Title: The Standard Model â€“ Theory: Altarelli-Parisi evolution

Date: Jul 07, 2015 09:00 AM

URL: http://pirsa.org/15070017

Abstract:

Collider Physics

figures for lecture 3

Altarelli-Parisi Evolution

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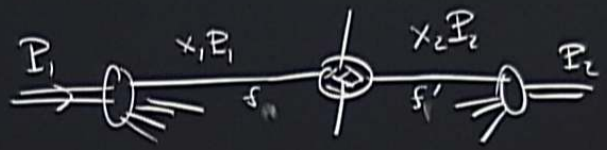
TRISEP School 2015

naive parton model



$0 < x_1, x_2 < 1$

$$\sigma(pp \to Y) = \int dx_1\, dx_2 \left[f_{f/p}(x_1)\, f_{f'/p}(x_2) + f_{f'/p}(x_1)\, f_{f/p}(x_2) \right] \sigma(ff' \to Y,\ \hat{s} = x_1 x_2 s)$$

$0 < x_1 x_2 < 1$

$$\sigma(pp \to Y) = \int dx_1\, dx_2 \left[f_{f/p}(x_1)\, f_{\bar{f}/\bar{p}}(x_2) + f_{f'/p}(x_1)\, f_{\bar{f}'/\bar{p}}(x_2) \right] \sigma(f\bar{f} \to Y,\ \hat{s} = x_1 x_2 s)$$

p_T of $f\bar{f}$: proton n.f. (non perturbative) $\left(\frac{m_p}{Q}\right)^2$

QCD pert. th. $\longrightarrow$ $\alpha_s \log(Q/m_p)$

$0 < z < 1$

$(E, 0, 0, E) \longrightarrow (zE, 0, 0, zE)$

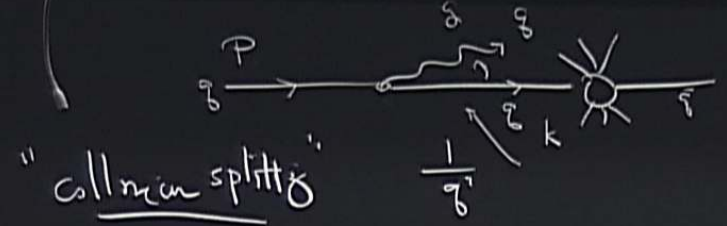
$\longrightarrow ((1-z)E, 0, 0, (1-z)E)$

"collinear splitting" $\frac{1}{q^2}$

$p = (E, 0, 0, E)$

$k = (zE, p_T, 0, zE)$

$q = ((1-z)E, -p_T, 0, (1-z)E - \frac{p_T^2}{2(1-z)E})$



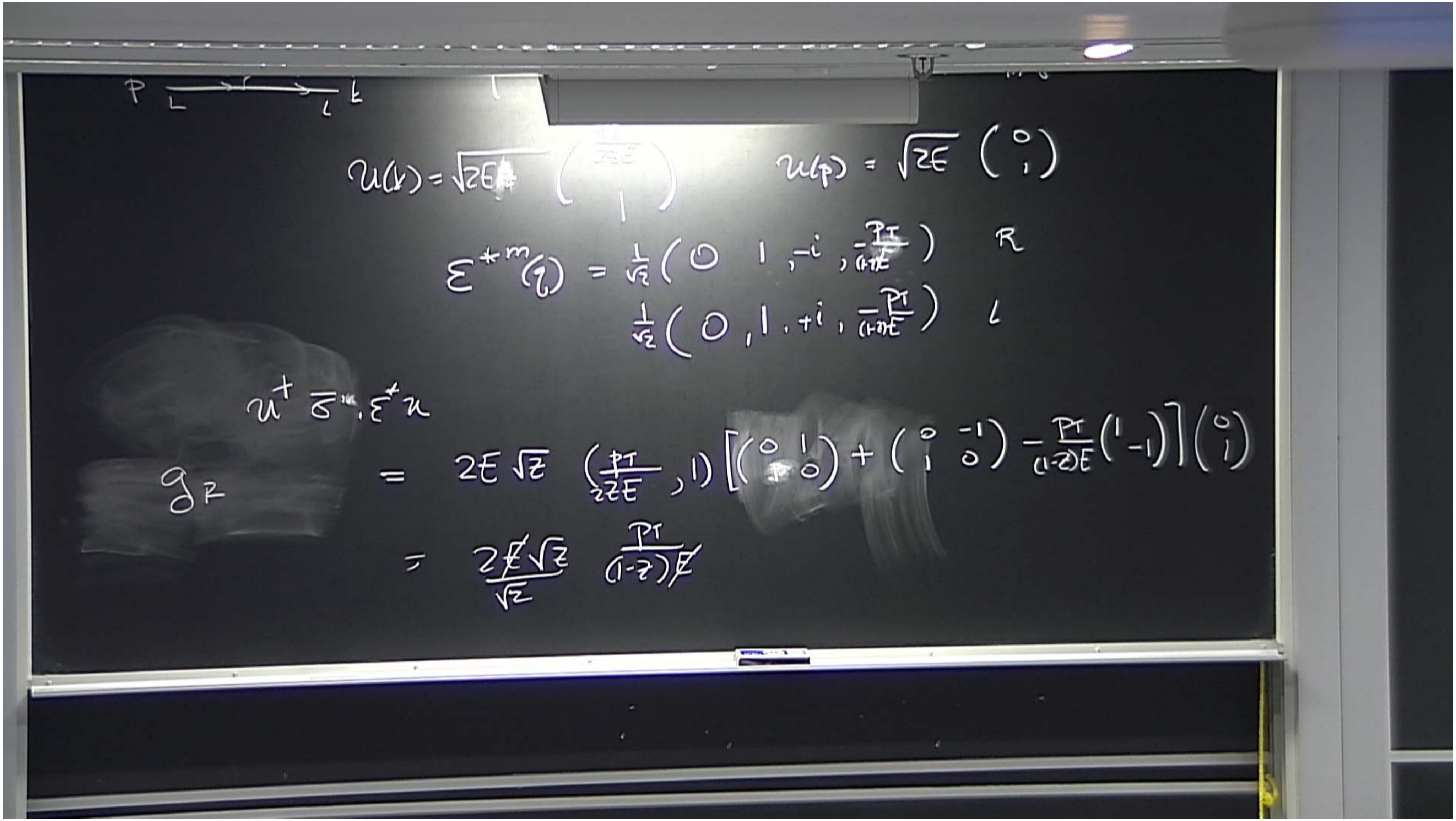
"collinear splitting" $\frac{1}{q^2}$

$p = (E, 0, 0, E)$

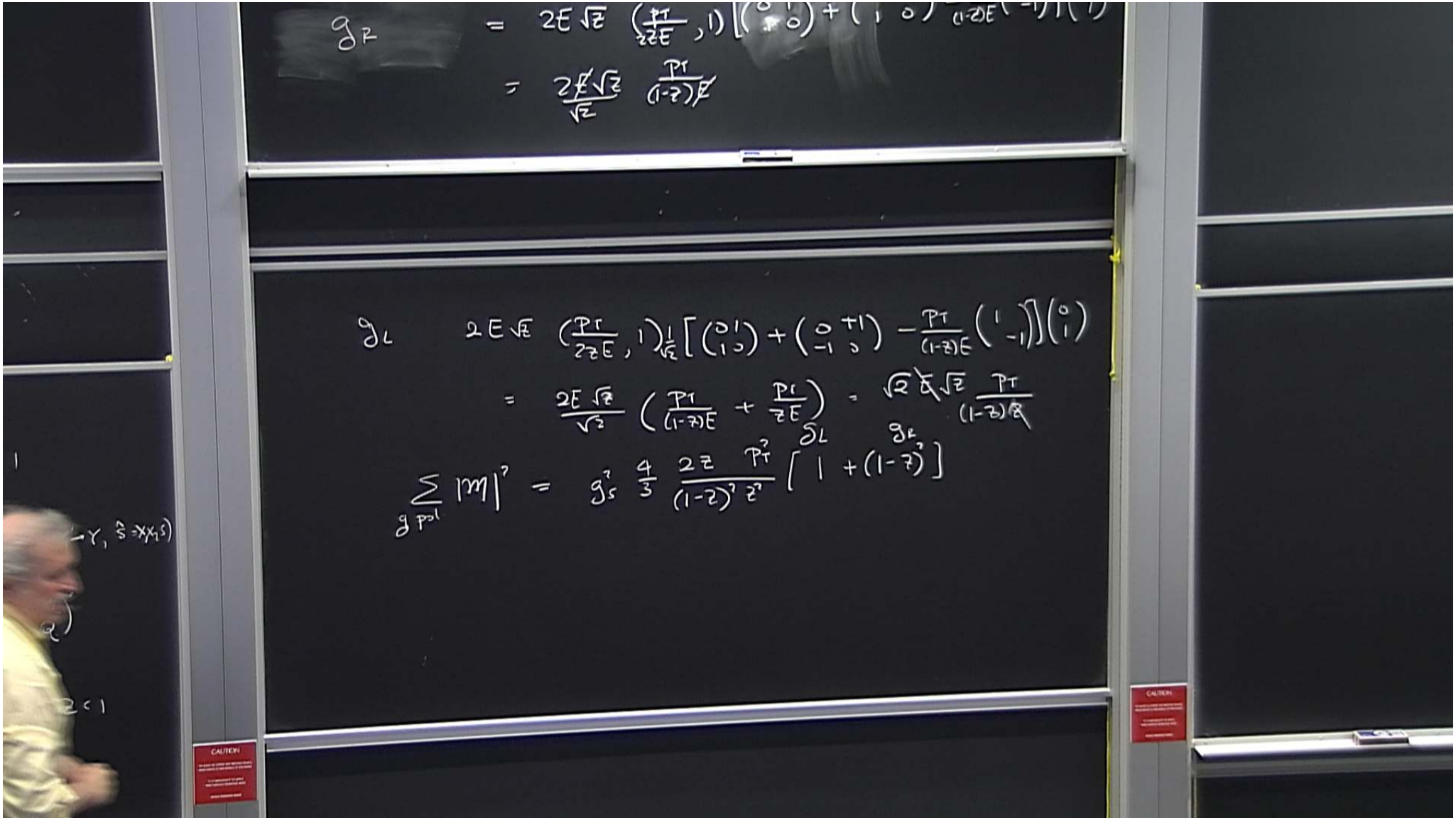
$k = \left(zE, p_T, 0, zE + \frac{p_T^2}{2(1-z)E}\right)$

