Title: The Standard Model â€“ Theory: The Goldstone Boson Equivalence Theorem

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URL: http://pirsa.org/15070021

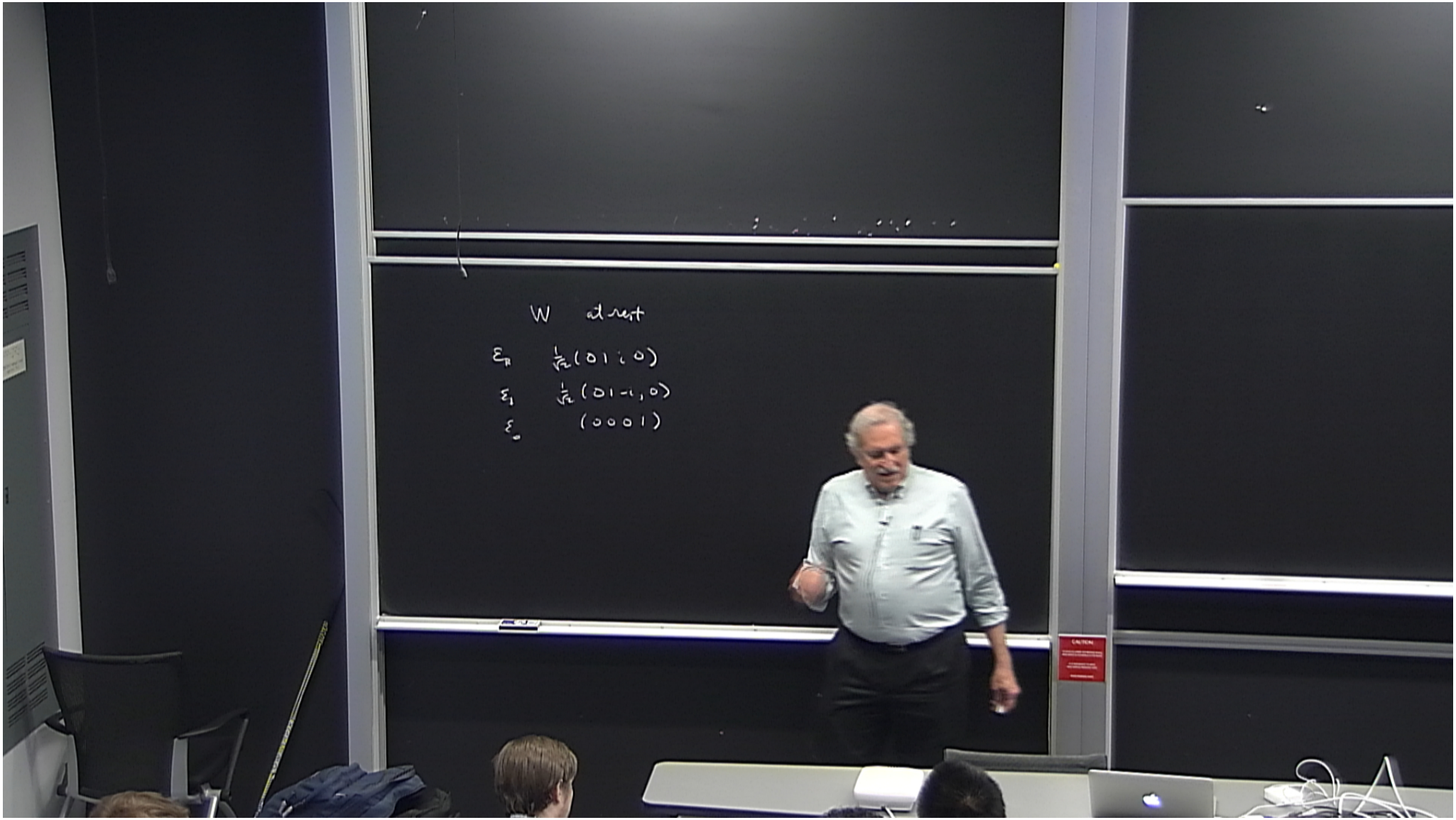
Abstract:

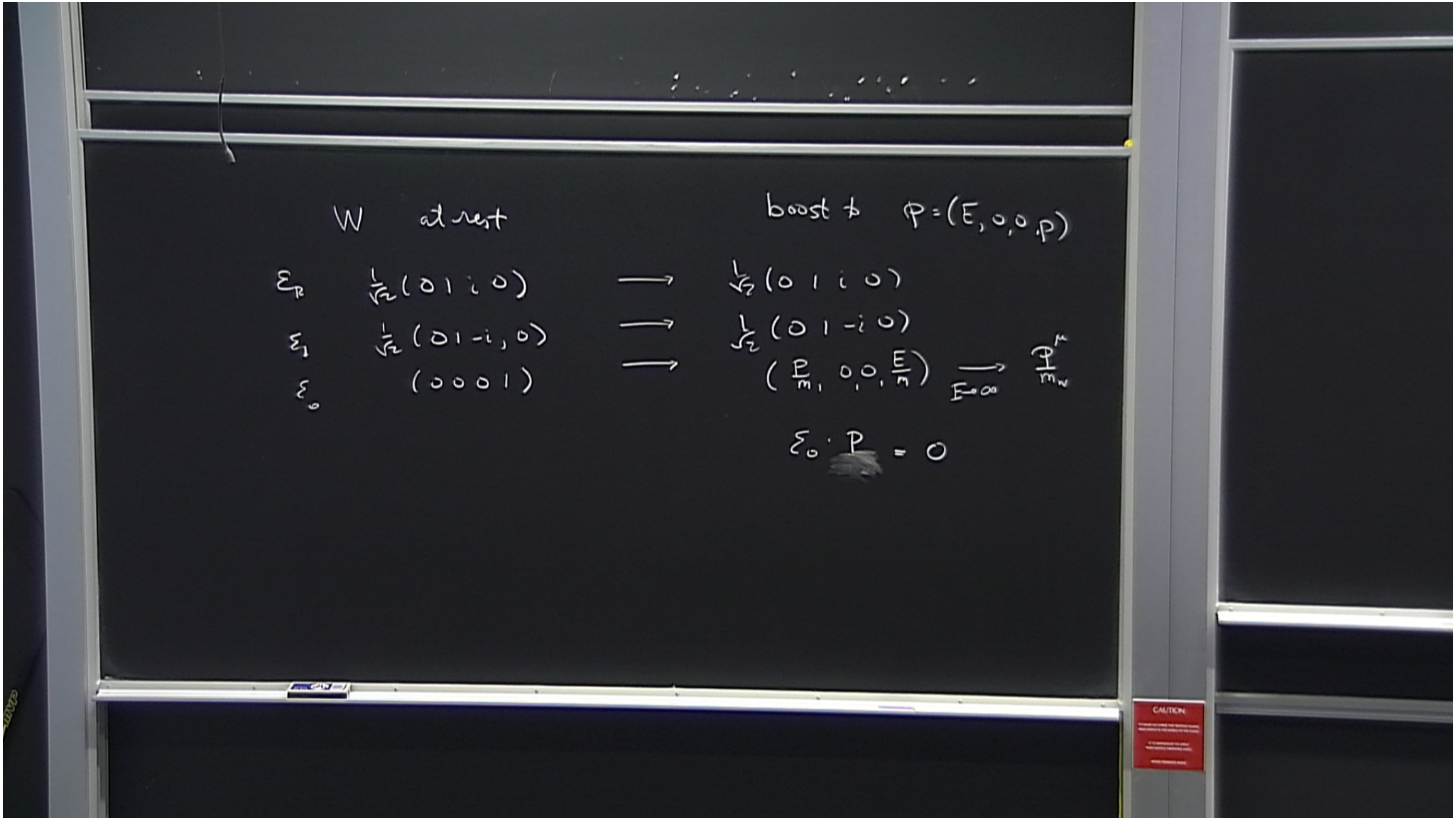
Collider Physics

figures for lecture 5

Goldstone Boson Equivalence Theorem

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TRISEP School 2015


W at rest
ε_π $\frac{1}{\sqrt{2}}(0\,1\,i\,0)$
ε_1 $\frac{1}{\sqrt{2}}(0\,1\,-i\,0)$
ε_0 $(0\,0\,0\,1)$

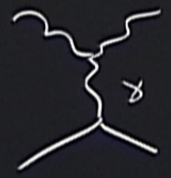

W at rest
boost to $p=(E,0,0,p)$
ε_R $\frac{1}{\sqrt{2}}(0,1,i,0)$ $\longrightarrow$ $\frac{1}{\sqrt{2}}(0,1,i,0)$
ε_1 $\frac{1}{\sqrt{2}}(0,1,-i,0)$ $\longrightarrow$ $\frac{1}{\sqrt{2}}(0,1,-i,0)$
ε_0 $(0,0,0,1)$ $\longrightarrow$ $(\frac{p}{m},0,0,\frac{E}{m})$ $\xrightarrow{E\to\infty}$ $\frac{p^\mu}{E}$
$\varepsilon_0 \cdot p = 0$
CAUTION

$e^+e^- \to \phi\phi$



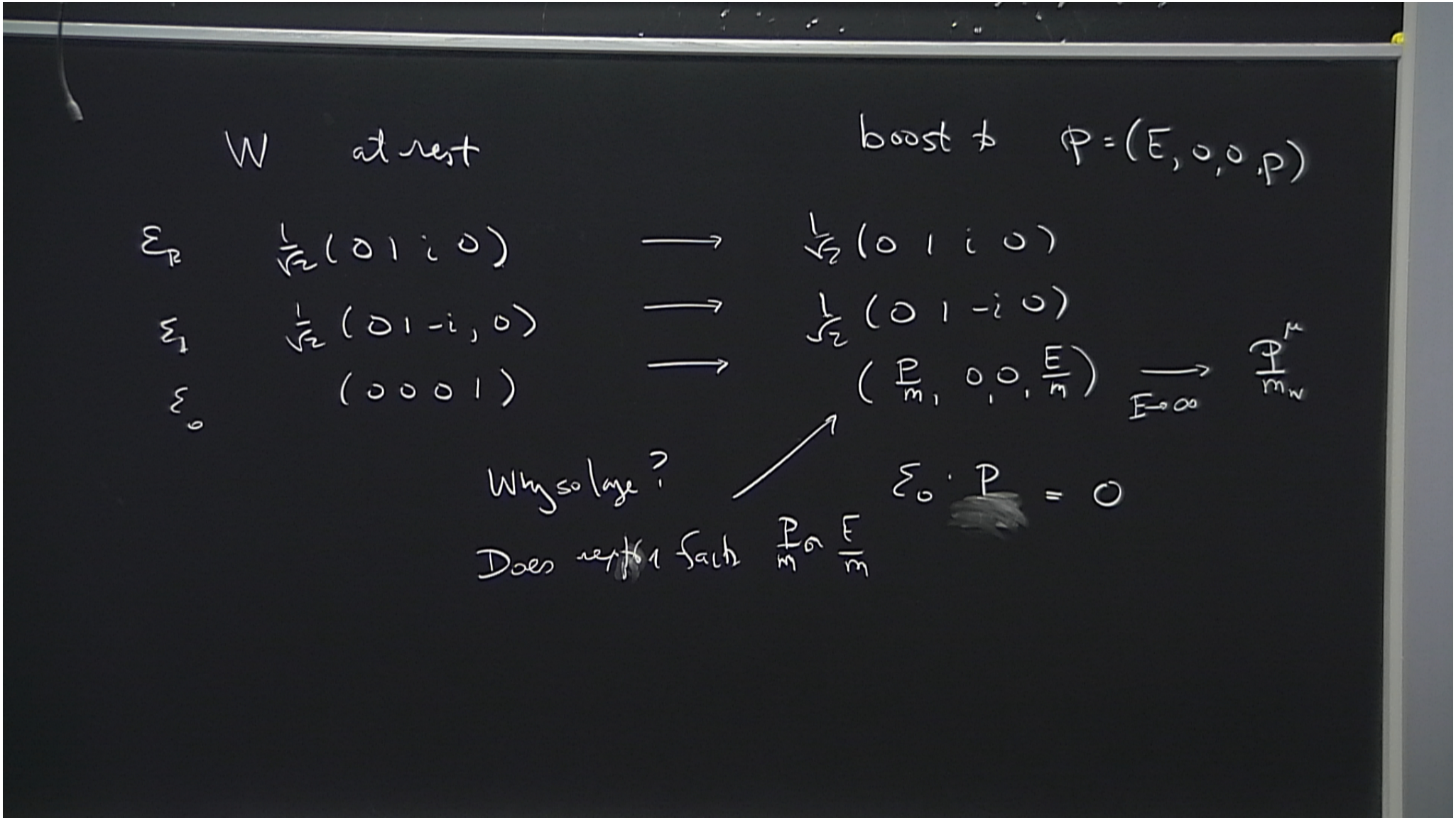
$e^2 \quad \varepsilon_\lambda(e) \cdot \frac{p_\phi}{2E}$

