Title: Cause and Effect in a Quantum World

Date: Jun 22, 2015 02:00 PM

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Abstract: Determining causal relationships is central to scientific understanding. Knowledge of such relations permit us not only to predict how a system will behave naturally, but also how it would behave under different hypothetical circumstances, including those where we exert control over some component. In the context of quantum theory, the problem of figuring out what causes what is particularly vexing. One of the central results in the foundations of quantum theory, Bell's theorem, can be understood as demonstrating that it is impossible to provide a causal explanation of the correlations that arise for entangled quantum systems without resorting to fine-tuning. Impossible, that is, using the standard framework of causal models. A new quantum notion of causal model, however, holds promise for achieving such an explanation. It also has practical applications, allowing one to infer causal relationships from observed correlations in scenarios where classically one could not. Correlation does not imply causation. Except in a quantum world, where sometimes it does.

Cause and Effect in a Quantum World

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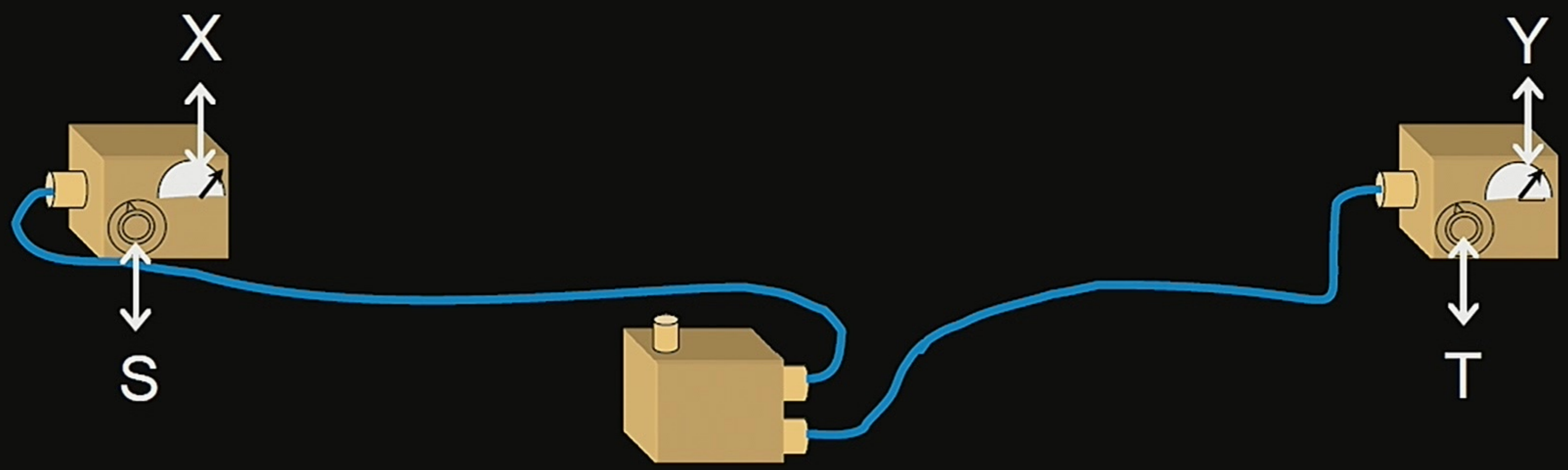
Perimeter Institute

Convergence
June 22, 2015

Causarum Investigatio
“Investigate the causes”

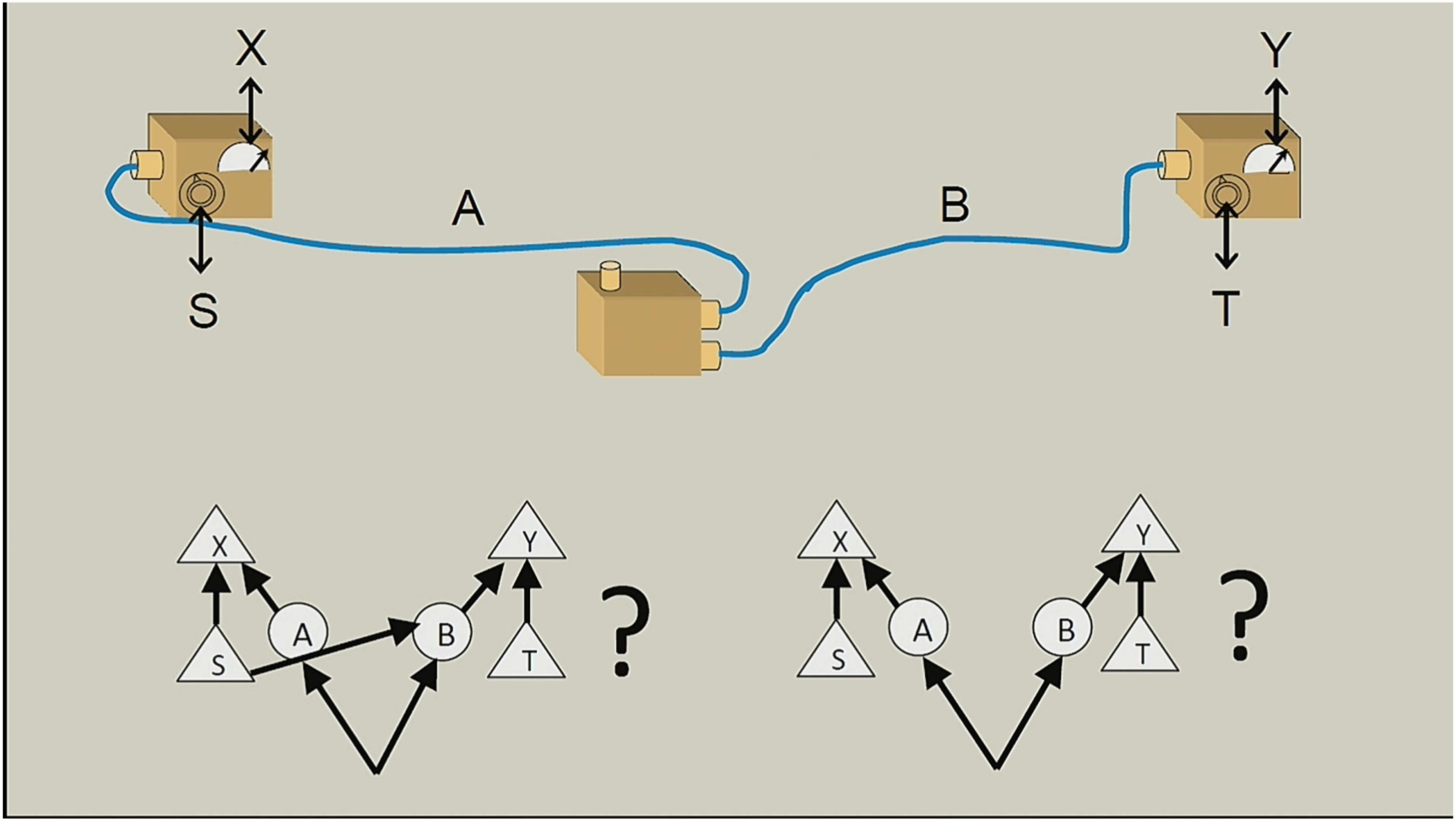


P(X,Y|S,T)

