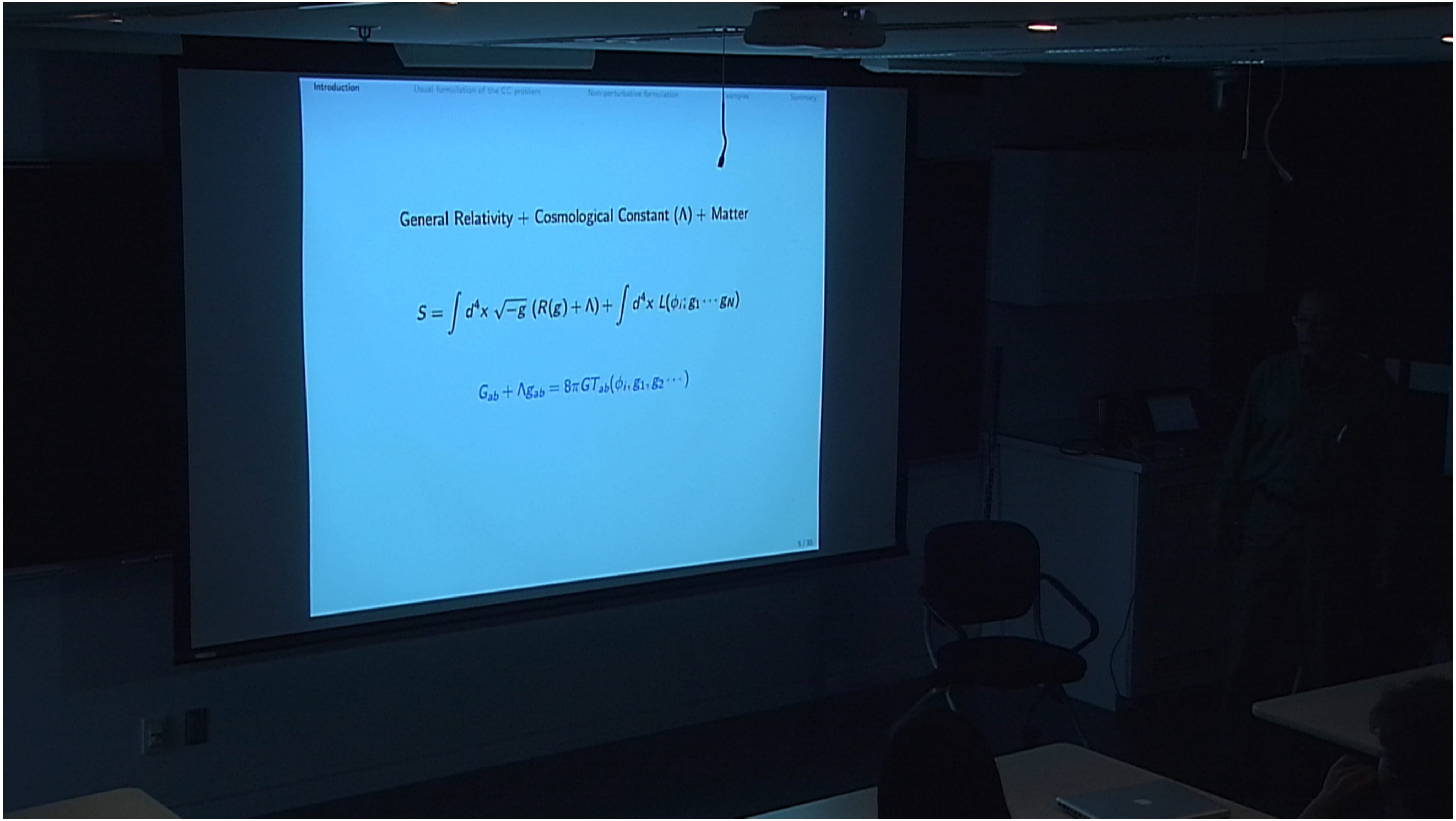
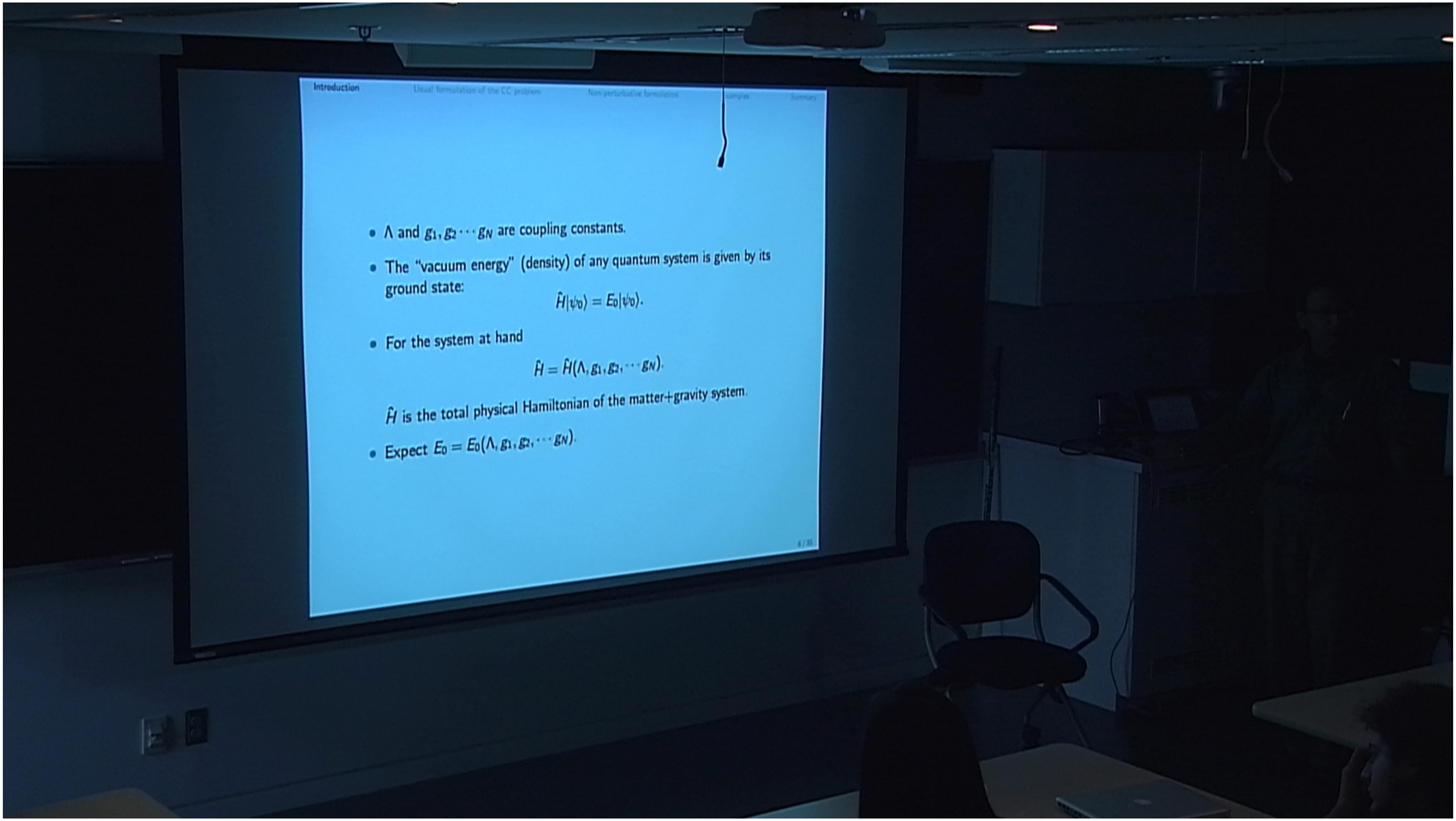
Title: A Non-Perturbative Approach to the Cosmological Constant Problem