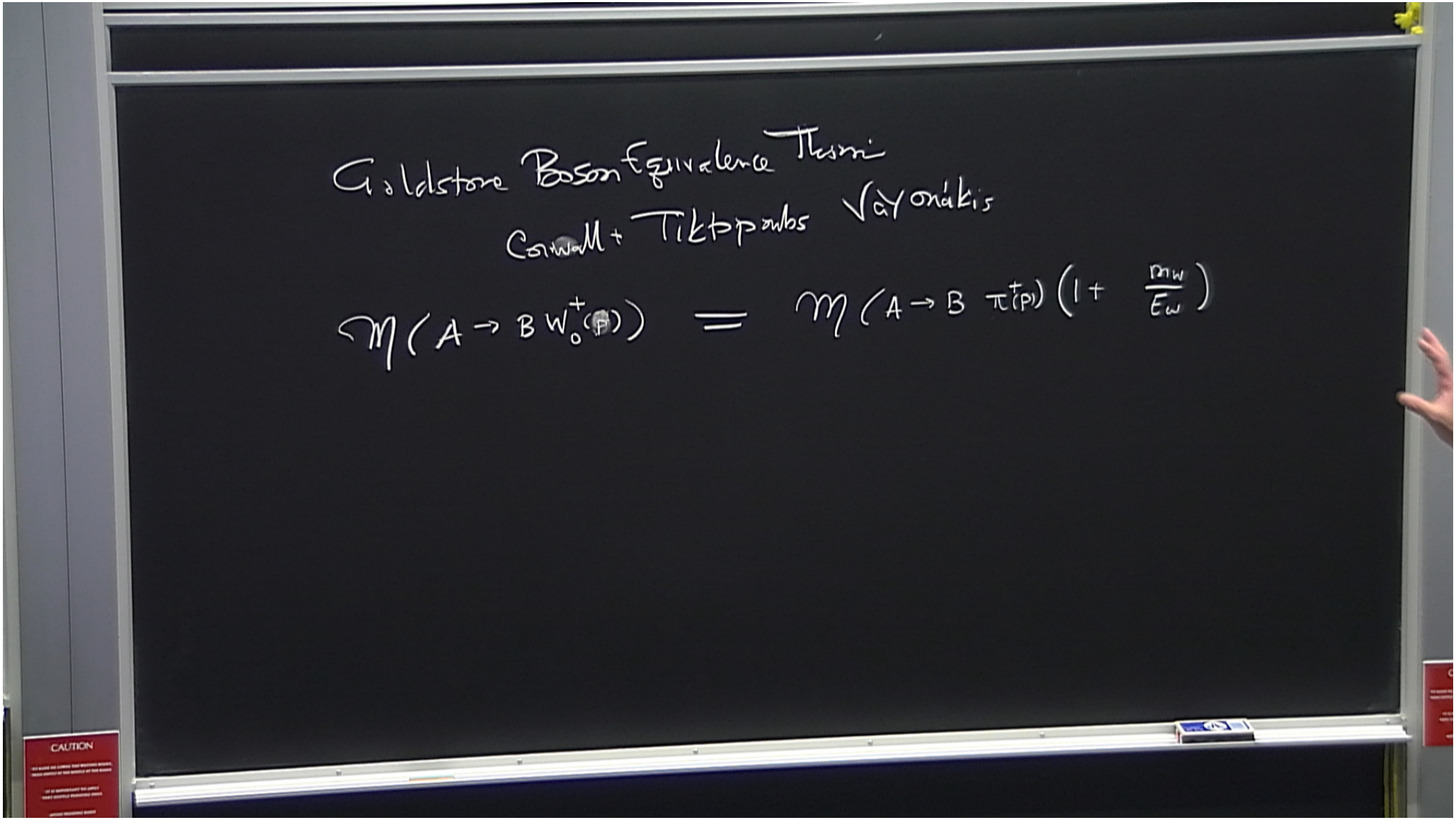
$e^+e^- \to W^+W^-$

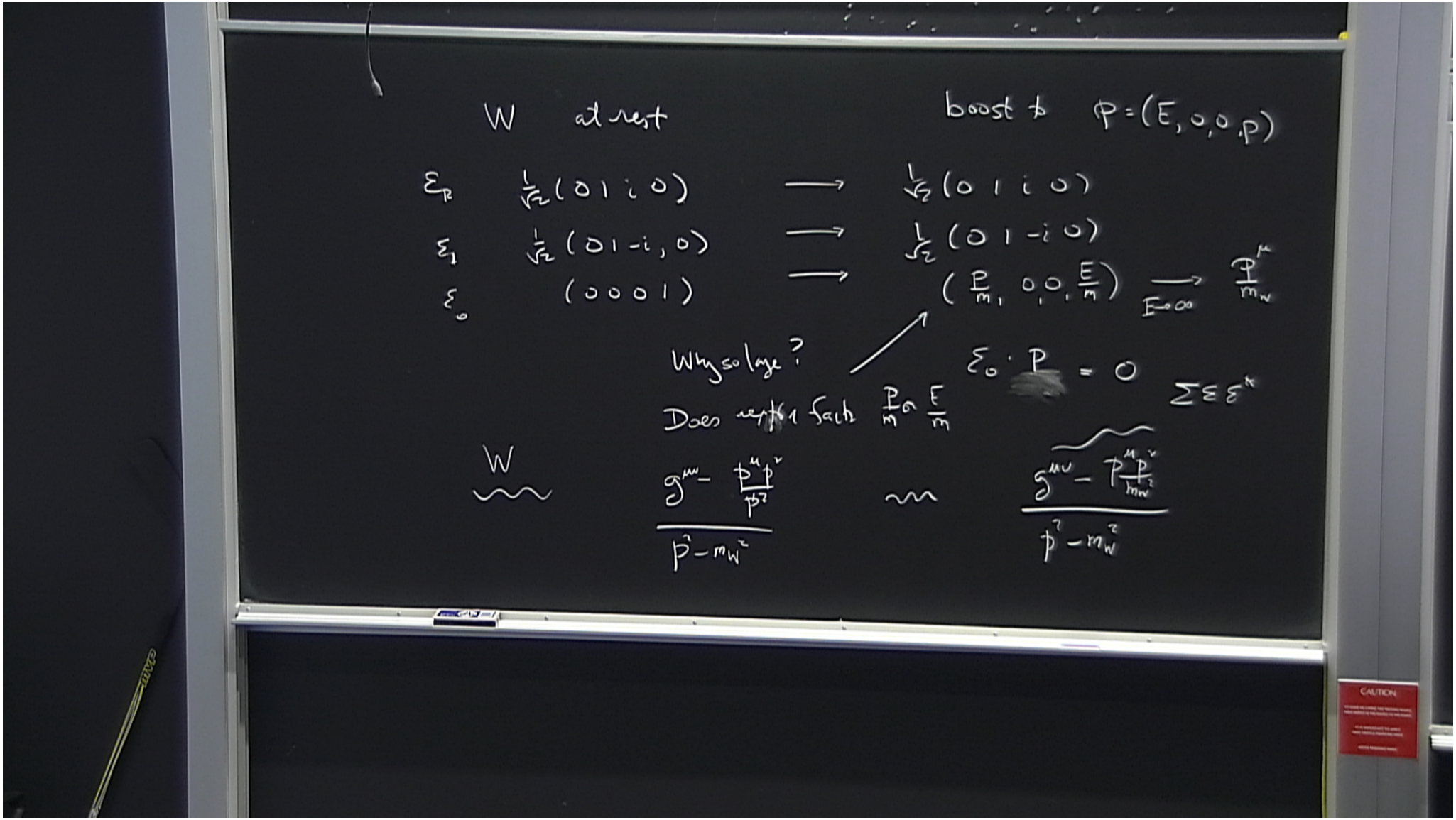


$e^2 \quad \varepsilon_\lambda(e^-) \cdot \frac{p_W}{E} \qquad \varepsilon_0^*(W^+) \cdot \varepsilon_0^*(W^-)$

$\frac{p_+}{m_W} \cdot \frac{p_-}{m_W} = \frac{s}{2m_W^2}$


W at rest
boost ∥ P = (E, 0, 0, p)
ε_R $\frac{1}{\sqrt{2}}(0\ 1\ i\ 0)$ ⟶ $\frac{1}{\sqrt{2}}(0\ 1\ i\ 0)$
ε_1 $\frac{1}{\sqrt{2}}(0\ 1\ -i\ 0)$ ⟶ $\frac{1}{\sqrt{2}}(0\ 1\ -i\ 0)$
ε_0 $(0\ 0\ 0\ 1)$ ⟶ $(\frac{p}{m}, 0, 0, \frac{E}{m})$ ⟶ $\frac{P^\mu}{m_W}$
$E \to \infty$
Why so large?
$\varepsilon_0 \cdot P = 0$
Does reflect fact $\frac{p}{m} \to \frac{E}{m}$


Goldstone Boson Equivalence Theorem
Cornell + Tiktopoulos Vayonakis
$\mathcal{M}(A \to B\, W_0^+(p)) = \mathcal{M}(A \to B\, \pi^+(p)) \left(1 + \frac{m_W}{E_W}\right)$
CAUTION


W at rest
boost ↑ p = (E, 0, 0, p)
ε_R $\frac{1}{\sqrt{2}}(0,1,i,0)$ ⟶ $\frac{1}{\sqrt{2}}(0,1,i,0)$
ε_1 $\frac{1}{\sqrt{2}}(0,1,-i,0)$ ⟶ $\frac{1}{\sqrt{2}}(0,1,-i,0)$
ε_0 $(0,0,0,1)$ ⟶ $(\frac{p}{m_W},0,0,\frac{E}{m_W})$ $\xrightarrow{E\to\infty}$ $\frac{p^\mu}{m_W}$
Why so large?
$\varepsilon_0 \cdot p = 0$
$\sum \varepsilon \varepsilon^*$
Does not scale
$3 \frac{p_0}{m_W} \frac{E}{m_W}$
W
$\frac{g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}}{p^2 - m_W^2}$
$\frac{g^{\mu\nu} - \frac{p^\mu p^\nu}{m_W^2}}{p^2 - m_W^2}$

$t \to W^+ b$

$\mathcal{M} = i \frac{g}{\sqrt{2}} u_L^\dagger(b) \bar{\sigma} \cdot \varepsilon(W) u_L(t)$

