Title: Welcome & Overview

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Abstract:


SD@ CONVERGENCE
A Shape Dynamics Workshop
PERIMETER INSTITUTE
Welcomes and Introduction
Flavio Mercati

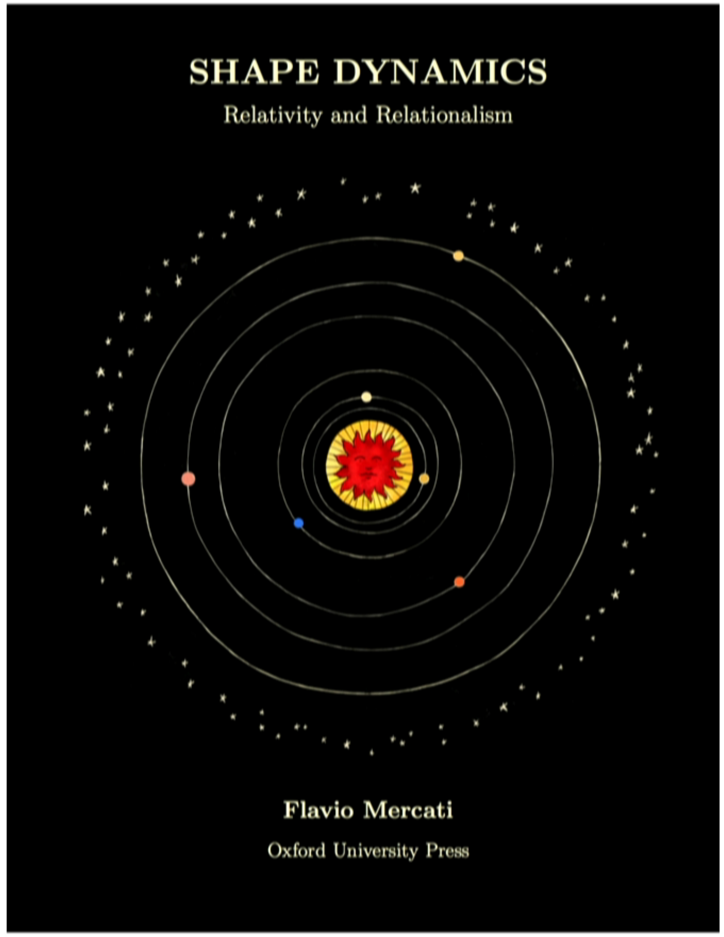
SD: A philosophy about the nature of space, time and scale which has condensed into a physical theory

"...This result has led me to doubt how fundamental the 4-dimensional requirement in physics is. A few decades ago it seemed quite certain that one had to express the whole of physics in 4D form. But now it seems that 4D symmetry is not of such overriding importance, since the description of nature sometimes gets simplified when one departs from it."

P. A. M. Dirac, 1963

"...The picture of dynamics that emerges is of the time-dependent geometry of shape ('transverse modes') interacting with the changing scale of space ('longitudinal mode')."

J. W. York, 1972


SHAPE DYNAMICS
Relativity and Relationalism
Flavio Mercati
Oxford University Press

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Principles

- Spatial Relationalism: 3D diffeo & 3D Weyl invariance ⇒ well-defined reduced configuration space,
- Temporal Relationalism: classical solutions are unparametrized curves in reduced configuration space (shape space).

spacetime is not fundamental: a simplification that applies to dynamics of small-backreaction probes. ⇒ requiring regular spacetime is wrong. Assuming regular 3D conformal geometry can lead to departures from GR.

Evidence in favour of SD: spacetime regularity impossible in GR (singularities), while regularity of conformal geometry leads to *singularity avoidance* (see talks by Andrea, Hamish and Gabriel).

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A message from the 60-70's

Arnowitt–Deser–Misner (60's)

Hamiltonian formulation of GR: $^{(4)}g_{\mu\nu} = \begin{pmatrix} -N^2 + g_{ij}\,\xi^i\,\xi^j & g_{ik}\,\xi^k \\ g_{jk}\,\xi^k & g_{ij} \end{pmatrix},$

Einstein action: $\int d^4x \sqrt{^{(4)}g}\;^{(4)}R = \int dt d^3x \left(\dot{g}_{ij}p^{ij} + N\,\mathcal{H}[g,p] + \xi^i \mathcal{H}_i[g,p]\right),$

3D-Diffeo constraint: $\boxed{\mathcal{D}_i = -2\,\nabla_j p^j{}_i \approx 0}$ (p^{ij} must be *transverse*)