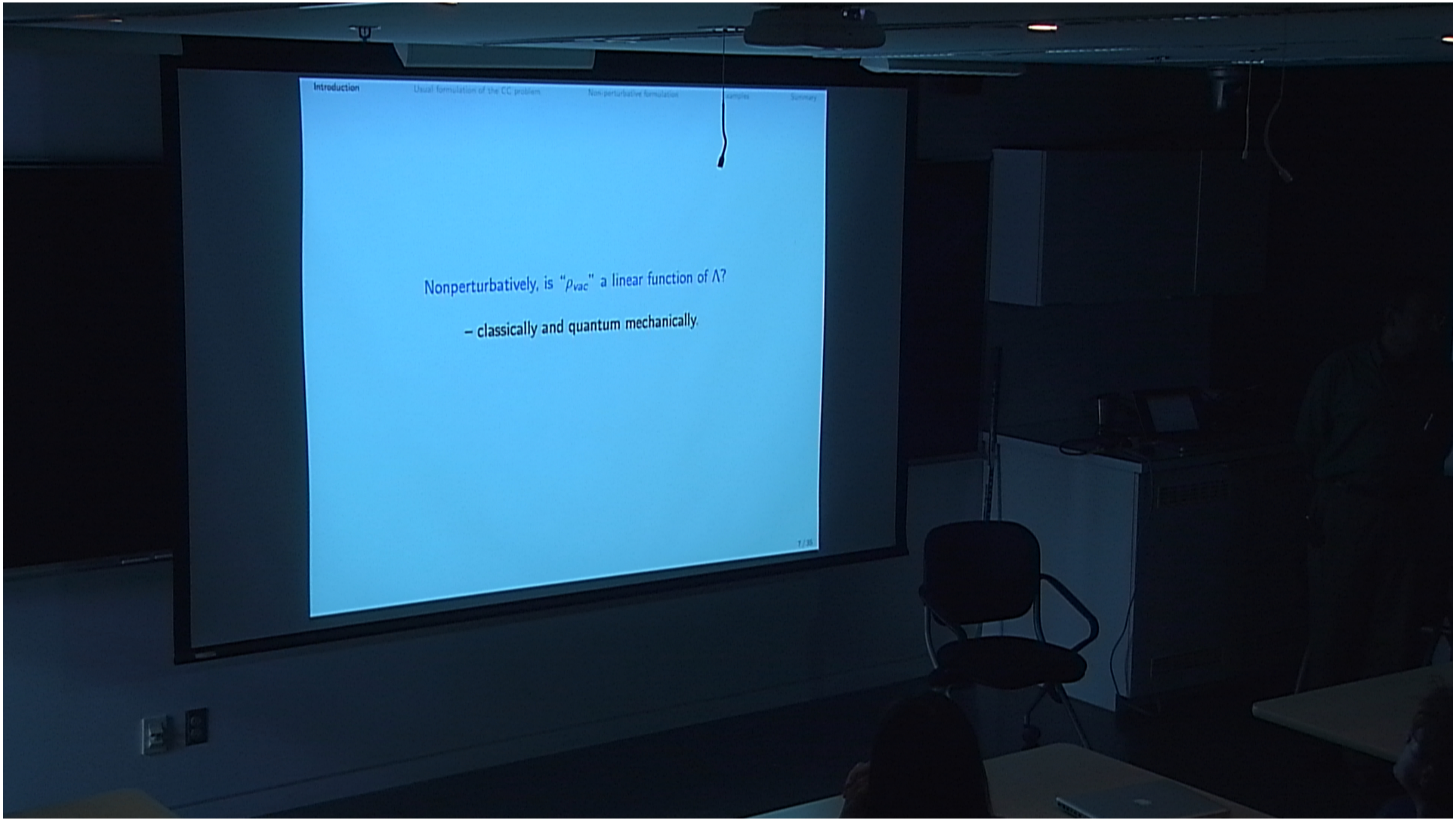
Date: Jun 26, 2015 02:00 PM

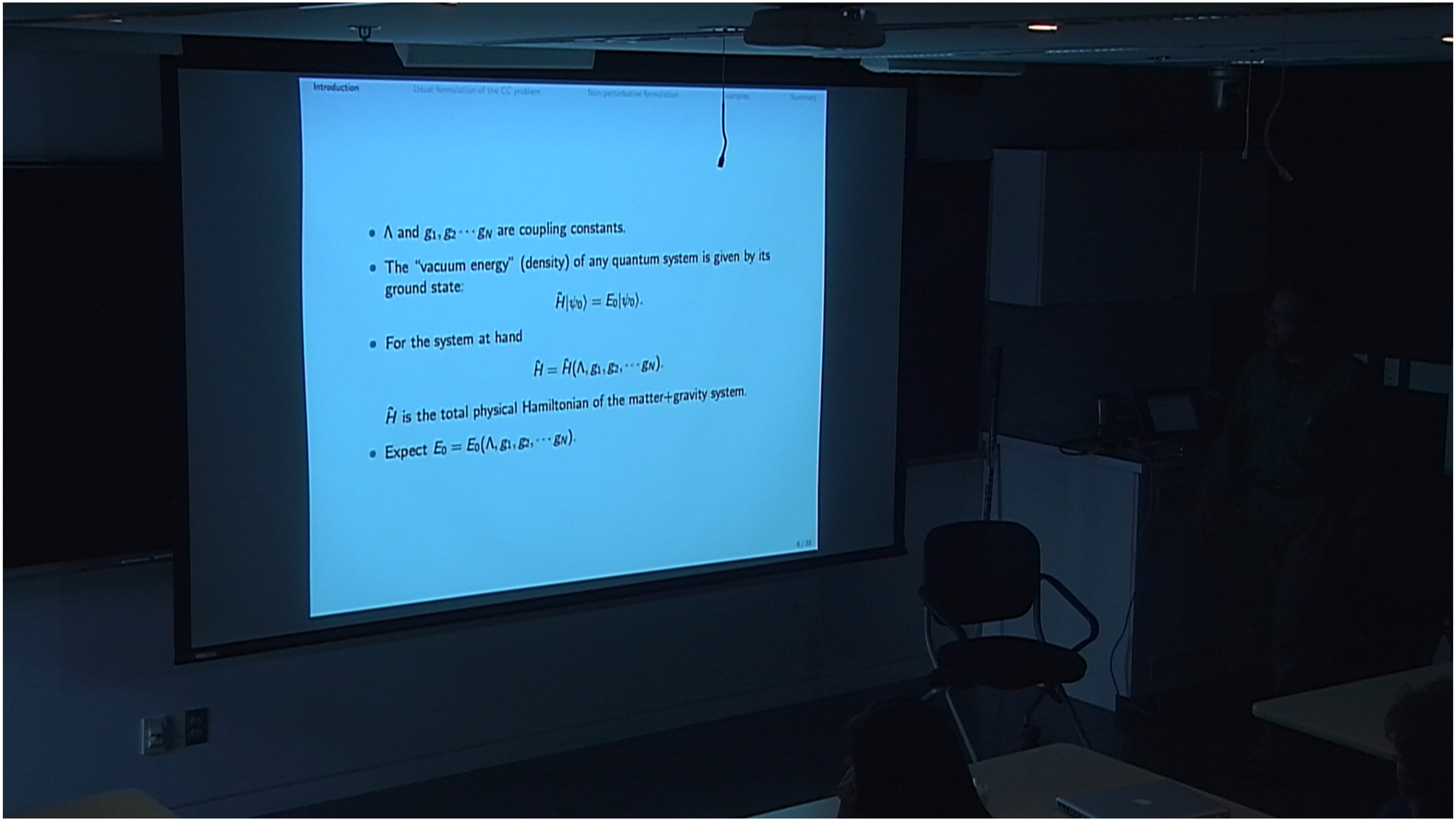
