Title: Aharonov meets Spekkens: What do quantum paradoxes tell us about the nature of reality?

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Abstract:

Aharonov meets Spekkens: What do quantum logical pre- and post-selection paradoxes tell us about the nature of reality?

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Perimeter Institute

24th June 2015

Aharonov

LPPS paradoxes
BS Contextuality
Non-BS contextual model
AS Contextuality
Discussion and Conclusions

- "Progress through paradox"[a]:
  - Three box paradox
  - Quantum pigeonhole principle
  - Quantum Cheshire cats
  - Anomalous weak values
  - Protective measurement

[a] Y. Aharonov and D. Rohrlich, "Quantum Paradoxes" (Wiley, 2005).

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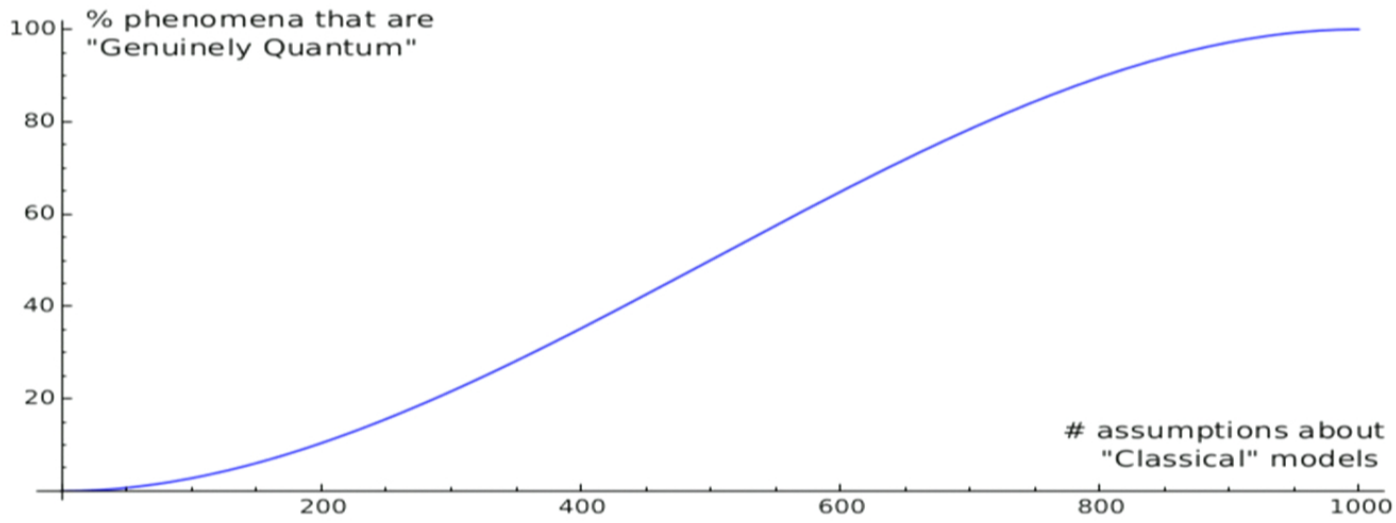
"Classical" "Quantum"



Three box paradox

LPPS paradoxes
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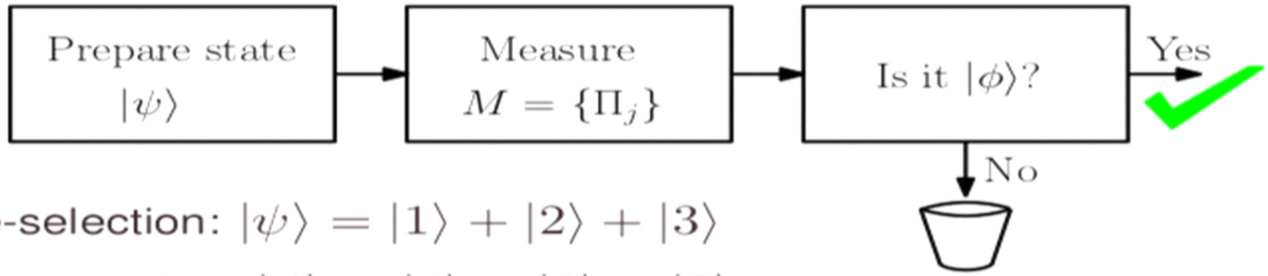

- Pre-selection: $|\psi\rangle = |1\rangle + |2\rangle + |3\rangle$
- Post-selection: $|\phi\rangle = |1\rangle + |2\rangle - |3\rangle$
- Two possible intermediate measurements:
  - M_1: Is ball in box 1? $\Pi_1 = |1\rangle\langle 1|, \quad \Pi_{2\vee 3} = |2\rangle\langle 2| + |3\rangle\langle 3|$

    $$\mathbb{P}(\Pi_1|\psi, M_1, \phi) = 1$$

  - M_2: Is ball in box 2? $\Pi_2 = |2\rangle\langle 2|, \quad \Pi_{1\vee 3} = |1\rangle\langle 1| + |3\rangle\langle 3|$

    $$\mathbb{P}(\Pi_2|\psi, M_2, \phi) = 1$$

Y. Aharonov and L. Vaidman, *J. Phys. A* 24 pp. 2315–2328 (1991).

Before Spekkens (BS) Noncontextuality

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BS Noncontextuality
Clifton's proof
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- *Outcome determinism*: At any given time, the system has a definite value for every observable.
  - For every orthonormal basis $\{|\psi_j\rangle\}$, precisely one of them is asigned the value 1, the rest 0.
- *Noncontextuality*: The outcome assigned to an observable does not depend on which other (commuting) observables it is measured with.
  - The value assigned to a basis vector does not depend on which basis it occurs in, e.g.