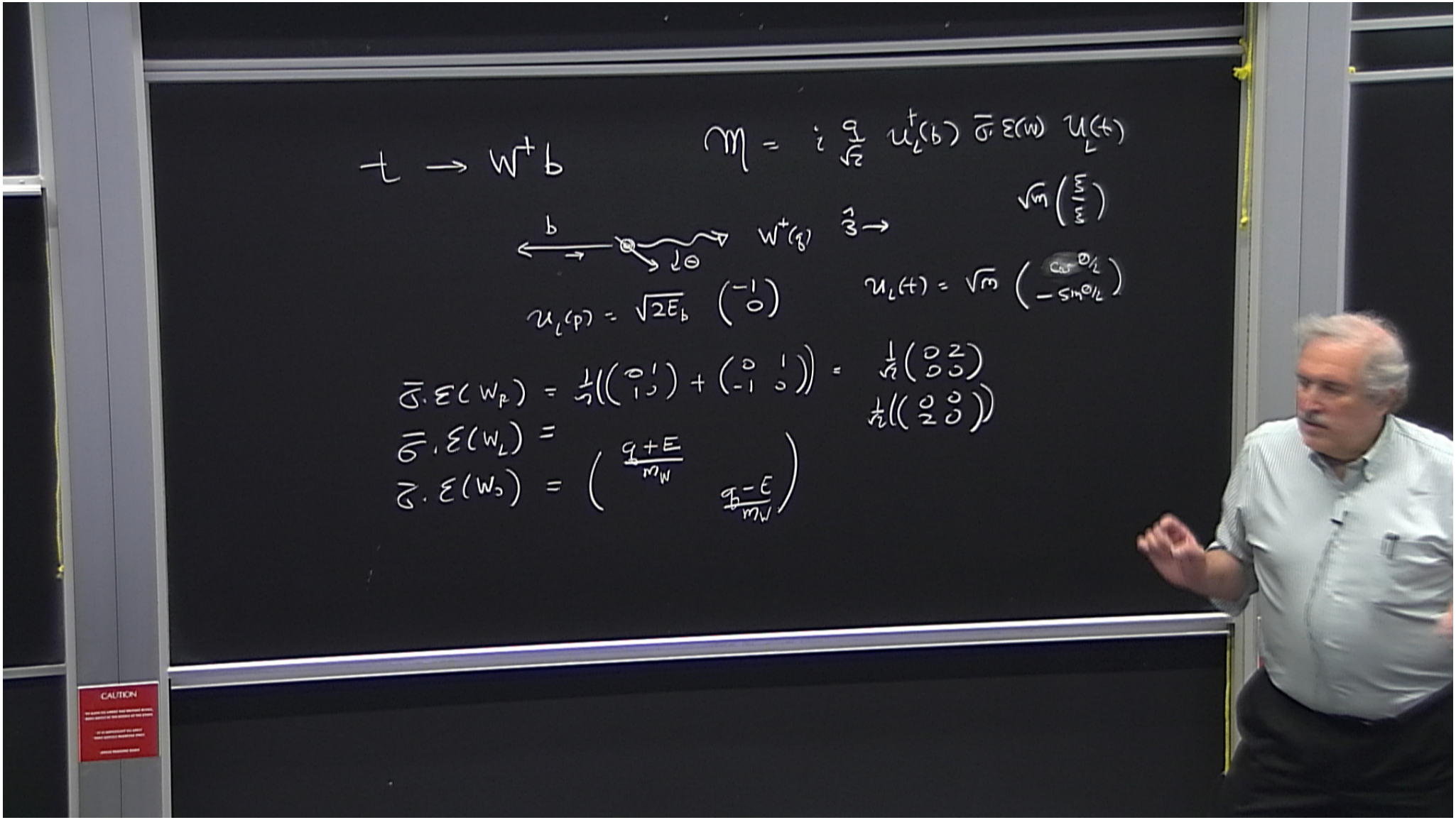


$\vec{\beta} \to$

$\sqrt{m}\begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$

$u_L(p) = \sqrt{2E_b}\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$u_L(t) = \sqrt{m}\begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$


$t \to W^+ b$
$\mathcal{M} = i \frac{g}{\sqrt{2}} u_L^\dagger(b) \bar{\varepsilon}\cdot\varepsilon(W) u_L(t)$
b
$W^+(q)$
$\vec{3} \to$
$\sqrt{m}\begin{pmatrix} m/m \\ \end{pmatrix}$
$u_L(p) = \sqrt{2E_b}\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
$u_L(t) = \sqrt{m}\begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$
$\bar{\varepsilon}\cdot\varepsilon(W_R) = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$
$\frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}\right)$
$\bar{\varepsilon}\cdot\varepsilon(W_L) =$
$\varepsilon\cdot\varepsilon(W_0) = \begin{pmatrix} \frac{q+E}{m_W} & \\ & \frac{q-E}{m_W} \end{pmatrix}$
CAUTION

$t \to W^+ b$

$$\mathcal{M} = i \frac{g}{\sqrt{2}} u_L^\dagger(b)\, \bar{\sigma}\cdot\varepsilon(W)\, u_L(t)$$

$$E_W = \frac{m_t^2 + m_W^2}{2m_t}$$

$$p_W = E_b = \frac{m_t^2 - m_W^2}{2m_t}$$



$\vec{3} \to$

$$\sqrt{m}\begin{pmatrix} m/m \end{pmatrix}\begin{matrix} \}L \\ \}R \end{matrix}$$

$$u_L(p) = \sqrt{2E_b}\begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$u_L(t) = \sqrt{m}\begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$$

$$\bar{\sigma}\cdot\varepsilon^*(W_R) = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\right)$$

$$\bar{\sigma}\cdot\varepsilon^*(W_L) =$$

$$\bar{\sigma}\cdot\varepsilon(W_0) = \begin{pmatrix} \frac{q+E}{m_W} & \\ & \frac{q-E}{m_W} \end{pmatrix}$$

$$\frac{g}{\sqrt{2}} \frac{p^2}{p^2 - m_W^2} \qquad \frac{}{p^2 - m_W^2}$$

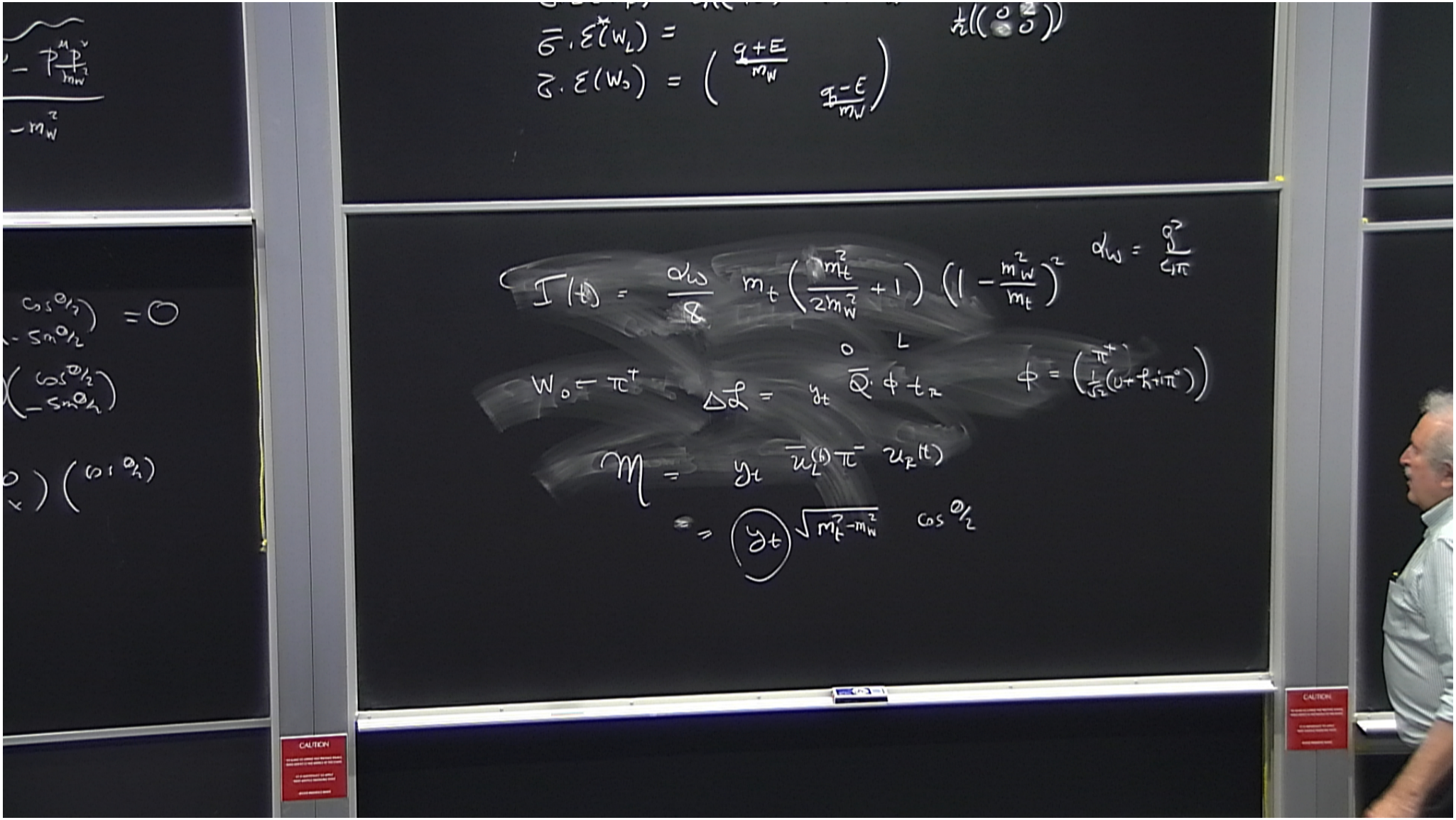
$$\mathcal{M}(t \to b W_R^+) = \frac{g}{\sqrt{2}} \sqrt{2E_b m_t}\ (-1\ 0) \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix} = 0$$

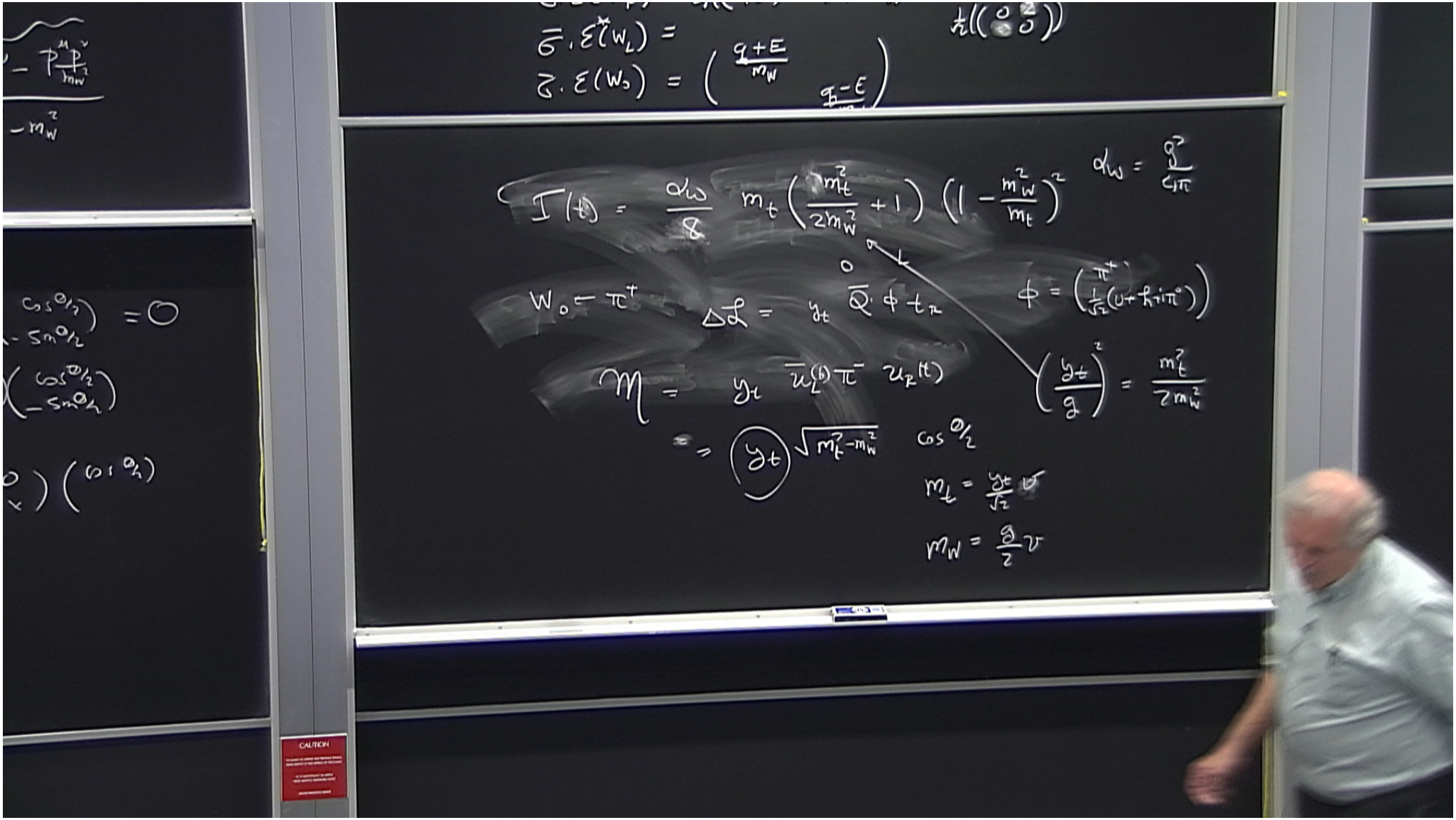
$$\mathcal{M}(t \to b W_L^+) = \frac{g}{\sqrt{2}} \sqrt{2E_b m_t}\ (-1\ 0) \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$$