URL: http://pirsa.org/15060032

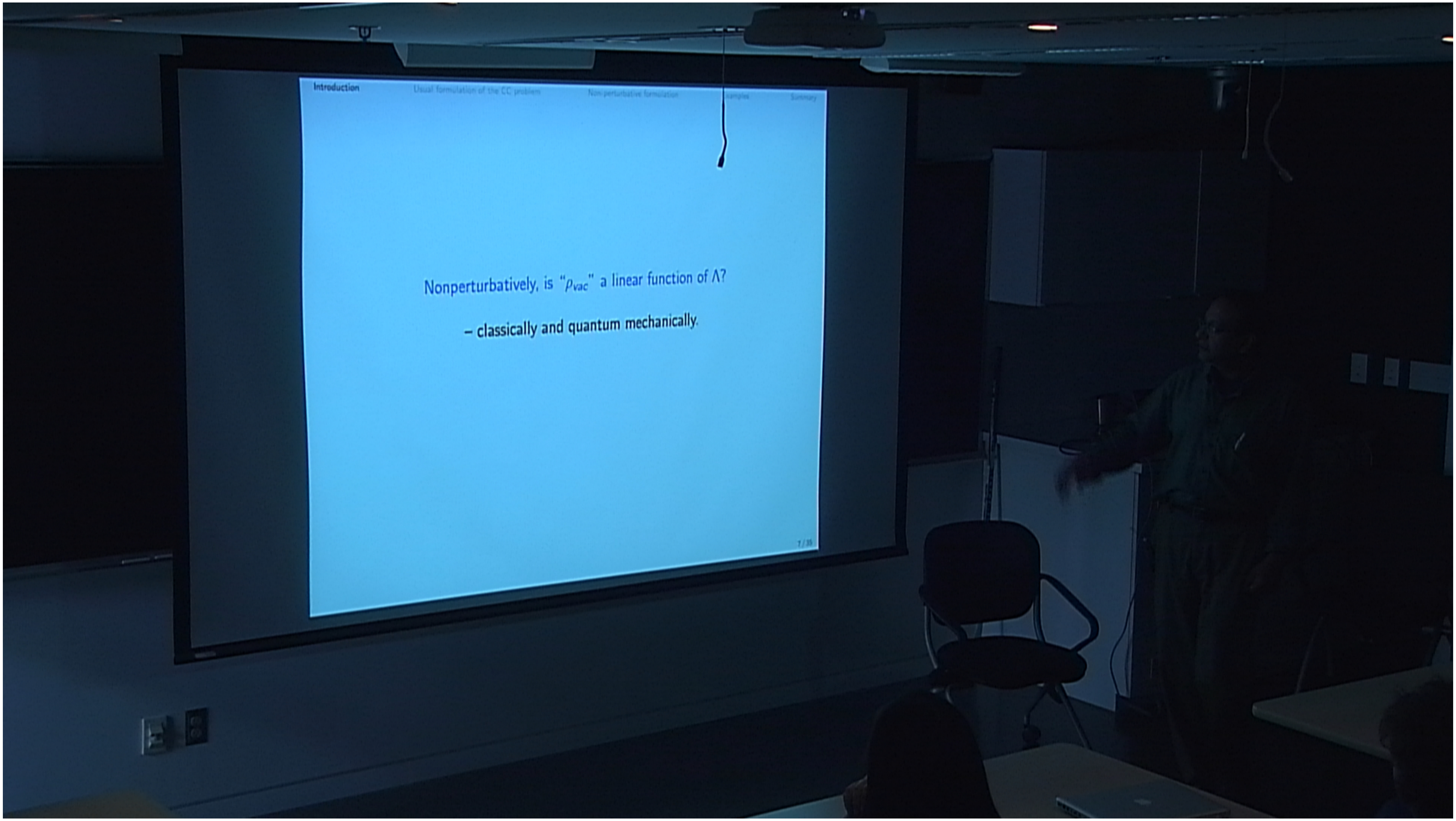
Abstract:

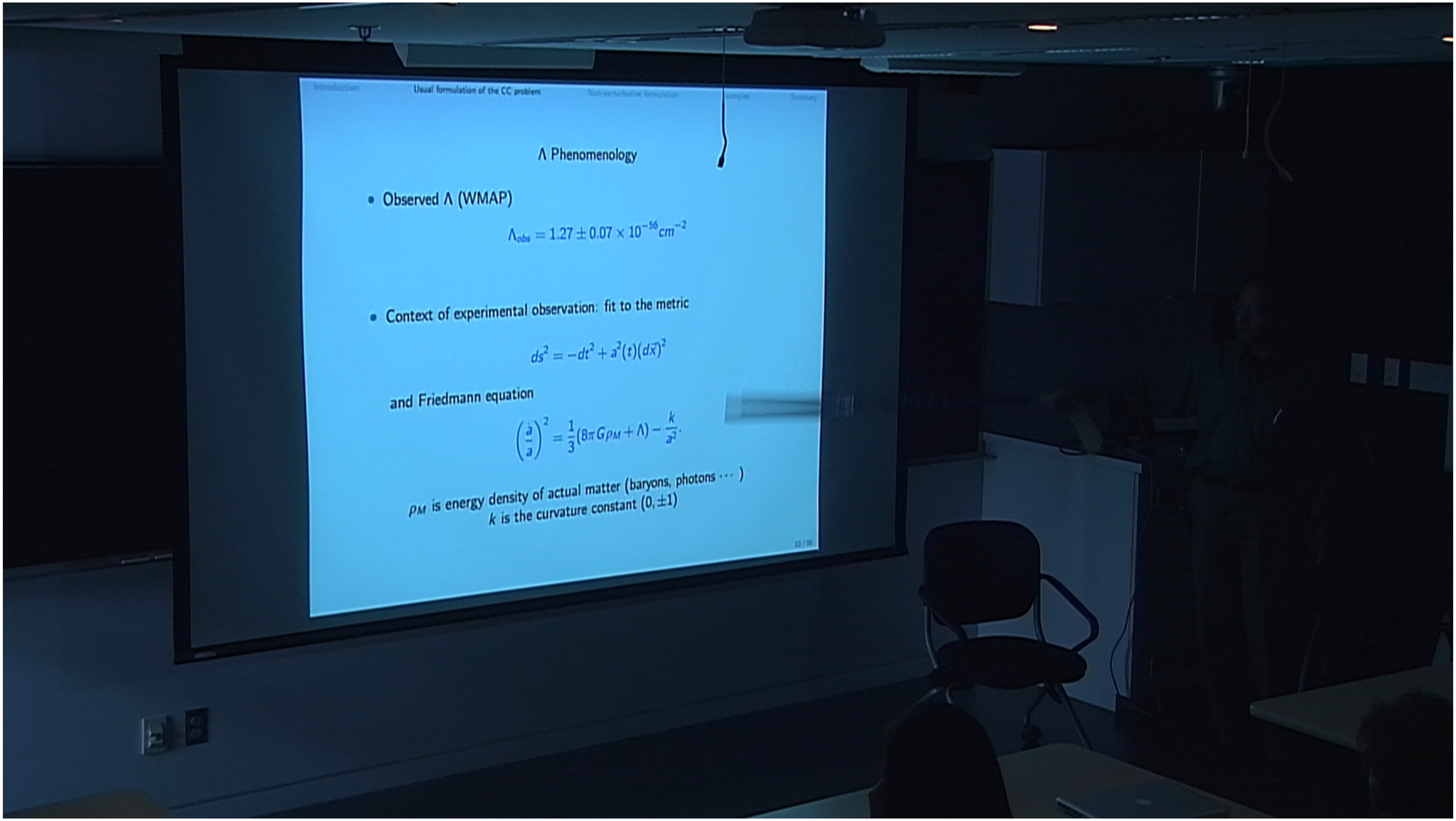

Introduction
Usual formulation of the CC problem
Non-perturbative formulation
Summary
General Relativity + Cosmological Constant (Λ) + Matter
$S = \int d^4x \sqrt{-g}\,(R(g) + \Lambda) + \int d^4x\, L(\phi_i; g_1 \cdots g_N)$
$G_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}(\phi_i, g_1, g_2 \cdots)$


Introduction
Usual formulation of the CC problem
Non-perturbative formulation
Summary
• Λ and $g_1, g_2 \cdots g_N$ are coupling constants.
• The "vacuum energy" (density) of any quantum system is given by its ground state:
$\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle.$
• For the system at hand
$\hat{H} = \hat{H}(\Lambda, g_1, g_2, \cdots g_N).$
$\hat{H}$ is the total physical Hamiltonian of the matter+gravity system.
• Expect $E_0 = E_0(\Lambda, g_1, g_2, \cdots g_N).$


Introduction
Nonperturbatively, is “ρ_{vac}” a linear function of Λ?
– classically and quantum mechanically.