$$|1\rangle\,,\,|2\rangle\,,\,|3\rangle$$

vs.

$$|1\rangle\,,\,|2\rangle + |3\rangle\,,\,|2\rangle - |3\rangle\,.$$

S. Kochen and E. Specker, *J. Math. Mech.* 1 pp. 59–87 (1967).

Clifton's contextuality proof

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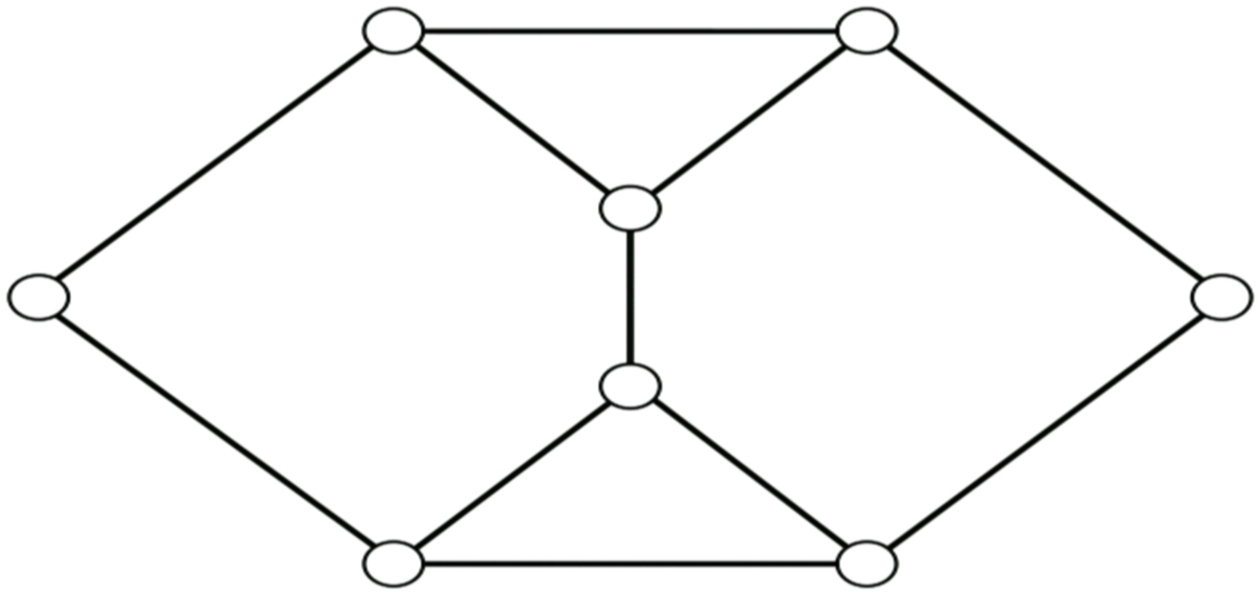

R. Clifton, *Am. J. Phys.* 61 443 (1993).

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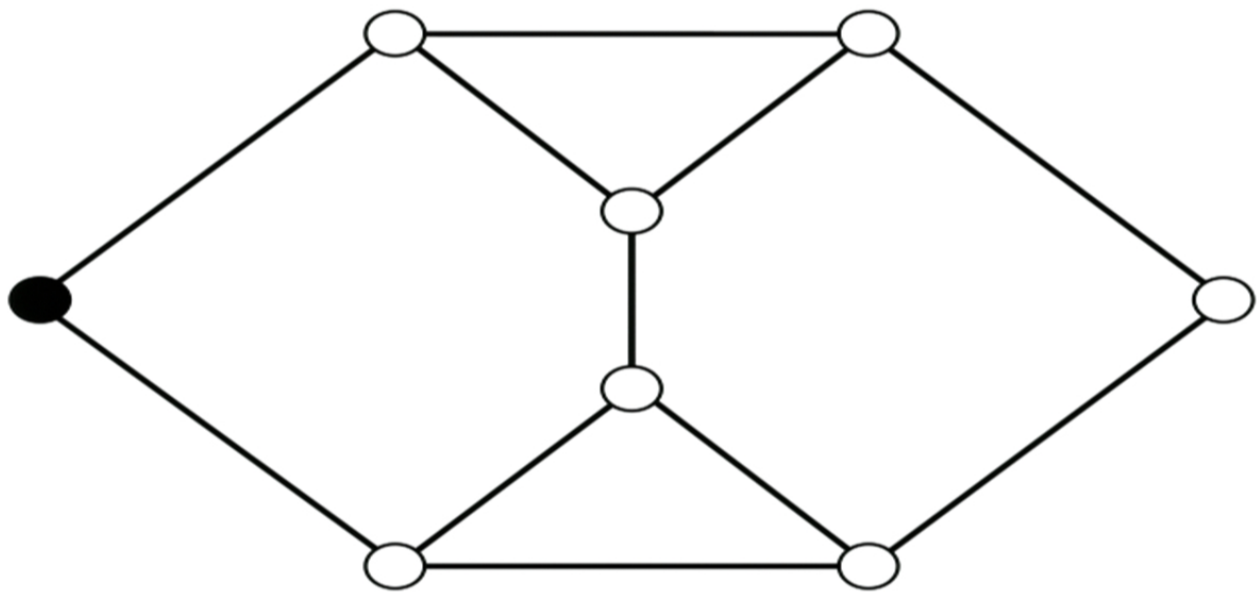

