Title: Gravitational Collapse in SD

Date: Jun 25, 2015 09:45 AM

URL: http://pirsa.org/15060026

Abstract:


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SD@Convergence Workshop
Gravitational Collapse in SD
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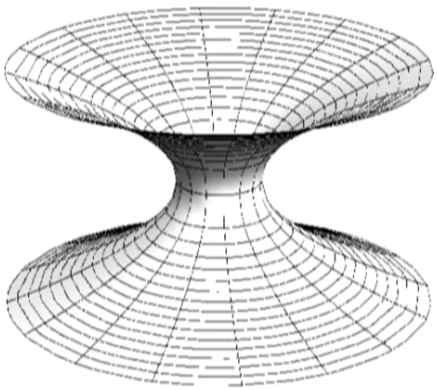
Isotropic Wormhole Solution

$$ds^2 = \left(\frac{1-\frac{\alpha}{4r}}{1+\frac{\alpha}{4r}}\right)^2 dt^2 + \left(1+\frac{\alpha}{4r}\right)^4 \left(dr^2 + r^2 d\Omega_2\right)$$



- No Singularity
- Symmetry under inversion $r \to \frac{\alpha^2}{16r}$
- Two Schwarzschild exteriors glued together at the throat

THE ISOTROPIC SOLUTION IS ETERNAL

"A Birkhoff theorem for Shape Dynamics" H. Gomes arXiv:1305.0310

The Thin Shell Model

WHAT?

Dynamical creation of the isotropic solution through gravitational collapse

HOW?

Spherically symmetric infinitely thin shell of dust

The Thin Shell Model: Starting Point

Metric tensor and conjugate momentum

$$g_{ij} = \begin{pmatrix} \mu^2 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma sin(\theta)^2 \end{pmatrix} \quad p^{ij} = \sin(\theta) \begin{pmatrix} \frac{f}{\mu} & 0 & 0 \\ 0 & \frac{1}{2}s & 0 \\ 0 & 0 & \frac{1}{2}s\, sin(\theta)^{-2} \end{pmatrix}$$

Hamiltonian, diffeo and conformal constraints

$$\frac{1}{2\sigma\mu^2}\left[2f\sigma\mu^2 s - f^2\mu^3 + \mu(\sigma')^2 + 4\sigma\mu^3 + 4\sigma\mu'\sigma' - 4\sigma\mu\sigma''\right] = \delta(r - R)\sqrt{\frac{P^2}{\mu^2} + M^2}$$

$$\mu f' - \frac{1}{2}s\sigma' = -\frac{P}{2}\delta(r - R)$$

$$\mu f + s\sigma = 0$$

Solution to Constraints

Solutions

- $f = p^{rr}\mu = \frac{A}{\sqrt{\sigma}}$
- $\mu^2 = g_{rr} = \frac{(\sigma')^2}{4\alpha\sqrt{\sigma}+4\sigma+\frac{A^2}{\sigma}}$

Integration Constants

- $A_{\text{in}}, A_{\text{out}}$
- $\alpha_{\text{in}}, \alpha_{\text{out}}$

4 spatial integration constants
2 inside the shell, 2 outside the shell

Metric EoM and LFE

Metric Tensor

$$\dot{g}_{ij} = \left(\frac{2f\mu^2 N}{\sigma} + 2\mu\xi\mu' + 2\mu^2\xi' \right) \delta^r{}_i \delta^r{}_j + \left(\frac{\sigma s N}{\mu} + \xi\sigma' \right) \left(\delta^\theta{}_i \delta^\theta{}_j + \delta^\phi{}_i \delta^\phi{}_j \sin^2\theta \right)$$