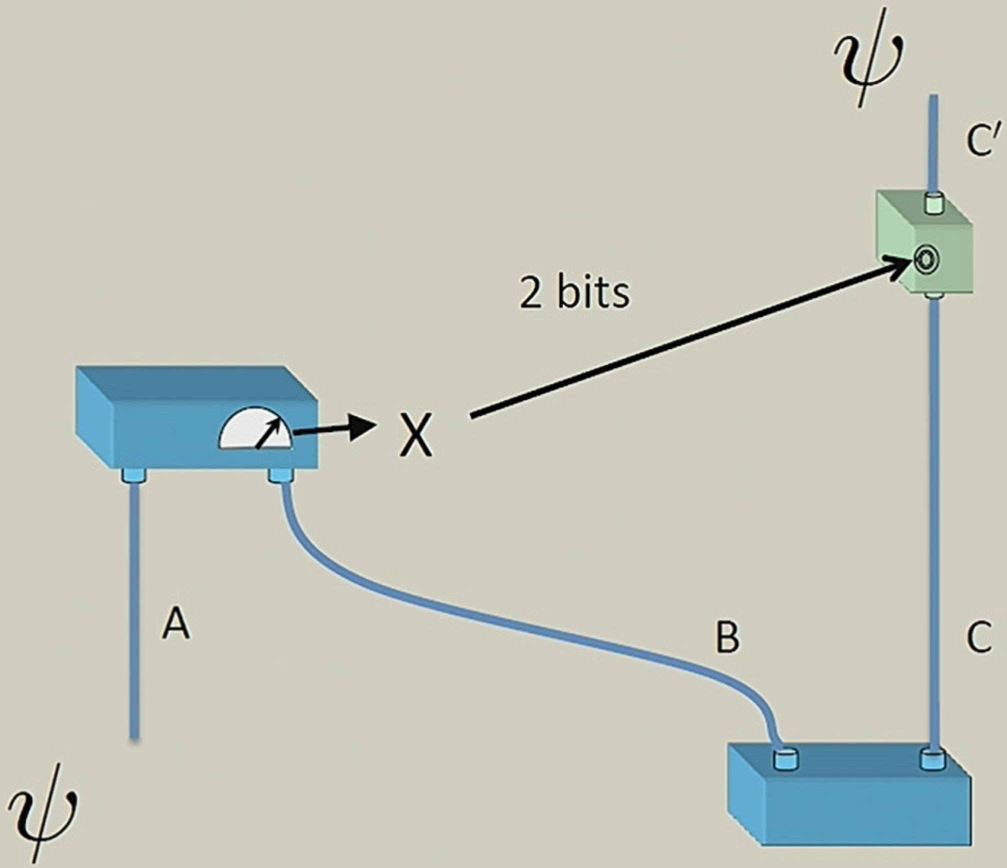
	X=0, Y=0	X=0, Y=1	X=1, Y=0	X=1, Y=1
S=0, T=0	0.427	0.073	0.073	0.427
S=0, T=1	0.427	0.073	0.073	0.427
S=1, T=0	0.427	0.073	0.073	0.427
S=1, T=1	0.073	0.427	0.427	0.073

Simpson's Paradox

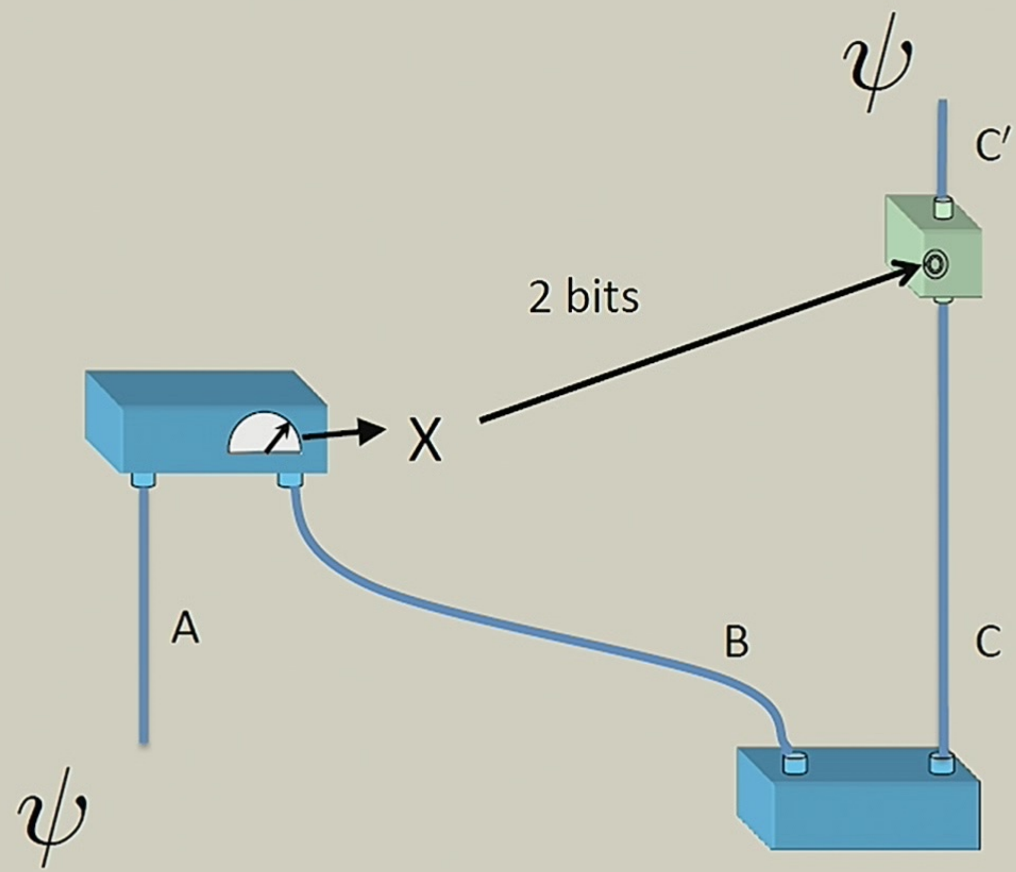
P(recovery | drug) > P(recovery | no drug)

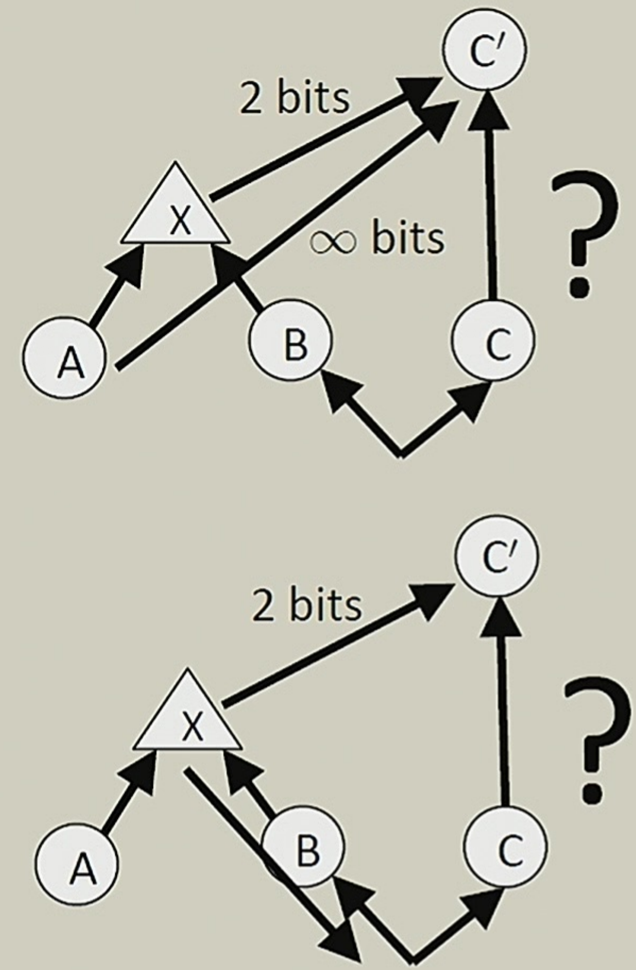

X
Y
A
B
S
T
X
A
B
Y
S
T
?
X
A
B
Y
S
T
?