$q = \left((1-z)E, -p_T, 0, (1-z)E - \frac{p_T^2}{2(1-z)E}\right)$

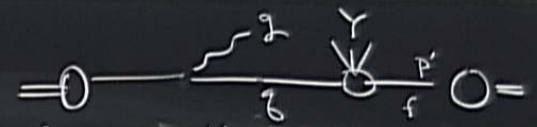
$k^2 = (zE)^2 - p_T^2 - (zE)^2 - \frac{p_T^2 z}{(1-z)} = -\frac{p_T^2}{(1-z)}$



P L L k
u(k) = √2E (/2zE ; 1)
u(p) = √2E (0 ; 1)
ε*m(q) = 1/√2 (0 1 , −i , −pT/(1+z)E) R
1/√2 (0 , 1 , +i , −pT/(1+z)E) L
u† σ̄m ε*m u
gR = 2E √z (pT/2zE , 1) [(0 1 ; 1 0) + (0 −1 ; i 0) − pT/(1−z)E (1 −1)] (0 ; 1)
= 2E √z/√z pT/(1−z)E



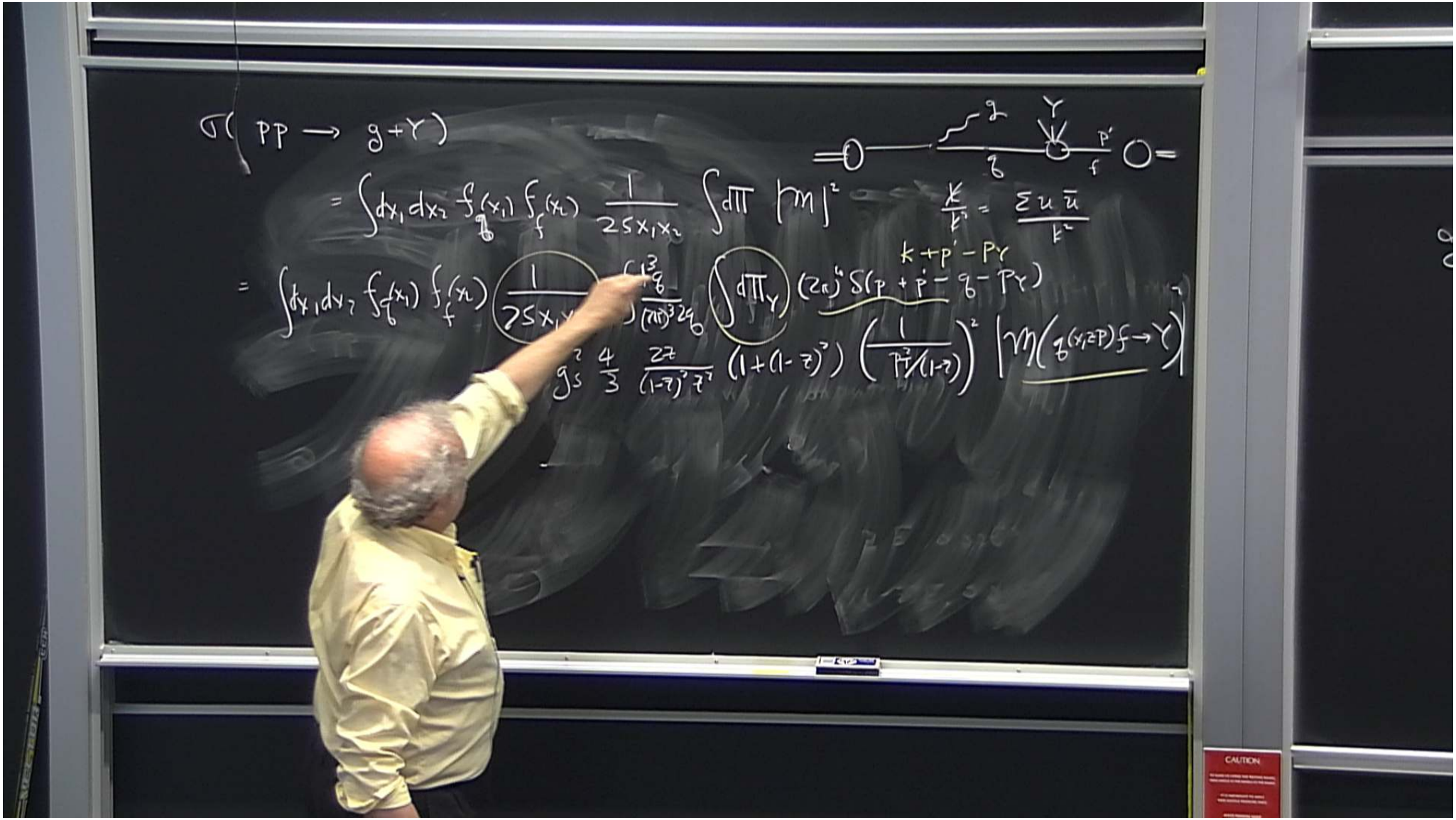
$\sigma(pp \to g + \gamma)$

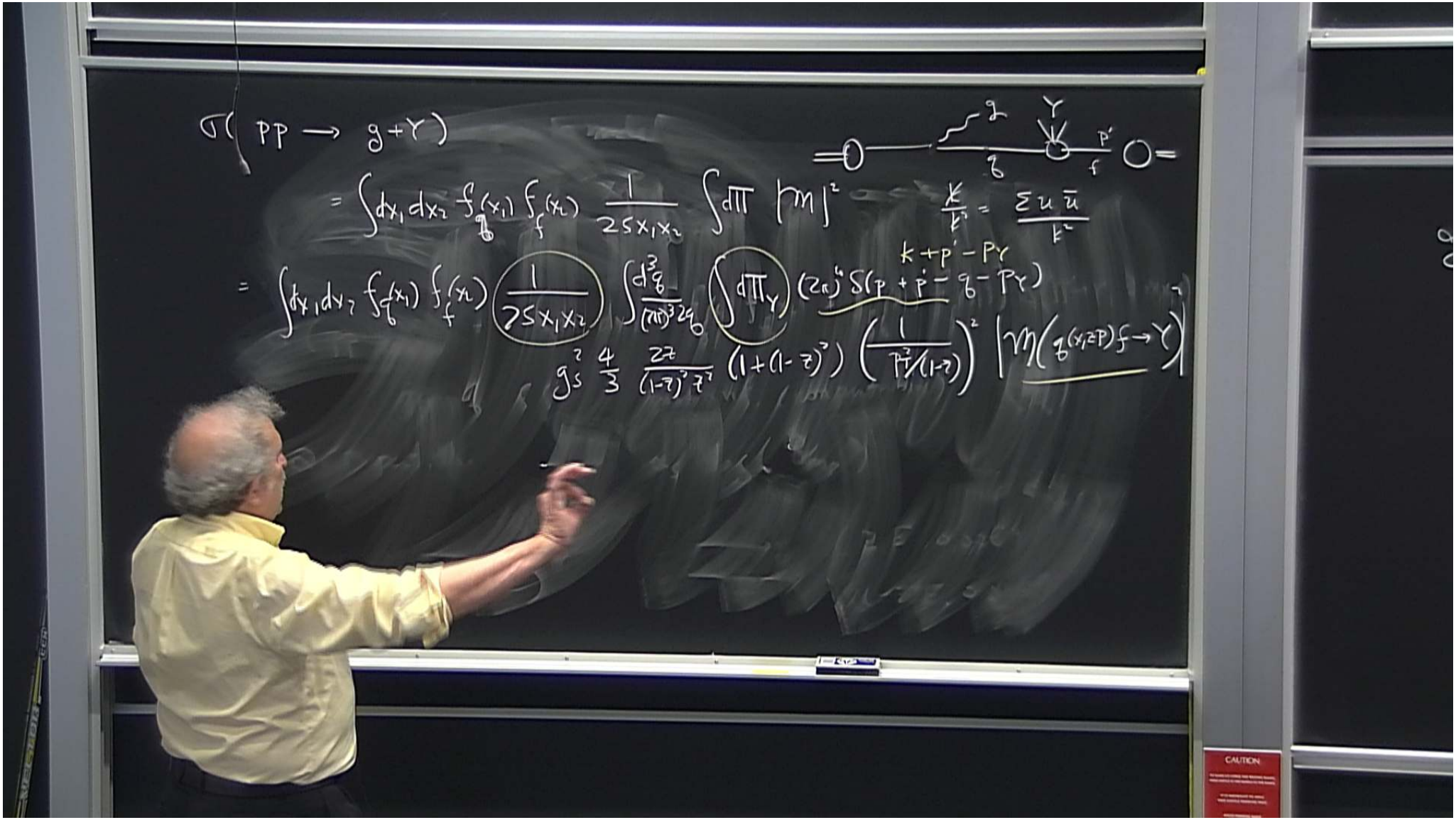


$$= \int dx_1 dx_2 \, f_g(x_1) f_f(x_2) \frac{1}{2 s x_1 x_2} \int d\Pi \, |M|^2 \qquad \frac{k}{k^2} = \frac{\varepsilon u \bar{u}}{k^2}$$