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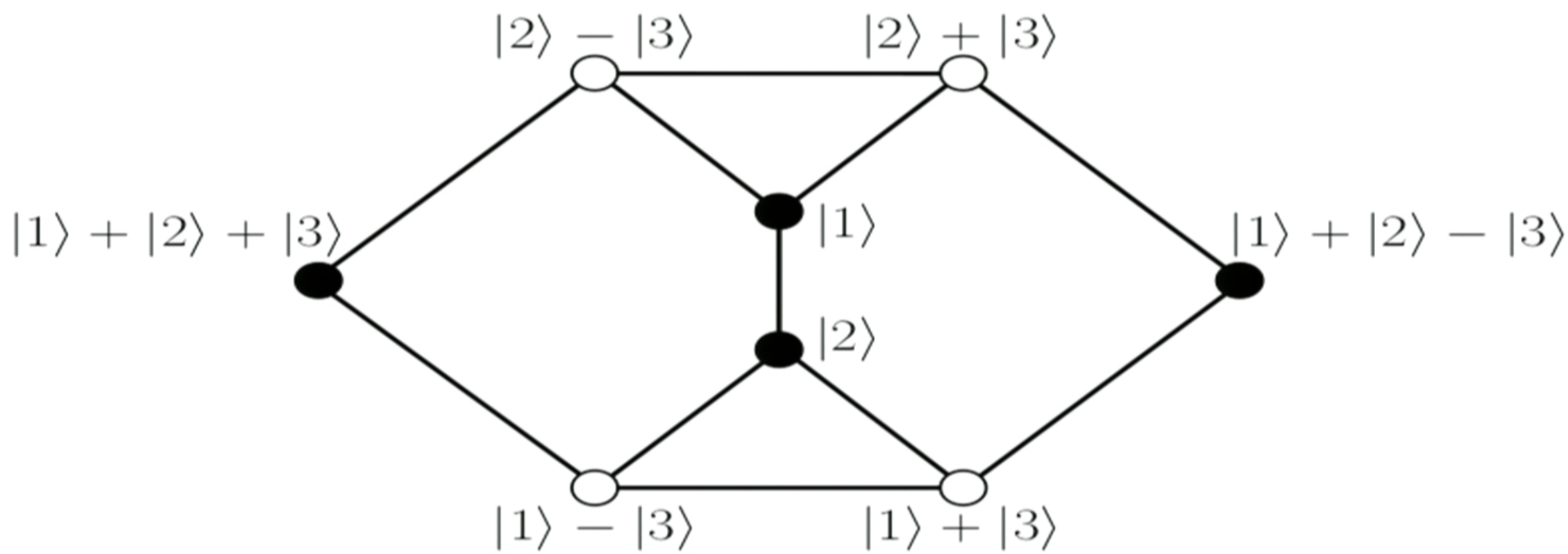

- All logical pre- and post-selection paradoxes are related to a proof of (BS) contextuality in the same way[3].

R. Clifton, *Am. J. Phys.* 61 443 (1993).

[3]M. Leifer and R. Spekkens, *Phys. Rev. Lett.* 95 200405 (2005).