Quantum teleportation

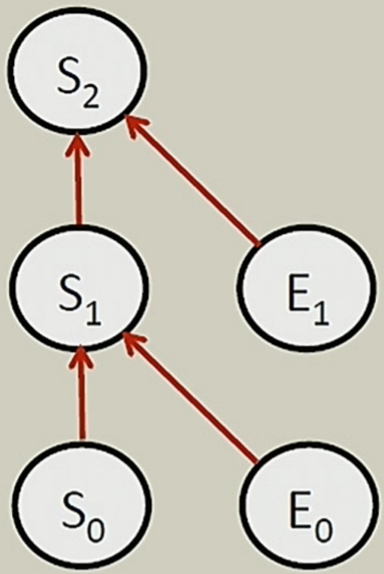


Quantum teleportation

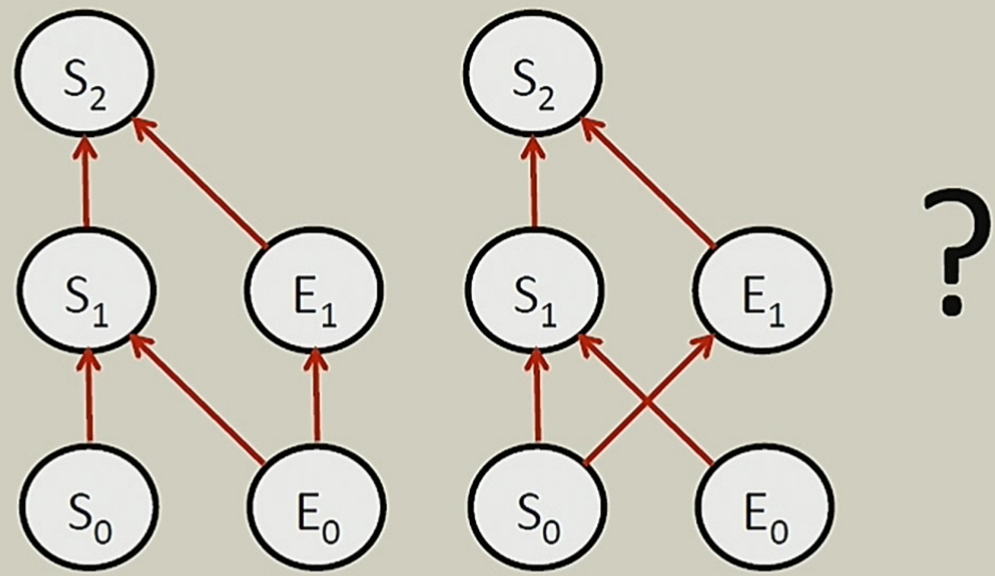




Markovian vs. nonMarkovian dynamics

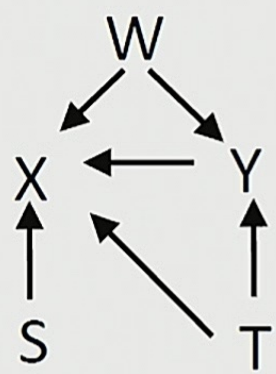


OR



Causal Model

Causal Structure



Statistical Parameters

$P(W)$

$P(S)$

$P(T)$

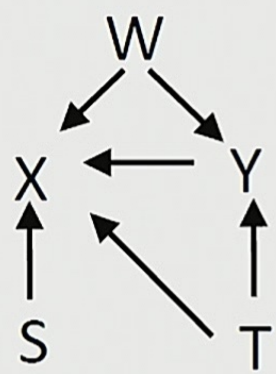
$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal Model

Causal Structure	Statistical Parameters



$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

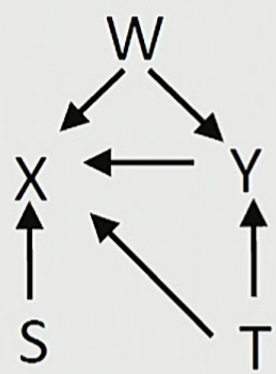
$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

The inverse problem is the "problem of causal inference"

Causal Model

Causal Structure



Statistical Parameters

$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

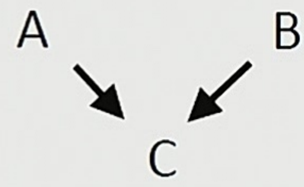
$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

