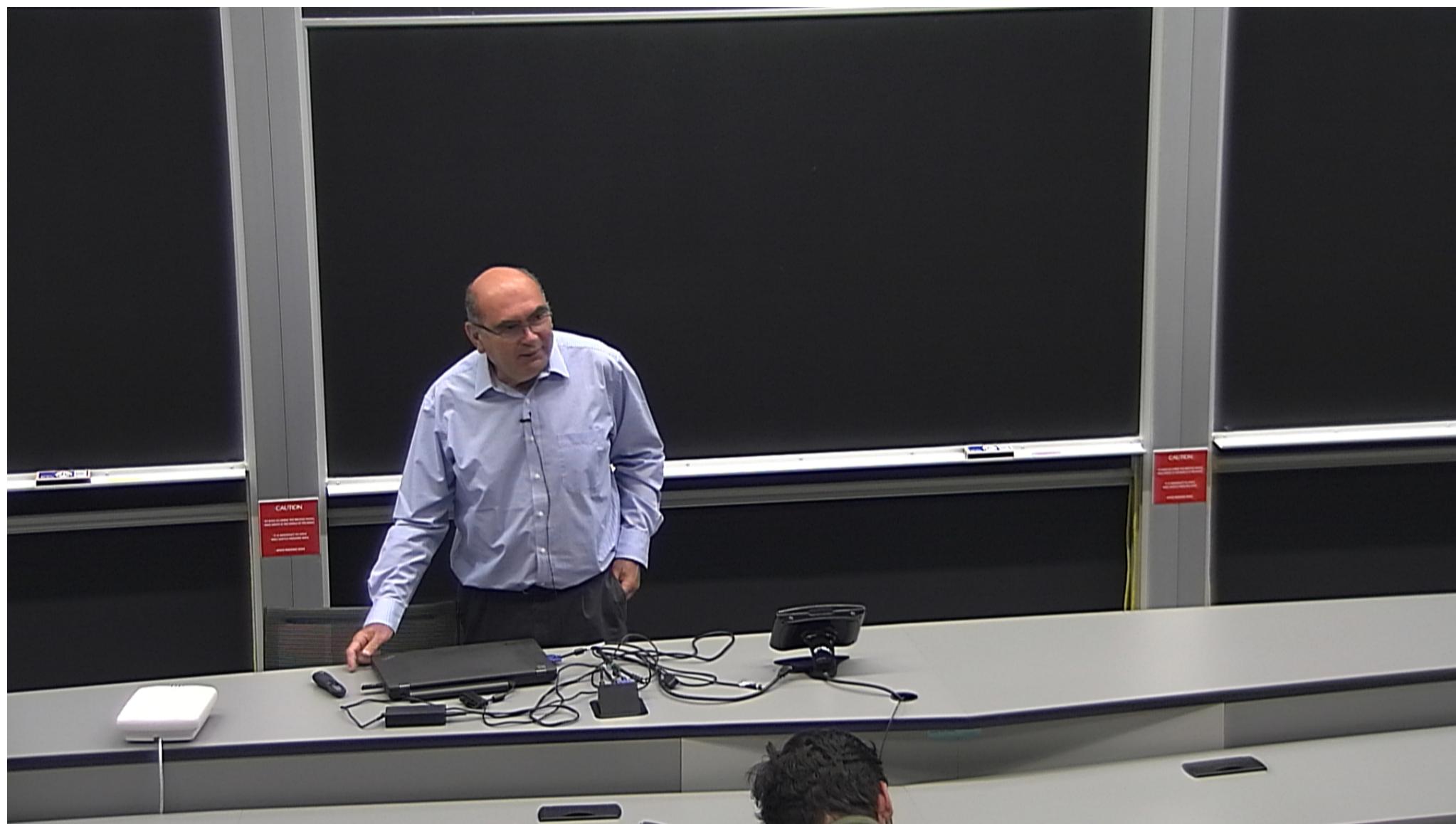


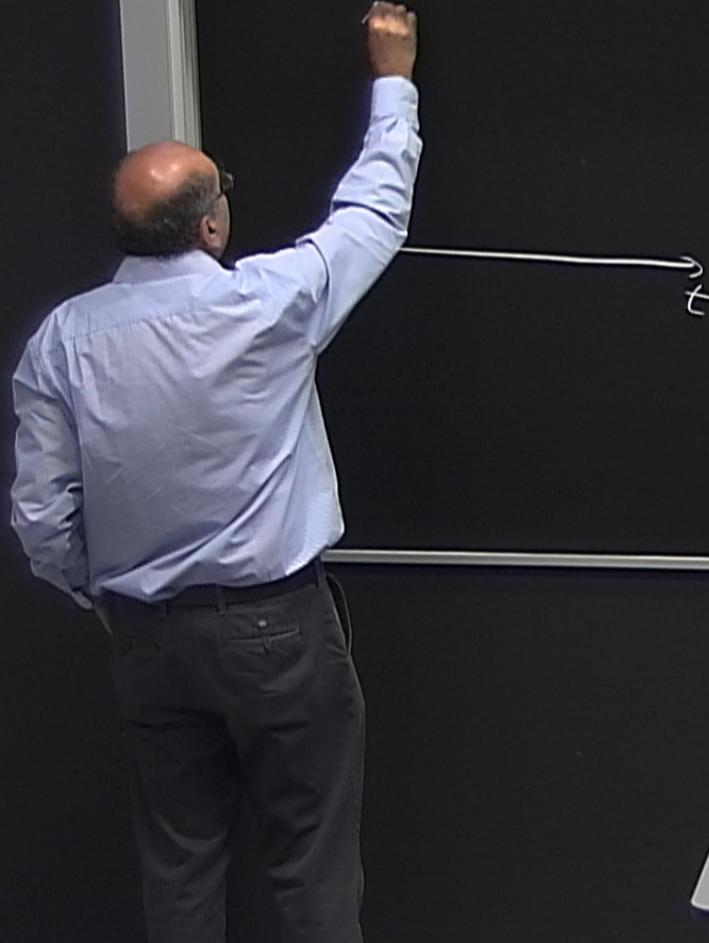
Title: Every Moment of Time a New Universe

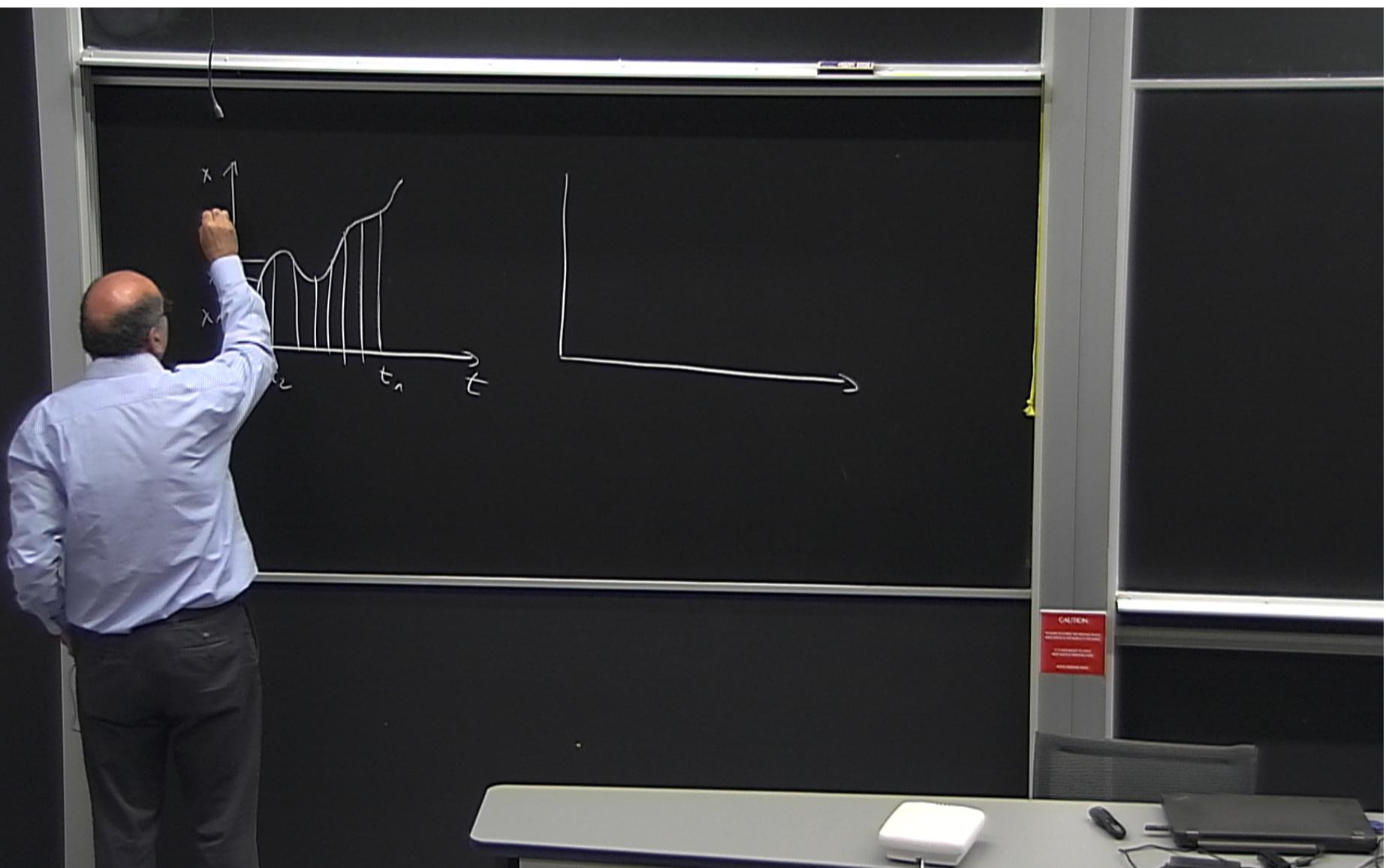
Date: Jun 24, 2015 02:00 PM

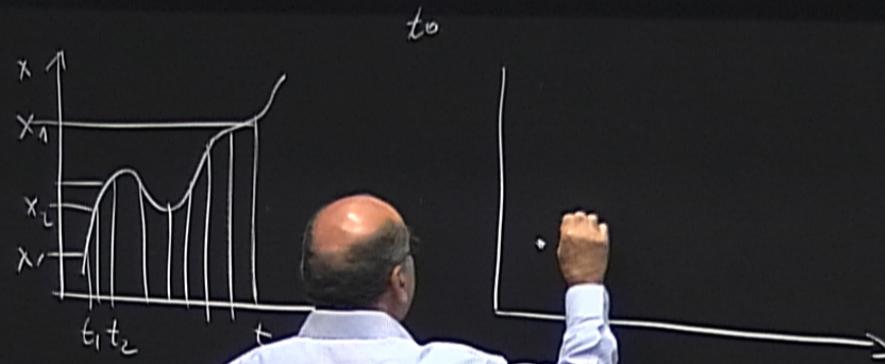
URL: <http://pirsa.org/15060035>

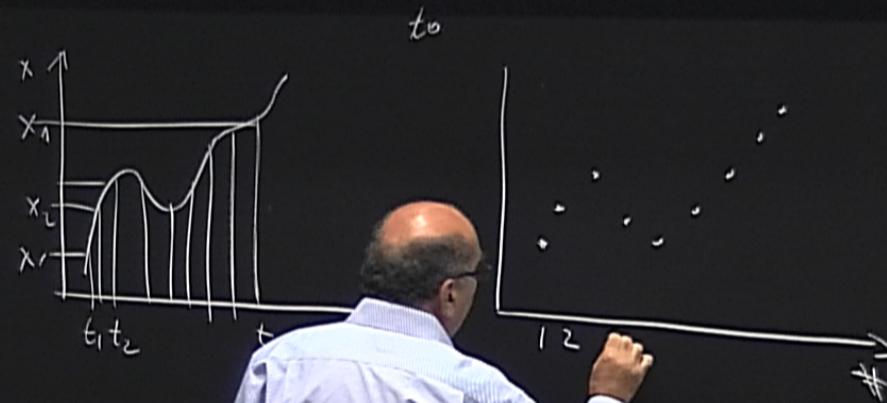
Abstract:



