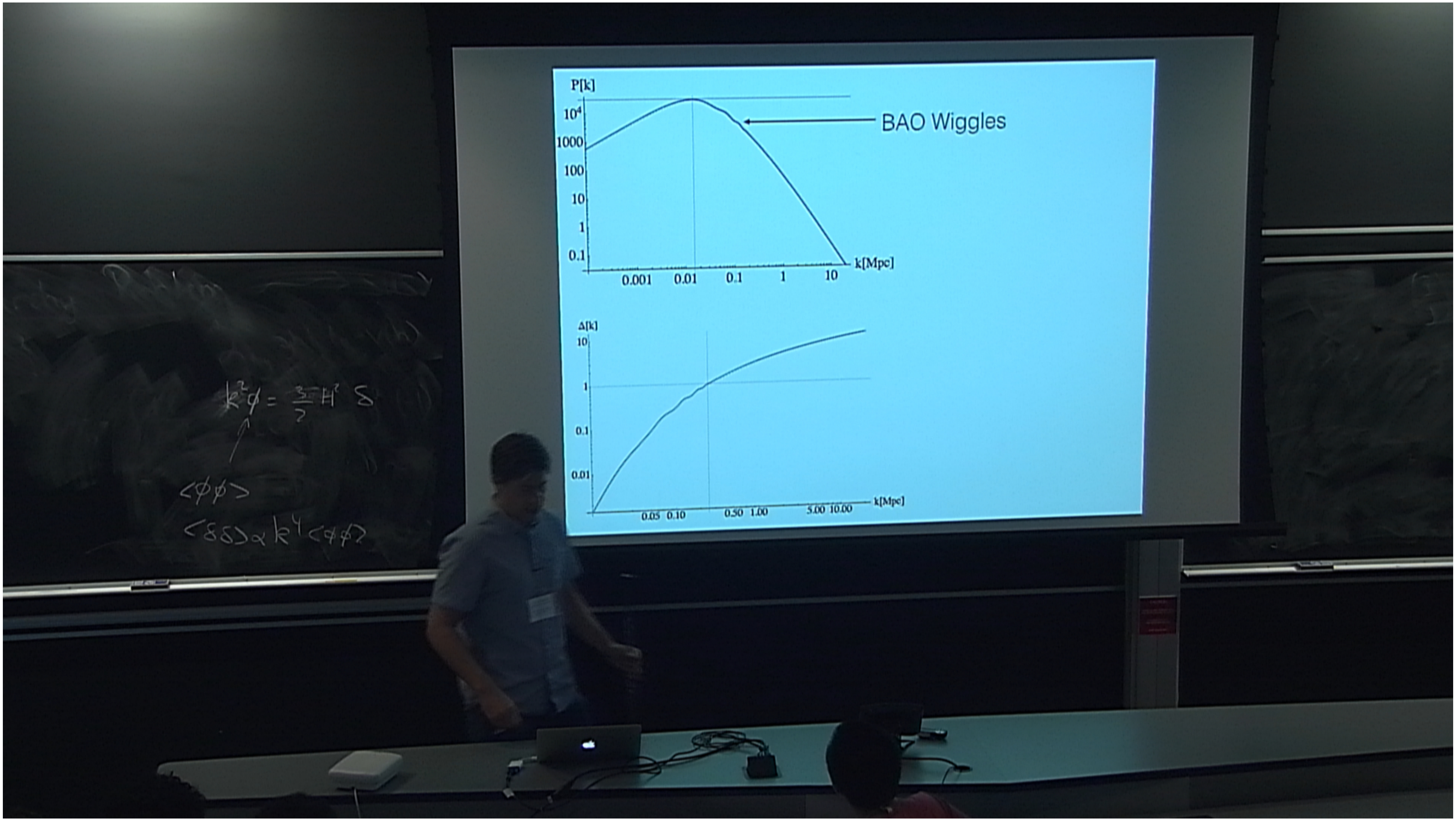
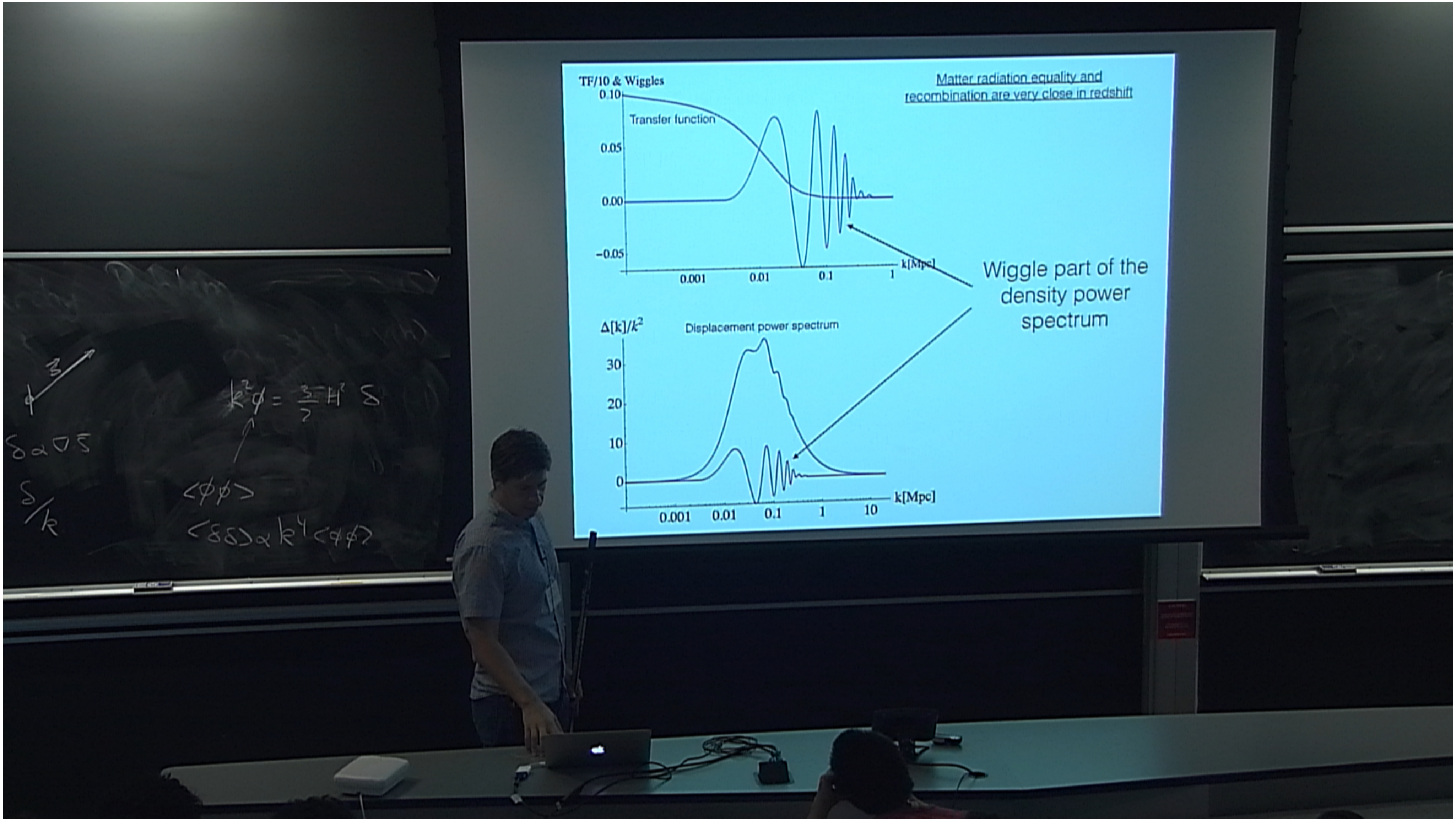
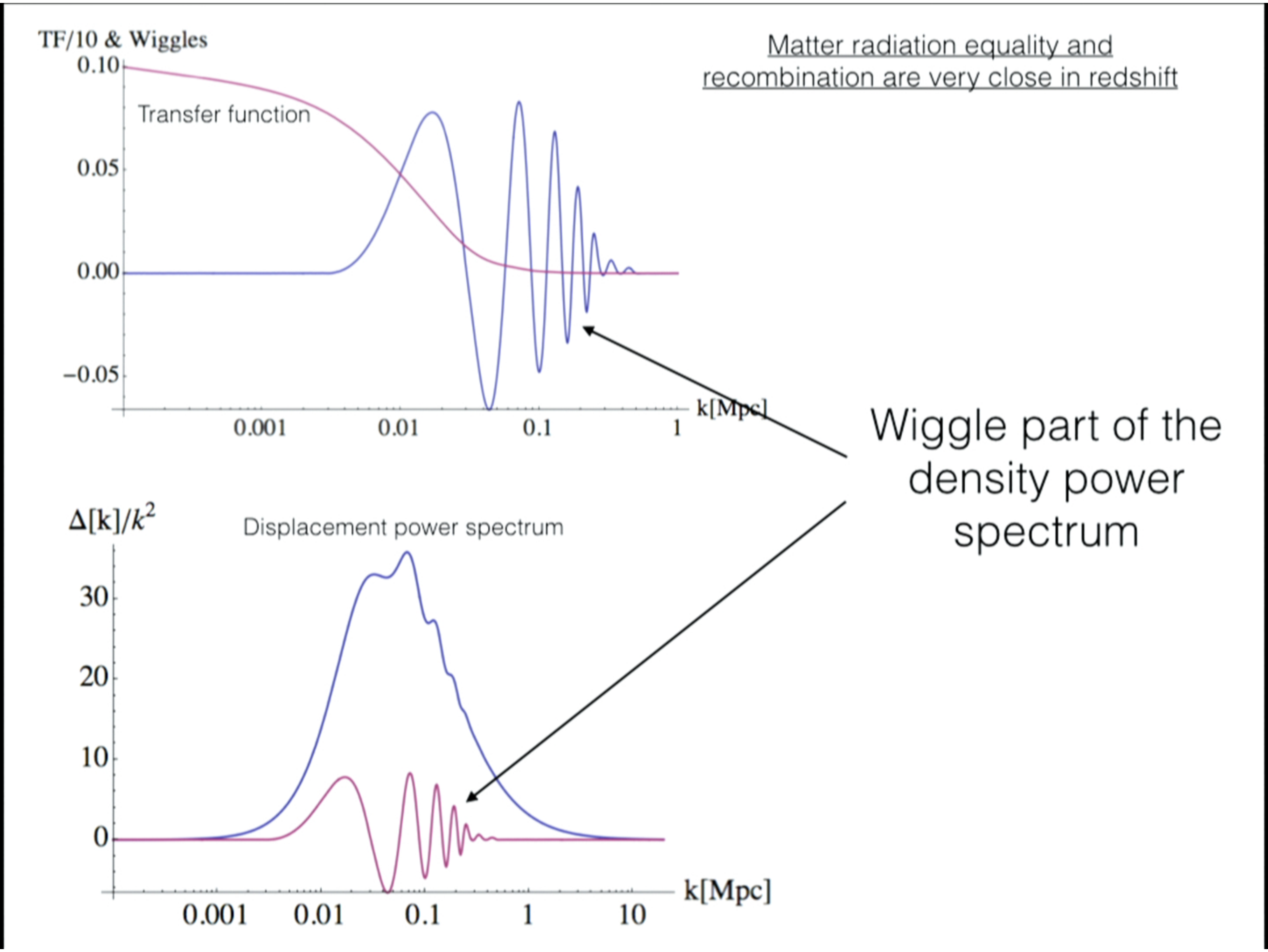