$$P(A, B, C)$$

A is independent of B

$$P(A, B) = P(A)P(B)$$

no other independence relations

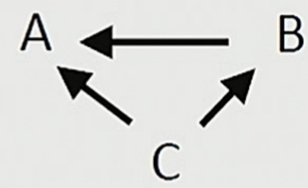


$P(A)$

$P(B)$

$P(C|A, B)$

$$P(A, B, C) = P(C|A, B)P(A)P(B)$$



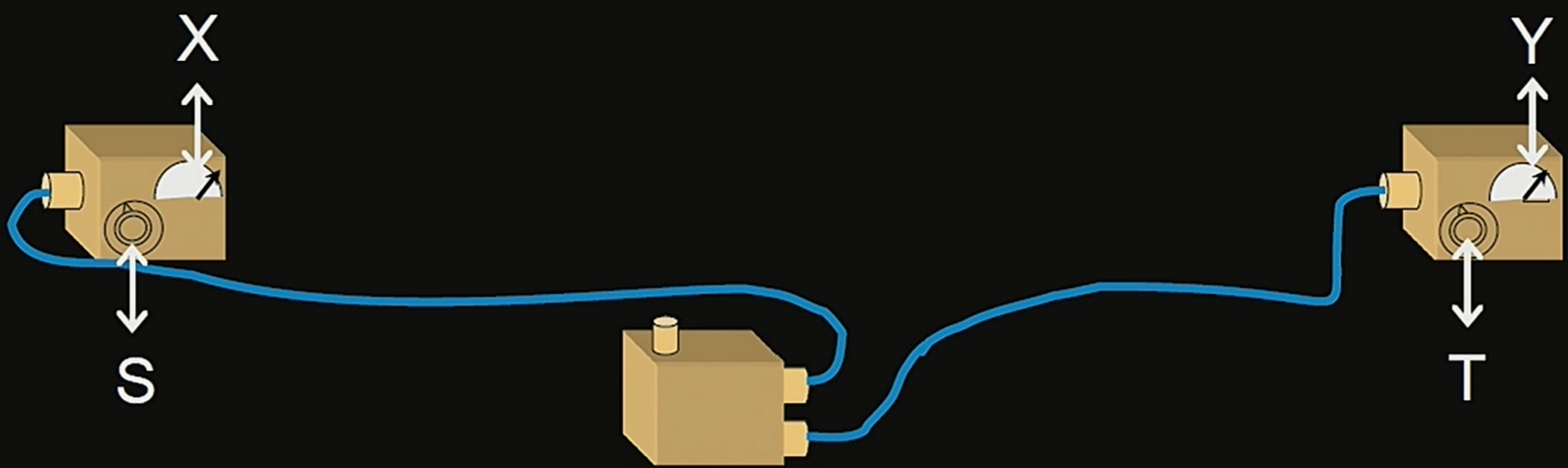
$P(C)$

$P(B|C)$

$P(A|B, C)$

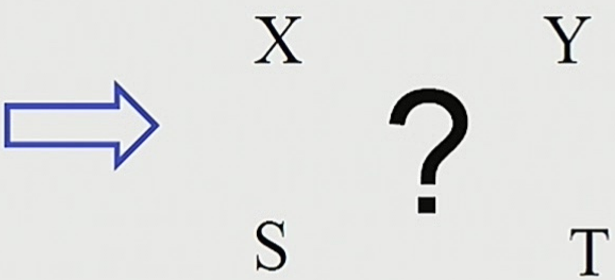
$$P(A, B, C) = P(A|B, C)P(B|C)P(C)$$

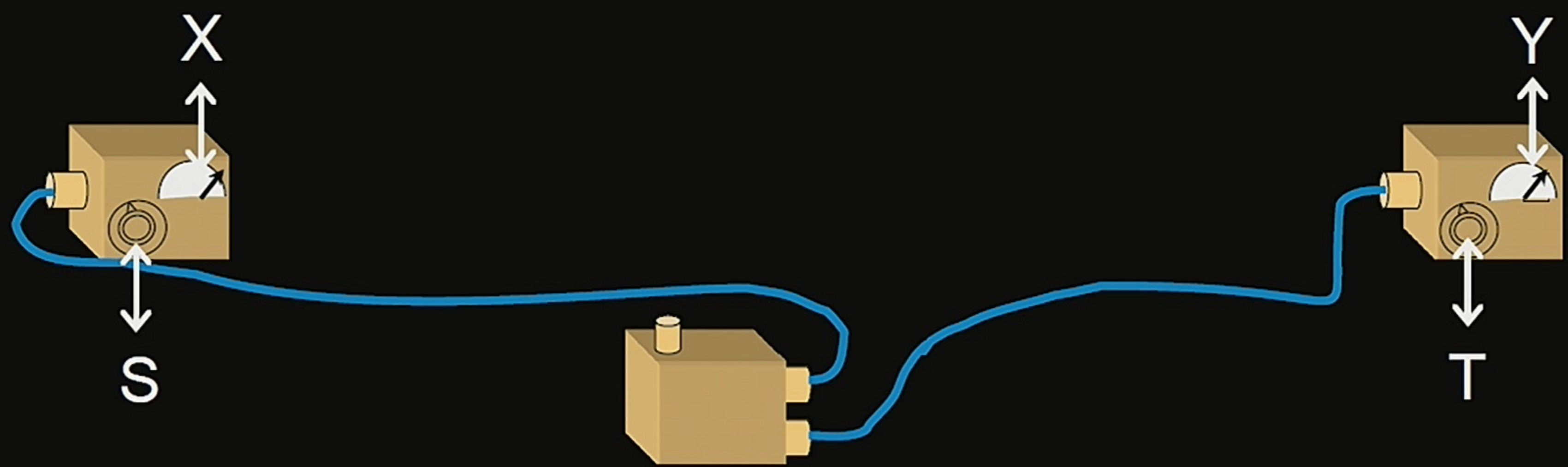
This model is fine-tuned



P(X,Y|S,T)

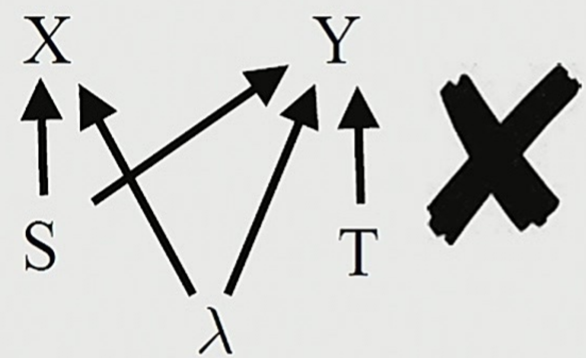
	X=0, Y=0	X=0, Y=1	X=1, Y=0	X=1, Y=1
S=0, T=0	1/2	0	0	1/2
S=0, T=1	1/2	0	0	1/2
S=1, T=0	1/2	0	0	1/2
S=1, T=1	0	1/2	1/2	0





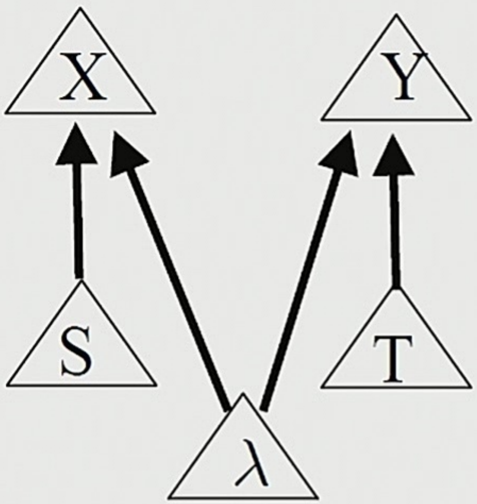
$P(X,Y|S,T)$

	X=0, Y=0	X=0, Y=1	X=1, Y=0	X=1, Y=1
S=0, T=0	1/2	0	0	1/2
S=0, T=1	1/2	0	0	1/2
S=1, T=0	1/2	0	0	1/2
S=1, T=1	0	1/2	1/2	0



Quantum Causal Models

Leifer, RWS (2013)



