Title: Spherically Symmetric Solutions of SD

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Abstract:

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Institute for Theoretical Physics

Comparing General Relativity with Shape Dynamics

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supervised by
Prof. Dr. Gerard 't Hooft
Dr Sean Gryb.

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- Introduce Hamiltonian formalism of GR

- Spherically symmetric, time dependent, vacuum

- Thin shell collapse

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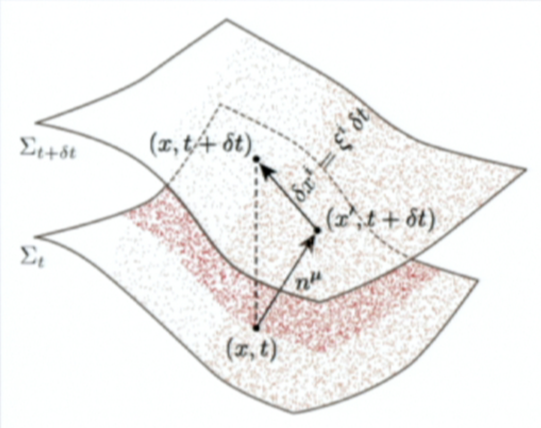
GR in ADM Hamiltonian formalism

- 3+1 space and time split

$$(g_{\mu\nu}) = \begin{pmatrix} -N^2 + \xi_k \xi^k & \xi_j \\ \xi_i & g_{ij} \end{pmatrix}$$



A Shape Dynamics Tutorial - Flavio Mercati
http://arxiv.org/abs/1409.0105

$$\left\{ g_{\mu\nu}(x), p^{\alpha\beta}(x') \right\}_{t=t'} = \frac{1}{2} \left(\delta^\alpha_\mu \delta^\beta_\nu + \delta^\beta_\mu \delta^\alpha_\nu \right) \delta(\mathbf{x} - \mathbf{x}')$$

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- Einstein's equations in vacuum

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0$$

$$\mathcal{H} = N\mathcal{H}_\perp + \xi^j \mathcal{H}_j \qquad H = \int_{\Sigma_t} \mathcal{H}\, d^3x$$

$$\mathcal{H}_\perp = g^{-1/2}(g_{ik}g_{jm} - \tfrac{1}{2} g_{ij} g_{km}) p^{ij} p^{km} - {}^3R g^{1/2} = 0$$

$$\mathcal{H}_j = -2 g_{ij} p^{ik}{}_{|k} = 0$$

$$\dot{g}_{ij} = \{g_{ij}, H\} \qquad \dot{p}^{ij} = \{p^{ij}, H\}$$

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SD Motivation

- Conformal 3-geometry = Unconstrained gravitational degrees of freedom
- Scalar constraint gives an equation for the conformal factor

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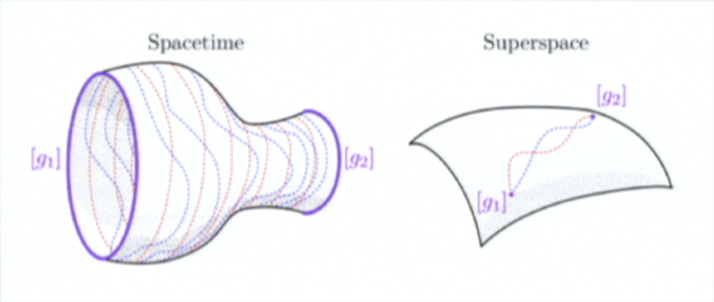
Configuration space

- Riem



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- Superspace

- Conformal Superspace

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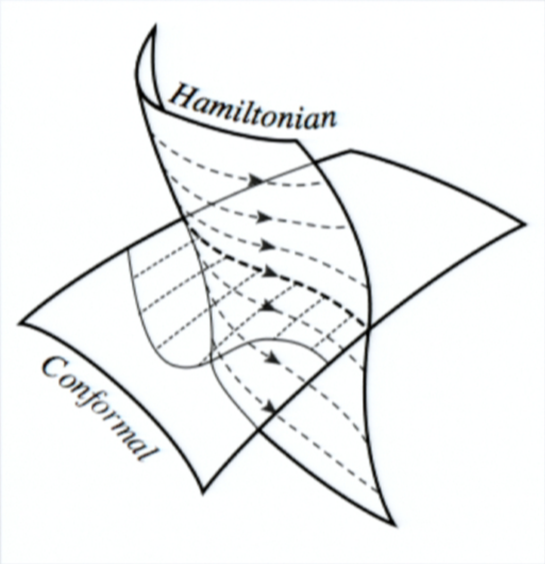
Phase space