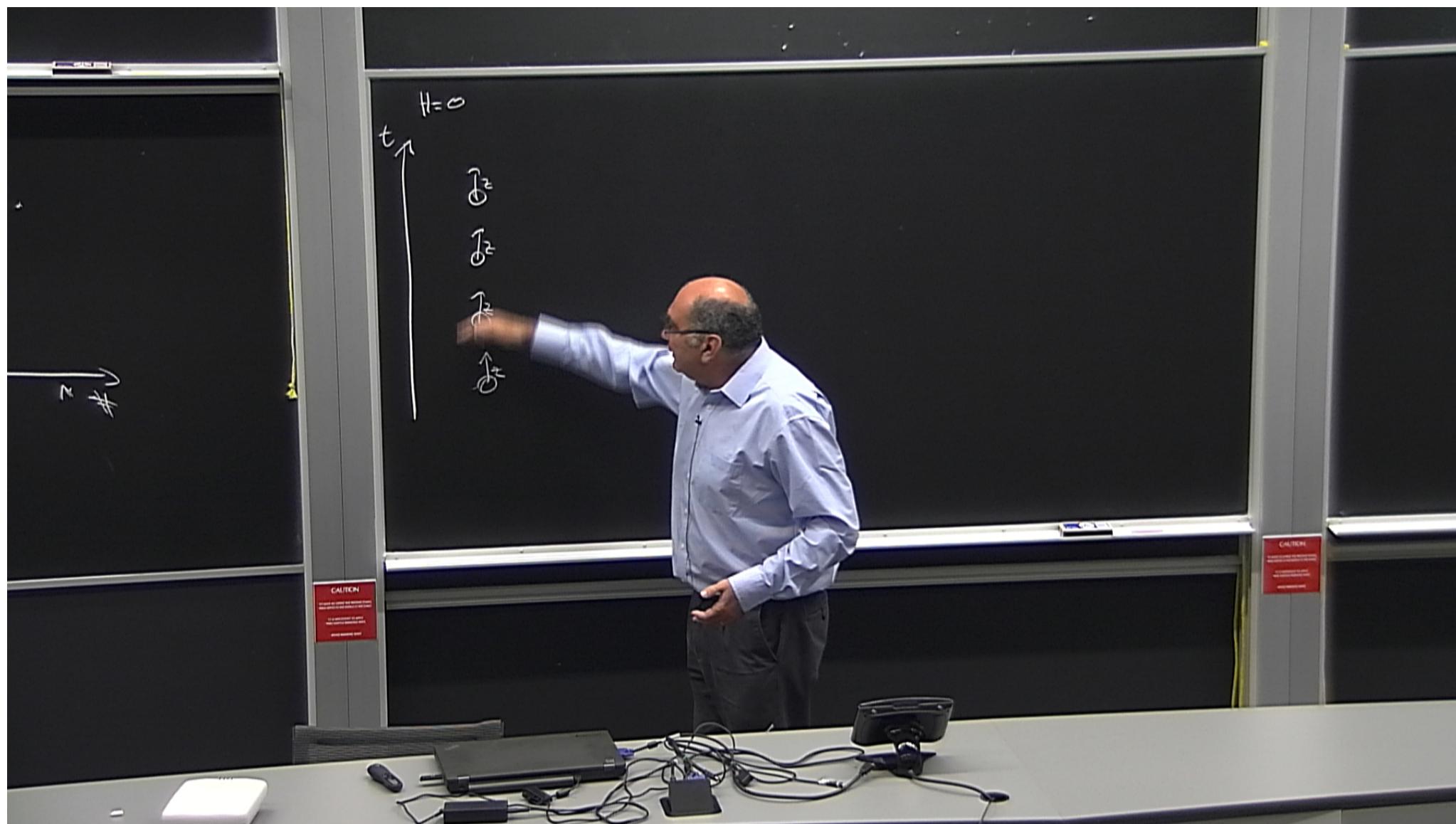


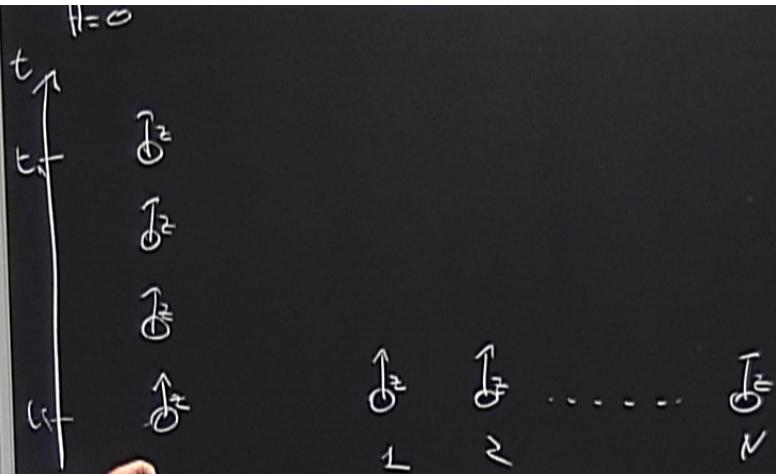


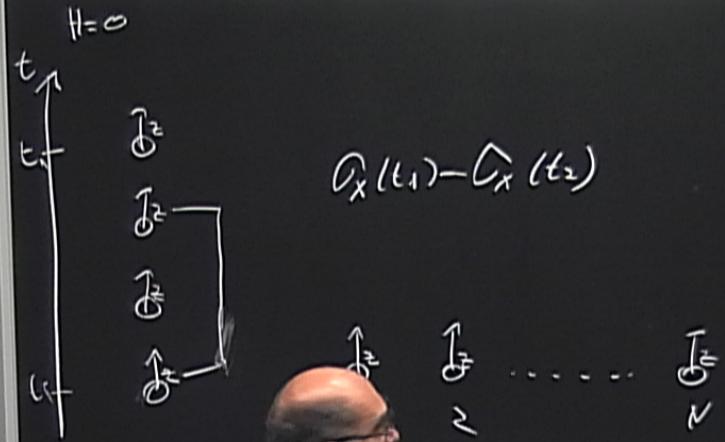


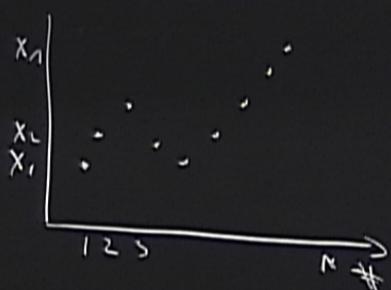
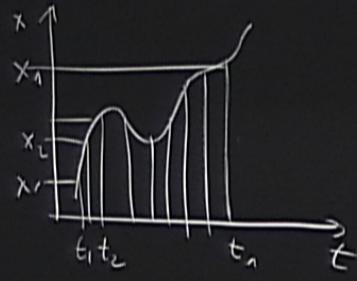


CAUTION:
DO NOT OPERATE EQUIPMENT
OR TURN ON LIGHTS
WHILE IN THE ROOM.
THIS CAN DAMAGE
THE PROJECTOR AND
LIGHTING EQUIPMENT.

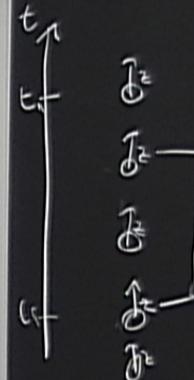








$$H=0$$

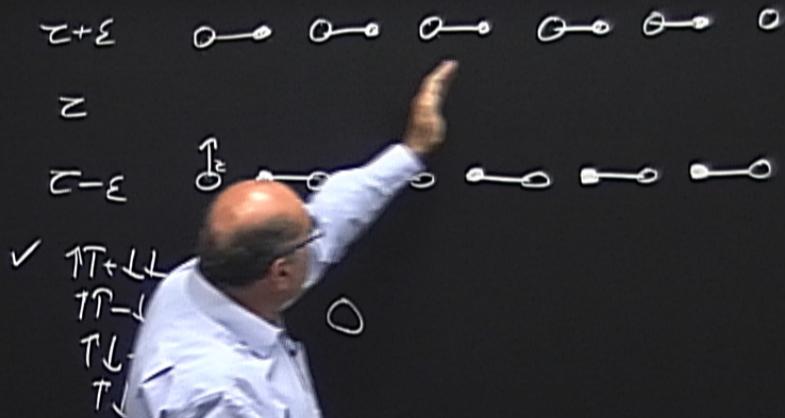


$$\hat{G}_X(t_1) - \hat{G}_X(t_2) = 0$$

$$\hat{G}_Y(t_1) - \hat{G}_Y(t_2) = 0$$

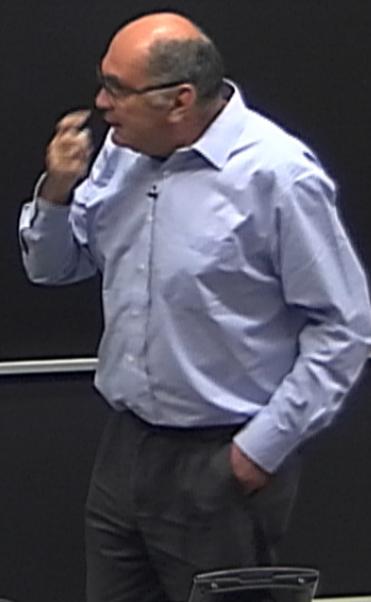
$$\hat{I}^z_1 \quad \hat{I}^z_2 \quad \dots \quad \hat{I}^z_{N-1} \quad \hat{I}^z_N$$





$$t_f \xrightarrow{\vec{B} \cdot \vec{b}} |\psi\rangle$$

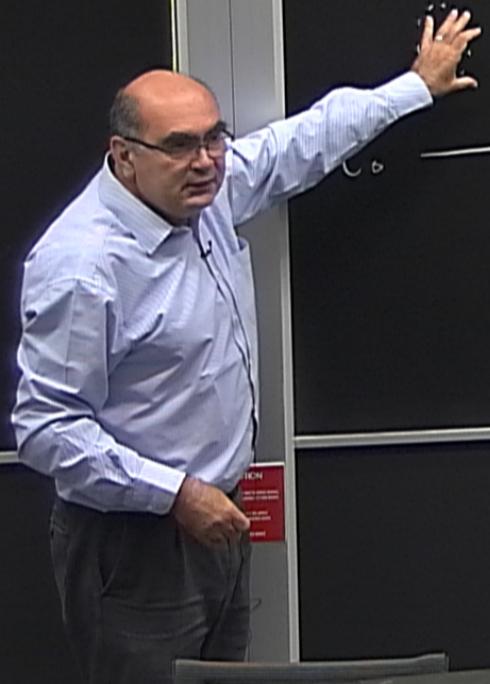
$$t_0 \longrightarrow |\psi\rangle$$



N $\#$

CAUTION

CAUTION



$$e_f \xrightarrow{B_b} | \emptyset \rangle$$

$$e_o \xrightarrow{\quad} | \psi \rangle$$

$$t_f \longrightarrow |\psi\rangle \quad \{|\psi\rangle, |\bar{\psi}\rangle\}$$

$$t \longrightarrow A_{(a_n)}^{|\psi\rangle} \quad \text{Prob}(a_k | \psi, \bar{\psi})$$

$$t_0 \longrightarrow |\psi\rangle = \frac{K\bar{\psi}|U|^2|U(t,t_0)|\psi\rangle}{\sqrt{K}}$$