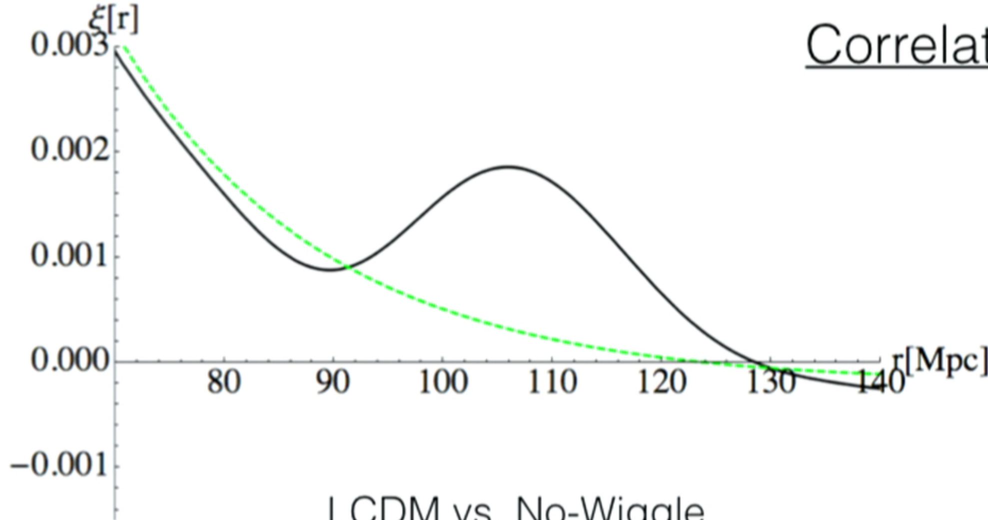
$\Delta_\phi[k]$
$10^{-10}$
$10^{-11}$
$10^{-12}$
$10^{-13}$
$10^{-14}$
Modes that entered the horizon in the matter era
in the radiation era
k[Mpc]
0.001
0.01
0.1
1
$\Delta[k]$
10
1
0.1
0.01
k[Mpc]
5.00 10.00
$\langle\delta(k_1)\delta(k_2)\rangle = (2\pi)^3\delta^D(k_1+k_2)P(k_1)$
$\Delta(k) = \frac{k^3P(k)}{2\pi^2}$
$\sigma^2 = \int\frac{d^3k}{(2\pi)^3}P(k) = \int d\ln k\Delta(k)$


P[k]
10^4
1000
100
10
1
0.1
0.001
0.01
0.1
1
10
k[Mpc]
BAO Wiggles
Δ[k]
10
1
0.1
0.01
0.05 0.10
0.50 1.00
5.00 10.00
k[Mpc]

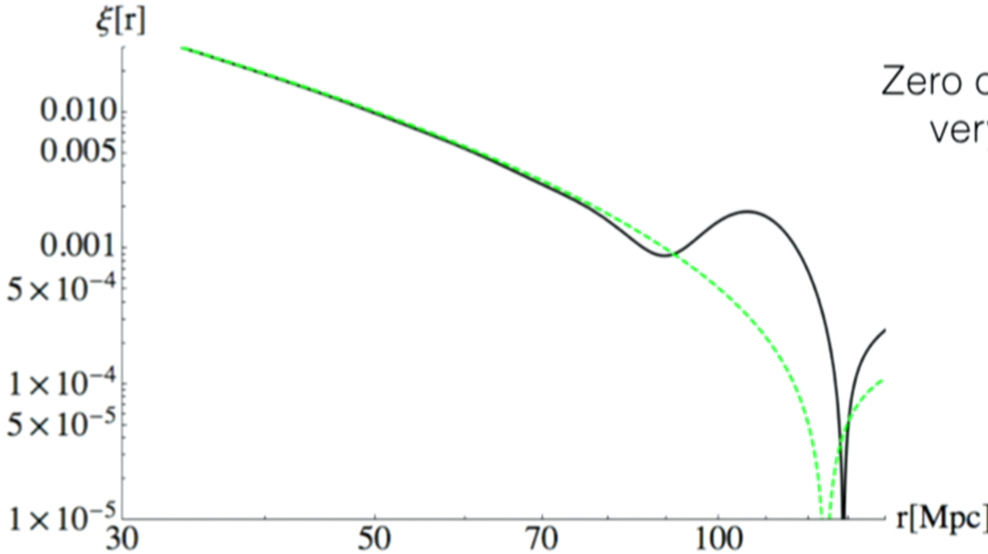

TF/10 & Wiggles
Matter radiation equality and recombination are very close in redshift
Transfer function
k[Mpc]
Wiggle part of the density power spectrum
Δ[k]/k²
Displacement power spectrum
k[Mpc]


TF/10 & Wiggles
Transfer function
k[Mpc]
$\Delta[k]/k^2$
Displacement power spectrum
k[Mpc]
Matter radiation equality and recombination are very close in redshift
Wiggle part of the density power spectrum