- ADM GR – Relative simultaneity
- SD – Relative local scale



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Spherically symmetric time dependent vacuum solution

- Conformal decomposition

$$\omega = \ln \left(\frac{g}{\gamma}\right)^{1/3} \qquad p = g_{ij} p^{ij}$$

$$\tilde{g}_{ij} = \left(\frac{g}{\gamma}\right)^{-1/3} g_{ij} \qquad \tilde{p}^{ij} = \left(\frac{g}{\gamma}\right)^{1/3} \left(p^{ij} - \tfrac{1}{3} p h^{ij}\right)$$

$$g_{ij} h^{jk} = \delta_i^k$$

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- Isotropic radial coordinate

$$g_{ij} = e^{\omega}\gamma_{ij} \qquad \gamma_{ij} = \mathrm{diag}(1, r^2, r^2\sin^2\theta)$$

$$\tilde{g}_{ij} = \gamma_{ij} \qquad \tilde{p}^{ij} = 0$$

- Conformal constraint ≈ maximal slicing

$$p = 0 \qquad N_{|j}{}^{j} - {}^{3}RN = 0$$

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- Static solution

$$\xi^i = 0 \qquad \dot{g}_{ij} = 0 \qquad p^{ij} = 0$$

- Isotropic line element

$$ds^2 = -\left(\frac{1 - m/2r}{1 + m/2r}\right)^2 dt^2 + (1 + m/2r)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

$$R = r\left(1 + \frac{m}{2r}\right)^2 \qquad \bar{r} = \frac{m^2}{4r}$$

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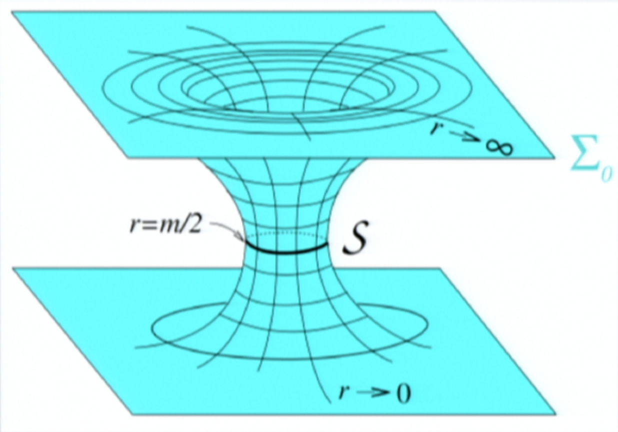
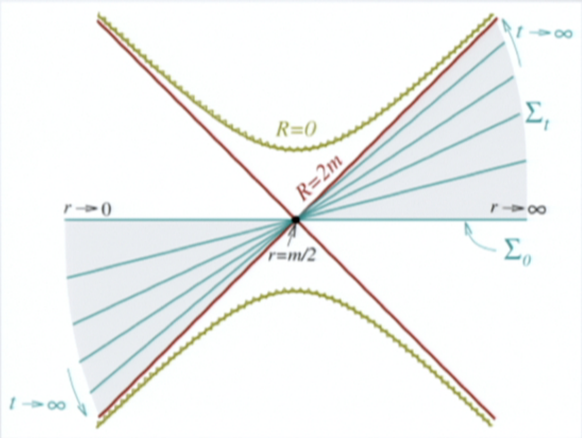
- Static spacetime



3+1 Formalism and Bases of Numerical Relativity
Eric Gourgoulhon
http://arxiv.org/abs/gr-qc/0703035