Introduction
Usual formulation of the CC problem
Non-perturbative formulation
Summary
• Λ and $g_1, g_2 \cdots g_N$ are coupling constants.
• The "vacuum energy" (density) of any quantum system is given by its ground state:
$\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle$.
• For the system at hand
$\hat{H} = \hat{H}(\Lambda, g_1, g_2, \cdots g_N)$.
$\hat{H}$ is the total physical Hamiltonian of the matter+gravity system.
• Expect $E_0 = E_0(\Lambda, g_1, g_2, \cdots g_N)$.

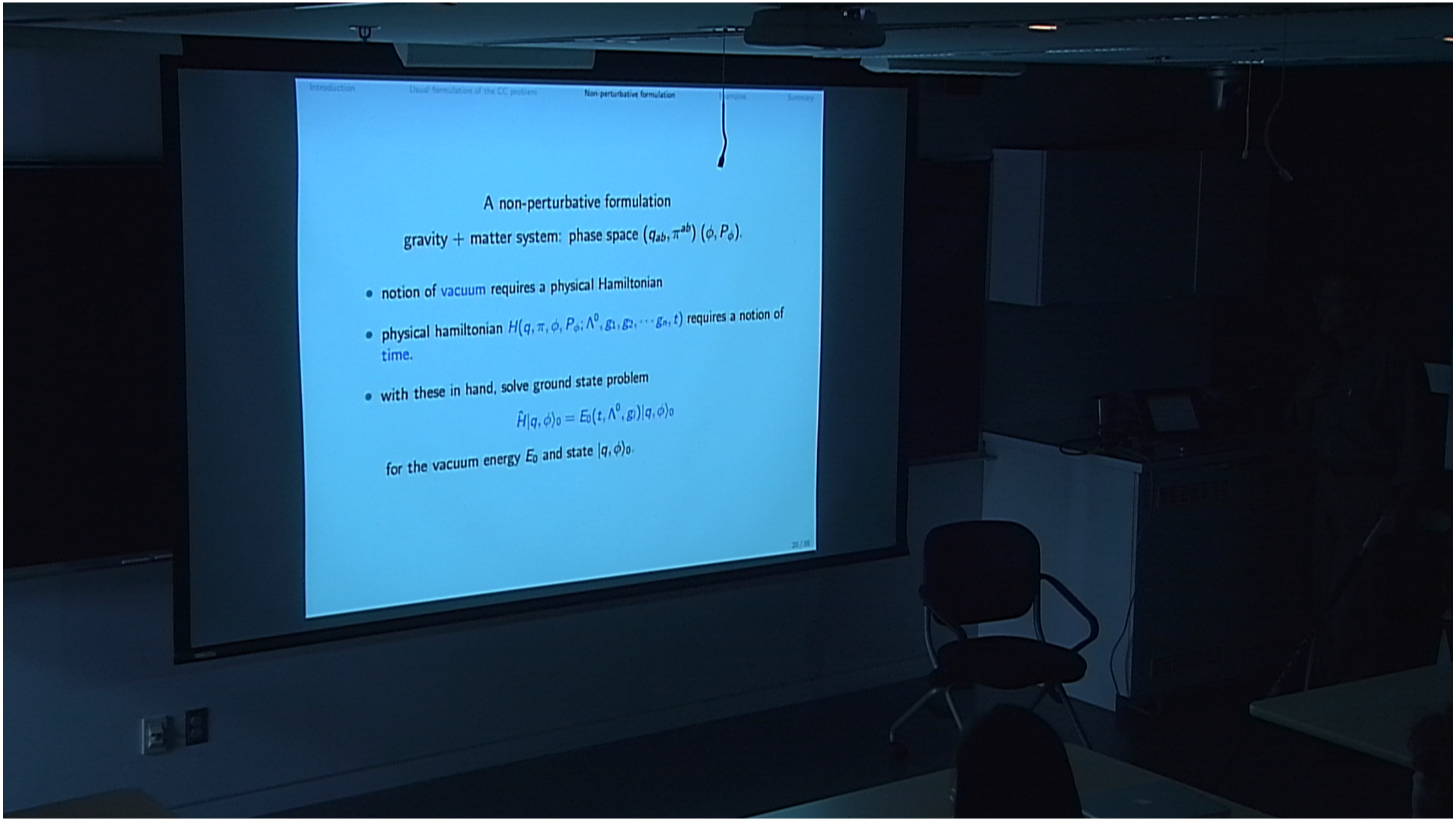

Introduction
Usual formulation of the CC problem
Non-perturbative formulation
Examples
Summary
Nonperturbatively, is "ρ_{vac}" a linear function of Λ?
– classically and quantum mechanically.
7 / 35