# Correlation function

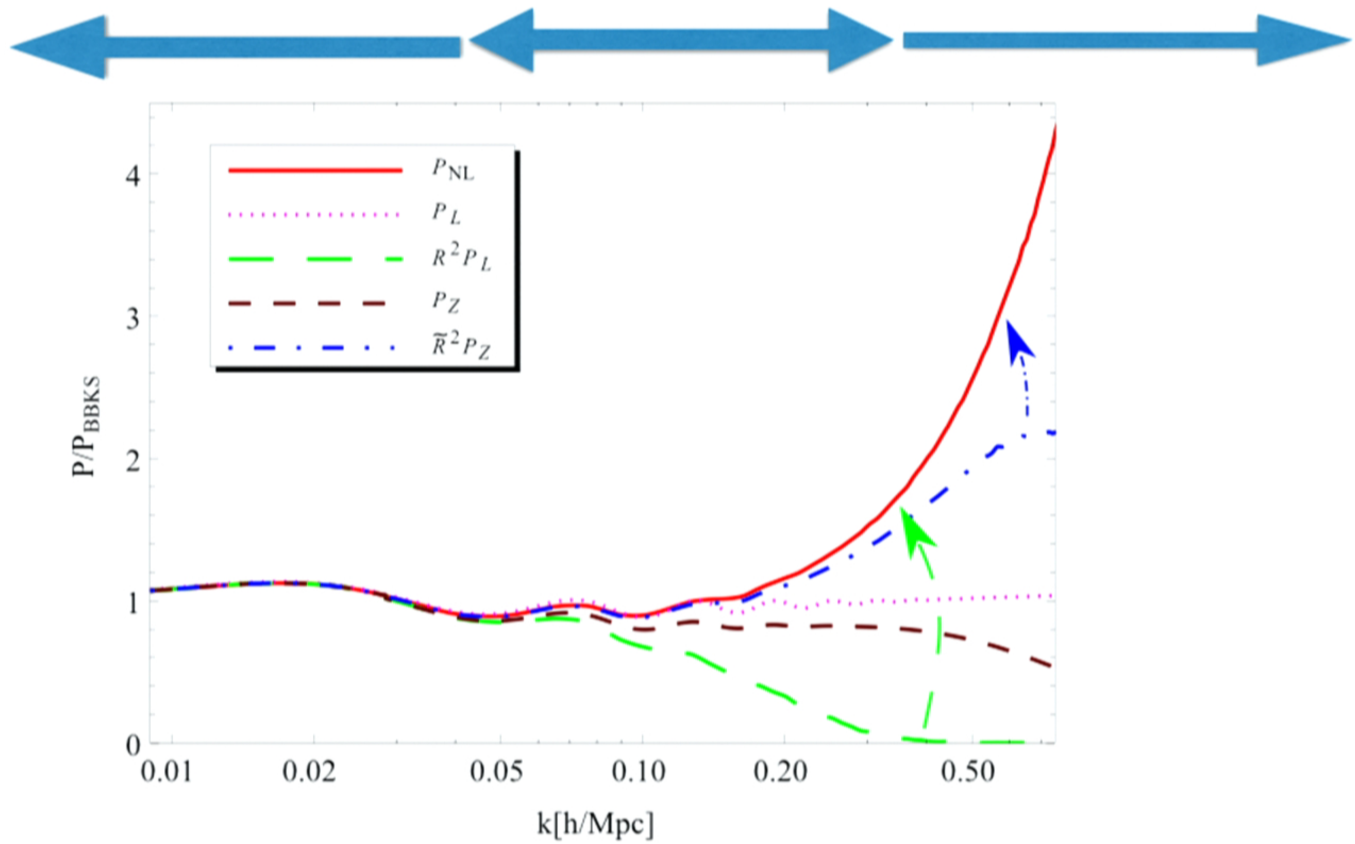


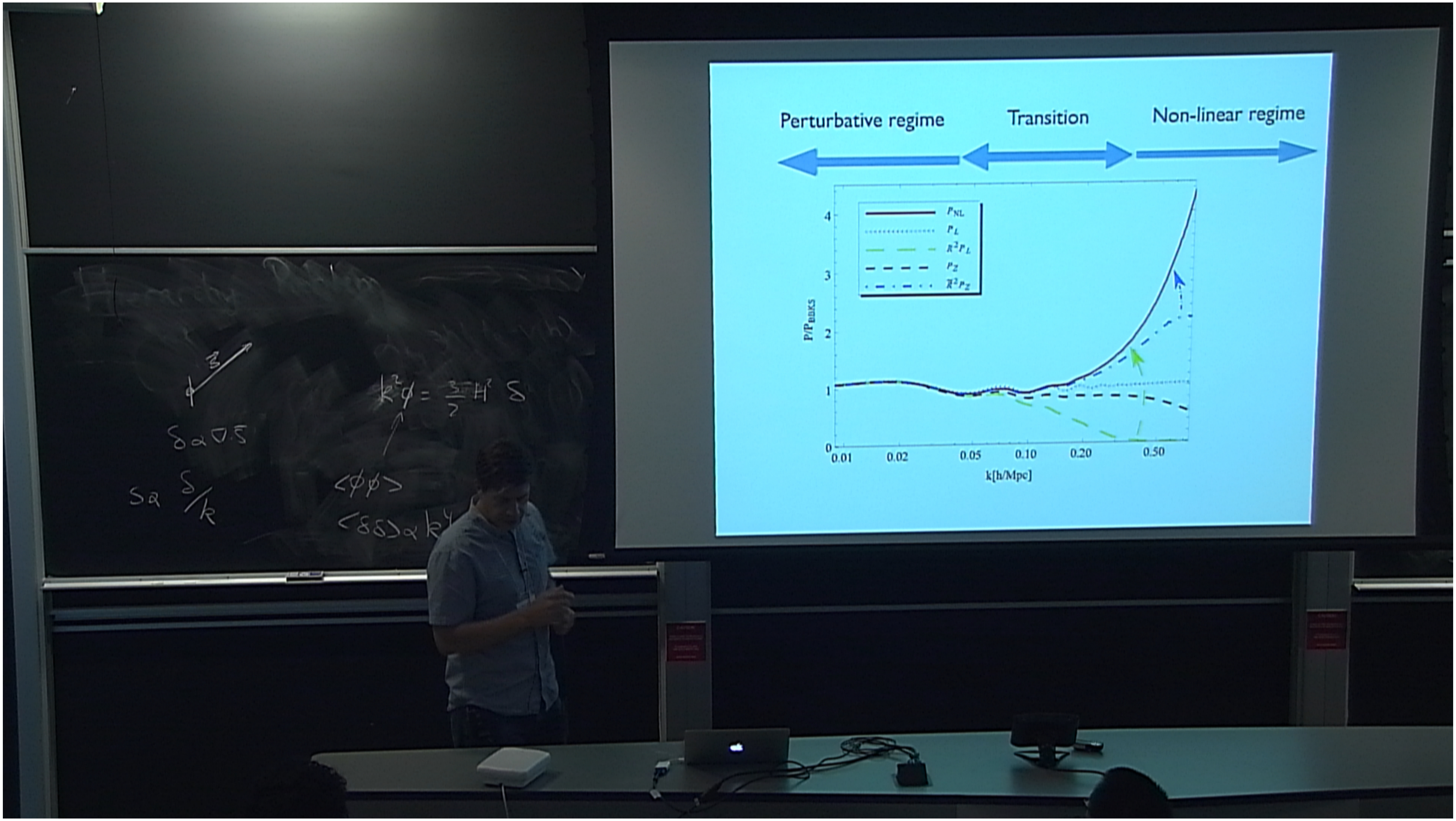

LCDM vs. No-Wiggle

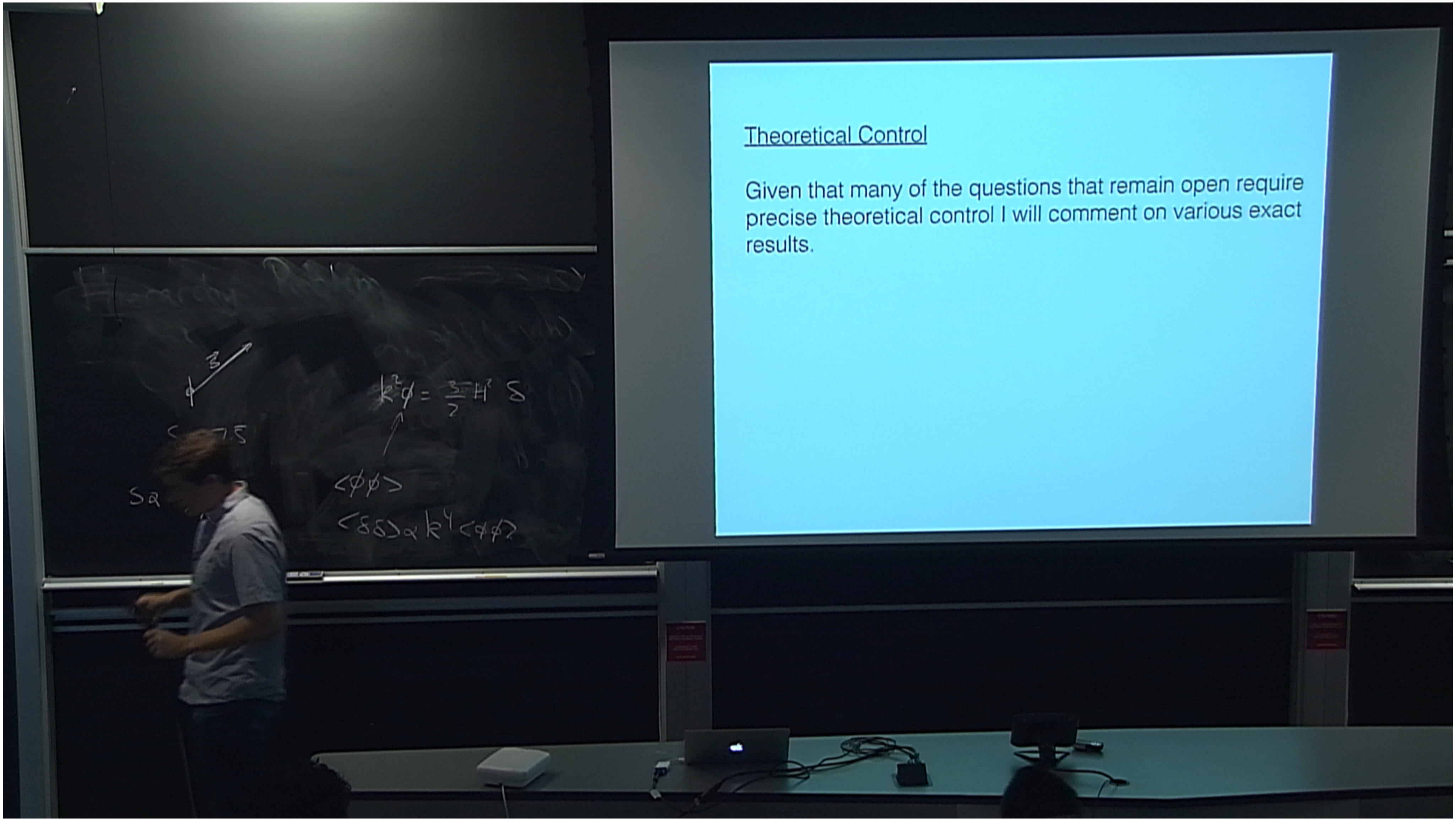


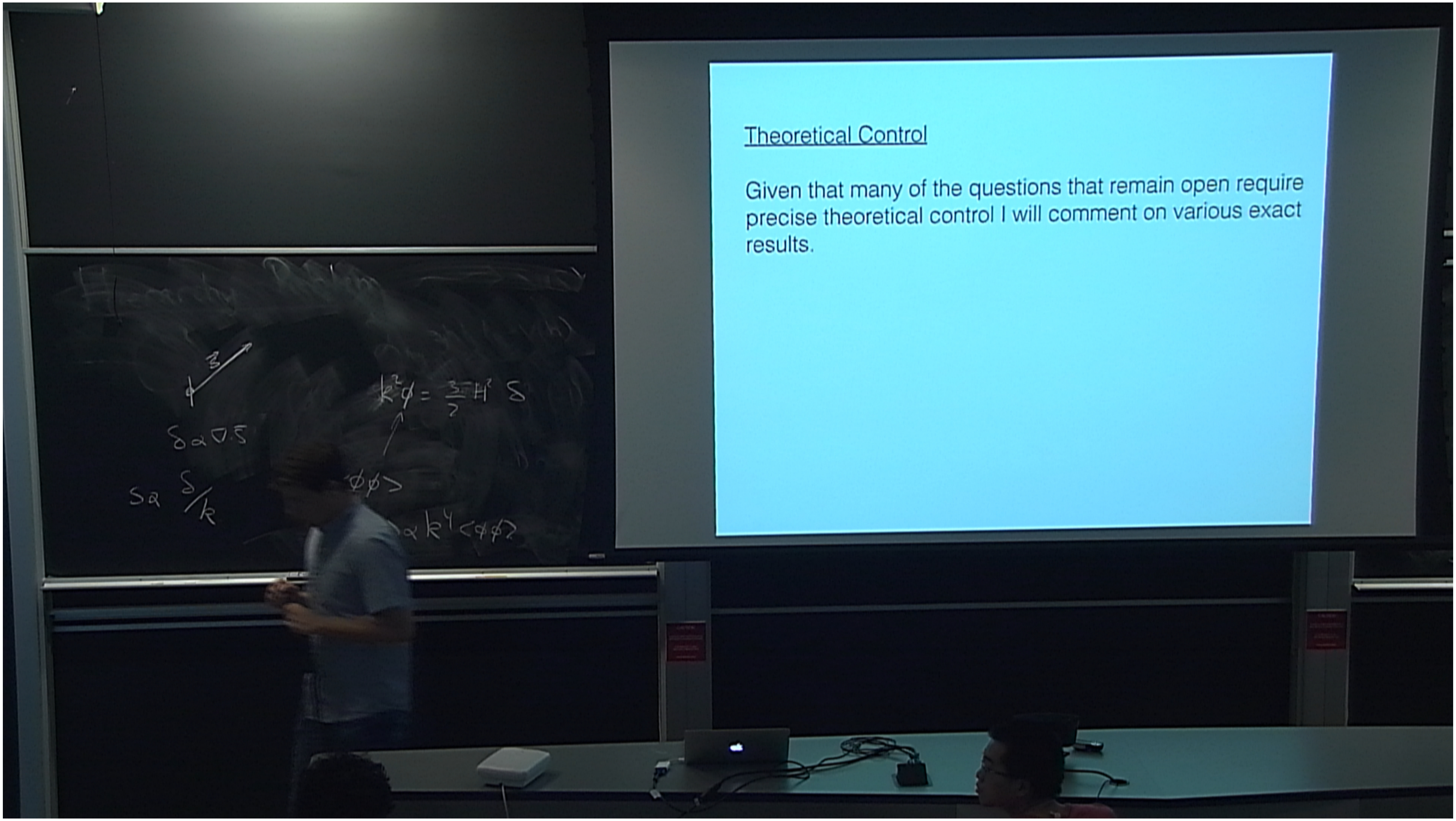

Zero of correlation function is very close to BAO peak

# Perturbative regime　Transition　Non-linear regime




Perturbative regime
Transition