Clifton's contextuality proof

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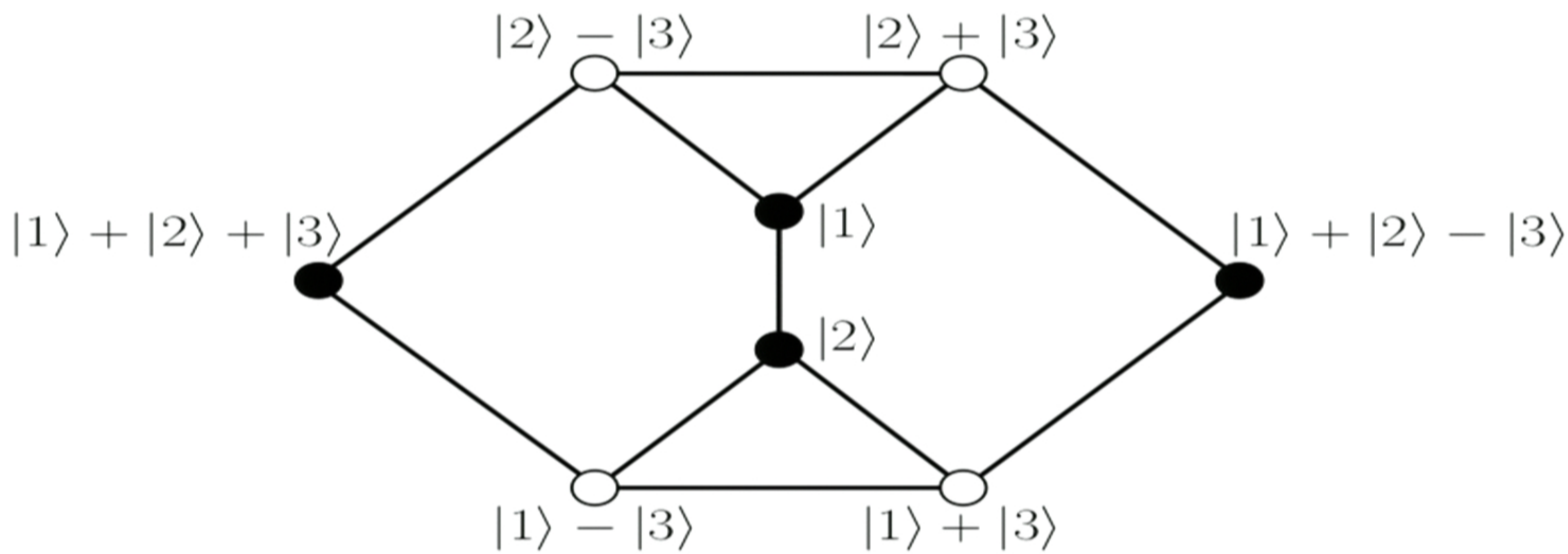

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A non-BS contextual model

The partitioned box paradox

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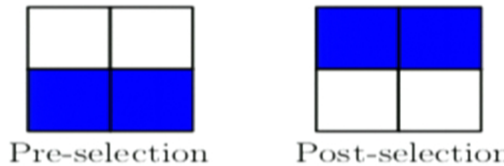
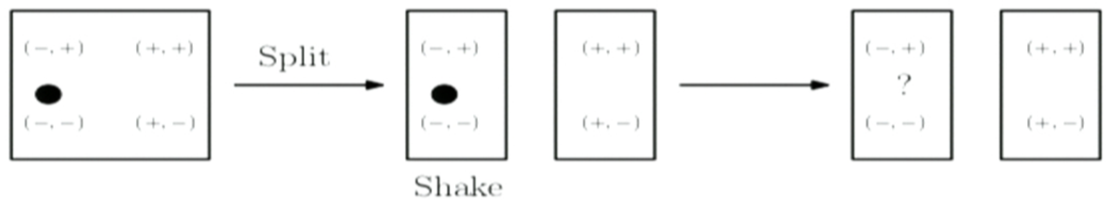


- "Left"-measurement:



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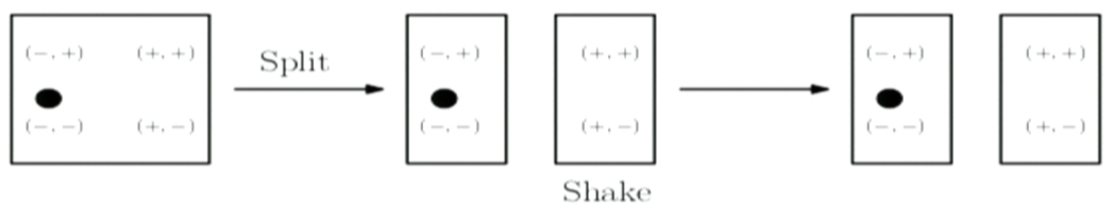
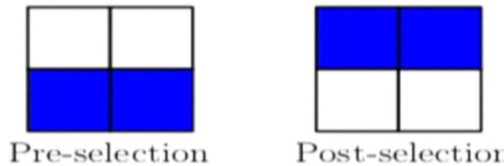


M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

The partitioned box paradox



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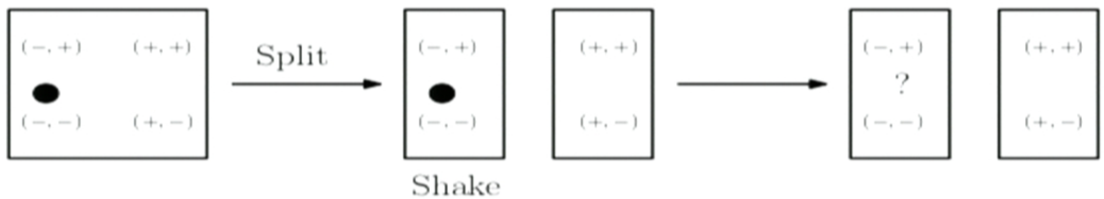