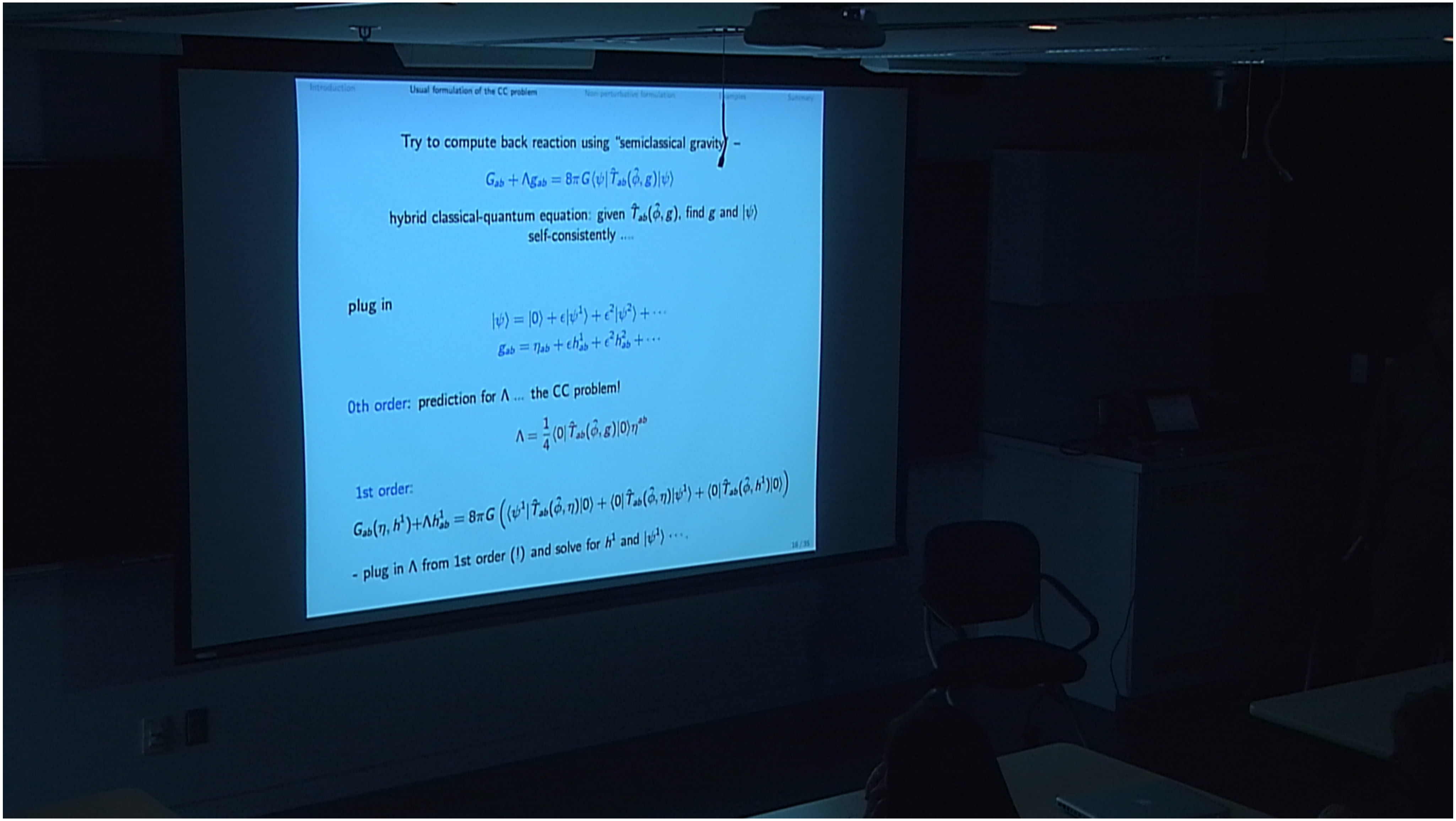

Usual formulation of the CC problem
Λ Phenomenology
Observed Λ (WMAP)
$\Lambda_{obs} = 1.27 \pm 0.07 \times 10^{-56} cm^{-2}$
Context of experimental observation: fit to the metric
$ds^2 = -dt^2 + a^2(t)(d\vec{x})^2$
and Friedmann equation
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}(8\pi G\rho_M + \Lambda) - \frac{k}{a^2}.$
ρ_M is energy density of actual matter (baryons, photons ⋯)
k is the curvature constant $(0, \pm 1)$

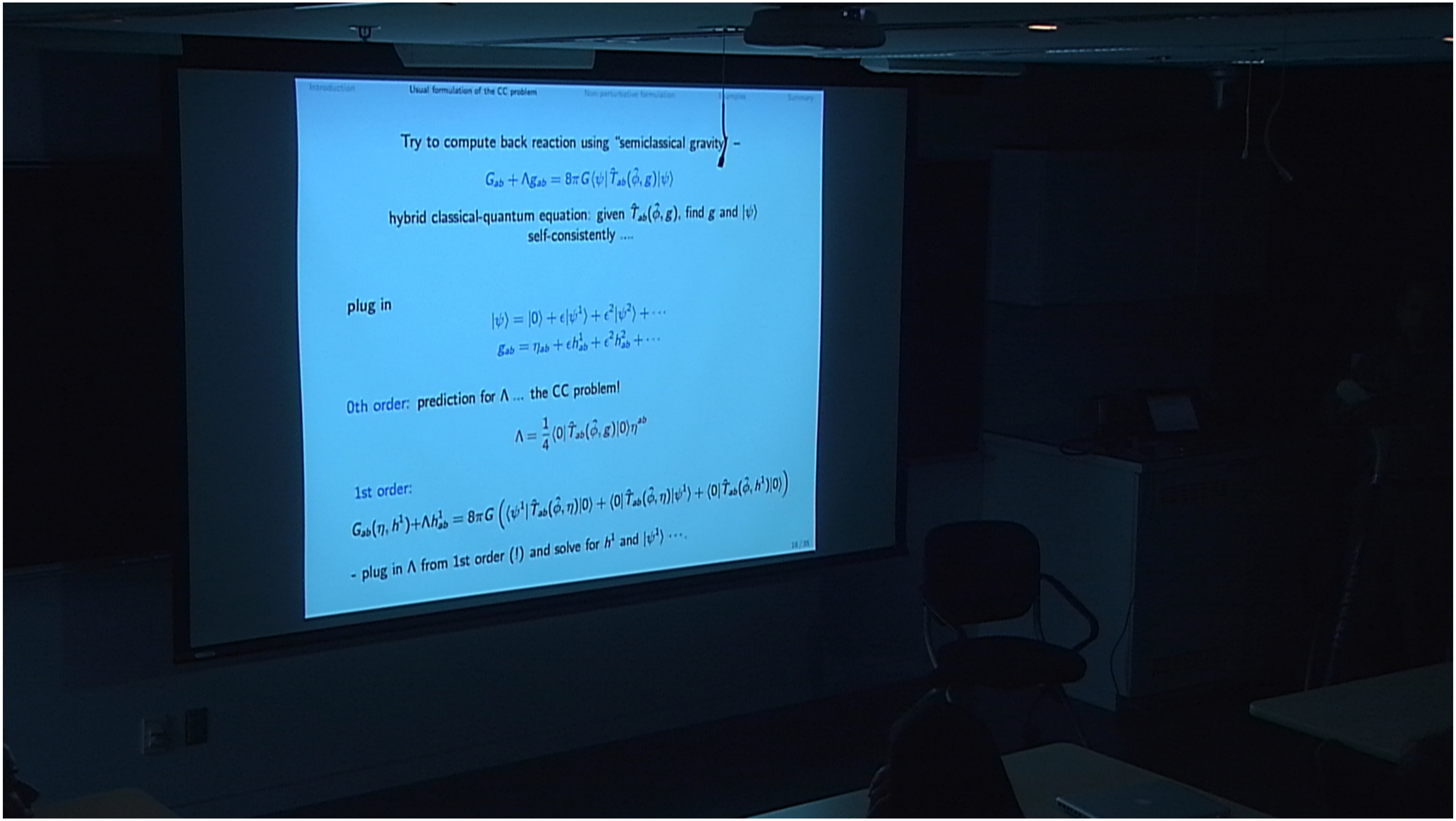
What is the microscopic origin of Λ?