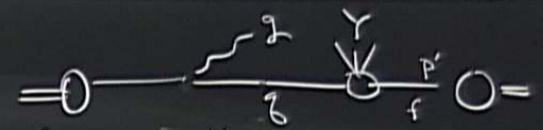
$$= \int dx_1 dx_2 \, f_g(x_1) f_f(x_2) \frac{1}{2 s x_1 x_2} \int \frac{d^3 q}{(2\pi)^3 2q} \int d\Pi_\gamma \, (2\pi)^4 \delta(p + \bar{p} - q - p_\gamma)$$

$$g_s^2 \frac{4}{3} \frac{z^2}{(1-z)^2 z^2} (1 + (1-z)^2) \left(\frac{1}{p_T^2 / (1-z)} \right)^2$$


$\sigma(pp \to g+\gamma)$
$= \int dx_1 dx_2 f_g(x_1) f_f(x_2) \frac{1}{2 s x_1 x_2} \int d\Pi |m|^2$
$\frac{k}{k^2} = \frac{\varepsilon u \bar{u}}{k^2}$
$k + p' - p_\gamma$
$(2\pi)^4 \delta(p + p' - q - p_\gamma)$
$g_s^2 \frac{4}{3} \frac{z_2}{(1-z)^2 z^2} (1+(1-z)^2) \left(\frac{1}{p_T^2/(1-z)}\right)^2 |m(g(x_1 z_2 p) f \to \gamma)|^2$
CAUTION


$\sigma(pp \to g+\gamma)$
$= \int dx_1 dx_2 f_g(x_1) f_f(x_2) \frac{1}{2sx_1x_2} \int d\Pi |\mathcal{M}|^2$
$\frac{k}{k^2} = \frac{\varepsilon u \bar{u}}{k^2}$
$= \int dx_1 dx_2 f_g(x_1) f_f(x_2) \frac{1}{2sx_1x_2} \int \frac{d^3q}{(2\pi)^3 2q} \int d\Pi_\gamma (2\pi)^4 \delta(p + p' - q - p_\gamma)$
$k + p' - p_\gamma$
$g_s^2 \frac{4}{3} \frac{z^2}{(1-z)^2 z^2} (1+(1-z)^2) \left(\frac{1}{p_T^2/(1-z)}\right)^2 |\mathcal{M}(g(x_1 z p) f \to \gamma f)|^2$
CAUTION

$\sigma(pp \to g+\gamma)$



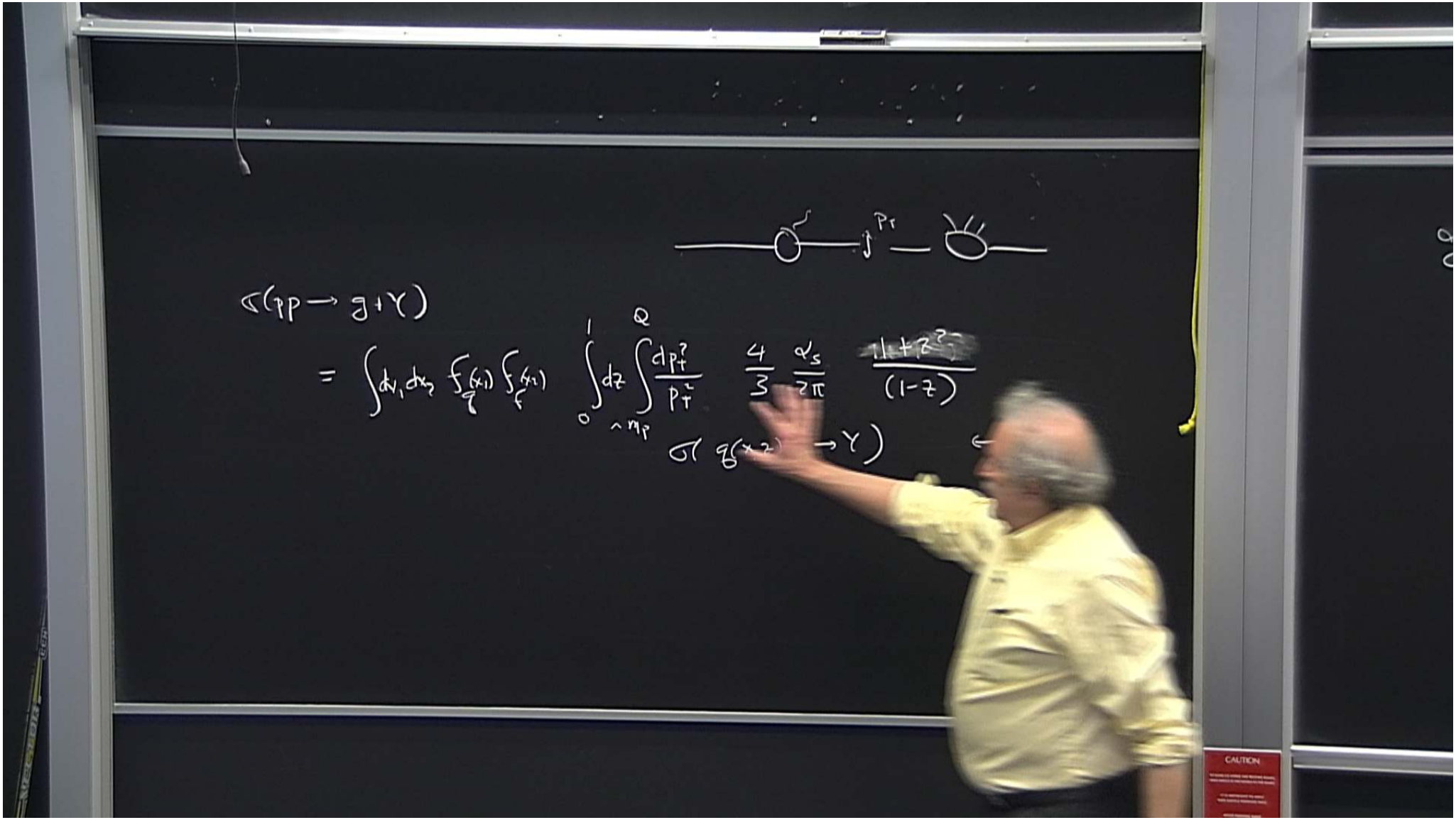
$$= \int dx_1 dx_2 f_g(x_1) f_f(x_2) \frac{1}{2 s x_1 x_2} \int d\Pi \, |\mathcal{M}|^2 \qquad \frac{k}{k^2} = \frac{\varepsilon u \bar{u}}{k^2}$$

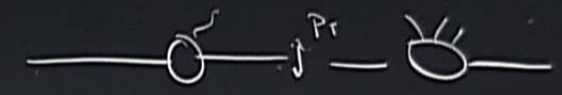
$$= \int dx_1 dx_2 f_g(x_1) f_f(x_2) \frac{1}{2 s x_1 x_2} \int \frac{d^3 q}{(2\pi)^3 2 q_0} \left(\int d\Pi_\gamma \right) (2\pi)^4 \delta(p + p' - q - p_\gamma) \qquad k = p' - p_\gamma$$

$$g_s^2 \frac{4}{3} \frac{z z_1 p_T^2}{(1-z)^2 z^2} (1 + (1-z)^2) \left(\frac{1}{p_T^2/(1-z)} \right)^2 |\mathcal{M}(g(x_1 z p) f \to \gamma f)|^2$$

$$\frac{z \, dz \, |\vec{p}| \, d^2 p_T}{(2\pi)^3 \, 2 (1-z) |\vec{p}|}$$

$$\int dx_1 dx_2 f_g(x_1) f_f(x_2) \int dz \int \frac{d|\vec{p}_T| \pi \, p_T^2}{p_T^4 \, 16\pi^3} \frac{1}{(1-z)} g_s^2 \frac{4}{3} \frac{2 z (1-z) p_T^2}{z^2} (1-z)^2 \sigma(q(z x_1 p) + f \to \gamma)$$