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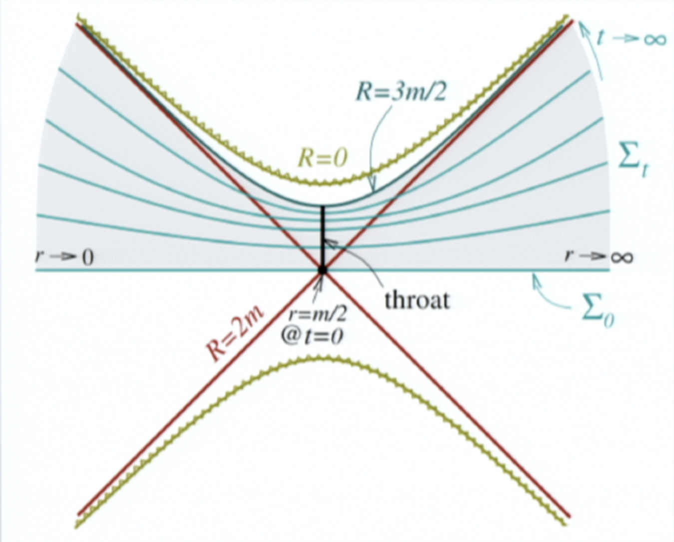
- Non-stationary maximally sliced spacetime



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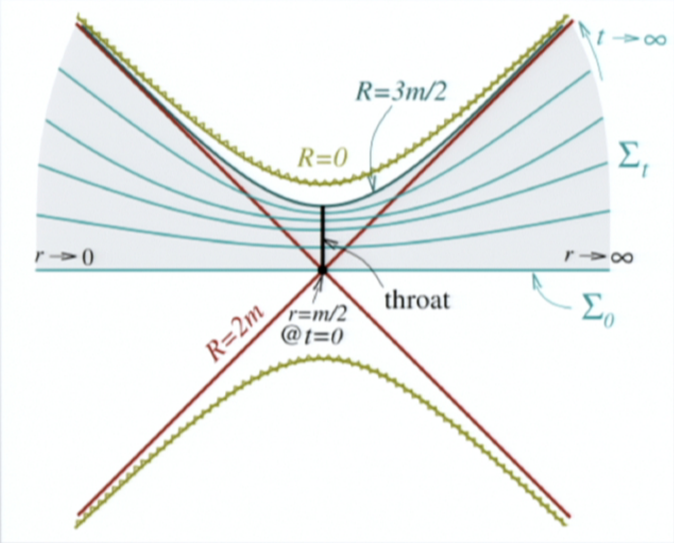

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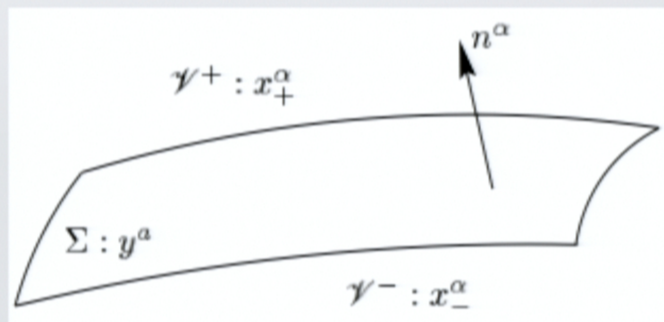
Thin shell dynamics

- Junction conditions

$$[h_{ij}] = h^{+}_{ij} - h^{-}_{ij} = 0 \qquad h^{\pm}_{ij} \equiv h_{ij}(V^{\pm})|_{\Sigma}$$

$$8\pi S_{ij} = h_{ij}[K] - [K_{ij}]$$



An Advanced course in General Relativity- Eric Poisson
http://www.physics.uoguelph.ca/poisson/research/agr.pdf

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- Spherical symmetry

$$ds_\pm^2 = -f_\pm\, dt_\pm^2 + f_\pm^{-1} dr^2 + r^2\, d\Omega^2$$

$$\mathcal{E}_\pm \equiv f_\pm(R)\frac{dt_\pm}{d\tau} = \lambda\sqrt{f_\pm(R) + \left(\frac{dR}{d\tau}\right)^2}$$