Λ "theory" (usual story): assume origin is the vacuum energy of QFT on a *fixed* background

- $$E_0 = \sum_k \hbar k \longrightarrow \frac{V}{(2\pi)^3} \int d^3k \; \hbar k.$$
- $$\rho_\Lambda := \frac{E_0}{V} = \frac{\hbar}{(2\pi)^3} \int_0^{k_P} 4\pi k^3 dk = \frac{\hbar}{8\pi^2} k_p^4.$$
- $$\Longrightarrow \Lambda_{theory} \equiv 8\pi G \rho_\Lambda \sim 10^{54} cm^{-2}.$$


Introduction
Usual formulation of the CC problem
Non-perturbative formulation
A non-perturbative formulation
gravity + matter system: phase space (q_{ab}, π^{ab}) (ϕ, P_ϕ).
notion of vacuum requires a physical Hamiltonian
physical hamiltonian $H(q, \pi, \phi, P_\phi; \Lambda^0, g_1, g_2, \cdots g_n, t)$ requires a notion of time.
with these in hand, solve ground state problem
$\hat{H}|q,\phi\rangle_0 = E_0(t, \Lambda^0, g_i)|q,\phi\rangle_0$
for the vacuum energy E_0 and state $|q,\phi\rangle_0$.


Introduction
Usual formulation of the CC problem
Non-perturbative formulation
Examples
Summary
Try to compute back reaction using "semiclassical gravity" –
$G_{ab} + \Lambda g_{ab} = 8\pi G \langle\psi|\hat{T}_{ab}(\hat{\phi}, g)|\psi\rangle$
hybrid classical-quantum equation: given $\hat{T}_{ab}(\hat{\phi}, g)$, find g and $|\psi\rangle$ self-consistently ....
plug in
$|\psi\rangle = |0\rangle + \epsilon|\psi^1\rangle + \epsilon^2|\psi^2\rangle + \cdots$
$g_{ab} = \eta_{ab} + \epsilon h^1_{ab} + \epsilon^2 h^2_{ab} + \cdots$
0th order: prediction for Λ ... the CC problem!
$\Lambda = \frac{1}{4}\langle 0|\hat{T}_{ab}(\hat{\phi}, g)|0\rangle\eta^{ab}$
1st order:
$G_{ab}(\eta, h^1) + \Lambda h^1_{ab} = 8\pi G \left(\langle\psi^1|\hat{T}_{ab}(\hat{\phi}, \eta)|0\rangle + \langle 0|\hat{T}_{ab}(\hat{\phi}, \eta)|\psi^1\rangle + \langle 0|\hat{T}_{ab}(\hat{\phi}, h^1)|0\rangle\right)$
- plug in Λ from 1st order (!) and solve for h^1 and $|\psi^1\rangle \cdots$.


Introduction Usual formulation of the CC problem Non perturbative formulation Examples Summary
Try to compute back reaction using "semiclassical gravity" –
$G_{ab} + \Lambda g_{ab} = 8\pi G \langle \psi | \hat{T}_{ab}(\hat{\phi}, g) | \psi \rangle$
hybrid classical-quantum equation: given $\hat{T}_{ab}(\hat{\phi}, g)$, find g and $|\psi\rangle$ self-consistently ....
plug in
$|\psi\rangle = |0\rangle + \epsilon|\psi^1\rangle + \epsilon^2|\psi^2\rangle + \cdots$
$g_{ab} = \eta_{ab} + \epsilon h^1_{ab} + \epsilon^2 h^2_{ab} + \cdots$
0th order: prediction for Λ ... the CC problem!
$\Lambda = \frac{1}{4}\langle 0 | \hat{T}_{ab}(\hat{\phi}, g) | 0 \rangle \eta^{ab}$
1st order:
$G_{ab}(\eta, h^1) + \Lambda h^1_{ab} = 8\pi G \left(\langle \psi^1 | \hat{T}_{ab}(\hat{\phi}, \eta) | 0 \rangle + \langle 0 | \hat{T}_{ab}(\hat{\phi}, \eta) | \psi^1 \rangle + \langle 0 | \hat{T}_{ab}(\hat{\phi}, h^1) | 0 \rangle \right)$
- plug in Λ from 1st order (!) and solve for h^1 and $|\psi^1\rangle \cdots$.