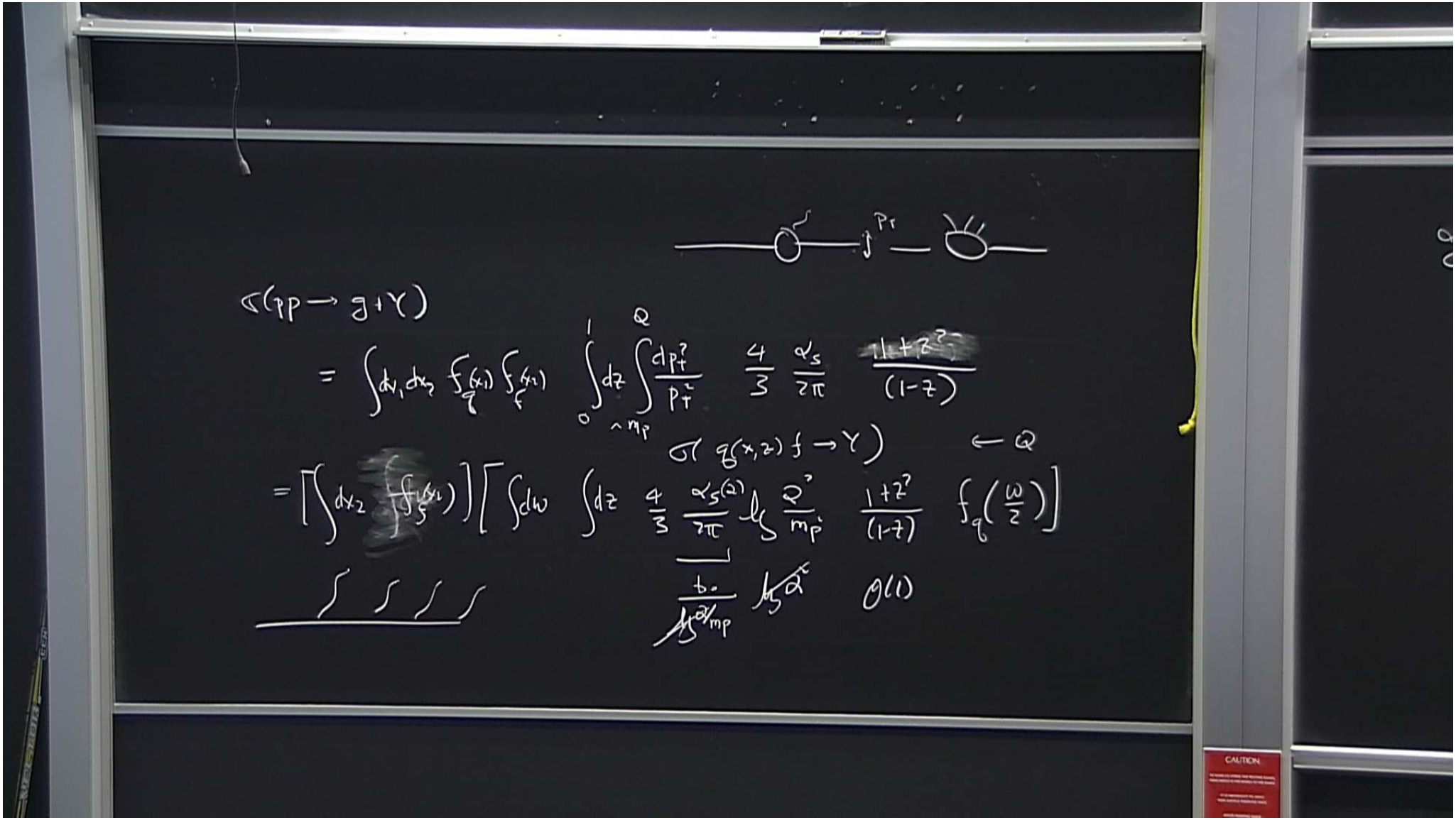

p_T
$\sigma(qp \to g+\gamma)$
$= \int dx_1 dx_2 f_{q}^{(x_1)} f_{\bar{c}}^{(x_2)} \int_0^1 dz \int_{\sim m_p}^{Q} \frac{dp_T^2}{p_T^2} \frac{4}{3} \frac{\alpha_s}{2\pi} \frac{1+z^2}{(1-z)}$
$\sigma(g(x_2) \to \gamma)$
CAUTION

