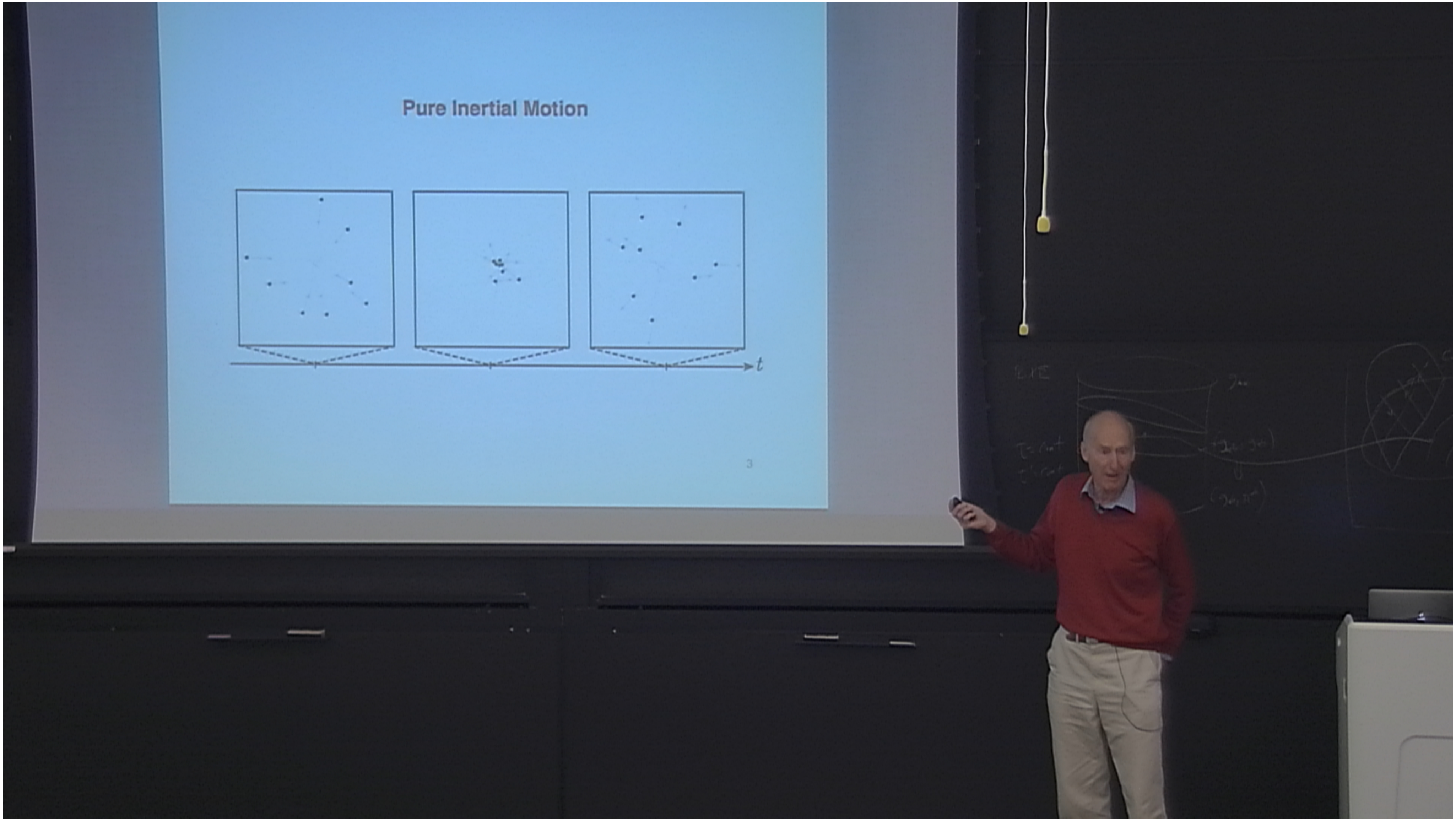
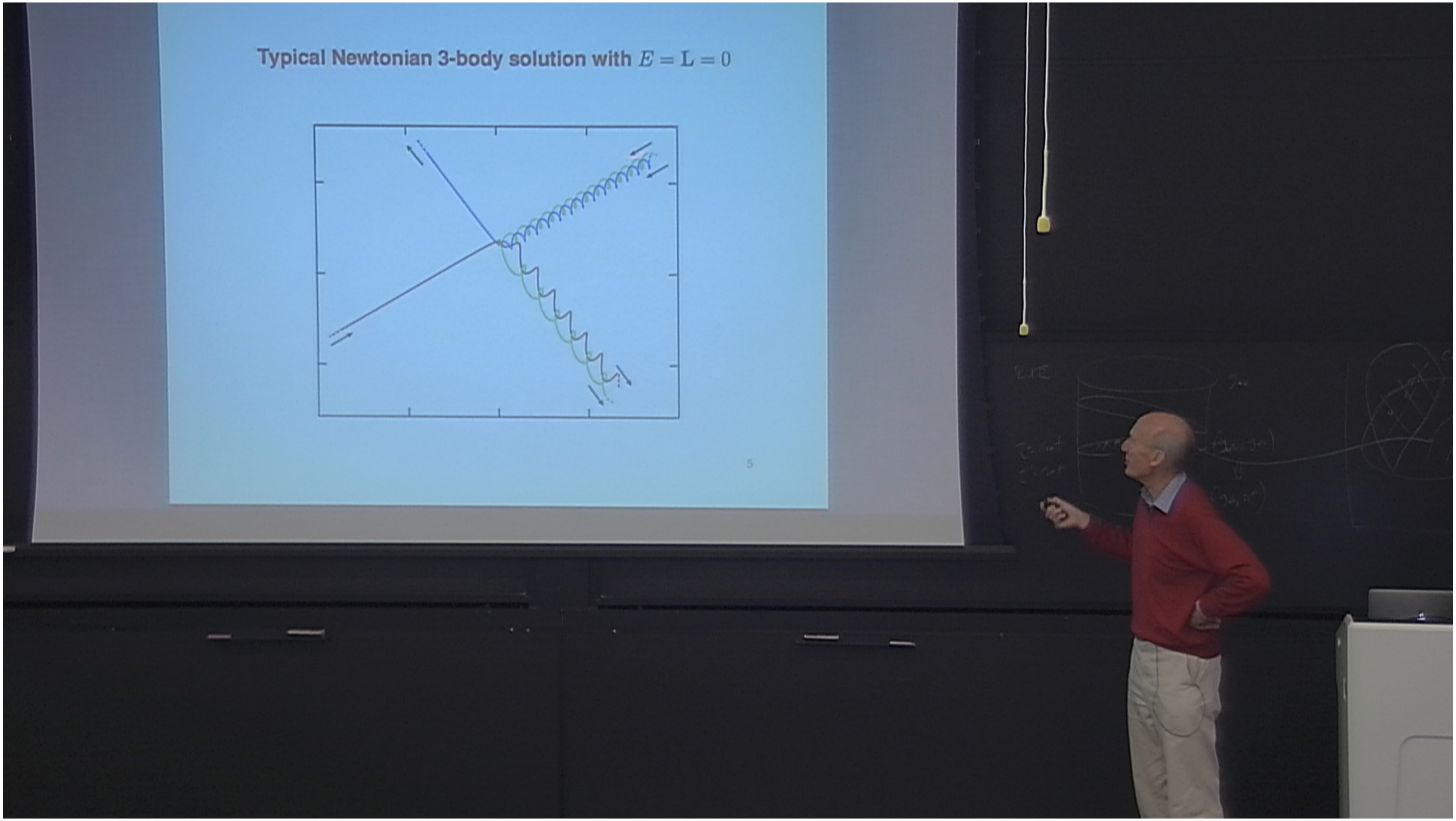
Title: Typicality of Universes and Discussion

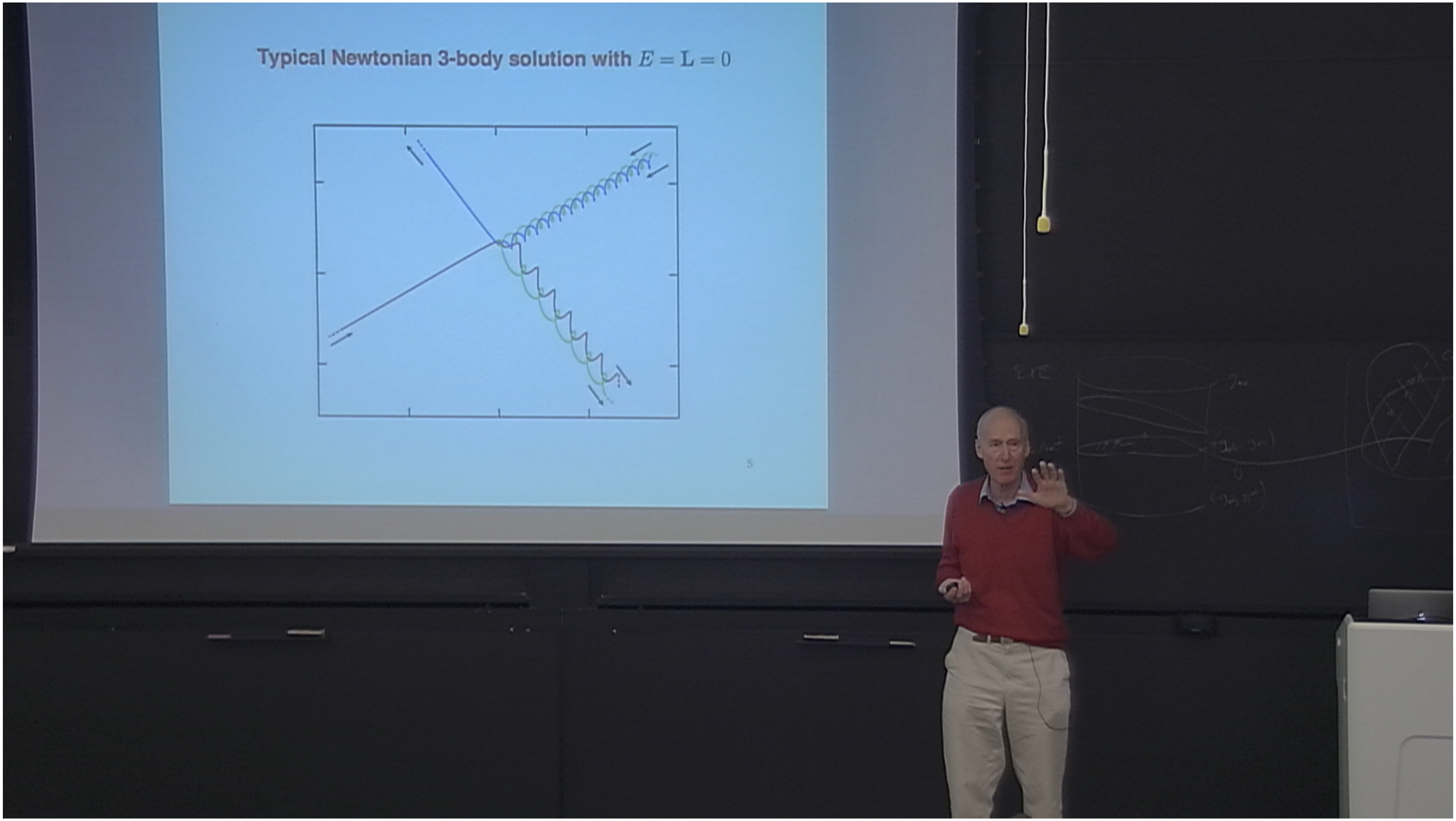
Date: Jun 26, 2015 09:30 AM

URL: http://pirsa.org/15060030

Abstract:


Pure Inertial Motion
t
3


Typical Newtonian 3-body solution with $E = \mathbf{L} = 0$
5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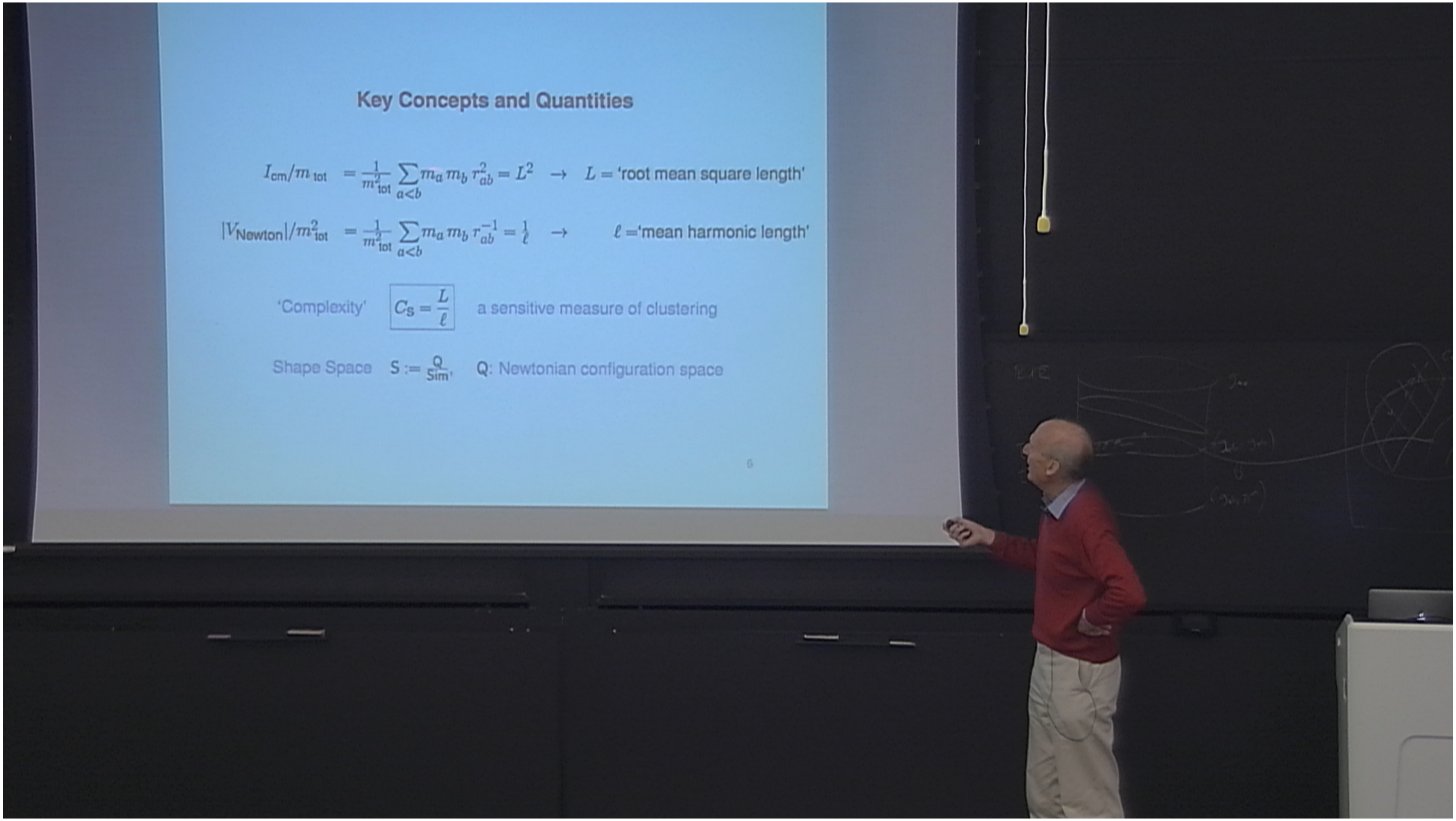
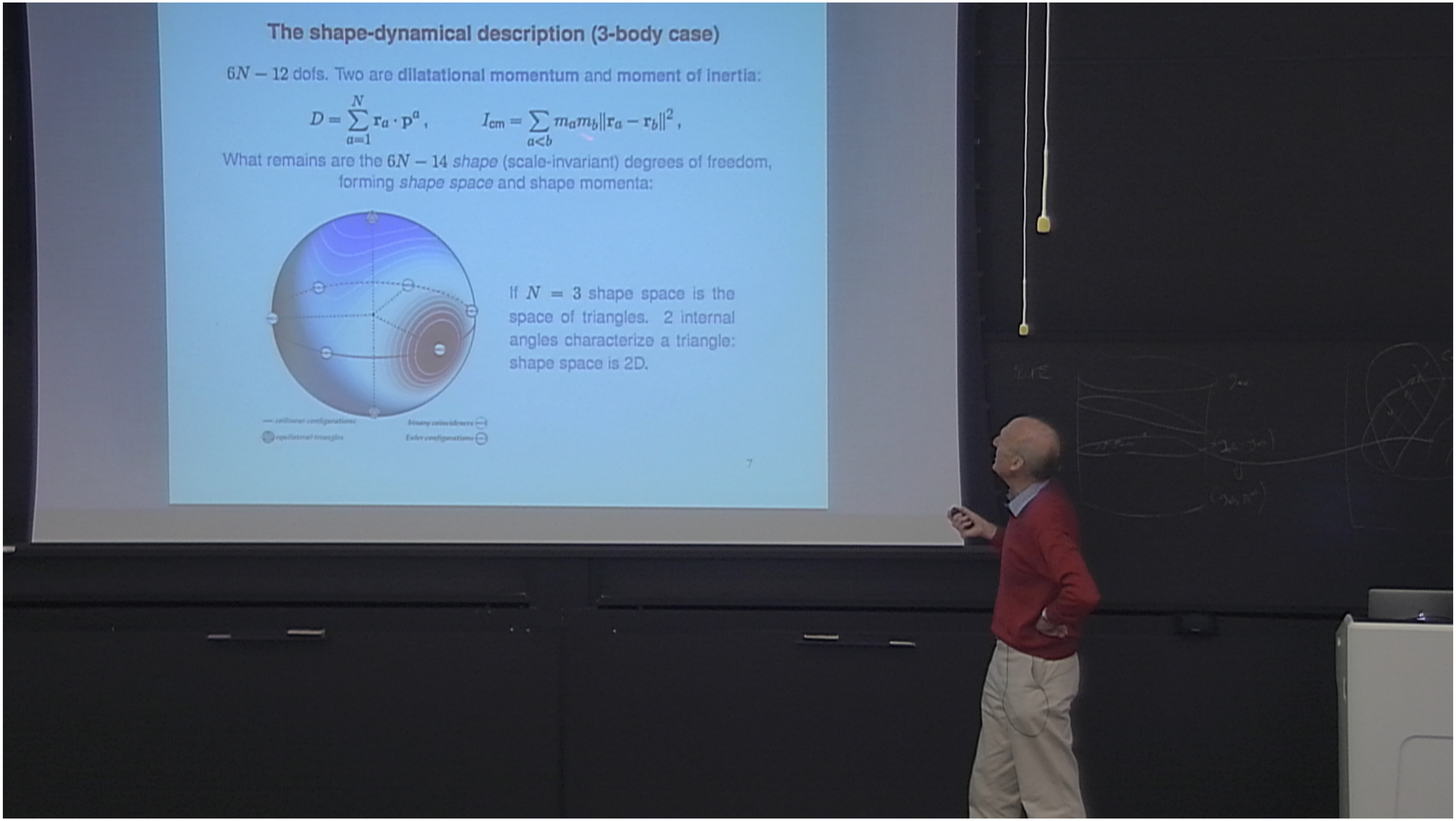

Key Concepts and Quantities
$I_{cm}/m_{tot} = \frac{1}{m_{tot}^2}\sum_{a<b} m_a m_b r_{ab}^2 = L^2 \rightarrow L =$ 'root mean square length'
$|V_{Newton}|/m_{tot}^2 = \frac{1}{m_{tot}^2}\sum_{a<b} m_a m_b r_{ab}^{-1} = \frac{1}{\ell} \rightarrow \ell =$ 'mean harmonic length'
'Complexity' $C_S = \frac{L}{\ell}$ a sensitive measure of clustering
Shape Space $S := \frac{Q}{Sim}$, Q: Newtonian configuration space
6


The shape-dynamical description (3-body case)
$6N-12$ dofs. Two are **dilatational momentum** and **moment of inertia**:
$$D=\sum_{a=1}^{N}\mathbf{r}_a\cdot\mathbf{p}^a\,,\qquad I_{\mathrm{cm}}=\sum_{a<b}m_a m_b\|\mathbf{r}_a-\mathbf{r}_b\|^2\,,$$
What remains are the $6N-14$ *shape* (scale-invariant) degrees of freedom, forming *shape space* and shape momenta:
If $N=3$ shape space is the space of triangles. 2 internal angles characterize a triangle: shape space is 2D.