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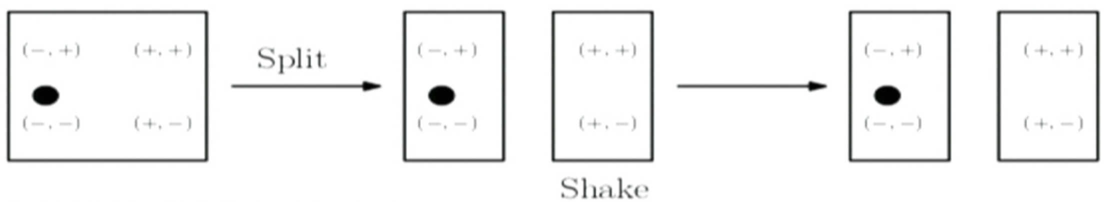

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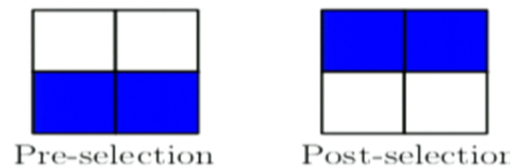

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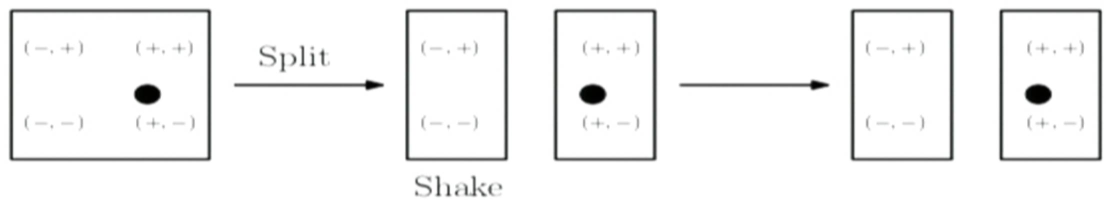

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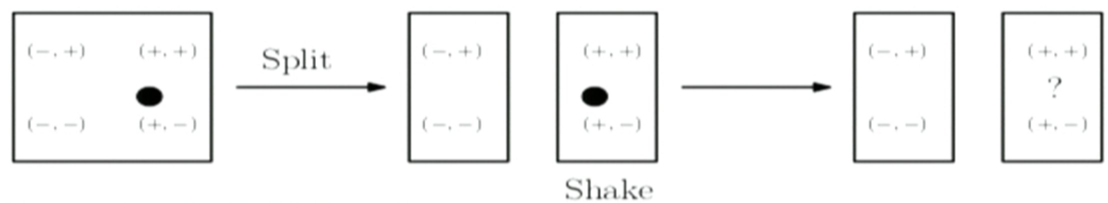

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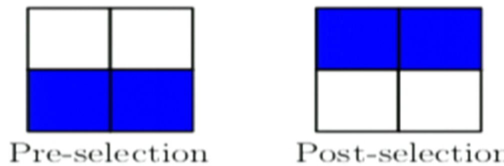

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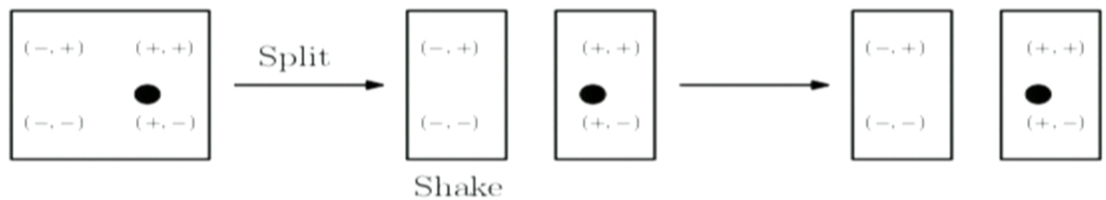

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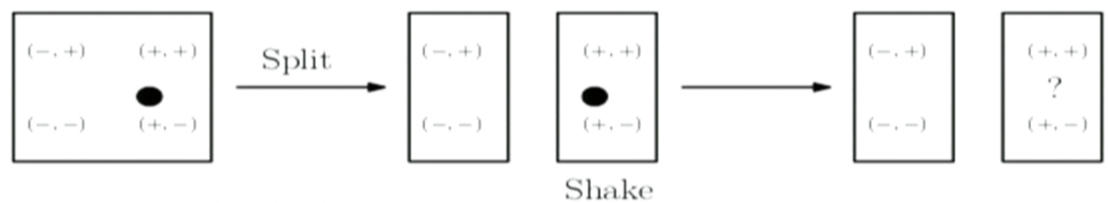
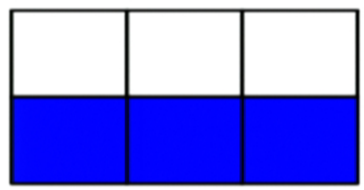


M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

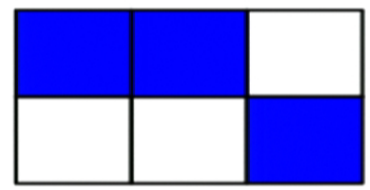
The partitioned box paradox

- We can reproduce the predictions of the three-box paradox exactly by adding more states and changing the update rule.