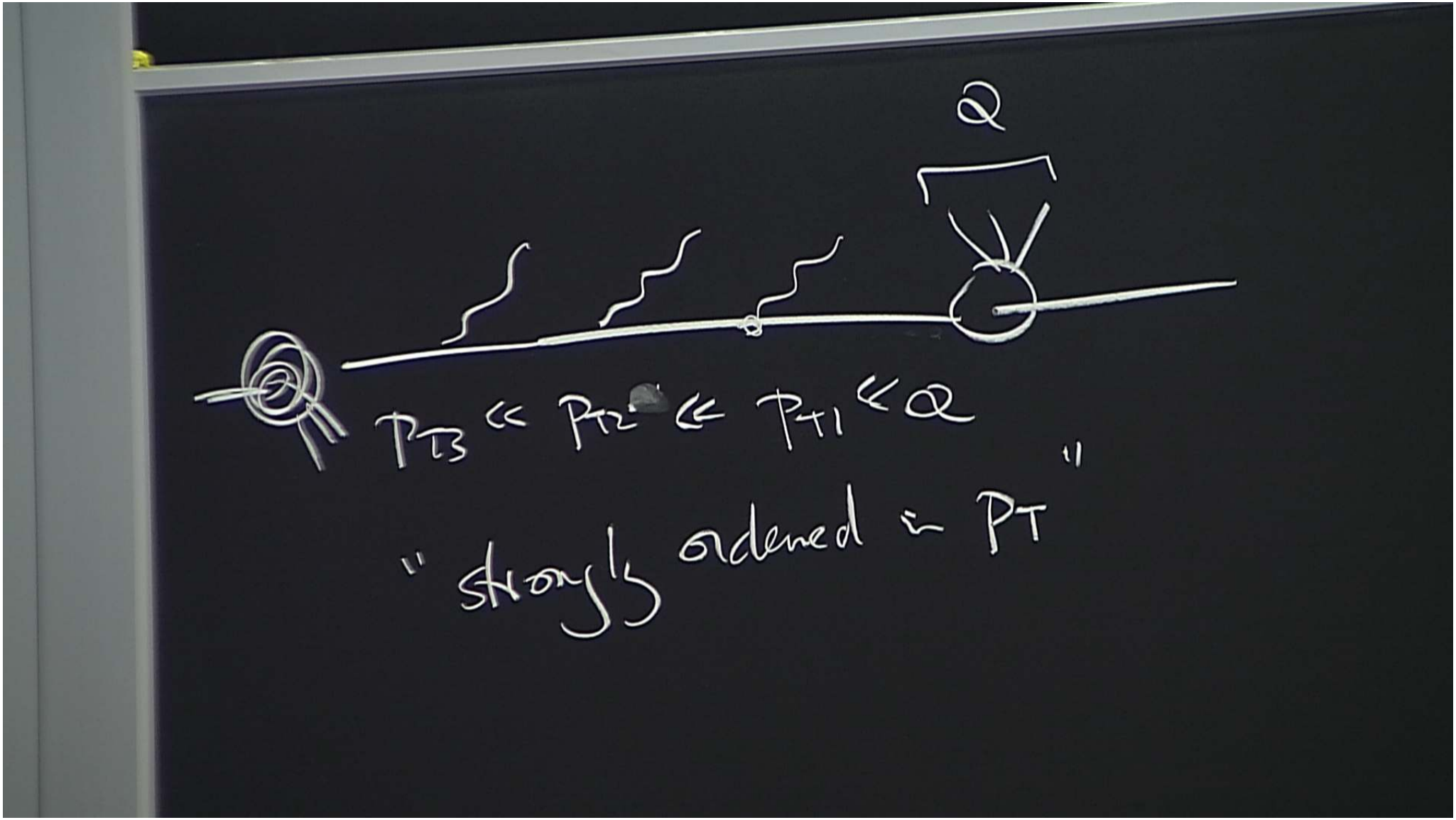


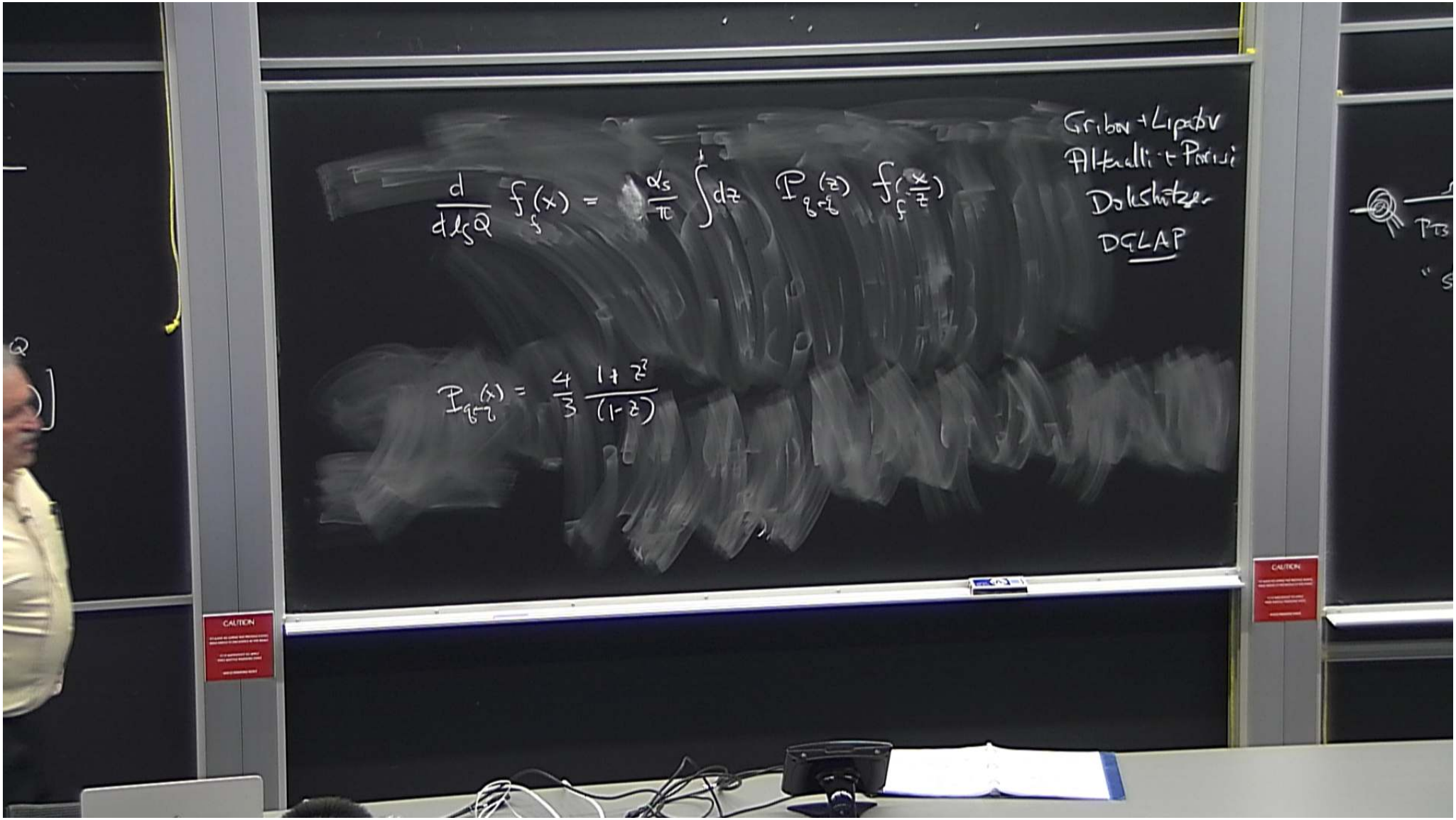
$$\sigma(pp \to g+\gamma)$$

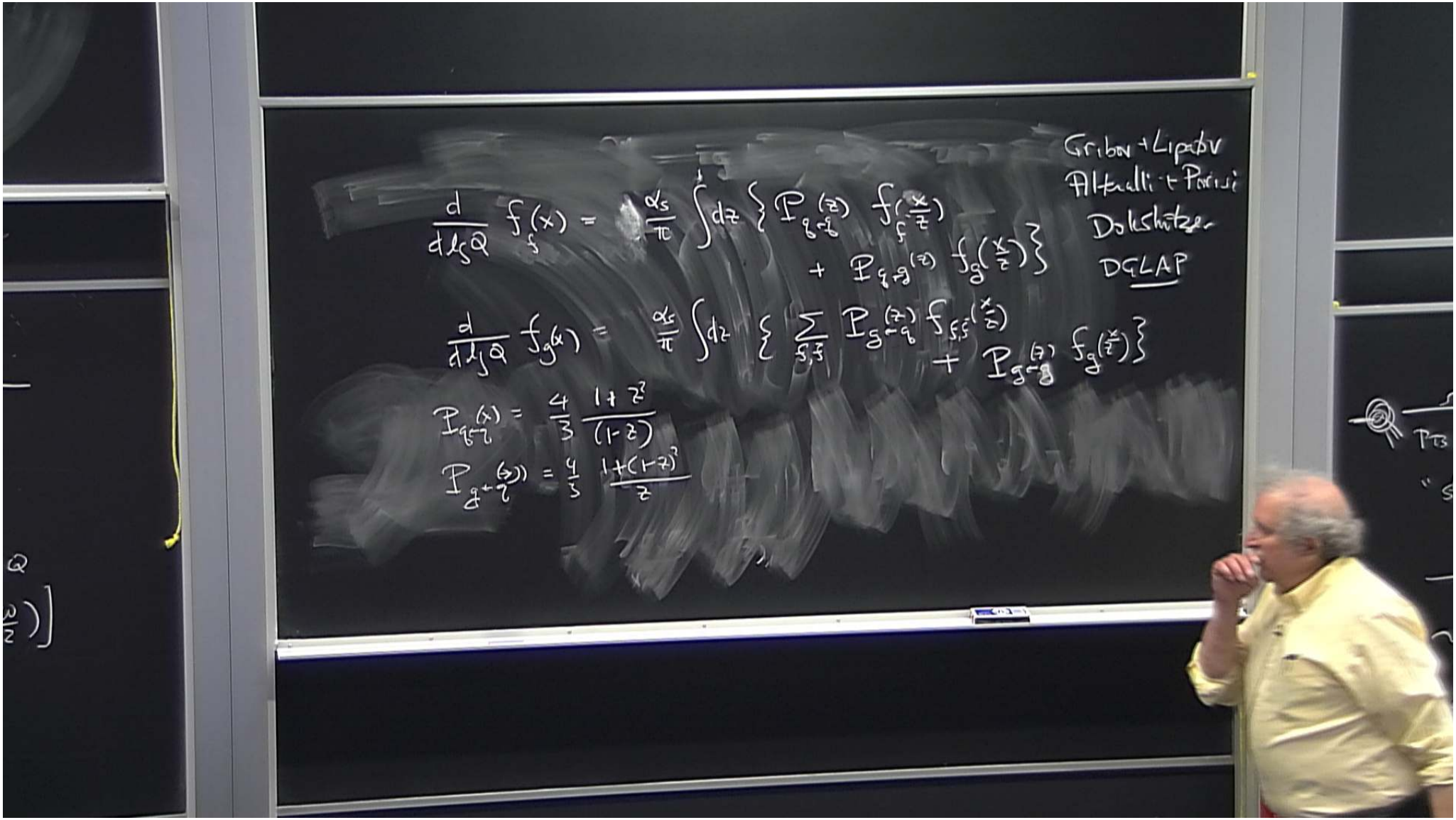
$$= \int dx_1 dx_2 \, f_q^{(x_1)} f_{\bar{q}}^{(x_2)} \int_0^1 dz \int_{\hat{m}_T}^{Q} \frac{dp_T^2}{p_T^2} \, \frac{4}{3} \frac{\alpha_s}{2\pi} \, \frac{1+z^2}{(1-z)}$$

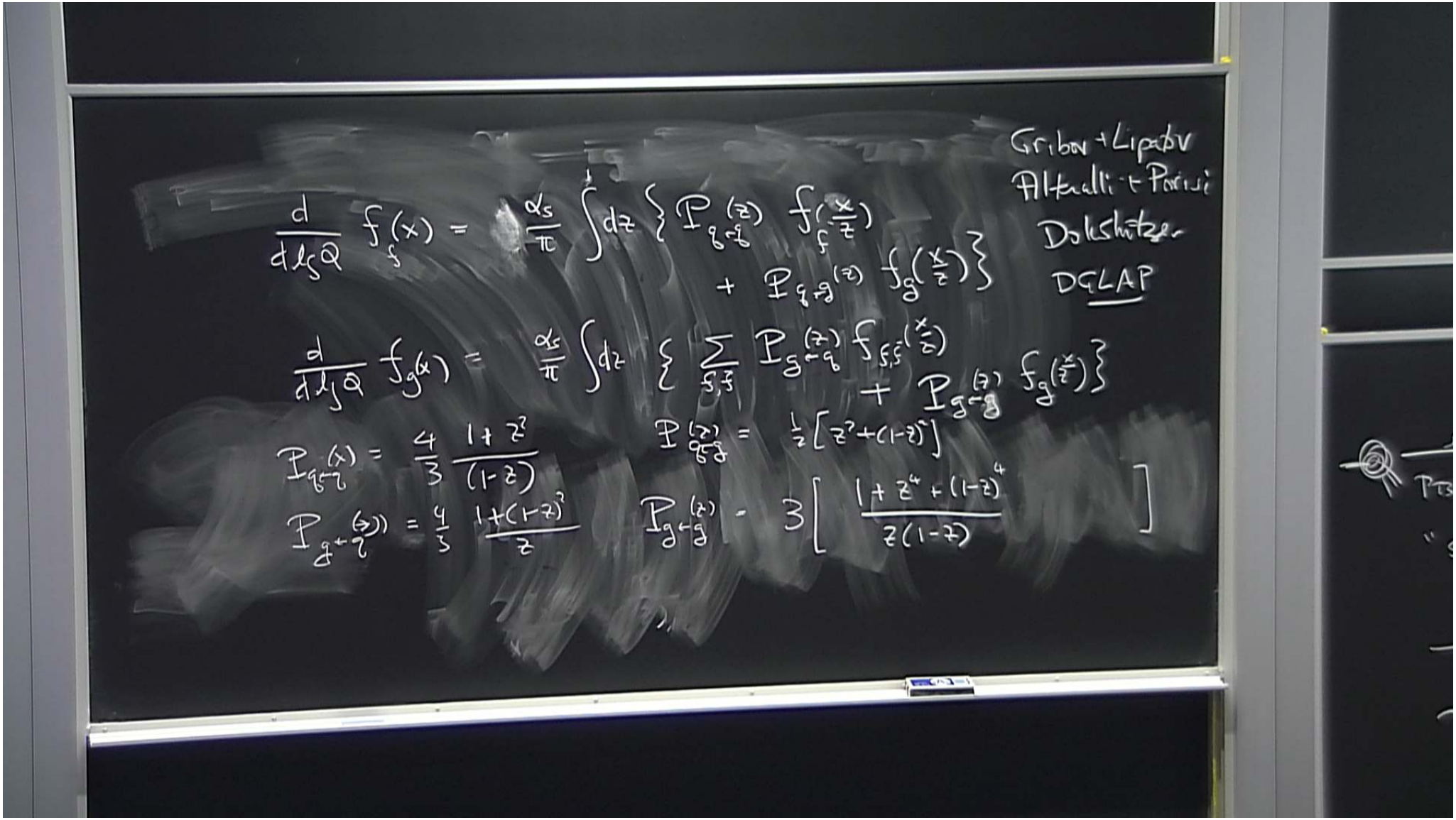
$$\sigma(q(x,z) f \to \gamma) \qquad \leftarrow Q$$