$$[\mathcal{E}] = -\frac{M}{R}$$

$$dM = -P\, d(4\pi R^2)$$

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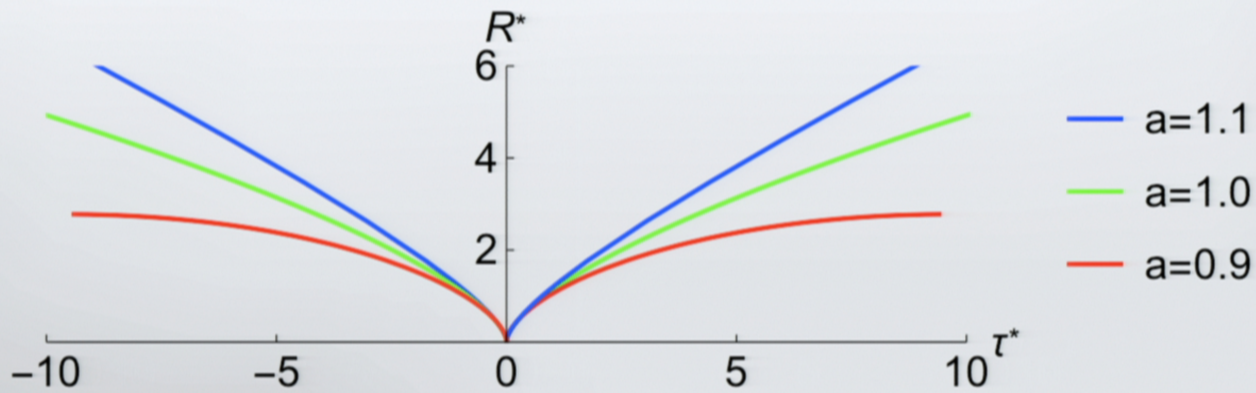
- Single dust shell in vacuum

$$m = M\sqrt{1 + \left(\frac{dR}{d\tau}\right)^2} - \frac{M^2}{2R} \qquad a = \frac{m}{M}$$



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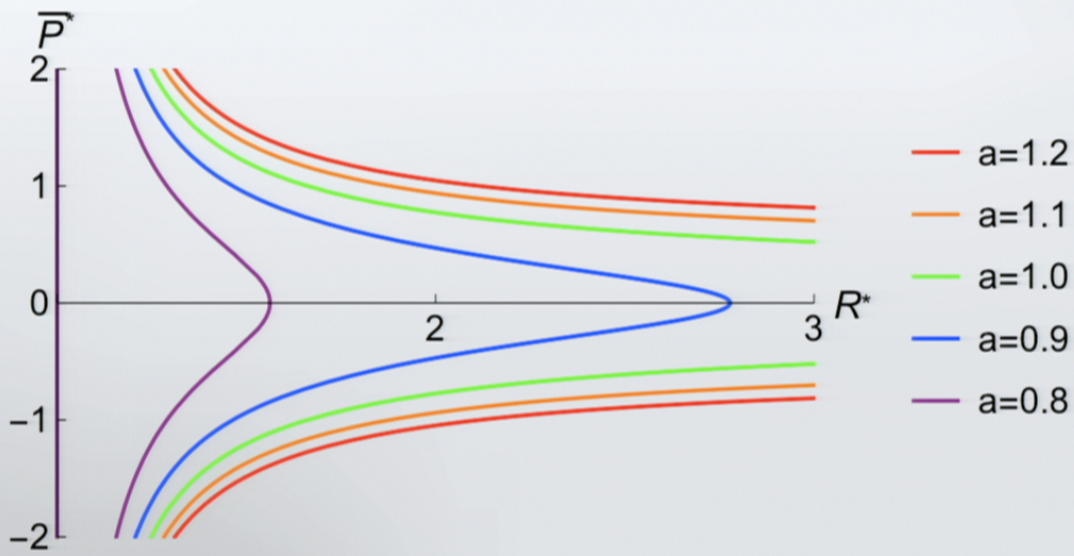
- Phase space

$$\bar{t} = \frac{t_+ + t_-}{2}$$

$$\bar{P} = \frac{m}{2a}\left(\frac{1}{f_+} + \frac{1}{f_-}\right)\frac{dR}{d\tau} \qquad \left(\frac{dR}{d\tau}\right)^2 = \left(a + \frac{m}{2aR}\right)^2 - 1$$