N

CAUTION

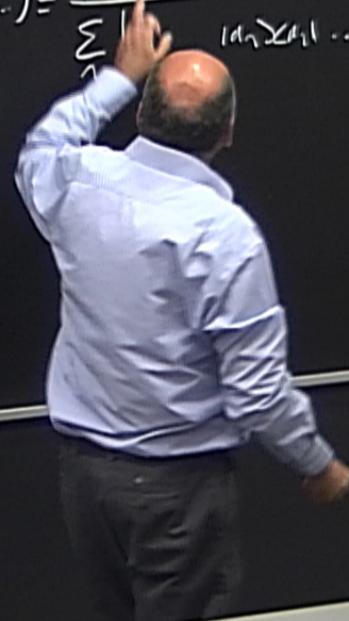
CAUTION

$$\langle \phi |_{\omega} | \psi \rangle_{t_1} \in \overset{\leftarrow}{\mathcal{H}}_{t_2} \otimes \overset{\rightarrow}{\mathcal{L}}_{t_1}$$

$$k|t, \epsilon) \\ \frac{(b(t)a_D)^2 |K_{AB}(t,t_0)|^2 |\psi\rangle|^2}{U(t_0)a_D^2 |\langle a_1| U(t,t_0)|t\rangle|^2}$$



$$P_{\text{hf}}(\alpha_k | \dots) = \frac{\left| \alpha \langle \Phi_1 | \alpha_k \rangle \langle \alpha_k | \psi \rangle + \beta \langle \Phi_2 | \alpha_k \rangle \langle \alpha_k | \psi \rangle \right|^2}{\sum_i \left| \alpha_i \langle \Phi_1 | \alpha_i \rangle \langle \alpha_i | \psi \rangle + \beta \langle \Phi_2 | \alpha_i \rangle \langle \alpha_i | \psi \rangle \right|^2}$$



, $|\psi\rangle\}$

$\alpha_k | \psi, \epsilon \rangle$

$$\langle \alpha_k | \psi, \epsilon \rangle = \frac{|K_{\alpha k} \langle \psi(t_0) | \psi \rangle|^2}{|\langle \alpha_k | \psi(t_0) | \psi \rangle|^2}$$

CAUTION



$$P_{\text{R}}(\alpha_2 | \dots) = \frac{\left| \alpha \langle \Phi_1 | \alpha_2 \rangle \langle \alpha_1 | \psi_2 \rangle + \beta \langle \Phi_2 | \alpha_2 \rangle \langle \alpha_1 | \psi_1 \rangle \right|^2}{\sum_n |\langle \alpha_n | \psi_1 \rangle|^2}$$

$$\begin{aligned} t_2 &= \sqrt{\alpha^2 + \beta^2} \\ t_1 &= \sqrt{2} \end{aligned}$$

$\xrightarrow{\alpha | \Psi_1 \rangle | \tilde{\alpha}_2 \rangle + \beta | \Psi_2 \rangle | \tilde{\alpha}_2 \rangle}$

$| \tilde{\alpha} \rangle \}$

$| \psi, \epsilon \rangle$

$K_{\alpha\beta} U(t, t_0) | \psi \rangle$

$U(t, t_0) | \alpha \rangle$

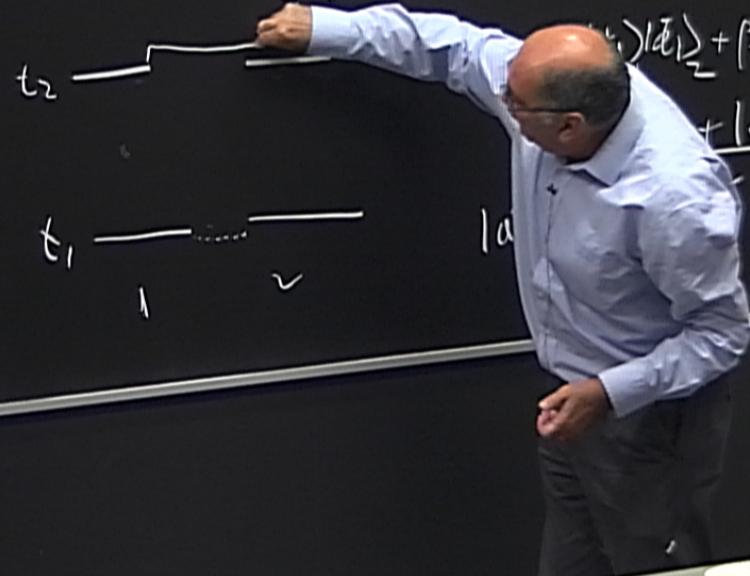
$\langle \alpha_1 | \psi \rangle$



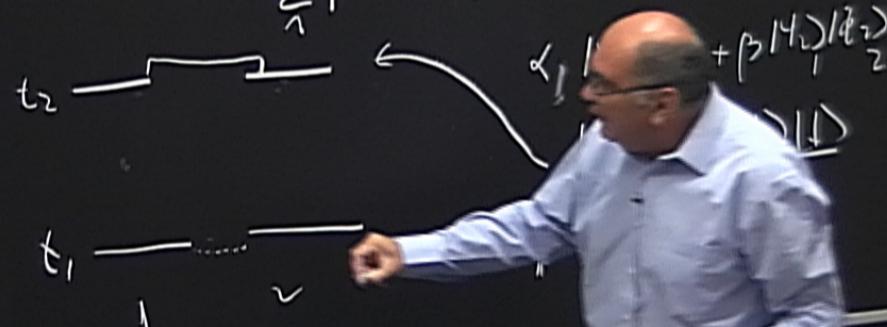
CAUTION



$$P_h(a_k | \dots) = \frac{|\alpha \langle \tilde{\psi}_1 | a_k \rangle \langle a_k | \psi_2 \rangle + \beta \langle \tilde{\psi}_2 | a_k \rangle \langle a_k | \psi_1 \rangle|^2}{\sum |\alpha_n \langle a_n | \dots | a_n \rangle|^2}$$
$$\begin{aligned} t_2 & \longrightarrow \quad \text{[Handwritten]} \\ t_1 & \longrightarrow \quad |a\rangle \end{aligned}$$



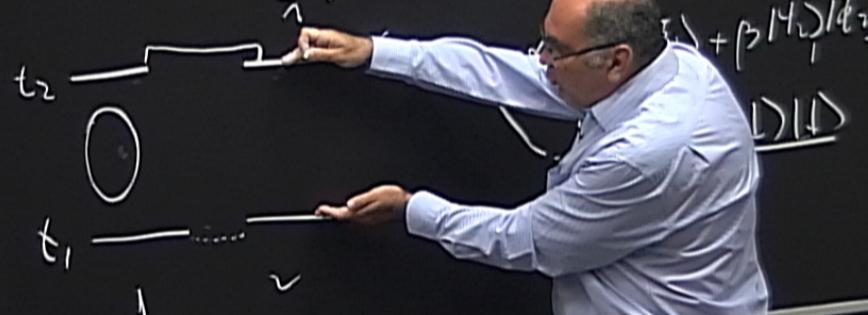
$$P_h(a_k | \dots) = \frac{\left| \alpha \langle \phi_1 | a_k \rangle \psi_1 | \psi_2 \rangle + \beta \langle \phi_2 | a_k \rangle \psi_1 | \psi_2 \rangle \right|^2}{\sum_n |\langle \phi_n | \psi_1 \rangle|^2}$$



$$\frac{|\psi(t+t_0)\rangle|^2}{|\psi(t-t_0)\rangle|^2}$$

CAUTION
ELECTRICAL EQUIPMENT
IN USE
DO NOT TURN OFF POWER

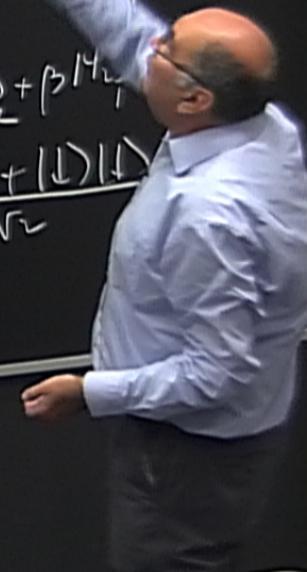
$$P_h(a_k | \dots) = \frac{\left| \alpha \sum_{t_2} \langle \phi_t | a_k \rangle \langle a_k | \psi_t \rangle + \beta \langle \bar{\phi}_t | a_k \rangle \langle a_k | \psi_t \rangle \right|^2}{\sum_{t_1} | \langle a_k | \psi_{t_1} \rangle |^2}$$