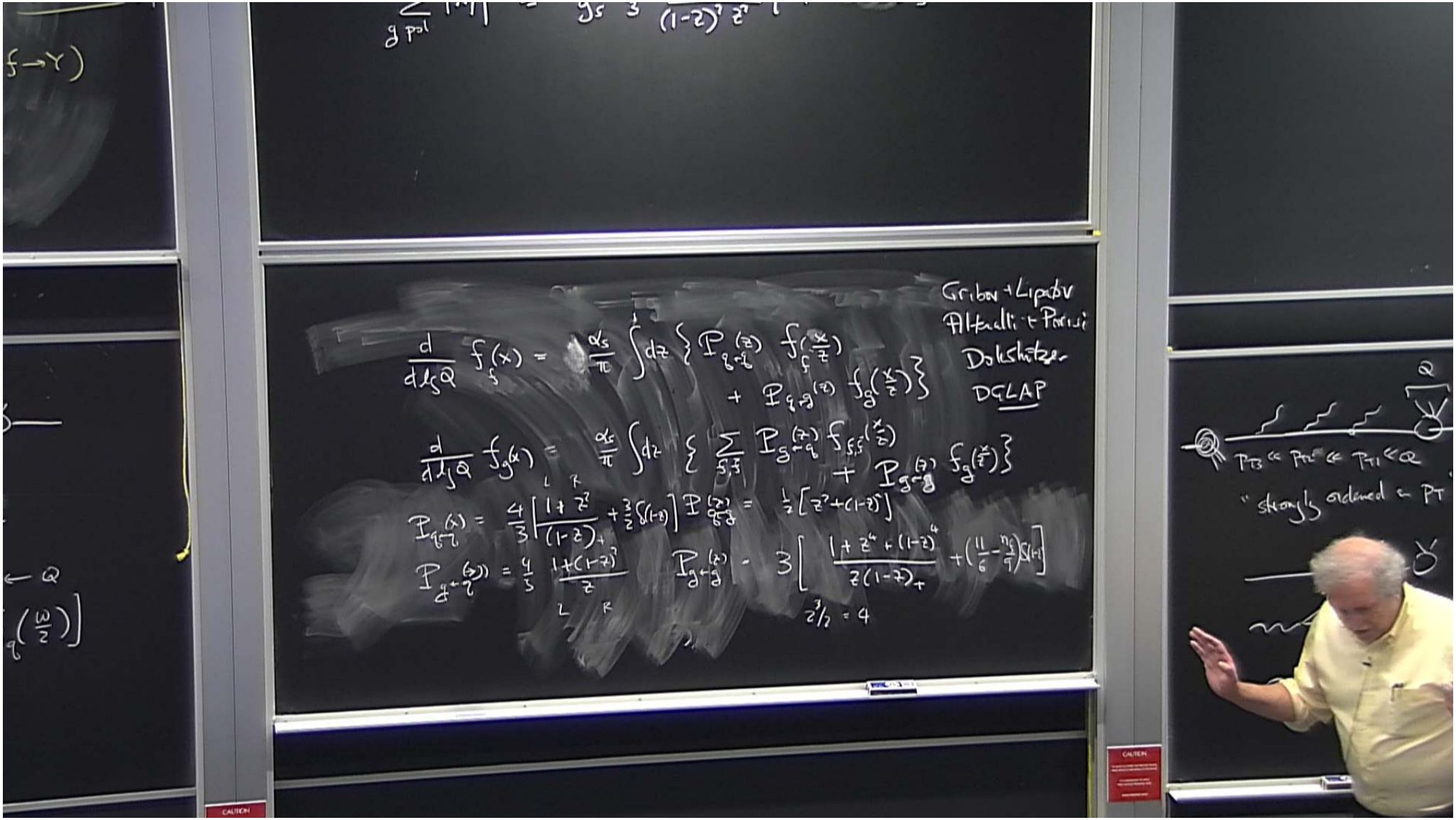

p_T
$\sigma(pp \to g + Y)$
$= \int dx_1 dx_2\, f_q(x_1) f_f(x_2) \int_0^1 dz \int_{\sim m_p}^{Q} \frac{dp_T^2}{p_T^2} \frac{4}{3} \frac{\alpha_s}{2\pi} \frac{1+z^2}{(1-z)}$
$\sigma(q(x,z) f \to Y)$
$\leftarrow Q$
$= \left[\int dx_2\, f_f(x_2)\right] \left[\int d\omega \int dz\, \frac{4}{3} \frac{\alpha_s(Q)}{2\pi} \log \frac{Q^2}{m_p^2} \frac{1+z^2}{(1-z)} f_q\left(\frac{\omega}{2}\right)\right]$
$\frac{b_0}{\log Q/m_p}$
$\log Q^2$
$O(1)$

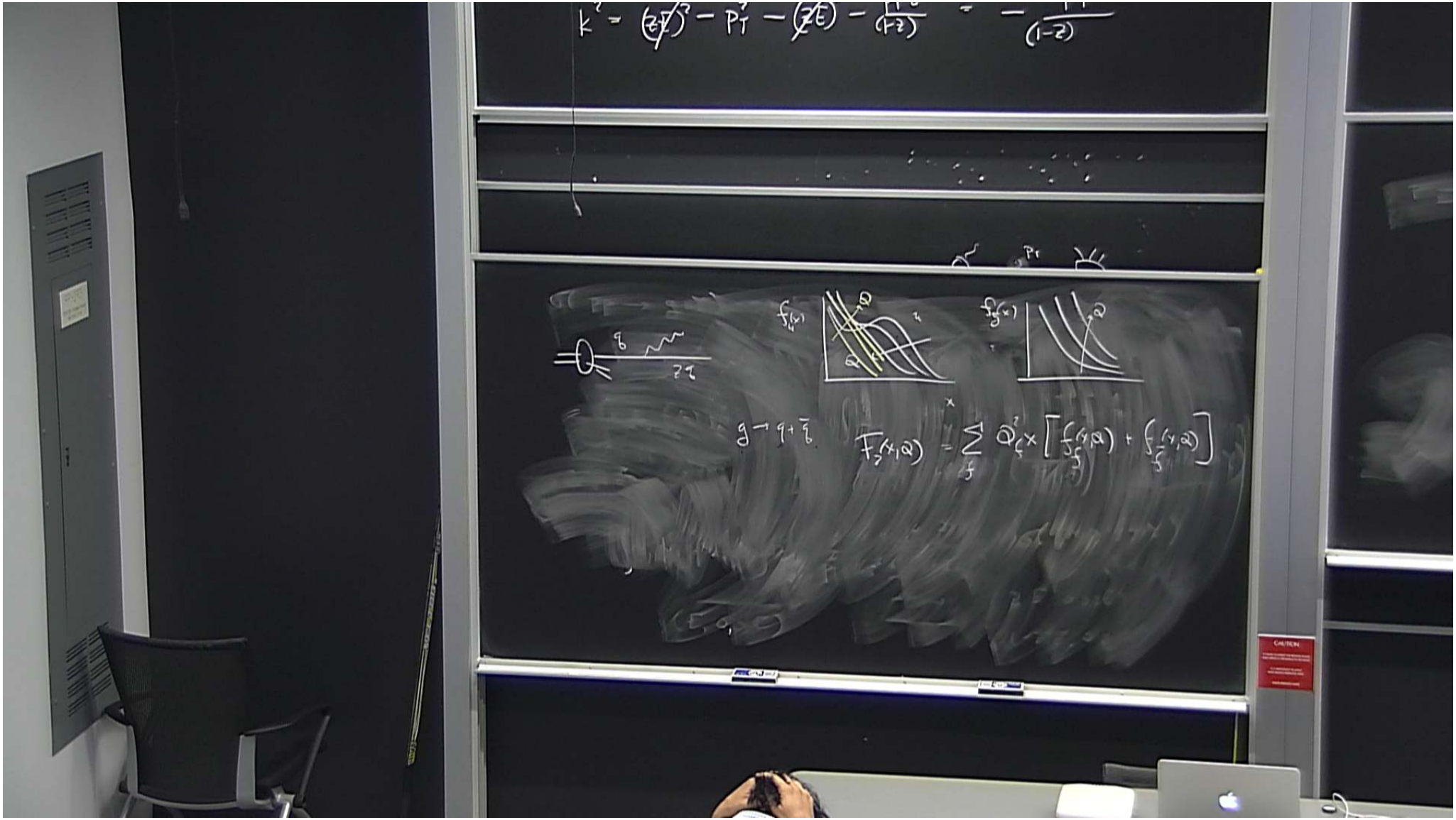

Q
$p_{T3} \ll p_{T2} \ll p_{T1} \ll Q$
"strongly ordered in p_T"

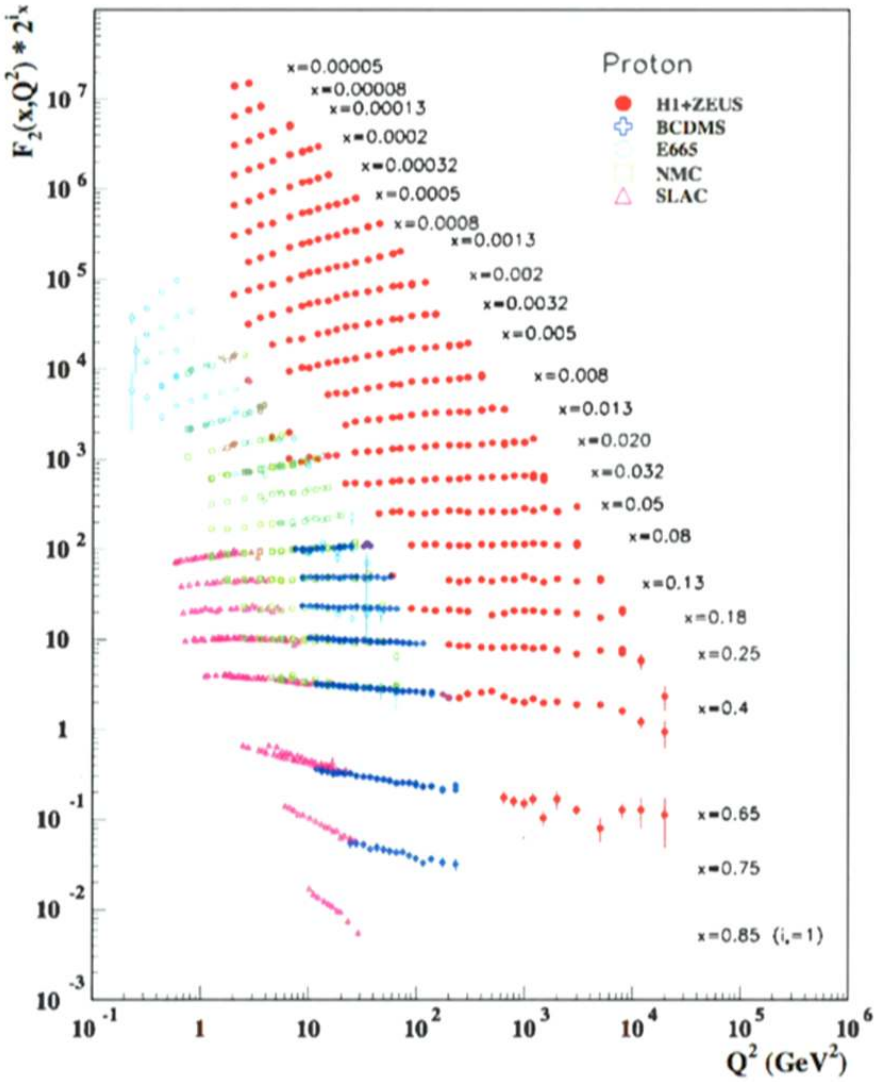

$\frac{d}{d\ln Q^2} f_i(x) = \frac{\alpha_s}{\pi} \int_x^1 dz \; P_{q\to q}^{(z)} \; f_i\left(\frac{x}{z}\right)$
$P_{q\to q}^{(x)} = \frac{4}{3} \frac{1+z^2}{(1-z)}$
Gribov + Lipatov
Altarelli + Parisi
Dokshitzer
DGLAP
CAUTION
CAUTION