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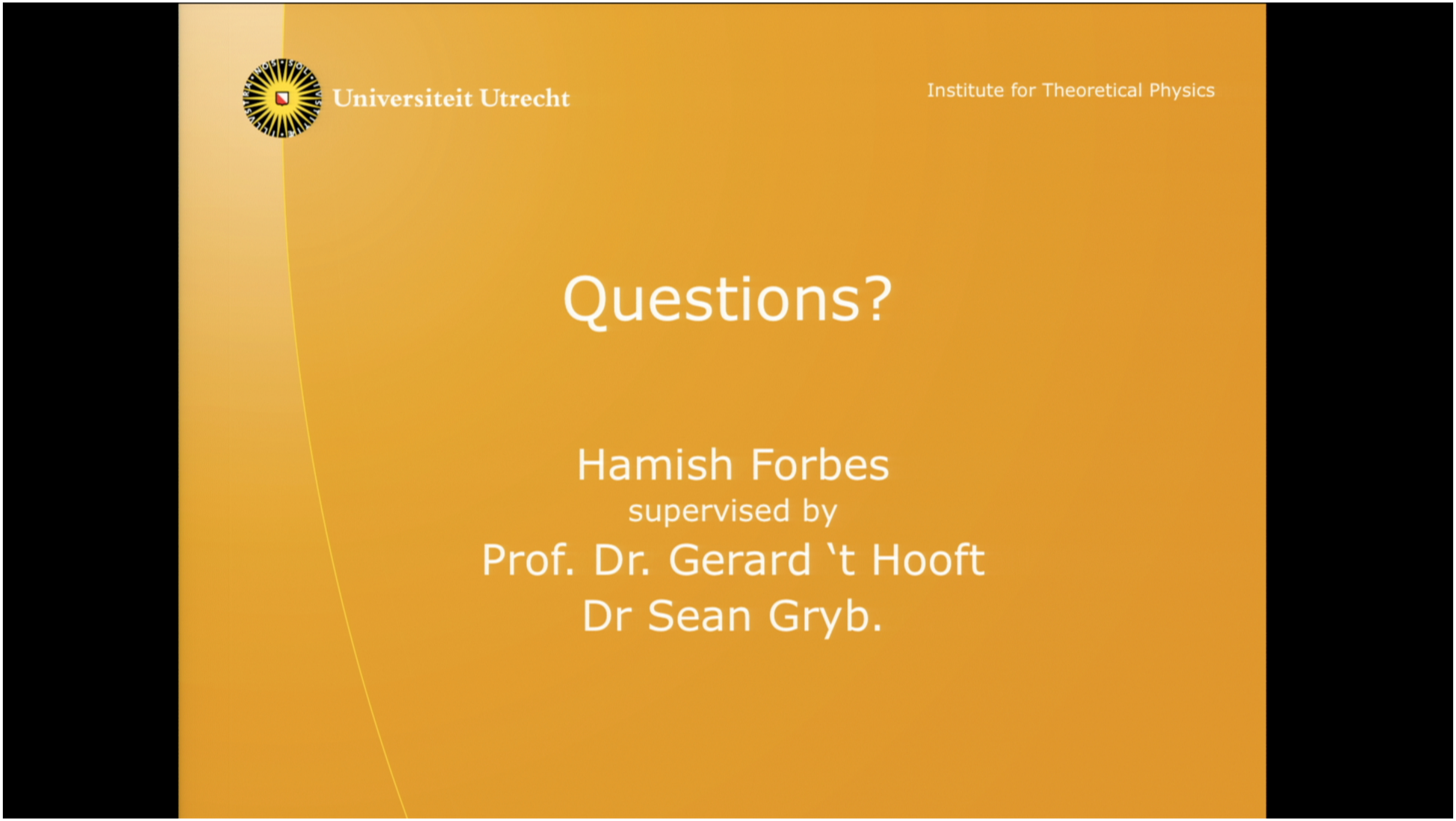
Summary

- Spherically symmetric time dependent vacuum solution in Hamiltonian formalism of SD.
- Thin shell dynamics, dust shell.
- Effective Hamiltonian, phase space.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Hamish Forbes
supervised by
Prof. Dr. Gerard 't Hooft
Dr Sean Gryb.