Conjugate Momentum

$$\begin{aligned}
\dot{p}^{ij} = &- \frac{\sin\theta}{4\sigma\mu^3} \left[5f^2 N\mu^2 + 4\sigma(N'\sigma' - \mu^2\xi f') + N(-4\sigma\mu^2 + (\sigma')^2) + 4f\sigma\mu(\xi\mu' + \mu\xi') \right] \delta^i{}_r \delta^j{}_r \\
&+ \frac{\sin\theta}{4\sigma^2\mu^2} \left[(f^2 N\mu^3 + N\mu(\sigma')^2 + 2\sigma^2(2N'\mu' + \mu^2(\xi s' + s\xi') - 2\mu N'') \right. \\
&\left. -2\sigma(-N\sigma'\mu' + \mu(N'\sigma' + N\sigma''))) \right] \left(\delta^i{}_\theta \delta^j{}_\theta + \delta^i{}_\phi \delta^j{}_\phi \sin^{-2}\theta \right) \\
&+ \sin\theta\, \delta^i{}_r \delta^j{}_r \frac{P^2/\mu^4}{2\sqrt{P^2/\mu^2 + M^2}} N(R)\delta(r - R)
\end{aligned}$$

Lapse Fixing Equation

$$\begin{aligned}
&\frac{1}{4\sigma\mu^2} \left[6f^2 N\mu^3 + 4\sigma \left(-\mu \left(2N'\sigma' + N\sigma'' \right) + N\sigma'\mu' + N\mu^3 \right) + N\mu(\sigma')^2 \right. \\
&\left. +\sigma^2 \left(\mu \left(3Ns^2 - 8N'' \right) + 8N'\mu' \right) \right] + \frac{P^2/\mu^2}{2\sqrt{P^2/\mu^2 + M^2}} N\delta(r - R) = 0
\end{aligned}$$

Solutions to EoM

Lapse function

$$N = \frac{\sigma'}{2\mu\sqrt{\sigma}}\left(c_1 + c_2\int_{r_a}^{r} dy\,\frac{\mu^3(y)}{(\sigma'(y))^2}\right)$$

Shift vector

$$\xi = \frac{\dot{\sigma}}{\sigma'} + \frac{A}{\sigma^{\frac{1}{2}}\sigma'}N(r)$$

Metric tensor and conjugate momentum

$$\dot{\alpha} = 0 \quad c_2 = -2\dot{A}$$

Assumptions and jump conditions

Integration Constants

$A_{\rm in}$ $A_{\rm out}$ $\alpha_{\rm in}$ $\alpha_{\rm out}$ $c_{1\,\rm in}$ $c_{1\,\rm out}$

Assumptions

- Continuity of the metric tensor
- No mass inside the shell
- Asymptotic flatness at infinity
- Continuity of the Lapse
- $N = 1$ at infinity

Jump Conditions

- Diffeo constraint
- Hamiltonian constraint
- LFE
- EoM for p_{ij}

$A_{\rm in}$ $A_{\rm out}$ $\alpha_{\rm out}$ $c_{1\,\rm out}$ $\alpha_{\rm in} = 0$

Assumptions and jump conditions

Integration Constants

$$A_{\text{in}}\ A_{\text{out}}\ \alpha_{\text{in}}\ \alpha_{\text{out}}\ c_{1\,\text{in}}\ c_{1\,\text{out}}$$

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- Continuity of the metric tensor
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- EoM for p_{ij}

$$A_{\text{in}}\ A_{\text{out}}\ \alpha_{\text{out}}\ c_{1\,\text{out}}\ \alpha_{\text{in}} = 0$$

Reduced Phase Space

On-Shell Relation

$$(\alpha_{out}+\rho){A_{in}}^2+\rho {A_{out}}^2+\left(\frac{M^2}{16\rho}-\alpha_{out}-2\rho\right)A_{in}A_{out}-\rho^3(\alpha_{out})^2-\frac{M^2\rho^2}{8}\left(\frac{M^2}{32\rho}-\alpha_{out}-2\rho\right)=0$$

Independent Variables

- $\rho=\sigma(R)=g_{\theta\theta}(R)$ Area of the Shell
- A_{in} A_{out} Related to the Momentum P
- α_{out} Related to the ADM mass