Non-linear regime
$P_{NL}$
$P_L$
$\bar{R}^2 P_L$
$P_Z$
$\bar{R}^2 P_Z$
P/P_BBKS
k[h/Mpc]
0.01
0.02
0.05
0.10
0.20
0.50


Theoretical Control
Given that many of the questions that remain open require precise theoretical control I will comment on various exact results.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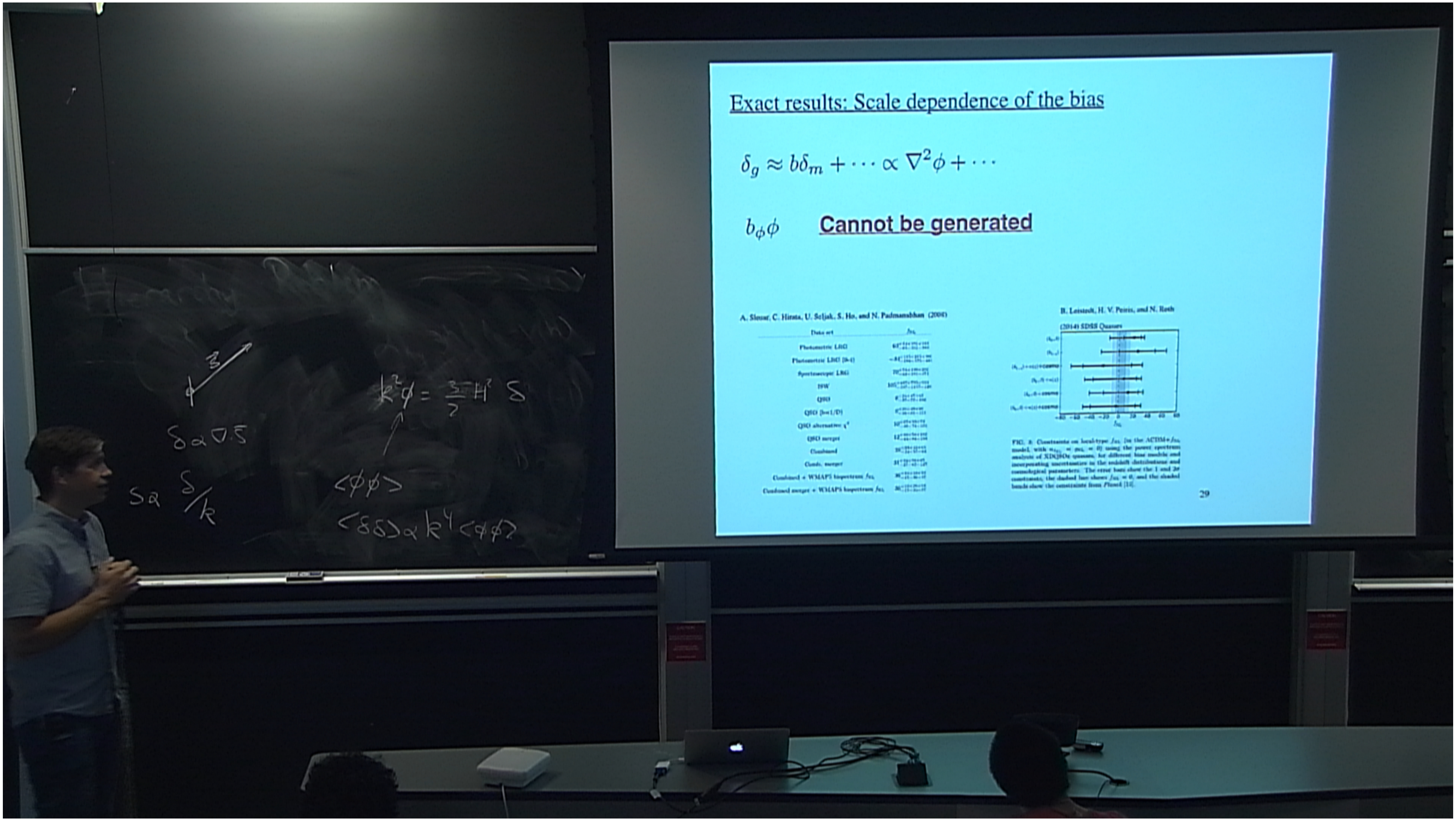