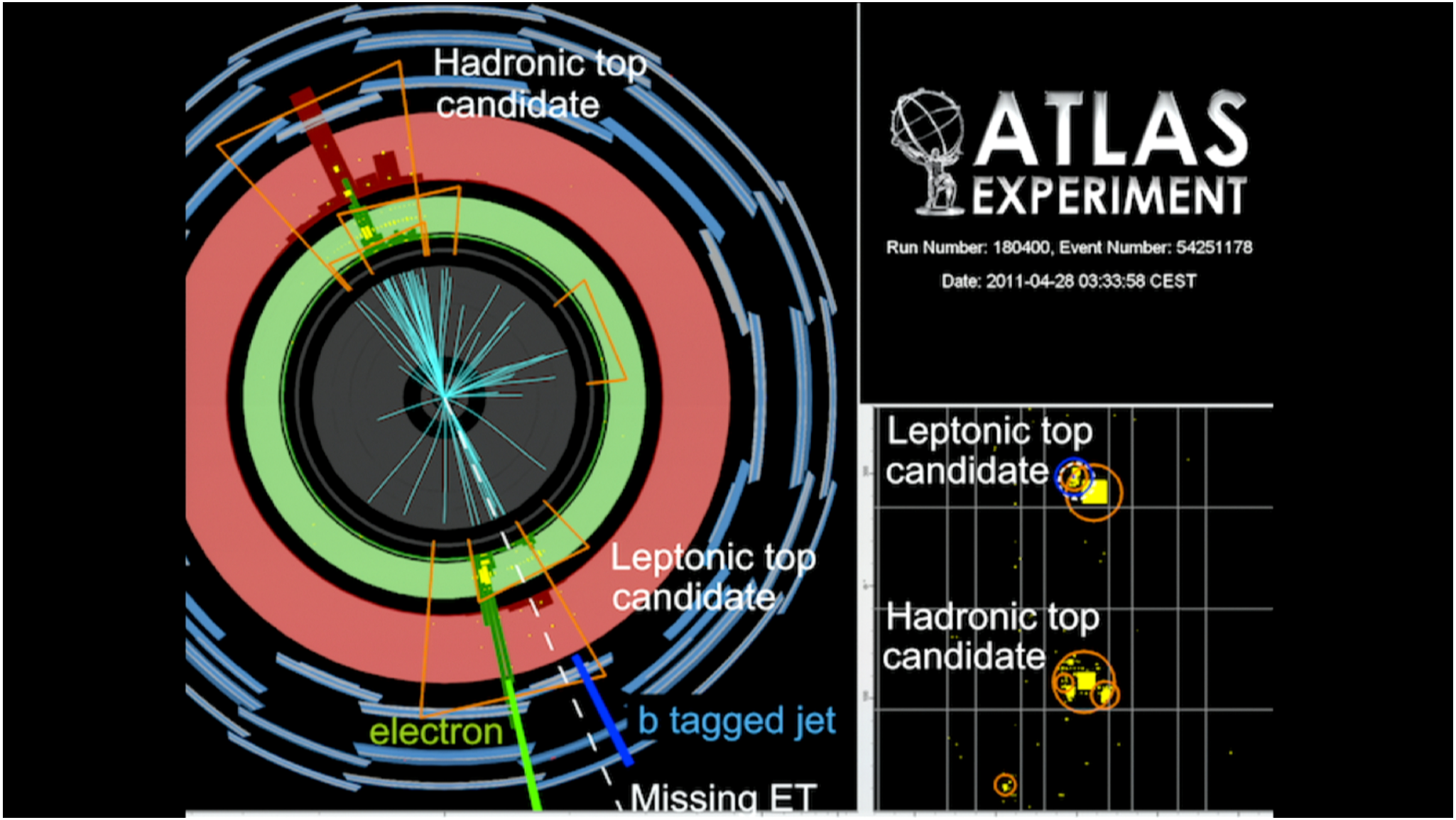
$$= g \sqrt{m_t^2 - m_W^2}\ \sin\theta/2$$

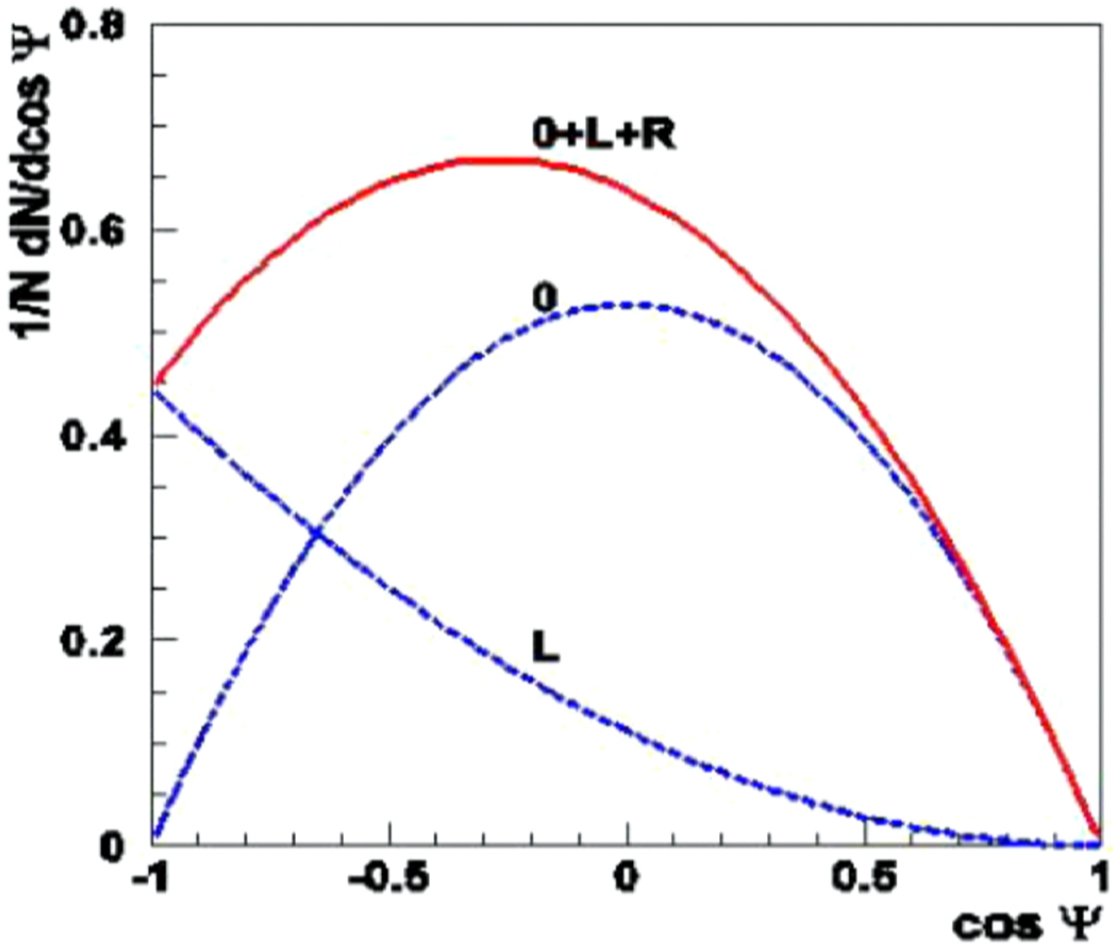
$$\mathcal{M}(t \to b W_0^+) = \frac{g}{\sqrt{2}} \sqrt{2E_b m_t}\ (-1\ 0) \begin{pmatrix} \frac{q + E_W}{m_W} & 0 \\ 0 & * \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$$

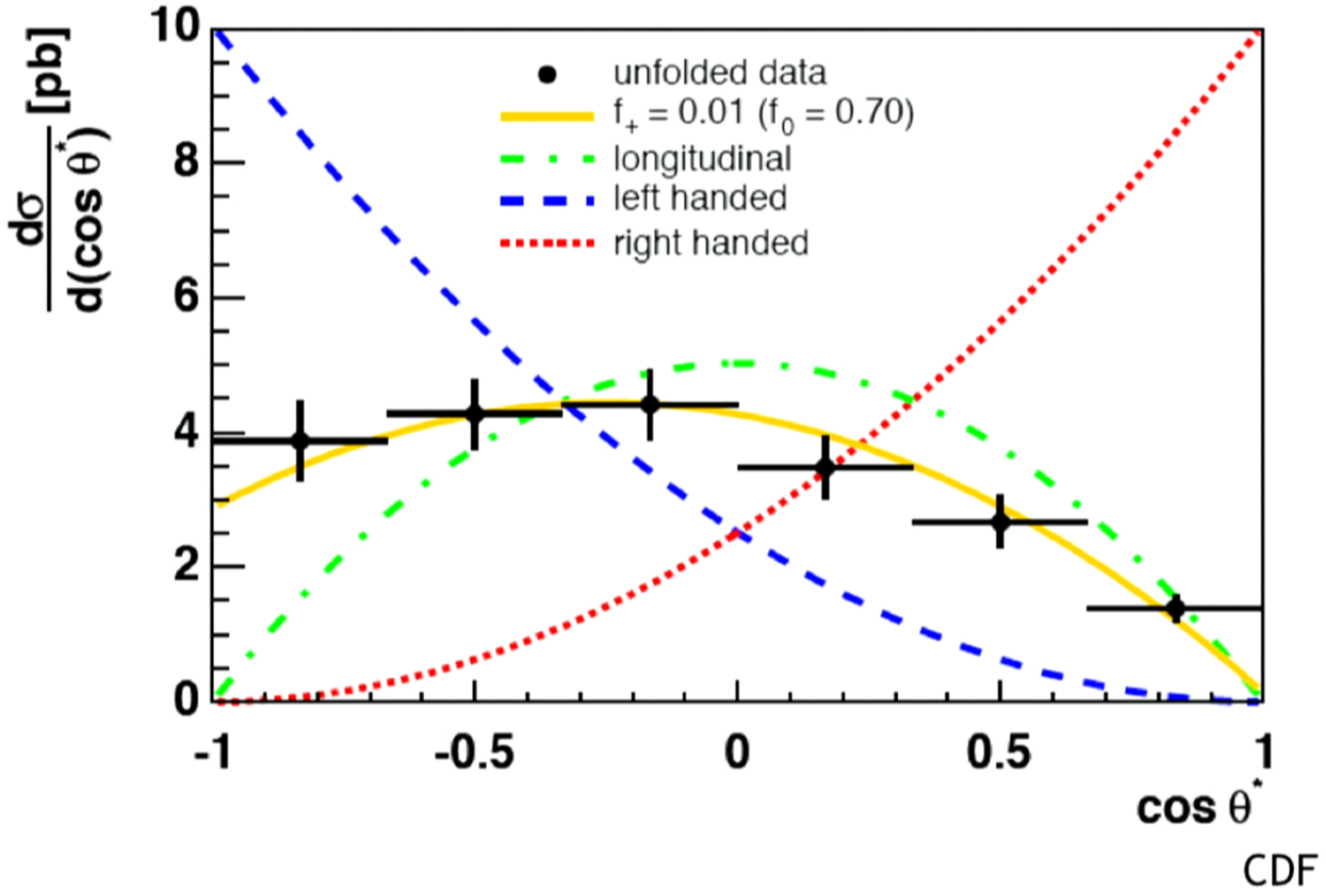
$$= \frac{g}{\sqrt{2}} \sqrt{m_t^2 - m_W^2} \left(\frac{m_t}{m_W}\right) \cos\theta/2$$