Hamiltonian constraint:

$$\boxed{\mathcal{H} = \frac{1}{\sqrt{g}}\left(p^{ij} - \tfrac{1}{3}g^{ij}\mathrm{tr}\,p\right)\left(p_{ij} - \tfrac{1}{3}g_{ij}\mathrm{tr}\,p\right) - \frac{1}{6}\frac{(\mathrm{tr}\,p)^2}{\sqrt{g}} - \sqrt{g}\,R}$$

Lichnerowicz, York, Choquet–Bruhat (70's)

Conformal method: in *CMC slicing* $\quad \mathrm{tr}\, p = g_{ij} p^{ij} = \frac{3}{2} \tau \sqrt{g}$

$\mathcal{H} \approx 0$ and $\mathcal{D}_i \approx 0$ **decouple** and turn into **elliptic equations**.

Closed-space solutions of GR specified by a *3d conformal geometry* and a symmetric *transverse-traceless (TT) tensor* + *'York time'* τ

$\boxed{-2\,\nabla_j p^j{}_i}$ = diffeomorphisms $\quad \delta g_{ij} = \nabla_i \xi_j + \nabla_j \xi_i$

$\boxed{\mathrm{tr}\, p = g_{ij}\, p^{ij}}$ = Weyl transforms $\quad \delta g_{ij} = \omega^4\, g_{ij}$

p^{ij}_{TT} generates changes of *shape*

conformal geometries represent the physical configuration space of GR, **TT-tensors** are their conjugate momenta.

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The exceptionality of CMC/maximal slicing

Marsden–Tipler [Phys. Rep. **3** 1980] argued that all physically relevant singularities are avoided by maximal slicing $\mathrm{tr}\, p = 0$.

In the CMC case $\mathrm{tr}\, p = \sqrt{g}\,\mathrm{const.}$ same holds except for crushing singularities $V = \int \sqrt{g}\, d^3x \to 0$ (big-bang-like).

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GR's constraint algebr*oid* vs. SD constraint algebra

Both GR's ($\mathcal{H}$, $\mathcal{D}_i$) and SD's ($\mathcal{C}$, $\mathcal{H}_i$) constraints close a first-class system:

GR

$$\{\mathcal{H}[f], \mathcal{H}[h]\} = \mathcal{D}_i[g^{ij} f \overleftrightarrow{\nabla}_j h]$$
$$\{\mathcal{D}_i[\xi^i], \mathcal{D}_j[\zeta^j]\} = \mathcal{D}_k[\pounds_\zeta \xi^k]$$
$$\{\mathcal{H}[f], \mathcal{D}[\xi^i]\} = \mathcal{H}[\pounds_\xi f]$$

SD

$$\{\mathcal{C}[f], \mathcal{C}[h]\} = 0$$
$$\{\mathcal{D}_i[\xi^i], \mathcal{D}_j[\zeta^j]\} = \mathcal{D}_k[\pounds_\zeta \xi^k]$$
$$\{\mathcal{C}[f], \mathcal{D}[\xi^i]\} = \mathcal{C}[\pounds_\xi f]$$

Structure constants of $\{\mathcal{H}, \mathcal{H}\}$ are *functions* on phase space (depend on g_{ij}).

Generator associated to refoliations $\mathcal{H}$ does not act as gauge transformations on a configuration space (Kuchar's battle for observables in GR).

SD's constraints admit interpretation of geometrical transformations. Reduced configuration space: *conformal superspace* (shape space). York Hamiltonian invariant under conformodiffeos.

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Local volume dofs: $\sqrt{g}$, conjugate dilatational momenta: $\mathrm{tr}\, p$.

SD repackages those in the following way:

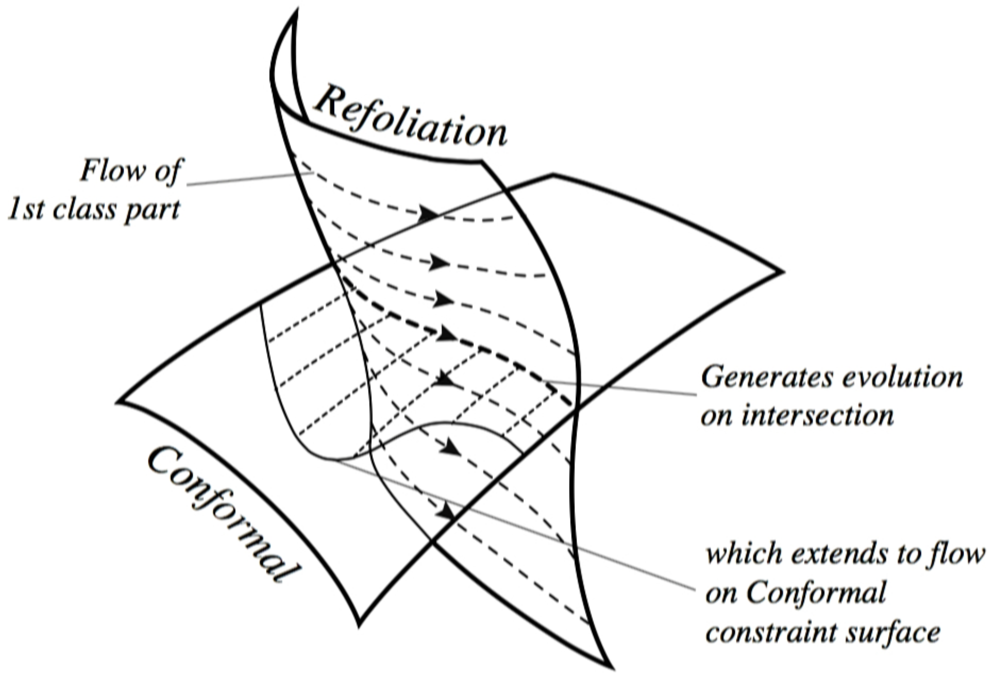
- local volume form $\sqrt{g}/V$: gauge,
- fluctuations of dilatational momentum $\mathrm{tr}\, p - \sqrt{g}\langle \mathrm{tr}\, p\rangle = 0$,
- global volume V: York Hamiltonian,
- global dilatational momentum $\langle \mathrm{tr}\, p\rangle$: CMC time.

York Hamiltonian: $H_{\mathrm{York}}(\tau) = \int d^3x \, \sqrt{g}\, \Omega^6[g, p; x, \tau)$

Ω solution of Lichnerowicz–York equation:

$$8\,\Omega\Delta\Omega + \frac{\left\| p - \frac{1}{3} g \,\mathrm{tr}\, p \right\|^2}{g\Omega^6} - \frac{3}{8}\tau^2\,\Omega^6 - \Omega^2\, R + \textit{(matter)} = 0\,.$$

The 'iconic diagram' of SD



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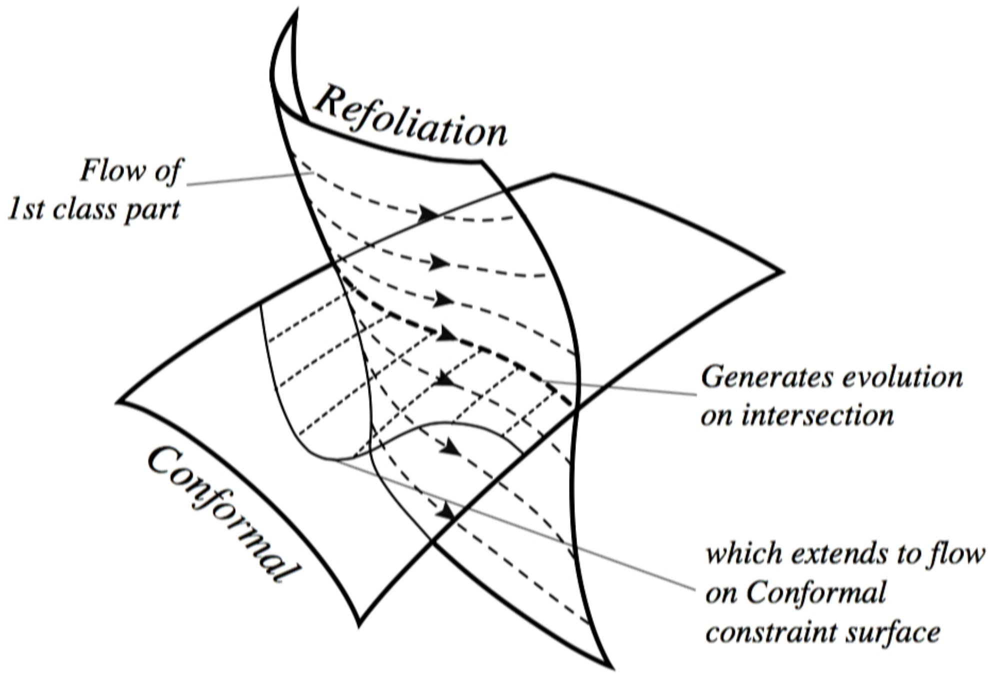
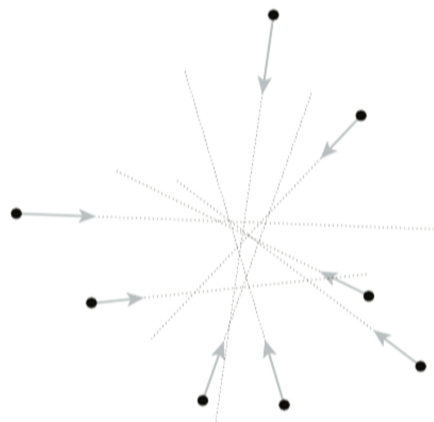