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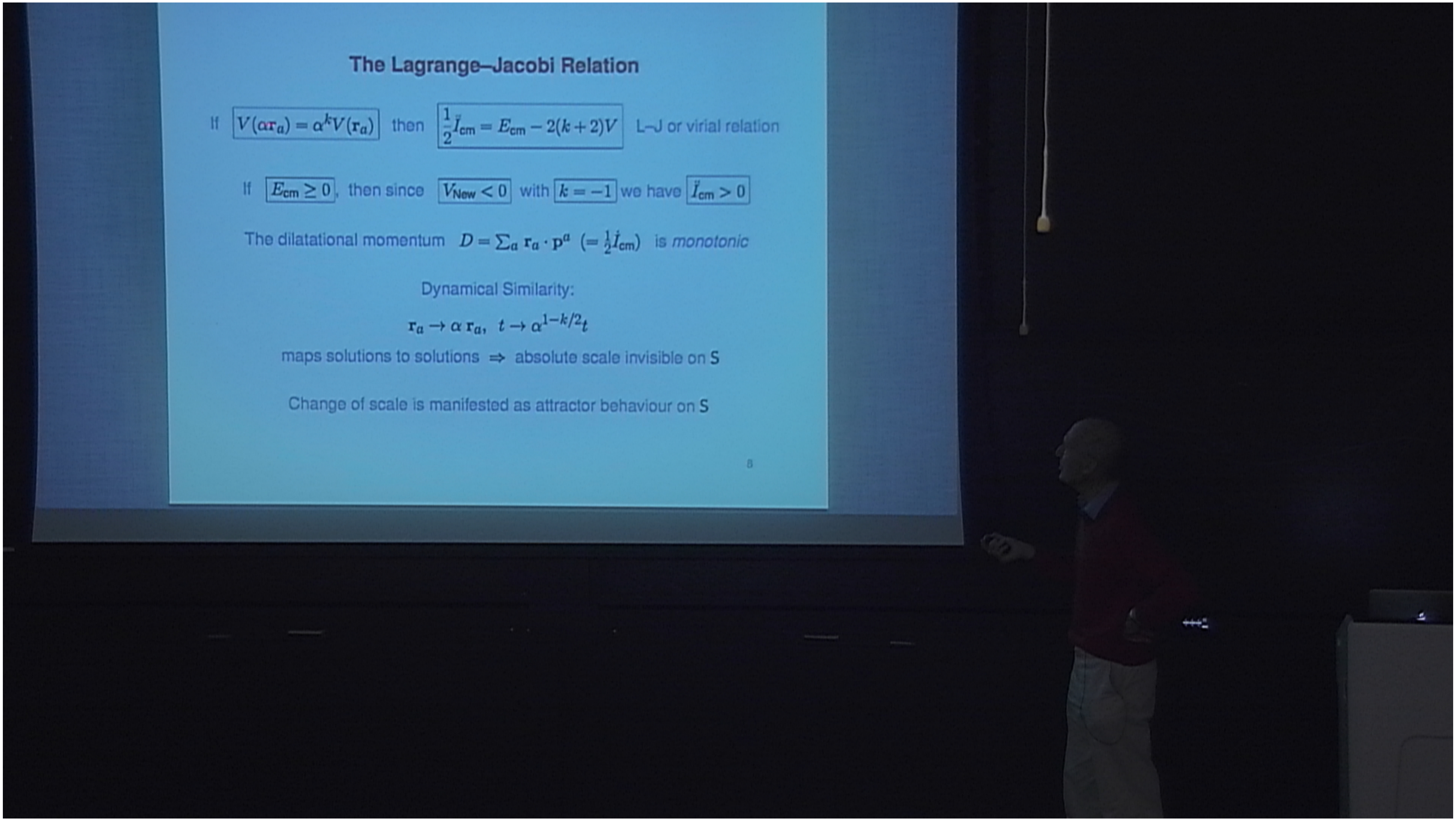
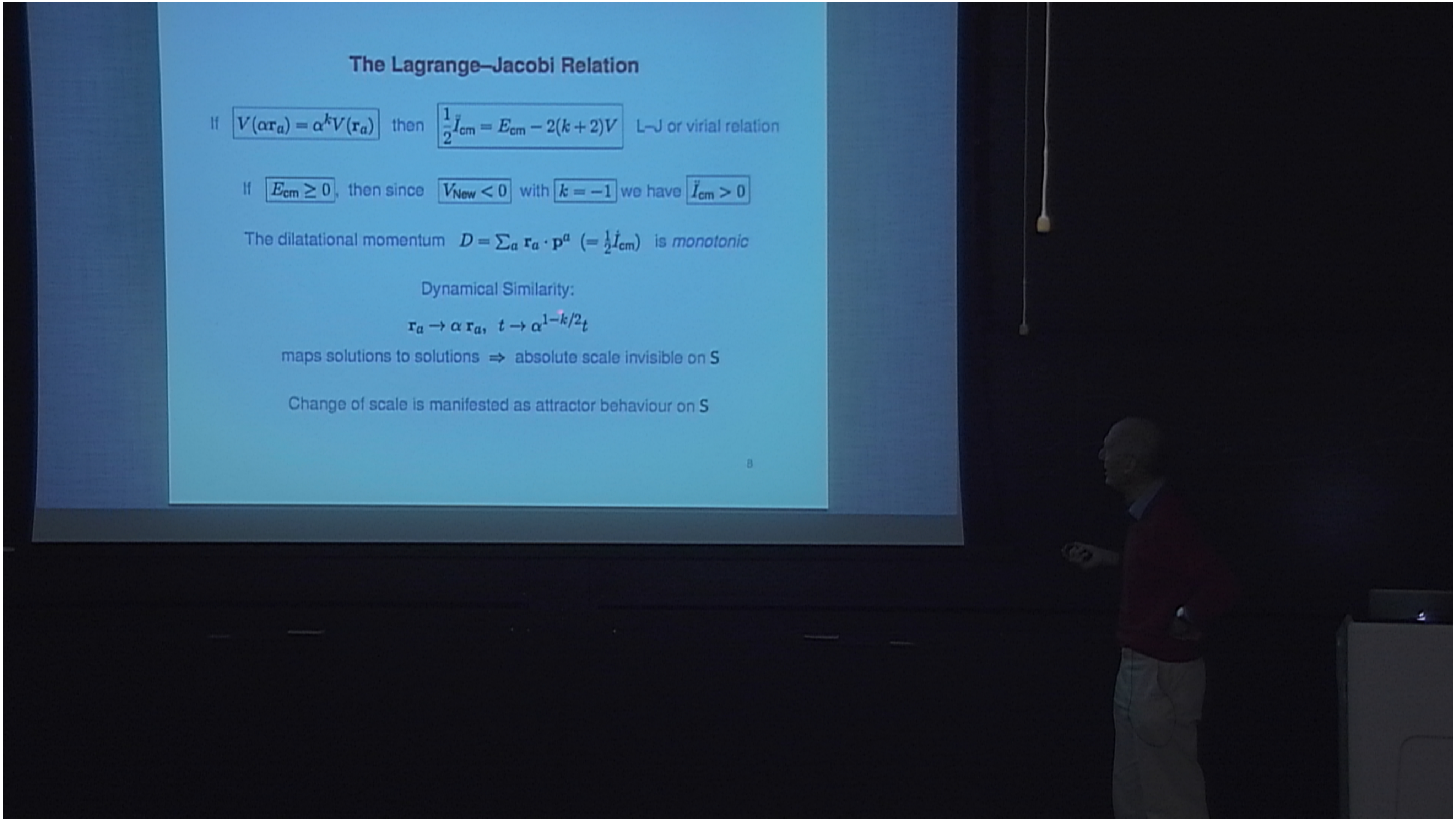

The Lagrange–Jacobi Relation
If $V(\alpha \mathbf{r}_a) = \alpha^k V(\mathbf{r}_a)$ then $\frac{1}{2}\ddot{I}_{\rm cm} = E_{\rm cm} - 2(k+2)V$ L–J or virial relation
If $E_{\rm cm} \geq 0$, then since $V_{\rm New} < 0$ with $k = -1$ we have $\ddot{I}_{\rm cm} > 0$