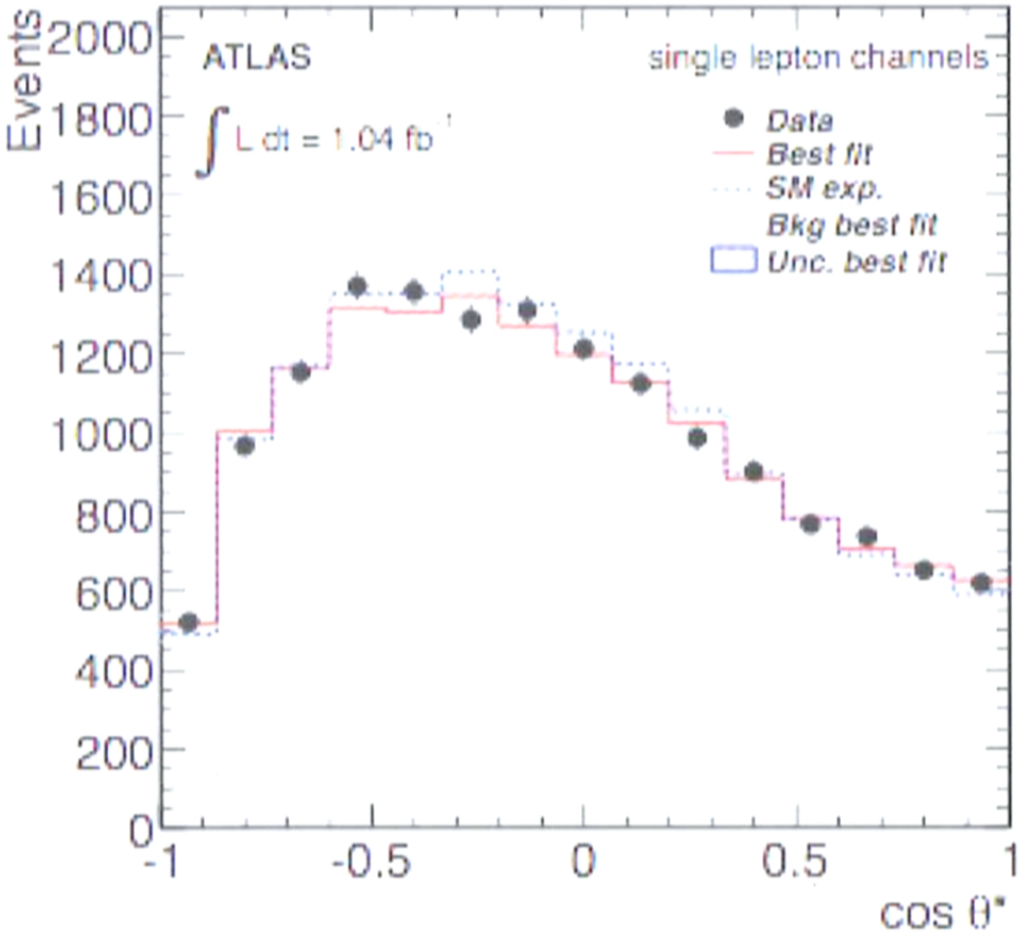

$\varepsilon(W_L) \cdot p =$
$\varepsilon(W_0) \cdot p = \left(\frac{q+E}{m_W} \quad \frac{q-E}{m_W}\right)$
$\Gamma(t) = \frac{\alpha_W}{8} m_t \left(\frac{m_t^2}{2m_W^2} + 1\right)\left(1 - \frac{m_W^2}{m_t}\right)^2$
$\alpha_W = \frac{g^2}{4\pi}$
$W_0 \to \pi^+$
$\Delta\mathcal{L} = y_t \, \bar{Q}_L \, \phi \, t_R$
$\phi = \begin{pmatrix} \pi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi^0) \end{pmatrix}$
$\mathcal{M} = y_t \, \bar{u}_L^{(1)} \pi^- u_R^{(t)}$
$= y_t \sqrt{m_t^2 - m_W^2} \cos\theta/2$

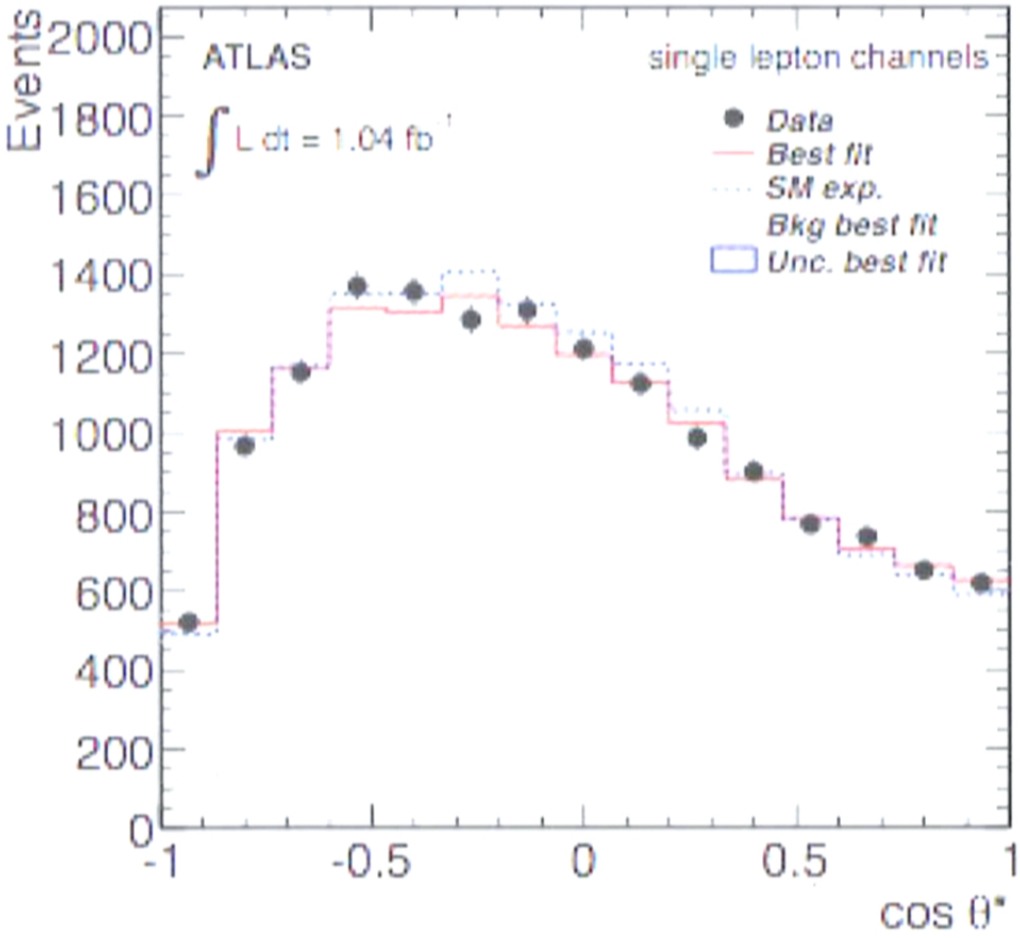

$\varepsilon(W_L) \cdot p =$
$\varepsilon(W_0) \cdot p = \left(\frac{p+E}{m_W}, \frac{p-E}{m_W}\right)$
$\Gamma(t) = \frac{\alpha_W}{8} m_t \left(\frac{m_t^2}{2m_W^2} + 1\right)\left(1 - \frac{m_W^2}{m_t^2}\right)^2$
$\alpha_W = \frac{g^2}{4\pi}$
$W_0 \to \pi^+$
$\Delta\mathcal{L} = y_t \bar{Q} \cdot \phi\, t_R$
$\phi = \begin{pmatrix} \pi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi^0) \end{pmatrix}$
$\mathcal{M} = y_t \bar{u}_L(b) \bar{\tau} u_R(t)$
$\left(\frac{y_t}{g}\right)^2 = \frac{m_t^2}{2m_W^2}$
$= y_t \sqrt{m_t^2 - m_W^2}$ $\cos\theta/2$
$m_t = \frac{y_t}{\sqrt{2}} v$
$m_W = \frac{g}{2} v$
CAUTION

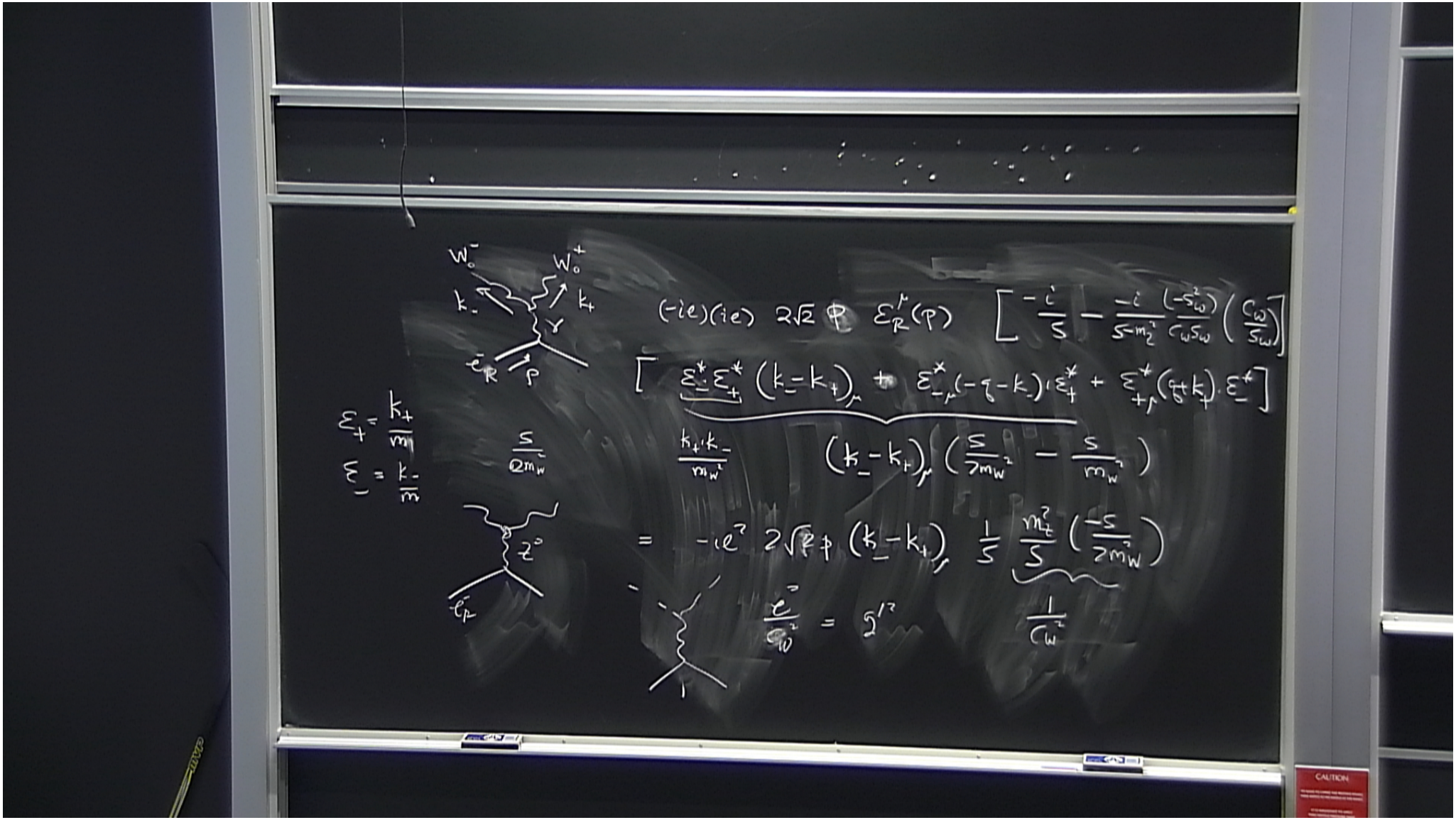

Hadronic top
candidate
Leptonic top
candidate
electron
b tagged jet
Missing ET
ATLAS
EXPERIMENT
Run Number: 180400, Event Number: 54251178
Date: 2011-04-28 03:33:58 CEST
Leptonic top
candidate
Hadronic top
candidate