  - New pre- and post-selection:

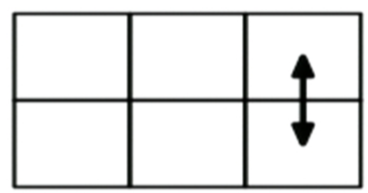


Pre-selection



Post-selection

  - Add this to state-update rule:



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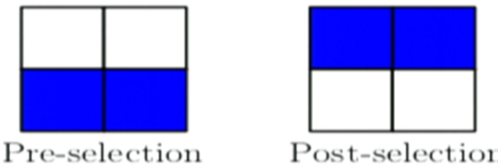

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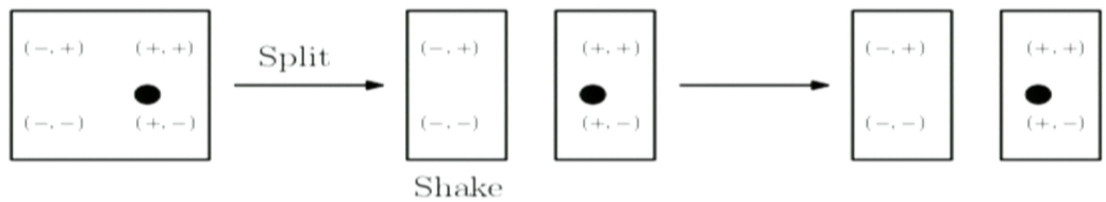

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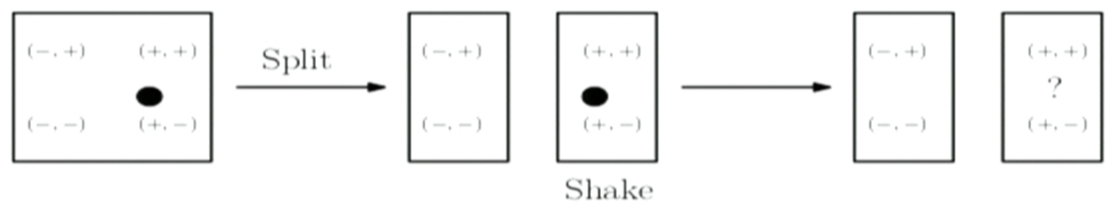

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After Spekkens Contextuality

After Spekkens (AS) Noncontextuality

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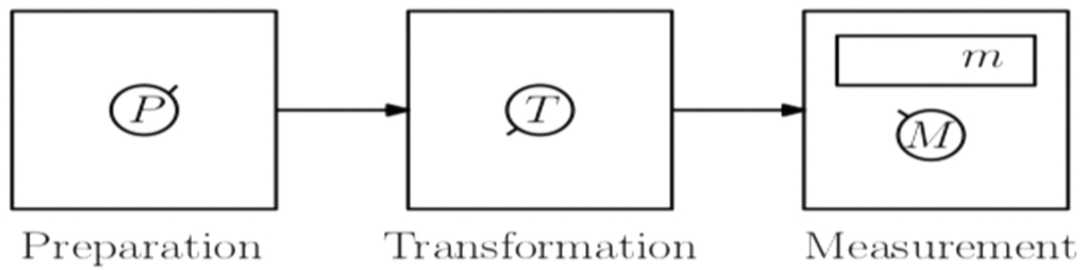
- Operational theory:



$$\mathbb{P}(m|P, M, T)$$

- In quantum theory:

$$\mathbb{P}(m|P, M, T) = \mathrm{Tr}\left(E_m^M \mathcal{E}_T(\rho_P)\right)$$

R. Spekkens, *Phys. Rev. A* 71:052108 (2005).

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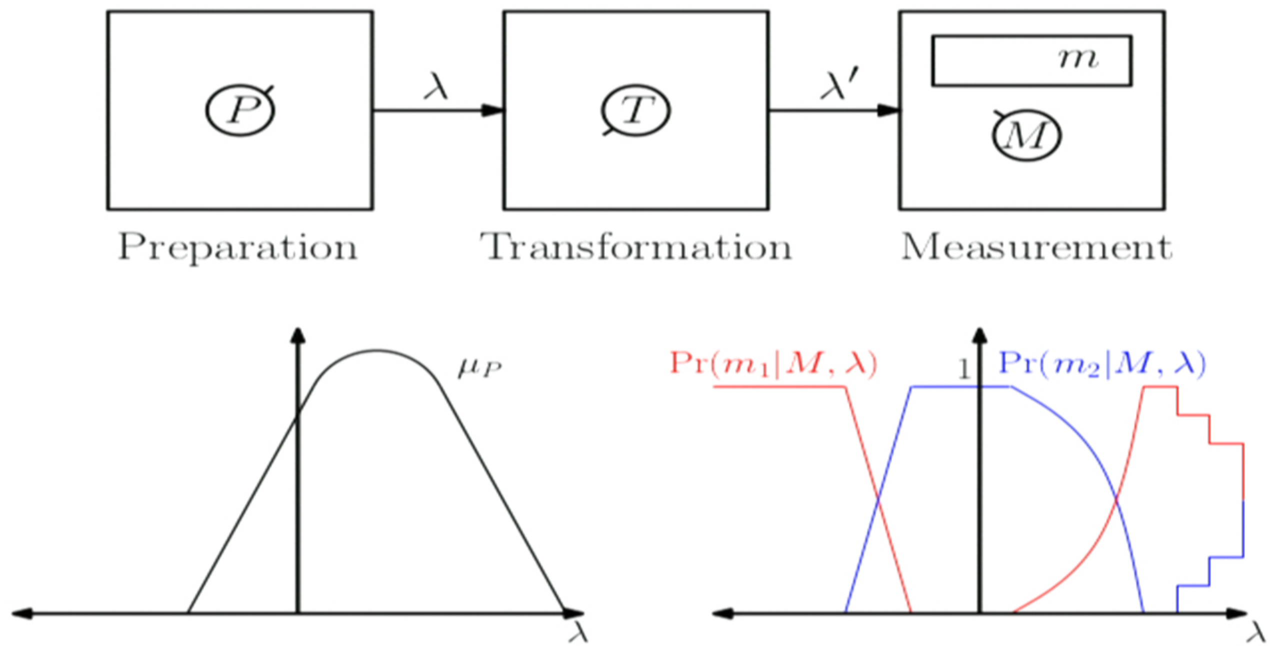
Ontological models



$$\mathbb{P}(m|P,M,T) = \int_{\Lambda'}\int_{\Lambda} \Pr(m|M,\lambda')d\Gamma_T(\lambda'|\lambda)d\mu_P(\lambda)$$

Implications for state-update rules

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Theorem. *Let* $\{\Pi_j\}$ *be a projective measurement and let* $\mathcal{E}$ *be the nonselective state-update rule*

$$\mathcal{E}(\rho) = \sum_j \Pi_j \rho \Pi_j .$$

Then,

$$\mathcal{E}(\rho) = p\rho + (1-p)\mathcal{C}(\rho),$$

where $\mathcal{C}$ *is a completely-positive, trace-preserving map and* $0 < p \leq 1$.

- Proof for special case $\{\Pi_1, \Pi_2\}$:

$$U_1 = \Pi_1 + \Pi_2 = I \qquad\qquad U_2 = \Pi_1 - \Pi_2$$

$$\mathcal{E}(\rho) = \frac{1}{2}U_1\rho U_1^\dagger + \frac{1}{2}U_2\rho U_2^\dagger = \frac{1}{2}\rho + \frac{1}{2}U_2\rho U_2^\dagger .$$

Proof of contextuality

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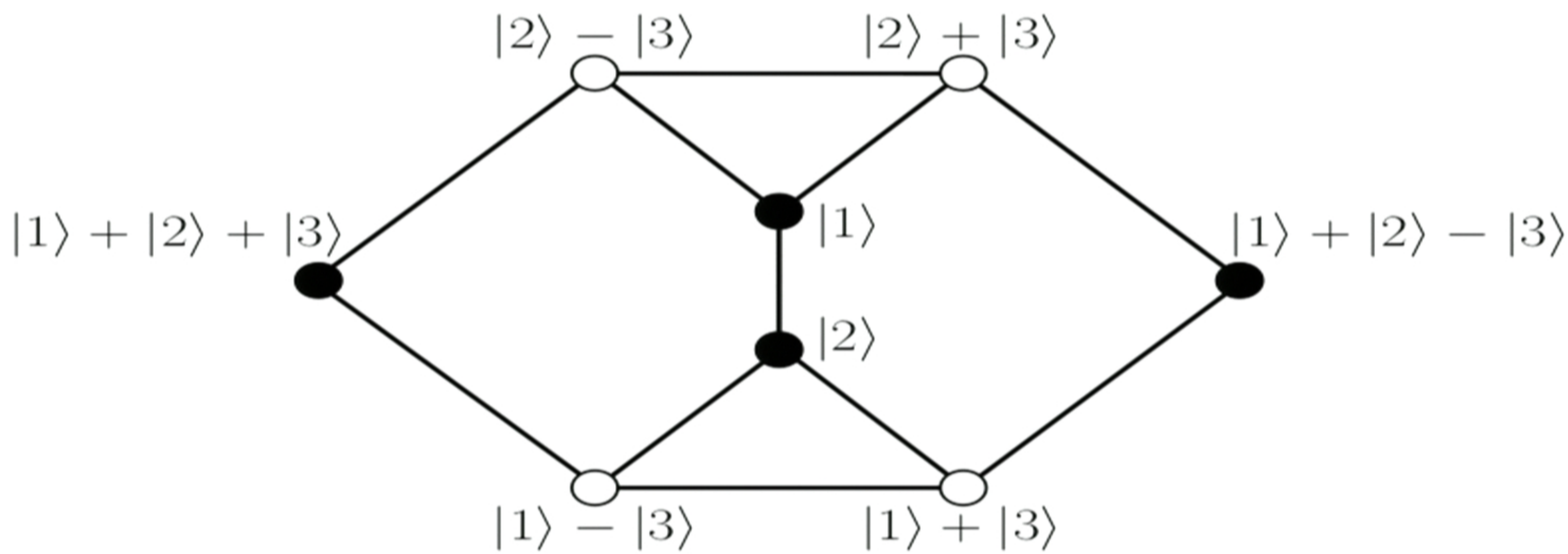

- All logical pre- and post-selection paradoxes are proofs of (PS) contextuality in a similar way.