The dilatational momentum $D = \sum_a \mathbf{r}_a \cdot \mathbf{p}^a$ $(= \frac{1}{2}\dot{I}_{\rm cm})$ is *monotonic*
Dynamical Similarity:
$\mathbf{r}_a \to \alpha\, \mathbf{r}_a,\;\; t \to \alpha^{1-k/2} t$
maps solutions to solutions ⇒ absolute scale invisible on S
Change of scale is manifested as attractor behaviour on S
8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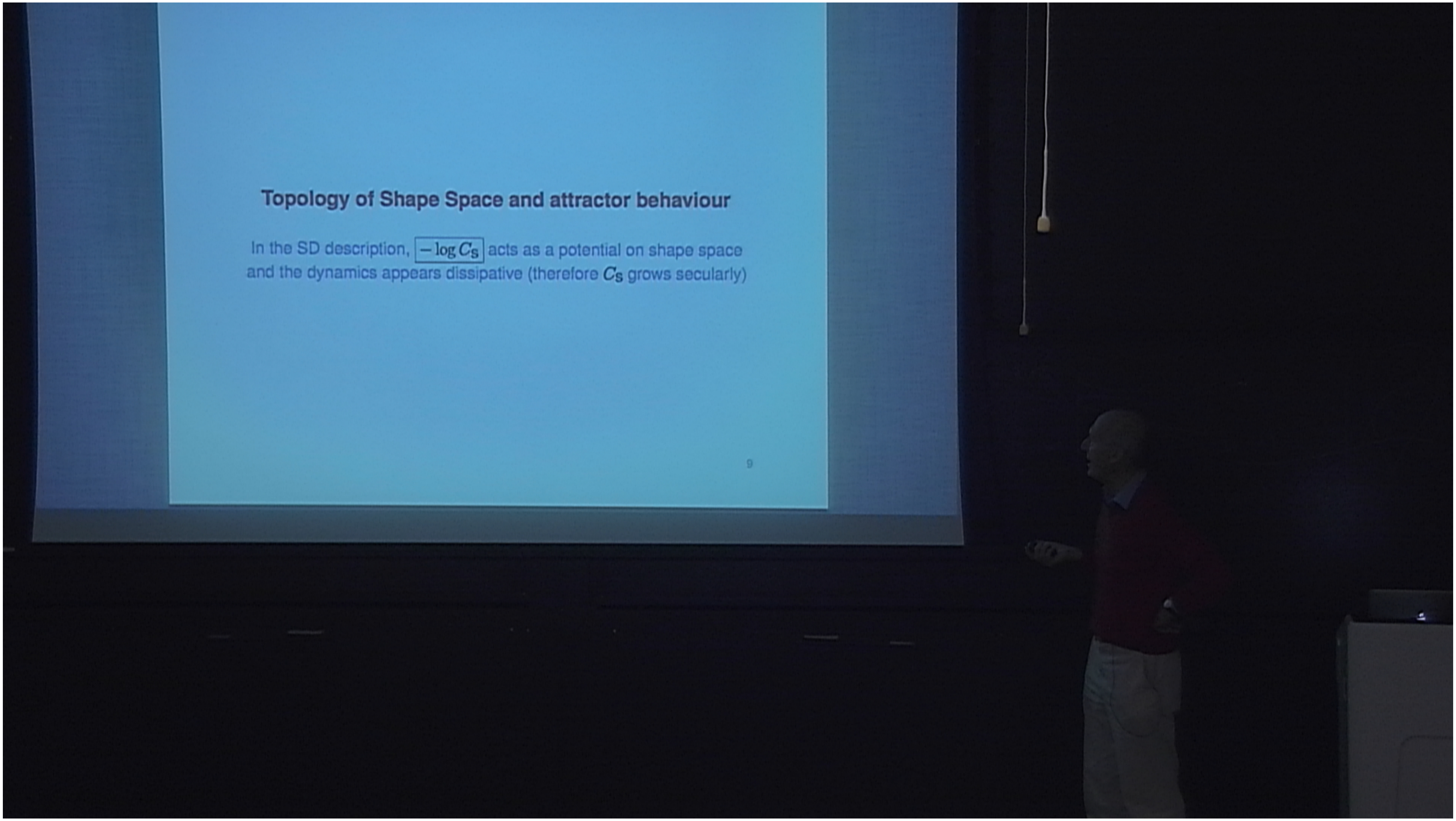
Topology of Shape Space and attractor behaviour
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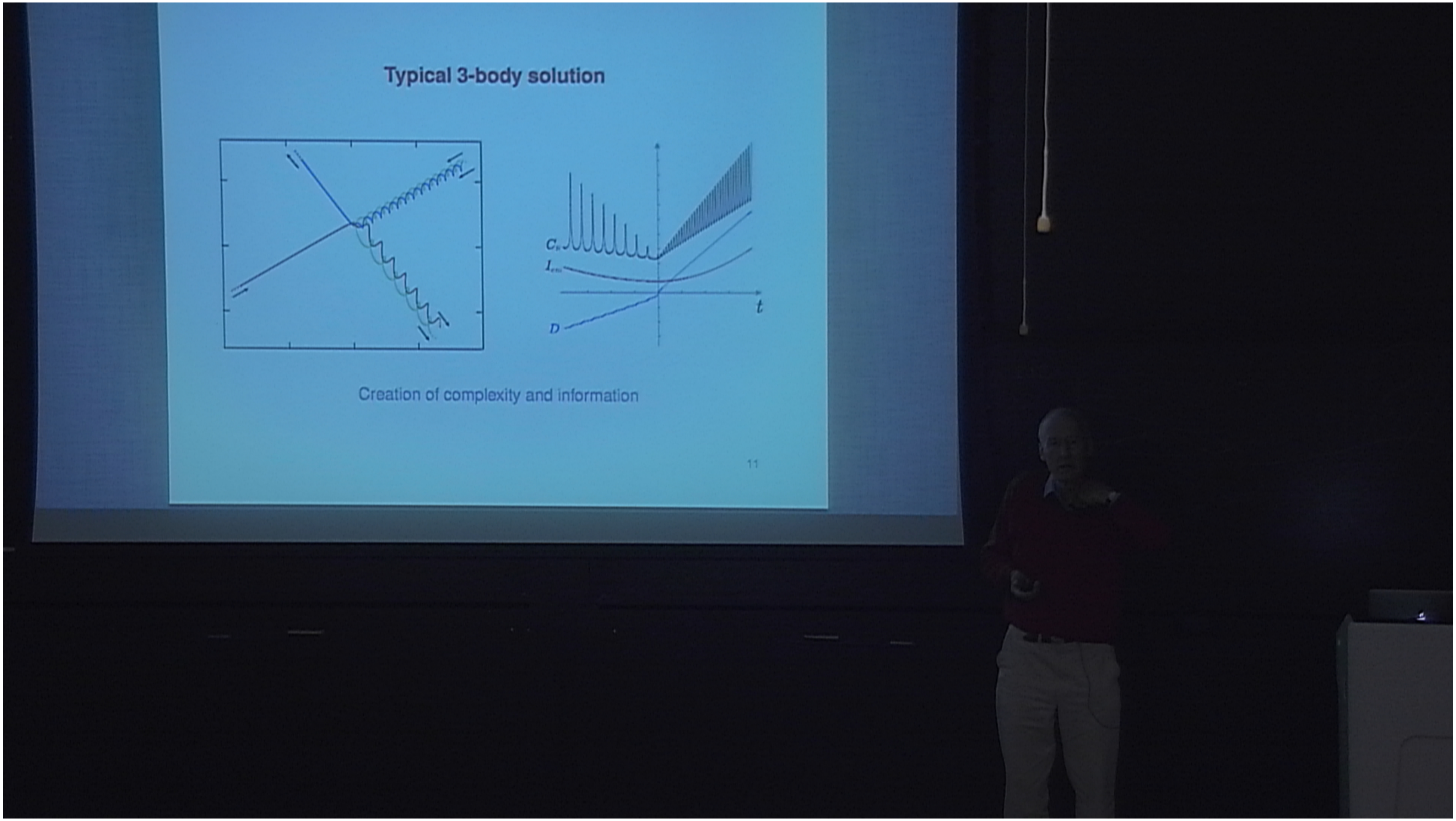
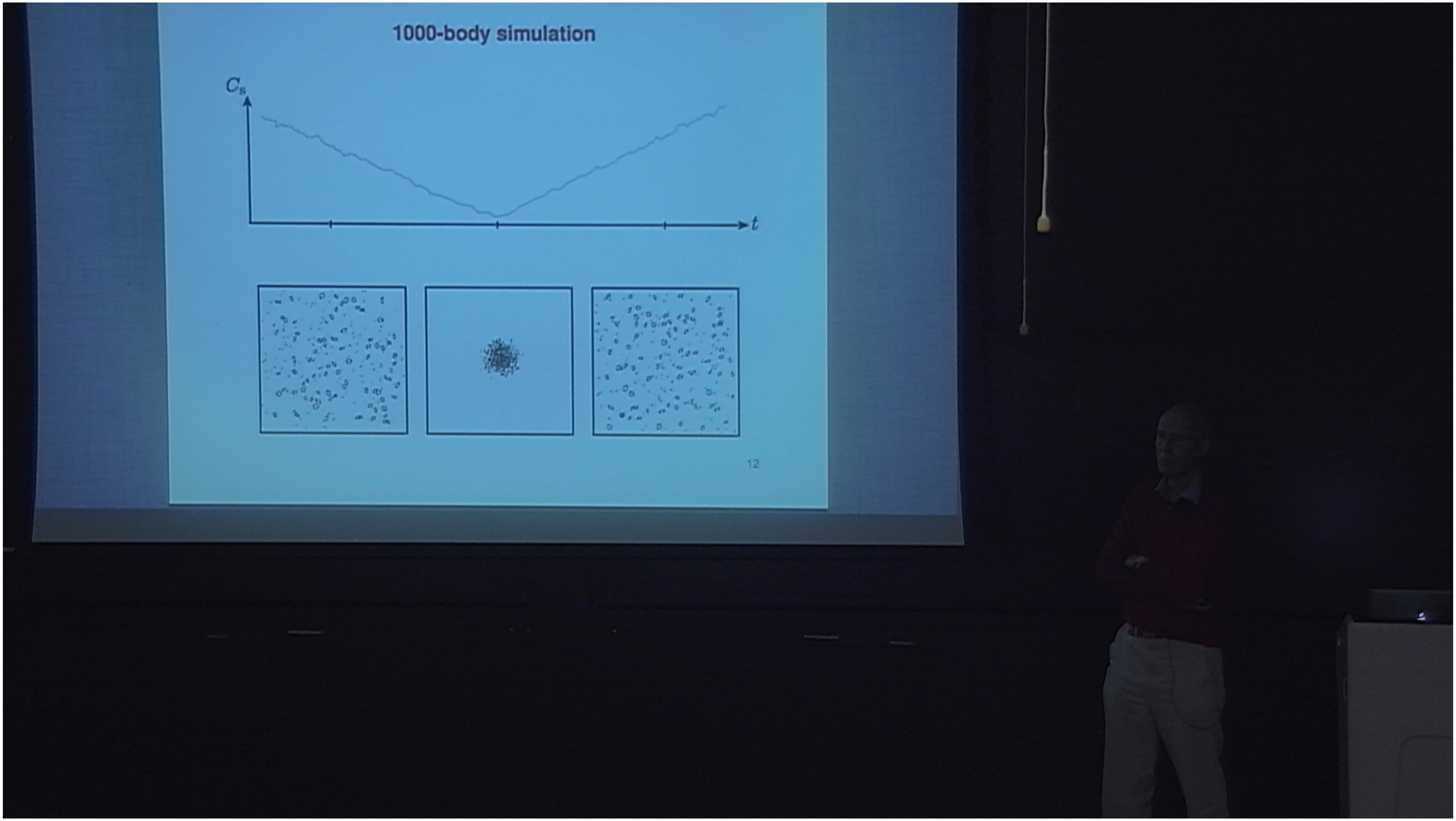
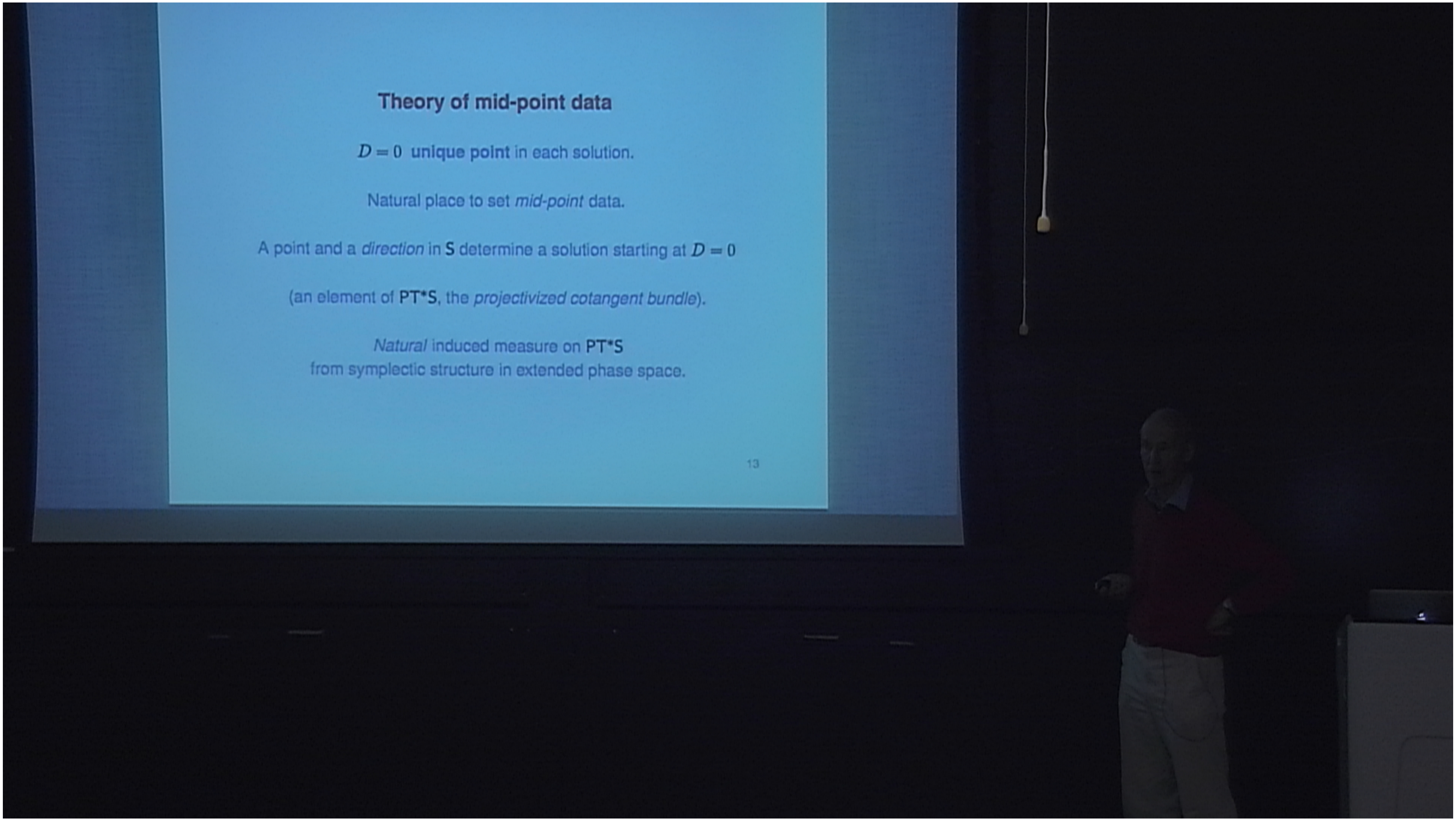
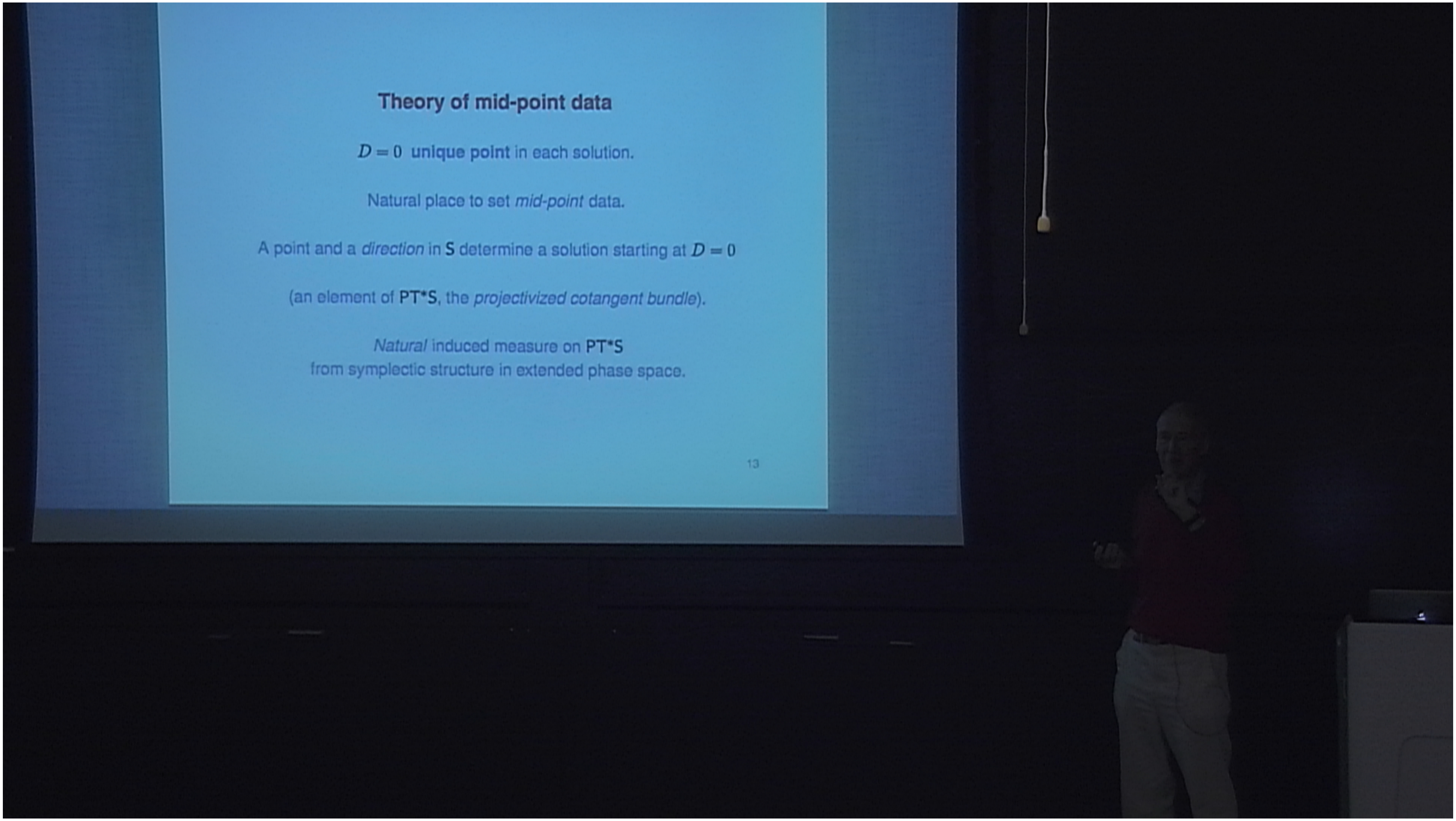

Typical 3-body solution
t
D
Creation of complexity and information
11



1000-body simulation
C_s
t
12


Theory of mid-point data
D = 0 unique point in each solution.
Natural place to set mid-point data.
A point and a direction in S determine a solution starting at D = 0
(an element of PT*S, the projectivized cotangent bundle).
Natural induced measure on PT*S
from symplectic structure in extended phase space.