UNDERDETERMINED SYSTEM

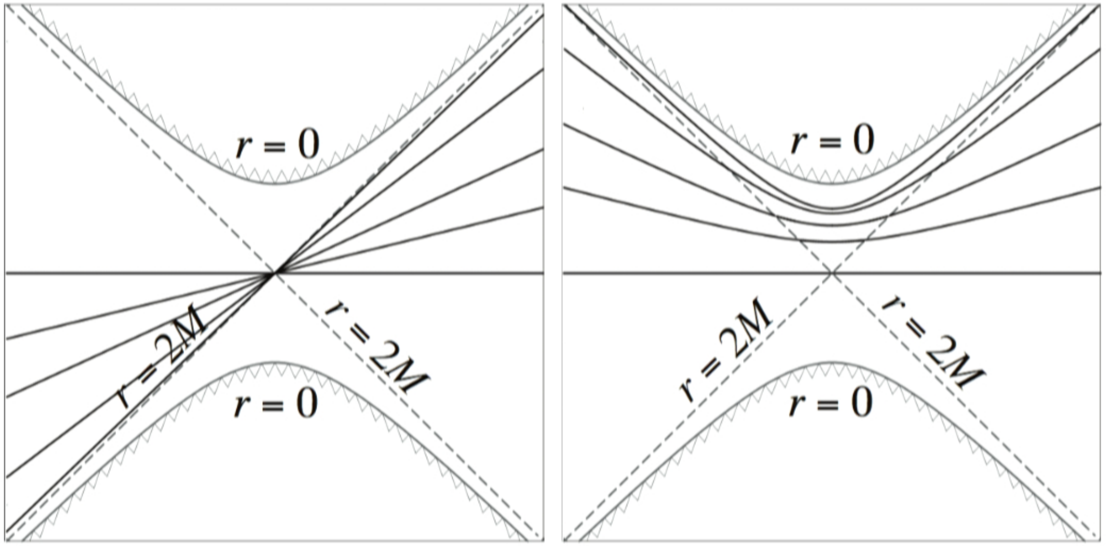
The on-shell relation and the condition $\alpha_{out}=const.$ define a 2-dimensional manifold - not a one dimensional curve

What is the under determination?

- Spherically symmetric thin shell
- Conformal constraint $g_{ij}p^{ij} = 0$
- General result: Birkhoff Theorem

Schwarzschild space-time in maximal foliation

MAXIMAL FOLIATION OF SCHWARZCHILD IS NOT UNIQUE



"3+1 Formalism in General Relativity" E.Gourgoulhon

GR point of view

Isotropic static foliation $A_{\text{out}} = 0$

$$ds^2 = \left(\frac{1 - \frac{\alpha}{4r}}{1 + \frac{\alpha}{4r}}\right)^2 dt^2 + \left(1 + \frac{\alpha}{4r}\right)^4 \left(dr^2 + r^2 d\Omega_2\right)$$

C foliation $C \leftrightarrow A_{\text{out}}$

$$ds^2 = f(C(\tau))\dot{C}(\tau)^2 d\tau^2 + g(C(\tau))\dot{C}(\tau) d\tau dr + \frac{1}{1 + \frac{\alpha}{y} + \frac{C(\tau)^2}{4y^2}} dy^2 + y^2 d\Omega_2^2.$$

A 4-dimensional diffeomorphism connects the two solutions

THERE IS NO AMBIGUITY IN GR

Why then the ambiguity in SD?

- SD rests on a **preferred notion of simultaneity**
- SD symmetries are **3d diffeomorphisms** and **3d conformal transformations**
- The 2 maximal foliations of Schwarzschild are not connected by a **3d conformodiffeo**

What fixes A_{out}?

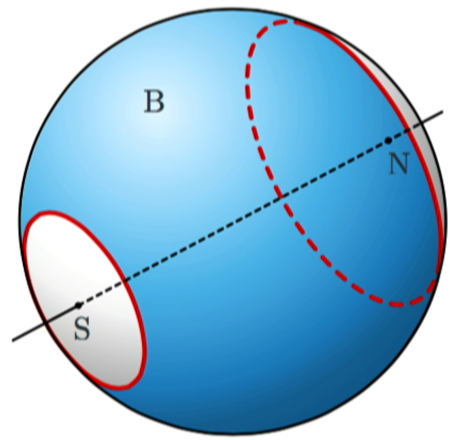
The rest of the universe

Twin Shell Model

SD is naturally defined in a **compact universe**

A **spherically symmetric** compact universe with S^3 **topology** is the simplest model

$$ds^2 = d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$



It requires at least **two thin shell** to not have any mass at the origin

The twin shell model: Starting point

Ansatz

$$g_{ij} = \text{diag}\left\{\mu^2, \sigma, \sigma \sin^2\theta\right\} \quad p^{ij} = \text{diag}\left\{\frac{f}{\mu}, \tfrac{1}{2}s, \tfrac{1}{2}s \sin^{-2}\theta\right\} \sin\theta \quad \xi^i = \{\xi, 0, 0\}$$