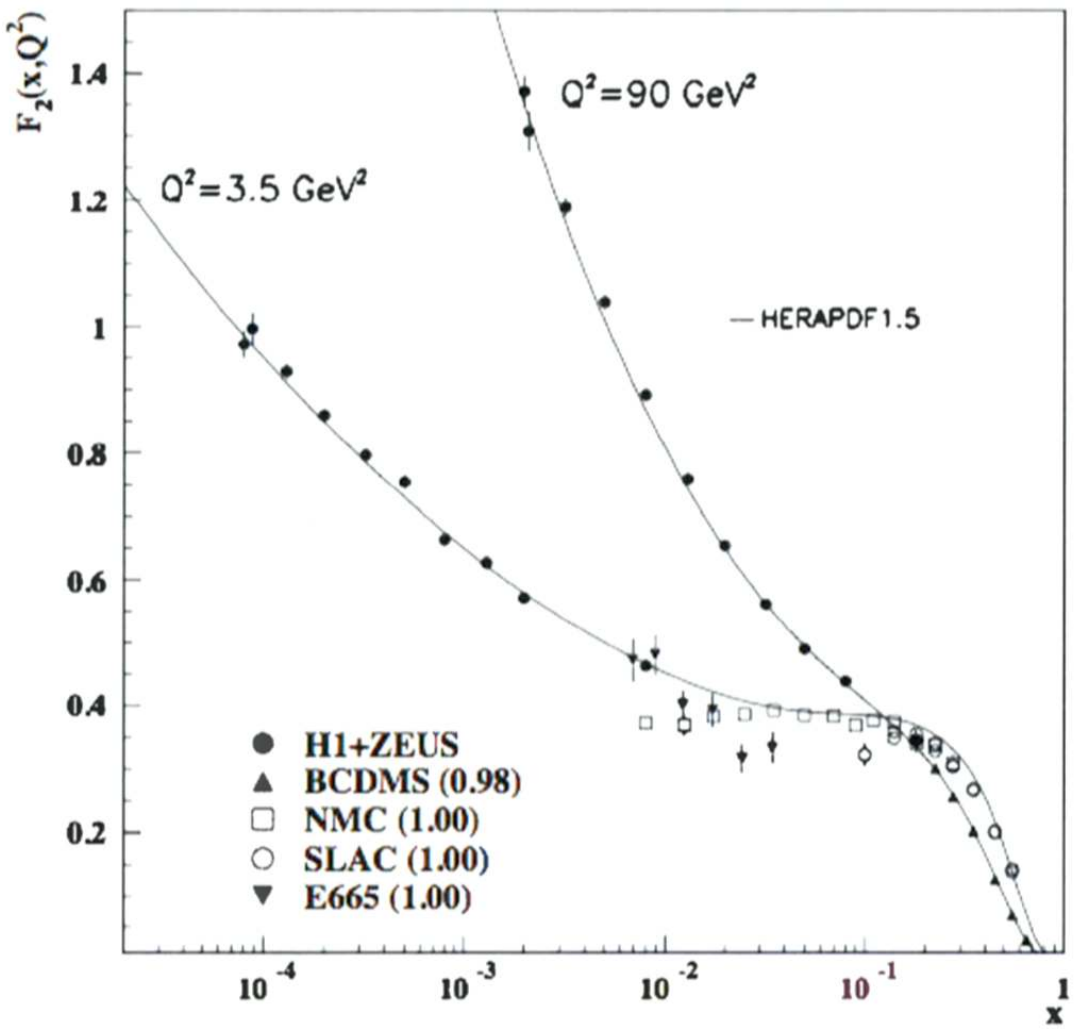

Gribov + Lipatov
Altarelli + Parisi
Dokshitzer-
DGLAP

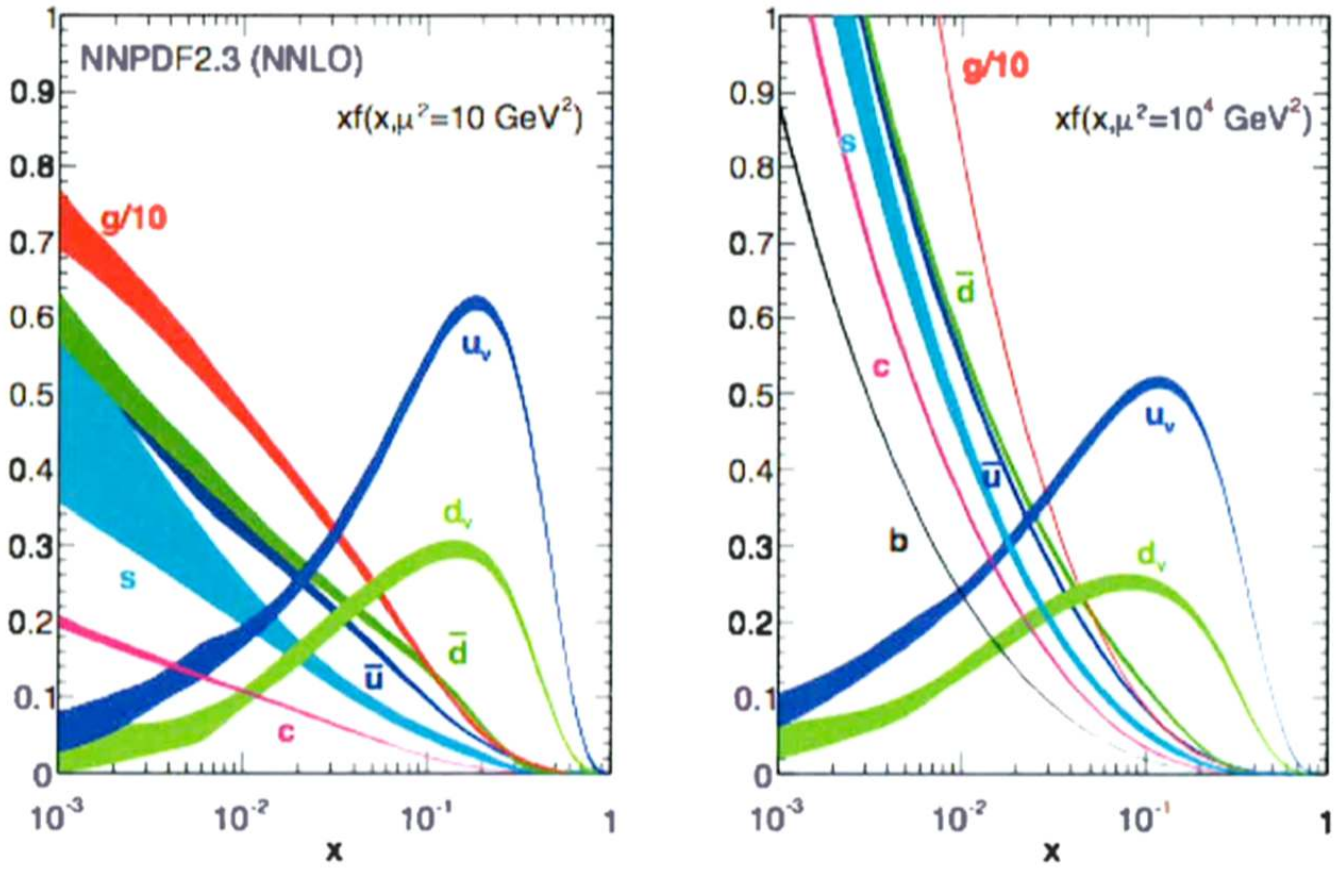

Gribov + Lipatov
Altarelli + Parisi
Dokshitzer
DGLAP

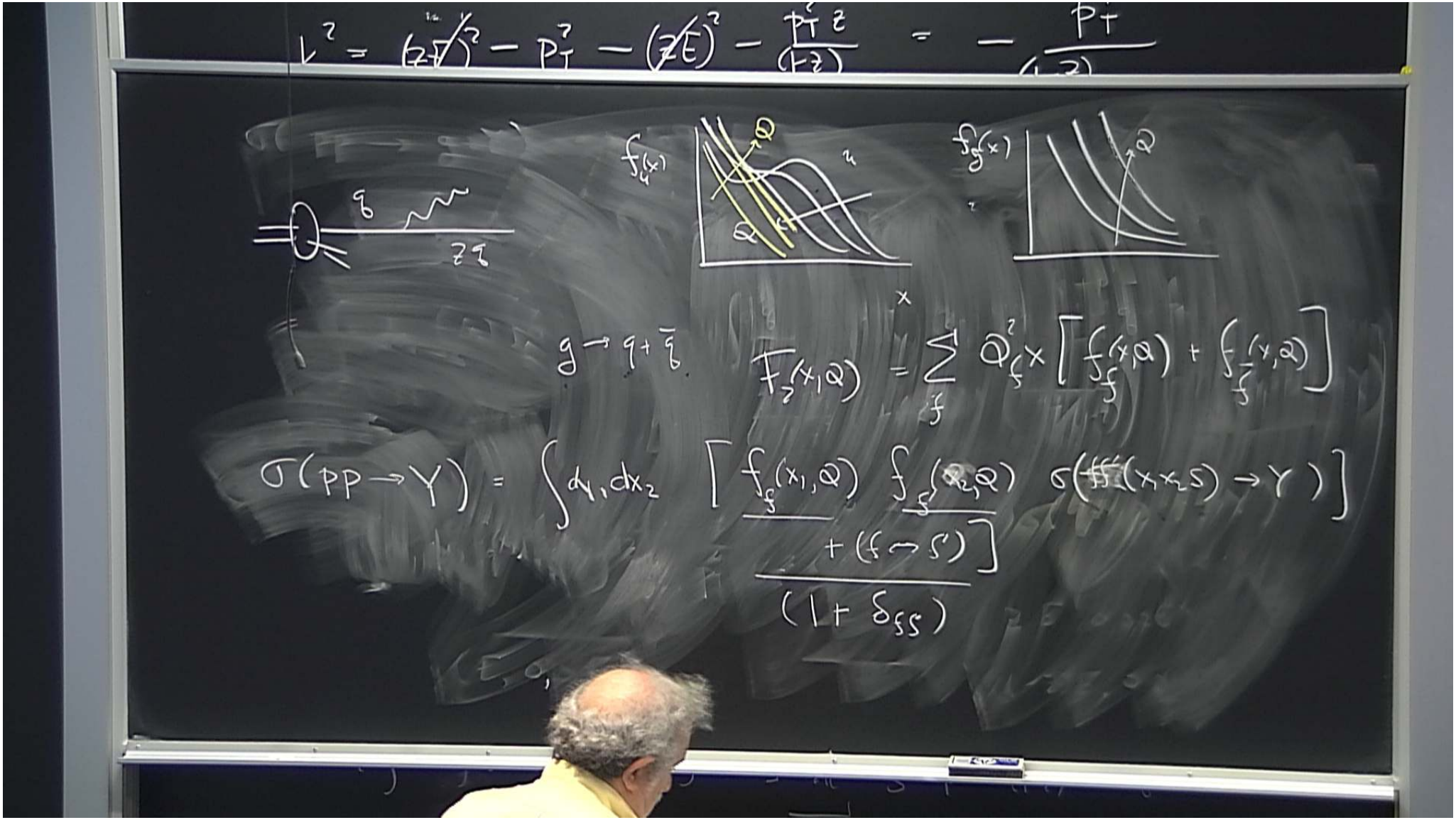

Gribov + Lipatov
Altarelli + Parisi
Dokshitzer-
DGLAP
$\frac{d}{d\ln Q^2} f_q(x) = \frac{\alpha_s}{2\pi} \int_x^1 dz \left\{ P_{q\to q}(z)\, f_q\left(\frac{x}{z}\right) + P_{g\to q}(z)\, f_g\left(\frac{x}{z}\right) \right\}$
$\frac{d}{d\ln Q^2} f_g(x) = \frac{\alpha_s}{2\pi} \int_x^1 dz \left\{ \sum_{q,\bar{q}} P_{q\to g}(z)\, f_{q,\bar{q}}\left(\frac{x}{z}\right) + P_{g\to g}(z)\, f_g\left(\frac{x}{z}\right) \right\}$
$P_{q\to q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$
$P_{q\to g}(z) = \frac{1}{2}\left[z^2+(1-z)^2\right]$
$P_{g\to q}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$
$P_{g\to g}(z) = 3\left[\frac{1+z^4+(1-z)^4}{z(1-z)_+} + \left(\frac{11}{6} - \frac{n_f}{9}\right)\delta(1-z)\right]$
$n_f = 4$
$p_{T3} \ll p_{T2} \ll p_{T1} \ll Q$
"strongly ordered in p_T"
Q


g → q + q̄