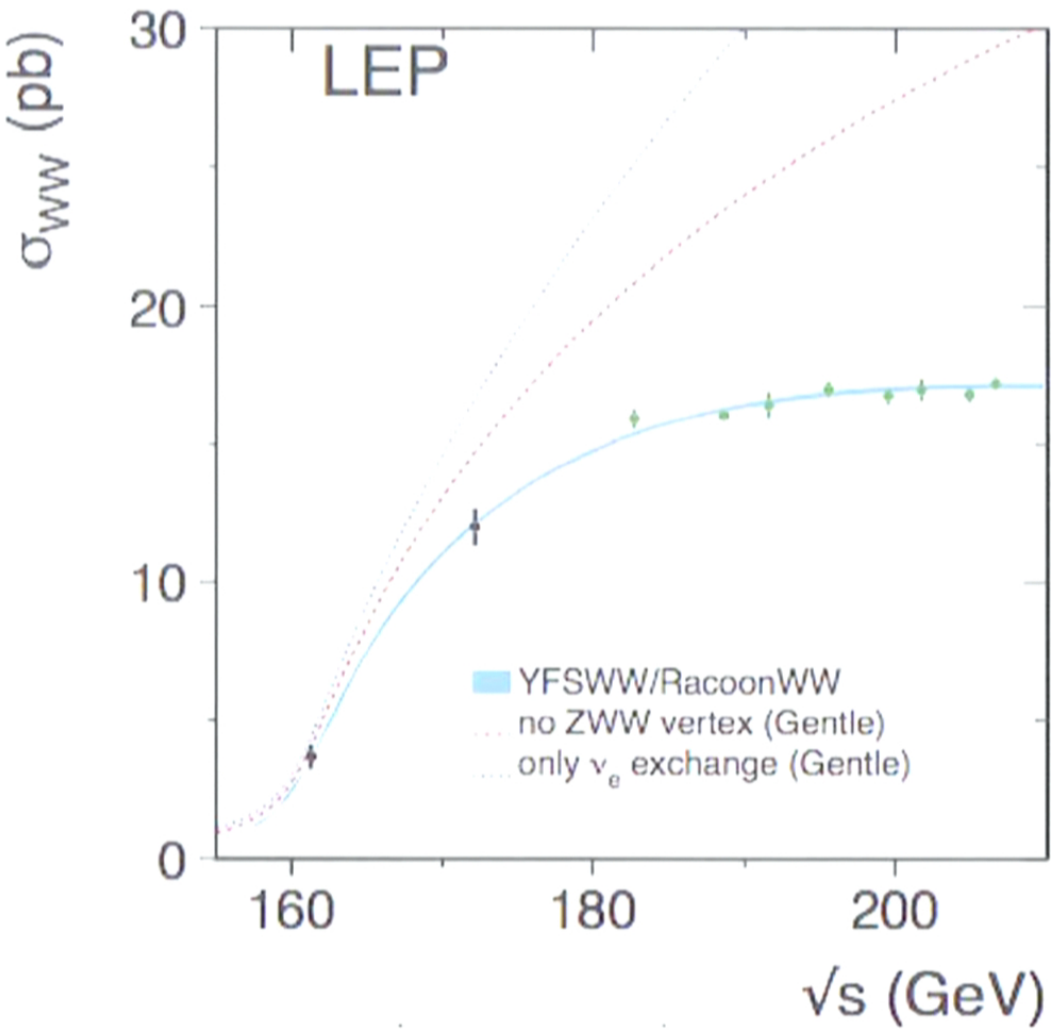

1/N dN/dcos Ψ
0.8
0.6
0.4
0.2
0
0+L+R
0
L
-1
-0.5
0
0.5
1
cos Ψ

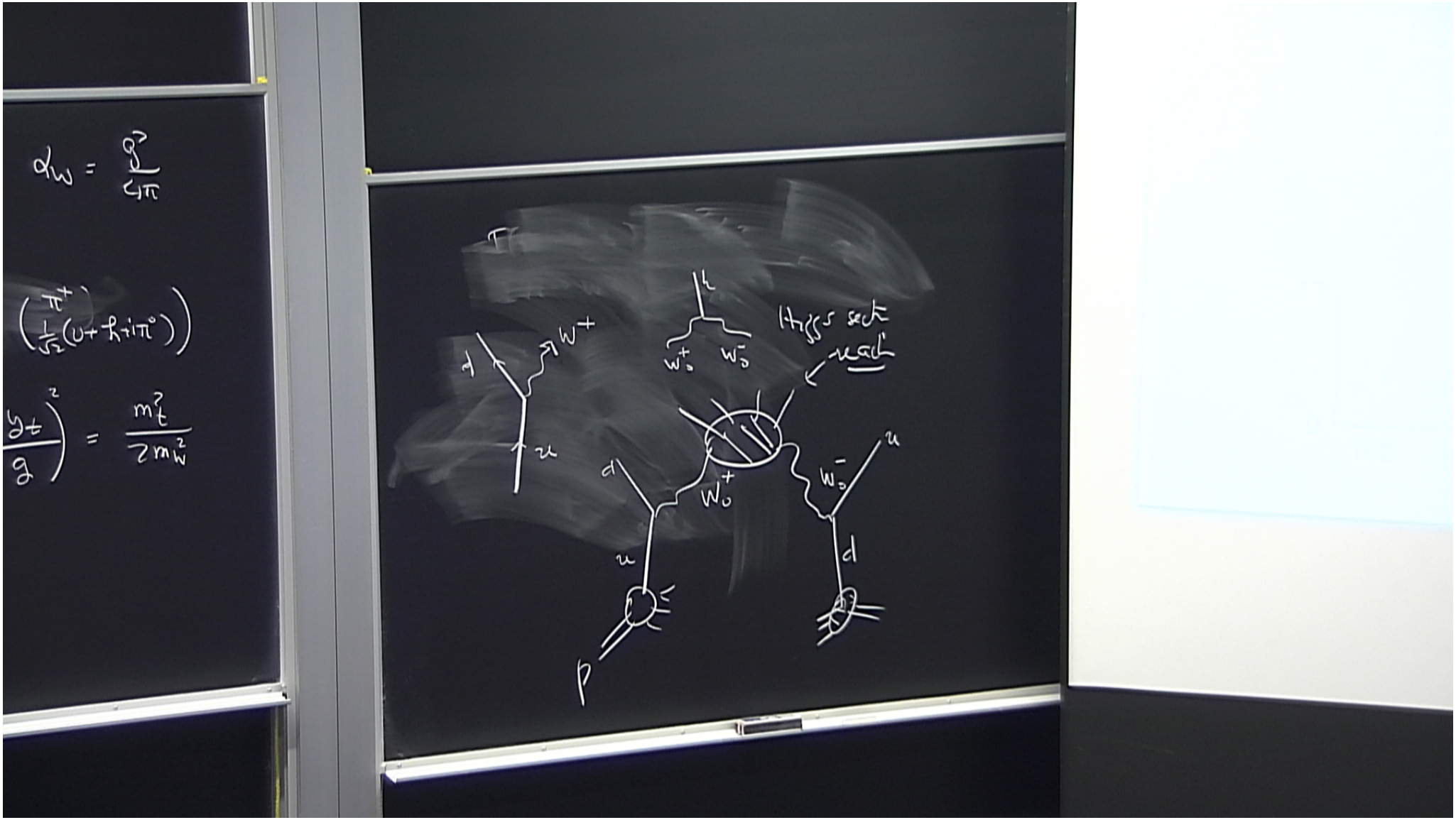

$\frac{d\sigma}{d(\cos\theta^*)}$ [pb]
10
8
6
4
2
0
-1
-0.5
0
0.5
1
cos θ*
unfolded data
f+ = 0.01 (f0 = 0.70)
longitudinal
left handed
right handed
CDF

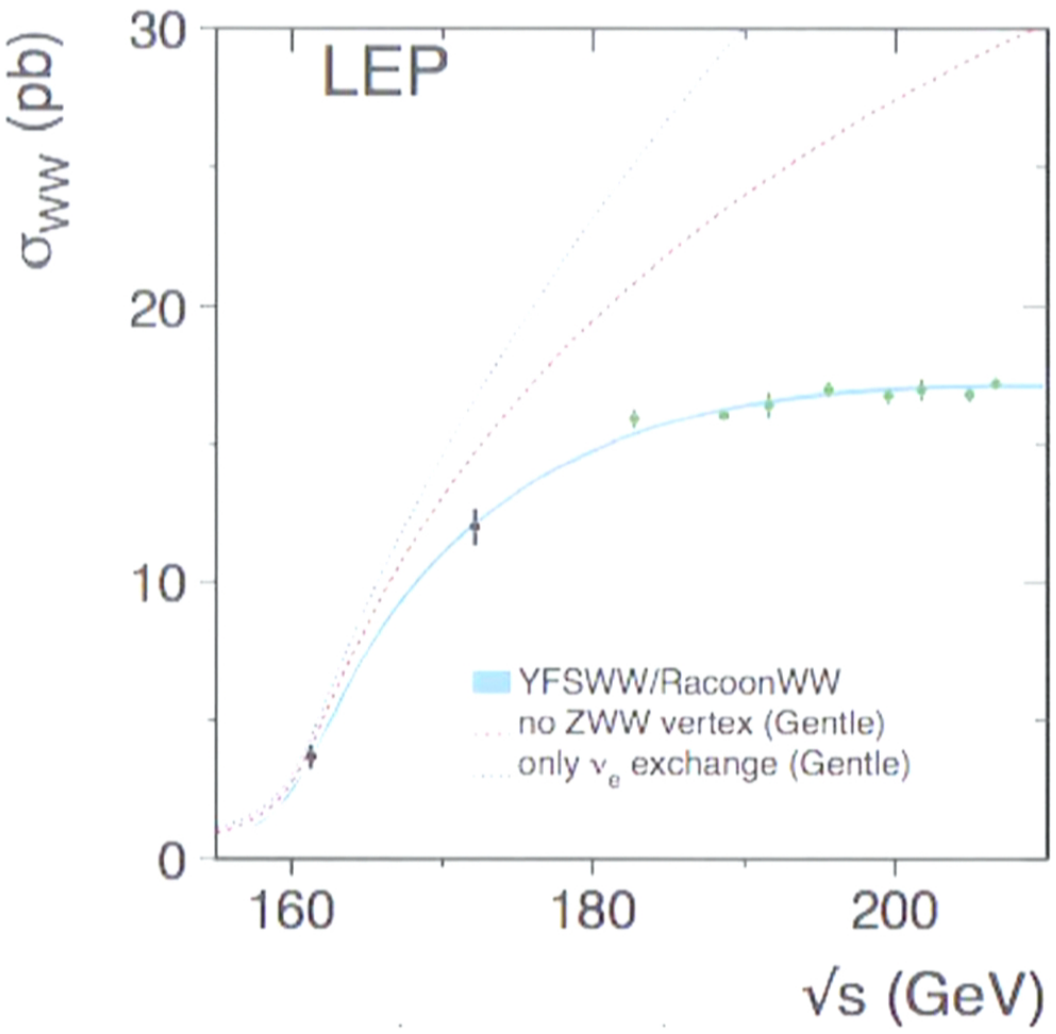

ATLAS
single lepton channels
∫ L dt = 1.04 fb⁻¹
Data
Best fit
SM exp.
Bkg best fit
Unc. best fit
Events
2000
1800
1600
1400
1200
1000
800
600
400
200
0
-1
-0.5
0
0.5
1
cos θ*