Exact results: Scale dependence of the bias
$\delta_g \approx b\delta_m + \cdots \propto \nabla^2\phi + \cdots$
$b_\phi \phi$
Cannot be generated
A. Slosar, C. Hirata, U. Seljak, S. Ho, and N. Padmanabhan (2008)
B. Leistedt, H. V. Peiris, and N. Roth
(2014) SDSS Quasars
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# Exact results: Scale dependence of the bias

$$\delta_g \approx b\delta_m + \cdots \propto \nabla^2\phi + \cdots$$

$b_\phi \phi$ **Cannot be generated**

A. Slosar, C. Hirata, U. Seljak, S. Ho, and N. Padmanabhan (2008)

| Data set | $f_{NL}$ |
|---|---|
| Photometric LRG | $63^{+54+101+143}_{-85-331-388}$ |
| Photometric LRG (0-4) | $-34^{+115+215+30\ldots}_{-194-375-44\ldots}$ |
| Spectroscopic LRG | $70^{+74+139+202}_{-83-191-371}$ |
| ISW | $105^{+647+755+933}_{-337-1157-128}$ |
| QSO | $8^{+26+47+65}_{-37-77-102}$ |
| QSO (b=1/D) | $8^{+28+49+69}_{-38-81-111}$ |
| QSO alternative $\chi^2$ | $10^{+27+52+72}_{-40-74-101}$ |
| QSO merger | $12^{+30+58+102}_{-44-94-138}$ |
| Combined | $28^{+23+42+65}_{-24-57-93}$ |
| Comb. merger | $31^{+16+39+65}_{-27-62-127}$ |
| Combined + WMAP5 bispectrum $f_{NL}$ | $36^{+18+33+52}_{-17-36-57}$ |
| Combined merger + WMAP5 bispectrum $f_{NL}$ | $36^{+13+29+53}_{-17-36-57}$ |

B. Leistedt, H. V. Peiris, and N. Roth

(2014) SDSS Quasars

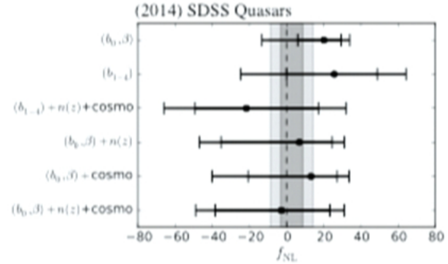
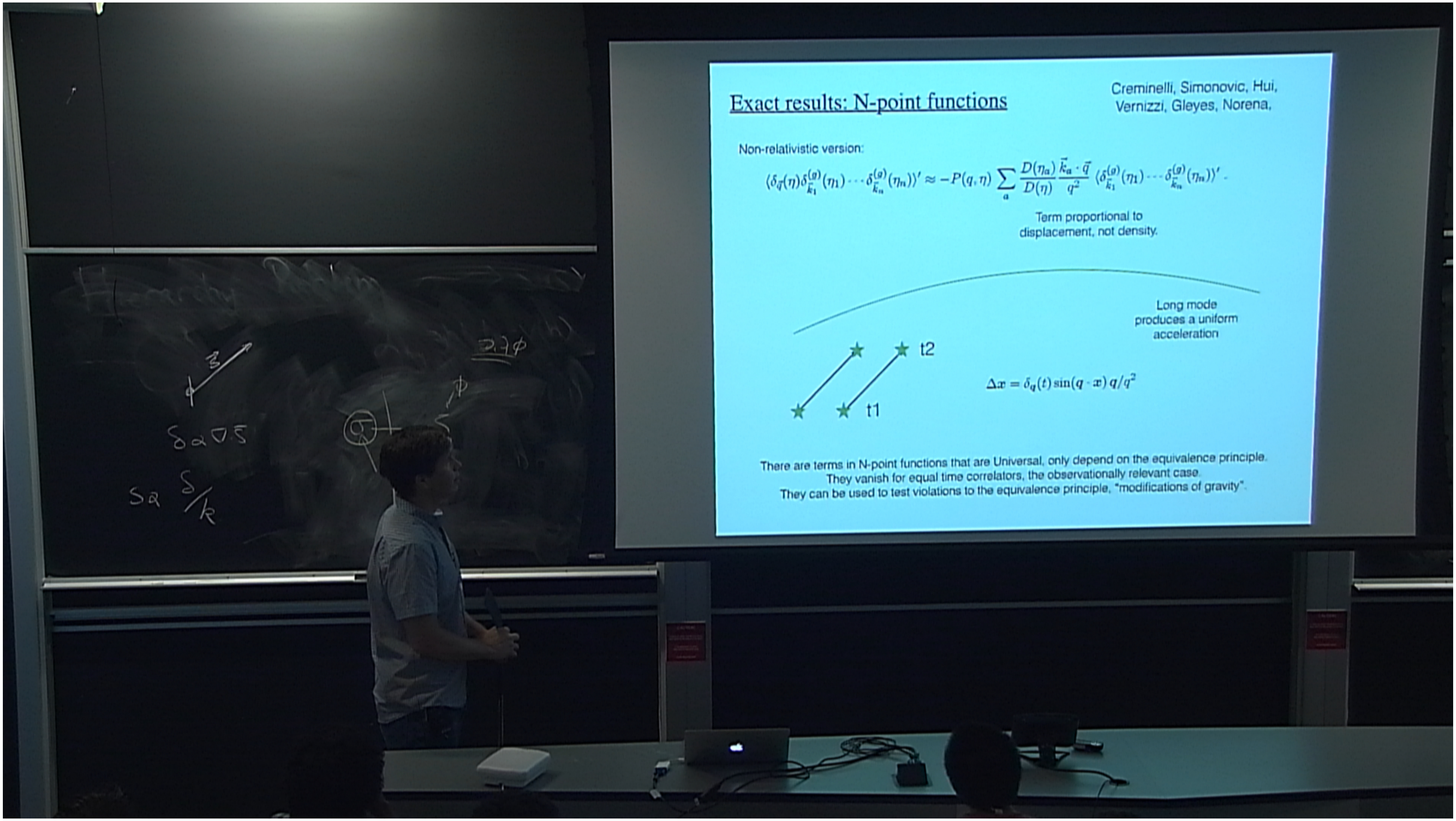



FIG. 3: Constraints on local-type $f_{NL}$ (in the ΛCDM+$f_{NL}$ model, with $n_{f_{NL}} = g_{NL} = 0$) using the power spectrum analysis of XDQSOz quasars, for different bias models and incorporating uncertainties in the redshift distributions and cosmological parameters. The error bars show the 1 and 2σ constraints, the dashed line shows $f_{NL} = 0$, and the shaded bands show the constraints from *Planck* [13].



Exact results: N-point functions
Creminelli, Simonovic, Hui,
Vernizzi, Gleyes, Norena,
Non-relativistic version:
$\langle \delta_{\vec{q}}(\eta) \delta^{(g)}_{\vec{k}_1}(\eta_1) \cdots \delta^{(g)}_{\vec{k}_n}(\eta_n) \rangle' \approx -P(q,\eta) \sum_a \frac{D(\eta_a)}{D(\eta)} \frac{\vec{k}_a \cdot \vec{q}}{q^2} \langle \delta^{(g)}_{\vec{k}_1}(\eta_1) \cdots \delta^{(g)}_{\vec{k}_n}(\eta_n) \rangle'$ .
Term proportional to
displacement, not density.
Long mode
produces a uniform
acceleration
t2
t1
$\Delta \boldsymbol{x} = \delta_{\boldsymbol{q}}(t) \sin(\boldsymbol{q} \cdot \boldsymbol{x})\, \boldsymbol{q}/q^2$
There are terms in N-point functions that are Universal, only depend on the equivalence principle.
They vanish for equal time correlators, the observationally relevant case.
They can be used to test violations to the equivalence principle, "modifications of gravity".

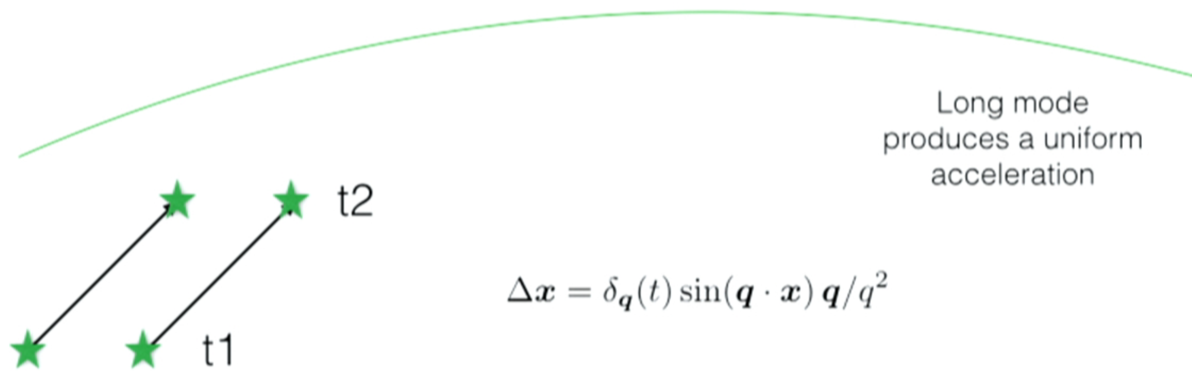
# Exact results: N-point functions

Creminelli, Simonovic, Hui,
Vernizzi, Gleyes, Norena,

Non-relativistic version:

$$\langle \delta_{\vec{q}}(\eta)\delta^{(g)}_{\vec{k}_1}(\eta_1)\cdots\delta^{(g)}_{\vec{k}_n}(\eta_n)\rangle' \approx -P(q,\eta)\sum_a \frac{D(\eta_a)}{D(\eta)}\frac{\vec{k}_a\cdot\vec{q}}{q^2}\,\langle \delta^{(g)}_{\vec{k}_1}(\eta_1)\cdots\delta^{(g)}_{\vec{k}_n}(\eta_n)\rangle' \,.$$

Term proportional to
displacement, not density.