13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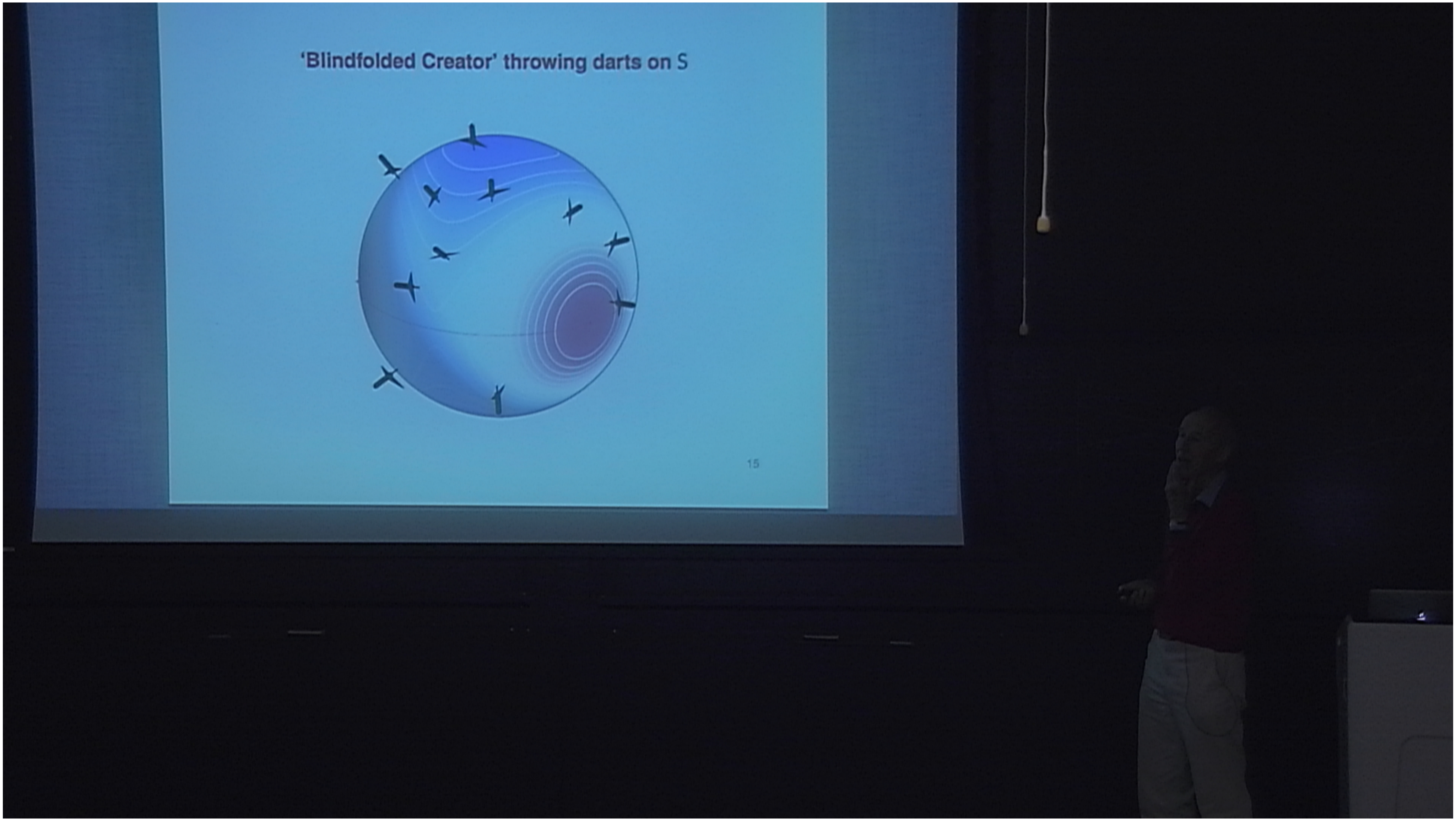
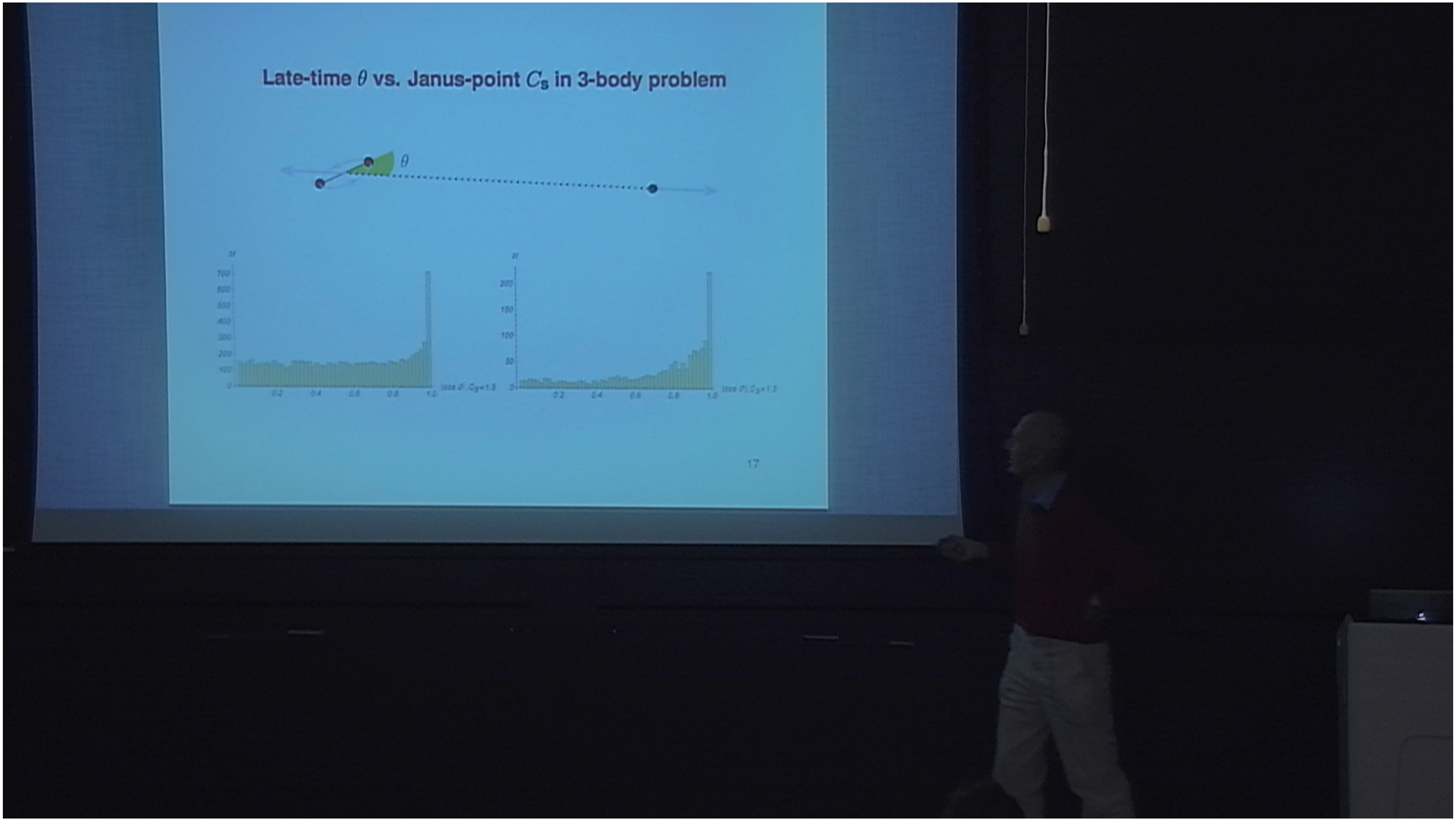

'Blindfolded Creator' throwing darts on S
15


Complexity vs. shape space volume
Vol(C)
20
15
10
5
1.0
1.1
1.2
1.3
1.4
1.5
N=3
N=10
N=15
N=25
N=50
N=100
16


Late-time θ vs. Janus-point C_s in 3-body problem
θ
17