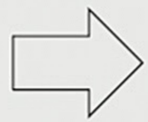
$P(S)$

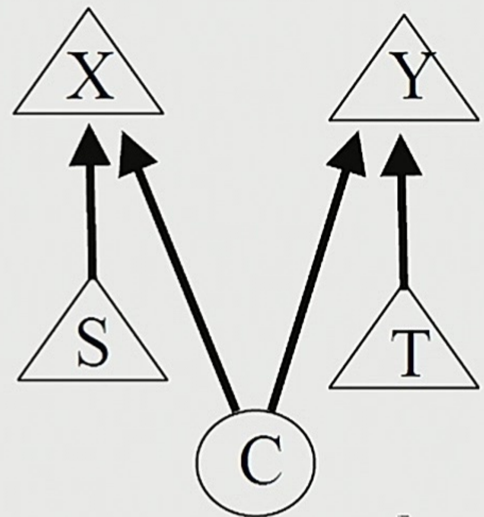
$P(T)$

$P(\lambda)$

$P(X|\lambda, S)$

$P(Y|\lambda, T)$





ρ_S

ρ_T

ρ_C

$\rho_{X|SC}$

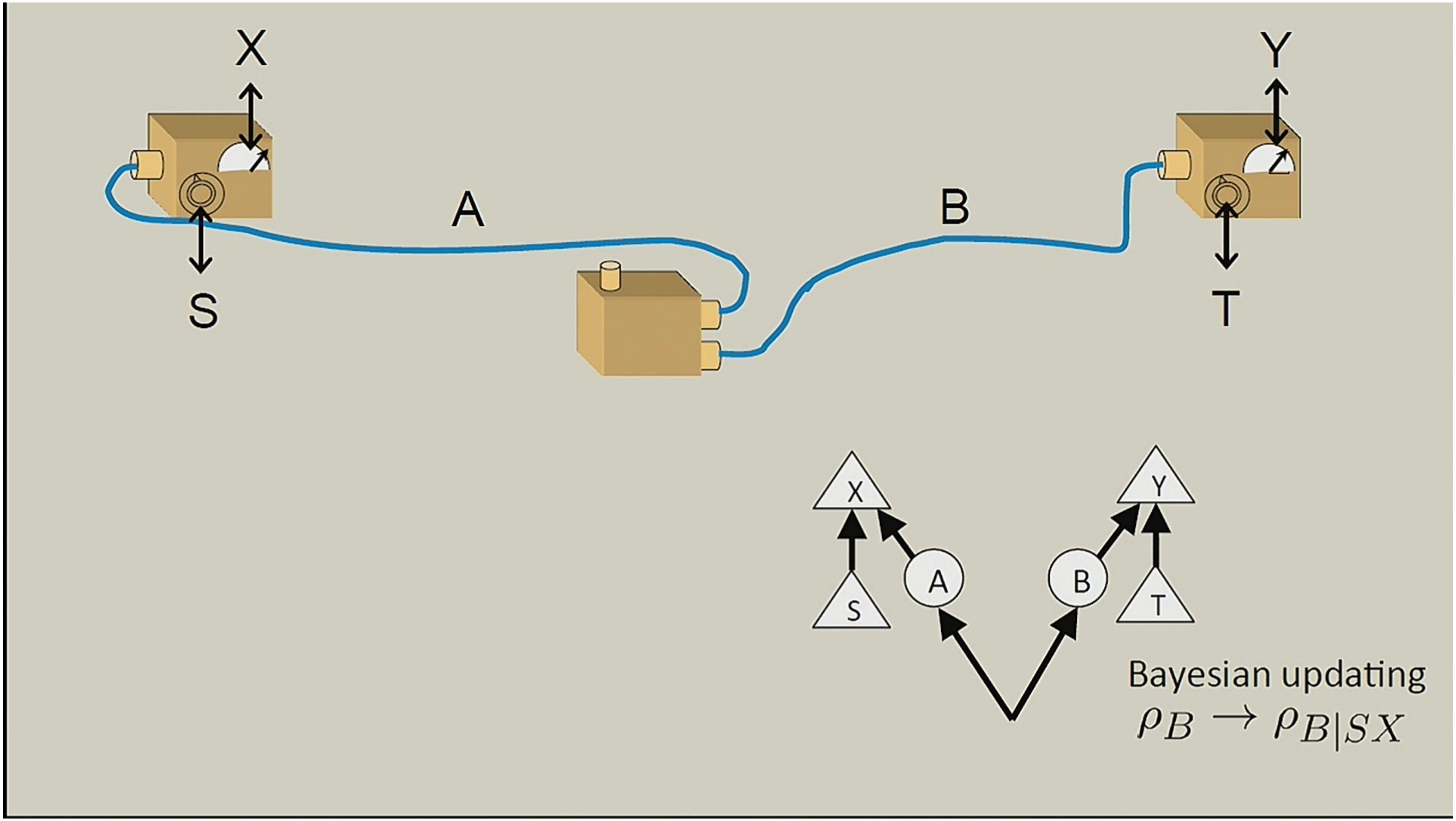
$\rho_{Y|TC}$

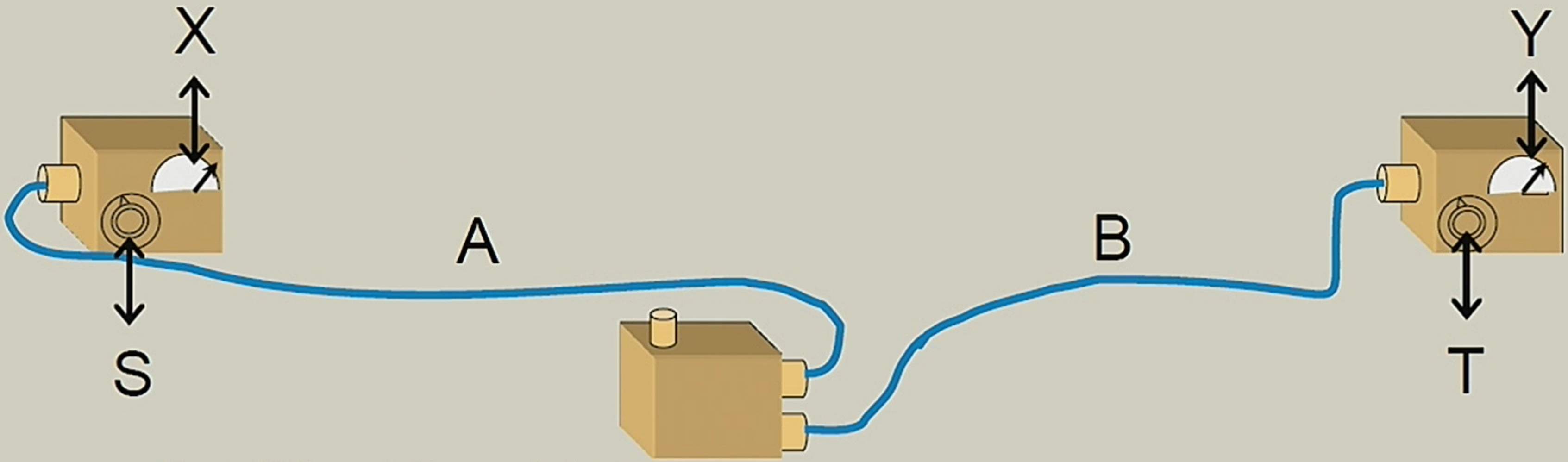
where

$[\rho_{X|SC}, \rho_{Y|TC}] = 0$

$$P(X,Y|S,T) = \sum_\lambda P(X|S,\lambda)P(Y|T,\lambda)P(\lambda)$$

$$P(X,Y|S,T) = \mathrm{Tr}_C(\rho_{X|SC}\rho_{Y|TC}\rho_C)$$


X
A
B
Y
S
T
X
Y
A
B
S
T
Bayesian updating
$\rho_B \to \rho_{B|SX}$



Conditional from joint

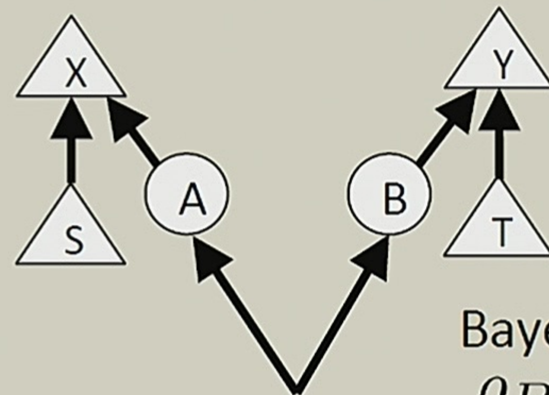
$$\rho_{B|A} = \rho_{AB} \star \rho_A^{-1}$$

Bayesian inversion

$$\rho_{A|XS} = \rho_{X|AS} \star \rho_A \rho_{X|S}^{-1}$$

Belief propagation

$$\rho_{B|SX} = \mathrm{tr}_A(\rho_{B|A}\rho_{A|SX})$$



Given:

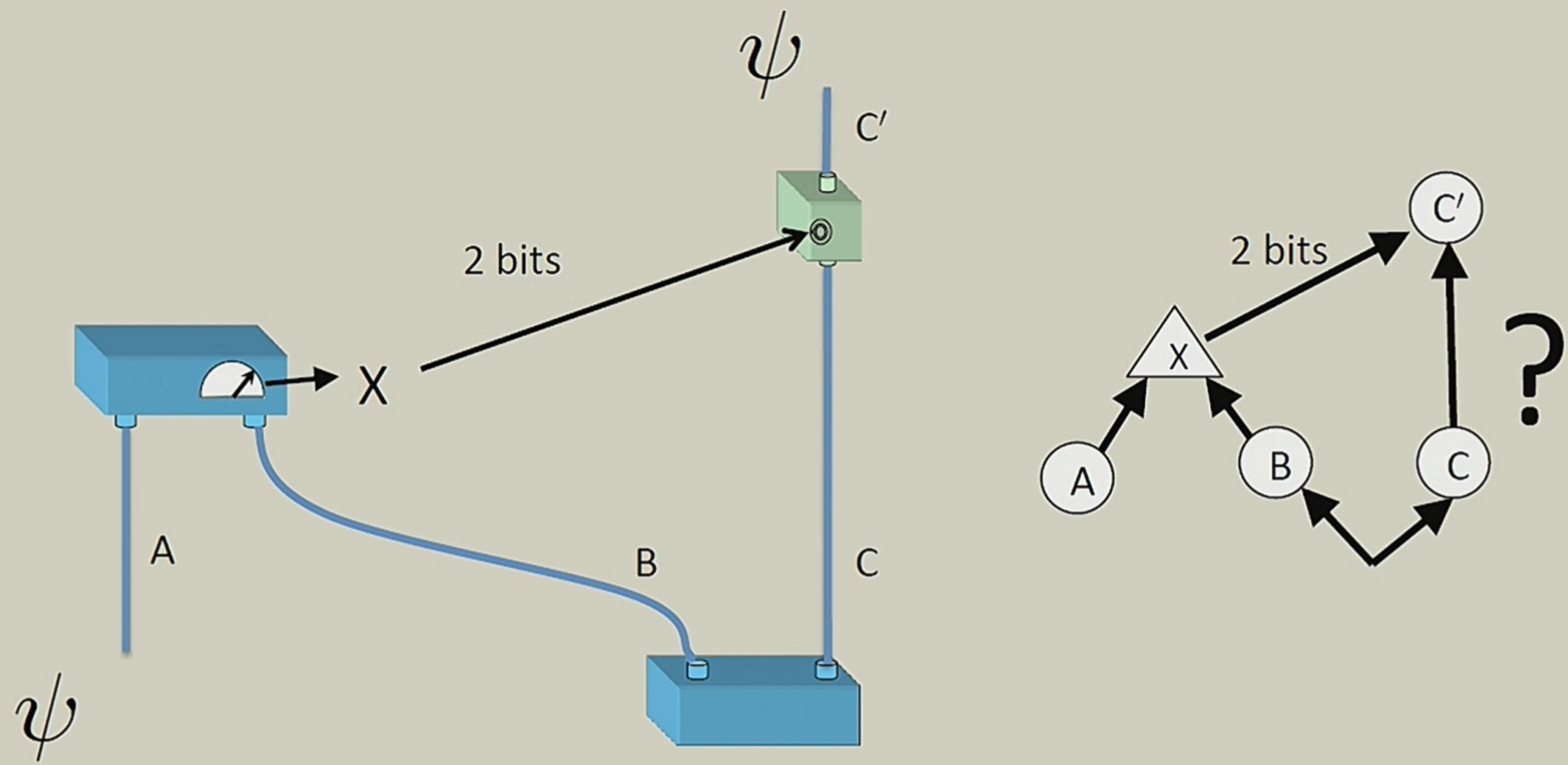
ρ_{AB}

$\rho_{X|SA}$

Bayesian updating

$$\rho_B \rightarrow \rho_{B|SX}$$

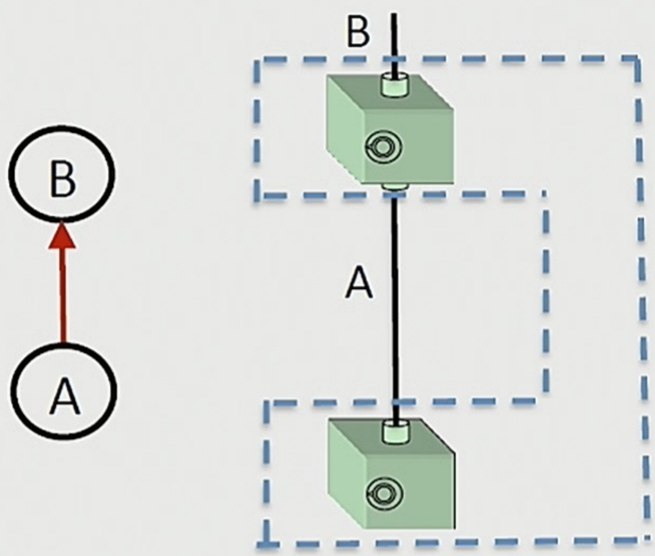
Quantum teleportation



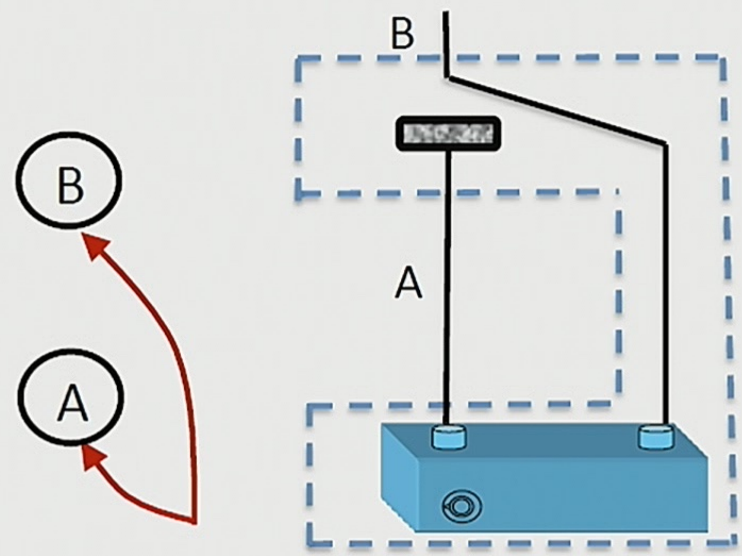
Quantum advantage for causal inference

Ried, Agnew, Vermeyden, Janzing, RWS, Resch (2015)