Hamiltonian Constraint

$$\int d\theta d\phi \left[\frac{p^{ij}_{\mathrm{T}} p^{\mathrm{T}}_{ij}}{\sqrt{g}} - \tfrac{1}{6}\left(\langle p\rangle^2 - 12\Lambda\right)\sqrt{g} - \sqrt{g}\,R\right] + 4\pi \sum_{a\in\{\mathrm{S,N}\}} \delta(\psi - \Psi_a)\sqrt{g^{rr}\, P_a^2 + M_a^2} = 0$$

Diffeo constraint

$$-2\int d\theta d\phi\, \nabla_j p^j{}_i - 4\pi\, \delta^\psi{}_i \sum_{a\in\{\mathrm{S,N}\}} \delta(\psi - \Psi_a)\, P_a = 0$$

Conformal constraint: CMC Foliation

$$g_{ij}p^{ij} - \sqrt{g}\,\langle p\rangle = 0$$

Solutions to EoM and OS Relation

Diffeo and Hamiltonian constraints

$$f = \frac{1}{3}\langle p\rangle\sigma + \frac{A}{\sigma^{\frac{1}{2}}} \quad \mu^2 = \frac{(\sigma')^2}{(\frac{2}{3}A\langle p\rangle + 4\alpha)\sigma^{\frac{1}{2}} + 4\sigma + \frac{A^2}{\sigma} + \frac{1}{9}\left(\langle p\rangle^2 - 12\Lambda\right)\sigma^2}$$

We get two on shell relations

$$\frac{M_S^4}{16\rho_S{}^2} - M_S^2\left(\frac{A_B A_S}{\rho_S{}^4} + 4 - T\rho_S{}^2\right) - 4\left(A_B - A_S\right)^2\left(\frac{16}{\rho_S{}^2} - 4T\right)$$
$$-\left(\frac{4}{\rho_S{}^3}A_S\left(A_S - A_B\right) + \frac{M_S^2}{2\rho_S}\right)X + X^2 = 0$$

$$\frac{M_N^4}{16\rho_N{}^2} - M_N^2\left(\frac{A_B A_N}{\rho_N{}^4} + 4 - T\rho_N{}^2\right) - 4\left(A_B - A_N\right)^2\left(\frac{16}{\rho_N{}^2} - 4T\right)$$
$$-\left(\frac{4}{\rho_N{}^3}A_N\left(A_N - A_B\right) + \frac{M_N^2}{2\rho_N}\right)X + X^2 = 0$$

$$\rho^2 = \sigma(\Psi) \quad X = \left(\frac{2}{3}A_B\langle p\rangle + 4\alpha_B\right) \quad T = \frac{1}{9}\left(12\Lambda - \langle p\rangle^2\right)$$

Twin Shell DoF

- ρ_S ρ_N
- A_S A_N A_B
- α_S α_N α_B
- $\langle p \rangle$ V

- 2 on shell relations
- $\alpha_S = 0$ $\alpha_N = 0$; $\alpha_B = const$

4 matter DoF (2 for each shell)
2 scale DoF (volume and momentum)

THE SYSTEM IS FULLY DETERMINED

Towards Asymptotic Flatness

- Take the limit $A_N \to \infty$
- Set to 0 each of the coefficients of A_N
- 3 equations that can be solved in terms of ρ_n α_B A_B