Transformation noncontextuality

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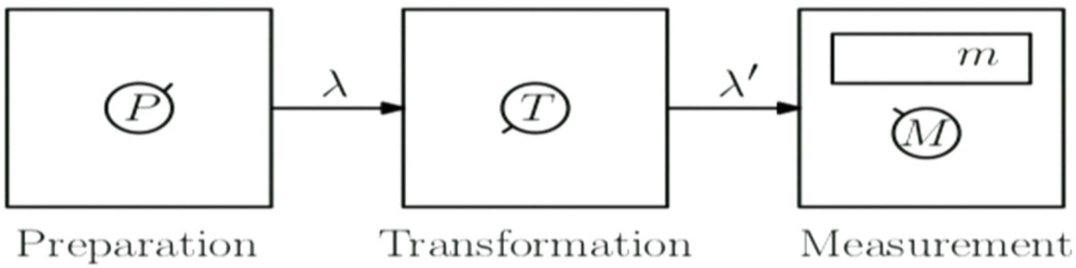

Definition. An ontological model is *transformation noncontextual* if, whenever

$$\mathbb{P}(m|P,M,T) = \mathbb{P}(m|P,M,S)$$

for all P, M, m, we have

$$\Gamma_T = \Gamma_S.$$

- In quantum theory, Γ_T only depends on $\mathcal{E}_T$.

Proof of contextuality

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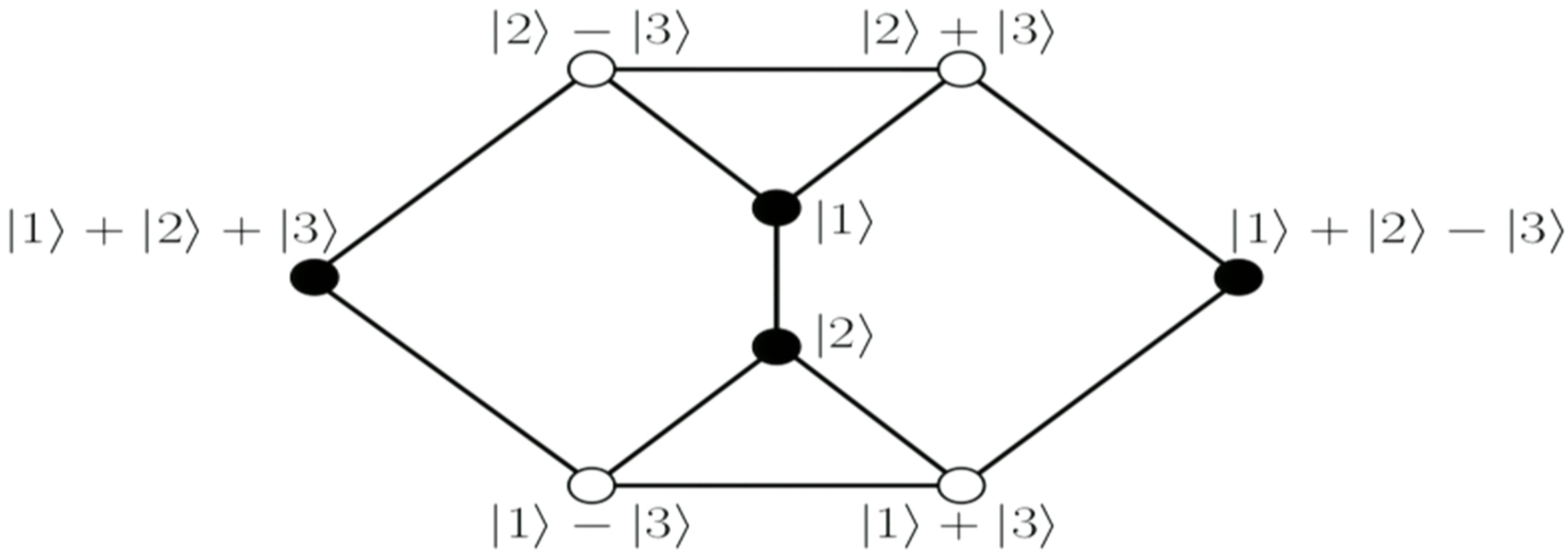

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Conclusions

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Weak measurements

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
  - Specifying assumptions for “classical” models.
  - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
  - Understandable in an AS noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
  - AS Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum”.

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