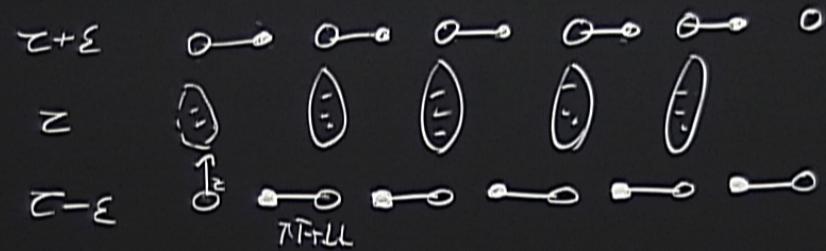
$$P_{hl}(\alpha_k | \dots) = \frac{\left| \alpha \langle \tilde{\psi}_2 | \alpha_k \rangle \langle \alpha_l | \psi_2 \rangle + \beta \langle \tilde{\psi}_2 | \alpha_k \rangle \langle \alpha_l | \psi_1 \rangle \right|^2}{\sum_n \left| \alpha_n \langle \tilde{\psi}_2 | \alpha_n \rangle \right|^2}$$

t_2 ————— \nearrow
 \bigcirc
 t_1 ————— \searrow

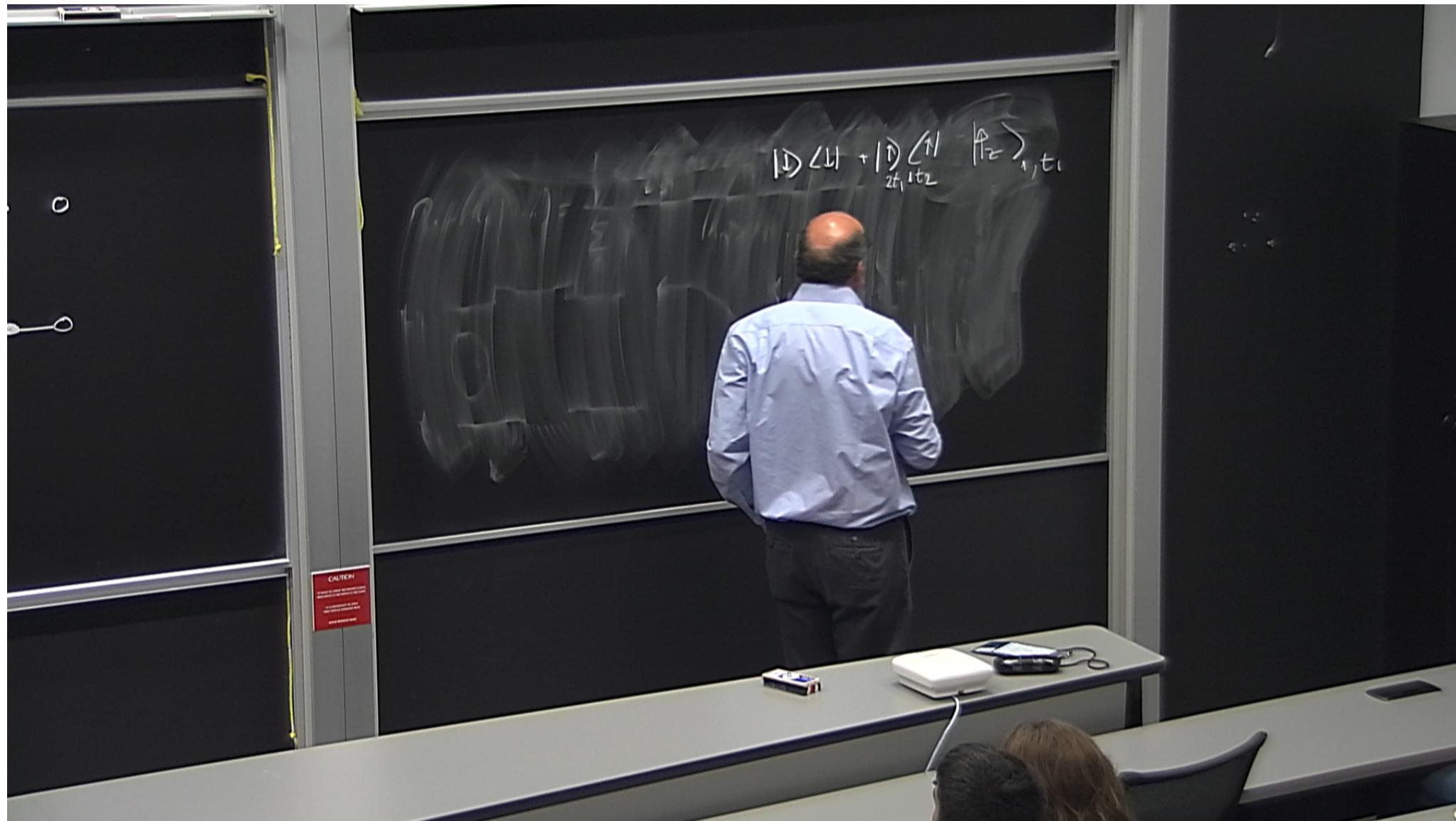
$\langle \tilde{\psi}_1 | \alpha_l \rangle + \beta \langle \tilde{\psi}_2 | \alpha_l \rangle$
 $| \uparrow \rangle | \uparrow \rangle + | \downarrow \rangle | \downarrow \rangle$
 $| \alpha_l \rangle \langle \alpha_l | \tilde{\psi}_2 \rangle$



CAUTION
ELECTRICAL OUTLET AND POWER STRIP
DO NOT USE FOR HIGH-POWER EQUIPMENT
NO PLUGS ARE TO BE USED ON THIS OUTLET



\checkmark $T\uparrow+LL$
 $T\uparrow-LL$
 $T\downarrow+LR$
 $T\downarrow-LR$



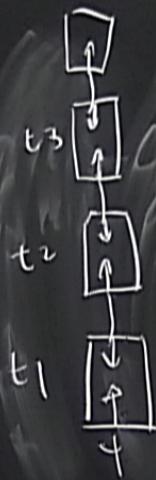
$$\left(\left| \downarrow \right\rangle_{3t_1} \left\langle \downarrow \right| + \left| \uparrow \right\rangle_{3t_1} \left\langle \uparrow \right| \right) \left(\left| \downarrow \right\rangle_{2t_1} \left\langle \downarrow \right| + \left| \uparrow \right\rangle_{2t_1} \left\langle \uparrow \right| \right) \left| \Psi_z \right\rangle_{1,t_1}$$



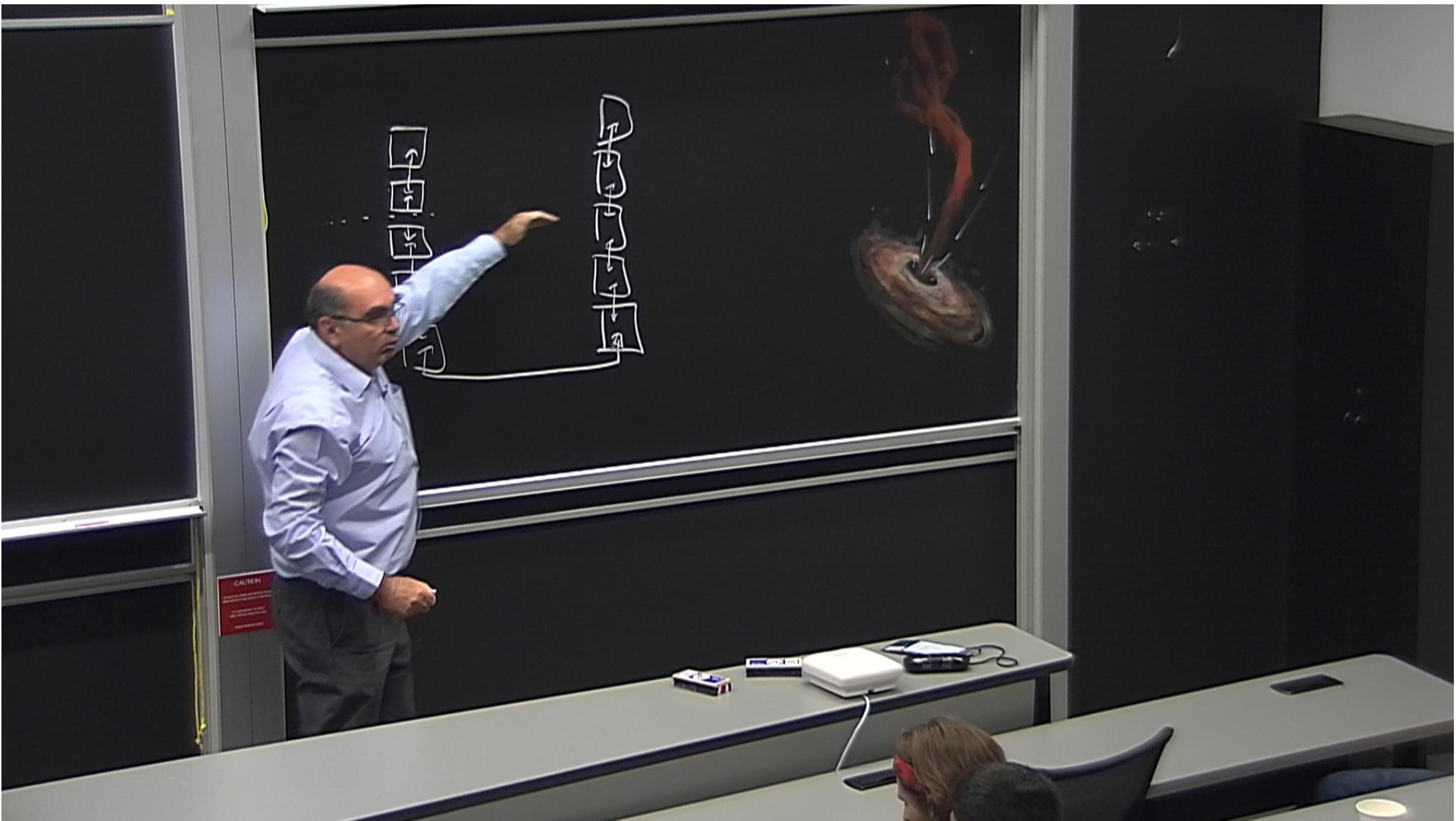
CAUTION
TO AVOID FIRE AND EXPLOSION HAZARD
DO NOT SPILL LIQUID
ON THE CHALKBOARD SURFACE

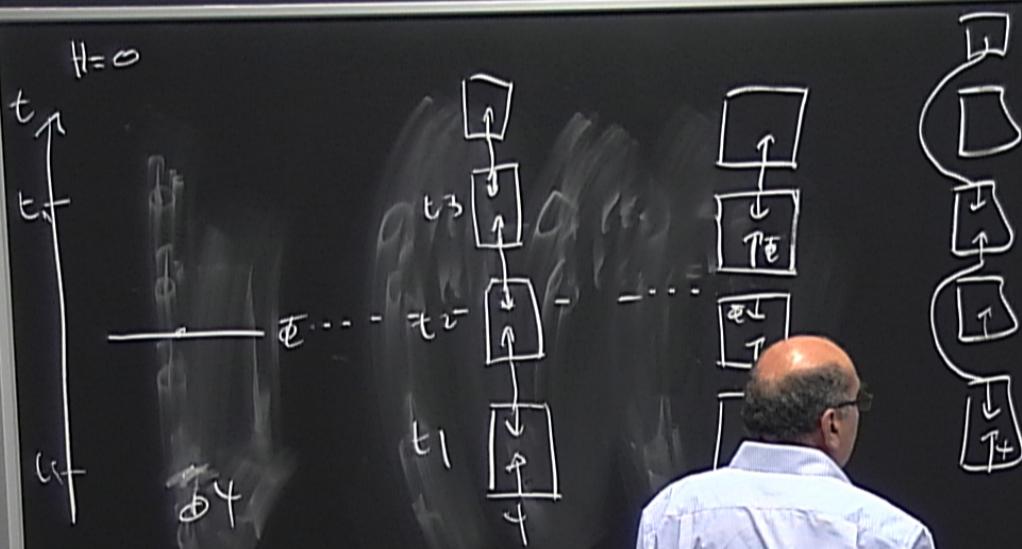
$$H=0$$

t_1



CAUTION
TO AVOID SERIOUS INJURY OR DEATH:
DO NOT CLIMB ON THE CHALKBOARD.
DO NOT JUMP ON THE CHALKBOARD.
DO NOT SIT ON THE CHALKBOARD.





CAUTION

Aharonov meets Spekkens: What do quantum logical pre- and post-selection paradoxes tell us about the nature of reality?