2 possible solutions

$$\rho_N = \sqrt{\frac{4-\sqrt{16-M_N^2 T}}{2T}}\,, \quad \alpha_B = -\sqrt{\frac{4-\sqrt{16-M_N^2 T}}{2T}}\left(\frac{1}{8}\sqrt{16-M_N^2 T}+\frac{1}{2}\right)\,, \quad A_B = 0$$

$$\rho_N = \sqrt{\frac{\sqrt{16-M_N^2 T}+4}{2T}}\,, \quad \alpha_B = \left(\frac{1}{8}\sqrt{16-M_N^2 T}-\frac{1}{2}\right)\sqrt{\frac{\sqrt{16-M_N^2 T}+4}{2T}} \quad A_B = 0$$

Asymptotic flatness becomes a good approximation for $T \to 0$

$$V \propto \frac{1}{T} \Leftrightarrow \lim_{T\to 0} V = \infty$$

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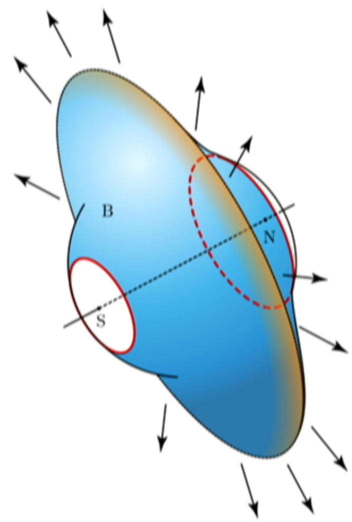
Deriving $A_{out} = 0$

Late time limit $T \to 0$

$\rho_N \to \frac{M_N}{4}$ $\alpha_B \to -\frac{M_N}{4}$ $A_B \to 0$

$\rho_N \sim \frac{2}{\sqrt{T}} \to \infty$ $\alpha_B \sim -\frac{M_N^2}{32}\sqrt{T} \to 0^-$ $A_B \to 0$

$$\rho_N \to \frac{M_N}{4} \quad A_{out} = A_B = 0 \quad \alpha_{out} = \alpha_B = -\frac{M_N}{4}$$



Deriving $A_{\text{out}} = 0$

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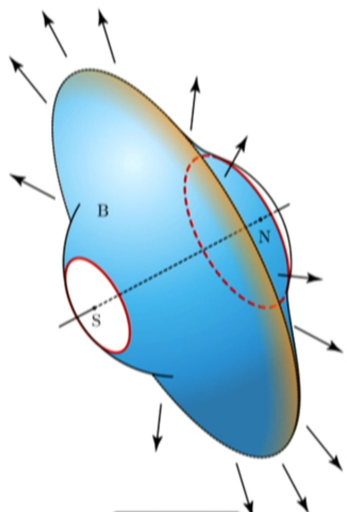
$$\rho_N \to \frac{M_N}{4} \qquad A_{\text{out}} = A_B = 0 \qquad \alpha_{\text{out}} = \alpha_B = -\frac{M_N}{4}$$



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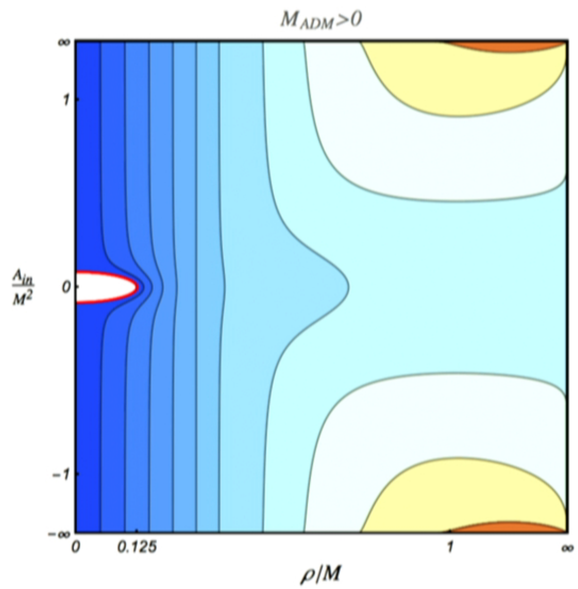
Reduced phase space $A_{out} = 0$

On Shell Relation for the Thin Shell

$$(\alpha_{out} + \rho)A_{in}{}^2 - \rho^3(\alpha_{out})^2 - \frac{M^2\rho^2}{8}\left(\frac{M^2}{32\rho} - \alpha_{out} - 2\rho\right) = 0$$