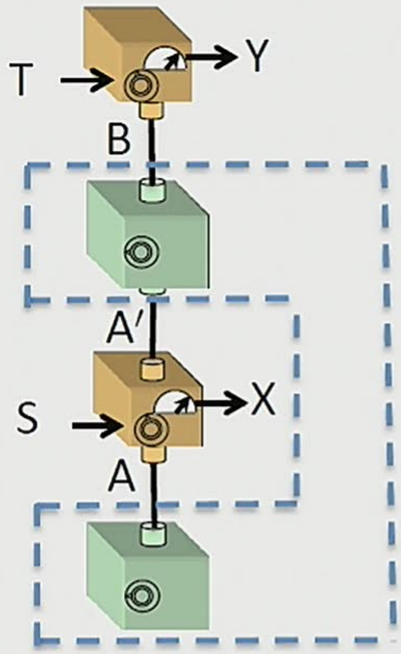
Direct cause



Common cause



Classical



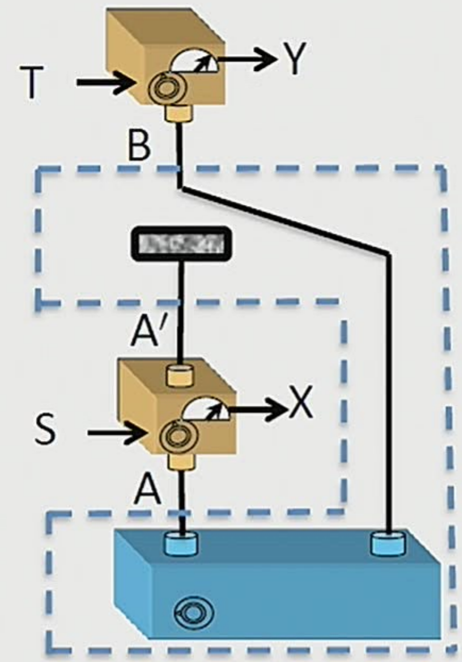
Same scope of possibilities for conditionals

$$\{p(B|A')\} = \{p(B|A)\}$$

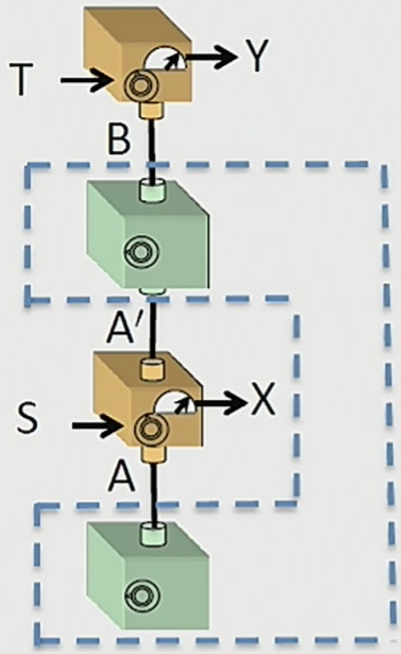
For probing schemes with informational symmetry

$$\forall S, X:\ p(A'|S,X) = p(A|S,X)$$

→ Same scope of possibilities for correlations

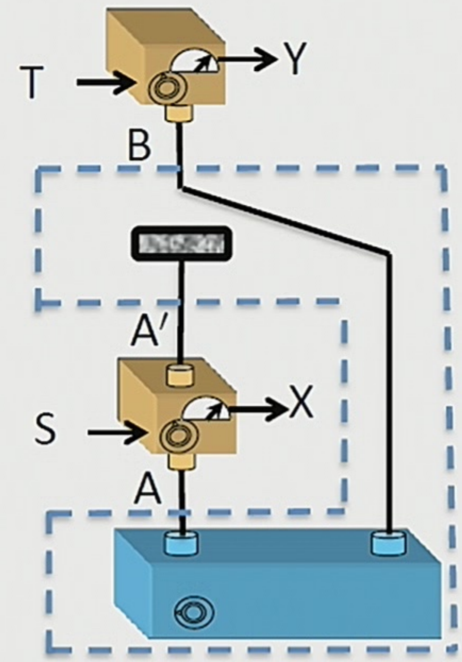


Quantum



Different scope of possibilities for conditionals

$$\{\rho_{B|A'}\} \neq \{\rho_{B|A}\}$$

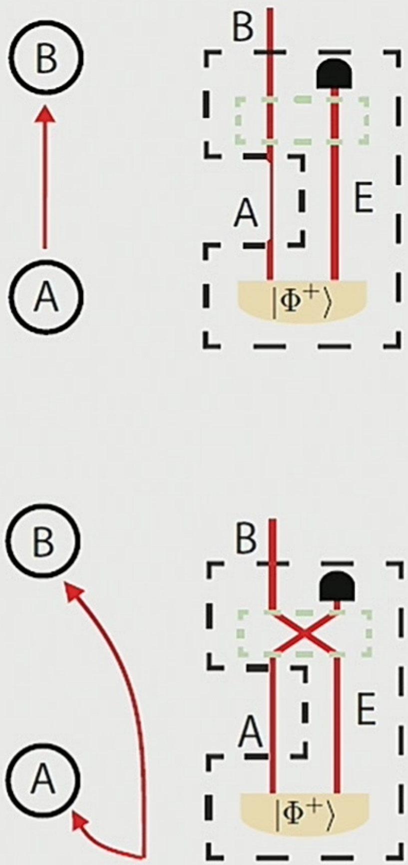


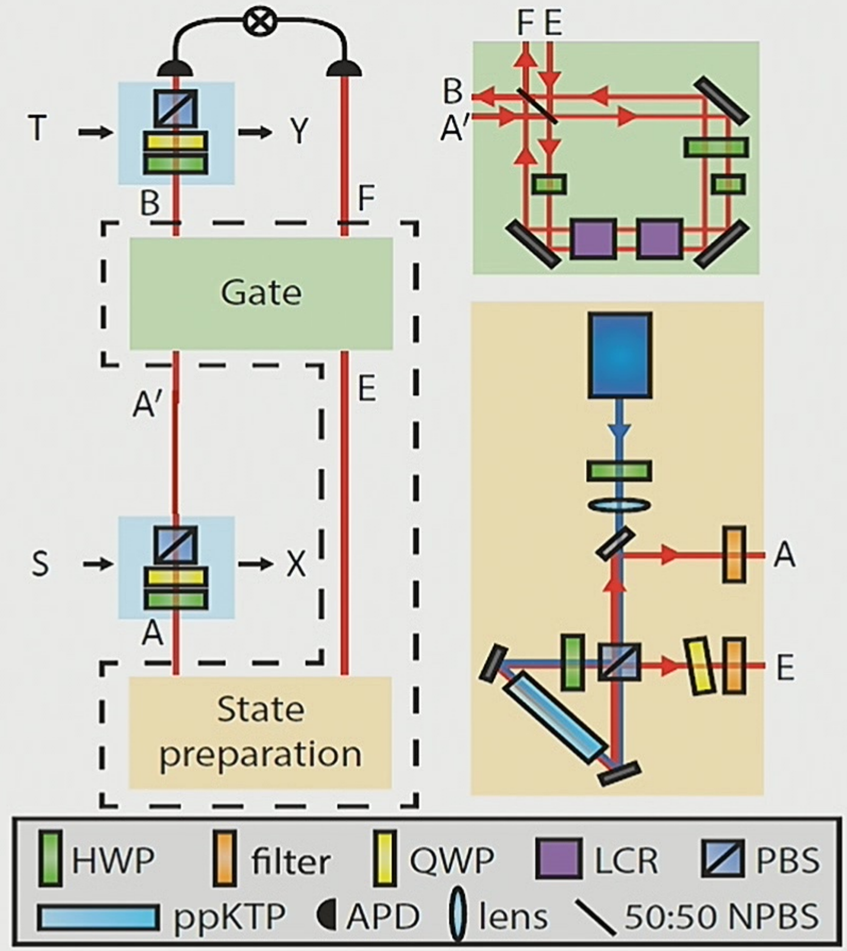
For probing schemes with informational symmetry

$$\forall S, X: \; \rho_{A|SX} = \rho_{A'|SX}$$

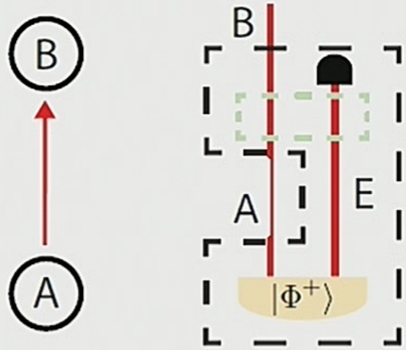
→ Different scope of possibilities for correlations