Matthew Leifer
Perimeter Institute

24th June 2015

Convergence: QF Workshop 6/24/2015 – 1 / 47

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



■ “Progress through paradox”^a:

- Three box paradox
- Quantum pigeonhole principle
- Quantum Cheshire cats
- Anomalous weak values
- Protective measurement

^aY. Aharonov and D. Rohrlich, “Quantum Paradoxes” (Wiley, 2005).

The two most meaningless words in physics

LPPS paradoxes

BS Contextuality

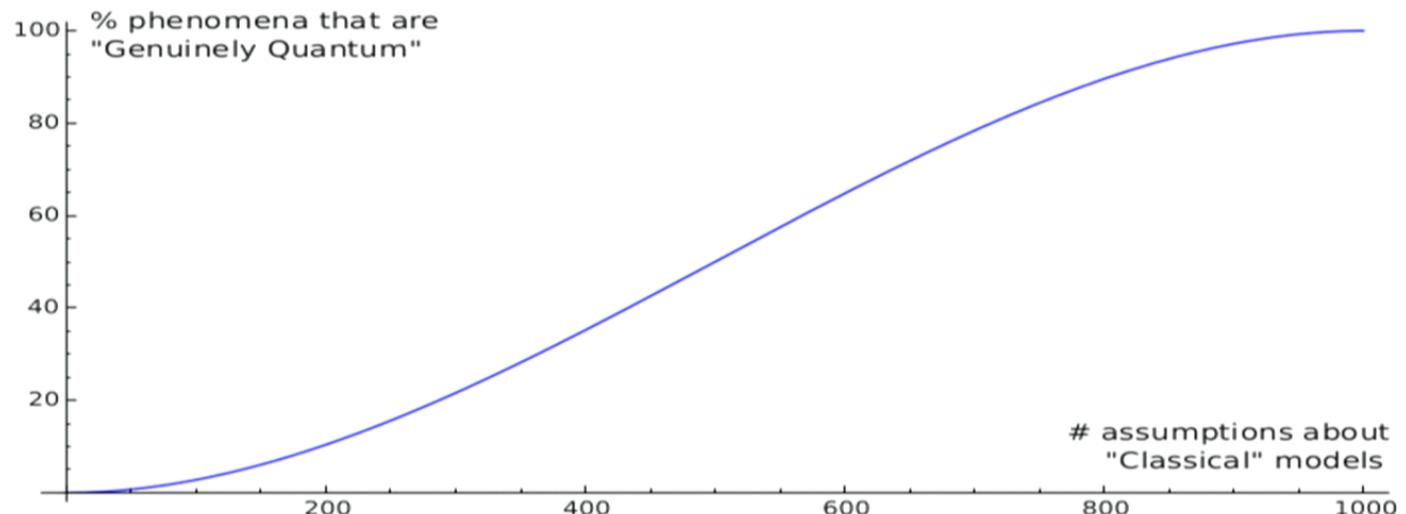
Non-BS contextual model

AS Contextuality

Discussion and Conclusions

“Classical”

“Quantum”



Convergence: QF Workshop 6/24/2015 – 4 / 47

Three box paradox

LPPS paradoxes

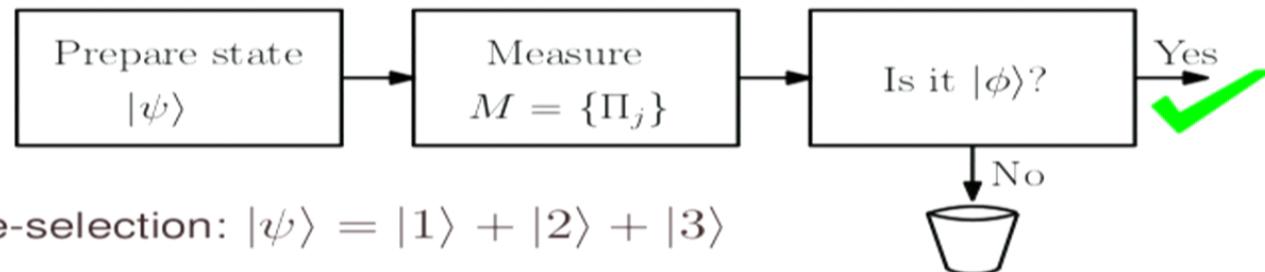
Three box paradox

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



- Pre-selection: $|\psi\rangle = |1\rangle + |2\rangle + |3\rangle$
- Post-selection: $|\phi\rangle = |1\rangle + |2\rangle - |3\rangle$
- Two possible intermediate measurements:
 - M_1 : Is ball in box 1? $\Pi_1 = |1\rangle\langle 1|$, $\Pi_{2\vee 3} = |2\rangle\langle 2| + |3\rangle\langle 3|$
 $\mathbb{P}(\Pi_1|\psi, M_1, \phi) = 1$
 - M_2 : Is ball in box 2? $\Pi_2 = |2\rangle\langle 2|$, $\Pi_{1\vee 3} = |1\rangle\langle 1| + |3\rangle\langle 3|$
 $\mathbb{P}(\Pi_2|\psi, M_2, \phi) = 1$

Y. Aharonov and L. Vaidman, *J. Phys. A* 24 pp. 2315–2328 (1991).

Convergence: QF Workshop 6/24/2015 – 7 / 47

Before Spekkens (BS) Noncontextuality

LPPS paradoxes

BS Contextuality

BS Noncontextuality

Clifton's proof

Non-BS contextual model

AS Contextuality

Discussion and Conclusions

- *Outcome determinism:* At any given time, the system has a definite value for every observable.
 - For every orthonormal basis $\{|\psi_j\rangle\}$, precisely one of them is assigned the value 1, the rest 0.
- *Noncontextuality:* The outcome assigned to an observable does not depend on which other (commuting) observables it is measured with.
 - The value assigned to a basis vector does not depend on which basis it occurs in, e.g.

$$|1\rangle, |2\rangle, |3\rangle$$

vs.