A_{out} diverges when the dynamics freezes or the shell escapes

Thin Shell Symplectic Reduction

Symplectic potential

$$\theta = \int dr d\theta d\phi p^{ij} \delta g_{ij} + 4\pi P \delta R$$

$$- 8\pi \left(\delta A_{\text{in}} \int_0^R dr \frac{\mu}{\sqrt{\sigma}} + \delta A_{\text{out}} \int_R^\infty dr \frac{\mu}{\sqrt{\sigma}} \right) + 4\pi \, P \delta R$$

Symplectic form

$$\omega = \delta\theta = -8\pi \left(\delta A_{\text{in}} \frac{\mu(R)}{\sqrt{\sigma(R)}} - \delta A_{\text{out}} \frac{\mu(R)}{\sqrt{\sigma(R)}} \right) \wedge \delta R,$$

$$\boxed{\omega = 4\pi \, \delta P \wedge \delta R}$$

P and R are the conjugate dynamical variables

ADM Mass

Definition

$$M_{\text{ADM}} = \frac{1}{16\pi} \lim_{r\to\infty} \int d\theta d\phi \left(\bar{\nabla}^j g_{rj} - \bar{\nabla}_r \bar{g}^{ij} g_{ij} \right) r^2 \sin\theta$$

$$M_{\text{ADM}} = \frac{1}{2} \lim_{r\to\infty} \left(r\,\mu^2 - \sigma' + \frac{1}{r}\sigma \right)$$

Assumption: Asymptotic flatness $\lim_{r\to\infty} \sigma = r^2$

$$\boxed{M_{\text{ADM}} = -\frac{\alpha_{\text{out}}}{2}}$$

α_{out} plays the role of the ADM Hamiltonian

Thin Shell $A_{out} = 0$: Isotropic Gauge

Isotropic gauge condition & $A_{out} = 0$

$$\frac{\mu^2 r}{\sigma(r)} = \frac{1}{r^2} \quad \Rightarrow \quad \sigma(r) = r^2 \left(1 - \frac{\alpha_{out}}{4r}\right)^4$$

EoM in terms of R and P: $\alpha_{out}(R, P)$

- On shell relation

$$(\alpha_{out} + \rho)A_{in}{}^2 - \rho^3(\alpha_{out})^2 - \frac{M^2\rho^2}{8}\left(\frac{M^2}{32\rho} - \alpha_{out} - 2\rho\right) = 0$$

- Isotropic relation

$$\rho^2 = R^2\left(1 - \frac{\alpha_{out}}{4R}\right)^4$$

- Diffeo jump condition

$$A_{in} - \frac{RP}{2} = 0$$

The solution of this system $\alpha_{out} = \alpha_{out}(R, P)$

Thin Shell EoM

$$(\dots)d\alpha_{out} + (\dots)dA_{in} + (\dots)d\rho + (\dots)dR + (\dots)dP = 0$$
$$(\dots)d\alpha_{out} + (\dots)dA_{in} + (\dots)d\rho + (\dots)dR + (\dots)dP = 0$$
$$(\dots)d\alpha_{out} + (\dots)dA_{in} + (\dots)d\rho + (\dots)dR + (\dots)dP = 0$$

$$d\alpha_{out} = \left(\frac{\partial \alpha_{out}}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dR + \left(\frac{\partial \alpha_{out}}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dP$$

$$dA_{in} = \left(\frac{\partial A_{in}}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dR + \left(\frac{\partial A_{in}}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dP$$

$$d\rho = \left(\frac{\partial \rho}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dR + \left(\frac{\partial \rho}{\partial R}\bigg|_{A_{in}=\frac{RP}{2},\ \rho=R\left(1-\frac{\alpha_{out}}{4R}\right)^2} \right) dP$$

$$\boxed{\dot{R} = -\frac{1}{2}\frac{d\alpha_{out}}{dP}(R, P, \alpha_{out}) \quad \dot{P} = +\frac{1}{2}\frac{d\alpha_{out}}{dR}(R, P, \alpha_{out})}$$

Thin Shell Numerical Simulations