Experimental set-up

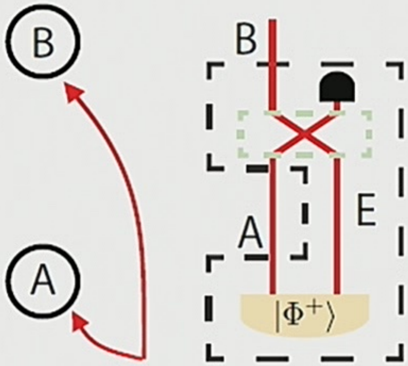




prob. (1-p)

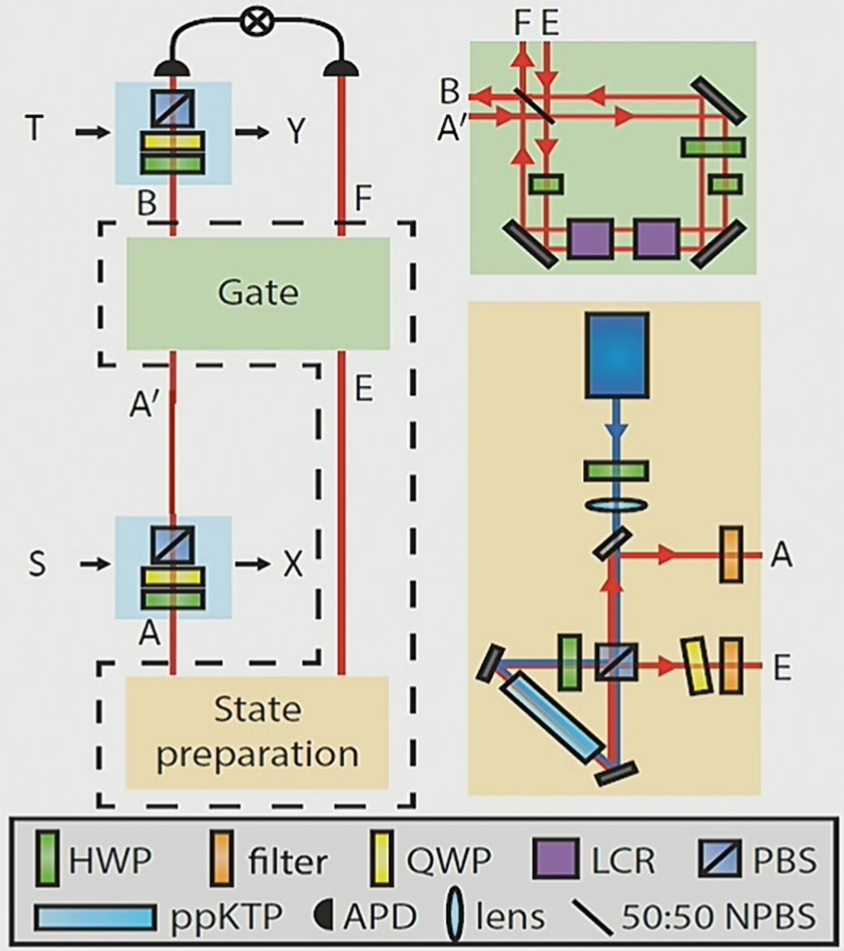


prob. p

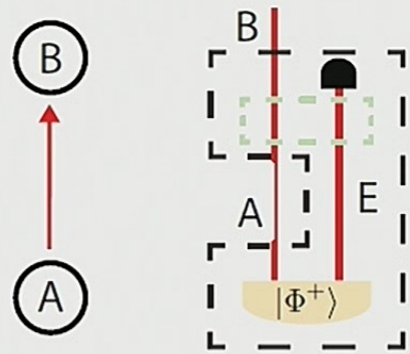


$p \in [0,1]$ a controlled parameter

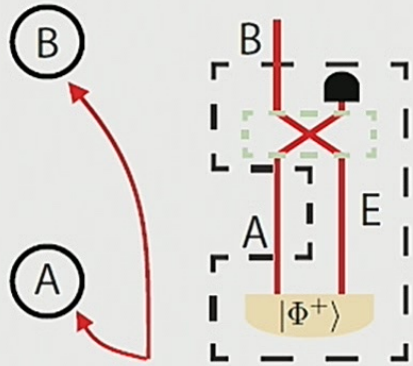
Experimental set-up



prob. (1-p)

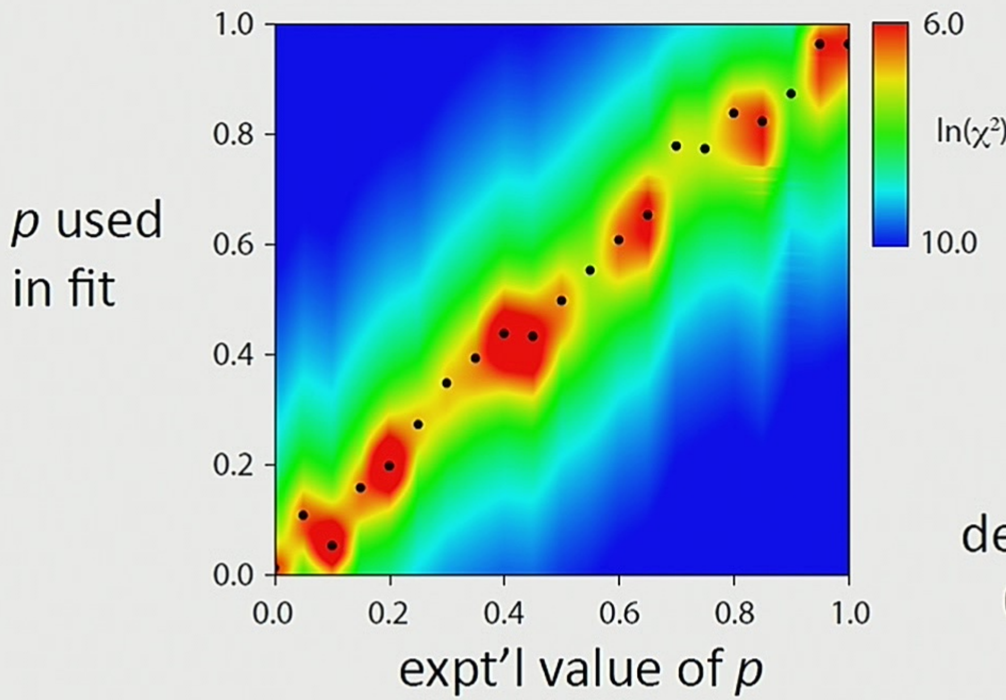


prob. p



$p \in [0, 1]$ a controlled parameter

Experimental Results



rms
deviation
0.032