$$|1\rangle, |2\rangle + |3\rangle, |2\rangle - |3\rangle.$$

S. Kochen and E. Specker, *J. Math. Mech.* 1 pp. 59–87 (1967).

Convergence: QF Workshop 6/24/2015 – 9 / 47

Clifton's contextuality proof

LPPS paradoxes

BS Contextuality

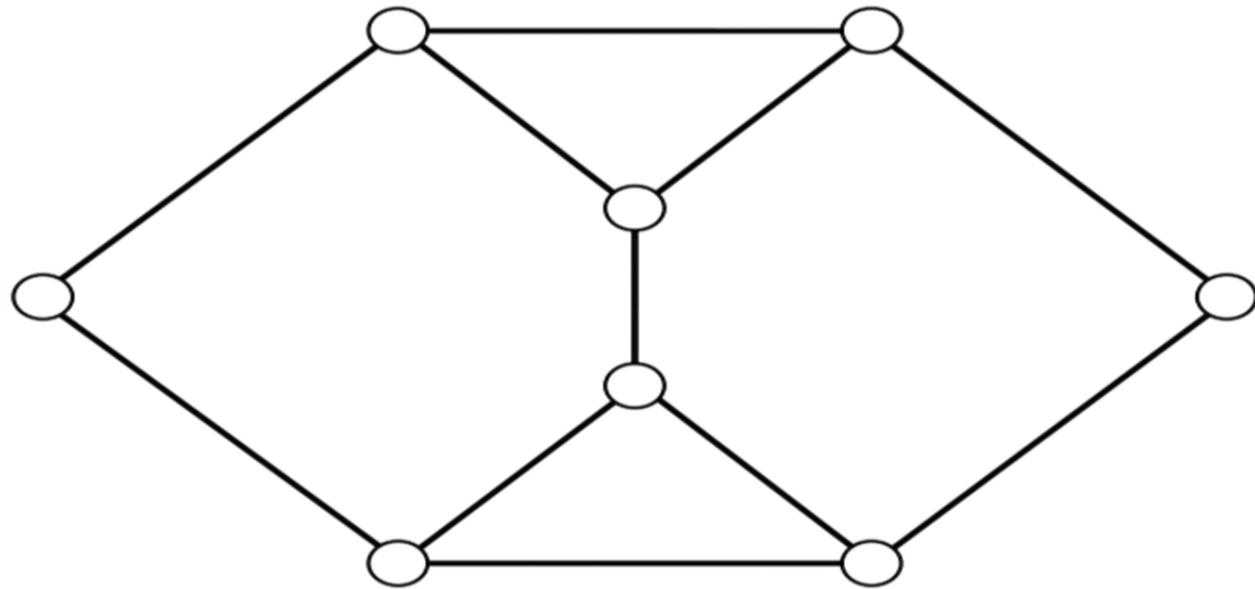
BS Noncontextuality

Clifton's proof

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



R. Clifton, *Am. J. Phys.* 61 443 (1993).

Convergence: QF Workshop 6/24/2015 – 10 / 47

Clifton's contextuality proof

LPPS paradoxes

BS Contextuality

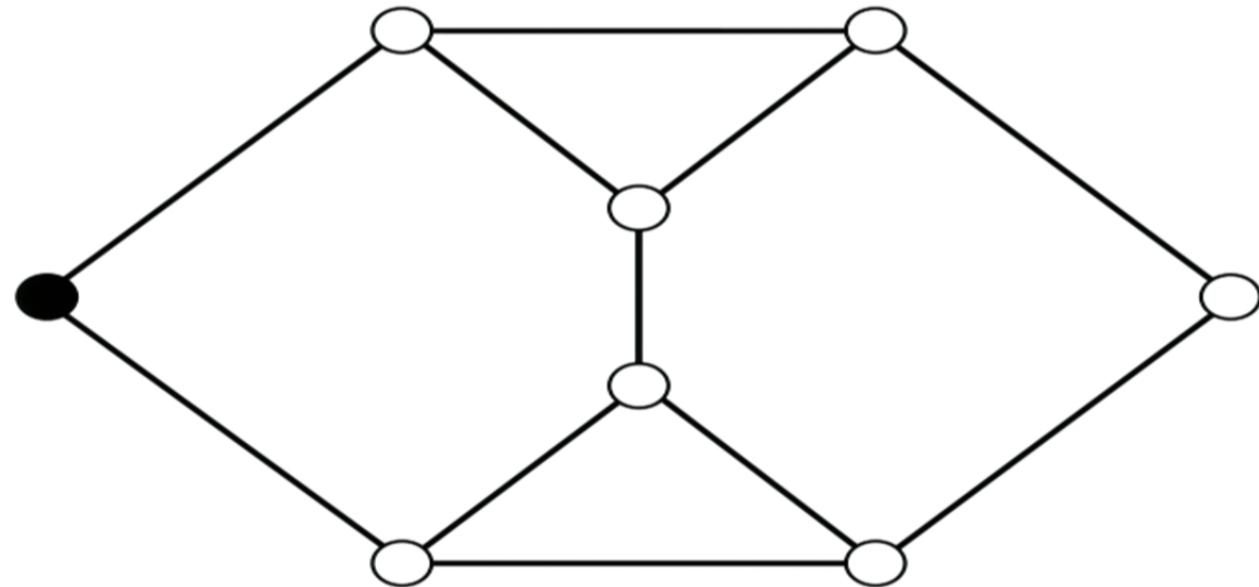
BS Noncontextuality

Clifton's proof

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



R. Clifton, *Am. J. Phys.* 61 443 (1993).

Convergence: QF Workshop 6/24/2015 – 11 / 47

Clifton's contextuality proof

LPPS paradoxes

BS Contextuality

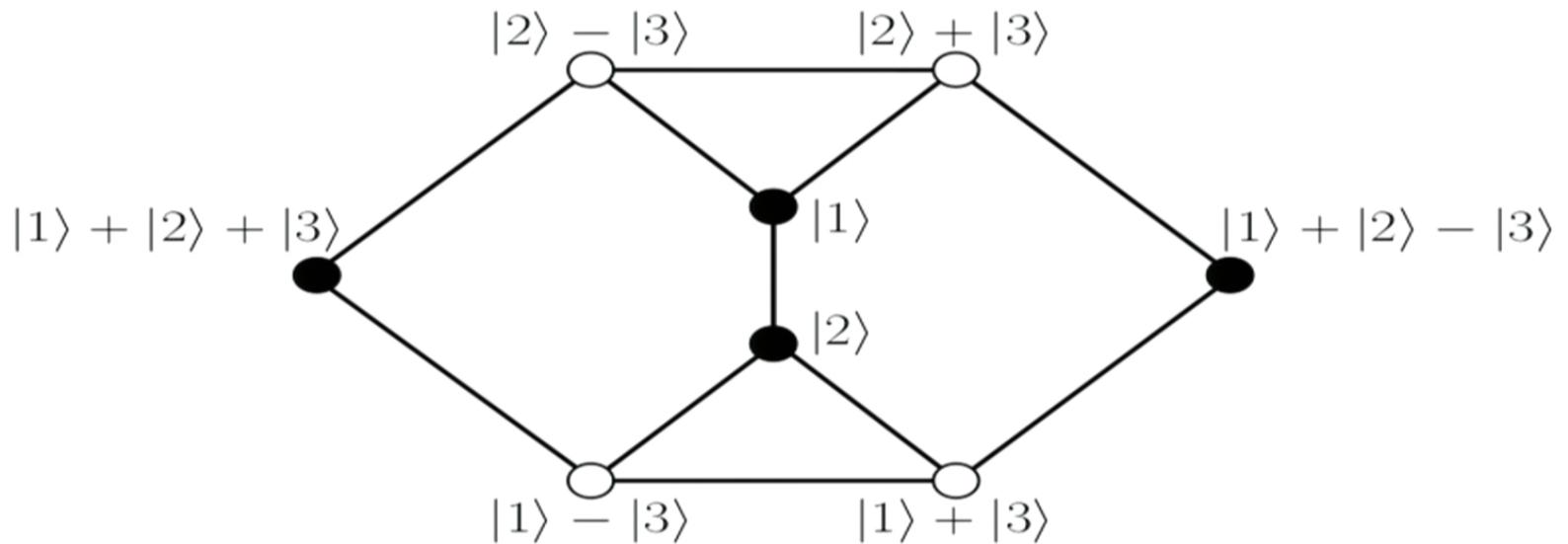
BS Noncontextuality

Clifton's proof

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



- All logical pre- and post-selection paradoxes are related to a proof of (BS) contextuality in the same way³.

R. Clifton, *Am. J. Phys.* 61 443 (1993).

³M. Leifer and R. Spekkens, *Phys. Rev. Lett.* 95 200405 (2005).

Convergence: QF Workshop 6/24/2015 – 15 / 47

Clifton's contextuality proof

LPPS paradoxes

BS Contextuality

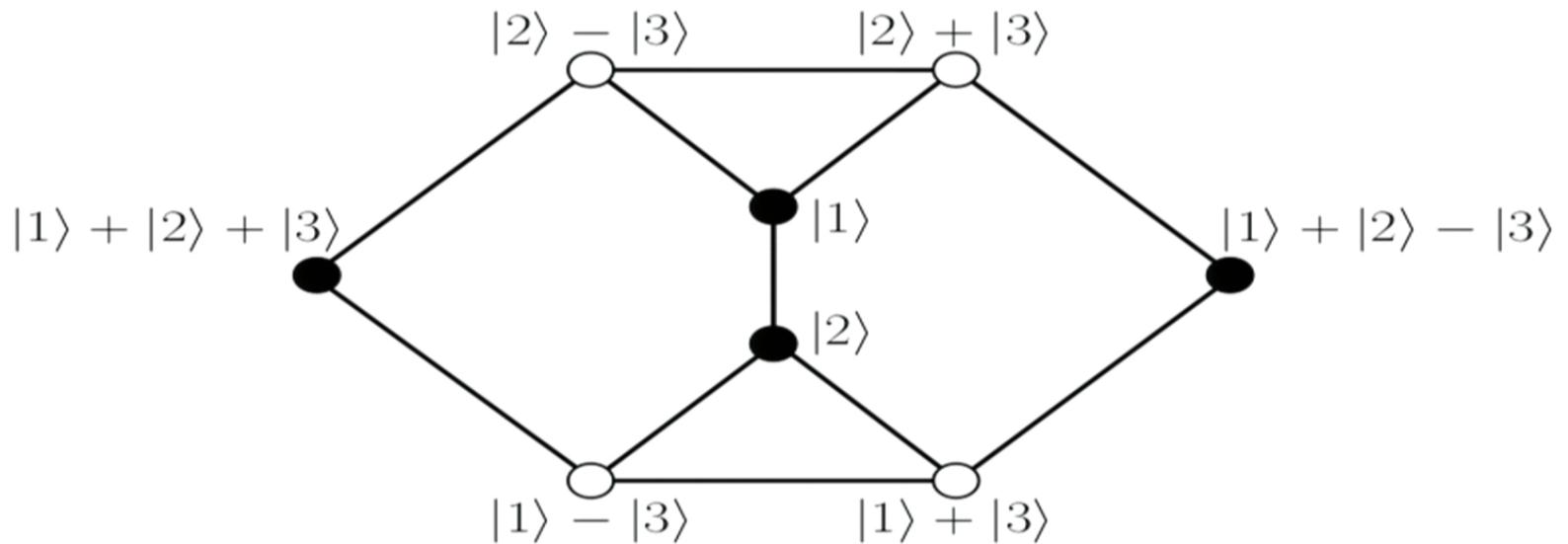
BS Noncontextuality

Clifton's proof

Non-BS contextual model

AS Contextuality

Discussion and Conclusions



- All logical pre- and post-selection paradoxes are related to a proof of (BS) contextuality in the same way³.

R. Clifton, *Am. J. Phys.* 61 443 (1993).

³M. Leifer and R. Spekkens, *Phys. Rev. Lett.* 95 200405 (2005).

Convergence: QF Workshop 6/24/2015 – 15 / 47

- [LPPS paradoxes](#)
- [BS Contextuality](#)
- [Non-BS contextual model](#)
- [Partitioned box](#)
- [AS Contextuality](#)
- [Discussion and Conclusions](#)

A non-BS contextual model

Convergence: QF Workshop 6/24/2015 – 16 / 47

The partitioned box paradox

LPPS paradoxes

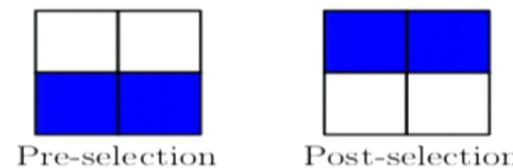
BS Contextuality

Non-BS contextual model

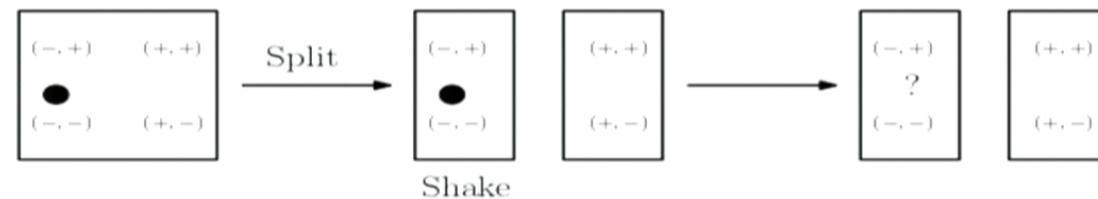
Partitioned box

AS Contextuality

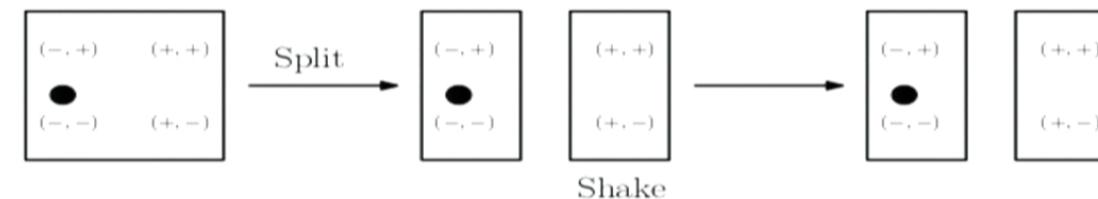
Discussion and Conclusions



■ “Left”-measurement:



■ “Right”-measurement:



M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

Convergence: QF Workshop 6/24/2015 – 17 / 47

The partitioned box paradox

LPPS paradoxes

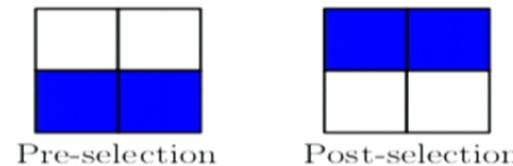
BS Contextuality

Non-BS contextual model

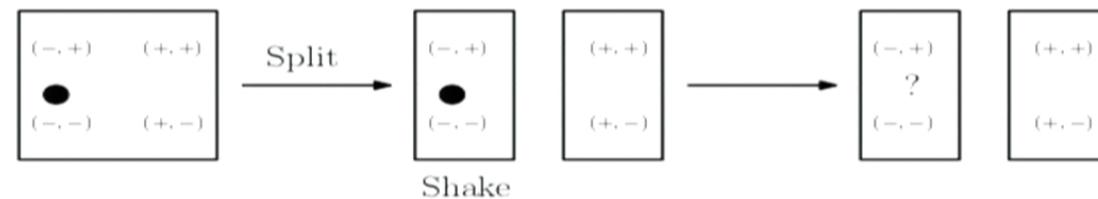
Partitioned box

AS Contextuality

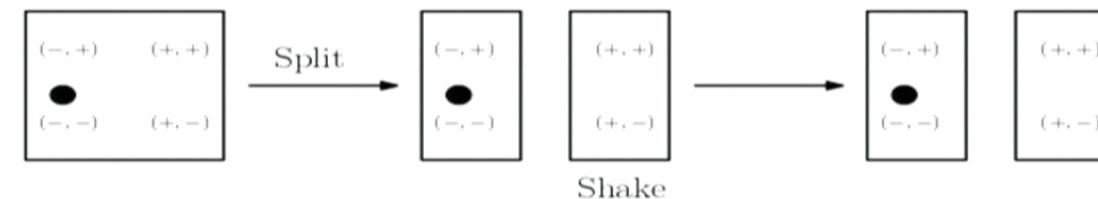
Discussion and Conclusions



■ “Left”-measurement:



■ “Right”-measurement:



M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

Convergence: QF Workshop 6/24/2015 – 17 / 47

The partitioned box paradox

LPPS paradoxes

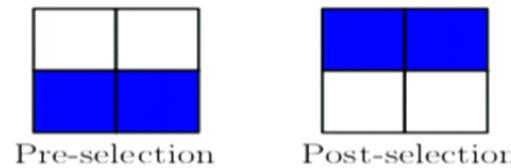
BS Contextuality

Non-BS contextual model

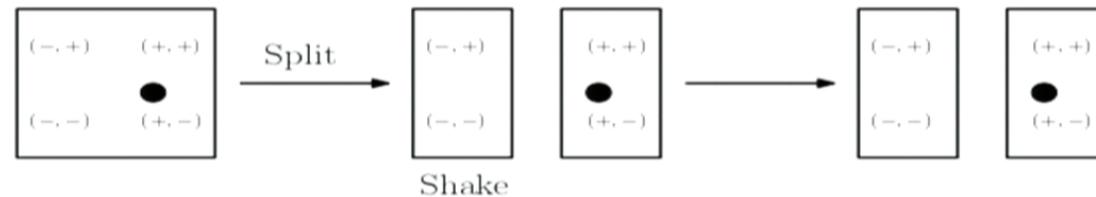
Partitioned box

AS Contextuality

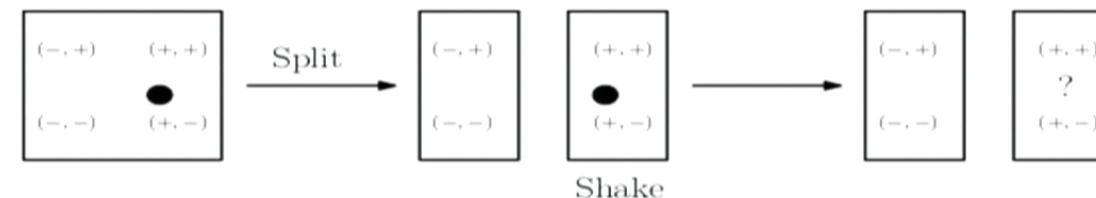
Discussion and Conclusions



■ “Left”-measurement:



■ “Right”-measurement:



M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

Convergence: QF Workshop 6/24/2015 – 18 / 47

The partitioned box paradox

LPPS paradoxes

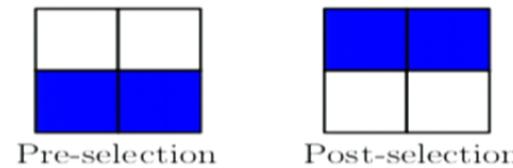
BS Contextuality

Non-BS contextual model

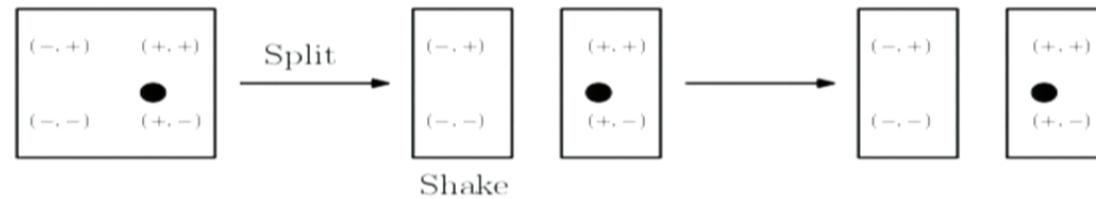
Partitioned box

AS Contextuality

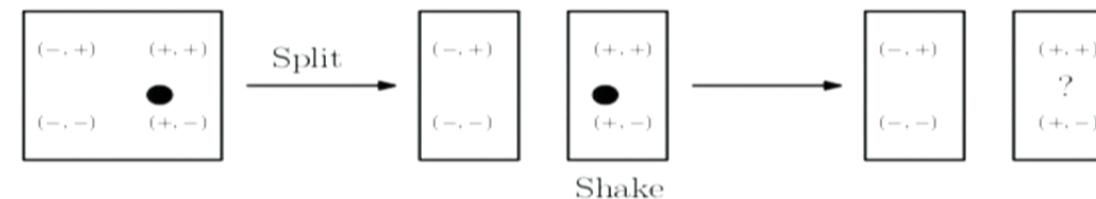
Discussion and Conclusions



■ “Left”-measurement:



■ “Right”-measurement:



M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

Convergence: QF Workshop 6/24/2015 – 18 / 47

The partitioned box paradox

LPPS paradoxes

BS Contextuality

Non-BS contextual model

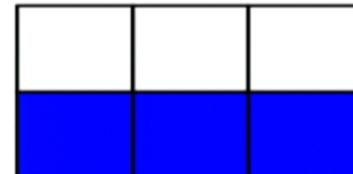
Partitioned box

AS Contextuality

Discussion and Conclusions

- We can reproduce the predictions of the three-box paradox exactly by adding more states and changing the update rule.

- New pre- and post-selection:

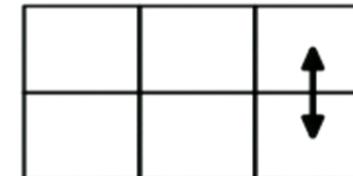


Pre-selection



Post-selection

- Add this to state-update rule:



Convergence: QF Workshop 6/24/2015 – 19 / 47

The partitioned box paradox

LPPS paradoxes

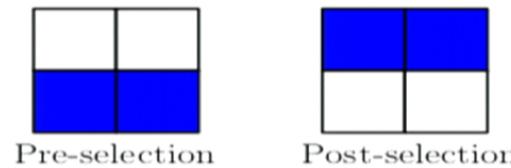
BS Contextuality

Non-BS contextual model

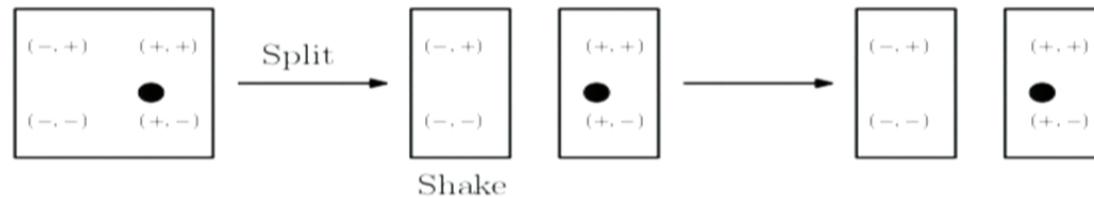
Partitioned box

AS Contextuality

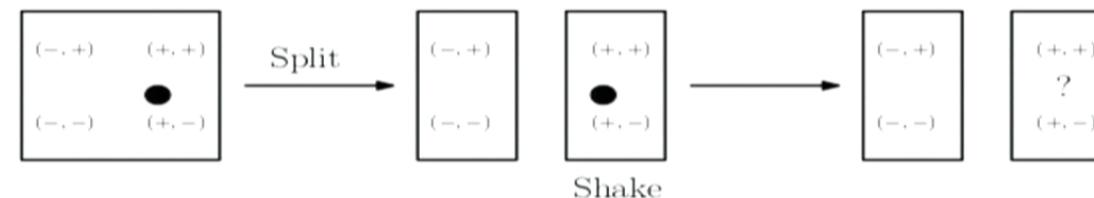
Discussion and Conclusions



■ “Left”-measurement:



■ “Right”-measurement:



M. Leifer and R. Spekkens, Int. J. Theor. Phys. 44 pp. 1977–1987 (2005).

Convergence: QF Workshop 6/24/2015 – 18 / 47

LPPS paradoxes
BS Contextuality
Non-BS contextual model
AS Contextuality
Operational theories
Ontological models
Trans. Contextuality
State-update rules
Proof of contextuality
Discussion and Conclusions

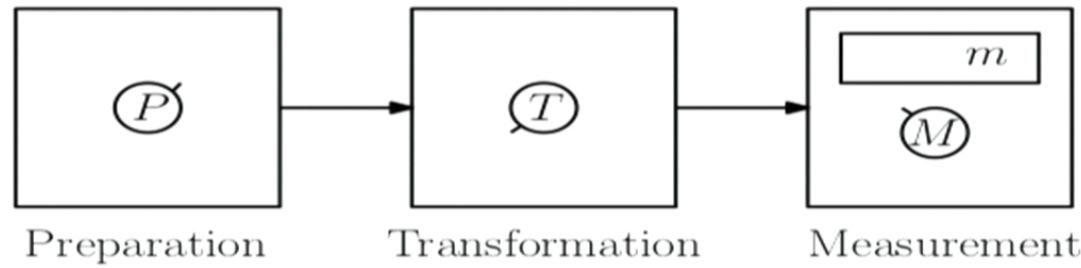
After Spekkens Contextuality

Convergence: QF Workshop 6/24/2015 – 20 / 47

After Spekkens (AS) Noncontextuality

LPPS paradoxes
BS Contextuality
Non-BS contextual model
AS Contextuality
Operational theories
Ontological models
Trans. Contextuality
State-update rules
Proof of contextuality
Discussion and Conclusions

■ Operational theory:



$$\mathbb{P}(m|P, M, T)$$

■ In quantum theory:

$$\mathbb{P}(m|P, M, T) = \text{Tr} (E_m^M \mathcal{E}_T(\rho_P))$$

R. Spekkens, *Phys. Rev. A* 71:052108 (2005).

Convergence: QF Workshop 6/24/2015 – 21 / 47

Ontological models

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Operational theories

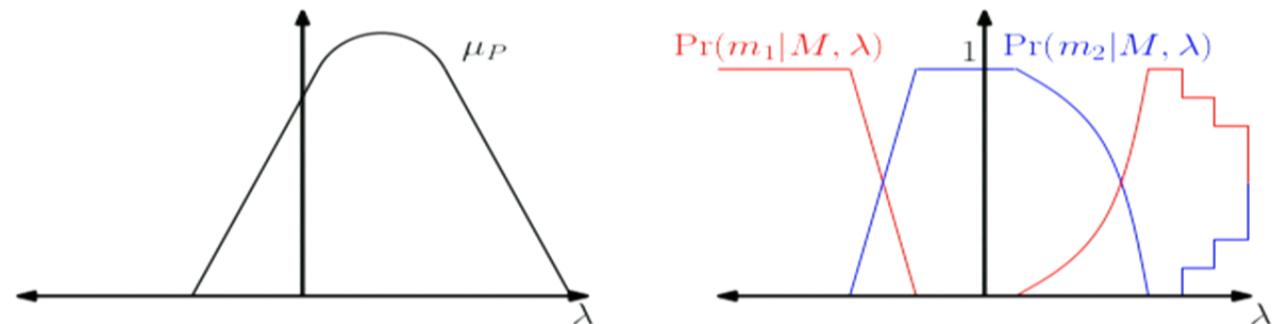
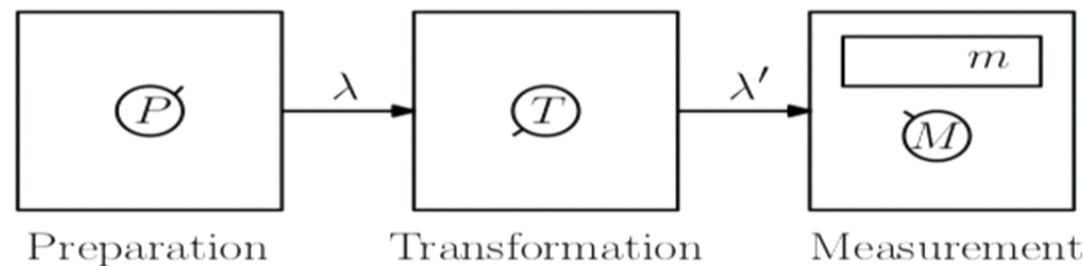
Ontological models

Trans. Contextuality

State-update rules

Proof of contextuality

Discussion and Conclusions



$$\mathbb{P}(m|P, M, T) = \int_{\Lambda'} \int_{\Lambda} \Pr(m|M, \lambda') d\Gamma_T(\lambda'|\lambda) d\mu_P(\lambda)$$

Convergence: QF Workshop 6/24/2015 – 22 / 47

Implications for state-update rules

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Operational theories

Ontological models

Trans. Contextuality

State-update rules

Proof of contextuality

Discussion and Conclusions

Theorem. Let $\{\Pi_j\}$ be a projective measurement and let \mathcal{E} be the nonselective state-update rule

$$\mathcal{E}(\rho) = \sum_j \Pi_j \rho \Pi_j.$$

Then,

$$\mathcal{E}(\rho) = p\rho + (1 - p)\mathcal{C}(\rho),$$

where \mathcal{C} is a completely-positive, trace-preserving map and $0 < p \leq 1$.