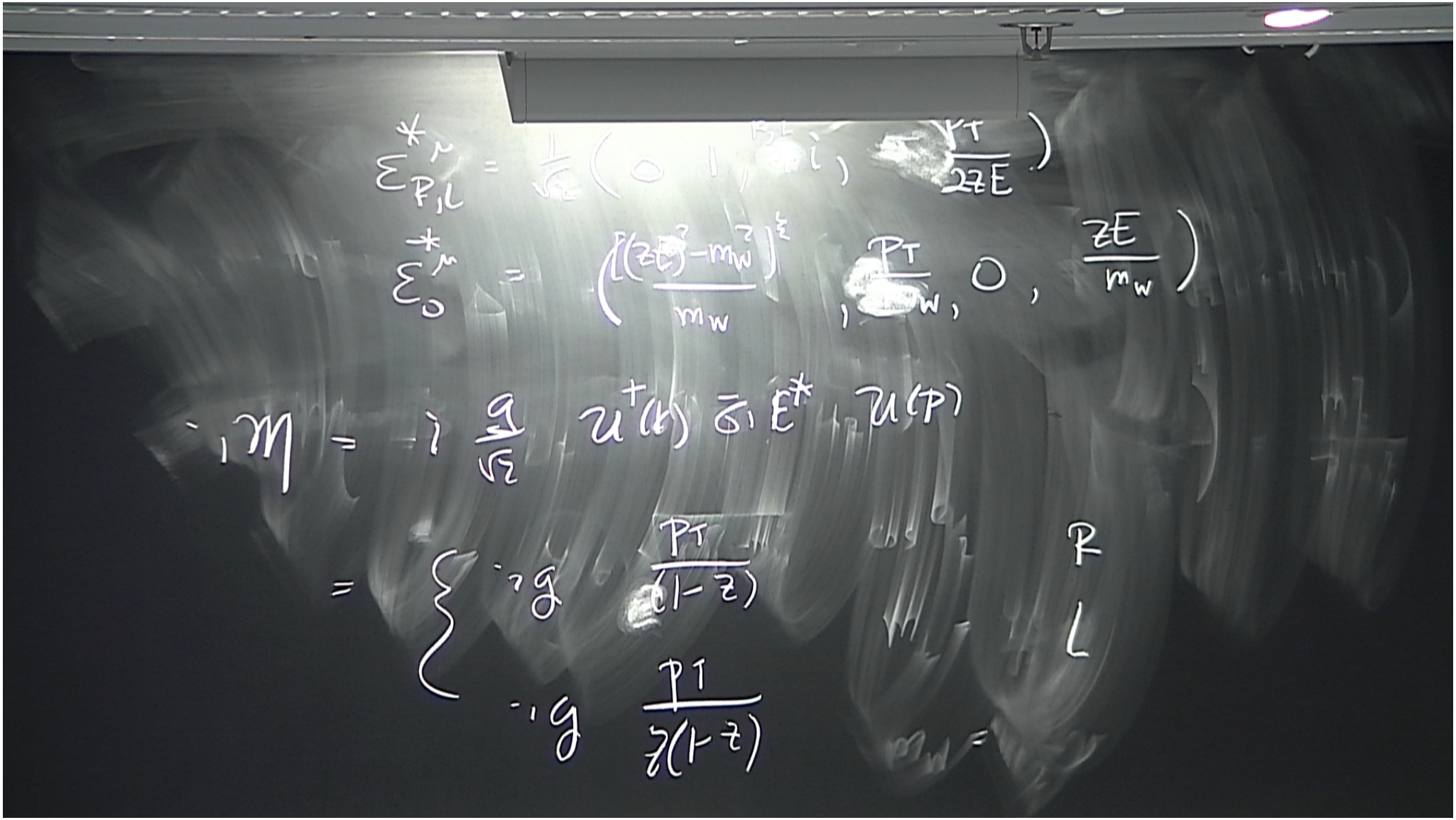

ATLAS
single lepton channels
∫ L dt = 1.04 fb⁻¹
Data
Best fit
SM exp.
Bkg best fit
Unc. best fit
Events
2000
1800
1600
1400
1200
1000
800
600
400
200
0
-1
-0.5
0
0.5
1
cos θ*

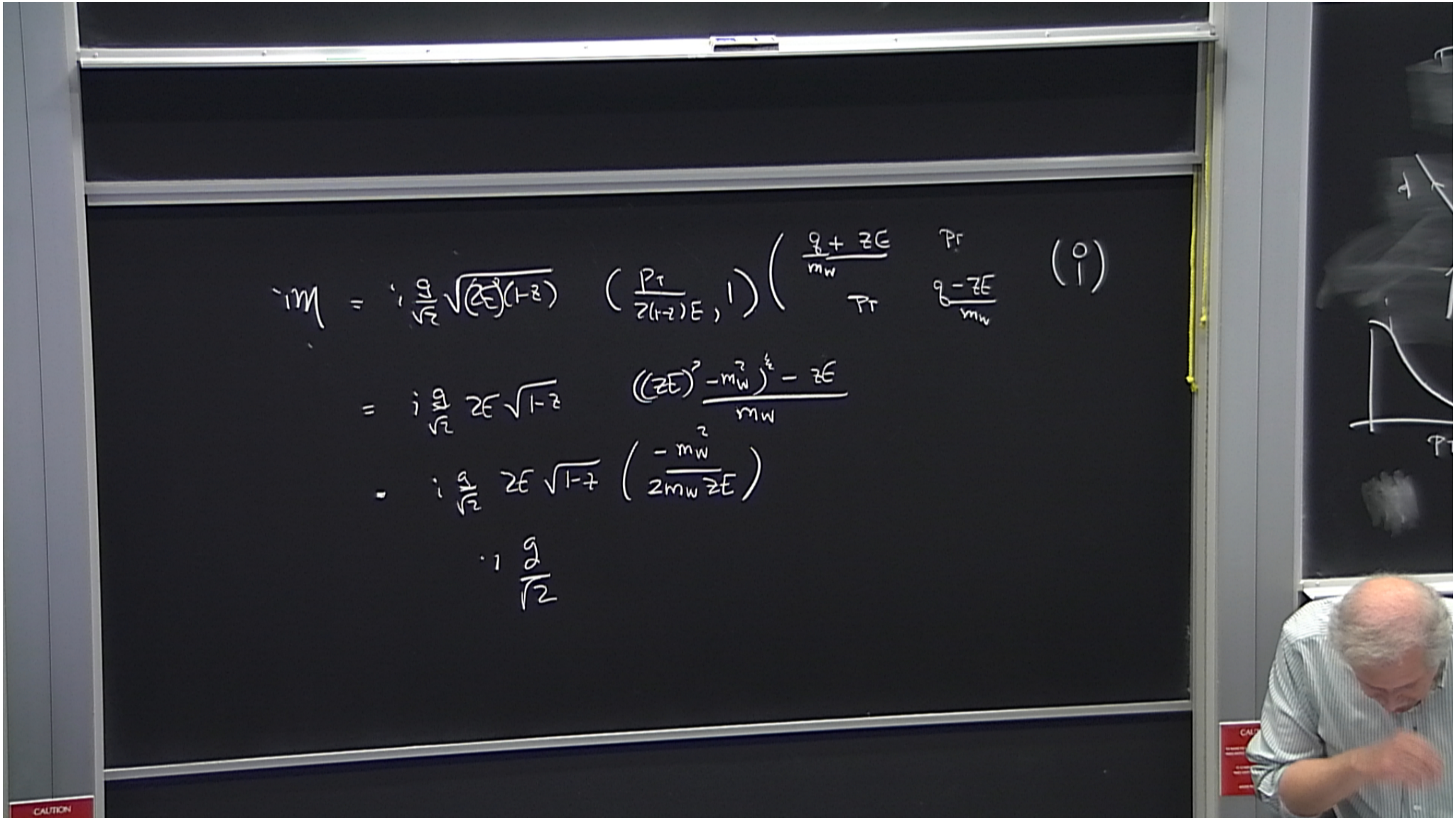

W^-
W^+
k_-
k_+
e_R^-
p
γ
Z^0
e_L^-
$\varepsilon_+ = \frac{k_+}{m_W}$
$\varepsilon_- = \frac{k_-}{m_W}$
$\frac{s}{2m_W^2}$
$(-ie)(ie)\ 2\sqrt{2}\ p\ \varepsilon_R^\mu(p)$
$\left[-\frac{i}{s} - \frac{-i}{s-m_Z^2} \frac{(-s_W^2)}{c_W s_W} \left(\frac{c_W}{s_W} \right) \right]$
$\left[\varepsilon_-^* \varepsilon_+^* (k_- - k_+)_\mu + \varepsilon_{-\mu}^* (-2 - k_-) \varepsilon_+^* + \varepsilon_{+\mu}^* (2 + k_+) \varepsilon_-^* \right]$
$\frac{k_+ \cdot k_-}{m_W^2}$
$(k_- - k_+)_\mu \left(\frac{s}{2m_W^2} - \frac{s}{m_W^2} \right)$
$= -ie^2\ 2\sqrt{2}\ p\ (k_- - k_+)_\mu\ \frac{1}{s}\ \frac{m_t^2}{s} \left(\frac{-s}{2m_W^2} \right)$
$\frac{e^2}{s_W^2} = g'^2$
$\frac{1}{c_W^2}$


LEP
σ_{WW} (pb)
30
20
10
0
160
180
200
√s (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)


$\alpha_W = \frac{g^2}{4\pi}$
$\left(\frac{\pi^+}{\frac{1}{\sqrt{2}}(v+h+i\pi^0)}\right)$
$\left(\frac{g_t}{g}\right)^2 = \frac{m_t^2}{2m_W^2}$
Higgs sector
W^+
W_0^+
W_0^-
W_0^+
W_0^-
u
d
u
d
u
d
p


LEP
σ_{WW} (pb)
30
20
10
0
160
180
200
√s (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)