$$\Delta \boldsymbol{x} = \delta_{\boldsymbol{q}}(t)\sin(\boldsymbol{q}\cdot\boldsymbol{x})\,\boldsymbol{q}/q^2$$

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Term proportional to
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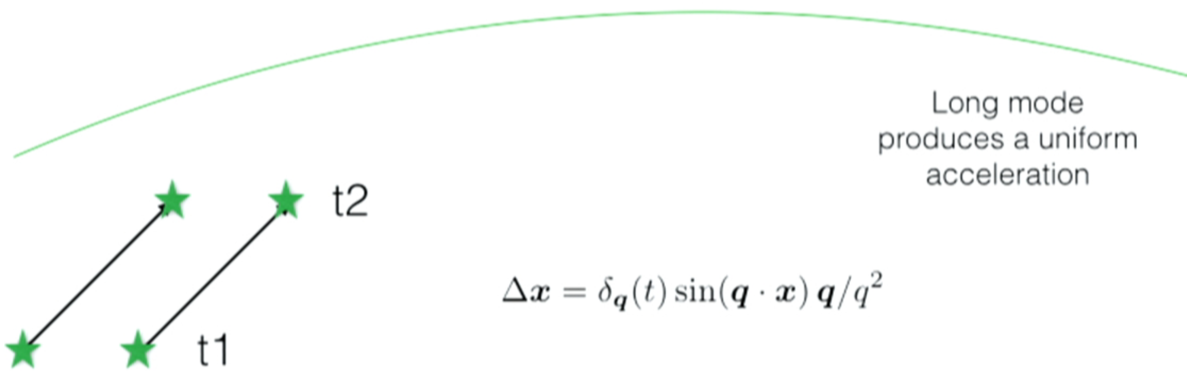



Long mode
produces a uniform
acceleration

$$\Delta \boldsymbol{x} = \delta_{\boldsymbol{q}}(t)\sin(\boldsymbol{q}\cdot\boldsymbol{x})\,\boldsymbol{q}/q^2$$

There are terms in N-point functions that are Universal, only depend on the equivalence principle.
They vanish for equal time correlators, the observationally relevant case.
They can be used to test violations to the equivalence principle, "modifications of gravity".

# Exact results: N-point functions

There are Universal pieces in N-point functions of LSS correlators determined solely by symmetries. Universal in the sense that the relations apply to arbitrary tracers.

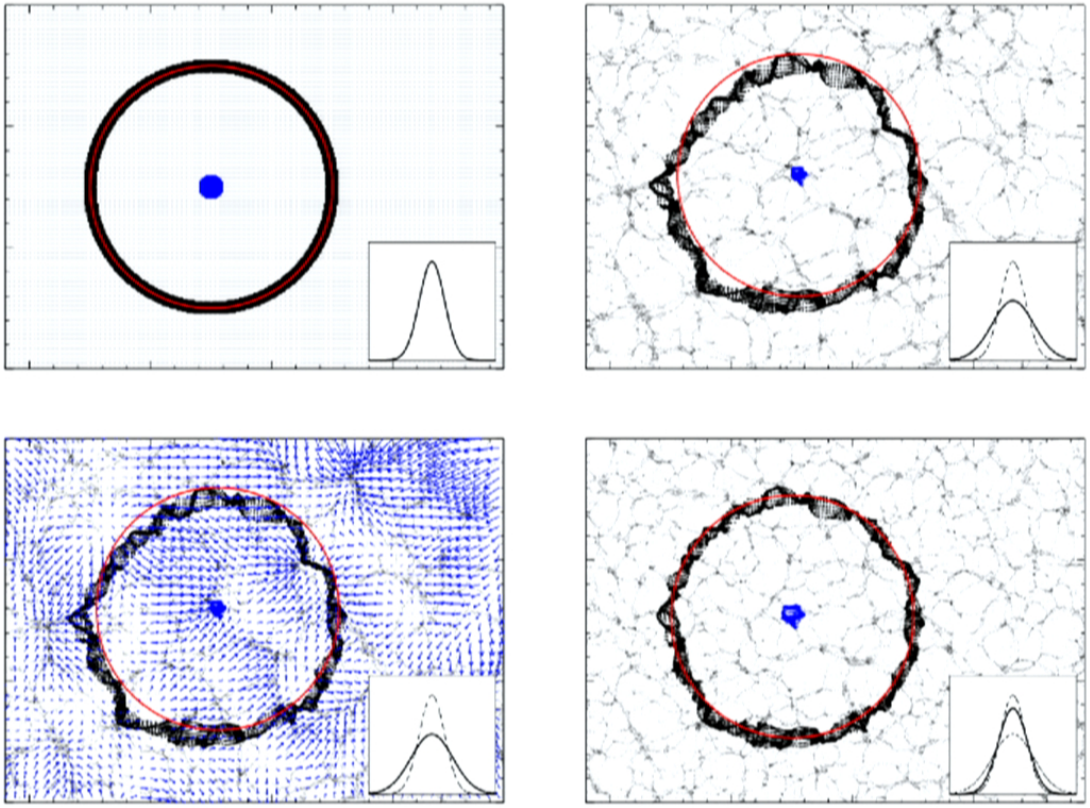
They are based only on the equivalence principle and the absence of primordial non-Gaussianities.

$$\langle \delta(\boldsymbol{q},t)\delta_g(\boldsymbol{k}_-,t_1)\delta_g(-\boldsymbol{k}_+,t_2)\rangle \simeq \frac{\boldsymbol{q}\cdot\boldsymbol{k}}{q^2}$$

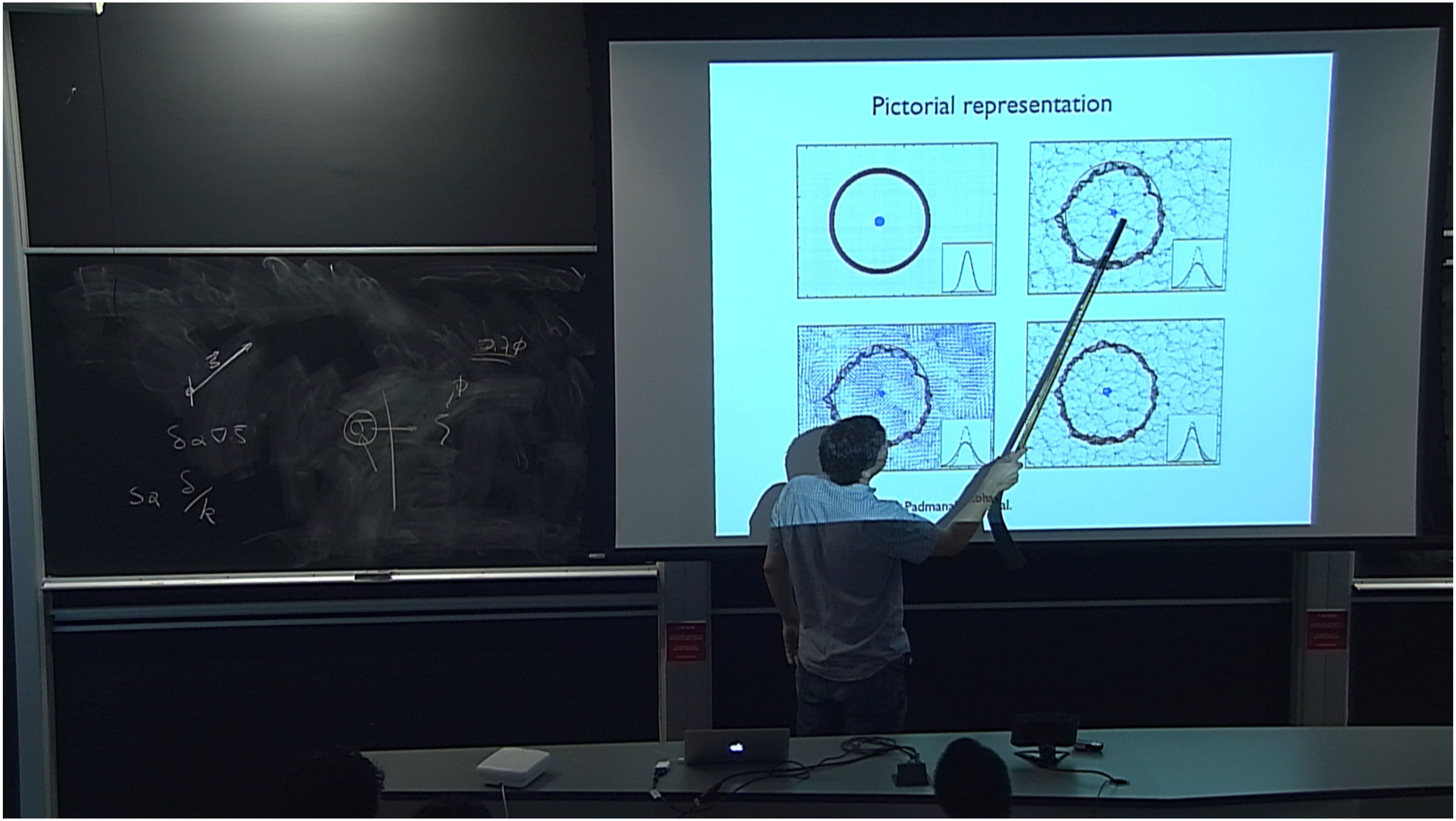
$$P_{\rm lin}(q,t)\Big[\frac{D(t_1)}{D(t)}P_g(k_-,t_1) - \frac{D(t_2)}{D(t)}P_g(k_+,t_2)\Big]$$