■ Proof for special case $\{\Pi_1, \Pi_2\}$:

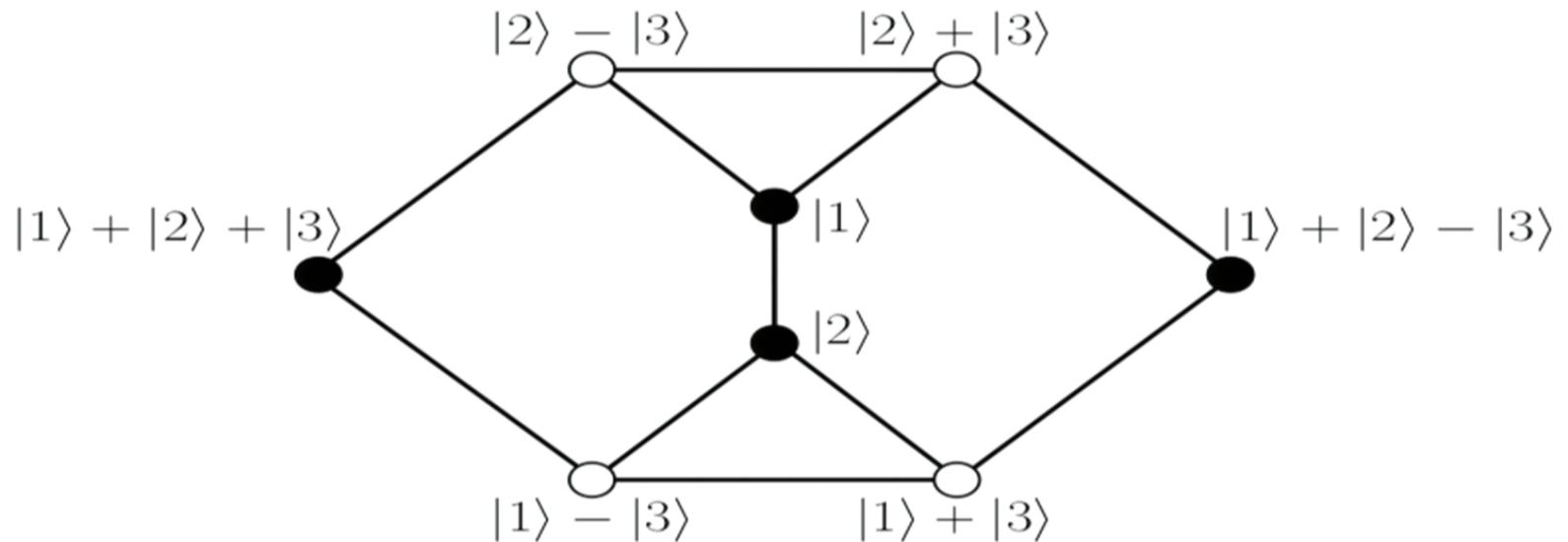
$$U_1 = \Pi_1 + \Pi_2 = I \quad U_2 = \Pi_1 - \Pi_2$$

$$\mathcal{E}(\rho) = \frac{1}{2}U_1\rho U_1^\dagger + \frac{1}{2}U_2\rho U_2^\dagger = \frac{1}{2}\rho + \frac{1}{2}U_2\rho U_2^\dagger.$$

Convergence: QF Workshop 6/24/2015 – 24 / 47

Proof of contextuality

LPPS paradoxes
BS Contextuality
Non-BS contextual model
AS Contextuality
Operational theories
Ontological models
Trans. Contextuality
State-update rules
Proof of contextuality
Discussion and Conclusions



- All logical pre- and post-selection paradoxes are proofs of (PS) contextuality in a similar way.

Convergence: QF Workshop 6/24/2015 – 25 / 47

Transformation noncontextuality

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Operational theories

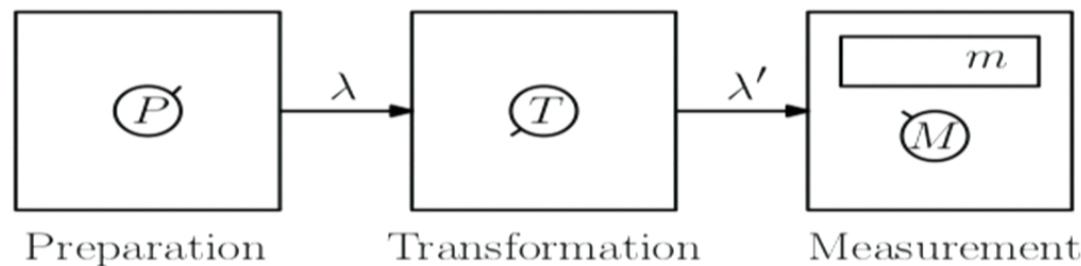
Ontological models

Trans. Contextuality

State-update rules

Proof of contextuality

Discussion and Conclusions



Definition. An ontological model is *transformation noncontextual* if, whenever

$$\mathbb{P}(m|P, M, T) = \mathbb{P}(m|P, M, S)$$

for all P, M, m , we have

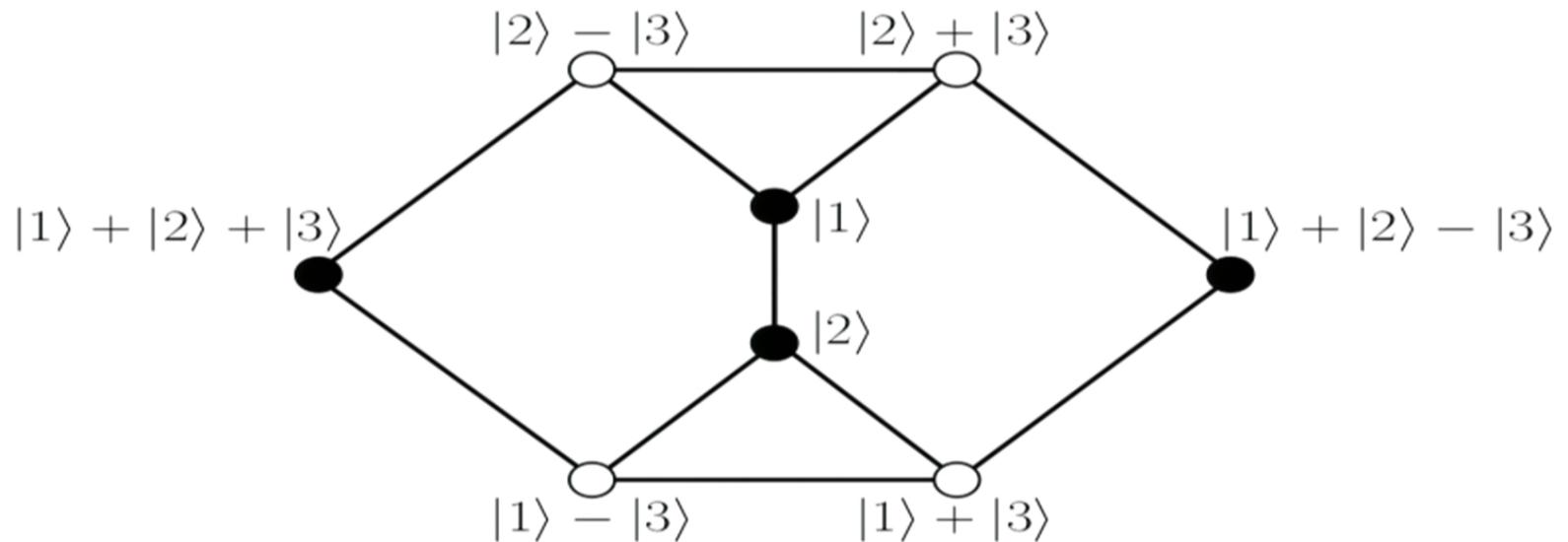
$$\Gamma_T = \Gamma_S.$$

- In quantum theory, Γ_T only depends on \mathcal{E}_T .

Convergence: QF Workshop 6/24/2015 – 23 / 47

Proof of contextuality

LPPS paradoxes
BS Contextuality
Non-BS contextual model
AS Contextuality
Operational theories
Ontological models
Trans. Contextuality
State-update rules
Proof of contextuality
Discussion and Conclusions



- All logical pre- and post-selection paradoxes are proofs of (PS) contextuality in a similar way.

Convergence: QF Workshop 6/24/2015 – 25 / 47

Conclusions

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions

Conclusions

Weak measurements

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
 - Specifying assumptions for “classical” models.
 - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
 - Understandable in an AS noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
 - AS Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum”.

Convergence: QF Workshop 6/24/2015 – 27 / 47

Conclusions

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions

Conclusions

Weak measurements

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
 - Specifying assumptions for “classical” models.
 - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
 - Understandable in an AS noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
 - AS Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum”.

Convergence: QF Workshop 6/24/2015 – 27 / 47

Conclusions

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions

Conclusions

Weak measurements

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
 - Specifying assumptions for “classical” models.
 - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
 - Understandable in an AS noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
 - AS Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum”.

Convergence: QF Workshop 6/24/2015 – 27 / 47

Conclusions

LPPS paradoxes

BS Contextuality

Non-BS contextual model

AS Contextuality

Discussion and Conclusions

Conclusions

Weak measurements

- There is no such thing as a “classical” or “genuinely quantum” phenomenon without
 - Specifying assumptions for “classical” models.
 - Specifying which aspects of the phenomenon you want to reproduce.
- A well-motivated set of assumptions is:
 - Understandable in an AS noncontextual classical probabilistic theory with restriction on knowledge = “classical”.
 - AS Contextual = “quantum”.
- On this classification LPPS paradoxes are “quantum”.

Convergence: QF Workshop 6/24/2015 – 27 / 47