F2(x,Q) = Σ Q_f^2 x [f_f(x,Q) + f_f̄(x,Q)]

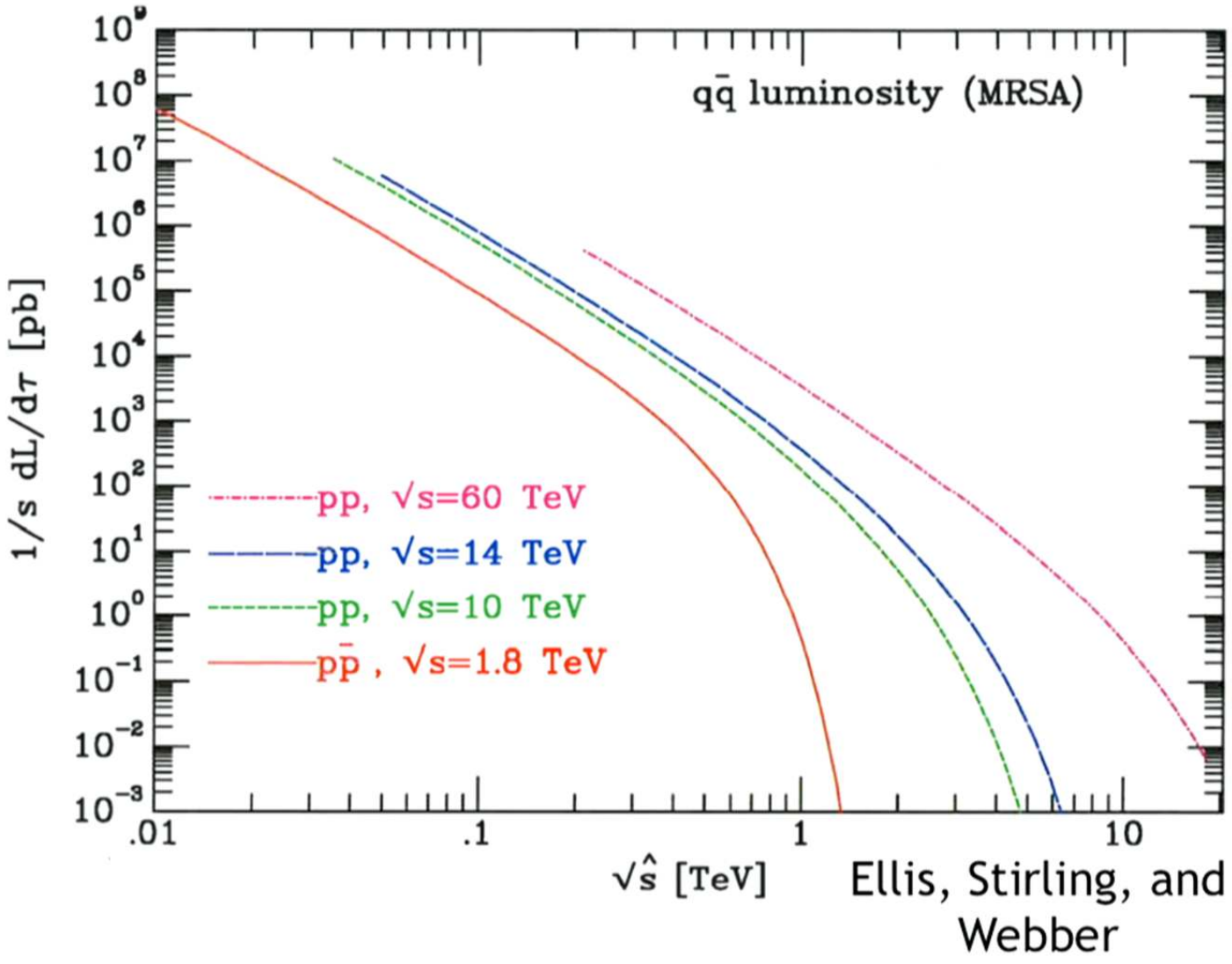

Proton
H1+ZEUS
BCDMS
E665
NMC
SLAC
x=0.00005
x=0.00008
x=0.00013
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.020
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (i=1)
$F_2(x,Q^2) * 2^{ix}$
Q^2 (GeV^2)


$F_2(x,Q^2)$
1.4
1.2
1
0.8
0.6
0.4
0.2
Q^2=90 GeV^2
Q^2=3.5 GeV^2
— HERAPDF1.5
H1+ZEUS
BCDMS (0.98)
NMC (1.00)
SLAC (1.00)
E665 (1.00)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x


NNPDF2.3 (NNLO)
xf(x,μ²=10 GeV²)
g/10
u_v
d_v
s
ū
d̄
c
x
xf(x,μ²=10⁴ GeV²)
g/10
s
d̄
c
ū
b
u_v
d_v
x



$k^2 = (zE)^2 - p_T^2 - (zE)^2 - \frac{p_T^2 z}{(1-z)} = -\frac{p_T}{(1-z)}$
$f_u(x)$
$f_g(x)$
$g \to q + \bar{q}$
$F_2(x,Q) = \sum_f Q_f^2 x \left[f_f(x,Q) + f_{\bar{f}}(x,Q) \right]$
$\sigma(pp \to Y) = \int dx_1 dx_2 \left[\frac{f_f(x_1,Q)\, f_{f'}(x_2,Q)\, \sigma(ff'(x_1 x_2 s) \to Y) + (f \leftrightarrow f')}{(1+\delta_{ff'})} \right]$


qq̄ luminosity (MRSA)
1/s dL/dτ [pb]
pp, √s=60 TeV
pp, √s=14 TeV
pp, √s=10 TeV
pp̄ , √s=1.8 TeV
√ŝ [TeV]
Ellis, Stirling, and Webber