Only seems relevant if consider different times

# Pictorial representation



1202.0090 Padmanabhan et al.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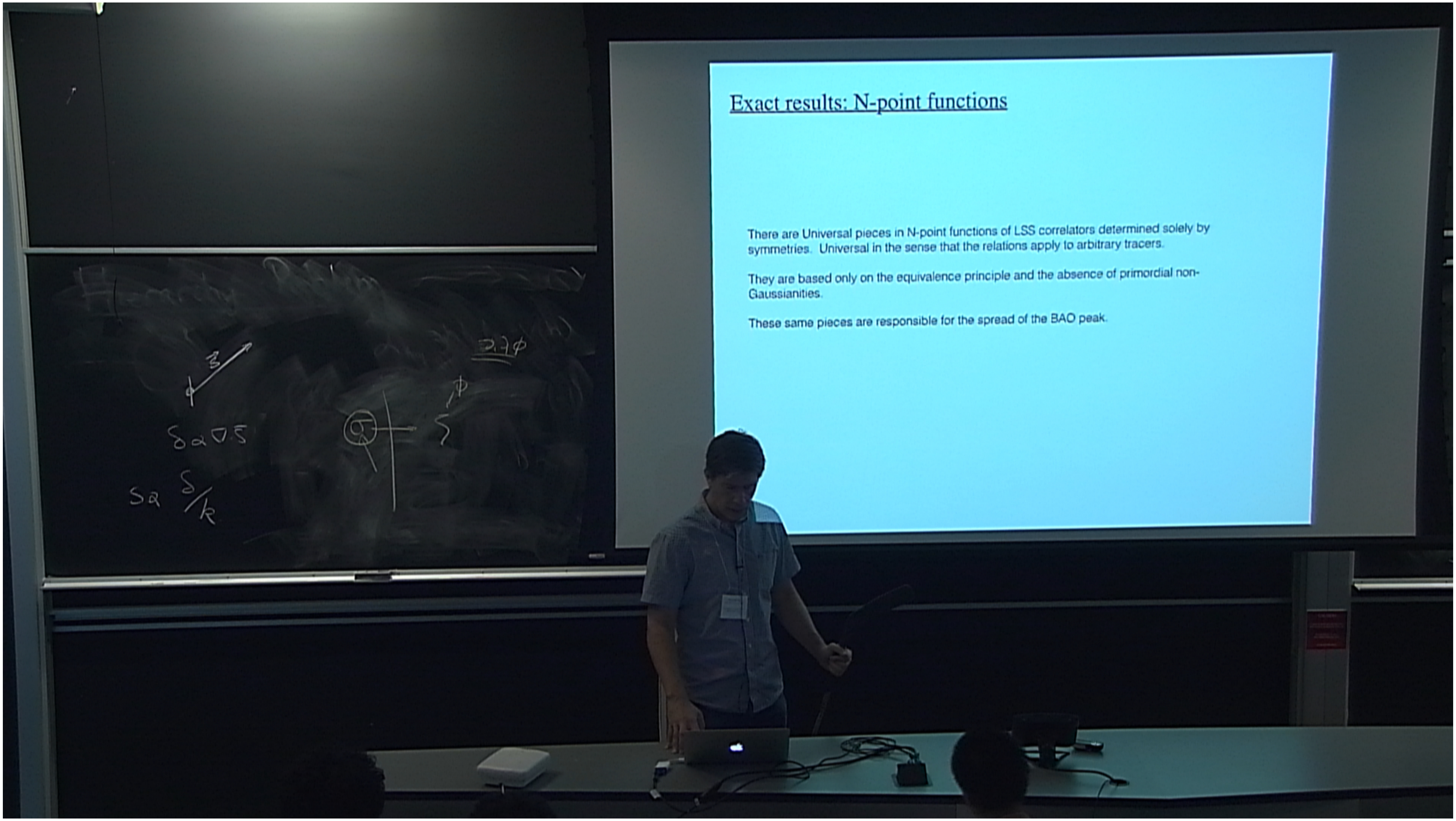
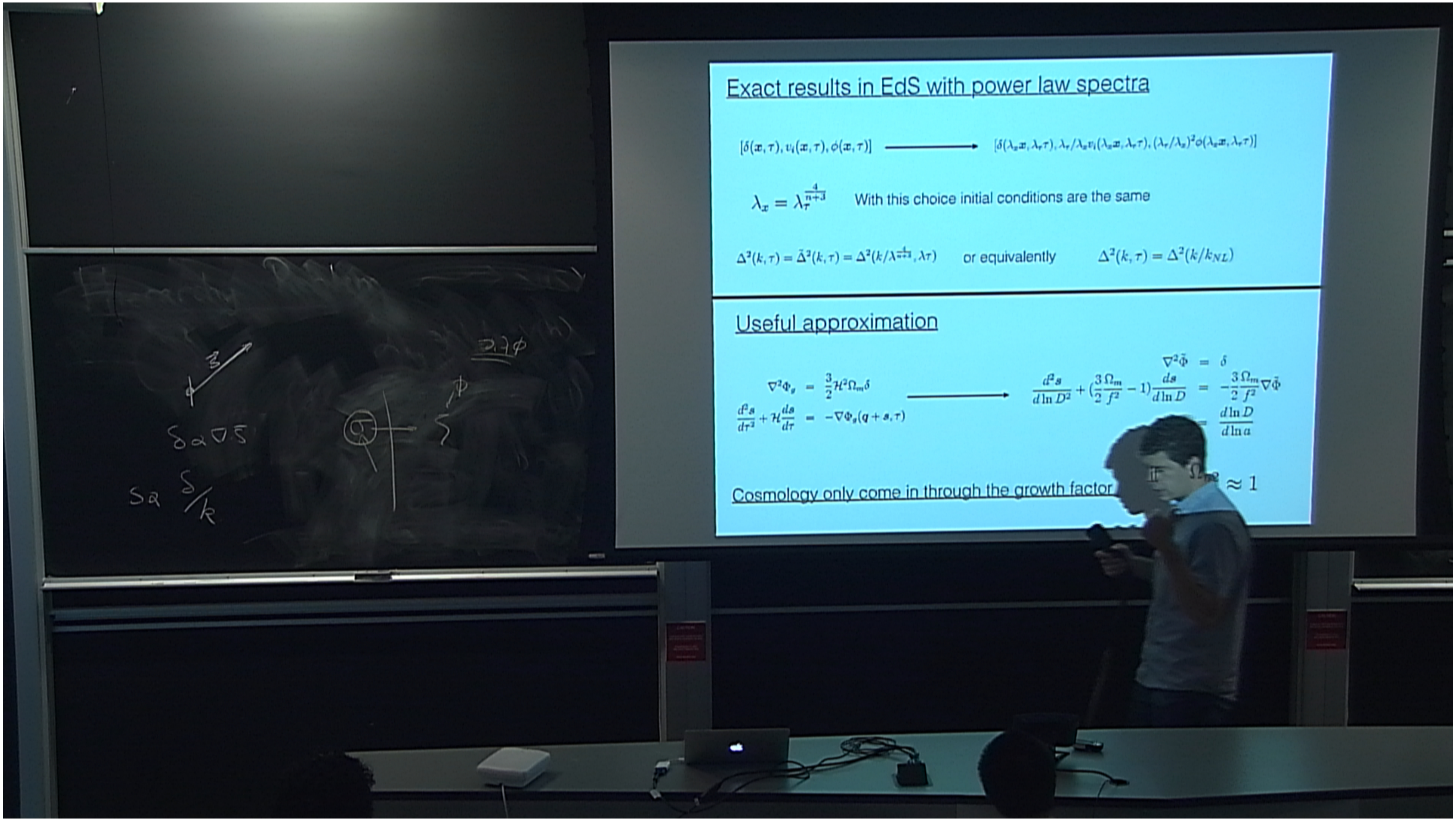
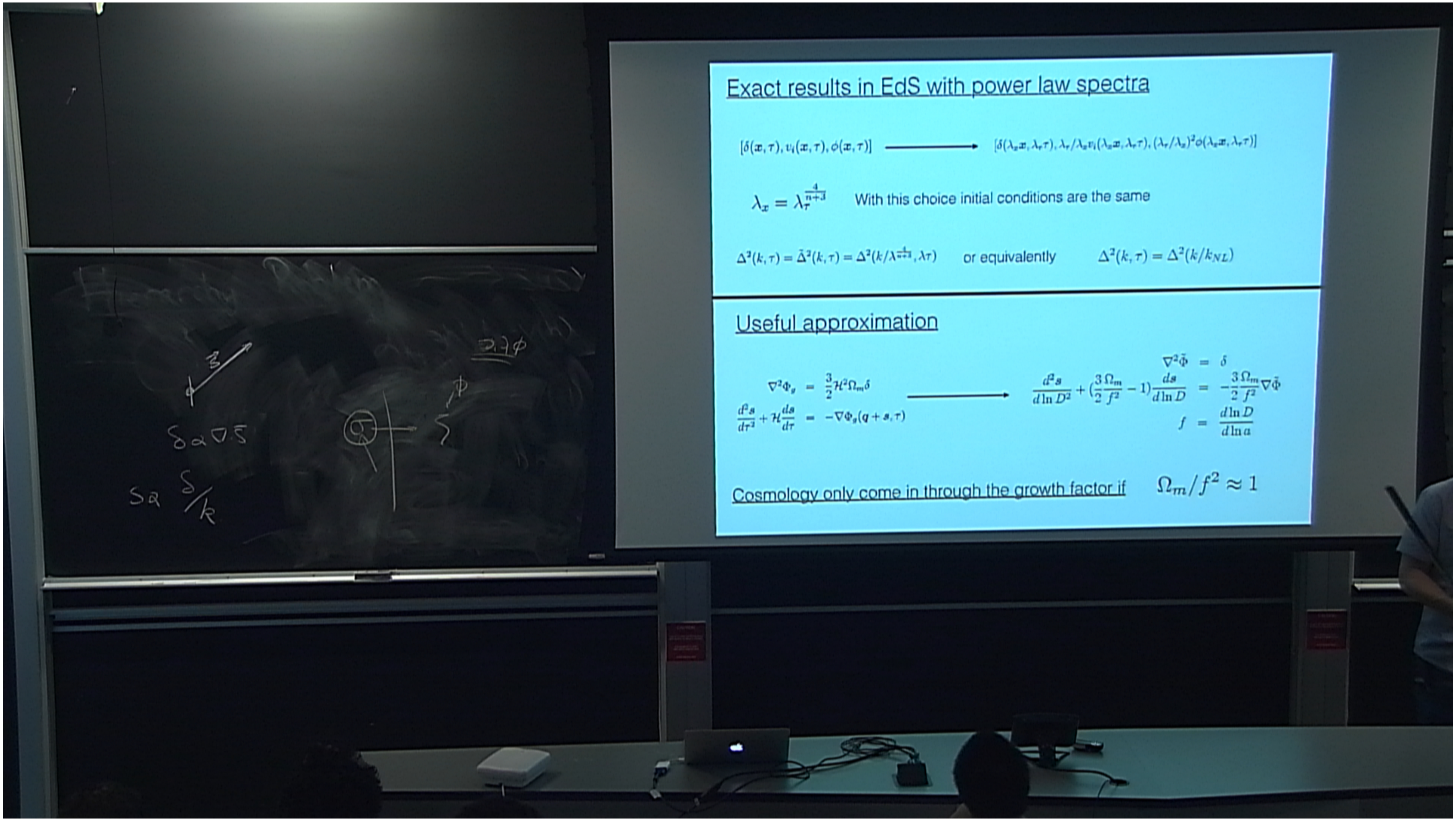