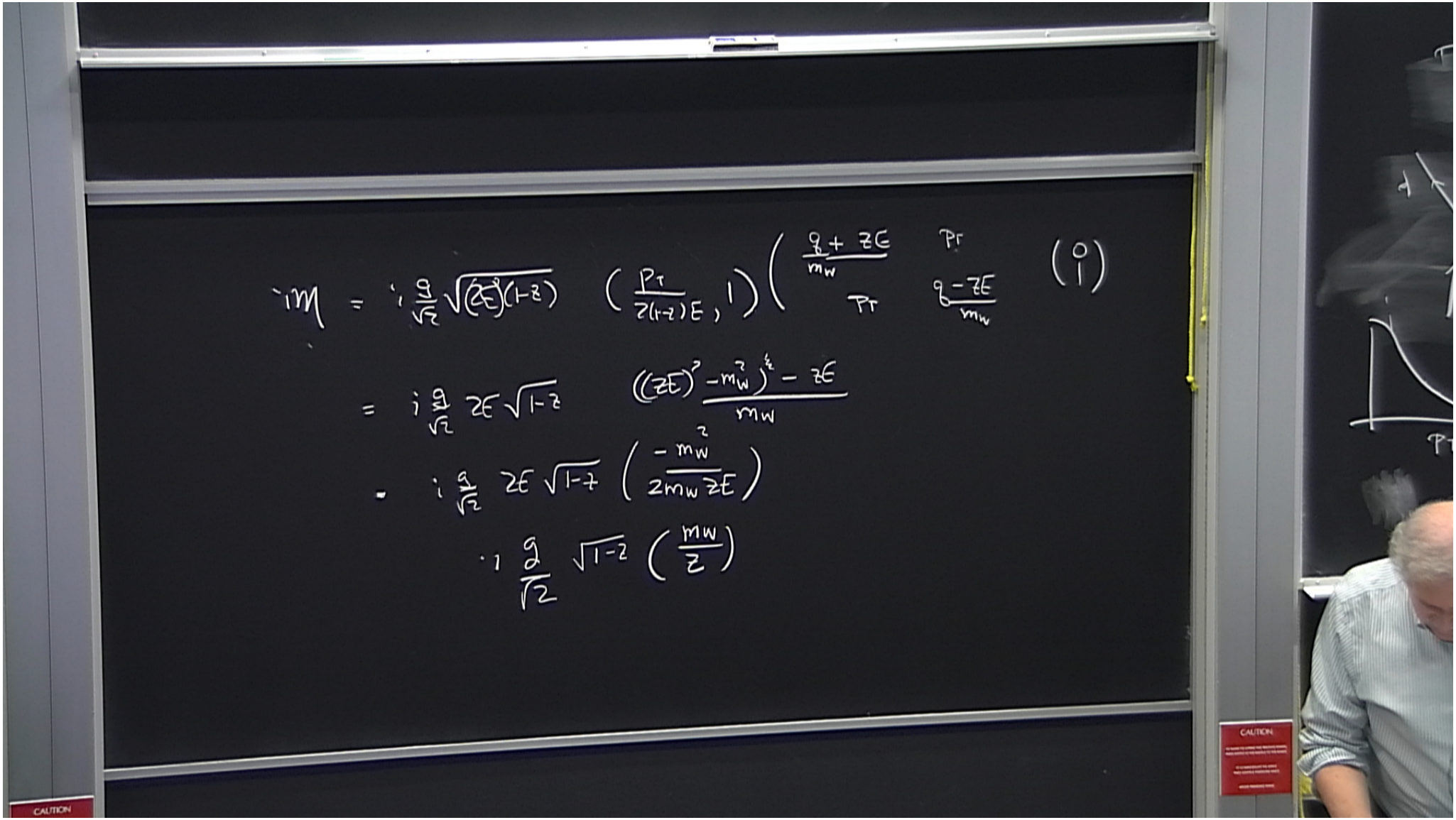


$$M_i = i\frac{g}{\sqrt{2}}\sqrt{(zE)^2(1-z)}\;\left(\frac{p_T}{z(1-z)E},1\right)\begin{pmatrix} \frac{g+zE}{m_W} & p_T \\ p_T & \frac{g-zE}{m_W} \end{pmatrix} \quad (i)$$

$$= i\frac{g}{\sqrt{2}}\,zE\sqrt{1-z}\;\;\frac{(zE)^2-m_W^2)^{\frac{1}{2}}-zE}{m_W}$$

$$= i\frac{g}{\sqrt{2}}\,zE\sqrt{1-z}\left(\frac{-m_W^2}{2m_W zE}\right)$$

$$\cdot\, i\frac{g}{\sqrt{2}}$$

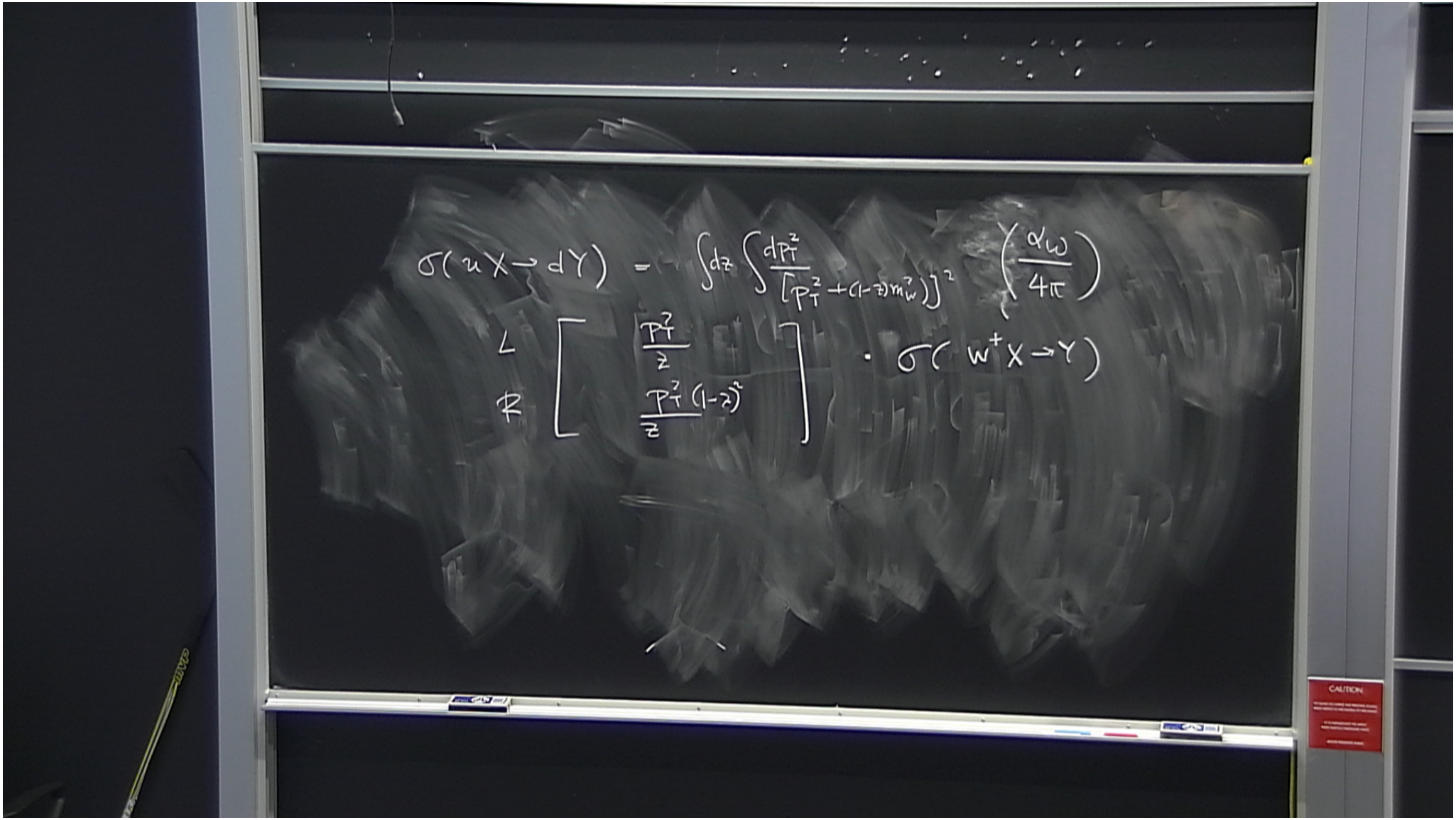


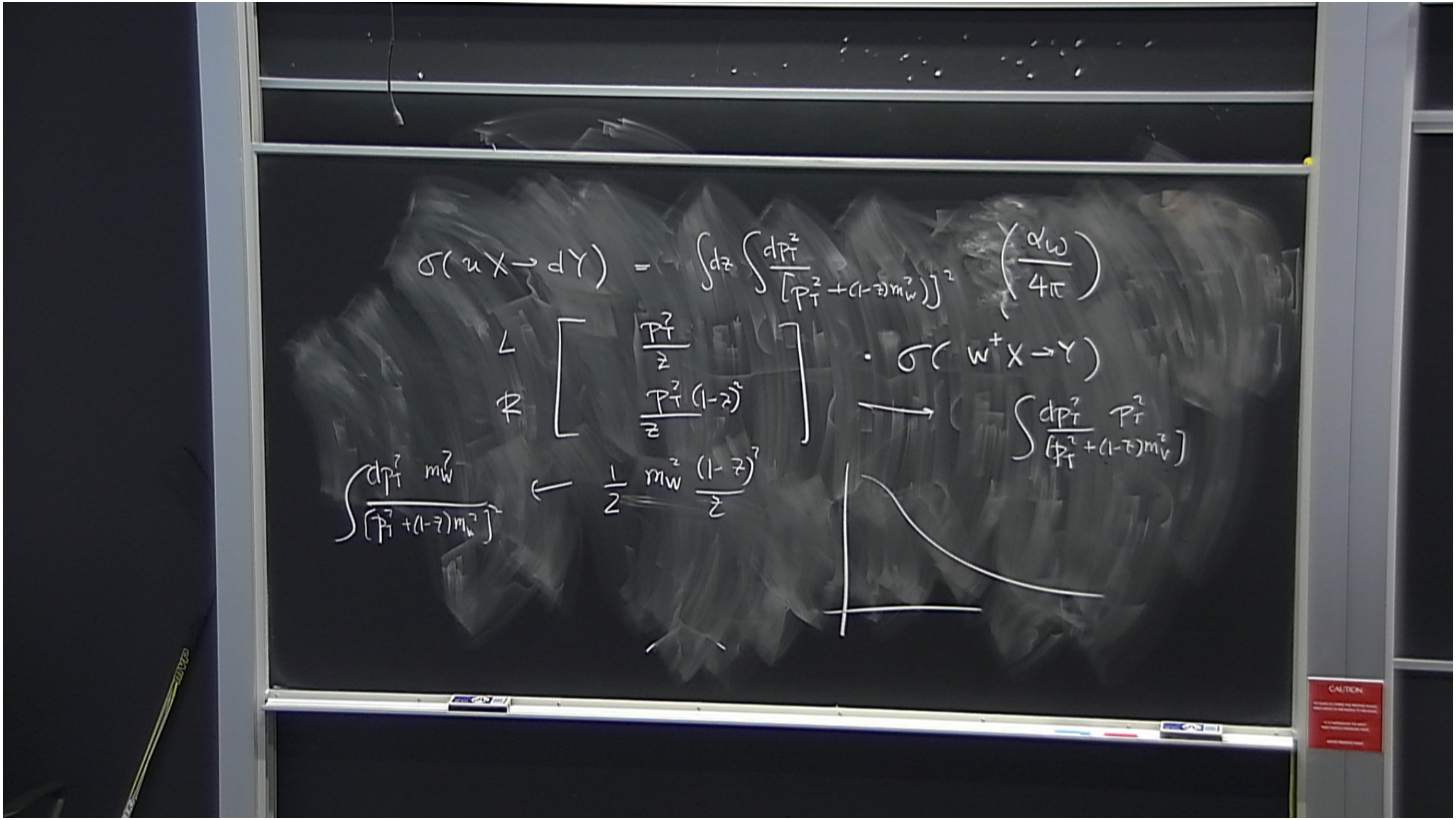
$$M_{fi} = i\frac{g}{\sqrt{2}}\sqrt{(2E)^2(1-z)}\;\left(\frac{p_T}{2(1-z)E},1\right)\begin{pmatrix} \frac{g+2E}{m_W} & p_T \\ p_T & \frac{g-2E}{m_W} \end{pmatrix} \quad (i)$$

$$= i\frac{g}{\sqrt{2}}\,2E\sqrt{1-z}\;\frac{((2E)^2-m_W^2)^{\frac{1}{2}}-2E}{m_W}$$

$$= i\frac{g}{\sqrt{2}}\,2E\sqrt{1-z}\left(\frac{-m_W^2}{2m_W 2E}\right)$$

$$\cdot i\frac{g}{\sqrt{2}}\sqrt{1-z}\left(\frac{m_W}{2}\right)$$


$\sigma(uX \to dY) = \int dz \int \frac{dp_T^2}{[p_T^2 + (1-z)m_W^2)]^2} \left(\frac{d\omega}{4\pi}\right)$
$L \quad \frac{p_T^2}{2}$
$R \quad \frac{p_T^2(1-z)^2}{2}$
$\cdot \sigma(W^+X \to Y)$
CAUTION


$\sigma(uX \to dY) = \int dz \int \frac{dp_T^2}{[p_T^2 + (1-z)m_W^2]^2} \left(\frac{d\omega}{4\pi}\right)$
$\propto \left[\frac{p_T^2}{2} \quad \frac{p_T^2(1-z)^2}{2}\right] \cdot \sigma(W^+X \to Y)$
$\int \frac{dp_T^2 \; p_T^2}{[p_T^2 + (1-z)m_V^2]}$
$\int \frac{dp_T^2 \; m_W^2}{[p_T^2 + (1-z)m_W^2]^2} \leftarrow \frac{1}{2} m_W^2 \frac{(1-z)^2}{2}$