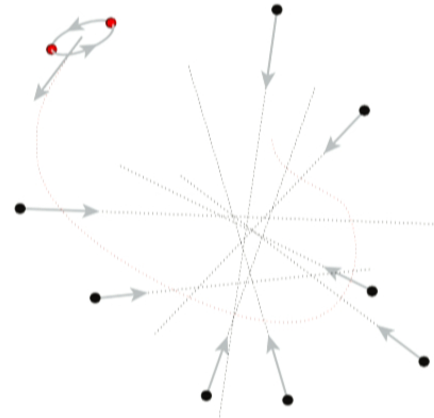
Example: homothetic fall in N-body problem

Alone:



just a point in shape space,

With a very light Kepler pair:



a curve in shape space.

There is a distinguished representation (time-dependent notion of scale and time parametrization) in extended phase space, where Kepler pair has stable major axis (rod property) and isochronous orbits (clock property).
From the Kepler pair's perspective, this universe is collapsing.

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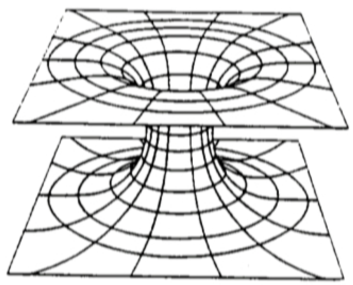
What you're going to see in this workshop

Towards the horizon (black holes & co.)

Henrique (ca. 2013):

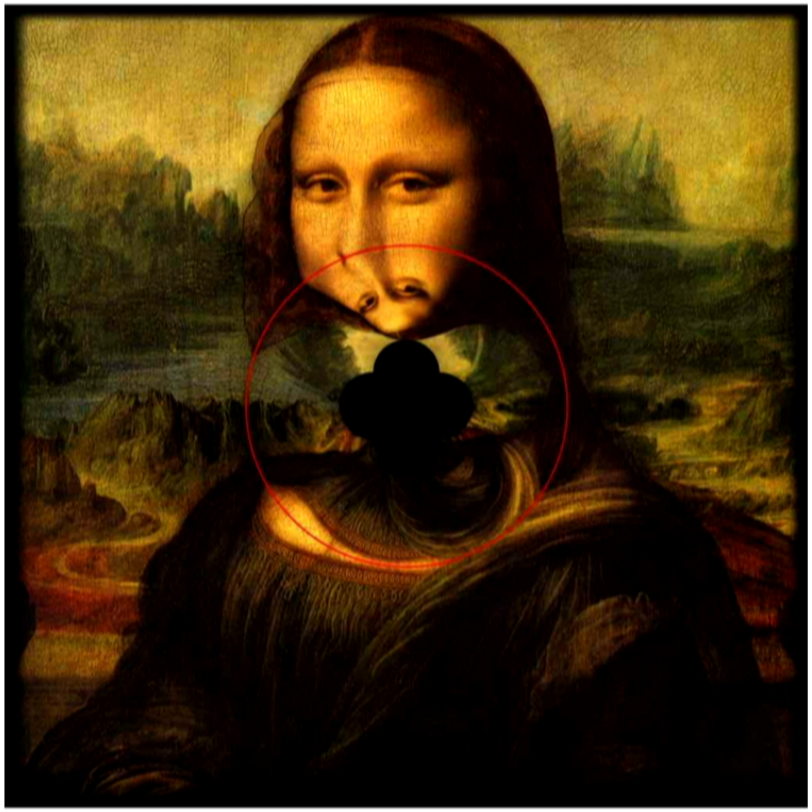


But
$\xi = 0$
and

- Andrea will present our efforts (in collaboration with Henrique and Tim) to study gravitational collapse of thin shells in SD.

- Hamish will talk about his work (with Sean) on comparing SD and GR solutions in presence of horizons.

- Gabriel will talk about his proposal for 'parity horizons':



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