Nonperturbative quantum vacuum for this model with

$$V(\phi) = \frac{1}{2} m^2 \phi^2.$$

Argument of the square root can be diagonalized – S.H.O. with a shift:

$$\left[\frac{8}{3}\Lambda + 8\pi G \left(\frac{\hat{p}_\phi^2}{2t^2} + \frac{1}{2} m^2 \hat{\phi}^2\right)\right] \Psi(\phi) = E^2 \Psi(\phi)$$

$$E_n^2 = \frac{8}{3}\Lambda + \left(n + \frac{1}{2}\right)\omega, \qquad \omega = 8\pi G \frac{m}{t}$$

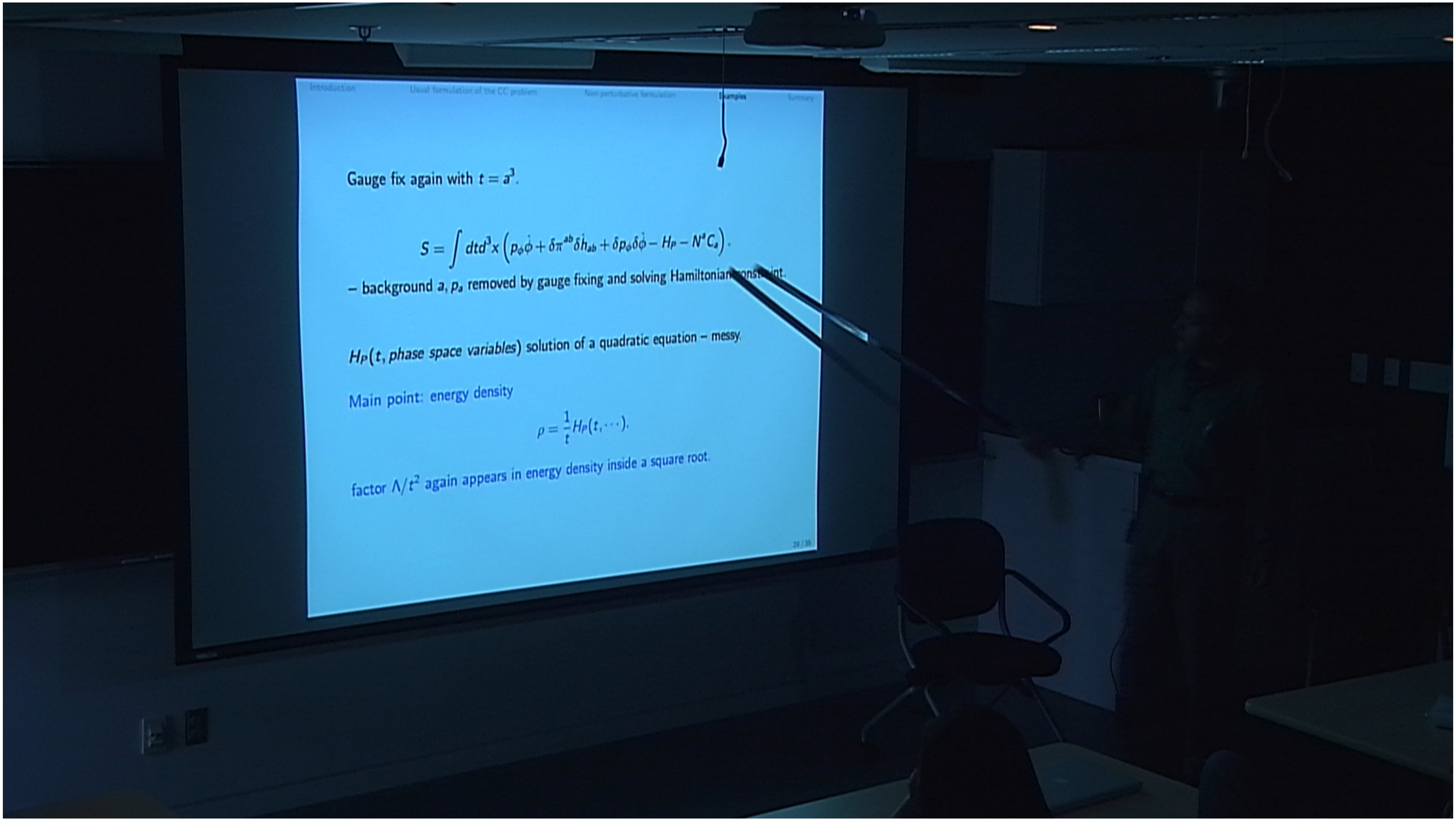
... therefore lowest energy density eigenvalue "vacuum" is

$$\rho_0 = \sqrt{\frac{8}{3t^2}\Lambda + 8\pi G \ \frac{m}{2t^3}}$$

damps to zero at late times – no CC problem!

Comments:

- We have looked at only one time gauge, in a homogeneous setting
- The result for vacuum energy density necessarily *depends* on choice of time $\rightarrow - \rho(\Lambda, t)$ not linear function of Λ.
- Other gauges: Hubble time, matter time, ...
- Inhomogeneous case? – next


Gauge fix again with $t = a^3$.
$S = \int dtd^3x \left(p_\phi \dot{\phi} + \delta\pi^{ab}\delta\dot{h}_{ab} + \delta p_\phi \delta\dot{\phi} - H_P - N^a C_a \right).$
– background a, p_a removed by gauge fixing and solving Hamiltonian
$H_P(t, \textit{phase space variables})$ solution of a quadratic equation – messy.
Main point: energy density
$\rho = \frac{1}{t} H_P(t, \cdots).$
factor Λ/t^2 again appears in energy density inside a square root.

As the problem really involves quantum gravity, string theory is the only framework for addressing it, at least with our present state of knowledge. Moreover, in string theory, the question is very sharply posed, as there is no dimensionless parameter. Assuming that the dynamics gives a unique answer for the vacuum, there will be a unique prediction for the cosmological constant. But that is, at best, a futuristic way of putting things. We are not anywhere near, in practice, to understanding how there would be a unique solution for the dynamics. In fact, with what we presently know, it seems almost impossible for this to be true...

E. Witten, The Cosmological Constant From the Viewpoint of String Theory hep-ph/0002297