Exact results: N-point functions
There are Universal pieces in N-point functions of LSS correlators determined solely by symmetries. Universal in the sense that the relations apply to arbitrary tracers.
They are based only on the equivalence principle and the absence of primordial non-Gaussianities.
These same pieces are responsible for the spread of the BAO peak.



Exact results in EdS with power law spectra
$[\delta(\boldsymbol{x},\tau), v_i(\boldsymbol{x},\tau), \phi(\boldsymbol{x},\tau)] \longrightarrow [\delta(\lambda_x \boldsymbol{x}, \lambda_\tau \tau), \lambda_\tau/\lambda_x v_i(\lambda_x \boldsymbol{x}, \lambda_\tau \tau), (\lambda_\tau/\lambda_x)^2 \phi(\lambda_x \boldsymbol{x}, \lambda_\tau \tau)]$
$\lambda_x = \lambda_\tau^{\frac{4}{n+3}}$
With this choice initial conditions are the same
$\Delta^2(k,\tau) = \tilde{\Delta}^2(k,\tau) = \Delta^2(k/\lambda^{\frac{4}{n+3}}, \lambda\tau)$
or equivalently
$\Delta^2(k,\tau) = \Delta^2(k/k_{NL})$
Useful approximation
$\nabla^2\Phi_g = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$
$\frac{d^2 s}{d\tau^2} + \mathcal{H}\frac{ds}{d\tau} = -\nabla\Phi_g(q+s,\tau)$
$\nabla^2\tilde{\Phi} = \delta$
$\frac{d^2 s}{d\ln D^2} + (\frac{3}{2}\frac{\Omega_m}{f^2} - 1)\frac{ds}{d\ln D} = -\frac{3}{2}\frac{\Omega_m}{f^2}\nabla\tilde{\Phi}$
$\frac{d\ln D}{d\ln a}$
Cosmology only come in through the growth factor
$\approx 1$



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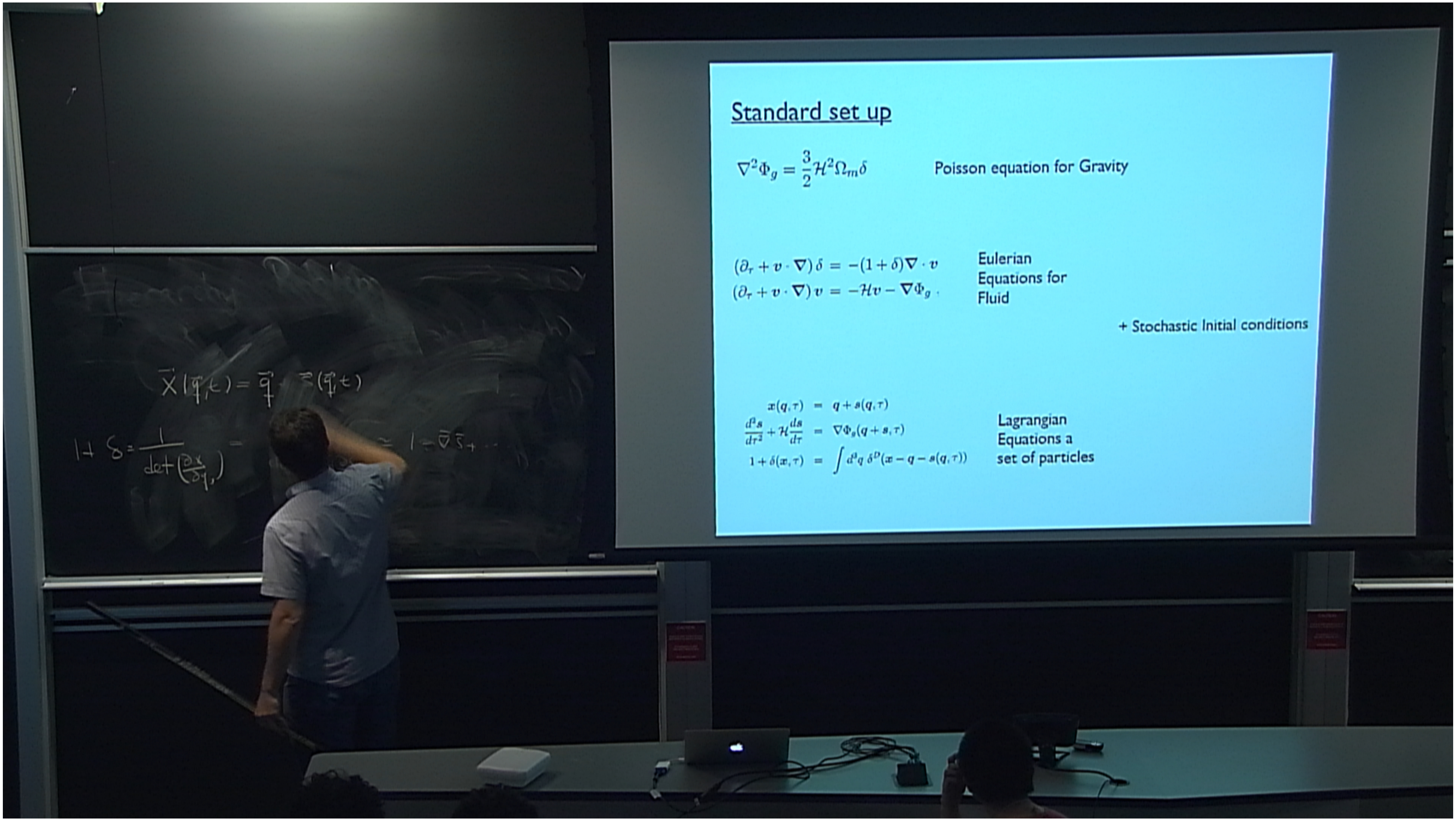
$f = \frac{d\ln D}{d\ln a}$
Cosmology only come in through the growth factor if
$\Omega_m/f^2 \approx 1$


Standard set up
$\nabla^2\Phi_g = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$
Poisson equation for Gravity
$(\partial_\tau + v\cdot\nabla)\delta = -(1+\delta)\nabla\cdot v$
$(\partial_\tau + v\cdot\nabla)v = -\mathcal{H}v - \nabla\Phi_g$ .
Eulerian Equations for Fluid
+ Stochastic Initial conditions
$x(q,\tau) = q + s(q,\tau)$
$\frac{d^2 s}{d\tau^2} + \mathcal{H}\frac{ds}{d\tau} = \nabla\Phi_g(q+s,\tau)$
$1+\delta(x,\tau) = \int d^3q\, \delta^D(x - q - s(q,\tau))$
Lagrangian Equations a set of particles